

Challenges with Internal Photons in Constructive QED

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Problem with Feynman Diagrams

# External Gluons	4	5	6	7	8	9	10
# Feynman Diagrams	4	25	220	2,485	34,300	559,405	10,525,900

Consider the scattering of 7 gluons

```
ch_3.6.27_nnnext — less results/f1.c — 78x25
tmp[0]=+DP[20]*(-200*(-DP[1]-DP[10]-DP[15])+260*(DP[11]+DP[16])+76*(DP[12]+
DP[17])+252*(DP[13]+DP[18])+400*DP[2]-760*DP[0]-680*DP[3]+1120*DP[4]-1440*
DP[5]+240*DP[6]-286*DP[7]-86*DP[8]-262*DP[9]-552*DP[20])+DP[6]*(18*(DP[10]+
DP[15])+342*(DP[11]+DP[16])+90*(DP[12]+DP[17])+306*(DP[13]+DP[18])+14*
DP[6]+532*DP[7]+140*DP[8]+476*DP[9])+DP[7]*(342*(DP[10]+DP[15])+684*(-
DP[11]-DP[16])+360*(DP[12]+DP[17])+144*(DP[13]+DP[18])+560*DP[8]-532*DP[7]+
224*DP[9])+DP[8]*(90*(DP[10]+DP[15])+360*(DP[11]+DP[16])+180*(-DP[12]-
DP[17])+216*(-DP[13]-DP[18])-140*DP[8]-336*DP[9])+DP[9]*(306*(DP[10]+
DP[15])+144*(DP[11]+DP[16])+216*(-DP[12]-DP[17])+180*(DP[13]+DP[18])+140*
DP[9])+DP[10]*(40*(DP[12]+DP[15])+4*DP[10]+152*DP[11]+136*DP[13]+760*
DP[16]+200*DP[17]+680*DP[18])+DP[11]*(160*DP[12]-152*DP[11]+64*DP[13]+760*
DP[15]-1520*DP[16]+800*DP[17]+320*DP[18])+DP[12]*(200*DP[15]-40*DP[12]-96*
DP[13]+800*DP[16]-400*DP[17]-480*DP[18])+DP[13]*(40*DP[13]+680*DP[15]+320*
DP[16]-480*DP[17]+400*DP[18])+DP[15]*(4*DP[15]+152*DP[16]+40*DP[17]+136*
DP[18])+DP[16]*(160*DP[17]-152*DP[16]+64*DP[18])+DP[17]*(-40*DP[17]-96*
DP[18])+40*S[1];
tmp[1]=+DP[19]*(380*(DP[6]-DP[0])+340*(-DP[1]-DP[10])+100*(-DP[3]-DP[15])+
256*(DP[4]-DP[19])+848*(-DP[5]-DP[20])+431*(DP[11]-DP[8])+135*(DP[16]-
DP[9])+127*(DP[13]+DP[17])+407*DP[12]-447*DP[7]+39*DP[18])+DP[20]*(200*(-
DP[3]-DP[10]-DP[15])+260*(DP[11]+DP[16])+252*(DP[12]+DP[17])+76*(DP[13]+
DP[18])+400*DP[4]-760*DP[0]-680*DP[1]-1440*DP[5]+240*DP[6]-286*DP[7]-262*
DP[8]-86*DP[9]-552*DP[20])+DP[6]*(18*(DP[10]+DP[15])+342*(DP[11]+DP[16])+
306*(DP[12]+DP[17])+90*(DP[13]+DP[18])+14*DP[6]+532*DP[7]+476*DP[8]+140*
DP[9])+DP[7]*(342*(DP[10]+DP[15])+684*(-DP[11]-DP[16])+144*(DP[12]+DP[17])+
144*(DP[13]+DP[18]));
```

It takes on the order of an hour to produce the code for all the diagrams. The full expression for this one diagram requires ~3500 lines of code.

BCFW Recursion Relations

Parke and Taylor [“An Amplitude for n Gluon Scattering,” Phys. Rev. Lett. 56, 2459 (1986)] noticed that the “maximally helicity violating” gluon amplitudes could be written with **only one mathematical term no matter how many gluons were present** (at tree level).

$$\mathcal{M}^{1,1,-1,\dots,-1} = \frac{[12]^4}{[12][23]\cdots[n1]}$$

Britto, Cachazo, Feng and Witten [“Direct proof of tree-level recursion relation in Yang-Mills theory,” Phys. Rev. Lett. 94, 181602 (2005) (hep-th/0501052)] found **recursion relations** that gave **all gluon amplitudes** (at tree level) without Feynman diagrams!

