On two-body and three-body spin correlations in leptonic $t\bar{t}Z$ production and anomalous couplings at the LHC

Rafiqul Rahaman

RECAPP
HRI, Prayagraj, India

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Outline

1. Introduction
2. Polarization and spin correlations
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4. Summary

Outline

Introduction

- Precision measurements at the electroweak (EW) scale are essential to look for the remnant of BSM physics probably sitting at a high energy scale.
- Top quark interactions play a key role in exploring BSM theories because of their heavy mass, which is the same order as the electroweak scale.
- Polarizations of top quark and massive gauge bosons are interesting tools for precision measurement, as they are sensitive to the modification of the interactions involved.
- The top quark, Z and W bosons being massive, they decay immediately after they are produced, letting their decay products carry their polarization as well as spin correlation information.
- Polarization and spin correlation parameters can be calculated from the production process as well as from the angular distributions of the decay products.

Introduction

- Here, we focus on the top quark interaction with the Z boson in the $t\bar{t}Z$ production process, sensitive for direct measurement.
- The $t\bar{t}Z$ is important background for search for BSM phenomena in multi-lepton and b-quark final state, and $t\bar{t}H$.
- We study the potential of polarization of top quarks and Z, two-body (t-$\bar{t}$, $t/\bar{t}$-Z) and three-body (t-$\bar{t}$-Z) spin correlation in probing $t\bar{t}Z$ anomalous couplings.

Outline

Polarizations

$$P_f(\lambda,\lambda') \;=\; \frac{1}{2}\Big[\mathbb{I}_{2\times 2} + \vec{p}\cdot\vec{\tau}\Big], \quad \big(\lambda,\lambda' \in [+1,-1]\big)\,,$$

$$P_V(\lambda,\lambda') = \frac{1}{3}\left[\mathbb{I}_{3\times 3} + \frac{3}{2}\vec{p}.\vec{S} + \sqrt{\frac{3}{2}}\,T_{ij}\big(S_iS_j + S_jS_i\big)\right], \;\; \big(\lambda,\lambda' \in [+1,0,-1]\big)$$

C. Bourrely, J. Soffer, E. Leader, Phys. Rep. 59 (1980) 95–297

$$\frac{1}{\sigma_f}\frac{d\sigma_f}{d\Omega} = \frac{1}{4\pi}\Big[1 + \alpha p_x \sin\theta\cos\phi + \alpha p_y \sin\theta\sin\phi + \alpha p_z \cos\theta\Big],$$

$$\begin{aligned}
\frac{1}{\sigma_V}\frac{d\sigma_V}{d\Omega} \quad = \quad & \frac{3}{8\pi}\left[\left(\frac{2}{3} - (1-3\delta)\ \frac{T_{zz}}{\sqrt{6}}\right) + \alpha\ p_z\cos\theta + \sqrt{\frac{3}{2}}(1-3\delta)\ T_{zz}\cos^2\theta\right. \\
+ \quad & \left(\alpha\ p_x + 2\sqrt{\frac{2}{3}}(1-3\delta)\ T_{xz}\cos\theta\right)\sin\theta\ \cos\phi \\
+ \quad & \left(\alpha\ p_y + 2\sqrt{\frac{2}{3}}(1-3\delta)\ T_{yz}\cos\theta\right)\sin\theta\ \sin\phi \\
+ \quad & (1-3\delta)\left(\frac{T_{xx}-T_{yy}}{\sqrt{6}}\right)\sin^2\theta\cos(2\phi) \\
+ \quad & \left.\sqrt{\frac{2}{3}}(1-3\delta)\ T_{xy}\ \sin^2\theta\ \sin(2\phi)\right].
\end{aligned}$$

