



UNIVERSITÀ DEGLI STUDI
DI MILANO



Understanding the W boson mass

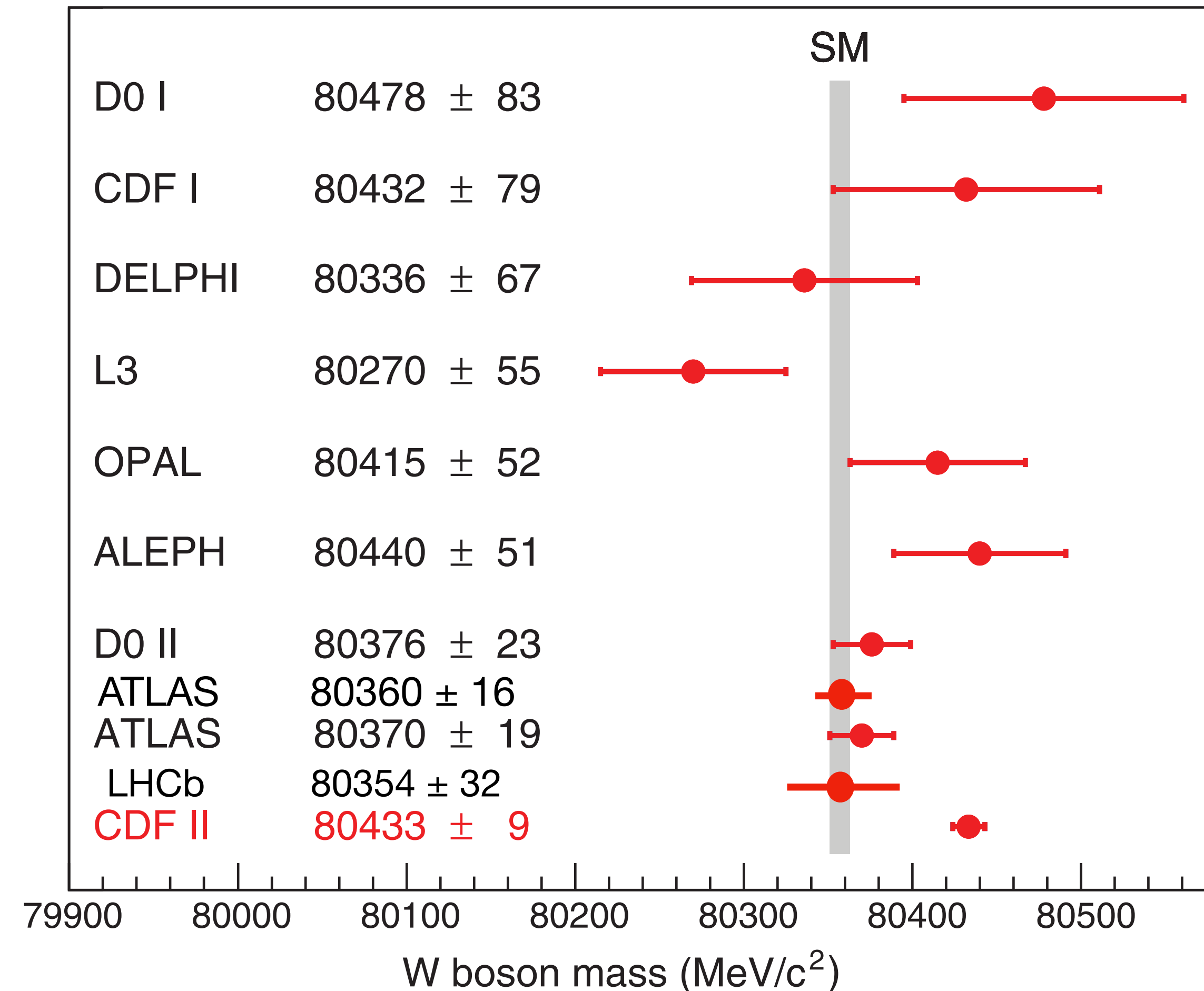
Alessandro Vicini

University of Milano, INFN Milano

PHENO 2023, Pittsburgh, May 8th 2023

Outline of the talk

- The m_W prediction and its relevance in the precision tests of the Standard Model
- The Drell-Yan kinematical distributions and the associated uncertainties
- The m_W determination from the kinematical distributions and the propagation of the theory uncertainties
- Proposal of a new observable, suitable for a transparent discussion of the QCD uncertainties on m_W
- The real challenge:
determination of a SM parameter at the 10^{-4} level at a hadron collider



The renormalisation of the SM and a framework for precision tests

- The Standard Model is a **renormalizable** gauge theory based on $SU(3) \times SU(2)_L \times U(1)_Y$
- The gauge sector of the SM lagrangian is assigned specifying (g, g', v, λ) in terms of 4 measurable inputs
- More observables can be computed and expressed in terms of the input parameters, including the available radiative corrections, at any order in perturbation theory
- The validity of the SM can be tested comparing these predictions with the corresponding experimental results

- The input choice $(g, g', v, \lambda) \leftrightarrow (\alpha, G_\mu, m_Z, m_H)$ **minimises the parametric uncertainty** of the predictions

$$\alpha(0) = 1/137.035999139(31)$$

$$G_\mu = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$$

$$m_Z = 91.1876(21) \text{ GeV}/c^2$$

$$m_H = 125.09(24) \text{ GeV}/c^2$$

- **with these inputs**, m_W and the **weak mixing angle** are **predictions** of the SM, to be tested against the experimental data

The W boson mass: theoretical prediction

Sirlin, 1980, 1984; Marciano, Sirlin, 1980, 1981;
 van der Bij, Veltman, 1984; Barbieri, Ciafaloni, Strumia 1993;
 Djouadi, Verzeqanassi 1987; Consoli, Hollik, Jegerlehner, 1989;
 Chetyrkin, Kühn, Steinhauser, 1995;
 Barbieri, Beccaria, Ciafaloni, Curci, Viceré, 1992, 1993; Fleischer, Tarasov, Jegerlehner, 1993;
 Degrassi, Gambino, AV, 1996; Degrassi, Gambino, Sirlin, 1997;
 Freitas, Hollik, Walter, Weiglein, 2000, 2003;
 Awramik, Czakon, 2002; Awramik, Czakon, Onishchenko, Veretin, 2003; Onishchenko, Veretin, 2003

The best available prediction includes
 the full 2-loop EW result, leading higher-order EW and QCD corrections,
 resummation of reducible terms
 Missing 3-loop and 4-loop terms needed to reduce the uncertainties.

$$m_W = w_0 + w_1 dH + w_2 dH^2 + w_3 dh + w_4 dt + w_5 dH dt + w_6 da_s + w_7 da^{(5)}$$

$$dt = [(M_t/173.34 \text{ GeV})^2 - 1]$$

$$da^{(5)} = [\Delta\alpha_{\text{had}}^{(5)}(m_Z^2)/0.02750 - 1]$$

$$dH = \ln\left(\frac{m_H}{125.15 \text{ GeV}}\right)$$

$$dh = [(m_H/125.15 \text{ GeV})^2 - 1]$$

$$da_s = \left(\frac{\alpha_s(m_Z)}{0.1184} - 1\right)$$

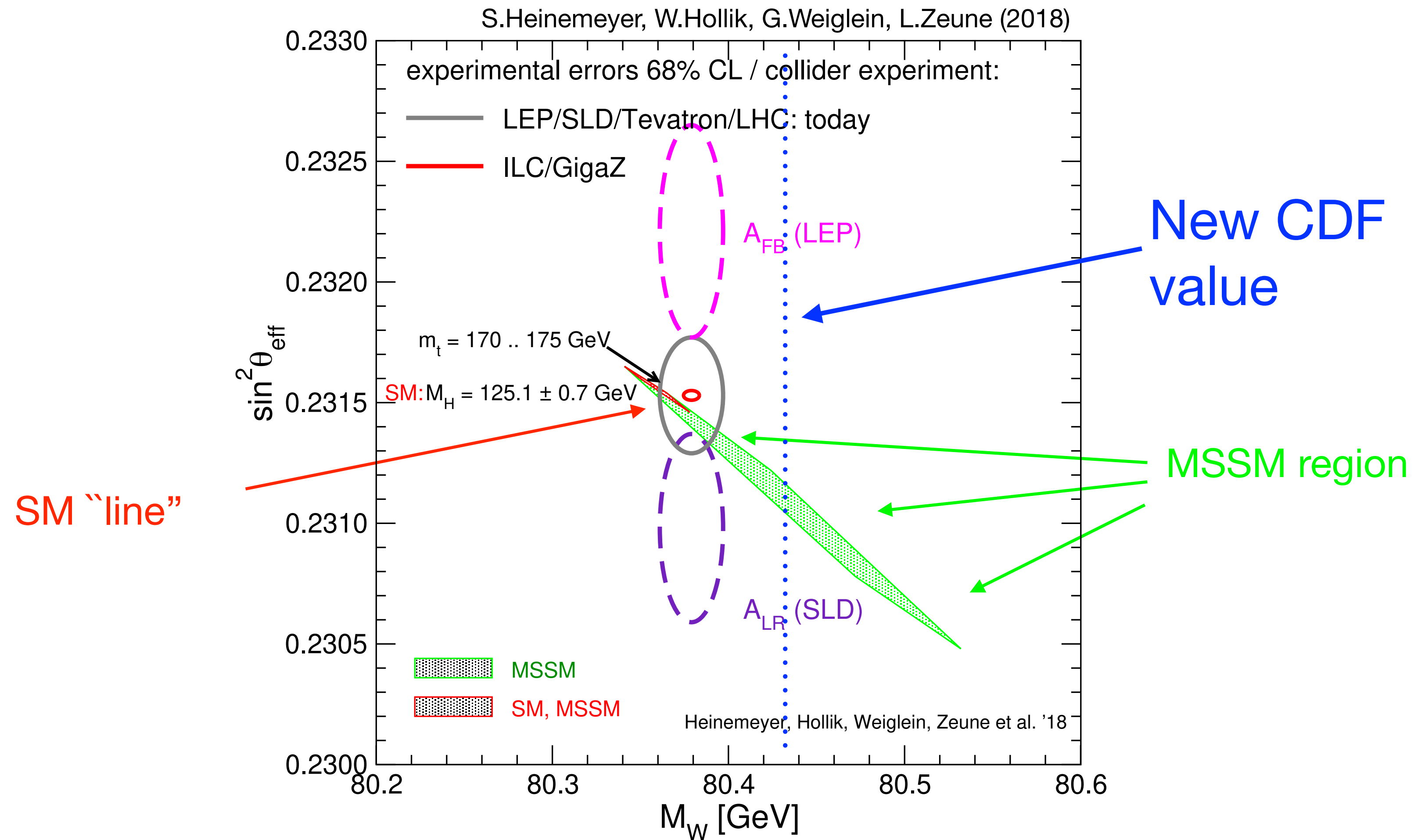
	$124.42 \leq m_H \leq 125.87 \text{ GeV}$	$50 \leq m_H \leq 450 \text{ GeV}$
w_0	80.35712	80.35714
w_1	-0.06017	-0.06094
w_2	0.0	-0.00971
w_3	0.0	0.00028
w_4	0.52749	0.52655
w_5	-0.00613	-0.00646
w_6	-0.08178	-0.08199
w_7	-0.50530	-0.50259

on-shell scheme $m_W^{os} = 80.353 \pm 0.004 \text{ GeV}$ (Freitas, Hollik, Walter, Weiglein)

MSbar scheme. $m_W^{\overline{MS}} = 80.351 \pm 0.003 \text{ GeV}$ (Degrassi, Gambino, Giardino)

parametric uncertainties $\delta m_W^{par} = \pm 0.005 \text{ GeV}$ due to the $(\alpha, G_\mu, m_Z, m_H, m_t)$ values

Relevance of a simultaneous study of m_W and of the weak mixing angle



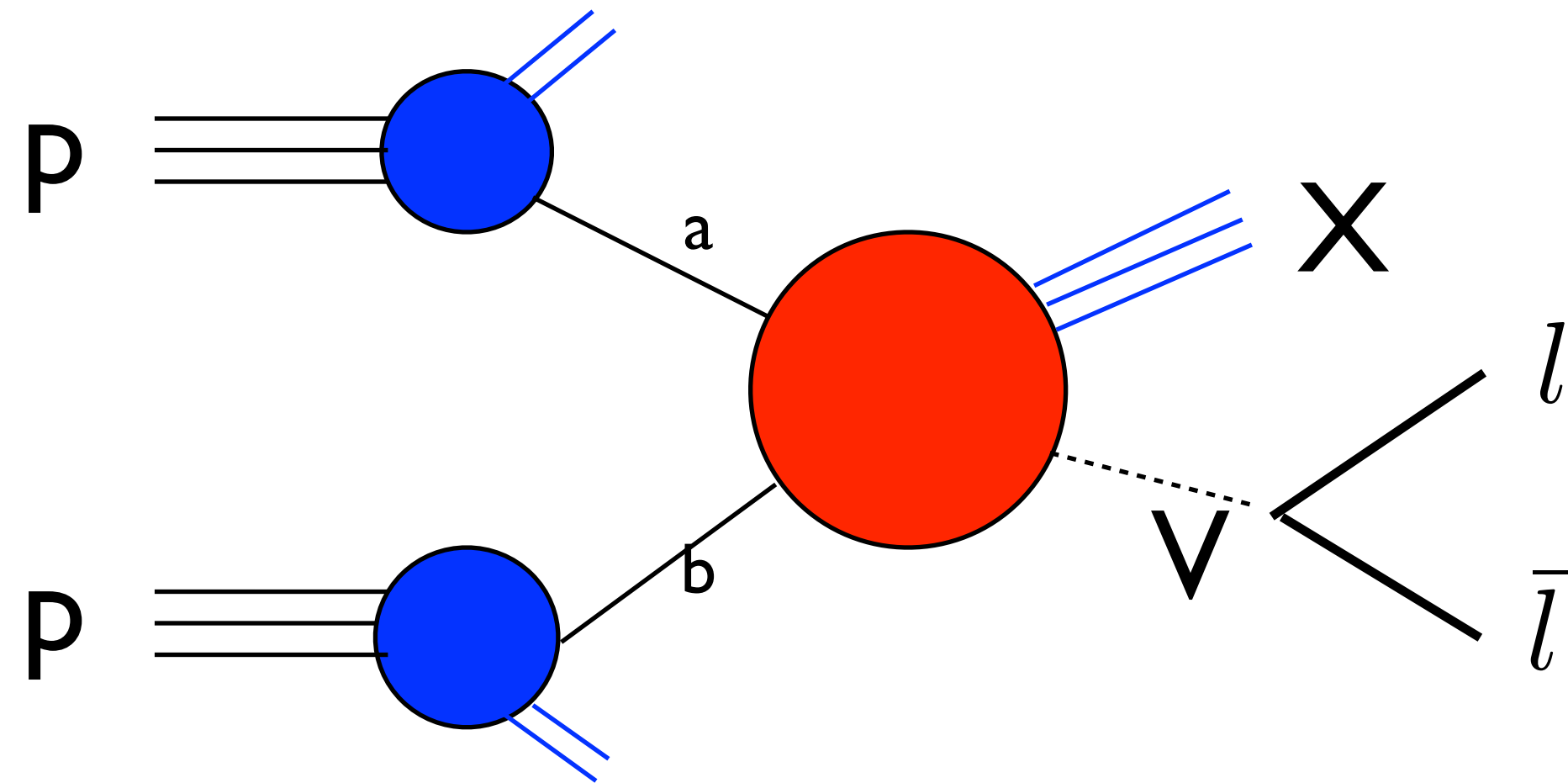
sensitivity to different sets of oblique corrections, i.e. to different combinations of gauge boson self-energies

independent determinations of these two parameters crucial for testing different New Physics alternatives

