Effects of EFT operators on off-shell production of the Higgs boson

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Pheno - Higgs - May 9, 2023

Lawrence Hall - Room 121

University of Pittsburgh

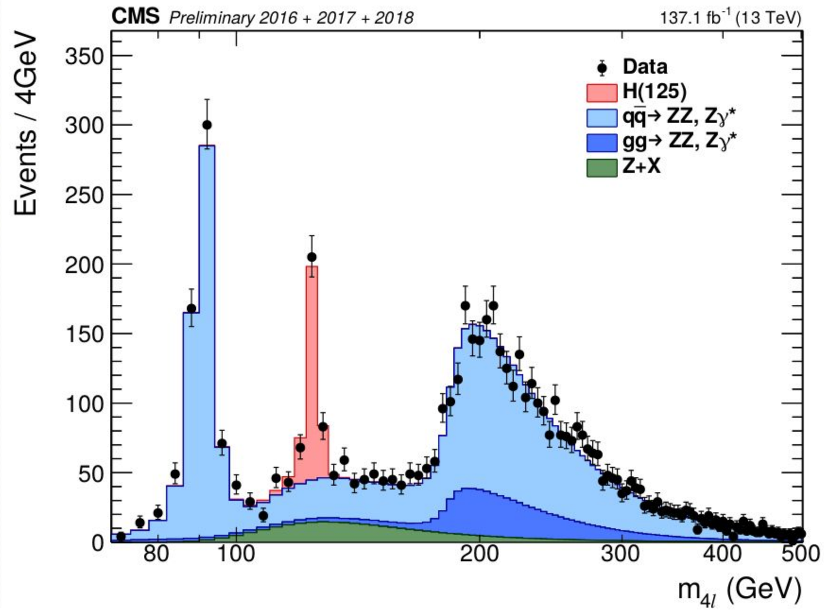
Outline

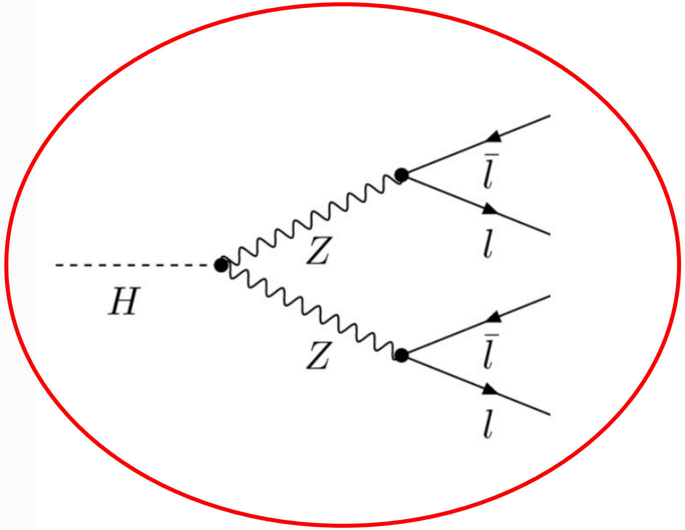
1. **The Higgs boson in the Standard Model**
    1.1. What are we studying?
2. **Exploring SMEFT through the Higgs boson**
    2.1. How do we probe anomalous behavior?
3. **Simulating anomalous couplings in signal and background**
    3.1. How can we make the best measurement?
4. **Searching for deviations from the Standard Model**
    4.1. What would they look like at the LHC?
5. **Conclusions and Future**

Part 1: Starting with the Standard Model Higgs

Targeting the Higgs boson

- The Higgs portal currently provides an opportunity to possibly probe new physics or anomalous behavior
- Focus on the Higgs decay via $H \rightarrow ZZ \rightarrow 4\ell$: a well-understood decay channel which can be accurately simulated
- Named the “Golden channel” for its good experimental resolution and clear separation of signal to background



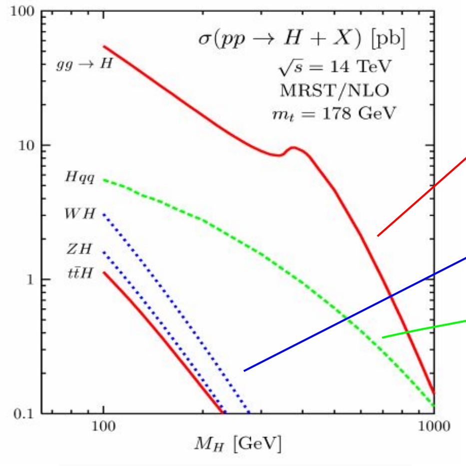


Higgs Production

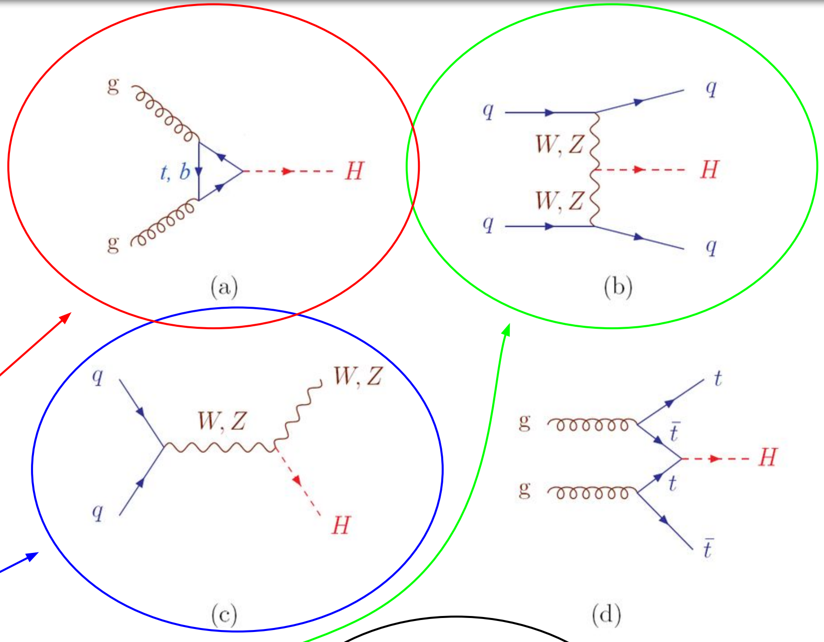
Primary production channels are:

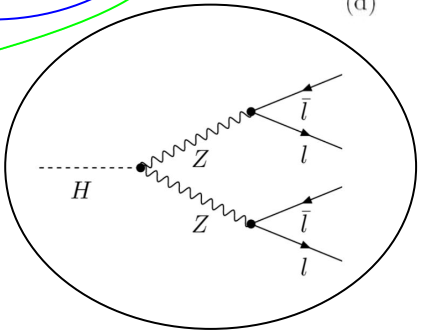
- gg fusion
- VBF (vector boson fusion)
- VH (Higgs-strahlung)
- tt̄H

Decay occurs through an HVV vertex to two di-lepton pairs



https://doi.org/10.1007/s12043-009-0002-2

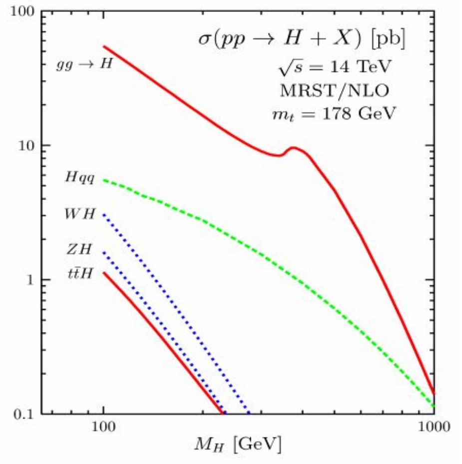




Measuring the Higgs boson

One way to make precise measurements of Higgs properties (like the Higgs width Γ_H) is to utilize off-shell bosons in addition to on-shell production.

Signal strength μ_{vvH} term cancels out when comparing between off-shell and on-shell events, giving us a precise measurement of the Higgs width.