F. Boudjema and R. K. Singh JHEP **0907**, 028 (2009)

$$\mathscr{A}_i[p/T] = \frac{\sigma\left(\mathscr{C}_i^{p/T} > 0\right) - \sigma\left(\mathscr{C}_i^{p/T} < 0\right)}{\sigma\left(\mathscr{C}_i^{p/T} > 0\right) + \sigma\left(\mathscr{C}_i^{p/T} < 0\right)} \propto p/T_{ij}$$

$$\mathscr{C}_i^{p} \in [c_x,\ c_y,\ c_z]$$

$$\mathscr{C}_i^{T} \in \left[c_x c_y,\ c_x c_z,\ c_y c_z,\ c_x^2 - c_y^2,\ |\sqrt{c_x^2 + c_y^2}|(4c_z^2 - 1)\right].$$

$$c_x = \sin\theta\cos\phi,\ c_y = \sin\theta\sin\phi,\ c_z = \cos\theta$$

R. Rahaman and R. K. Singh Eur.Phys.J.C 76 (2016) 10, 539

Spin correlation

Spin-1/2 – spin-1/2 correlations

$$P^{AB}\left(\lambda_A, \lambda'_A, \lambda_B, \lambda'_B\right)$$

$$= \quad \frac{1}{\left(2\times\frac{1}{2}+1\right)^2}\left[\underbrace{\mathbb{I}_{2\times2}\otimes\mathbb{I}_{2\times2}}_{\mathbb{I}_{4\times4}}+\vec{p}^A\cdot\vec{\tau}\otimes\mathbb{I}_{2\times2}+\mathbb{I}_{2\times2}\otimes\vec{p}^B\cdot\vec{\tau}+pp^{AB}_{ij}\tau_i\otimes\tau_j\right],$$

$$(i, j \in [x\equiv1, y\equiv2, z\equiv3]),$$

W. Bernreuther and A. Brandenburg, Phys. Lett. B 314 (1993) 104–111

Spin-1/2 – spin-1/2 correlations

$$P^{AB}\left(\lambda_A, \lambda'_A, \lambda_B, \lambda'_B\right)$$

$$= \frac{1}{\left(2\times\frac{1}{2}+1\right)^2}\left[\underbrace{\mathbb{I}_{2\times 2}\otimes\mathbb{I}_{2\times 2}}_{\mathbb{I}_{4\times 4}}+\vec{p}^A\cdot\vec{\tau}\otimes\mathbb{I}_{2\times 2}+\mathbb{I}_{2\times 2}\otimes\vec{p}^B\cdot\vec{\tau}+pp^{AB}_{ij}\tau_i\otimes\tau_j\right],$$

$$\left(i,j\in[x\equiv 1, y\equiv 2, z\equiv 3]\right),$$

W. Bernreuther and A. Brandenburg, Phys. Lett. B 314 (1993) 104–111

Spin-1/2 – spin-1 correlations

$$P^{AB}\left(\lambda_A, \lambda'_A, \lambda_B, \lambda'_B\right) =$$

$$\frac{1}{\left(2\times\frac{1}{2}+1\right)}\frac{1}{\left(2\times 1+1\right)}\left[\underbrace{\mathbb{I}_{2\times 2}\otimes\mathbb{I}_{3\times 3}}_{\mathbb{I}_{6\times 6}}+\vec{p}^A\cdot\vec{\tau}\otimes\mathbb{I}_{3\times 3}+\frac{3}{2}\mathbb{I}_{2\times 2}\otimes\vec{p}^B\cdot\vec{S}\right.$$

$$\left.+\quad\sqrt{\frac{3}{2}}\mathbb{I}_{2\times 2}\otimes T^B_{ij}\left(S_iS_j+S_jS_i\right)+pp^{AB}_{ij}\tau_i\otimes S_j+pT^{AB}_{ijk}\tau_i\otimes\left(S_jS_k+S_kS_j\right)\right]$$