The Drell-Yan processes: kinematical distributions

The degrees of freedom in charged-current Drell-Yan and the associated uncertainty sources

$$\sigma_{had}(pp \rightarrow l\bar{l} + X) = \sum_{a,b=q,g,\gamma} \int_0^1 dx_1 dx_2 f_{h_1,a}(x_1, \mu_F) f_{h_2,b}(x_2, \mu_F) \hat{\sigma}(ab \rightarrow l\bar{l} + X; \alpha_s(\mu_R), \alpha(\mu_R), \mu_F)$$



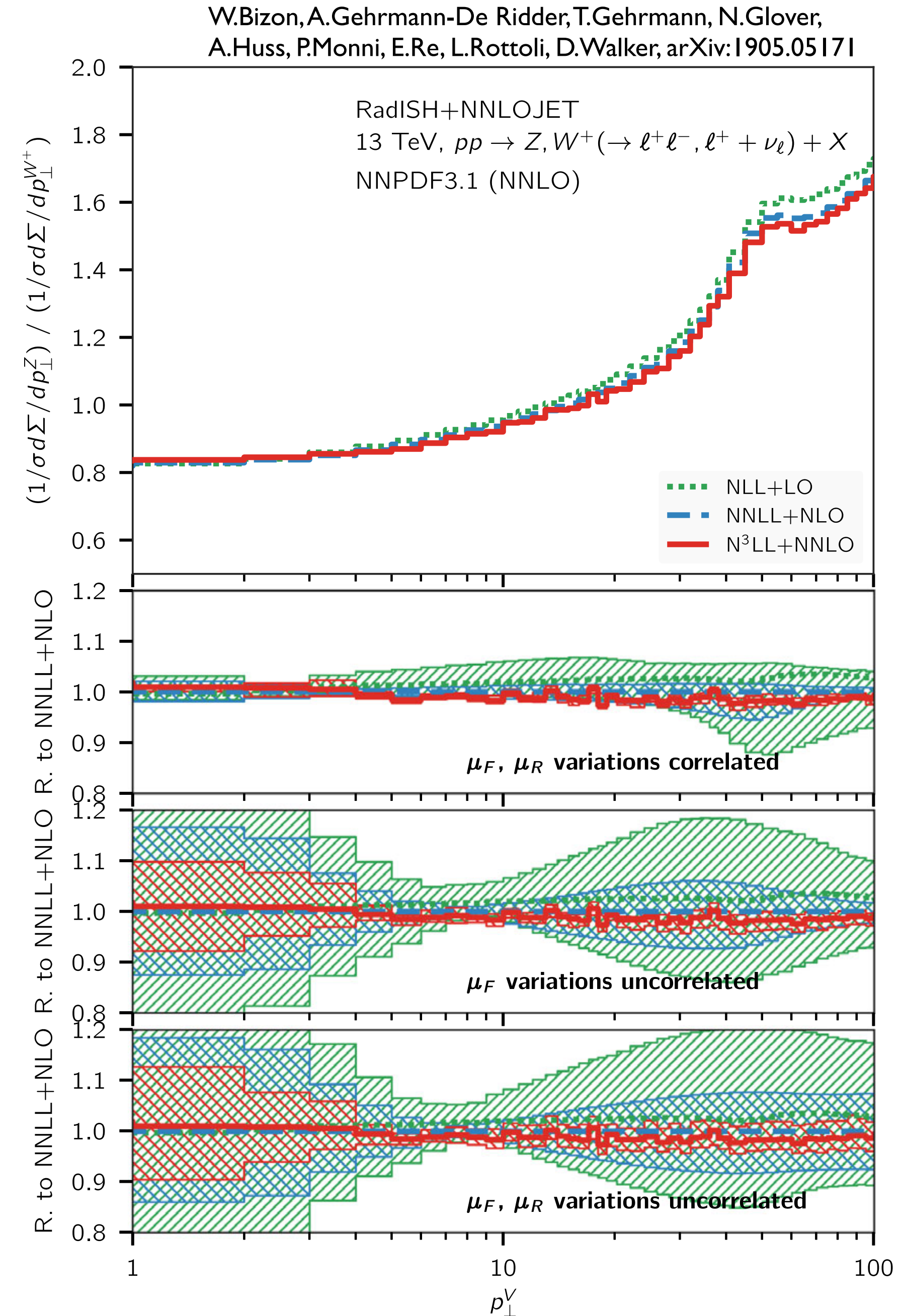
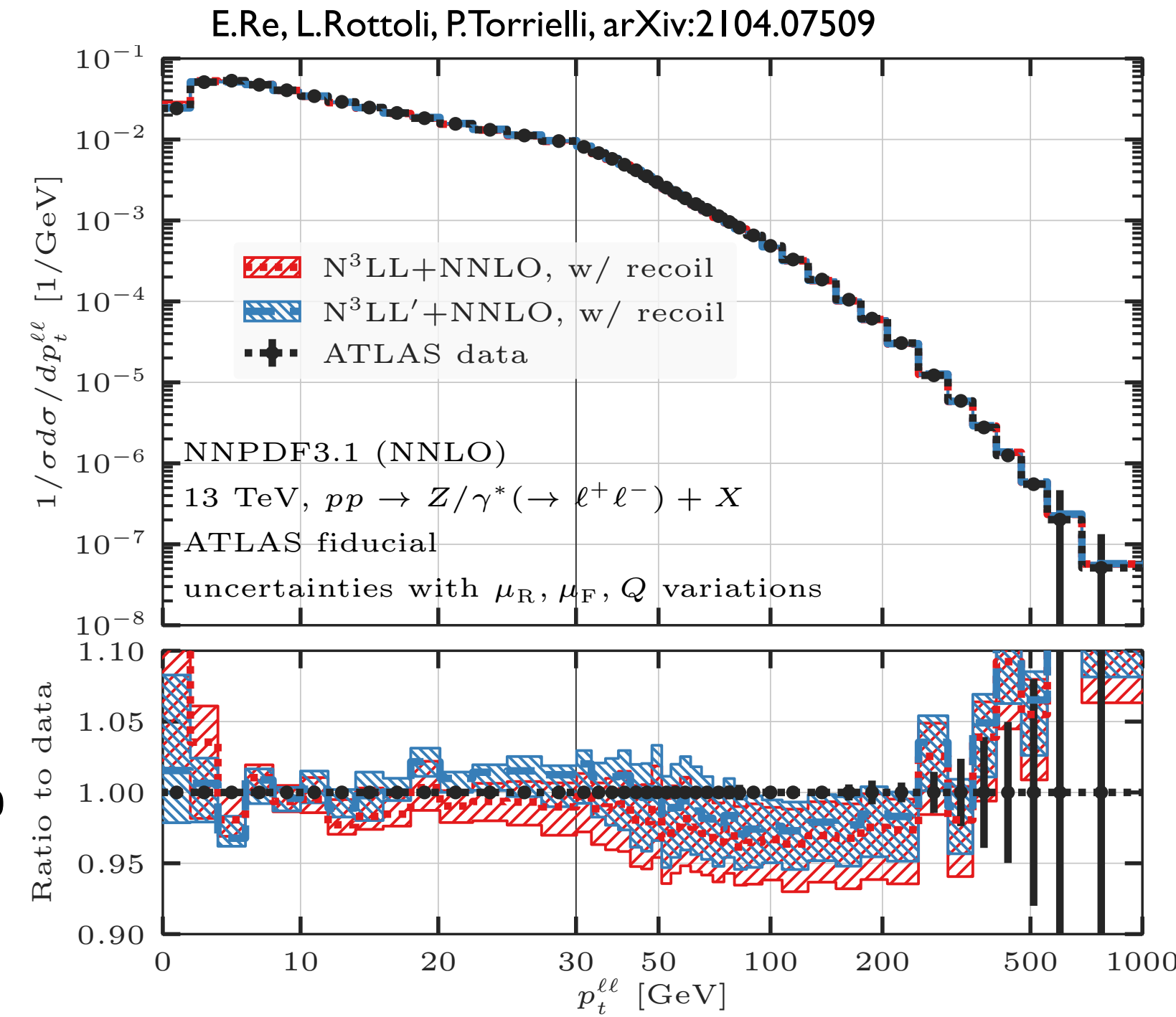
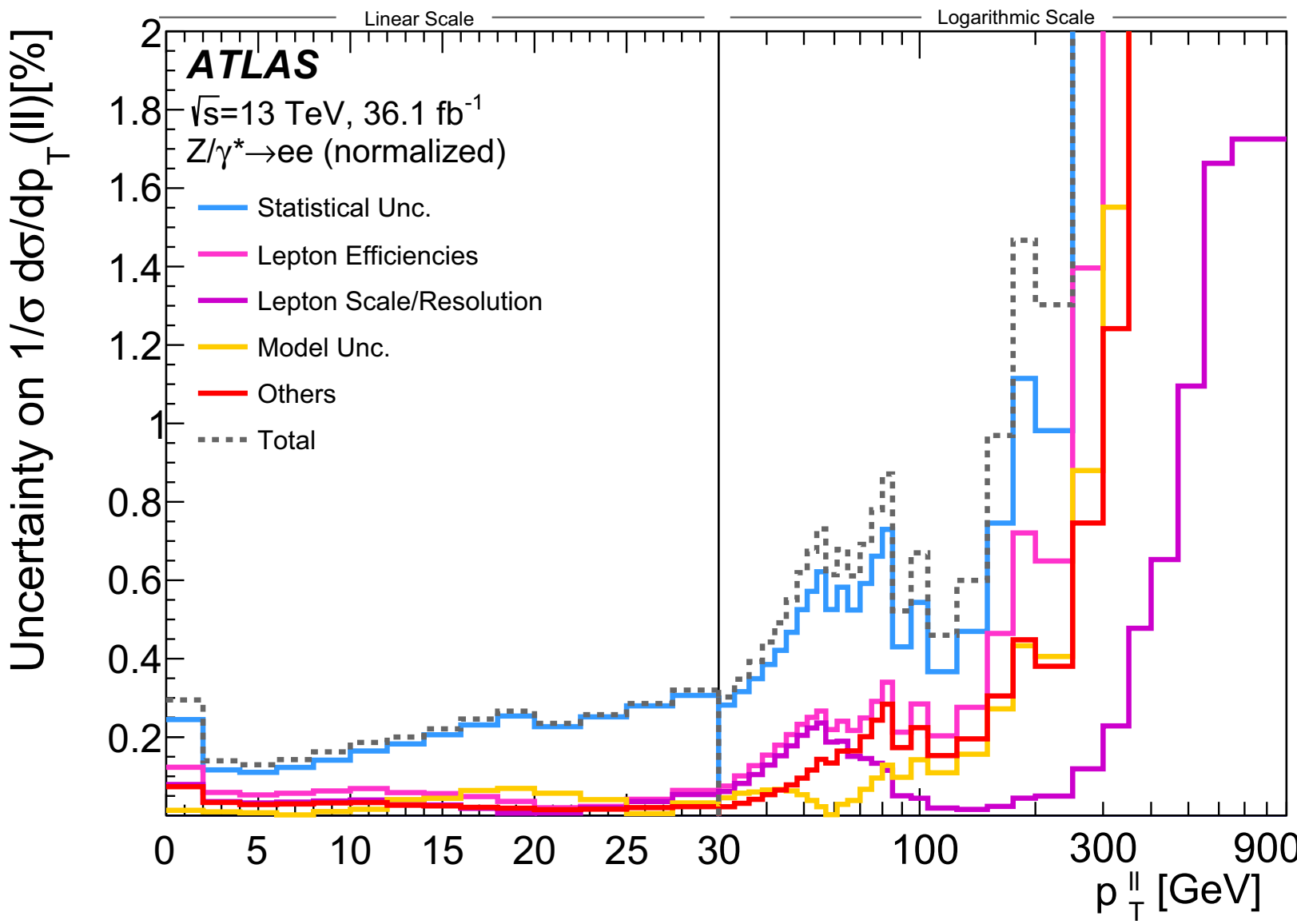
We need

- best description of the **partonic cross section** including fixed- and all-orders radiative corrections **QCD**, EW, mixed **QCDxEW**
- accurate and consistent description of the **QCD environment** including PDFs, intrinsic partonic k_\perp , QED DGLAP PDF evolution

- ▷ QCD modelling both perturbative and non-perturbative QCD contributions
 - transverse d.o.f. → gauge bosons p_\perp^V spectra; dependent on non-perturbative contributions at low p_\perp^Z
 - longitudinal d.o.f. → rapidity distributions ; affected by PDF uncertainties
- ▷ EW and mixed QCDxEW effects
 - important QED/EW corrections (mostly FSR) modulated by the underlying QCD dynamics

are our current tools adequate for the precision determination of EW parameters ?