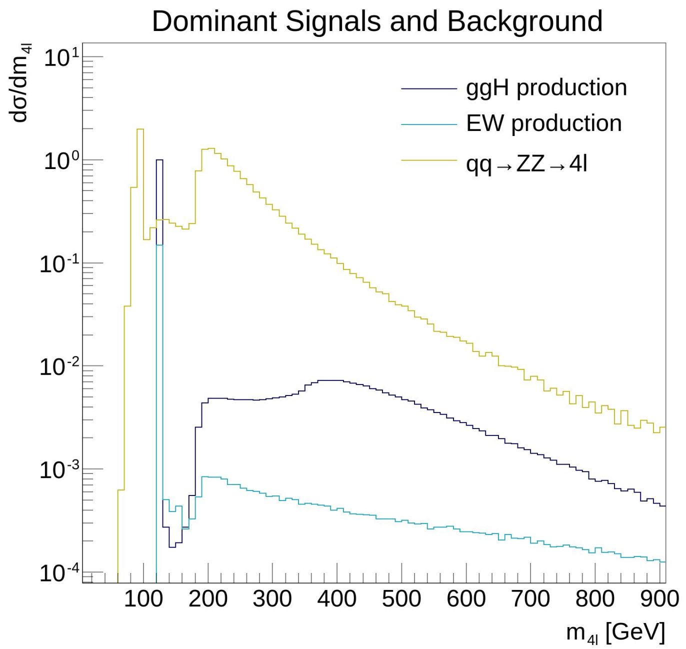


https://doi.org/10.1007/s12043-009-0002-2

$$\sigma^{\text{on-shell}}_{\mathrm{vv}\to\mathrm{H}\to 4\ell} \propto \mu_{\mathrm{vvH}}$$

$$\sigma^{\text{off-shell}}_{\mathrm{vv}\to\mathrm{H}\to 4\ell} \propto \mu_{\mathrm{vvH}} \cdot \Gamma_{\mathrm{H}}$$

https://arxiv.org/abs/1901.00174



These invariant mass distributions were made using JHUGen+MCFM

Part 2: Considering the Higgs in SMEFT

SM as an Effective Field Theory

- A commonly-discussed, developing description of the Standard Model is as an EFT
- Allowing for higher dimension operators in the SM Lagrangian results in new terms in our amplitudes

$$\mathcal{L}_{\text{SM}} = \mathcal{L}^{(0)} + \mathcal{L}^{(2)} + \mathcal{L}^{(4)}$$

$$\downarrow \quad \boxed{\Lambda \gg M_H}$$

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}^{(0)} + \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \ldots + \mathcal{L}^{(n)}$$

$$\mathcal{L}^{(n)} = \frac{1}{\Lambda^{n-4}} \Sigma_k C_k^{(n)} \mathcal{O}_k^{(n)}$$

Λ **scale of BSM**

$C_k^{(n)}$ **dimensionless Wilson coefficient**

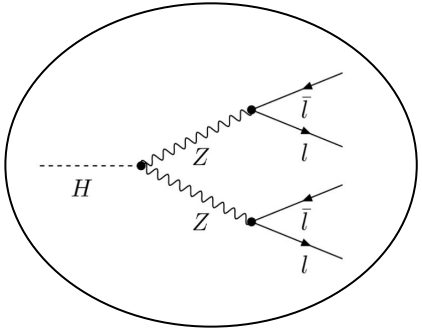
$\mathcal{O}_k^{(n)}$ **higher order operator**

Amplitude of the HVV vertex

- A general form of this HVV amplitude is shown below, quoted from a previous analysis
- Recall our VBF and VH processes have HVV vertices on the production side as well
    - Anomalous EW production modes can help in exaggerating the effect of any BSM terms in this decay amplitude
- We can also define f_{ai} values which describe the fractional size of the BSM contribution for our Higgs decays. For example, f_{a1} = 0 would indicate a pure SM Higgs boson, while f_{a2} = 1 would gives us a pure BSM particle with strict dependence on anomalous coupling strength a_2

$$A(\mathrm{HVV}) \sim \left[a_1^{\mathrm{HVV}} + \frac{\kappa_1^{\mathrm{HVV}} q_{\mathrm{V}_1}^2 + \kappa_2^{\mathrm{HVV}} q_{\mathrm{V}_2}^2}{\left(\Lambda_1^{\mathrm{HVV}}\right)^2} \right] m_{\mathrm{V}_1}^2 \epsilon_{\mathrm{V}_1}^* \epsilon_{\mathrm{V}_2}^* + a_2^{\mathrm{HVV}} f_{\mu\nu}^{*(1)} f^{*(2)\mu\nu} + a_3^{\mathrm{HVV}} f_{\mu\nu}^{*(1)} \tilde{f}^{*(2)\mu\nu}$$

$$f_{ai} = \frac{|a_i|^2 \sigma_i}{\sum_{j=1,2,3...} |a_j|^2 \sigma_j},$$



https://arxiv.org/abs/1707.00541
https://arxiv.org/abs/1507.06656

Part 3: Simulating the Higgs (including AC)

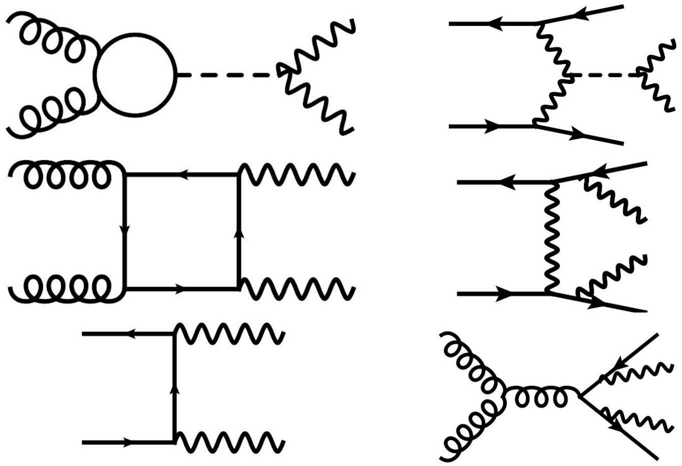
Simulating kinematic distributions

Gluon fusion | **Electroweak (VBF)**

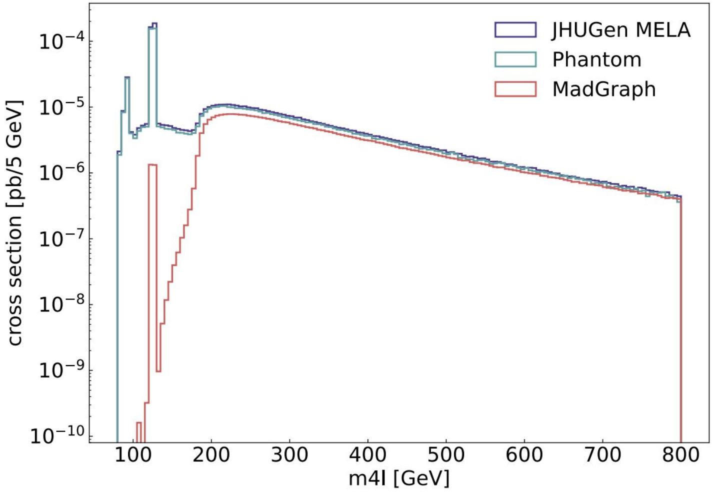
SIG

INT

BKG



https://arxiv.org/pdf/2002.09888.pdf

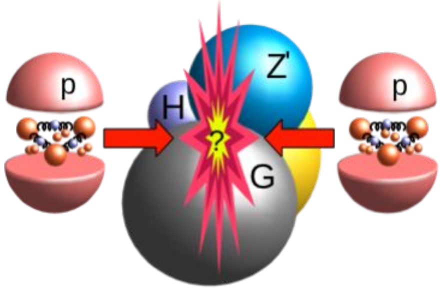


Comparison between generators—good agreement between JHUGen and Phantom.

MadGraph sample to be taken with a grain of salt here. Suboptimal sampling of phase space in simulation resulting in partial xsec.