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t–$\bar{t}$–Z spin correlations

$$
\begin{aligned}
& P^{t\bar{t}Z}\left(\lambda_t, \lambda'_t, \lambda_{\bar{t}}, \lambda'_{\bar{t}}, \lambda_Z, \lambda'_Z\right) = \\
& \frac{1}{\left(2\times\frac{1}{2}+1\right)^2}\frac{1}{(2\times 1+1)}\Big[\mathbb{I}_{12\times 12} + \vec{p}^{t}\cdot\vec{\tau}\otimes\mathbb{I}_{6\times 6} + \mathbb{I}_{2\times 2}\otimes\vec{p}^{\bar{t}}\cdot\vec{\tau}\otimes\mathbb{I}_{3\times 3} \\
+ \quad & \frac{3}{2}\mathbb{I}_{4\times 4}\otimes\vec{p}^{Z}\cdot\vec{S} + \sqrt{\frac{3}{2}}\mathbb{I}_{4\times 4}\otimes T^{Z}_{ij}\left(S_iS_j + S_jS_i\right) + pp^{t\bar{t}}_{ij}\tau_i\otimes\tau_j\otimes\mathbb{I}_{3\times 3} \\
+ \quad & pp^{tZ}_{ij}\tau_i\otimes\mathbb{I}_{2\times 2}\otimes S_j + pT^{tZ}_{ijk}\tau_i\otimes\mathbb{I}_{2\times 2}\otimes\left(S_jS_k + S_kS_j\right) \\
+ \quad & pp^{\bar{t}Z}_{ij}\mathbb{I}_{2\times 2}\otimes\tau_i\otimes S_j + pT^{\bar{t}Z}_{ijk}\mathbb{I}_{2\times 2}\otimes\tau_i\otimes\left(S_jS_k + S_kS_j\right) \\
+ \quad & \underbrace{ppp^{t\bar{t}Z}_{ijk}\tau_i\otimes\tau_j\otimes S_k + ppT^{t\bar{t}Z}_{ijkl}\tau_i\otimes\tau_j\otimes\left(S_kS_l + S_lS_k\right)}_{\text{Three body correlation}}\Big]
\end{aligned}
$$

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Angular distribution and asymmetries

Spin-1/2 – spin-1/2 case

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\Omega_a d\Omega_b} = \frac{1}{16\pi^2}\left[1 + \alpha_A p_i^A c_i^a + \alpha_B p_i^B c_i^b + \alpha_A \alpha_B pp_{ij}^{AB} c_i^a c_j^b\right]$$

$$c_x = \sin\theta\cos\phi,\;\; c_y = \sin\theta\sin\phi,\;\; c_z = \cos\theta$$

$$\mathscr{A}[pp_{ij}^{AB}] \equiv \frac{\sigma\left(c_i^a c_j^b > 0\right) - \sigma\left(c_i^a c_j^b < 0\right)}{\sigma\left(c_i^a c_j^b > 0\right) + \sigma\left(c_i^a c_j^b < 0\right)} = \frac{1}{4}\alpha_A \alpha_B pp_{ij}^{AB}.$$

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Spin-1/2 – spin-1 case

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\Omega_a d\Omega_b} = \frac{1}{16\pi^2}\Bigg[\alpha_A(1-3\delta_B)pT^{AB}_{ijk}c^a_i c^b_j c^b_k \ (j \neq k)$$
$$+ \frac{1}{2}\alpha_A(1-3\delta_B)\left(pT^{AB}_{i11} - pT^{AB}_{i22}\right)c^a_i\left((c^b_1)^2-(c^b_2)^2\right)$$
$$+ \frac{1}{2}\alpha_A(1-3\delta_B)pT^{AB}_{i33}c^a_i\left(3(c^b_3)^2-1\right)\Bigg]$$

$$\mathscr{A}[pT^{AB}] \equiv \frac{\sigma(\mathscr{C}^p_i\mathscr{C}^T_j > 0) - \sigma(\mathscr{C}^p_i\mathscr{C}^T_j < 0)}{\sigma(\mathscr{C}^p_i\mathscr{C}^T_j > 0) + \sigma(\mathscr{C}^p_i\mathscr{C}^T_j < 0)} \propto \alpha_A \times pT^{AB}$$

$$\mathscr{C}^p_i \in [c_x,\ c_y,\ c_z]$$

$$\mathscr{C}^T_i \in \left[c_x c_y,\ c_x c_z,\ c_y c_z,\ c_x^2 - c_y^2,\ |\sqrt{c_x^2+c_y^2}|(4c_z^2-1)\right].$$

