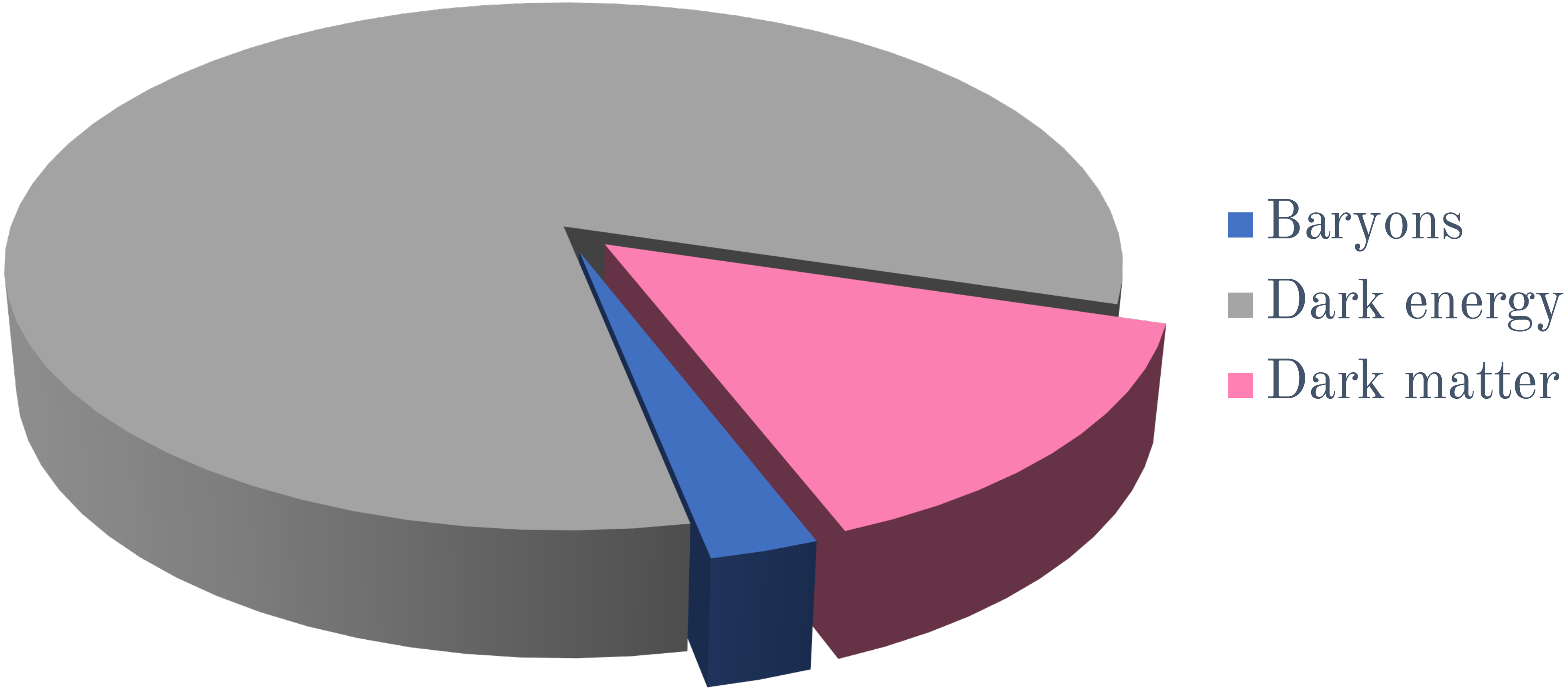
Magnetic Moments of Dark Baryons

Chester Mantel

Work in progress with Pouya Asadi and Graham Kribs

Dark Matter



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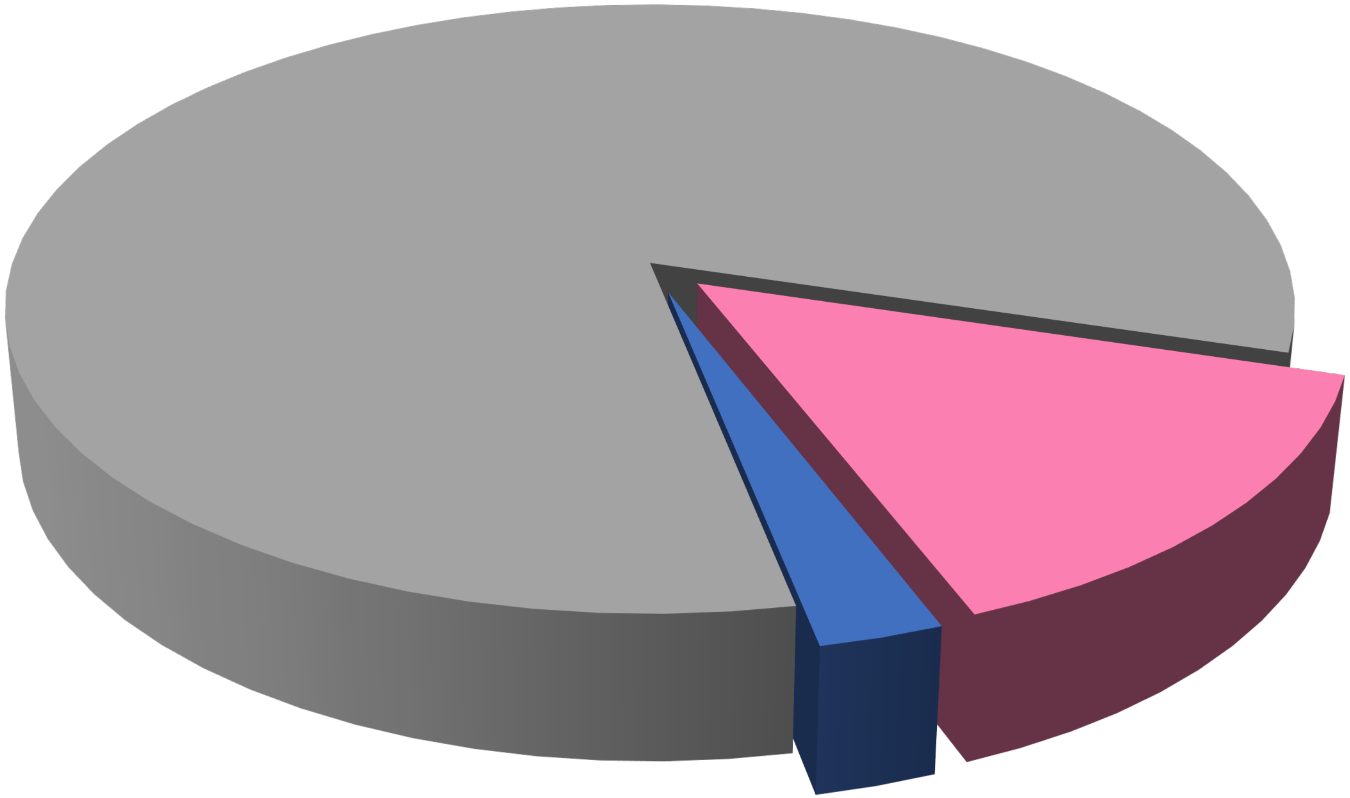