R. Wang @ APS 2023

Focus on the JHUGen Framework



JHUGenerator

- Simulate various processes involving spin 0,1,2 particles with a general coupling model
- Interfaces with modified version of MCFM for off-shell EW matrix elements

JHUGen MELA (Matrix Element Likelihood Approach)

- Reweight generated samples from one hypothesis to another
- Construct observables to isolate processes or operators

https://spin.pha.jhu.edu/index.html

JHUGenLexicon

- Translate between EFT bases and the JHUGen amplitude basis convention

M. Schulze @ Monte-Carlo reweighting (2015)
H. Roskes @ Pheno 2020
M. Xiao @ ICHEP 2020
A. Gritsan @ LHC HXS WG (2020)
U. Sarica @ Higgs 2020
H. Roskes @ LHC EFT WG (2020)
J. Davis @ Pheno 2022

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*******************************************************************************
*                          JHU Generator v7.5.2                               *
*******************************************************************************
*                                                                             *
*   Spin and parity determination of single-produced resonances at hadron colliders   *
*                                                                             *
*        I. Anderson, S. Bolognesi, F. Caola, J. Davis, Y. Gao, A. V. Gritsan,       *
*   L. S. Mandacaru Guerra, Z. Guo, C. B. Martin, T. Martini, K. Melnikov, R. Pan,   *
*R. Rontsch, J. Roskes, U. Sarica, M. Schulze, N. V. Tran, A. Whitbeck, M. Xiao, Y. Zhou*
*               Phys.Rev. D81 (2010) 075022;  arXiv:1001.3396  [hep-ph],      *
*               Phys.Rev. D86 (2012) 095031;  arXiv:1208.4018  [hep-ph],      *
*               Phys.Rev. D89 (2014) 035007;  arXiv:1309.4819  [hep-ph],      *
*               Phys.Rev. D94 (2016) 055023;  arXiv:1606.03107 [hep-ph],      *
*               Phys.Rev. D102 (2020) 056022; arXiv:2002.09888 [hep-ph],      *
*               Phys.Rev. D102 (2021) 055045; arXiv:2104.04277 [hep-ph].      *
*               Phys.Rev. D105 (2022) 096027; arXiv:2109.13363 [hep-ph].      *
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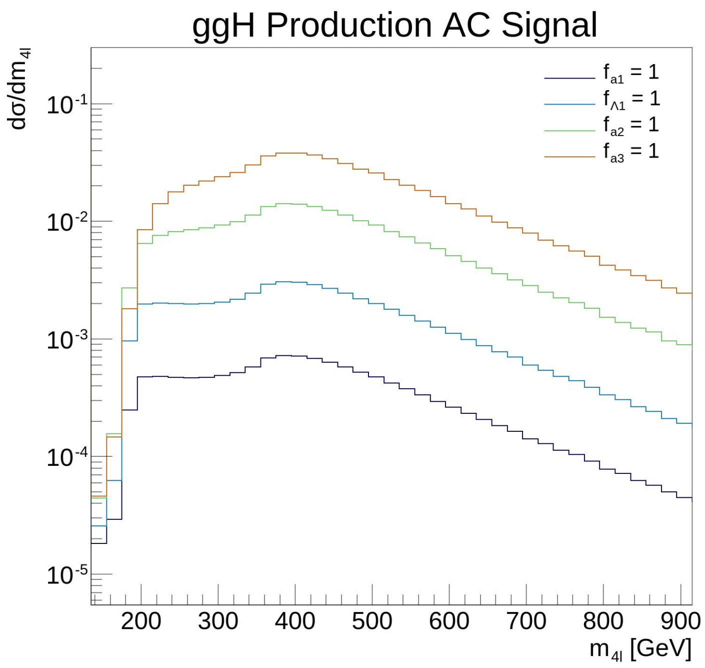
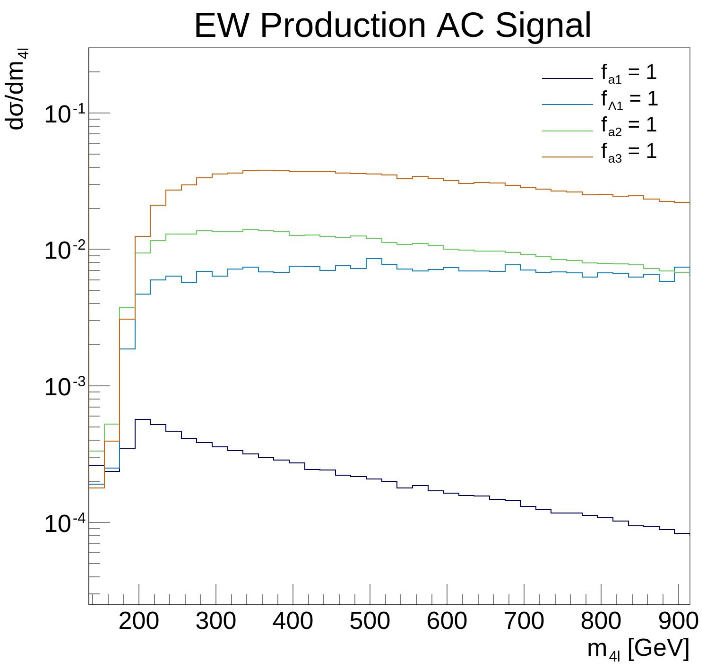
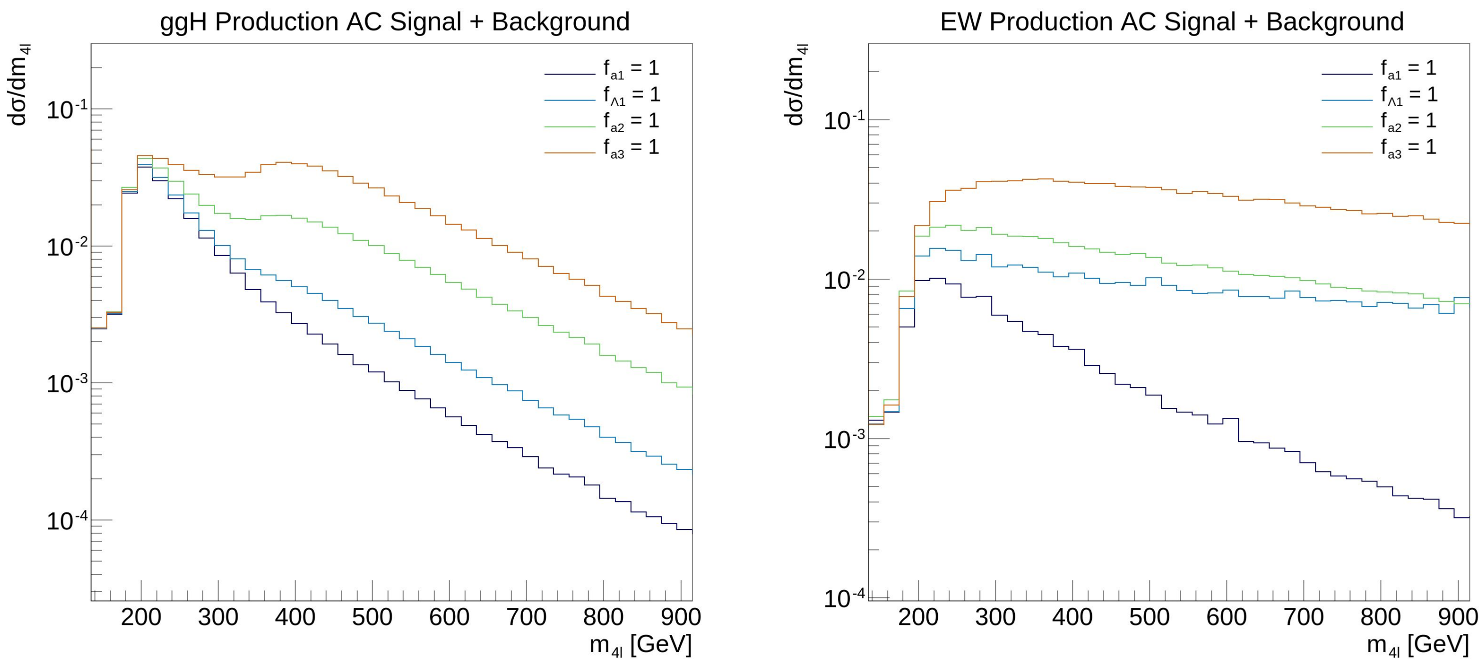
Effects of AC on off-shell production (SIG)