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t-$\bar{t}$-Z case

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$$\frac{1}{\sigma}\frac{d^3\sigma}{d\Omega_{l_t}d\Omega_{l_{\bar{t}}}d\Omega_{l_Z}} =$$

$$\frac{1}{64\pi^3}\Bigg[1+\alpha_t\alpha_{\bar{t}}\alpha_Z\ ppp^{t\bar{t}Z}_{ijk}\ \ c_i^{l_t}c_j^{l_{\bar{t}}}c_k^{l_Z}$$

$$+\quad \alpha_t\alpha_{\bar{t}}(1-3\delta_Z)ppT^{t\bar{t}Z}_{ijkl}c_i^{l_t}c_j^{l_{\bar{t}}}c_k^{l_Z}c_l^{l_Z}\ \ (k\neq l)$$

$$+\quad \frac{1}{2}\alpha_t\alpha_{\bar{t}}(1-3\delta_Z)\left(ppT^{t\bar{t}Z}_{ijxx}-pT^{t\bar{t}Z}_{ijyy}\right)c_i^{l_t}c_j^{l_{\bar{t}}}\left((c_x^{l_Z})^2-(c_y^{l_Z})^2\right)$$

$$+\quad \frac{1}{2}\alpha_t\alpha_{\bar{t}}(1-3\delta_Z)ppT^{t\bar{t}Z}_{ijzz}c_i^{l_t}c_j^{l_{\bar{t}}}\left(3(c_z^{l_Z})^2-1\right)\Bigg]$$

$$\mathscr{A}[ppp^{t\bar{t}Z}]\equiv\frac{\sigma(\mathscr{C}_i^p\mathscr{C}_j^p\mathscr{C}_k^p>0)-\sigma(\mathscr{C}_i^p\mathscr{C}_j^p\mathscr{C}_k^p<0)}{\sigma(\mathscr{C}_i^p\mathscr{C}_j^p\mathscr{C}_k^p>0)+\sigma(\mathscr{C}_i^p\mathscr{C}_j^p\mathscr{C}_k^p<0)}\propto\alpha_t\alpha_{\bar{t}}\alpha_Z\times ppp^{t\bar{t}Z}$$

$$\mathscr{A}[ppT^{t\bar{t}Z}]\equiv\frac{\sigma(\mathscr{C}_i^p\mathscr{C}_j^p\mathscr{C}_k^T>0)-\sigma(\mathscr{C}_i^p\mathscr{C}_j^p\mathscr{C}_k^T<0)}{\sigma(\mathscr{C}_i^p\mathscr{C}_j^p\mathscr{C}_k^T>0)+\sigma(\mathscr{C}_i^p\mathscr{C}_j^p\mathscr{C}_k^T<0)}\propto\alpha_t\alpha_{\bar{t}}\times ppT^{t\bar{t}Z}$$

Outline

$t\bar{t}Z$ anomalous couplings

$$\mathscr{L}_{t\bar{t}Z} = e\bar{t}\left[\gamma^{\mu}\left(C_1^V + \gamma_5 C_1^A\right) + \frac{i\sigma^{\mu\nu}q_{\nu}}{m_Z}\left(C_2^V + i\gamma_5 C_2^A\right)\right] tZ_{\mu},$$

$$\sigma^{\mu\nu} = \frac{1}{2}\left[\gamma^{\mu}, \gamma^{\nu}\right], \ \ q_{\nu} = (p_t - p_{\bar{t}})_{\nu}$$