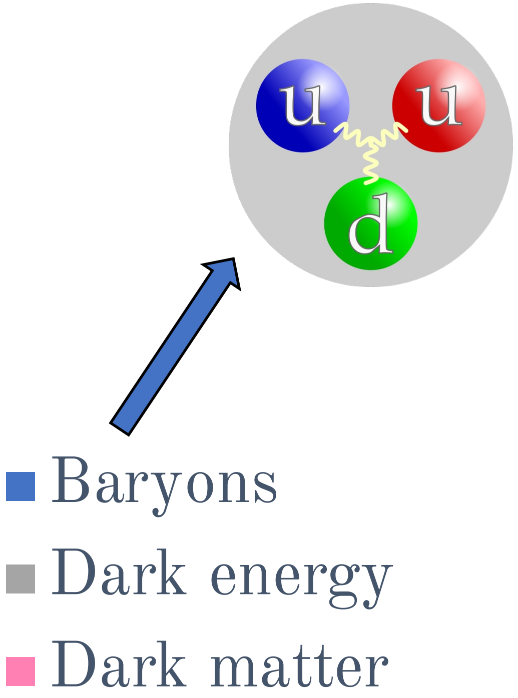

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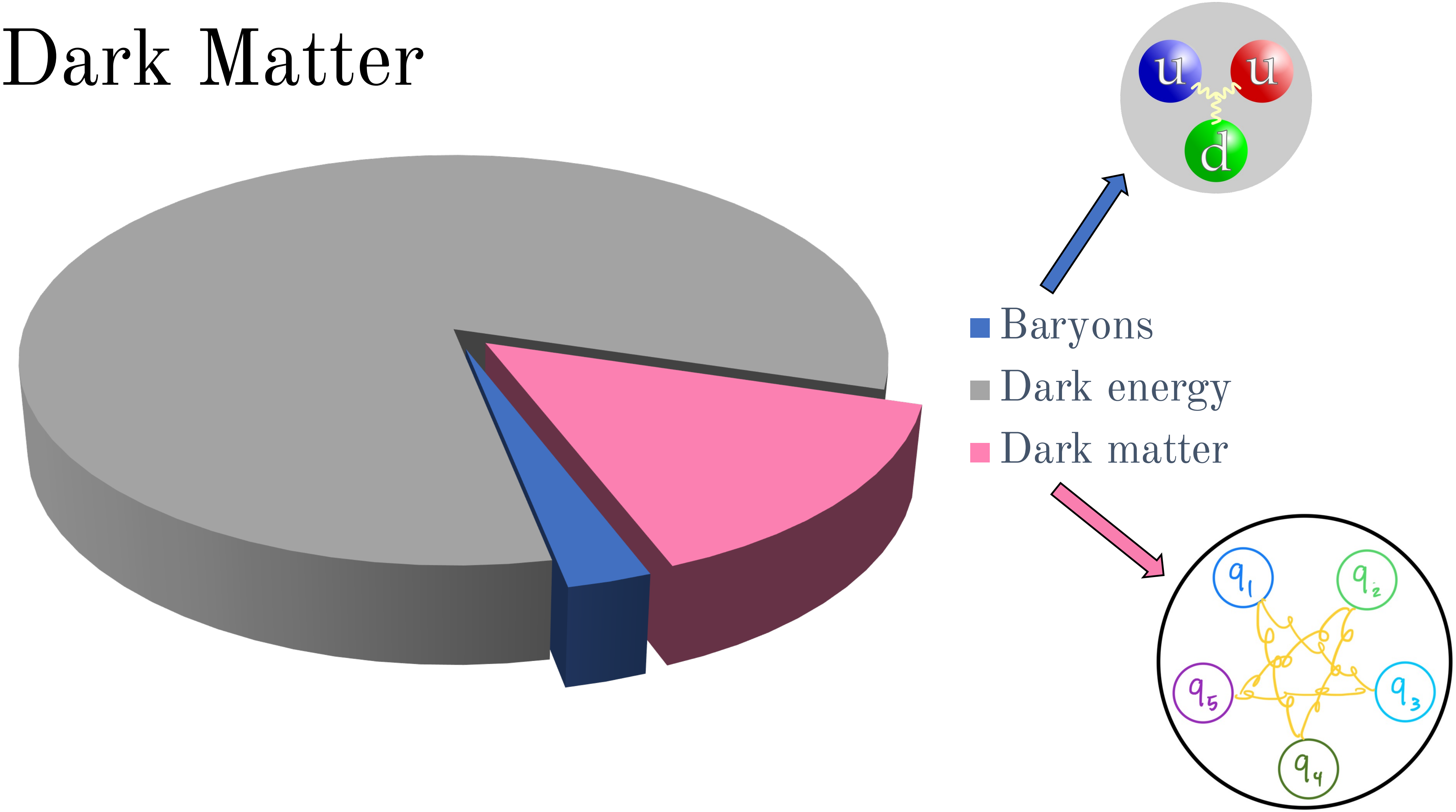

Model

<u>Dark quarks</u>

- Fermion with Dirac mass m_q
- $N_f = \mathbf{3, 5, \ldots}$ of $SU(2)_L$
- Hypercharge $Y=0$