Off-shell production is very sensitive to anomalous effects (as illustrated above)
These plots were made with the use of JHUGen+MCFM and MELA

Effects of AC on off-shell production (BSI)



Unit normalized in on-shell region (105-140 GeV)
These plots were made with the use of JHUGen+MCFM and MELA

Off-shell Higgs

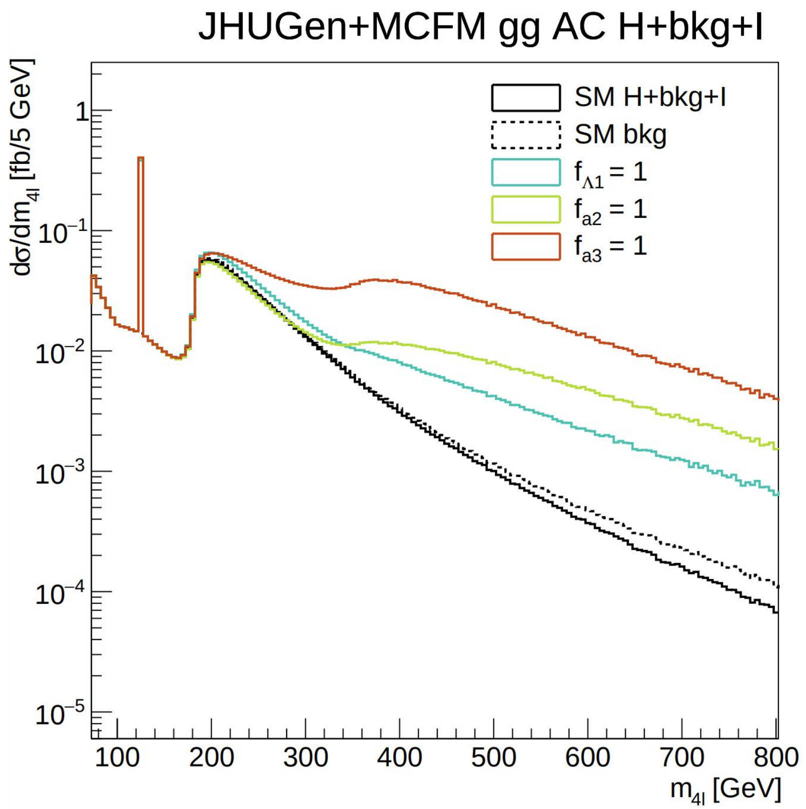
- Off-shell production is very sensitive to anomalous effects
- We can include probability distributions for off-shell Higgs simulations that have anomalous interactions
- Set precise constraints on potential BSM couplings
- Contributed to off-shell Higgs WG Report

https://arxiv.org/abs/2203.02418

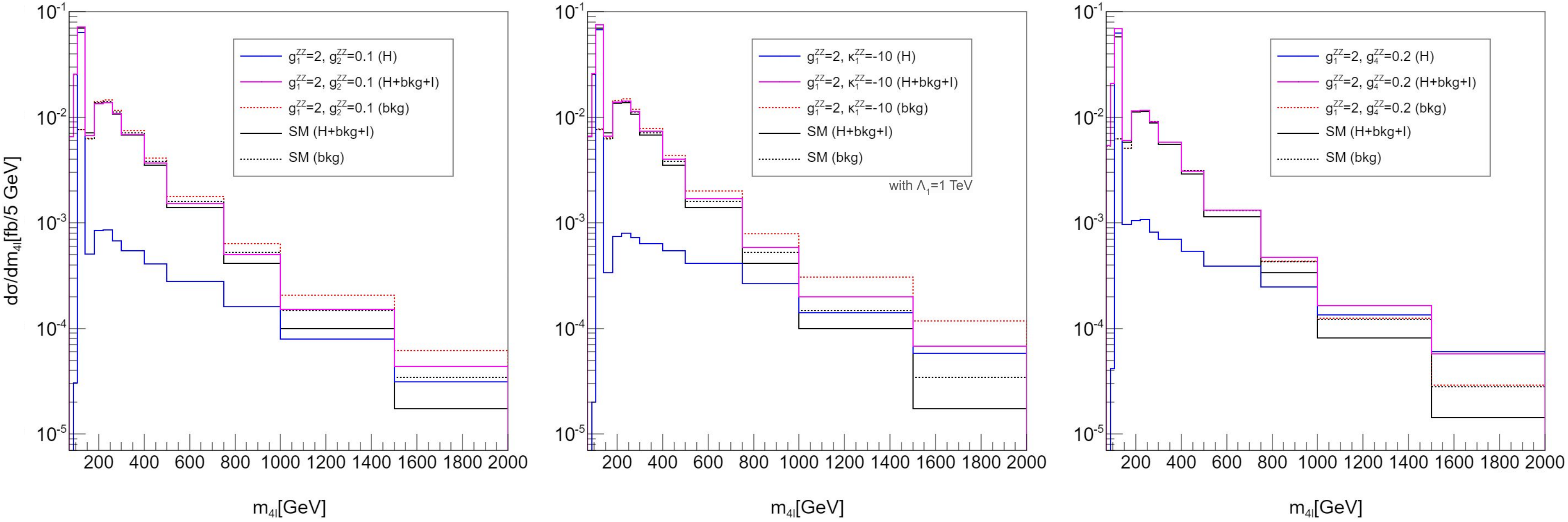
Off-shell Higgs Interpretations Task Force[b]

Models and Effective Field Theories Subgroup Report

Aleksandr Azatov [1,2,c], Jorge de Blas [3,d], Adam Falkowski[4,e], Andrei V. Gritsan[5,f], Christophe Grojean[6,7,g], Lucas Kang[5,h], Nikolas Kauer[8,i] (ed.), Ennio Salvioni[9,10,j], Ulascan Sarica[11,k], Marion Thomas[12,l] and Eleni Vryonidou[12,m]