Helicity Spinors

For massless particles,

$$p_{i\alpha\dot{\beta}} = E_i \begin{pmatrix} 1 + \cos \theta_i & \sin \theta_i e^{-i\phi_i} \\ \sin \theta_i e^{i\phi_i} & 1 - \cos \theta_i \end{pmatrix}$$

This can be written as the product of two related helicity spinors.

$$p_{i\alpha\dot{\beta}} = |i\rangle_\alpha [i|_{\dot{\beta}}.$$

where

$$|i\rangle_\alpha = \sqrt{2E} \begin{pmatrix} c \\ s \end{pmatrix}, \quad [i|_{\dot{\alpha}} = \sqrt{2E} \begin{pmatrix} c \\ s^* \end{pmatrix}$$

and

$$c = \cos \frac{\theta}{2} \quad \text{and} \quad s = \sin \frac{\theta}{2} e^{i\phi}$$

Spin Spinors

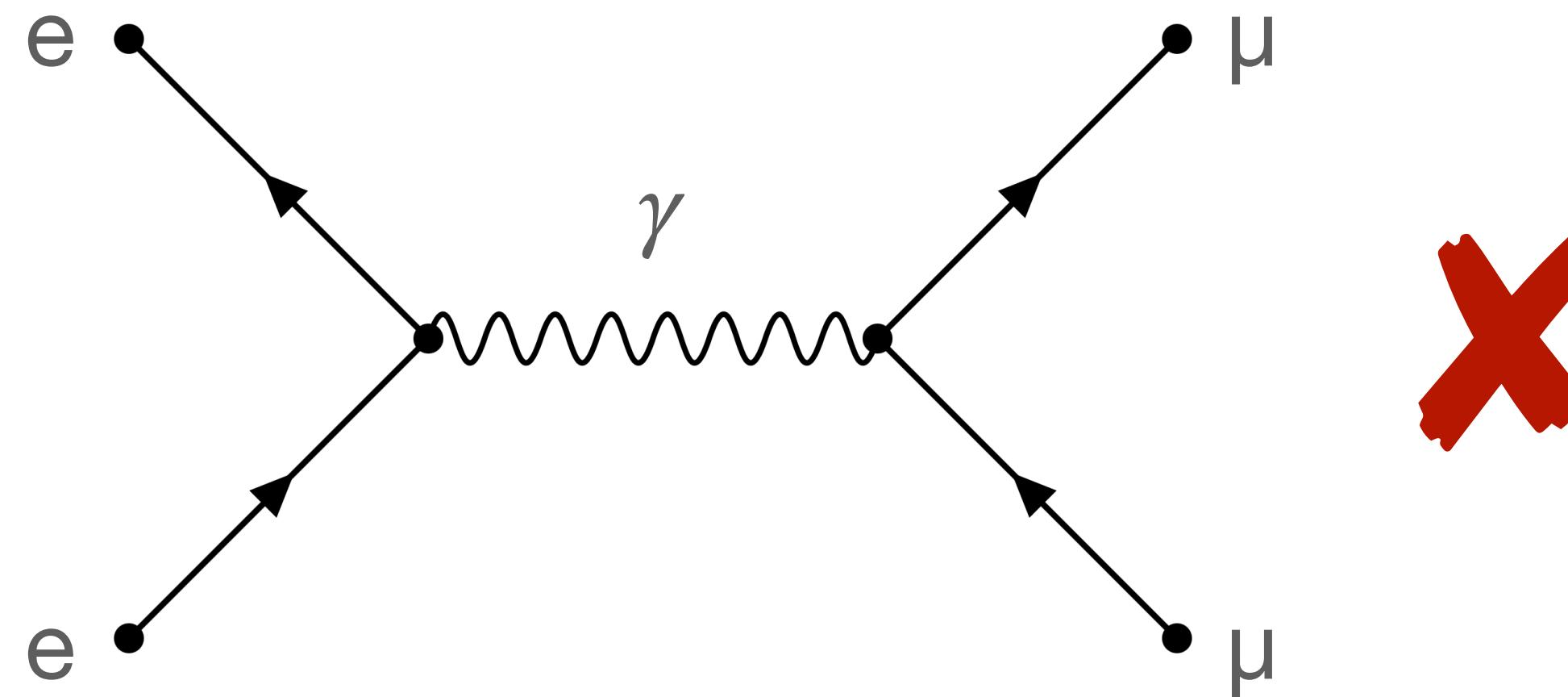
Arakani-Hamed, Huang and Huang (arXiv:1709.04891) generalized these to spin spinors for massive particles.