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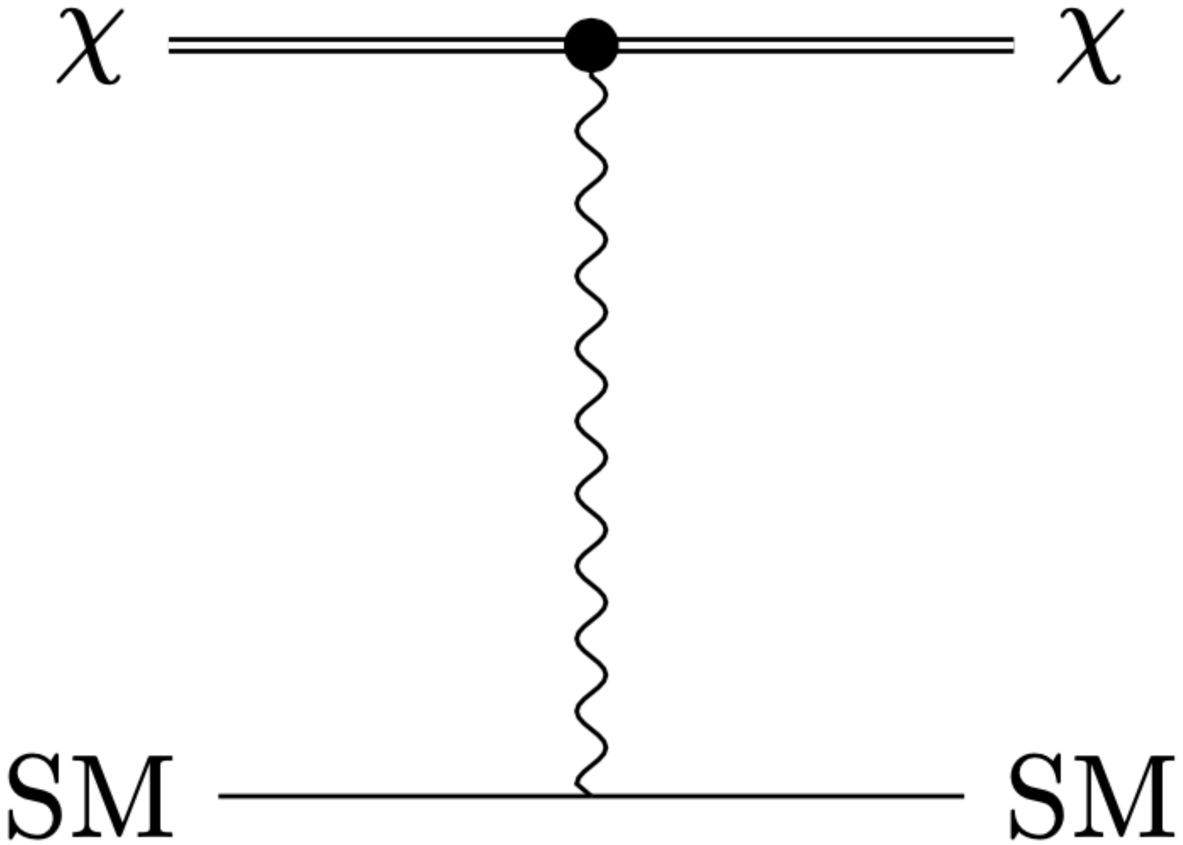
DM Candidate

- Lightest stable baryon
- Spin-½
- Zero charge, hypercharge

SM Coupling



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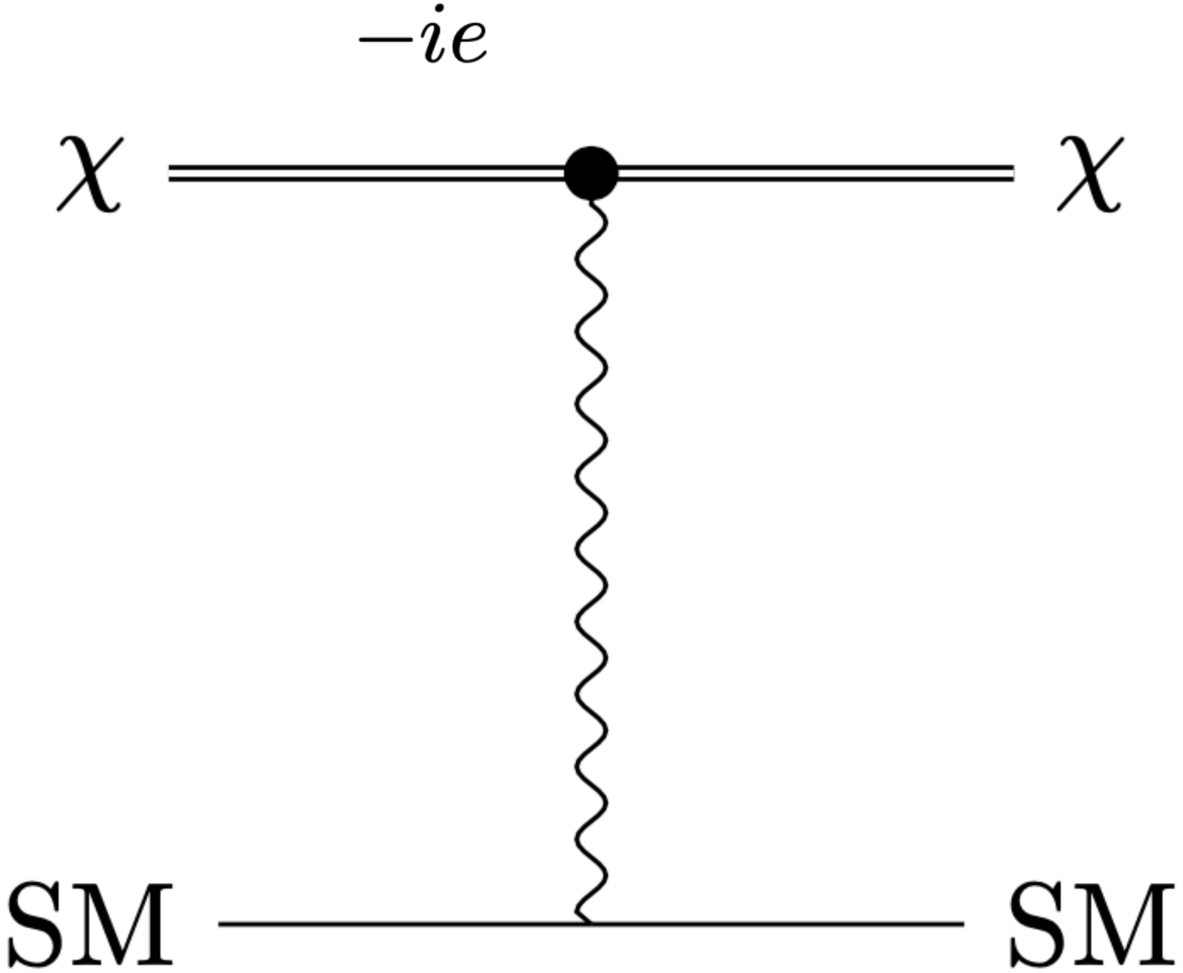