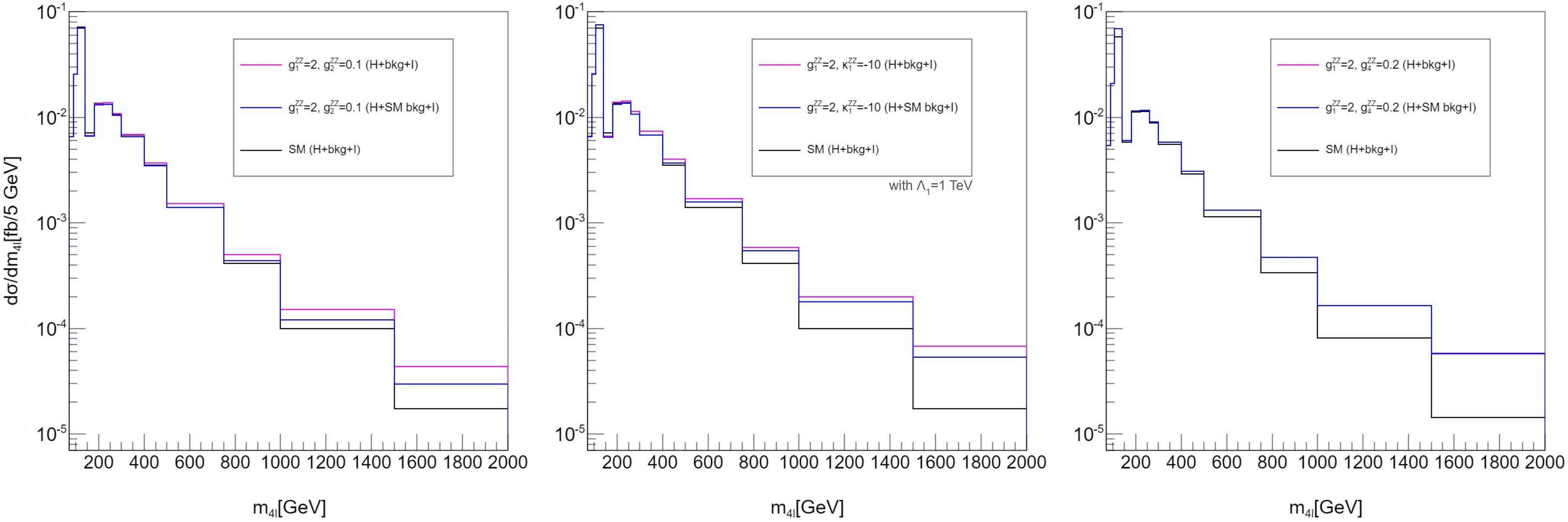
AC contributions to pure SM behavior



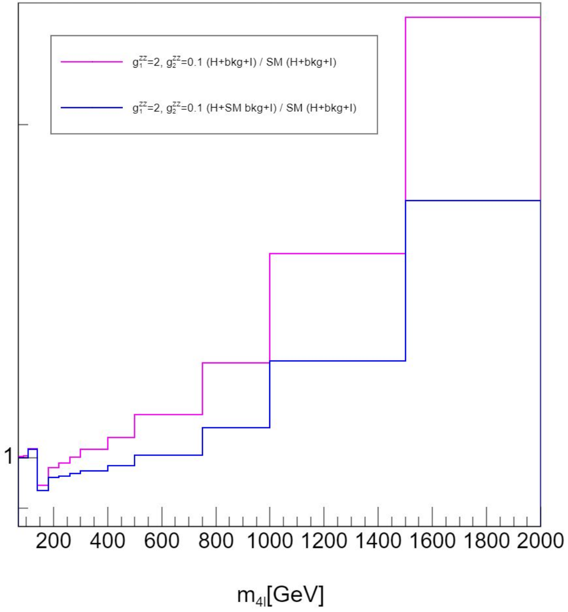
Anomalous coupling strengths chosen to be at the scale of expected experimental constraints from H^* off-shell data at LHC

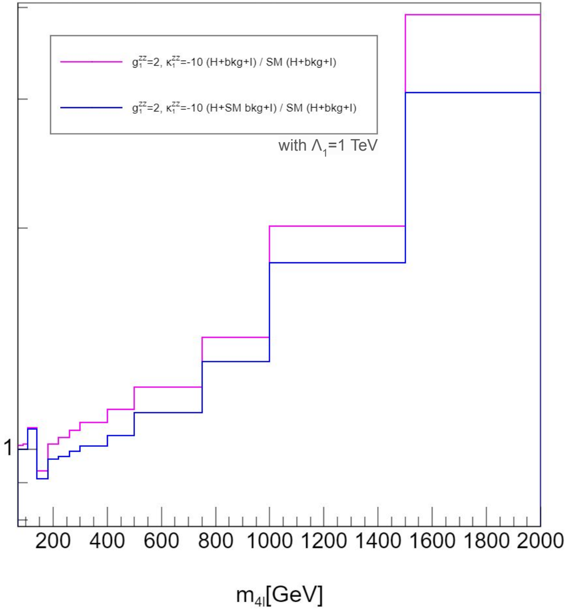
https://arxiv.org/abs/2202.06923
https://arxiv.org/abs/2002.09888
https://arxiv.org/abs/1901.00174

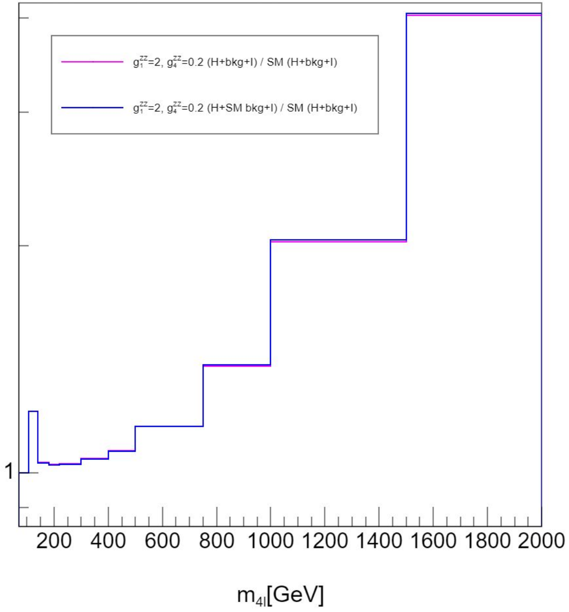
Effect of including AC in signal and background



Effect of including AC in signal and background (ratio to SM)



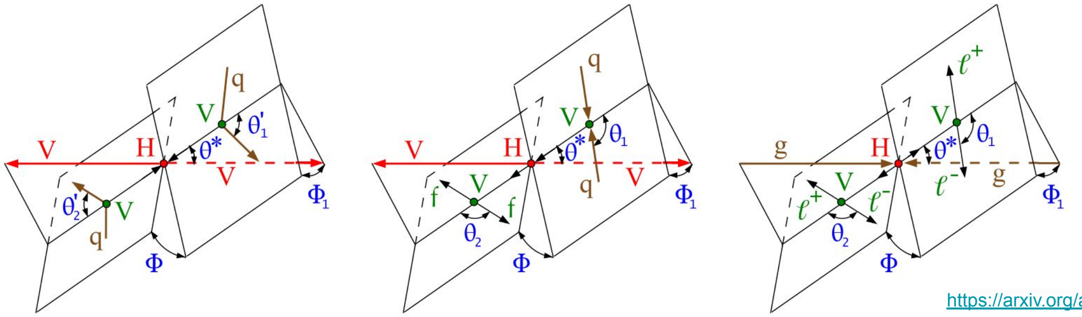




Part 4: Towards analysis

Kinematic Observables

- 6 observables in 1→4 process, but 13 observables in 2→6 process
  - $\Omega^{decay} = \{\theta_1, \theta_2, \Phi, m_1, m_2, m_{4f}\}$
  - $\Omega^{prod} = \{\theta^*, \Phi_1\}$
  - $\Omega^{assoc} = \{\theta_1^{(VBF, VH)}, \theta_2^{(VBF, VH)}, \Phi^{(VBF, VH)}, q_1^{2,(VBF, VH)}, q_2^{2,(VBF, VH)}\}$
- MELA (matrix element likelihood approach) packages this information into matrix elements
  - These can be used to reweight our events from one hypothesis to another (f_{a1}=1 to f_{a2}=1, for example)
  - This provides the benefit of letting us improve statistics for any given sample, as well as calculate cross-terms to create interference samples when we float the values of more than one anomalous coupling at a time
- Discriminants are constructed from MELA probabilities for event selection and to use as observable in the analysis



$$\mathcal{D}_{\text{alt}}\left(\vec{\Omega}\right) = \frac{\mathcal{P}_{\text{sig}}\left(\vec{\Omega}\right)}{\mathcal{P}_{\text{sig}}\left(\vec{\Omega}\right) + \mathcal{P}_{\text{alt}}\left(\vec{\Omega}\right)}$$

$$\mathcal{D}_{\text{int}}\left(\vec{\Omega}\right) = \frac{\mathcal{P}_{\text{int}}\left(\vec{\Omega}\right)}{2 \cdot \sqrt{\mathcal{P}_{\text{sig}}\left(\vec{\Omega}\right) \cdot \mathcal{P}_{\text{alt}}\left(\vec{\Omega}\right)}}$$

https://arxiv.org/abs/1901.00174

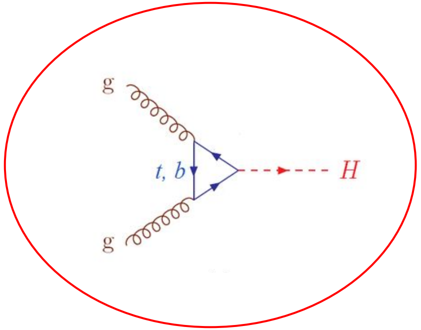
Observables and Tools

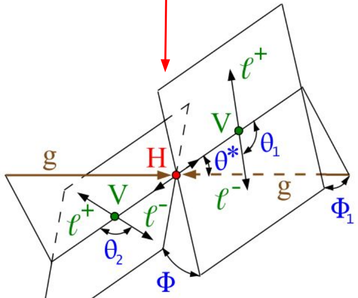
- JHUGen is used for simulating off-shell Higgs production (https://spin.pha.jhu.edu)
- MELA (matrix element likelihood approach) packages kinematic information
    - Avoid simulating MC for each hypothesis
    - Increase statistics of MC simulation
- Construct variables for event categorization
- Construct observables for analysis

$$\mathcal{D}_{\text{alt}}\left(\vec{\Omega}\right) = \frac{\mathcal{P}_{\text{sig}}\left(\vec{\Omega}\right)}{\mathcal{P}_{\text{sig}}\left(\vec{\Omega}\right) + \mathcal{P}_{\text{alt}}\left(\vec{\Omega}\right)}$$

$$\mathcal{D}_{\text{int}}\left(\vec{\Omega}\right) = \frac{\mathcal{P}_{\text{int}}\left(\vec{\Omega}\right)}{2 \cdot \sqrt{\mathcal{P}_{\text{sig}}\left(\vec{\Omega}\right) \cdot \mathcal{P}_{\text{alt}}\left(\vec{\Omega}\right)}}$$