$$|\dot{\mathbf{i}}\rangle_{\alpha}^I = \begin{pmatrix} \sqrt{E+p} c & -\sqrt{E-p} s^* \\ \sqrt{E+p} s & \sqrt{E-p} c \end{pmatrix}$$

$$[\dot{\mathbf{i}}]_{\dot{\alpha} I} = \begin{pmatrix} \sqrt{E+p} c & -\sqrt{E-p} s \\ \sqrt{E+p} s^* & \sqrt{E-p} c \end{pmatrix}$$

$$\begin{aligned} |\dot{\mathbf{j}}\rangle_{\alpha}^I [\dot{\mathbf{j}}]_{\dot{\beta} I} &= \begin{pmatrix} E_j + p_j \cos \theta_j & p_j \sin \theta_j e^{-i\phi_j} \\ p_j \sin \theta_j e^{i\phi_j} & E_j - p_j \cos \theta_j \end{pmatrix} \\ &= \begin{pmatrix} E_j + p_{jz} & p_{jx} - ip_{jy} \\ p_{jx} + ip_{jy} & E_j - p_{jz} \end{pmatrix} \\ &= \begin{pmatrix} p^0 + p^3 & p^1 - ip^2 \\ p^1 + ip^2 & p^0 - p^3 \end{pmatrix} \equiv p_{\alpha\dot{\beta}}, \end{aligned}$$

Constructive QED ee \rightarrow $\mu\mu$



AHH: JHEP11 (2021) 070

$$\frac{e^2}{s} \frac{\langle 12 \rangle^{2S} \langle 34 \rangle^{2S}}{m^{2S}} (p_1 - p_2) \cdot p_3 , \quad (5.44)$$