The lepton-pair transverse-momentum distribution in neutral-current Drell-Yan

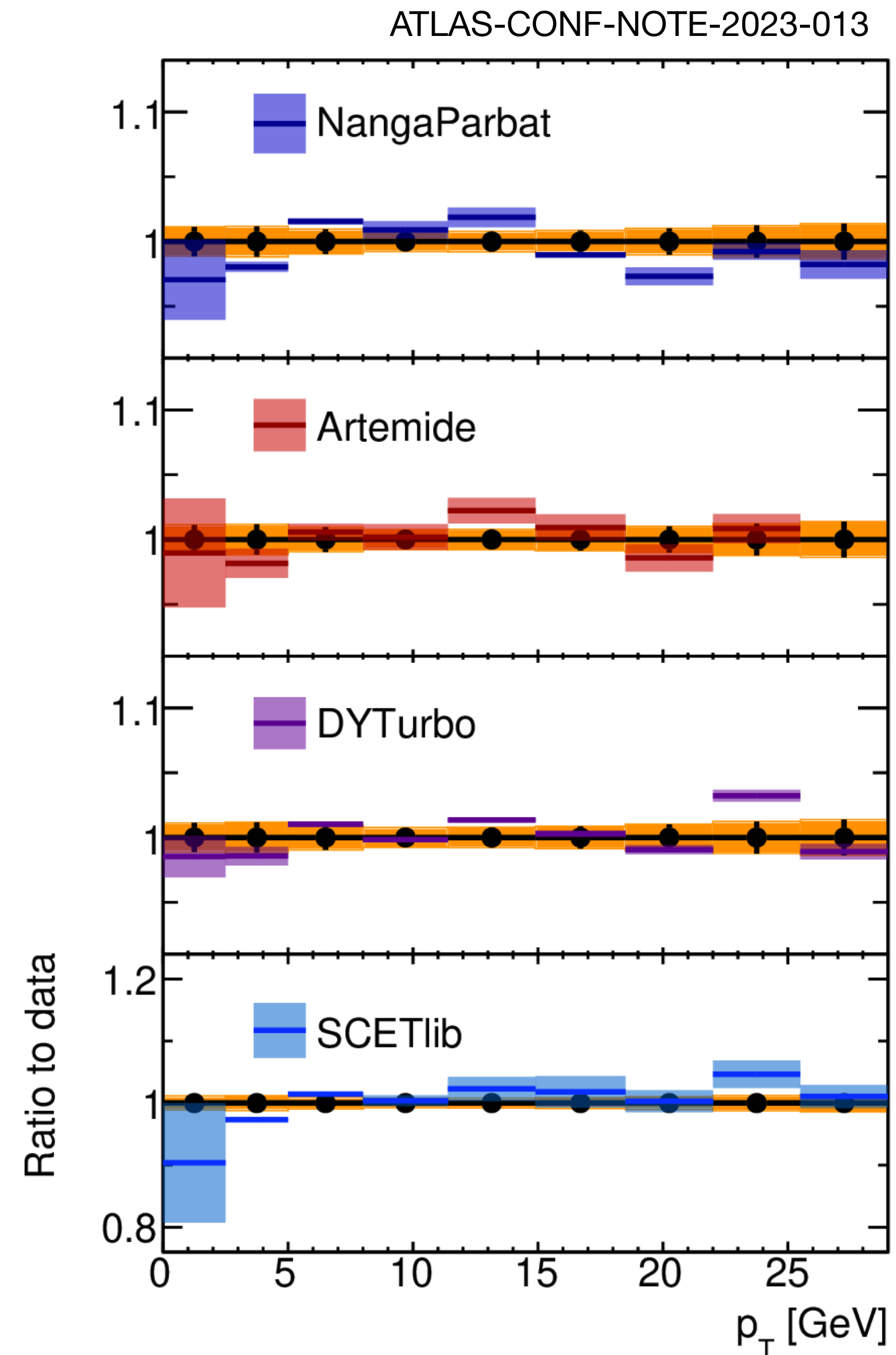
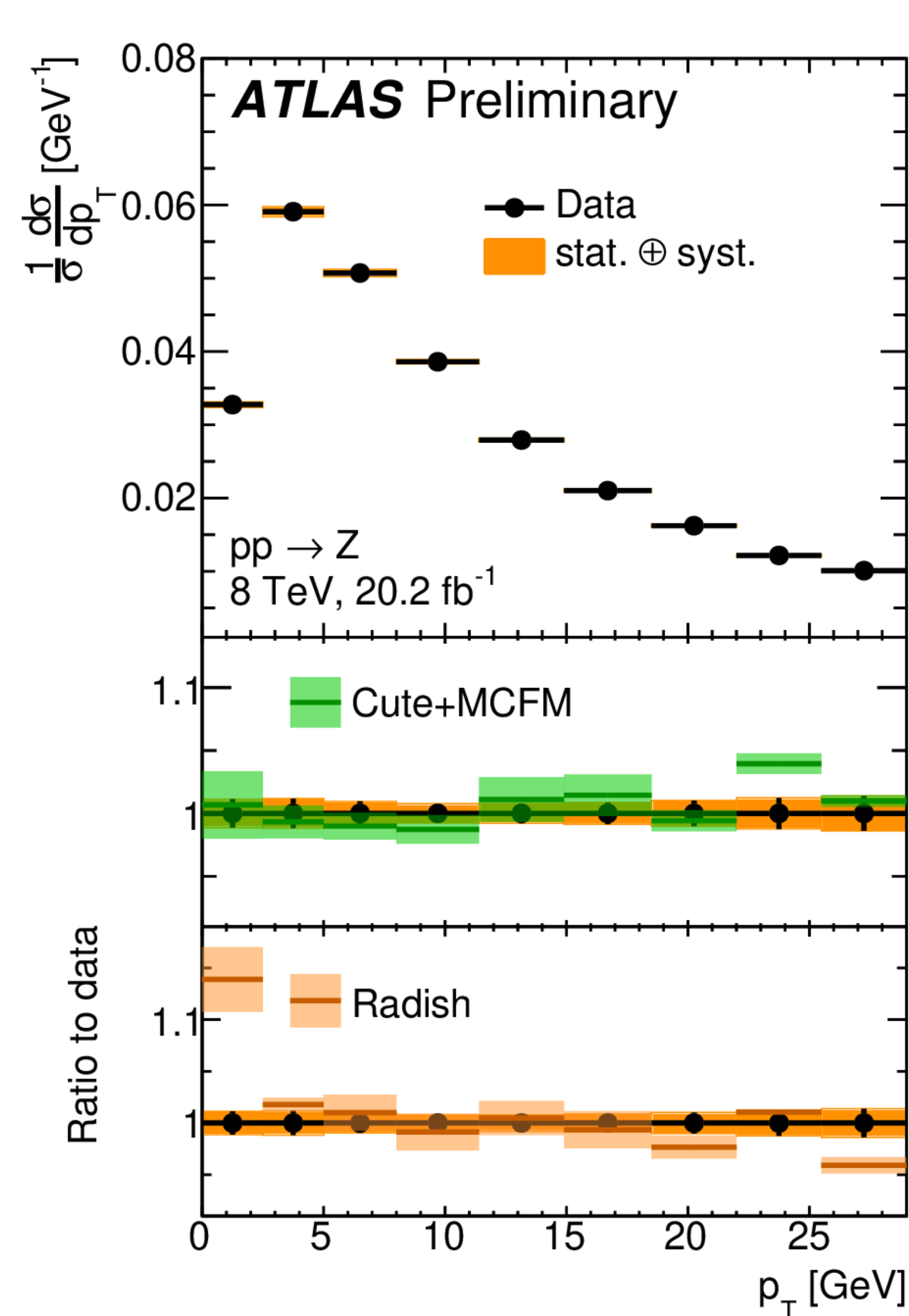


- very high experimental precision
- the p_T^Z distribution is responsible for a large component of the lepton transverse momentum \rightarrow relevance for m_W
- the resummation at N3LL and the matching with NNLO results has a strong impact on both central value and uncertainty band
- the description of the data based on a perturbative-QCD description at N3LL is quite accurate, with perturbative uncertainties in the 2-3% range
- \rightarrow the need for a non-perturbative contribution is reduced
- \rightarrow the reweighing factor needed to “reach the data” should become closer to 1

Comparison of theoretical predictions and p_{\perp}^Z data

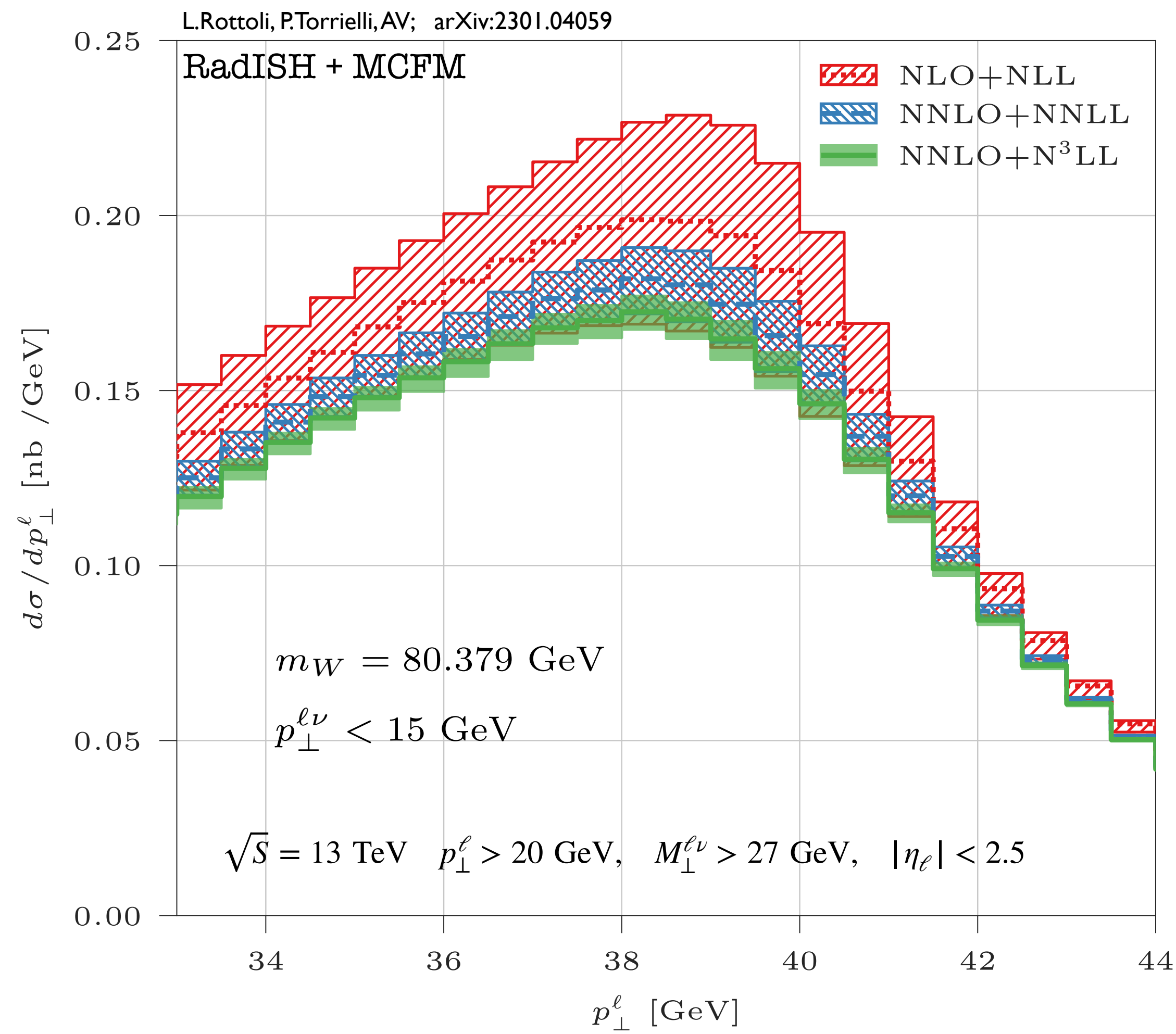
cfr. the p_{\perp}^Z benchmarking exercise in the CERN-EW-WG

all the codes share N3LL precision in the resummation



The charged lepton transverse momentum distribution in charged-current Drell-Yan

One of the main observables relevant for the m_W determination



Two mechanisms yield a lepton transverse momentum

- LO decay
- gauge-boson recoil against QCD radiation

→ resummation of $\log(p_{\perp}^V/m_V)$ factors is needed

Impressive progress in QCD calculations

X.Chen, T.Gehrmann, N.Glover, A.Huss, P.Monni, E.Re, L.Rottoli, P.Torrielli, arXiv:2203.01565
 X.Chen, T.Gehrmann, N.Glover, A.Huss, T.yang, H.Zhu, arXiv: 2205.11426
 J.Campbell, T.Neumann, arXiv:2207.07056

Logarithmic order counting for $p_{\perp}^{\ell\ell}$ resummation
 Fixed-order counting for the total DY cross section

Uncertainty band based on canonical scale variations

$$\mu_{R,F} = \xi_{R,F} \sqrt{(M^{\ell\nu})^2 + (p_{\perp}^{\ell\nu})^2}, \quad \mu_Q = \xi_Q M^{\ell\nu}$$

$\xi_{R,F} \in (1/2, 1, 2)$ excluding ratios=4 (7 variations)