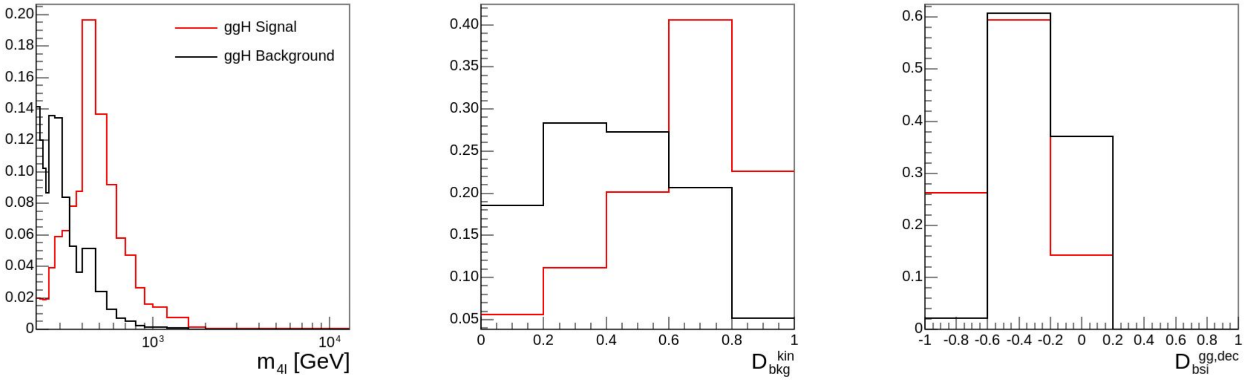
https://arxiv.org/abs/1901.00174





https://arxiv.org/abs/1309.4819

These templates were made using JHUGen for this study



Regarding MELA:

https://doi.org/10.1016/j.physletb.2012.08.021

https://doi.org/10.1103/PhysRevLett.110.081803

Conclusion

Motivated by setting better constraints on contributions to SMEFT, but want to stay model-agnostic and use generalized forms

Stay limited to dimension-6 operators and consider all BSM contributions to the Higgs Lagrangian (focusing on HVV vertex)

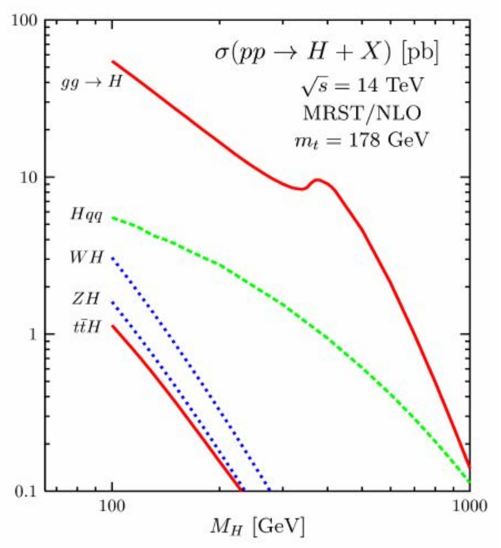
Simulated effects of anomalous couplings in Higgs off-shell production, and now also the VBS EW background

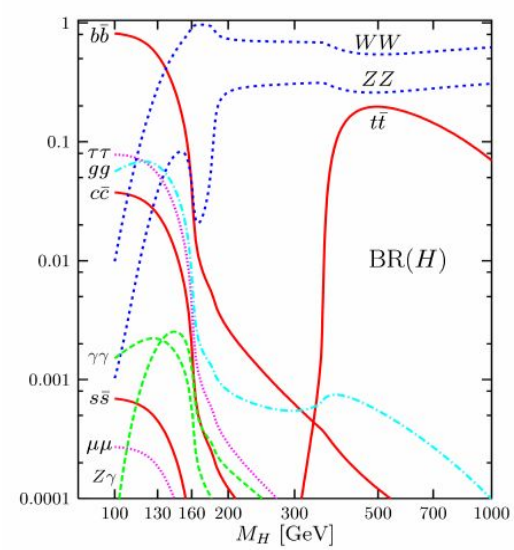
We show that both H-signal and VBS distributions are affected by the same EFT operators

To be taken into account in a SMEFT analysis of LHC data

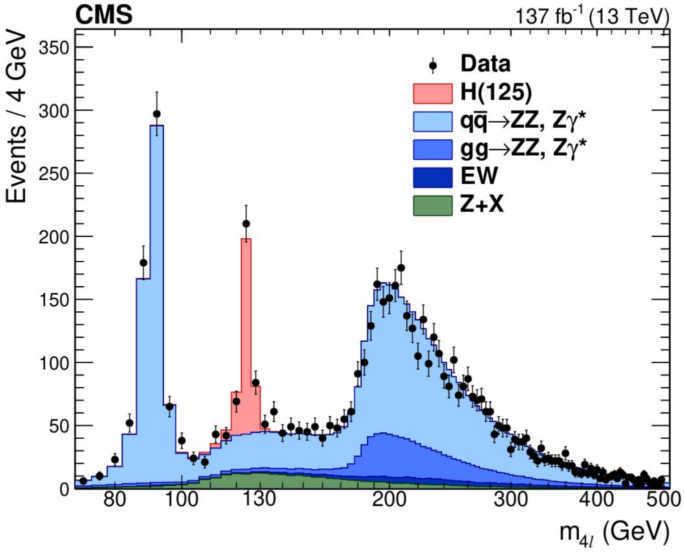
Thank you!

