## A PHENOMENOLOGICAL STUDY OF HIGGS JETS AT A MUON COLLIDER

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- Muon colliders
- Vector boson Fusion and collinear factorization


## OUTLINE

- Super-renormalizable splitting
- QCD di-jet events
- Comparison plots


## CASE FOR A MUON COLLIDER


image ref: Symmetry Magazine

- Muon colliders can extend the precision frontier and the energy frontier in comparison to $e^{+} e^{-}$colliders and $p p$ colliders.
- An $\mathcal{O}(10) \mathrm{TeV}$ muon collider with $\mathcal{O}(10 / a b)$ luminosity could produce an order of magnitude more Higgs bosons compared to $e^{+} e^{-}$"Higgs factories". It will additionally produce $\mathcal{O}\left(10^{4}\right)$ di-Higgs events.

Higgs production as a fraction of "total" cross-section


## VECTOR-BOSON FUSION \& COLLINEAR FACTORIZATION



- Vector boson fusion provides a dominant channel.
- We consider collinear factorization in final state splittings for High-energy electroweak processes.

- If the daughter particles $B$ and $C$ are approximately collinear to the offshell parent particle $A^{*}$, we have

$$
d \sigma_{Y, B C} \simeq d \sigma_{Y, A^{*}} \times d P_{A^{*} \rightarrow B+C}
$$

- To compute the hard cross-section, we convolute the partonic cross-section with the W -boson PDFs.


## SUPERRENORMALIZABLE SPLITTING



- All gauge and Yukawa splittings in the unbroken electroweak theory scale as $d k_{T}^{2} / k_{T}^{2}$, so typical splittings in the broken theory scale the same way.

- After SSB, we also have 'super-renormalizable’ splittings (or ultra-collinear) in the broken theory that scale as $m^{2} d k_{T}^{2} / k_{T}^{4}$
- In particular, we look at $h \rightarrow h h$ splitting. We can calculate the jet function cross-section for the process with an offshell initial Higgs at an observed invariant jet mass $m$



## QCD BACKGROUND



## PHENO '23

- The hard process that we study, at fixed jet mass $m$, for QCD backgrounds is

$$
W^{+} W^{-} \rightarrow q \bar{q}
$$

- The differential cross-section can be obtained by taking the inverse Laplace Transform of the NLL resummed crosssection

$$
\frac{d \sigma}{d m^{2}}=\sigma_{0} * \frac{d}{d m^{2}} R_{Q C D}\left(\frac{m^{2}}{Q^{2}}, Q\right)
$$

where, $m^{2}$ is the invariant mass squared of the final state jets.


## SUPER-RENORMALIZABLE HIGGS JET DISTRIBUTION

- Integrated cross-section for $\underset{2}{\mathrm{Higgs}}$ jets

$$
R_{H}\left(m^{2}\right)=\frac{\int_{4 m_{h}^{2}}^{m^{2}} J_{H}\left(m^{\prime 2}\right) d m^{\prime 2}}{\int_{4 m_{h}^{2}}^{\infty} J_{H}\left(m^{\prime 2}\right) d m^{\prime 2}}
$$

- The Higgs jet distribution is given by

$$
\begin{gathered}
\frac{1}{\sigma_{H H}} \frac{d \sigma}{d m^{2}}=\frac{d}{d m^{2}} R\left(m^{2}\right)=\frac{1}{\sigma_{t o t}} J_{H}\left(m^{2}\right) \\
\frac{d \sigma}{d m^{2}}=\frac{\sigma_{H H}}{\sigma_{t o t}} J_{H}\left(m^{2}\right)
\end{gathered}
$$

- This is the lowest order in $\alpha_{E W}$. What about higherorder corrections in $R_{H}$ ?



## ELECTRO-WEAK CORRECTIONS

- Typical higher order Electroweak corrections that go as double log will look like ( $\alpha_{E W} \simeq 0.033$ )

$$
\begin{aligned}
C_{E W}^{(1)} & =\frac{-2 \alpha_{E W}}{\pi} \ln ^{2}\left(Q^{2} / 4 m_{H}^{2}\right) \\
& \simeq-0.0866 * \ln ^{2}(4 * Q[\mathrm{TeV}])
\end{aligned}
$$



| $C_{E W}^{(1)}(10 \mathrm{TeV})$ |  | $C_{E W}^{(1)}(30 \mathrm{TeV})$ |
| :--- | :--- | :--- |
| $-I .178$ | $-I .984$ | -3.108 |

- While these corrections are not small, they can be resummed. This will be part of future work.