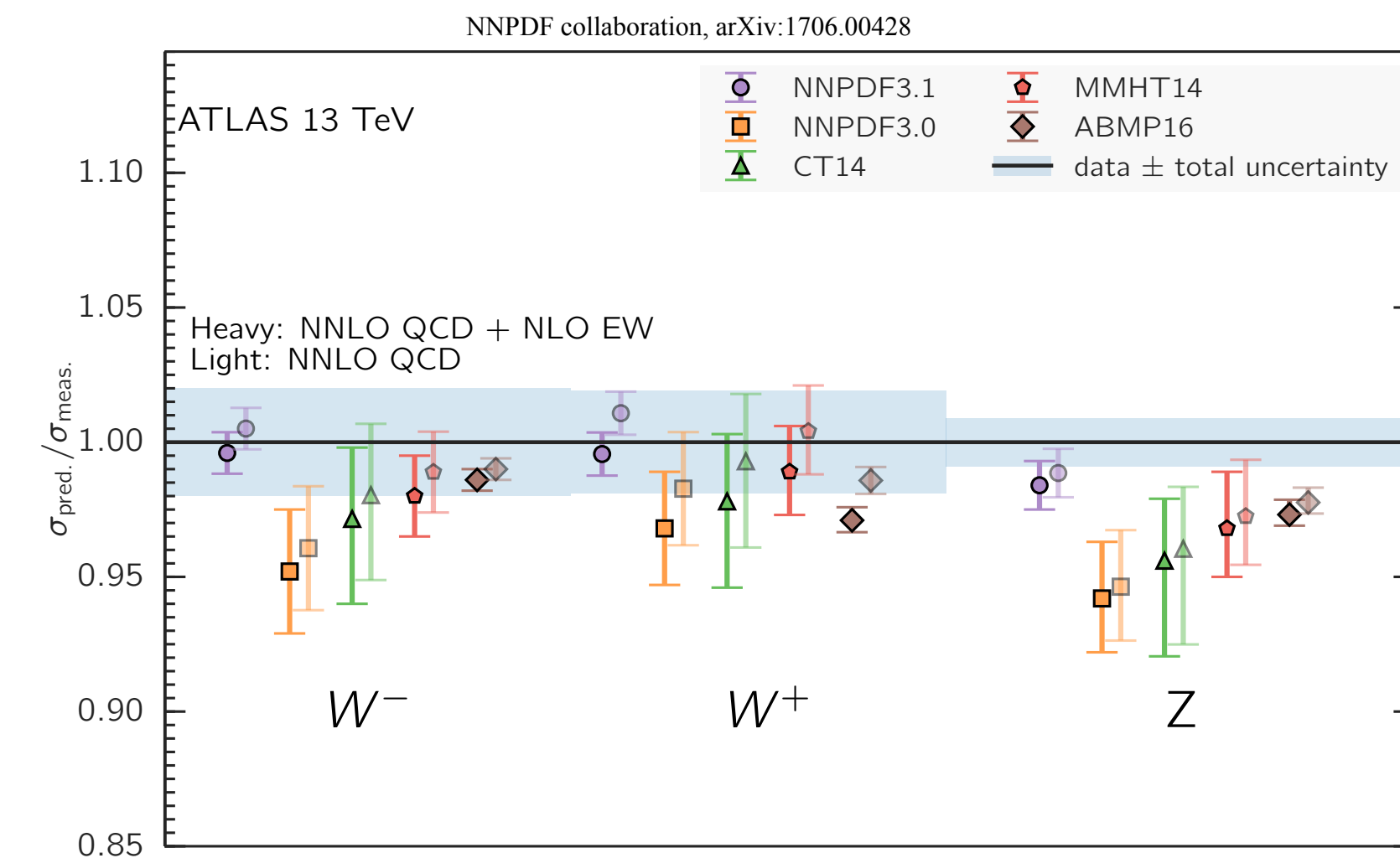
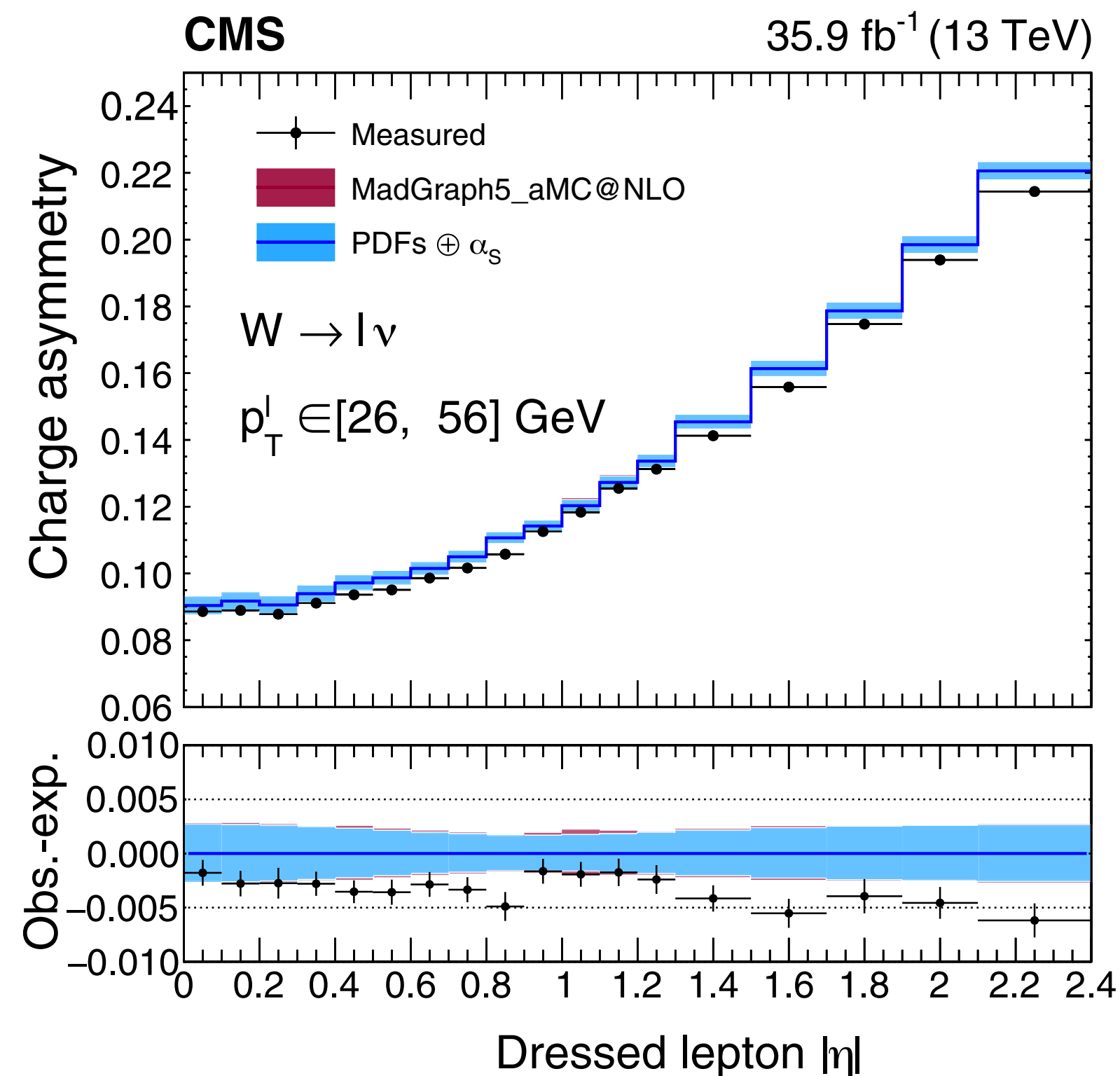
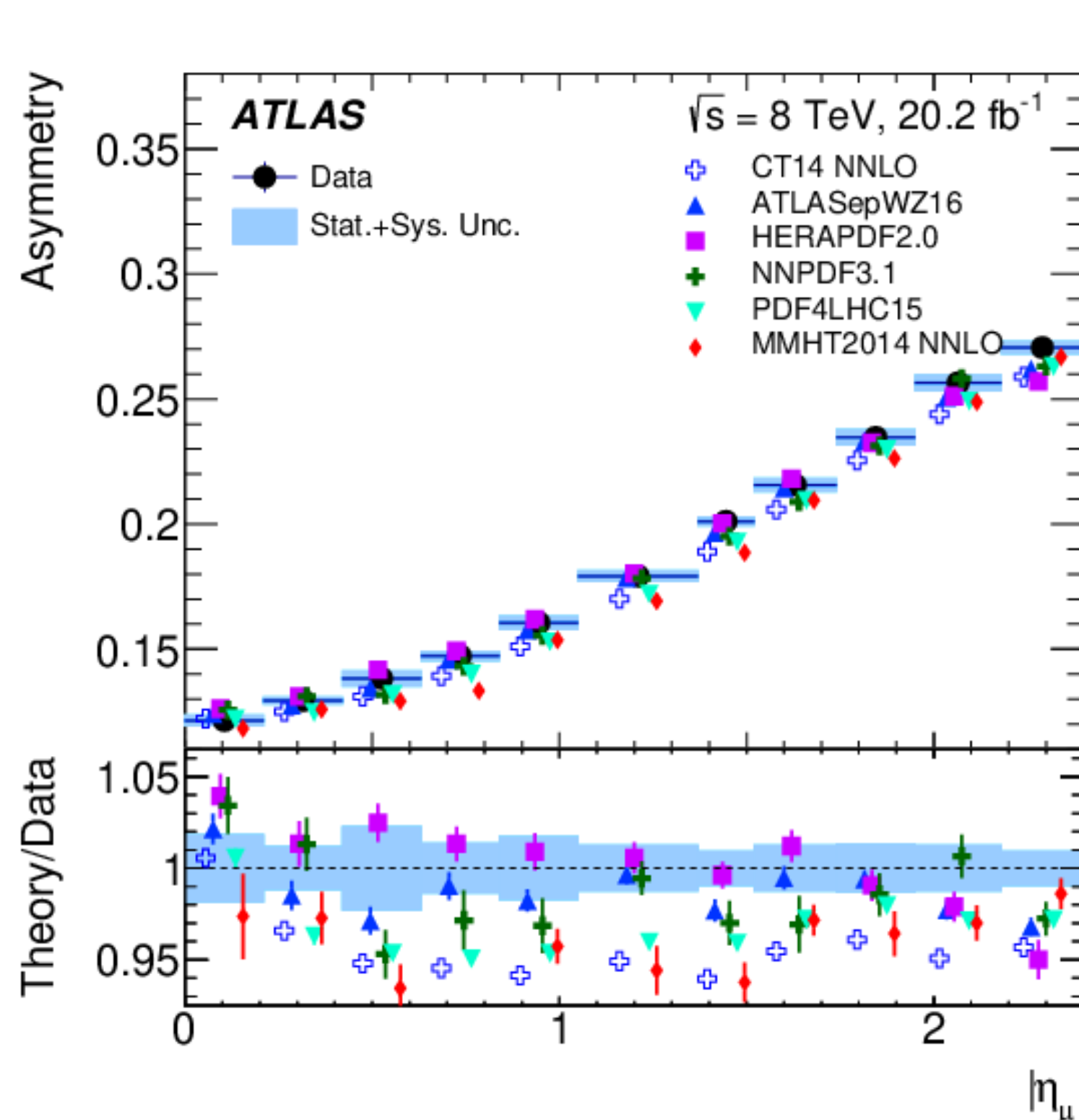
$(\xi_R, \xi_F) = (1, 1)$ and $\xi_Q = (1/4, 1)$ (2 variations)

At NNLO+N3LL, residual $\pm 2\%$ uncertainty

It benefits from the improvement of the resummation of the gauge boson transverse momentum distribution

Rapidity distributions in Drell-Yan processes and proton PDF determination

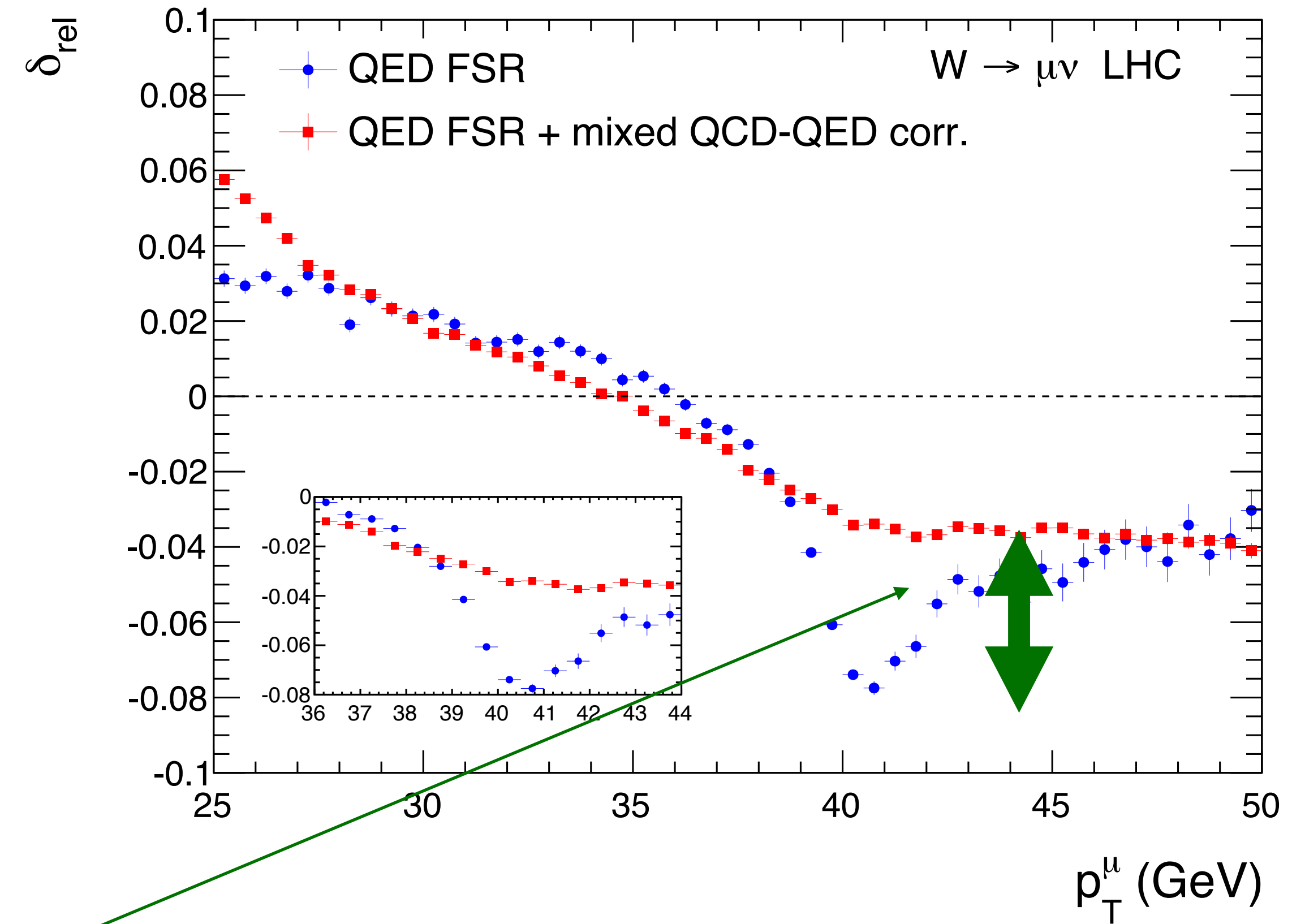
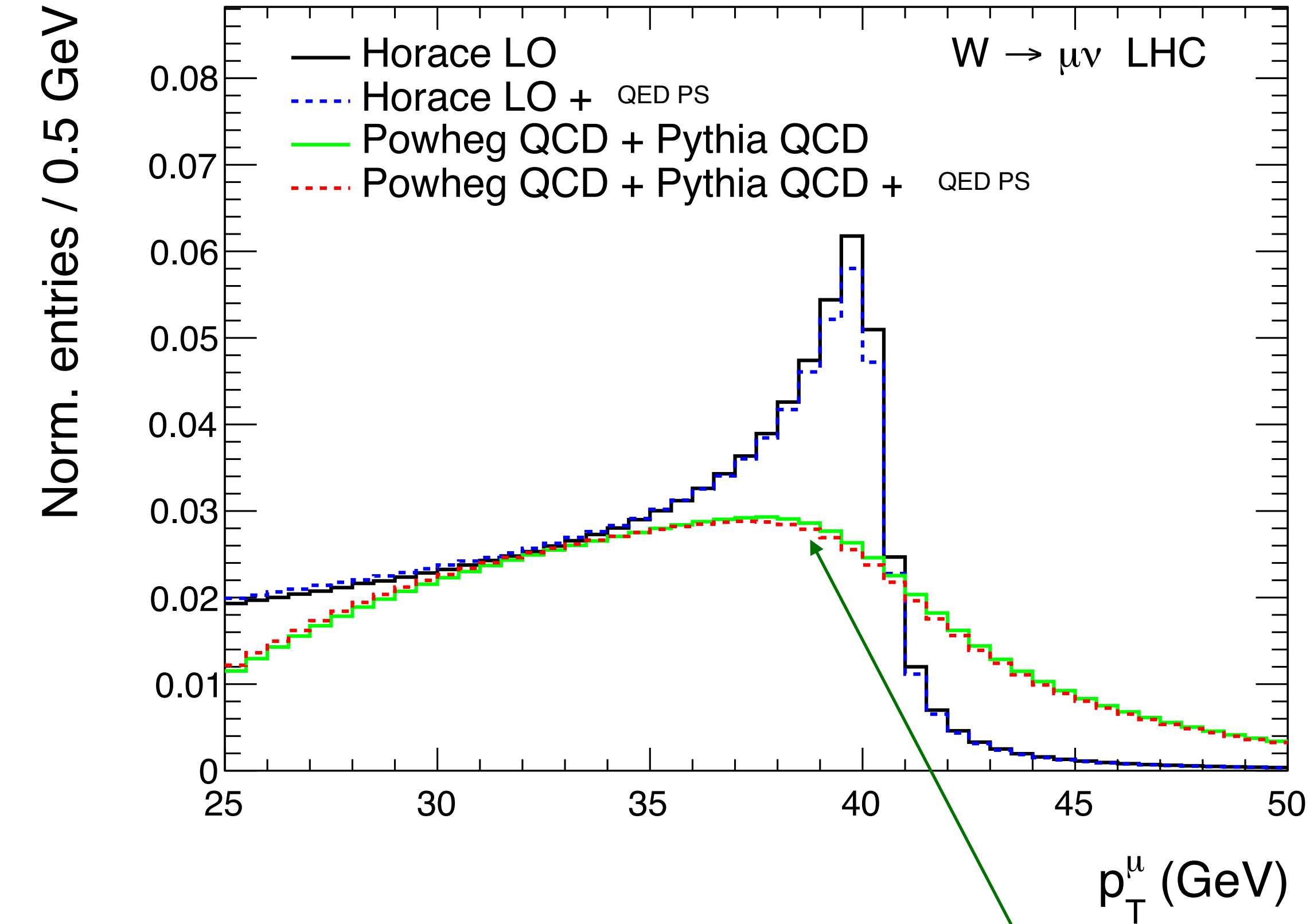
- An important role in the determination of proton structure is played by the rapidity distributions and the total sec
 - ▷ the lepton charge-asymmetry rapidity distribution is needed to improve the flavour separation
 - ▷ precise results at parton level for the total xsec make its contribution to the PDF fit more significant
 - importance of NNLO and N3LO calculations
 - ▷ in a fiducial volume the rapidity and transverse momentum dependencies are connected by kinematics
 - PDF uncertainties impact on the m_W determination