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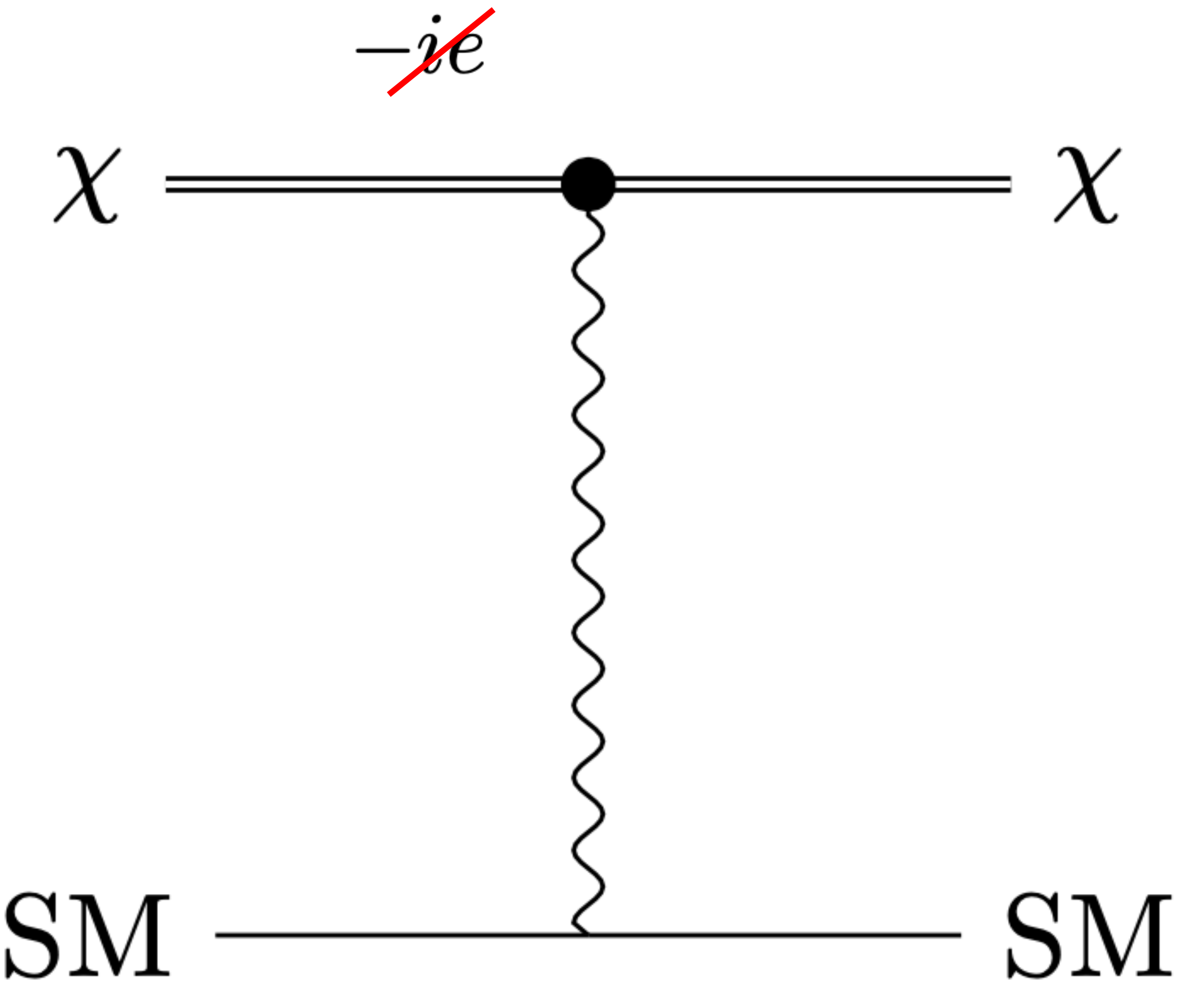
$Q_B = 0$