## COMPARISON PLOTS


$C_{E W}^{(1)}(10 \mathrm{TeV})=-1.178$

$C_{E W}^{(1)}(30 \mathrm{TeV})=-1.984$


## CONCLUSIONS

- We calculated the "Super-renormalizable" Higgs jet distribution at lowest order in the Jet function and the Hard cross section.
- We showed that at higher center of mass energies, the Higgs jet distribution shows a distinctive peak compared to QCD background jets.
- The higher order EW corrections, albeit not small, can be handled via resummation. They will give us Sudakov suppression but won't change the qualitative picture.
- Muon collider could provide an interesting opportunity to observe these jets with applications to test BSM physics.


## WORKS CITED

- Carola F. Berger and George Sterman JHEP09(2003)058,"Scaling rule for nonperturbative radiation in a class of event shapes"
- Tao Han, Yang Ma, and Keping Xie Phys. Rev. D I03, L03I30I,"High energy leptonic collisions and electroweak parton distribution functions"
- Chen, J., Han, T. \& Tweedie, B. Electroweak splitting functions and high energy showering.J. High Energ. Phys. 20I7, 93 (20I7). https://doi.org/I 0.I007/JHEPI I (2017)093
- S. Catani, L.Trentadue, G.Turnock, B.R.Webber,"Resummation of large logarithms in e+e- event shape distributions", Nuclear Physics B, I993, ISSN 0550-32I3,
- https://arxiv.org/abs/2103.14043, "The Muon Smasher's Guide"
- G. Cuomo, L.Vecchi, A.Wulzer, Goldstone equivalence and high energy electroweak physics, SciPost Phys. 8 (5) (2020) 078, http://dx.doi.org/I0.2I468/SciPostPhys.8.5.078, arXiv:I9|I.I2366


## THANK YOU FOR LISTENING

## Any Questions or Comments?

## BACKUP SLIDES

## ALTERNATE

## QCD BACKGROUND



- For a fixed jet mass, $m$, we consider a well-known infrared safe event shape variable, Thrust.

$$
1-\frac{\mathrm{m}^{2}}{\mathrm{Q}^{2}} \simeq 1-\tau \equiv T(N)=\max _{\hat{n}} \frac{\sum_{i}\left|\overrightarrow{p_{i}} \cdot \hat{n}\right|}{\sum_{j}\left|\overrightarrow{p_{j}}\right|}
$$

- The differential cross-section for such di-jet events at fixed values of $\tau$ is given by

$$
\frac{d \sigma(\tau, Q)}{d \tau}=\frac{1}{2 Q^{2}} \sum_{N}|M(N)|^{2} \delta(\tau-\tau(N))
$$

- We work with the integrated cross-section or Radiator calculated for the $e^{+} e^{-}$annihilation

$$
R(\tau, Q)=\frac{1}{\sigma_{t o t}} \int_{0}^{\tau} d \tau^{\prime} \frac{d \sigma\left(\tau^{\prime}, Q\right)}{d \tau^{\prime}}
$$

## QCD BACKGROUND



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- The differential cross-section can be obtained by taking the inverse Laplace Transform of the NLL resummed exponent to get the Radiator.

$$
\frac{1}{\sigma_{t o t}} \frac{d \sigma(\tau, Q)}{d \tau}=\frac{1}{\tau} \frac{d}{d \ln \tau} R(\tau, Q)
$$

- Using $\tau=m^{2} / Q^{2}$, we calculate $d \sigma / \mathrm{dm}^{2}$
- The hard process that we study, at fixed jet mass $m$, for QCD backgrounds is

$$
W^{+} W^{-} \rightarrow q \bar{q}
$$




## MUON COLLIDER



Equivalent cross-section at pp collider v/s muon collider


Higgs production as a fraction of "total" crosssection

## DI-JET CROSS SECTION

$$
e^{+}+e^{-} \rightarrow J_{1}(N)+J_{2}(N)
$$

- The differential cross section for such di-jet events at fixed values of $\tau_{a}$ is given by

$$
\frac{d \sigma\left(\tau_{a}, Q\right)}{d \tau_{a}}=\frac{1}{2 Q^{2}} \sum_{N}|M(N)|^{2} \delta\left(\tau_{a}-\tau_{a}(N)\right)
$$

- For di-jet cross sections, we look at $\tau_{a} \ll 1$, and we take the laplace transform of the crosssection

$$
\tilde{\sigma}(v, Q, a)=\int_{0}^{1} d \tau_{a} e^{-v \tau_{a}} \frac{d \sigma\left(\tau_{a}, Q\right)}{d \tau_{a}}
$$

- Large logarithms of $v$ need to be resummed.