$$\mathscr{O}^{ij}_{uB} = \left(\bar{Q}_i \sigma^{\mu\nu} u_j\right) \tilde{\varphi} B_{\mu\nu}, \qquad \mathscr{O}^{ij}_{uW} = \left(\bar{Q}_i \sigma^{\mu\nu} \tau^I u_j\right) \tilde{\varphi} W^I_{\mu\nu},$$

$$\mathscr{O}^{ij}_{\varphi u} = \left(\varphi^{\dagger} \overleftrightarrow{D}_{\mu} \varphi\right)\left(\bar{u}_i \gamma^{\mu} u_j\right), \qquad \mathscr{O}^{1(ij)}_{\varphi Q} = \left(\varphi^{\dagger} \overleftrightarrow{D}_{\mu} \varphi\right)\left(\bar{Q}_i \gamma^{\mu} Q_j\right),$$

$$\mathscr{O}^{3(ij)}_{\varphi Q} = \left(\varphi^{\dagger} \overleftrightarrow{D}^I_{\mu} \varphi\right)\left(\bar{Q}_i \gamma^{\mu} \tau^I Q_j\right).$$

Q: left-handed quark doublet;

u: right-handed singlet quark;

φ: Higgs doublet

$$\overleftrightarrow{D}_{\mu} = D_{\mu} - \overleftarrow{D}_{\mu}$$

J.A. Aguilar-Saavedra, Nucl. Phys. B 812 (2009) 181

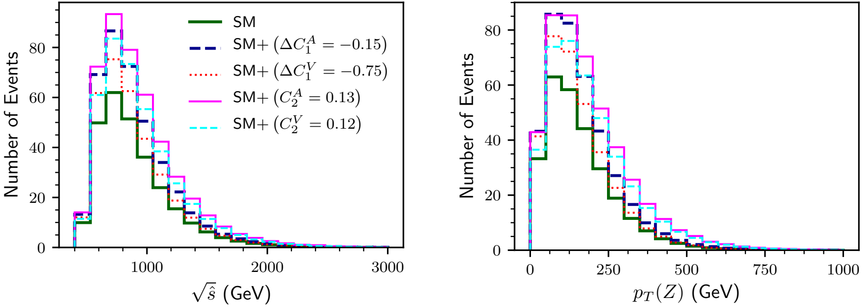
$t\bar{t}Z$ anomalous couplings

$$
\begin{aligned}
C_1^V &= C_{1,\mathrm{SM}}^V + \frac{v^2}{2\Lambda^2 \sin\theta_W \cos\theta_W} \mathrm{Re}\left[-c_{\varphi t} - c_{\varphi Q}^-\right], \\
C_1^A &= C_{1,\mathrm{SM}}^A + \frac{v^2}{2\Lambda^2 \sin\theta_W \cos\theta_W} \mathrm{Re}\left[-c_{\varphi t} + c_{\varphi Q}^-\right], \\
C_2^V &= \frac{\sqrt{2}v^2}{2\Lambda^2 \sin\theta_W \cos\theta_W} c_{tZ}, \quad C_2^A = \frac{\sqrt{2}v^2}{2\Lambda^2 \sin\theta_W \cos\theta_W} c_{tZ}^I
\end{aligned}
$$

$$
\begin{aligned}
c_{tZ} &= \mathrm{Re}[-\sin\theta_W C_{uB}^{33} + \cos\theta_W C_{uW}^{33}], \\
c_{tZ}^I &= \mathrm{Im}[-\sin\theta_W C_{uB}^{33} + \cos\theta_W C_{uW}^{33}], \\
c_{\varphi t} &= C_{\varphi u}^{33}, \\
c_{\varphi Q}^- &= C_{\varphi Q}^{1(33)} - C_{\varphi Q}^{3(33)}.
\end{aligned}
$$