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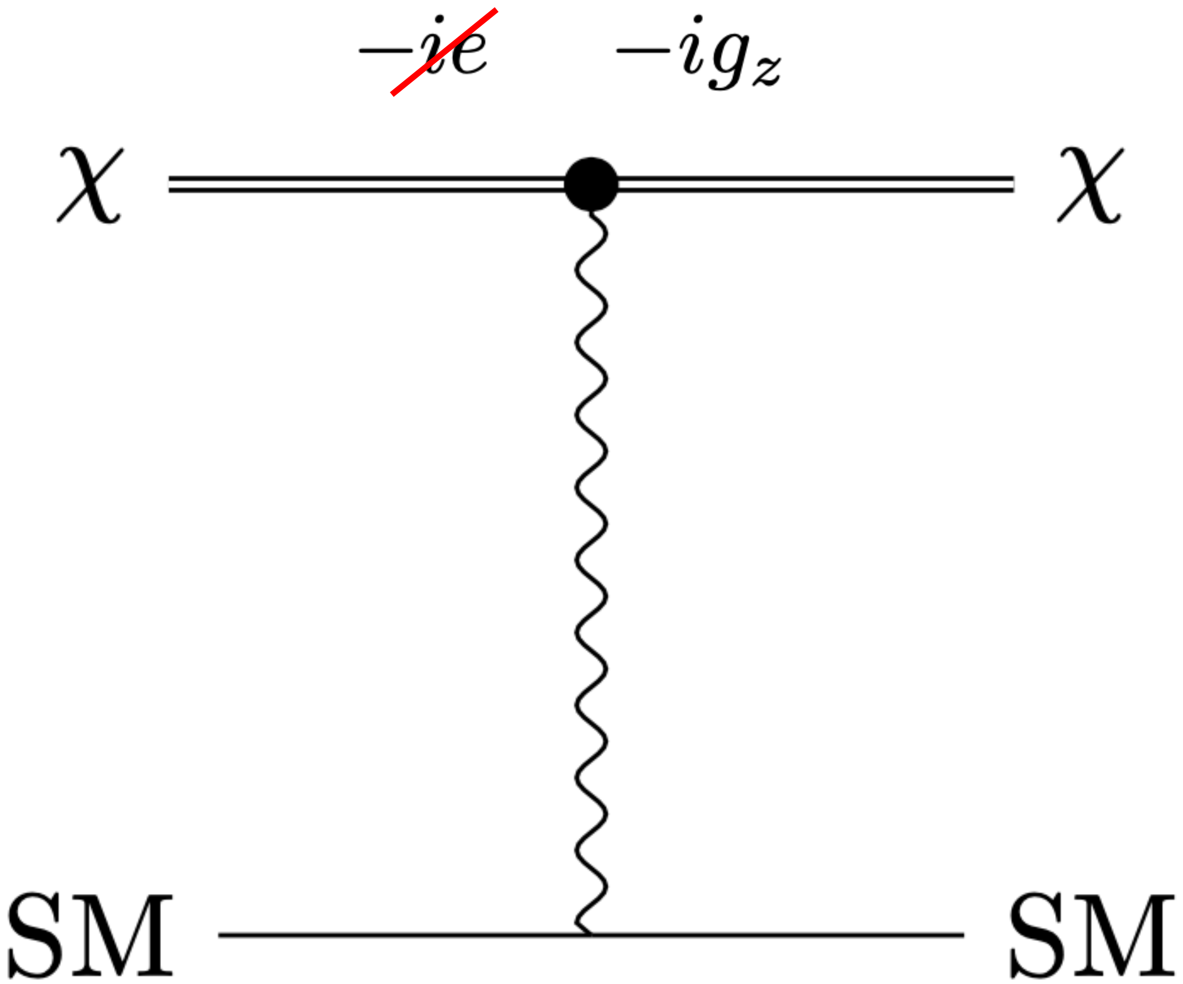
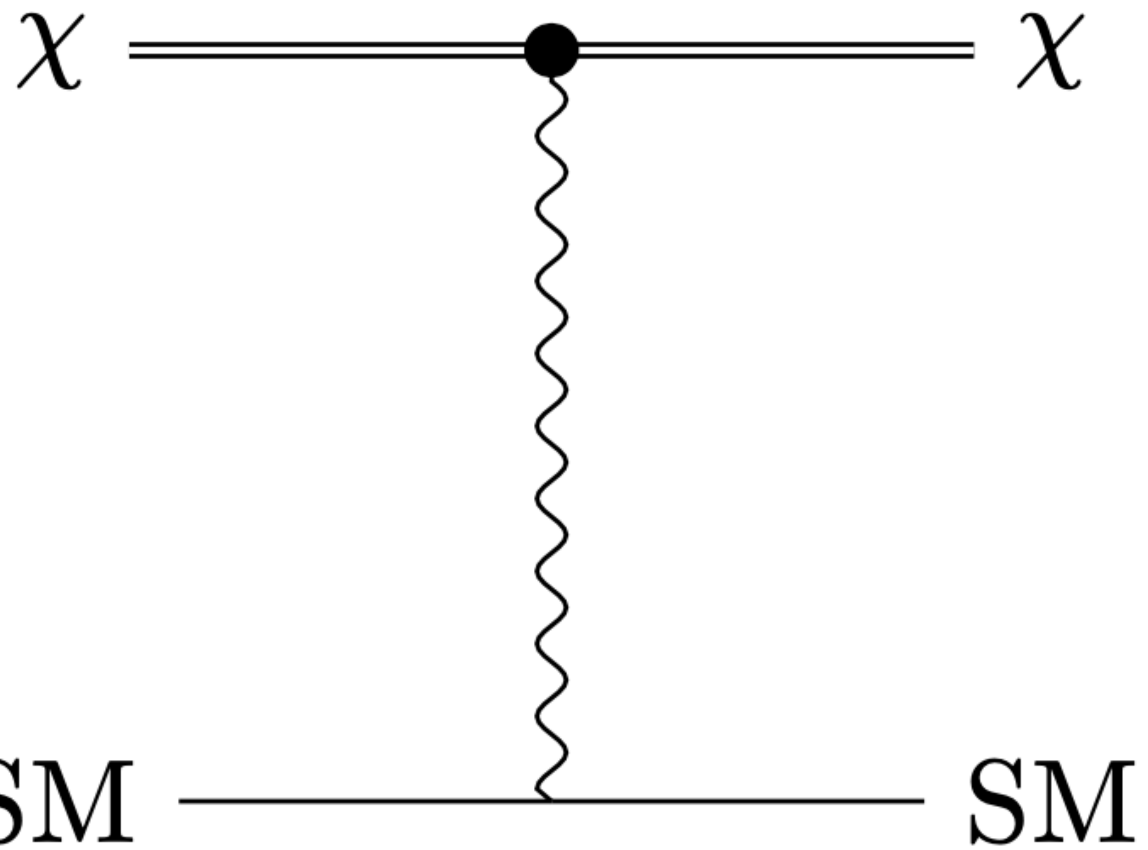