## RESUMMED CROSS-SECTION AT NLL

- The Next-to-leading-log (NLL) resummed cross-section for $a<1$ in moment space, can be written as

$$
\begin{aligned}
\frac{1}{\sigma_{t o t}} \tilde{\sigma}(v, Q, a)= & \exp \left\{2 \int _ { 0 } ^ { 1 } \frac { d u } { u } \left[\int_{u^{2} Q^{2}}^{u Q^{2}} \frac{d p_{T}^{2}}{p_{T}^{2}} A\left(\alpha_{S}\left(p_{T}\right)\right)\left(e^{-u^{1-a} v\left(p_{T} / Q\right)^{a}}-1\right)\right.\right. \\
& \left.\left.+\frac{1}{2} B\left(\alpha_{s}(Q \sqrt{u})\right)\left(e^{-u(v / 2)^{\frac{2}{2-a}}}-1\right)\right]\right\} \\
\equiv & {[\mathcal{J}(v, Q, a)]^{2} }
\end{aligned}
$$

## RESUMMED CROSS-SECTION AT NLL

- The resummation is in terms of anomalous dimensions $A\left(\alpha_{s}\right)$ and $B\left(\alpha_{s}\right)$ which have finite expressions in the running coupling,

$$
A\left(\alpha_{s}\right)=\sum_{n=1}^{\infty} A^{(n)}\left(\frac{\alpha_{s}}{\pi}\right)^{n}
$$

- The coefficients of the perturbative expansion are well known at NLL,

$$
\begin{gathered}
A^{(1)}=C_{F}, \quad B^{(1)}=-\frac{3}{2} C_{F} \\
A^{(2)}=\frac{1}{2} C_{F}\left[C_{A}\left(\frac{67}{18}-\frac{\pi^{2}}{6}\right)-\frac{10}{9} T_{F} N_{f}\right]
\end{gathered}
$$

## RADIATOR

- We work with integrated cross-section or the Radiator

$$
R\left(\tau_{a}, Q\right)=\frac{1}{\sigma_{t o t}} \int_{0}^{\tau_{a}} d \tau_{a}^{\prime} \frac{d \sigma\left(\tau_{a}^{\prime}, Q\right)}{d \tau_{a}^{\prime}}
$$

- The Radiator can be directly calculated from the jet function $\mathcal{J}(v, Q, a)$ in transform space by

$$
\begin{gathered}
R\left(\tau_{a}, Q\right)=\frac{1}{2 \pi i} \int_{C} \frac{d v}{v} e^{v \tau_{a}}[\mathcal{J}(v, Q, a)]^{2} \\
R\left(\tau_{a}, Q\right)=\frac{\exp \left\{2 \ln \left(\frac{1}{\tau_{a}}\right) g_{1}(x, a)+2 g_{2}(x, a)+2(2-a) x^{2} \ln \left(\frac{2 \mu}{Q}\right) g_{1}^{\prime}(x, a)\right\}}{\Gamma\left[1-2 g_{1}(x, a)-2 x g_{1}^{\prime}(x, a)\right]}
\end{gathered}
$$

## RADIATOR

$$
\begin{aligned}
& g_{1}(x, a)=-\frac{4}{\beta_{0}} \frac{1}{1-a} \frac{1}{x} A^{(1)}\left[\left(\frac{1}{2-a}-x\right) \ln (1-(2-a) x)-(1-x) \ln (1-x)\right] \\
& g_{2}(x, a)=\frac{2}{\beta_{0}} B^{(1)} \ln (1-x)-\frac{8}{\beta_{0}^{2}} \frac{1}{1-a} A^{(1)}[\ln (1-x)-\ln (1-(2-a) x)] \\
&+\frac{4}{\beta_{0}} \ln 2 \frac{1}{1-a} A^{(1)}\left[\left(\frac{1}{2-a}-x\right) \ln (1-(2-a) x)-(1-x) \ln (1-x)\right] \\
&-\frac{\beta_{1}}{\beta_{0}^{3}} \frac{1}{1-a} A^{(1)}[2 \ln (1-(2-a) x)-2(2-a) \ln (1-x) \\
&\left.+\ln ^{2}(1-(2-a) x)-(2-a) \ln ^{2}(1-x)\right]
\end{aligned}
$$

where,

$$
x=\frac{\alpha_{s}(\mu)}{\pi} \frac{\beta_{0}}{2(2-a)} \ln \left(1 / \tau_{a}\right)
$$

## COMPARISON PLOTS