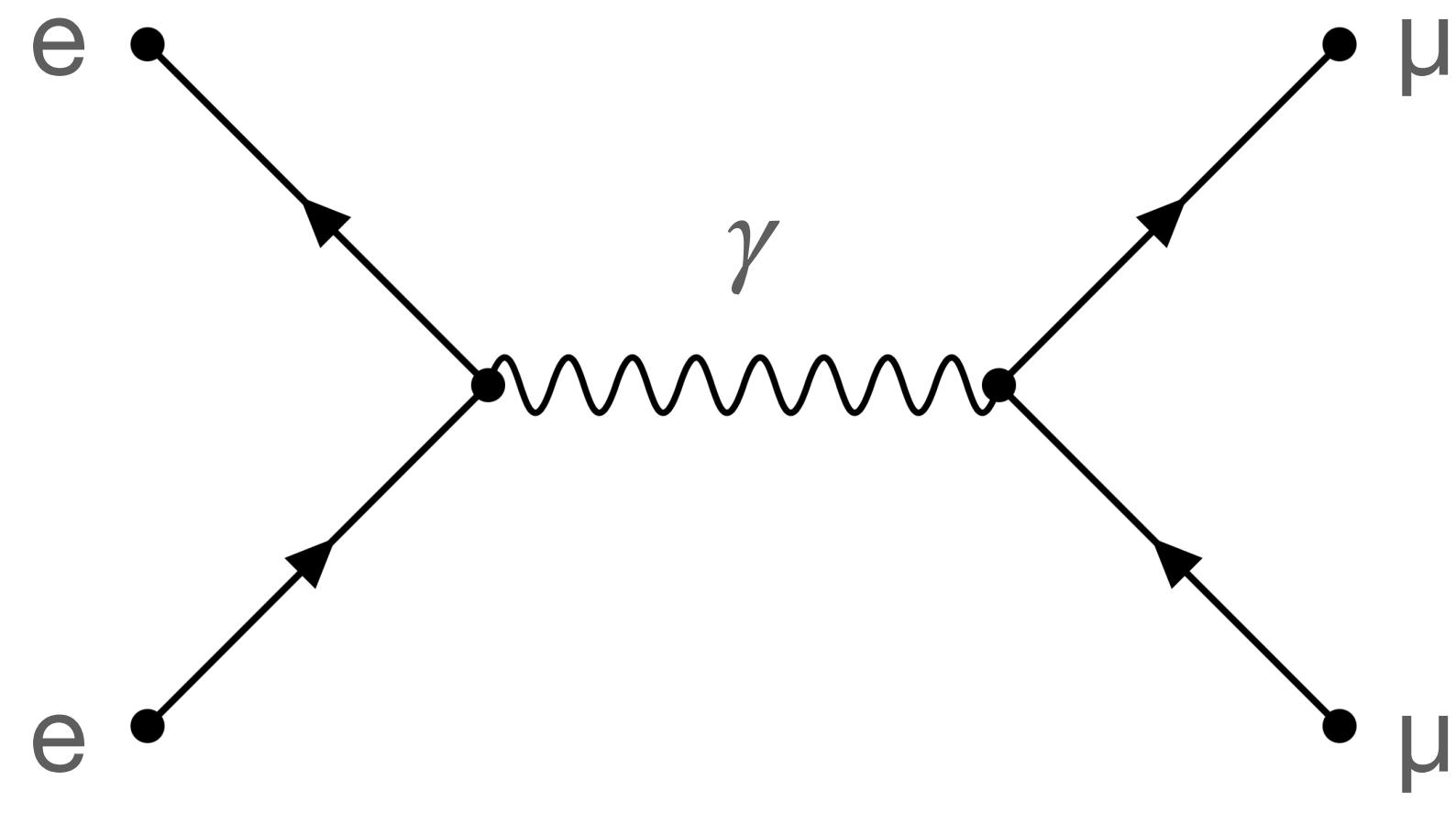
arXiv:2209.15018: App. C1

$$\mathcal{M} = -\frac{e^2}{s} \frac{\langle 12 \rangle \langle 34 \rangle}{2m_e m_\mu} (p_2 - p_1) \cdot (p_4 - p_3)$$

Does not agree with Feynman diagrams.

Bad quadratic high-energy growth.

Correct QED ee \rightarrow $\mu\mu$



Must be built from:

$$\langle 12 \rangle [34] + [12] \langle 34 \rangle$$

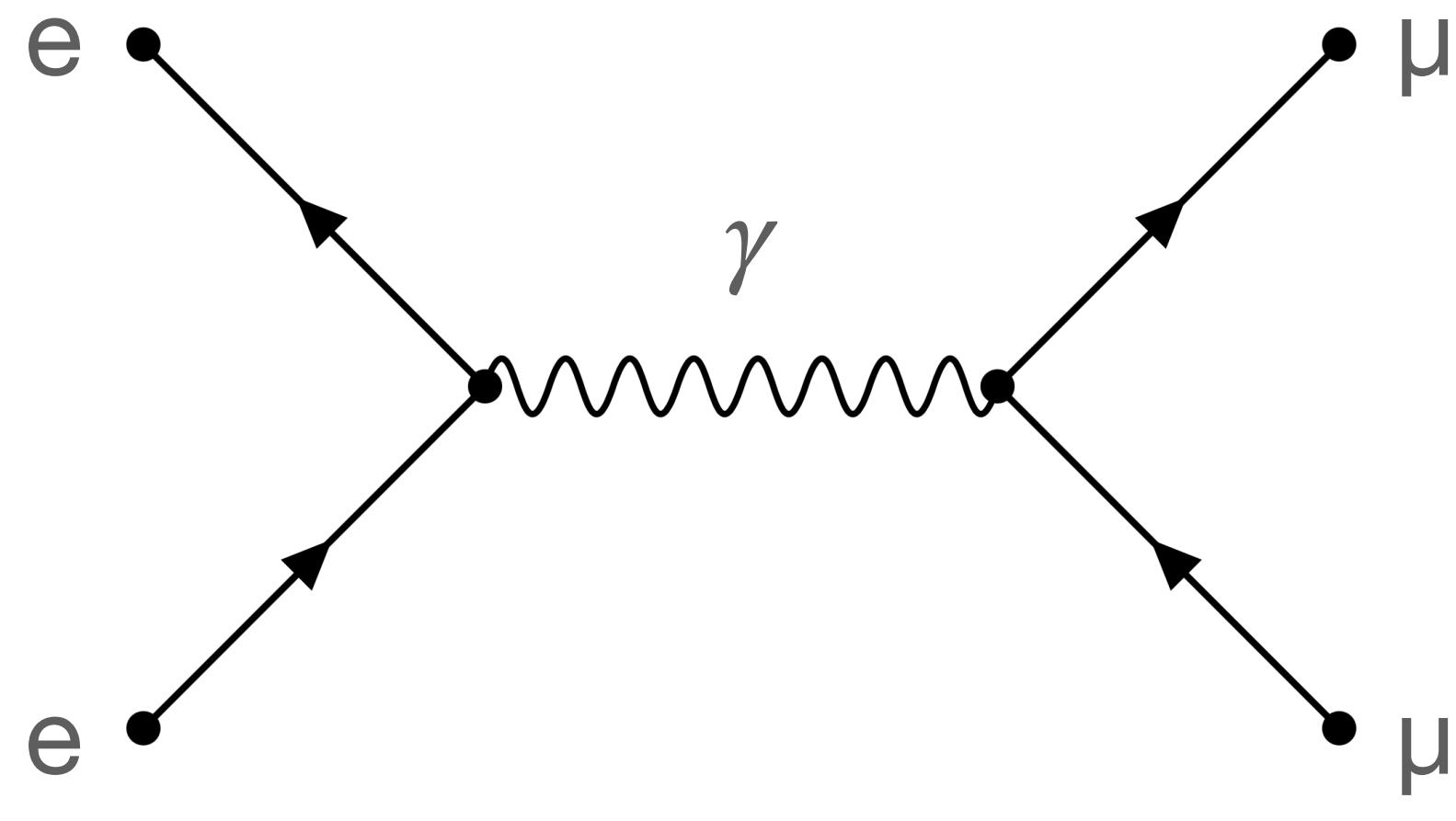
$$\langle 13 \rangle [24] + [13] \langle 24 \rangle$$

$$\langle 14 \rangle [23] + [14] \langle 23 \rangle$$

$$\langle 13 \rangle \langle 24 \rangle + [13][24]$$

$$\langle 14 \rangle \langle 23 \rangle + [14][23]$$

Correct QED ee \rightarrow $\mu\mu$



Must be built from:

$$\langle 13 \rangle [24] + [13] \langle 24 \rangle$$

$$\langle 14 \rangle [23] + [14] \langle 23 \rangle$$

$$\mathcal{M} = \frac{e^2}{s} (\langle 13 \rangle [24] + [13] \langle 24 \rangle + \langle 14 \rangle [23] + [14] \langle 23 \rangle)$$



Why Doesn't the Constructive Calculation Work for QED $e\bar{e} \rightarrow \mu\bar{\mu}$?

$$\mathcal{M} = \frac{e^2}{(p_1 + p_2)^2} (\langle 13 \rangle [24] + [13] \langle 24 \rangle + \langle 14 \rangle [23] + [14] \langle 23 \rangle)$$

Want:

$$\lim_{z \rightarrow \infty} M(z) = 0$$

$$\hat{\mathcal{M}}_{[1,3]} \xrightarrow[z \rightarrow \infty]{\sim} \frac{e^2 [34] \langle 21 \rangle}{q^2} \neq 0$$

Summary

- Constructive techniques are exciting.
 - Bypass fields
 - Trivially gauge invariant.
 - Potentially more efficient in ME generators.
- Constructive $M_{ee \rightarrow \mu\mu}$ disagrees with Feynman diagrams.
- Work arounds exist:
 - Enumerate possible terms and try different coefficients.
 - Massless limit of Massive Photon Amplitude.
- $\lim_{z \rightarrow \infty} M_{ee \rightarrow \mu\mu} \neq 0$
 - A clue to why constructive approach not working?
- Missing ingredient for processes like this?
 - Without workarounds.