SM Coupling

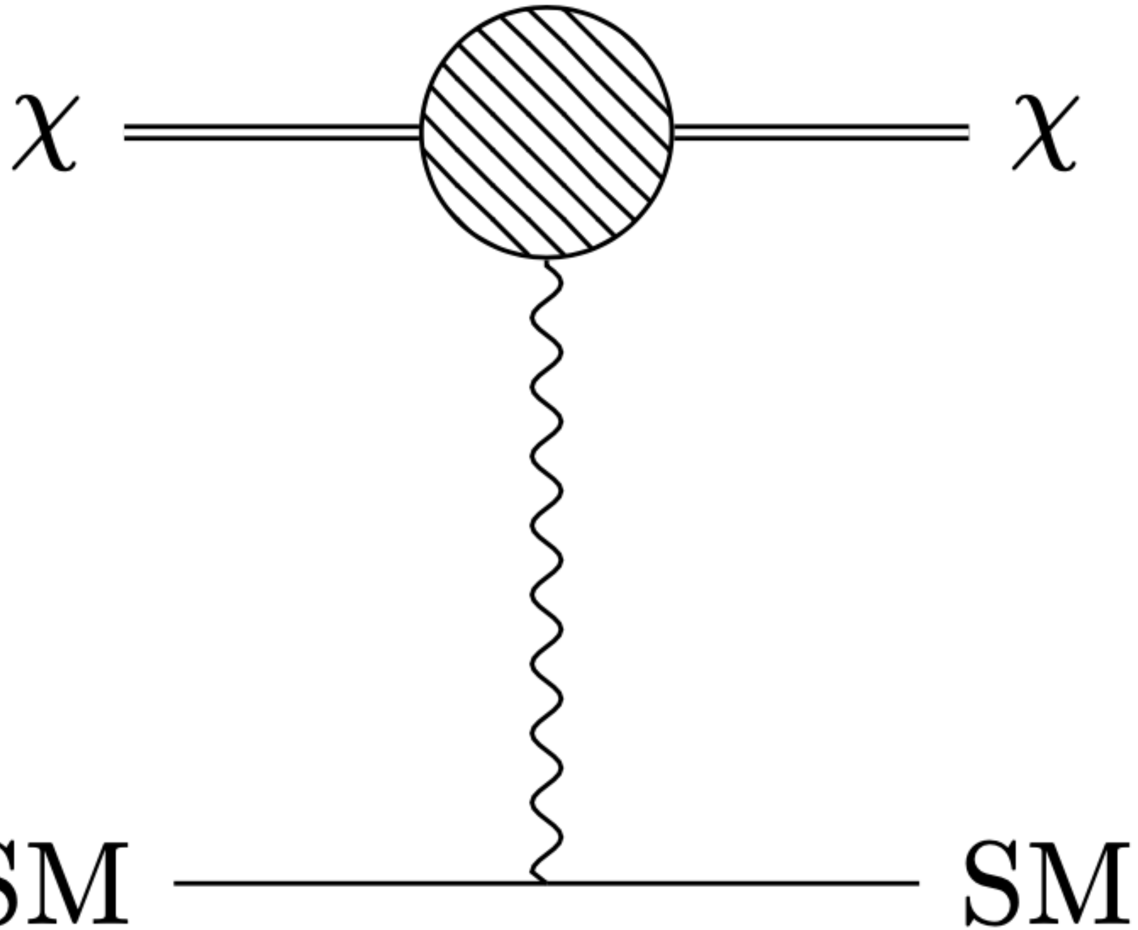
$Q_B = 0$ $\quad$ $T^3{}_B + Y_B \sin(\theta_w) = 0$

$\cancel{-ie}$ $\quad$ $\cancel{-ig_z}$



SM Coupling

$$\sum_n \mathcal{O}_n/\Lambda^n = \frac{1}{\Lambda}\overline{\chi}\sigma_{\mu\nu}\chi F^{\mu\nu} + \mathcal{O}(1/\Lambda^6)$$



Naïve Phenomenology

$$\mathcal{L} \supset \frac{1}{\Lambda}\overline{\chi}\sigma_{\mu\nu}\chi F^{\mu\nu}$$

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Direct detection constraints: $m_\chi < 47$ TeV for $g_M \sim 1$

see also J. Eby, P. Fox, G. Kribs, to appear

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What is the magnetic moment of the ground state baryon?

Quark Model

Baryon wavefunction: $\Psi_B = \xi_{\text{color}}\, \eta_{\text{space}}\, \phi_{\text{spin}} \psi_{\text{flavor}}$ ➡ Spin-flavor symmetric

Antisymmetric: Confinement   Symmetric for s-wave

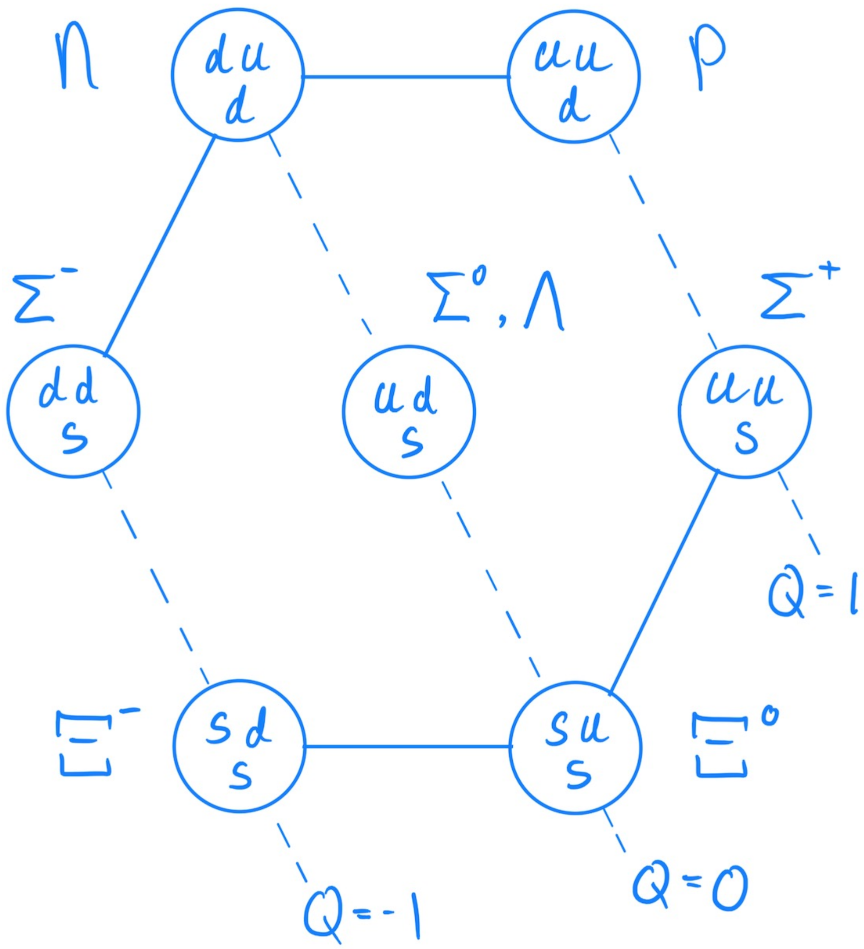
Quarks transform under fundamental rep of $SU(N_f)$ and $SU(2_{\text{spin}})$