on-shell gauge boson production
as a PDF benchmark

Interplay of QCD and QED corrections

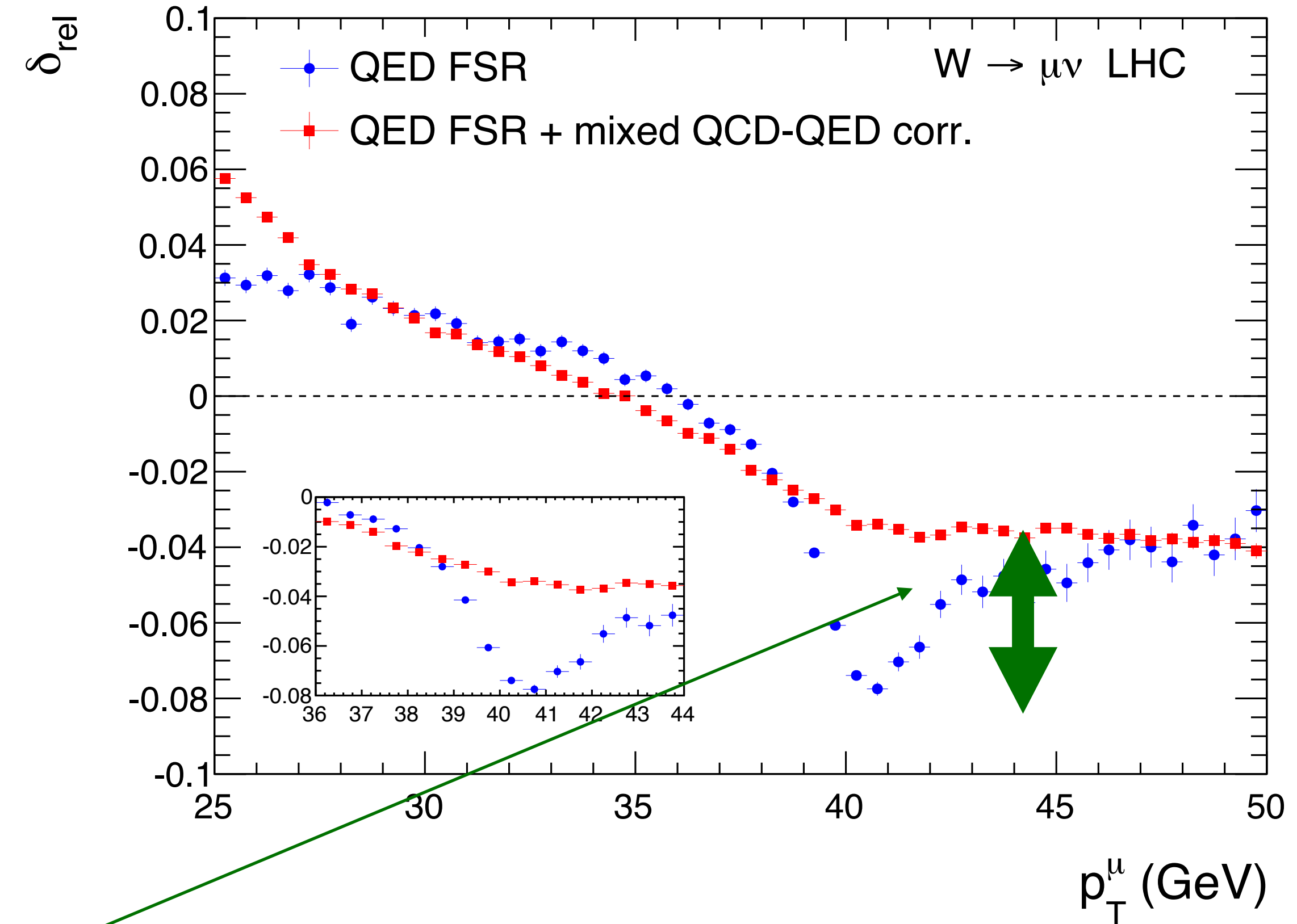
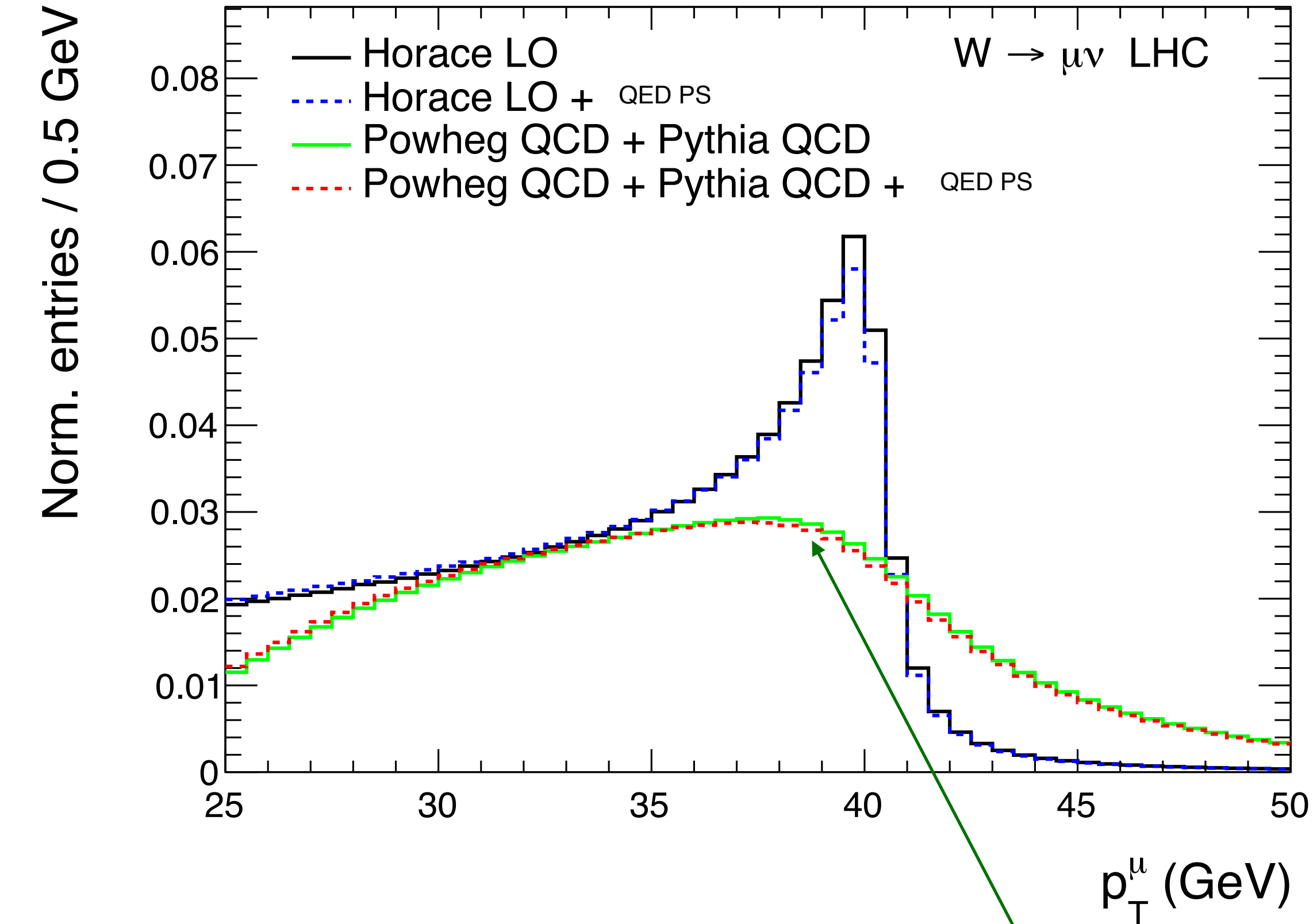
C.Carloni Calame, M.Chiesa, H.Martinez, G.Montagna, O.Nicrosini, F.Piccinini, AV, arXiv:1612.02841



- very large impact of initial-state QCD radiation on the p_{Tlep} distribution
- large radiative corrections due to QED final state radiation at the jacobian peak
- very large **interplay of QCD and QED corrections** redefining the precise shape of the jacobian peak

Interplay of QCD and QED corrections

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- very large **interplay of QCD and QED corrections** redefining the precise shape of the jacobian peak

NLO-QCD + QCDPS + QEDPS is the lowest order meaningful approximation of this observable

the precise size of the mixed QCDxQED corrections (and uncertainties) depends on the choice for the QCD modelling

MW determination at hadron colliders

History of the m_W determination

the latest m_W determinations at hadron colliders are dominated by modelling systematics

a few examples

ATLAS-2017

$$\begin{aligned} m_W &= 80369.5 \pm 6.8 \text{ MeV(stat.)} \pm 10.6 \text{ MeV(exp. syst.)} \pm 13.6 \text{ MeV(mod. syst.)} \\ &= 80369.5 \pm 18.5 \text{ MeV,} \end{aligned}$$