Backup Slides





https://doi.org/10.1007/s12043-009-0002-2

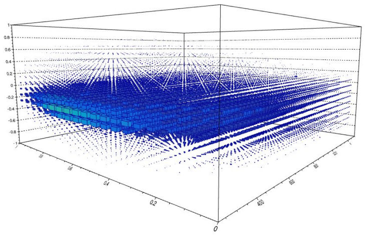


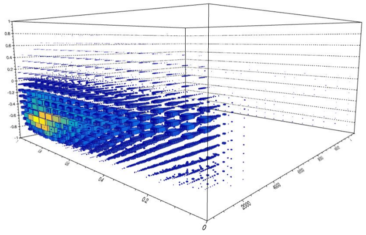
https://arxiv.org/abs/2103.04956

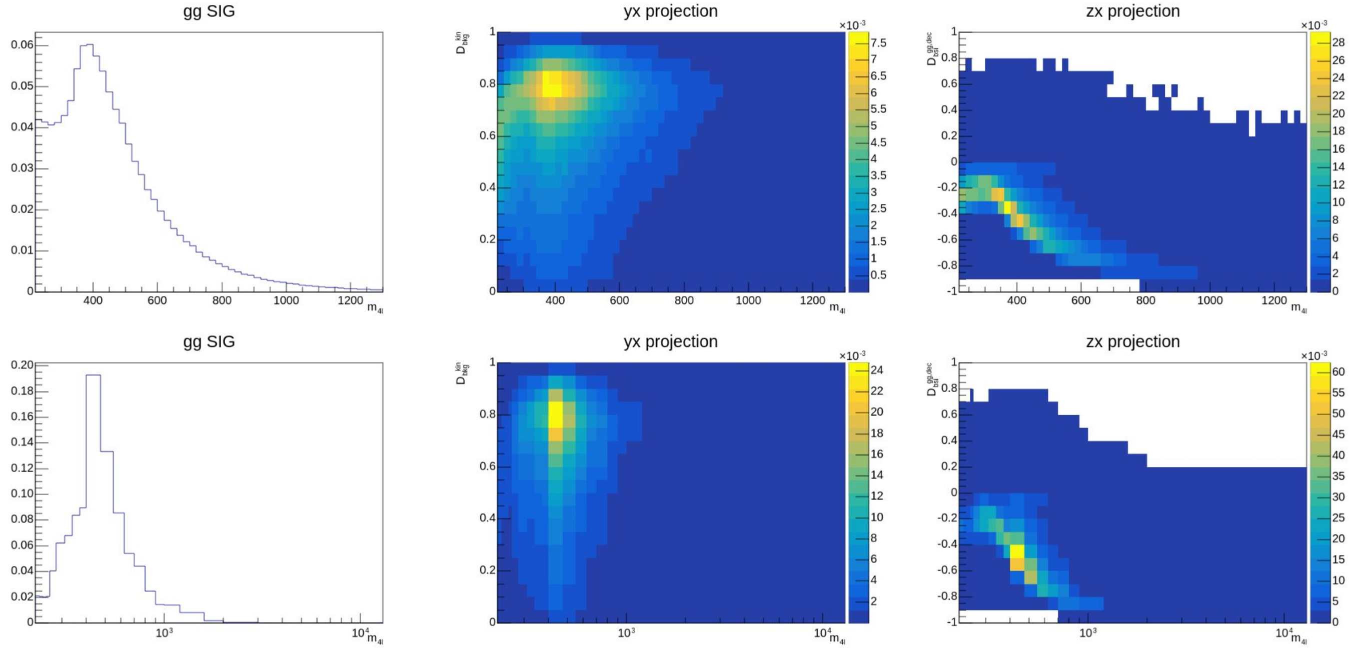
Example SM template with $m_{4\ell}, \mathcal{D}_{\rm bkg}^{\rm kin}, \mathcal{D}_{\rm bsi}^{\rm gg,dec}$

In the top row, we have a template filled with ggH events using regular binning.

We compare this to an arbitrary example of the same template with variable mass bins to exaggerate certain features.



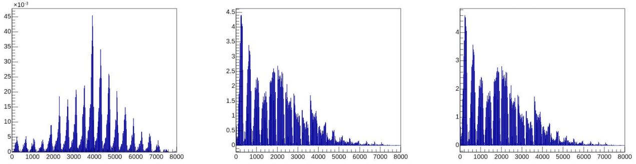




Example SM template with $m_{4\ell}, D^{\mathrm{kin}}_{\mathrm{bkg}}, D^{gg,\mathrm{dec}}_{\mathrm{bsi}}$

- Illustrative example of observables for SM
  - gg→4ℓ process
- Template for analysis is constructed from 3D histograms of observables for each sample
- Unrolled to 1D template
- Binning shown here is arbitrary

Unrolled templates illustrative of what is used in analysis

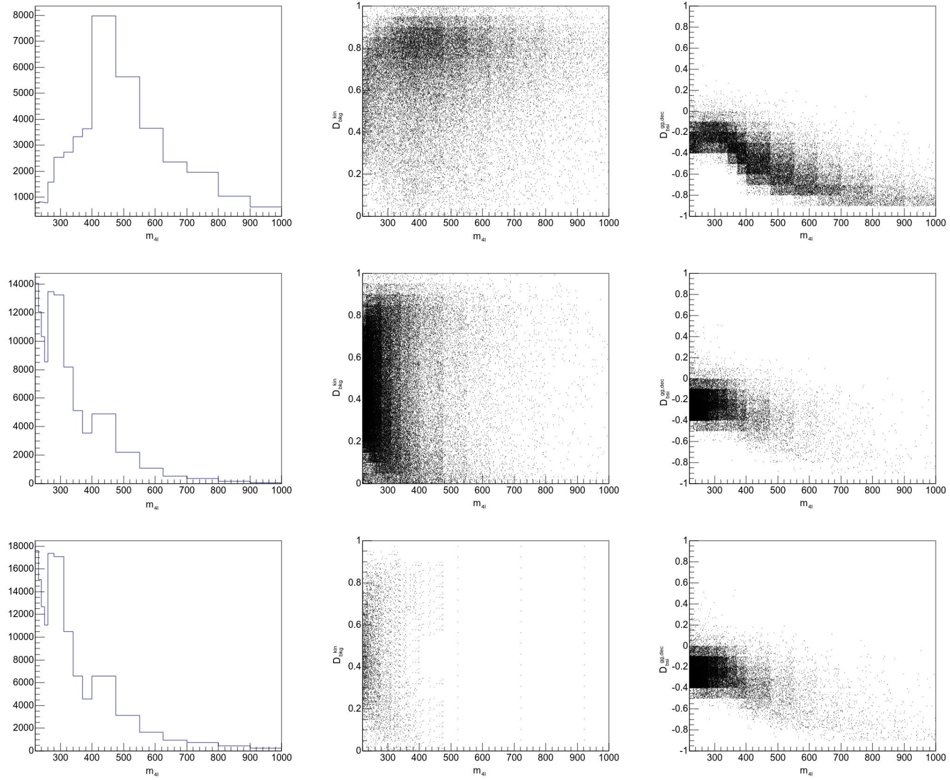


SIG BSI BKG

SIG

BSI

BKG



Parameterization

- Off-shell parameterization is performed for m_H = 125 GeV
- Very similar to on-shell parameterization but notably has a more significant contribution of interference between signal and background
- Higgs boson width Γ_0 is that of a reference model (SM)
- The parameterization of off-shell likelihoods with anomalous couplings is more involved for EW production (VBF and VH)
  - Unlike ggH process, there are two HVV vertices in EW, one in production and one in decay
  - Introduces cross terms between the individual production and decay amplitudes
- We parameterize our likelihood with the signal strength and ratio of Higgs widths
  - In the off-shell-only study, with simplifying SM assumptions, one can set the signal strength to 1
  - Solve for the most likely value of the width ratio, then scale by SM reference width

Parameter 1: $\sqrt{\mu_i}$

Parameter 2: $\sqrt{\Gamma_{\mathrm{H}}/\Gamma_0}$

$$\mathcal{P}^j(\vec{x}) = (\mu_i \cdot \Gamma_{\mathrm{H}}/\Gamma_0)\, \mathcal{P}^j_{\mathrm{sig}}(\vec{x}) + \sqrt{\mu_i \cdot \Gamma_{\mathrm{H}}/\Gamma_0}\, \mathcal{P}^j_{\mathrm{int}}(\vec{x}) + \mathcal{P}^j_{\mathrm{bkg}}(\vec{x})$$

https://arxiv.org/abs/1901.00174

A(HVV) general form

M_V is the vector boson pole mass, v is the SM Higgs field vacuum expectation value

At tree level in the SM, only the CP-even HZZ and HWW interactions contribute via $g_1^{ZZ} = g_1^{WW} = 2$

Loop induced interactions of HZγ, Hγγ, and Hgg contribute effectively via the CP-even g_2^{VV} terms and are parameterically suppressed by α or α_s

The CP-violating couplings g_4^{VV} are generated only at three-loop level in the SM