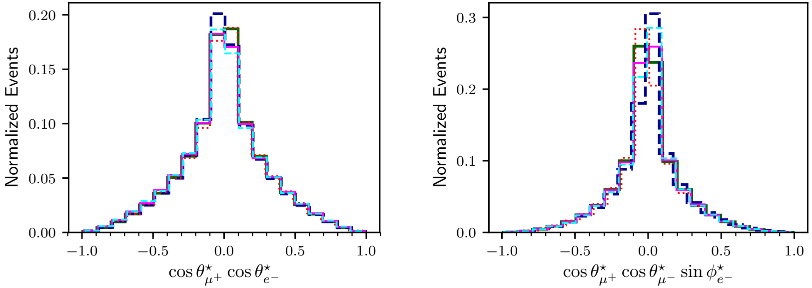
J.A. Aguilar-Saavedra, Nucl. Phys. B 812 (2009) 181

Distributions with anomalous $t\bar{t}Z$ couplings



$$
\begin{aligned}
Bin_1 &\equiv p_T(Z) < 250 \text{ GeV}, \\
Bin_2 &\equiv p_T(Z) \in [250, 500] \text{ GeV}, \\
Bin_3 &\equiv p_T(Z) \in [500, 750] \text{ GeV}, \\
Bin_4 &\equiv p_T(Z) > 750 \text{ GeV}.
\end{aligned}
$$

Distributions with anomalous $t\bar{t}Z$ couplings



$$pp \to t\bar{t}Z,\ t \to b(W^+ \to \mu^+\nu_\mu),\ \bar{t} \to \bar{b}(W^- \to \mu^-\bar{\nu}_\mu),\ Z \to e^-e^+,$$

Reconstructation of neutrinos

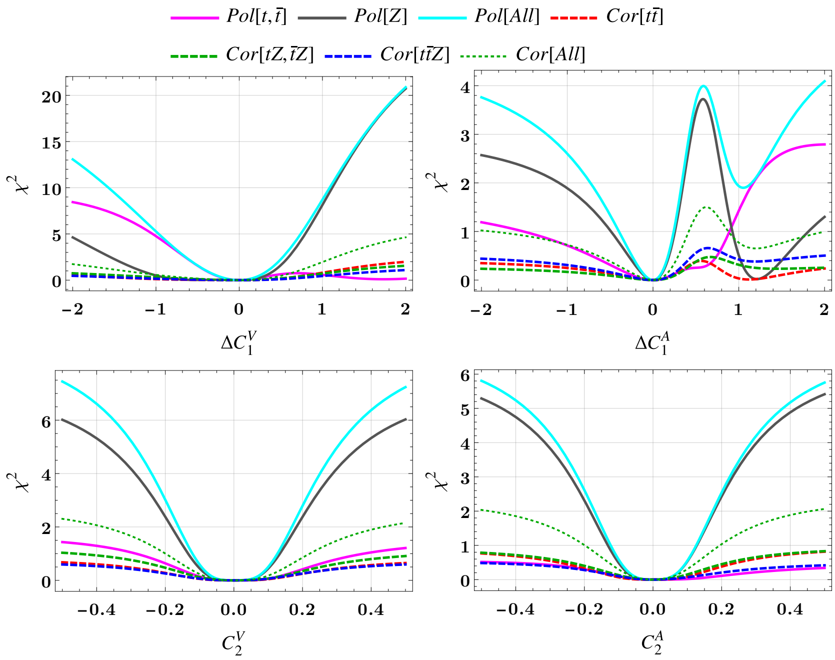
$$\vec{\not{p}}_T = \vec{p}_T(\nu_\mu) + \vec{p}_T(\bar{\nu}_\mu),$$
$$m^2_{\mu^+\nu_\mu} = m^2_W = m^2_{\mu^-\bar{\nu}_\mu},$$
$$m^2_{b\mu^+\nu_\mu} = m^2_t = m^2_{\bar{b}\mu^-\bar{\nu}_\mu}.$$