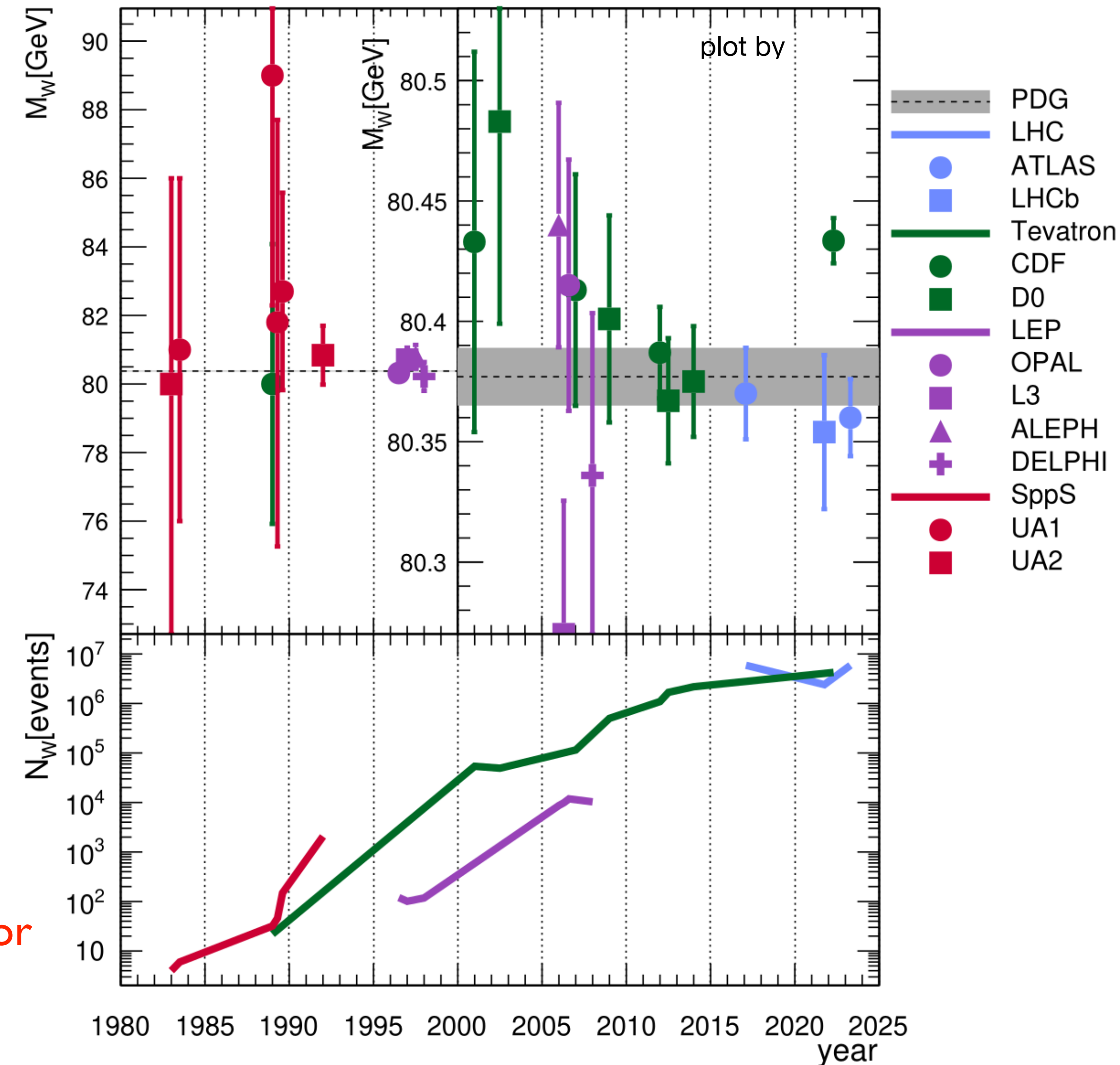
LHCb-2021

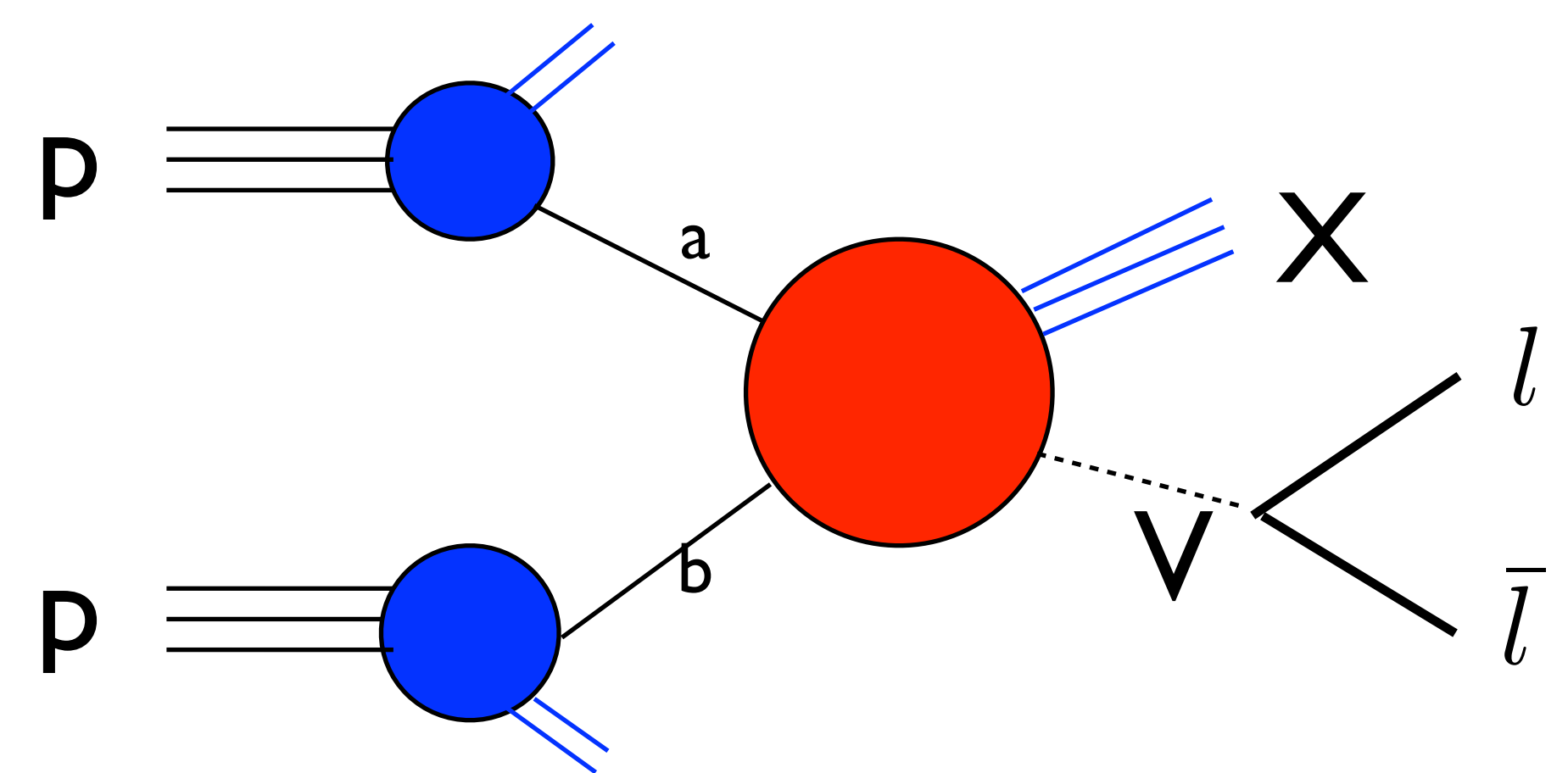
$$m_W = 80354 \pm 23_{\text{stat}} \pm 10_{\text{exp}} \pm 17_{\text{theory}} \pm 9_{\text{PDF}} \text{ MeV}$$

ATLAS-2023

$$m_W = 80360 \pm 5(\text{stat.}) \pm 15(\text{syst.}) = 80360 \pm 16 \text{ MeV}$$

the significance of the SM test depends on the size of the total error



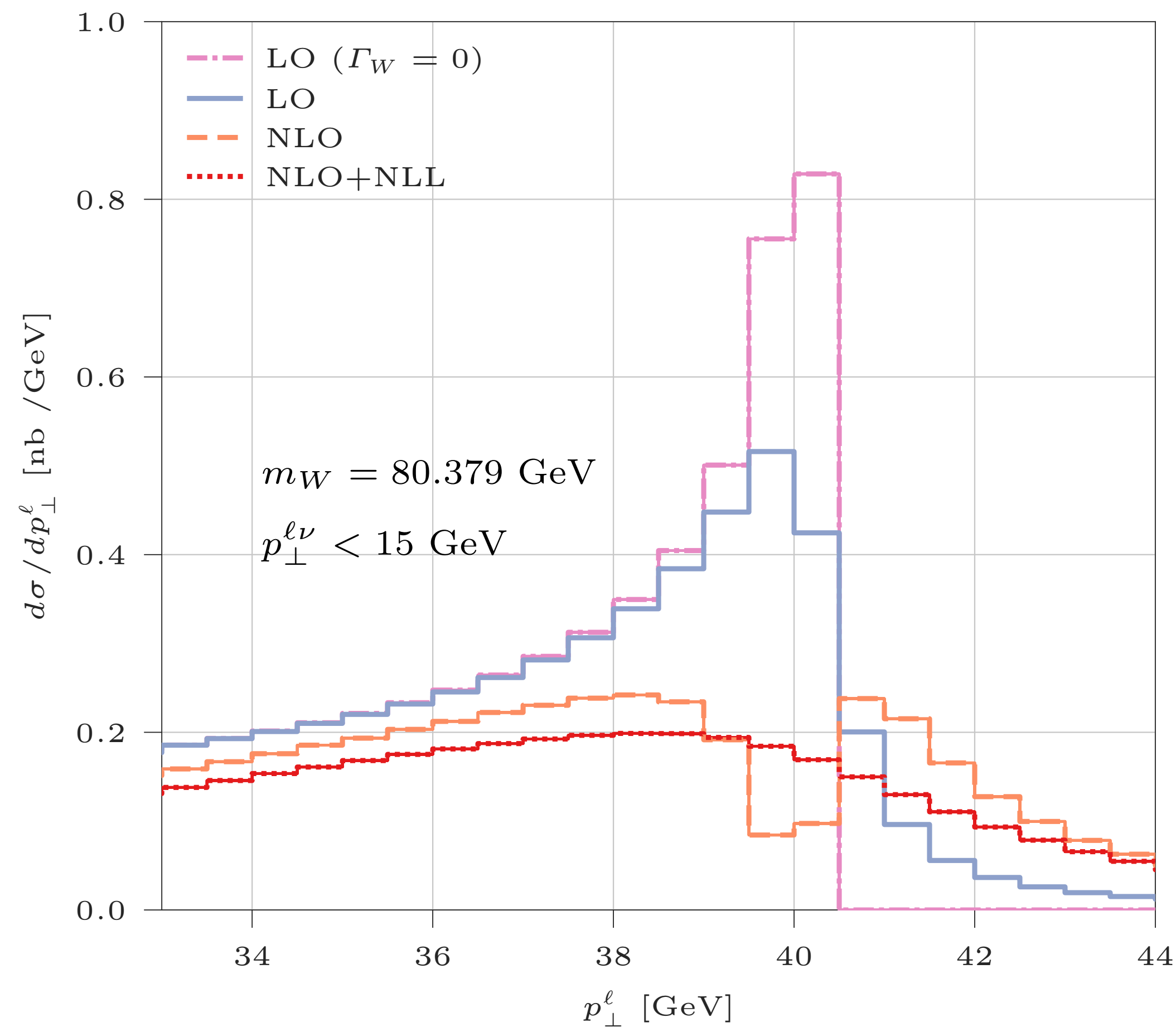


- In charged-current DY, it is **NOT** possible to reconstruct the lepton-neutrino invariant mass
Full reconstruction is possible (but not easy) only in the transverse plane
- A generic observable has a linear response to an m_W variation
With a goal for the relative error of 10^{-4} , the problem seems to be unsolvable
- m_W extracted from the study of the **shape** of the p_{\perp}^l , M_{\perp} and E_{\perp}^{miss} distributions in CC-DY
thanks to the **jacobian peak** that enhances the sensitivity to m_W

$$\frac{d}{dp_{\perp}^2} \rightarrow \frac{2}{s} \frac{1}{\sqrt{1 - 4p_{\perp}^2/s}} \frac{d}{d \cos \theta} \sim \frac{d}{dp_{\perp}^2} \rightarrow \frac{2}{s} \frac{1}{\sqrt{1 - 4p_{\perp}^2/m_W^2}} \frac{d}{d \cos \theta}$$

→ **enhanced sensitivity** at the 10^{-3} level (p_{\perp}^l distribution) or even at the 10^{-2} level (M_{\perp} distribution)

The lepton transverse momentum distribution in charged-current Drell-Yan



The lepton transverse momentum distribution has a jacobian peak induced by the factor $1/\sqrt{1 - \frac{s}{4p_{\perp}^2}}$.

When studying the W resonance region, the peak appears at $p_{\perp} \sim \frac{m_W}{2}$

Kinematical end point at $\frac{m_W}{2}$ at LO

The decay width allows to populate the upper tail of the distribution

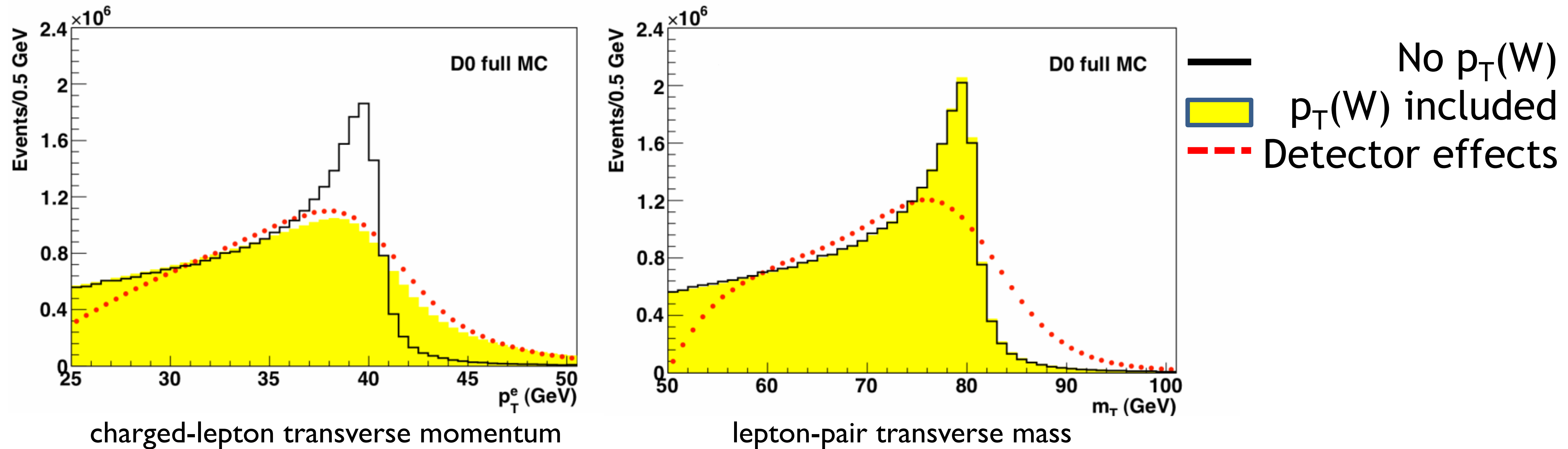
Sensitivity to soft radiation \rightarrow double peak at NLO-QCD

The QCD-ISR leading log resummation broadens the distribution and cures the sensitivity to soft radiation at the jacobian peak.