Beyond the SM, all of these couplings can receive additional contributions

$$A(HV_1V_2) = \frac{1}{v} \left\{ M_{V_1}^2 \left(g_1^{VV} + \frac{\kappa_1^{VV} q_1^2 + \kappa_2^{VV} q_2^2}{\left(\Lambda_1^{VV}\right)^2} + \frac{\kappa_3^{VV} (q_1 + q_2)^2}{\left(\Lambda_Q^{VV}\right)^2} + \frac{2 q_1 \cdot q_2}{M_{V_1}^2} g_2^{VV} \right) (\varepsilon_1 \cdot \varepsilon_2) \right.$$
$$\left. - 2 g_2^{VV} (\varepsilon_1 \cdot q_2)(\varepsilon_2 \cdot q_1) - 2 g_4^{VV} \varepsilon_{\varepsilon_1 \varepsilon_2 q_1 q_2} \right\},$$

https://arxiv.org/abs/2002.09888

Higgs in SMEFT

SU(3) × SU(2) × U(1) invariant Lagrangian for H boson interactions with gauge bosons (written in the mass eigenstate parameterization)

$$\begin{aligned}\mathcal{L}_{\rm hvv} = \; & \frac{h}{v}\Bigg[(1+\delta c_z)\,\frac{(g^2+g'^2)v^2}{4} Z_\mu Z_\mu + c_{zz}\,\frac{g^2+g'^2}{4} Z_{\mu\nu}Z_{\mu\nu} + c_{z\Box}g^2 Z_\mu \partial_\nu Z_{\mu\nu} + \tilde{c}_{zz}\,\frac{g^2+g'^2}{4} Z_{\mu\nu}\tilde{Z}_{\mu\nu} \\ & + (1+\delta c_w)\,\frac{g^2v^2}{2} W_\mu^+ W_\mu^- + c_{ww}\,\frac{g^2}{2} W_{\mu\nu}^+ W_{\mu\nu}^- + c_{w\Box}g^2\left(W_\mu^-\partial_\nu W_{\mu\nu}^+ + \text{h.c.}\right) + \tilde{c}_{ww}\,\frac{g^2}{2} W_{\mu\nu}^+ \tilde{W}_{\mu\nu}^- \\ & + c_{z\gamma}\,\frac{e\sqrt{g^2+g'^2}}{2} Z_{\mu\nu}A_{\mu\nu} + \tilde{c}_{z\gamma}\,\frac{e\sqrt{g^2+g'^2}}{2} Z_{\mu\nu}\tilde{A}_{\mu\nu} + c_{\gamma\Box}gg' Z_\mu\partial_\nu A_{\mu\nu} \\ & + c_{\gamma\gamma}\,\frac{e^2}{4} A_{\mu\nu}A_{\mu\nu} + \tilde{c}_{\gamma\gamma}\,\frac{e^2}{4} A_{\mu\nu}\tilde{A}_{\mu\nu} + c_{gg}\,\frac{g_s^2}{4} G^a_{\mu\nu}G^a_{\mu\nu} + \tilde{c}_{gg}\,\frac{g_s^2}{4} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}\Bigg]\end{aligned}$$

Relations to the amplitude parameterization:

https://arxiv.org/abs/2002.09888

$$\begin{aligned} & \delta c_z = \frac{1}{2}g_1^{ZZ} - 1\,, \quad c_{zz} = -\frac{2s_w^2c_w^2}{e^2}g_2^{ZZ}\,, \quad c_{z\Box} = \frac{M_Z^2 s_w^2}{e^2}\frac{\kappa_1^{ZZ}}{(\Lambda_1^{ZZ})^2}\,, \quad \tilde{c}_{zz} = -\frac{2s_w^2c_w^2}{e^2}g_4^{ZZ}\,, \\ & \delta c_w = \frac{1}{2}g_1^{WW} - 1\,, \quad c_{ww} = -\frac{2s_w^2}{e^2}g_2^{WW}\,, \quad c_{w\Box} = \frac{M_W^2 s_w^2}{e^2}\frac{\kappa_1^{WW}}{(\Lambda_1^{WW})^2}\,, \quad \tilde{c}_{ww} = -\frac{2s_w^2}{e^2}g_4^{WW}\,, \\ & c_{z\gamma} = -\frac{2s_wc_w}{e^2}g_2^{Z\gamma}\,, \quad \tilde{c}_{z\gamma} = -\frac{2s_wc_w}{e^2}g_4^{Z\gamma}\,, \quad c_{\gamma\Box} = \frac{s_wc_w}{e^2}\frac{M_Z^2}{(\Lambda_1^{Z\gamma})^2}\kappa_2^{Z\gamma}\,, \\ & c_{\gamma\gamma} = -\frac{2}{e^2}g_2^{\gamma\gamma}\,, \quad \tilde{c}_{\gamma\gamma} = -\frac{2}{e^2}g_4^{\gamma\gamma}\,, \quad c_{gg} = -\frac{2}{g_s^2}g_2^{gg}\,, \quad \tilde{c}_{gg} = -\frac{2}{g_s^2}g_4^{gg}\,. \end{aligned}$$

HVV coupling relationships

Not all of the EFT coefficients are independent when limiting the discussion to dimension-six interactions

The linear relations for the dependent coefficients can be translated into relations between our anomalous couplings

Enforcing these relations, we find the following for the couplings in HVV interactions

$$
\begin{aligned}
g_1^{WW} &= g_1^{ZZ} + \frac{\Delta M_W}{M_W}\,, \\
g_2^{WW} &= c_w^2 g_2^{ZZ} + s_w^2 g_2^{\gamma\gamma} + 2 s_w c_w g_2^{Z\gamma}\,, \\
g_4^{WW} &= c_w^2 g_4^{ZZ} + s_w^2 g_4^{\gamma\gamma} + 2 s_w c_w g_4^{Z\gamma}\,, \\
\frac{\kappa_1^{WW}}{(\Lambda_1^{WW})^2}(c_w^2 - s_w^2) &= \frac{\kappa_1^{ZZ}}{(\Lambda_1^{ZZ})^2} + 2 s_w^2 \frac{g_2^{\gamma\gamma} - g_2^{ZZ}}{M_Z^2} + 2\frac{s_w}{c_w}(c_w^2 - s_w^2)\frac{g_2^{Z\gamma}}{M_Z^2}\,, \\
\frac{\kappa_2^{Z\gamma}}{(\Lambda_1^{Z\gamma})^2}(c_w^2 - s_w^2) &= 2 s_w c_w \left(\frac{\kappa_1^{ZZ}}{(\Lambda_1^{ZZ})^2} + \frac{g_2^{\gamma\gamma} - g_2^{ZZ}}{M_Z^2} \right) + 2(c_w^2 - s_w^2)\frac{g_2^{Z\gamma}}{M_Z^2}\,.
\end{aligned}
$$

https://arxiv.org/abs/2002.09888
https://arxiv.org/abs/1610.07922

Coefficients in gauge boson self-interaction amplitudes

Given that M_W is experimentally measured to high precision, we assume $\Delta M_W \approx 0$

The couplings $e_\lambda^{V' f' f}$ for $HVf\bar{f}$ contact interactions are equal to the corresponding $Vf\bar{f}$ couplings $g_\lambda^{V f' f}$ in the SM, and are strongly constrained by electroweak precision measurement

The gauge boson self couplings are then determined by HVV couplings

$$
\begin{aligned}
d_1^\gamma &= 1 + (g_2^{\gamma\gamma} - g_2^{ZZ})c_w^2 + g_2^{Z\gamma}\left(\frac{c_w}{s_w} - 2s_w c_w\right) , \\
d_4^\gamma &= (g_4^{\gamma\gamma} - g_4^{ZZ})c_w^2 + g_4^{Z\gamma}\left(\frac{c_w}{s_w} - 2s_w c_w\right) , \\
d_1^Z &= 1 - 2\frac{s_w^2 c_w^2}{c_w^2 - s_w^2}\left(g_2^{\gamma\gamma} - g_2^{ZZ}\right) - 2s_w c_w g_2^{Z\gamma} - \frac{M_Z^2}{2(c_w^2 - s_w^2)}\frac{\kappa_1^{ZZ}}{(\Lambda_1^{ZZ})^2} , \\
d_2^Z &= d_3^Z = 1 - \frac{s_w^2}{c_w^2 - s_w^2}\left(g_2^{\gamma\gamma} - g_2^{ZZ}\right) - \frac{s_w}{c_w}g_2^{Z\gamma} - \frac{M_Z^2}{2(c_w^2 - s_w^2)}\frac{\kappa_1^{ZZ}}{(\Lambda_1^{ZZ})^2} , \\
d_4^Z &= -\frac{s_w^2}{c_w^2} d_4^\gamma , \\
d^{ZZWW} &= \frac{c_w^2}{s_w^2}\left(2d_2^Z - 1\right) , \qquad d^{Z\gamma WW} = \frac{c_w}{s_w} d_2^Z .
\end{aligned}
$$

https://arxiv.org/abs/2002.09888