e.g., $SU(2)_L$ triplet with $N_c = 3$,

Flavor: $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$

Spin: $2 \otimes 2 \otimes 2 = 2 \oplus 2 \oplus 4$

The Eightfold Way



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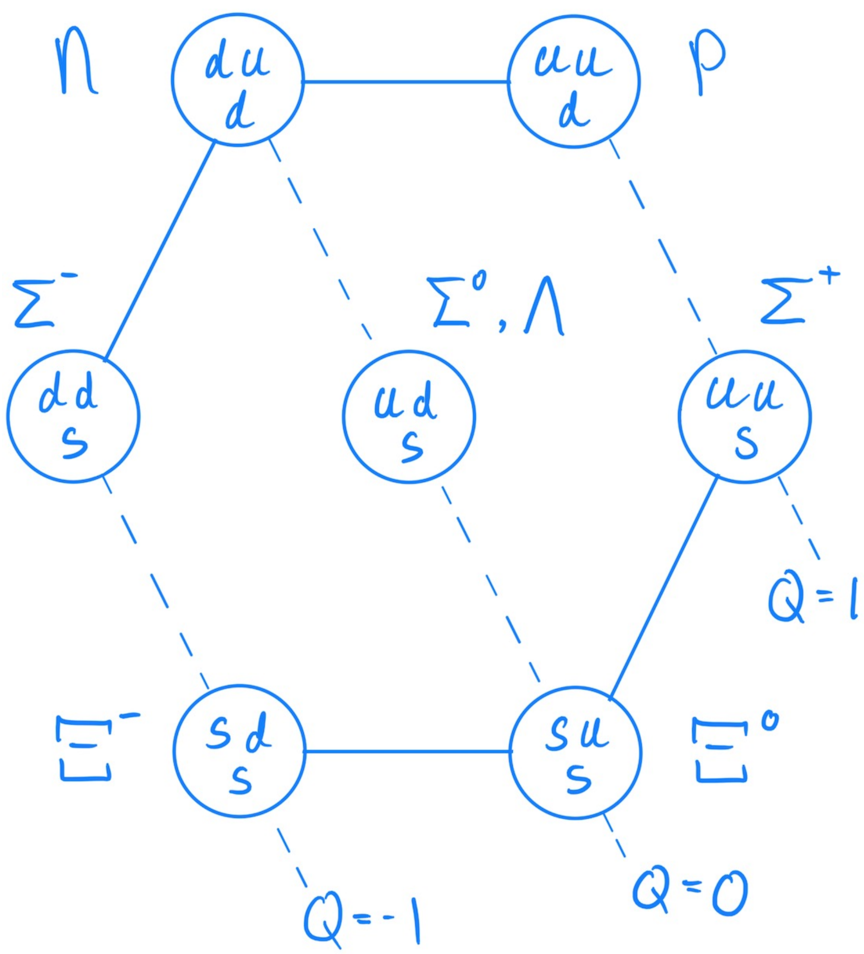
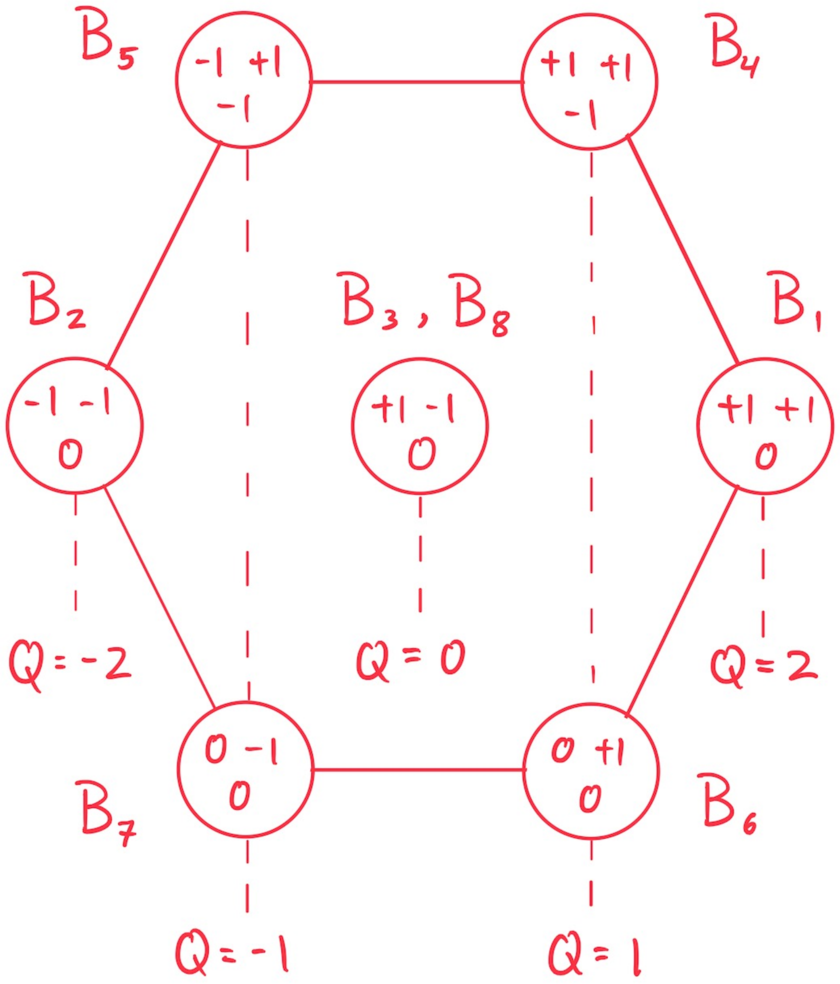


The Dark Eightfold Way



e.g. $\mathrm{N_c = N_f = 3}$

Wavefunction: $$|\Sigma^0\rangle = \frac{1}{\sqrt{2}}\Big[\frac{1}{\sqrt{12}}(2uds - usd - dsu + 2dus)\cdot\frac{1}{\sqrt{6}}(\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$$
$$+\frac{1}{2}(usd + dsu - sdu - sud)\cdot\frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)\Big]$$
$$= \frac{1}{6}\Big[(2\,u\uparrow d\uparrow s\downarrow - u\uparrow d\downarrow s\uparrow - u\downarrow d\uparrow s\uparrow) + \text{permuations}\Big]$$

Magnetic moment: $$\mu_{\Sigma^0} = \frac{1}{36}\left[4\left(\mu_u+\mu_d-\mu_s\right)+\left(\mu_u-\mu_d+\mu_s\right)+\left(-\mu_u+\mu_d+\mu_s\right)\right]\times 3$$
$$= \frac{1}{3}(2\mu_u + 2\mu_d - \mu_s)$$

Results

Neutral spin-½ baryons
with zero magnetic moment
with nonzero magnetic moment

N_f \ N_c	3	5	7	9
3	2	3	4	5
5	4 / 2	11 / 10	19 / 36	
7	6 / 6	24 / 50		
9	8 / 12			
11	10 / 20			

Dark Quark Magnetic Moment: $$\mu_i = \frac{Q_i e}{2m_q} S_i := Q_i \tilde{\mu}$$

Discussion

I. Some neutral spin-½ baryons have vanishing magnetic moments
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II. If baryon mass degeneracy is lifted by $\delta m_q \sim Q_q^2$, then the
dark matter magnetic moment always vanishes.