In the p_{\perp}^{ℓ} spectrum the sensitivity to m_W and important QCD features are closely intertwined

The position of the end-point is “hidden” by the smearing induced by the QCD (and QED) radiation

- problems are due to
- the smearing of the distributions due to difficult neutrino reconstruction
 - sensitivity to the modelling of initial state QCD effects



m_W determination at hadron colliders: template fitting via χ^2 minimisation

Given one experimental kinematical distribution,

given one theoretical simulation model,

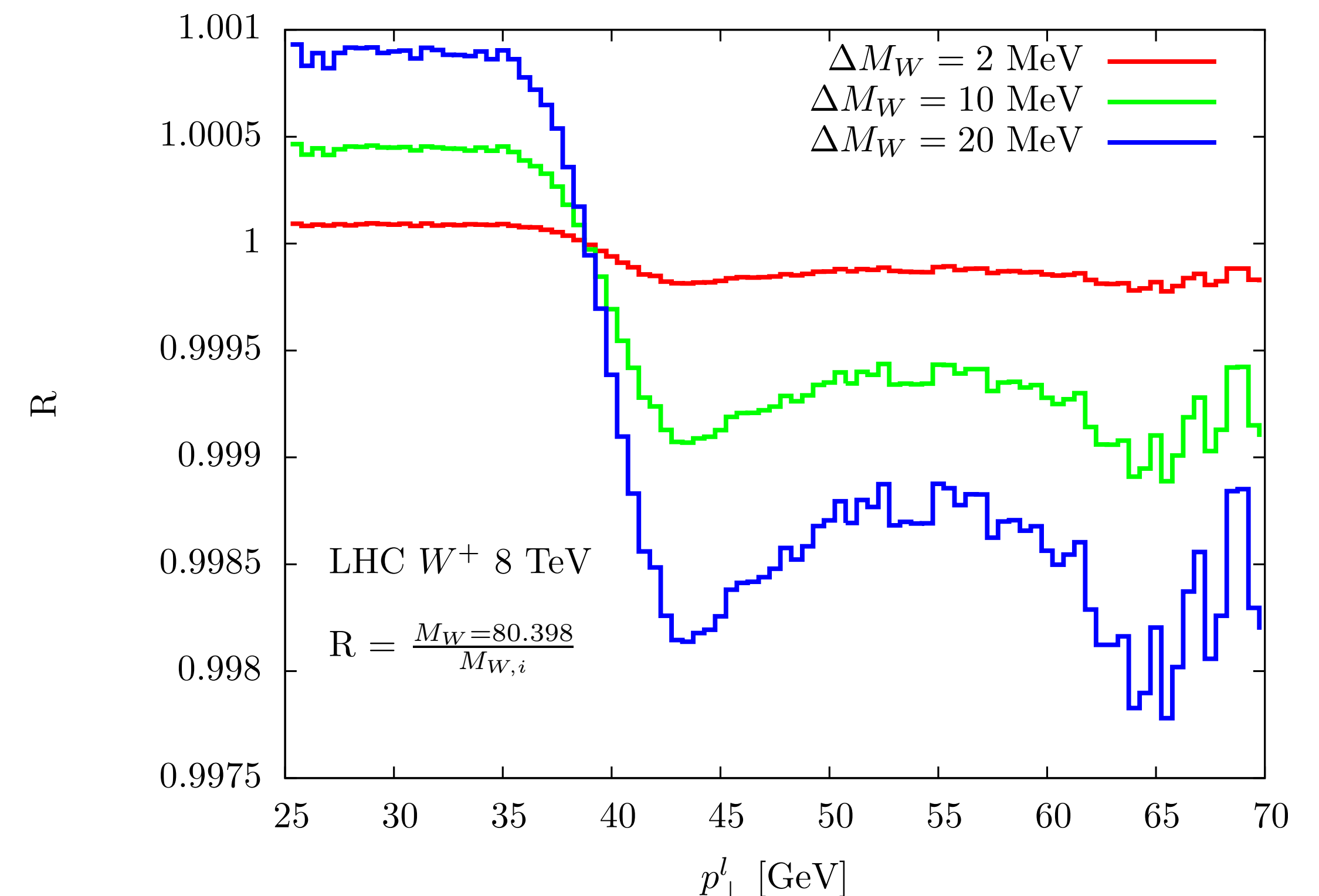
- we compute the corresponding theoretical distribution for several hypotheses of one Lagrangian input parameters (e.g. m_W)
- we compute, for each $m_W^{(k)}$ hypothesis, a χ_k^2 defined in a certain interval around the jacobian peak (fitting window)
- we look for the minimum of the χ^2 distribution

The m_W value associated to the position of the minimum is the experimental result

A determination at the 10^{-4} level requires
a control over the shape of the distributions at the per mille level

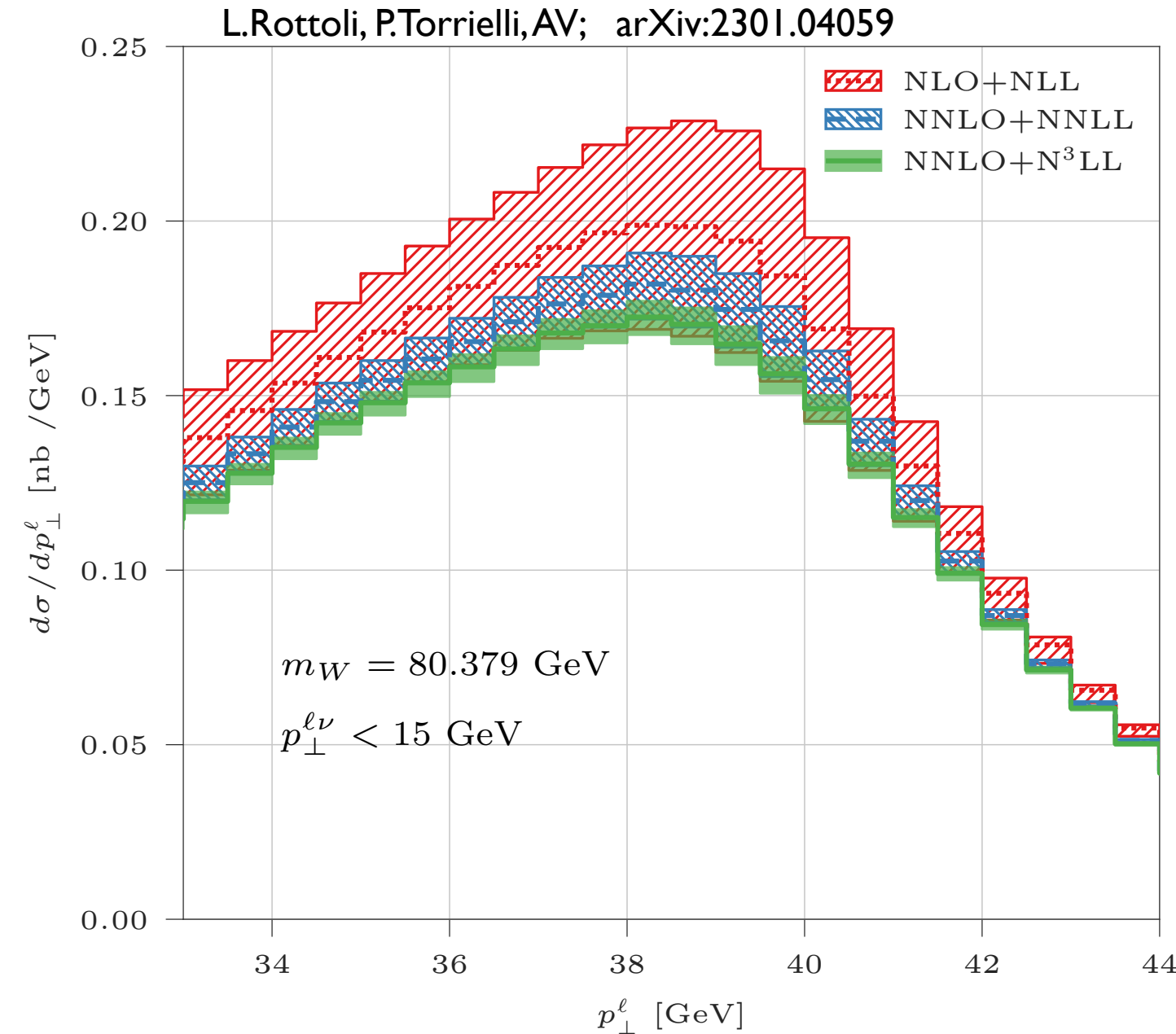
The theoretical uncertainties of the templates
contribute to the **theoretical systematic error on m_W**

→ we need the best available simulation tools



Template fitting: description of the single lepton transverse momentum distribution

The template fitting procedure is acceptable if the data are described by the theoretical distribution with high quality

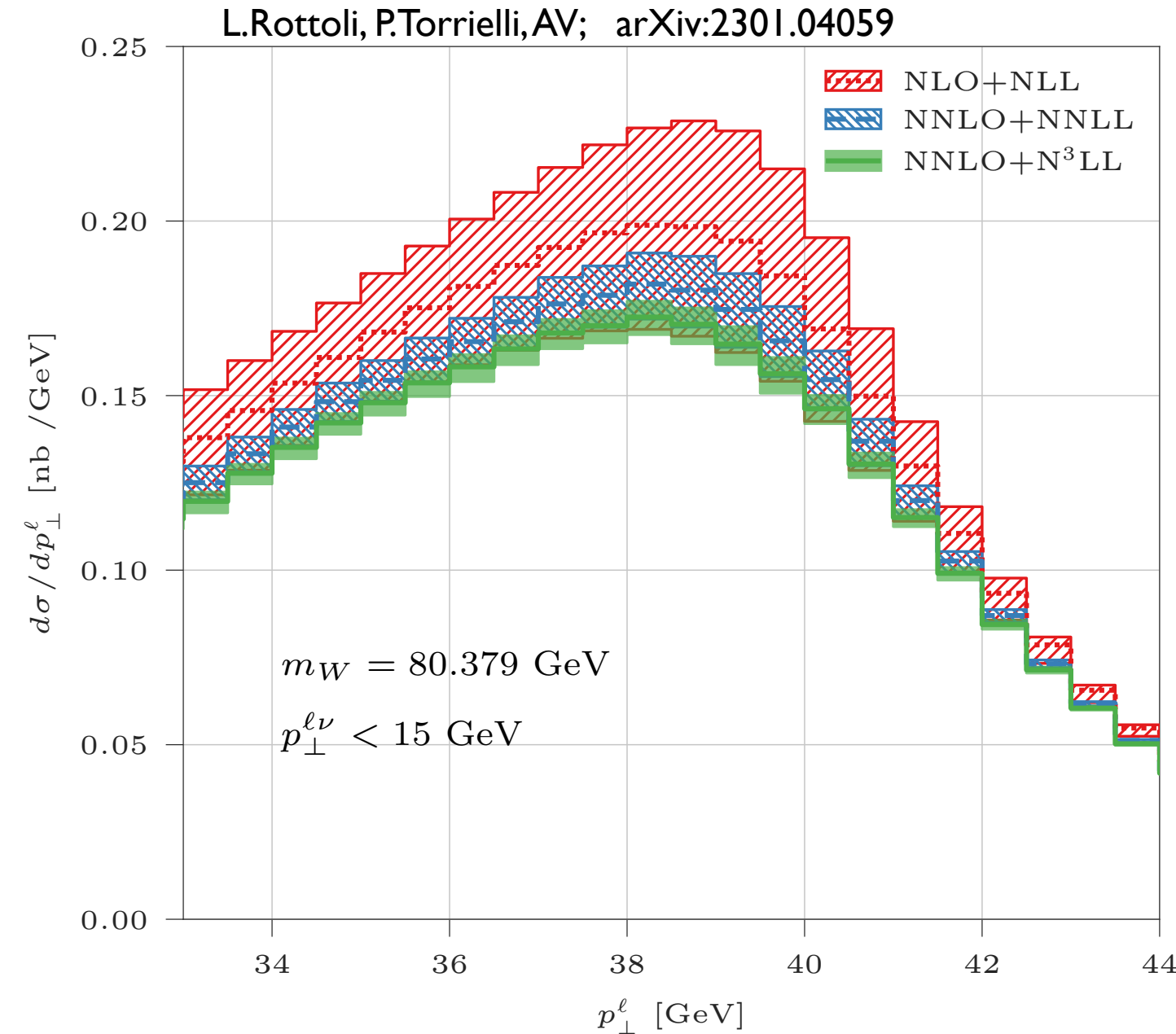


Scale variation of the NNLO+N³LL prediction for p_{tlep} provides a set of **equally good templates** but the width of the uncertainty band is at the few **percent** level **a factor 10 larger** than the naive estimate would require !
A χ^2 -fit in these conditions is impossible, very unstable

→ **data driven** approach
a Monte Carlo event generator is tuned to the data in NCDY (p_{\perp}^Z)
↓
the same parameters are then used to prepare the CCDY templates

Template fitting: description of the single lepton transverse momentum distribution