Comparing spin parameters

$$\chi^2(C_i) = \sum_{n=1}^{N} \left| \frac{\mathscr{O}_n(C_i) - \mathscr{O}_n(C_i = 0)}{\delta \mathscr{O}_n} \right|^2 ,$$

$$\delta\sigma_i = \sqrt{\frac{\sigma_i}{\mathscr{L}} + (\epsilon_{\sigma_i}\sigma_i)^2}, \quad \delta\mathscr{A}_i = \sqrt{\frac{1-\mathscr{A}_i^2}{\mathscr{L}\times\sigma} + \epsilon_{\mathscr{A}}^2},$$

$\epsilon_\sigma = 0.1$ [CMS, JHEP 03 (2020) 056]
$\epsilon_{\mathscr{A}} = 0.01$ [CMS, Phys. Rev. D 100 (2019) 072002].
$\mathscr{L} = 3\ \mathrm{ab}^{-1}$



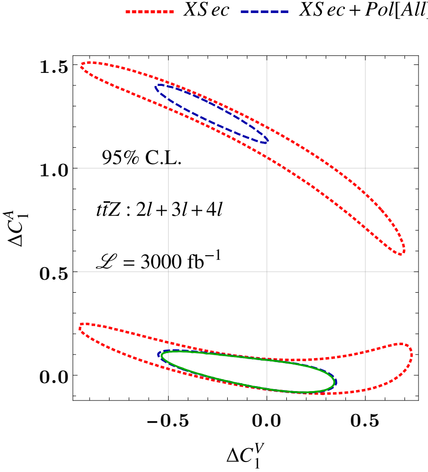
$Pol[t,\bar{t}]$
$Pol[Z]$
$Pol[All]$
$Cor[t\bar{t}]$
$Cor[tZ,\bar{t}Z]$
$Cor[t\bar{t}Z]$
$Cor[All]$
χ^2
ΔC_1^V
ΔC_1^A
C_2^V
C_2^A

Combining different channels $(2l + 3l + 4l)$

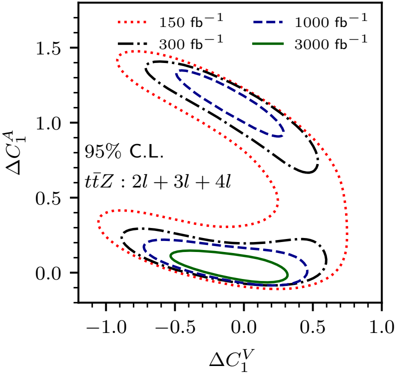
- $Z \to l^+ l^-$, $t/\bar{t} \to$ hadronic $(2l)$: Only the Z polarizations are obtained in this case,
- $Z \to l^+ l^-$, $t \to$ leptonic, $\bar{t}$ hadronic $(3l)$: Top quark polarizations are obtained along with the t-Z spin correlations by reconstructing the missing neutrino. The $\bar{t}$ polarizations and $\bar{t}$-Z spin correlations are obtained by reversing the top and anti-top decay,
- Fully leptonic $(4l)$: In this case, $t\bar{t}$ and $t\bar{t}Z$ spin correlations are obtained.

Systematic uncertainty $(\sigma) \Rightarrow$ 4l: 3l: 2l = 0.1: 0.13: 0.16

Constraints on couplings



Two-parameter fit



Simultaneous fit

Outline

Summary

- We bring the spin correlations of $t/\bar{t}$-Z and t-$\bar{t}$-Z in market to study new physics.
- The polarizations and spin correlations help in tightening the region of parameters space, especially for the vector and axial-vector couplings (ΔC_1^V–ΔC_1^A) in comparison to the cross section by a considerable amount.
- Spin-correlations further helps in finding direction to the anomalous couplings.
- Our strategy in this analysis can serve as an extra handle in interpreting anomalous interactions on the data at the high energy and high luminosity LHC.

Thank you