III. Quark model exact at large N_c ➡ $\mu_\chi = \mathcal{O}(1/N_c)$
Constraints on dim-6 operators dominate – weaker than constraints on magnetic dipole

Thank you!

Questions?

Acknowledgements:

Summer Undergraduate Research Fellowship

UNIVERSITY OF OREGON

cure

CENTER for UNDERGRADUATE RESEARCH and ENGAGEMENT

Backup: Quark Model Generalized

- Express baryon in quark bases of spin i and flavor j
- C_a are generalized Clebsch-Gordan Coefficient – Calculated in Mathematica with *GroupMath*

$$|\mathcal{B}_a\rangle = \sum_{i_1,j_1,\ldots,i_{N_c},j_{N_c}} (C_a)_{i_1,j_1,\ldots,i_{N_c},j_{N_c}} |i_1, j_1; \ldots ; i_{N_c}, j_{N_c}\rangle$$

- Calculate the magnetic moment from wavefunction

$$\langle\mathcal{B}_a|\,\hat{\mu}\,|\mathcal{B}_a\rangle = \mu_{\mathcal{B}_a} = \sum_{i_1,j_1,\ldots,i_{N_c},j_{N_c}} \left[(C_a)_{i_1,j_1,\ldots,i_{N_c},j_{N_c}}\right]^2 \left[(-1)^{j_1}\mu_{i_1} + \cdots + (-1)^{j_{N_c}}\mu_{i_{N_c}}\right]^2$$

Backup: Baryon wavefunctions

- Flavor wavefunctions are SU(N_f) tensors. Representation has multiplicity γ

$$\mathcal{F}_a^\gamma = (\mathcal{F}_a^\gamma)^{i_1,\ldots,\, i_{m+1}}_{i_{m+2},\ldots,\, i_{N_c}} \qquad \text{Odd } N_c = 2m+1$$

$$\mathcal{F}_a^\gamma = (\mathcal{F}_a^\gamma)^{i_1,\ldots,\, i_{m}}_{i_{m+1},\ldots,\, i_{N_c}} \qquad \text{Even } N_c = 2m$$

- Spin wavefunction identical to first flavor
    - Do so to match the multiplicities by the symmetries of their indices

$$\mathcal{S}^\gamma = \mathcal{F}_1^\gamma = (\mathcal{S}^\gamma)^{j_1,\ldots,\, j_{m+1}}_{j_{m+2},\ldots,\, j_{N_c}} \qquad \text{Odd } N_c$$

$$(\mathcal{S}^\gamma)^{j_1,\ldots,\, j_{m}}_{j_{m+1},\ldots,\, j_{N_c}} \qquad \text{Even } N_c.$$

- Baryons are product of spin and flavor, summed over multiplicity

$$\mathcal{B}_a = \sum_\gamma \mathcal{S}^\gamma \otimes \mathcal{F}_a^\gamma$$

$$= (\mathcal{B}_a^\gamma)^{j_1,\ldots,\, j_{m+1},\, i_1,\ldots,\, i_{m+1}}_{j_{m+2},\ldots,\, j_{N_c},\, i_{m+2},\ldots,\, i_{N_c}} \qquad \text{Odd } N_c,$$

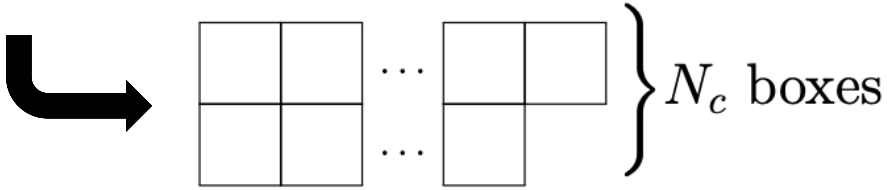
$$= (\mathcal{B}_a^\gamma)^{j_1,\ldots,\, j_{m},\, i_1,\ldots,\, i_{m}}_{j_{m+1},\ldots,\, j_{N_c},\, i_{m+1},\ldots,\, i_{N_c}} \qquad \text{Even } N_c.$$

Backup: Quark model generalized

$$(\mathbf{N_f}, \mathbf{2})_1 \otimes, \ldots, \otimes (\mathbf{N_f}, \mathbf{2})_{N_c} = \bigoplus_p M_p(\mathbf{R}_{p,\text{flavor}}, \mathbf{R}_{p,\text{spin}})$$

SU(N_f) irrep of spin-½ baryons: $\left(1, \frac{N_c - 1}{2}, \overbrace{0, \ldots, 0}^{N_f - 3}\right)$

- For $N_f = 3$



Backup: Proton Large N_c Scaling

- "Proton" magnetic moment
  - Two flavors, $I_3 = ½$
  - Neutron is C conjugate – charged when quarks are a double with zero hypercharge

Agrees with $\mu_{\text{nucleon}} = \mathcal{O}(N_c)$

N_c	μ_B
3	$(4\mu_+ - \mu_-)/3$
5	$(5\mu_+ - 2\mu_-)/3$
7	$2\mu_+ - \mu_-$
9	$(7\mu_+ - 4\mu_-)/3$
11	$(8\mu_+ - 5\mu_-)/3$

Backup: Results: $\mathrm{N_f} = 5$, $\mathrm{N_c} = 3$

Quark content	μ_B	$\mu_B(\mu_i \to Q_i)$
q_1, q_3, q_5	μ_3	0
q_2, q_3, q_4	μ_3	0
q_1, q_3, q_5	$\frac{1}{3}(2\mu_1 + 2\mu_5 - \mu_3)$	0
q_2, q_3, q_4	$\frac{1}{3}(2\mu_2 + 2\mu_4 - \mu_3)$	0
q_1, q_4, q_4	$\frac{1}{3}(4\mu_4 - \mu_1)$	-2
q_2, q_2, q_5	$\frac{1}{3}(4\mu_2 - \mu_5)$	2

$$Q_i = \begin{pmatrix} 2 \\ 1 \\ 0 \\ -1 \\ -2 \end{pmatrix}$$