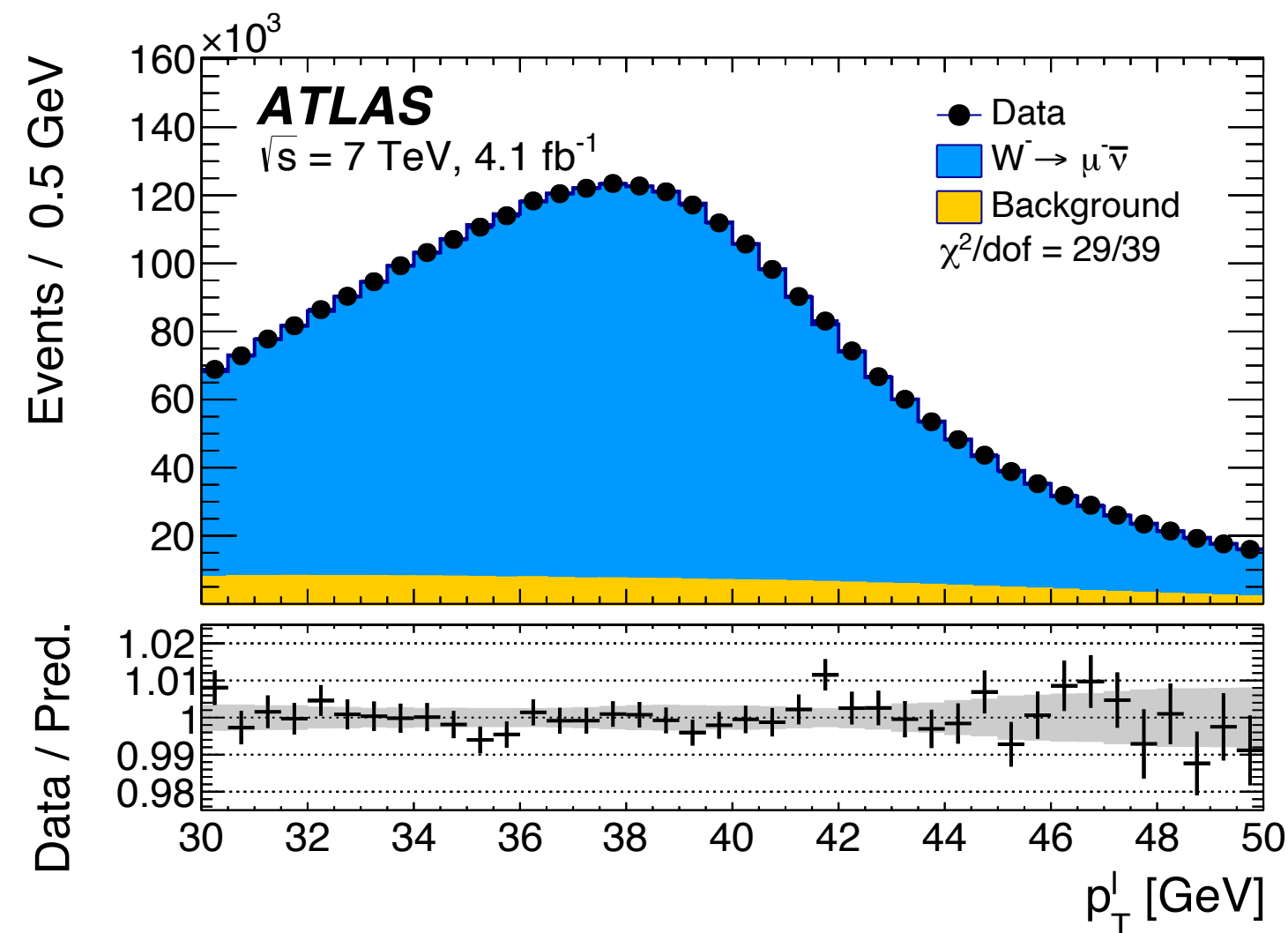
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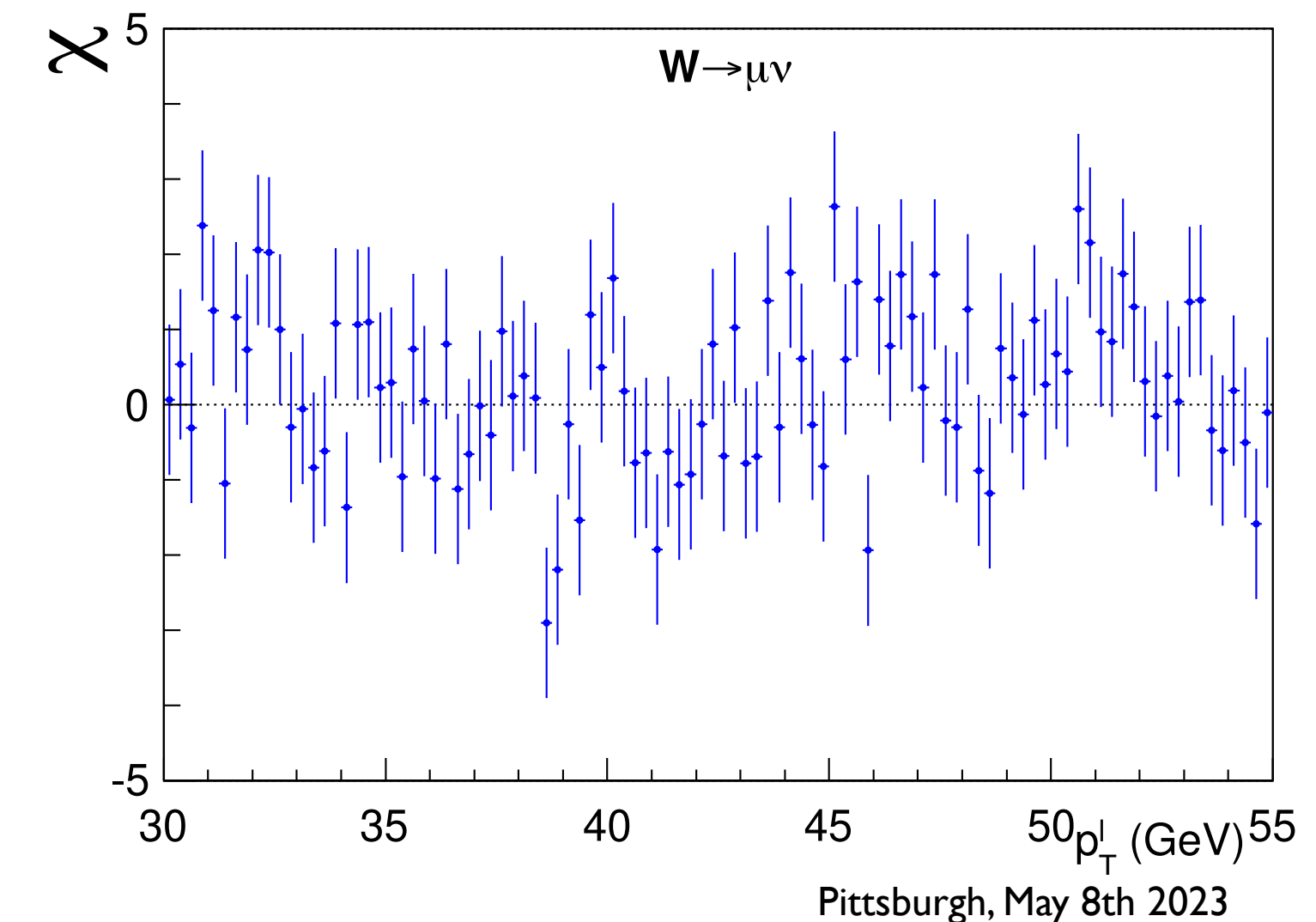
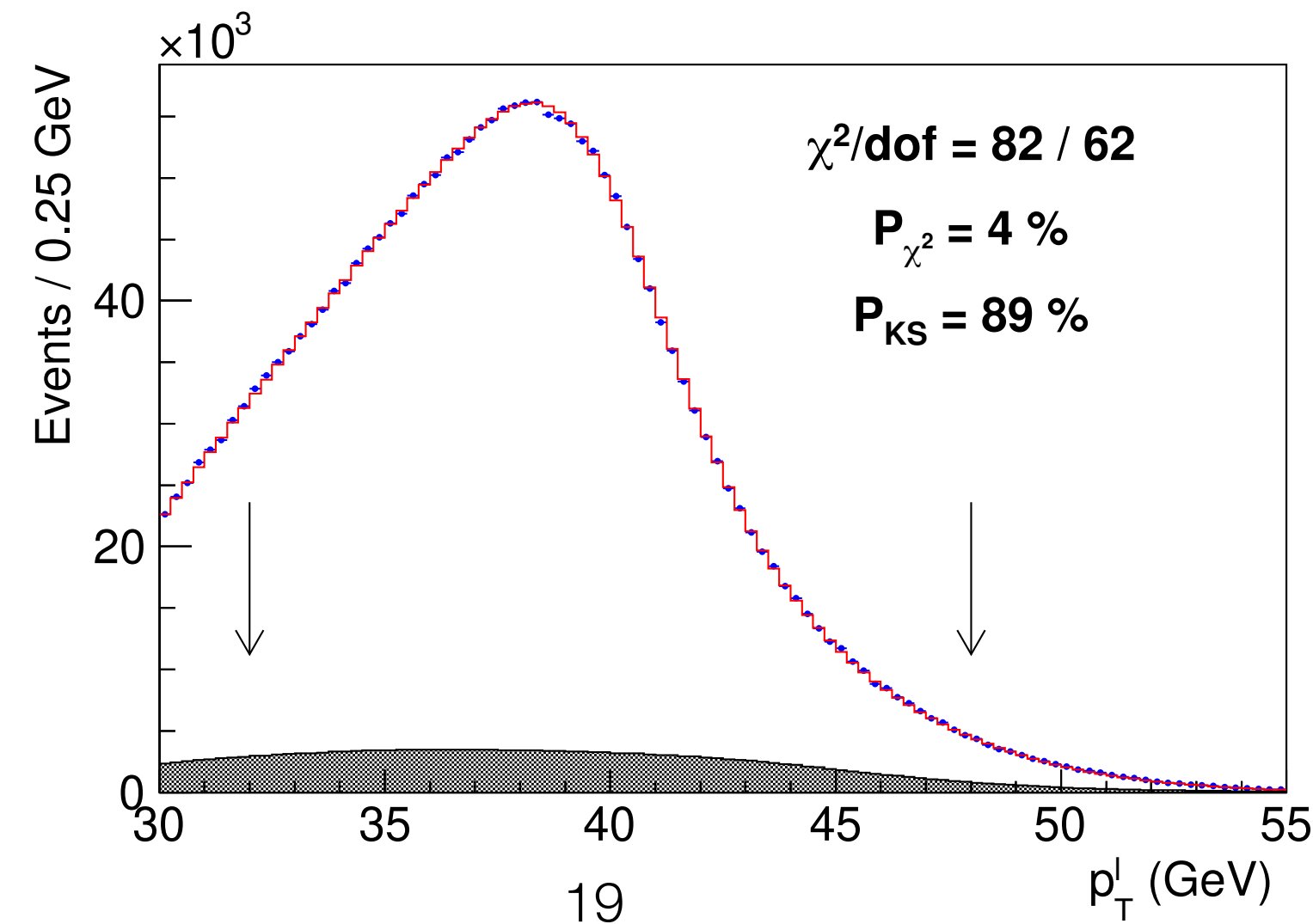
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Eur.Phys.J.C 78 (2018) 2, 110, *Eur.Phys.J.C* 78 (2018) 11, 898 (erratum)



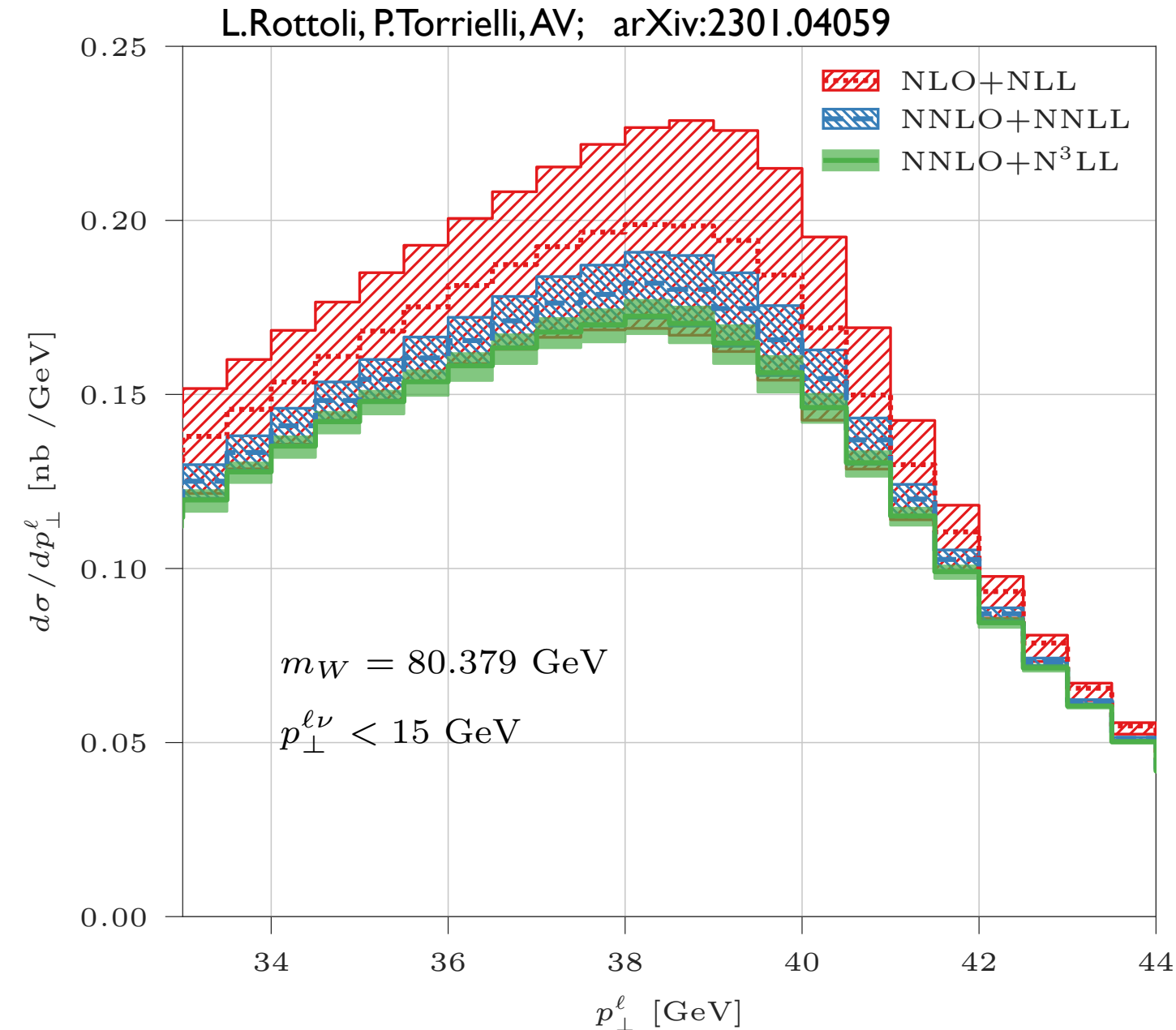
Alessandro Vicini - University of Milano

CDF collaboration, Science 376, 170-176 (2022)



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a Monte Carlo event generator is tuned to the data in NCDY (p_{\perp}^Z)
↓
the same parameters are then used to prepare the CCDY templates

What are the limitations of the transfer of information from NCDY to CCDY ?
Which (theory) uncertainty can we assign to the templates ?

Comments on the data driven approach

- The tuning assumes that the missing factor taken from the data is universal, i.e. identical for NCDY and CCDY but several elements of difference:
 - masses and phase-space factors, acceptances
 - different electric charges (QED corrections)
 - different initial states (\rightarrow PDFs, heavy quarks effects)
- The tuning assumes that the reweighing factor derived from p_{\perp}^Z applies equally well to the p_{\perp}^W and to the lepton transverse momentum in CCDY
- It is possible that BSM physics is reabsorbed in the tuning
- The interpretation of the fitted value is not necessarily the SM lagrangian parameter

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- The usage of NCDY data **increases the accuracy** of the templates
 \rightarrow the CCDY distributions are better described \rightarrow the χ^2 -fit is possible and stable
but
- The usage of NCDY data **does not improve the precision** of the templates
because the pQCD uncertainty of the fitting model is not modified
 \rightarrow how large are the pQCD uncertainties on m_W ?

Physics modelling of the cross section in the experimental analyses

$$\frac{d\sigma}{dM_V dy_V d\vec{p}_\perp^V d\cos\theta d\phi} = \frac{3}{16\pi} \frac{d\sigma^{unpol}}{d^4q} \left(1 + \cos^2\theta + \sum_{i=0}^7 A_i(p_\perp^V, y_V) P_i(\cos\theta, \phi) \right)$$

- The xsec is decomposed in terms of spherical harmonics
(exact **only** in pure QCD , the EW interaction introduces additional angular momentum components)
- Monte Carlo event generators including QED-FSR are the starting point of the templates preparation
The generators are tuned on the p_\perp^Z data, to improve the overall accuracy
Higher-order QCD corrections are applied via reweighting

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- Monte Carlo event generators are the starting point of the templates preparation
The generators are tuned on the p_\perp^Z data, to improve the overall accuracy
Higher-order QCD corrections are applied via reweighing
- ATLAS factorises the whole xsec into the product of 1D or 2D differential secs,
to better combine information from data and higher-order calculations

$$\frac{d\sigma}{dM_V dy_V dp_\perp^V d\cos\theta d\phi} = \frac{3}{16\pi} \frac{d\sigma(M_V)}{dM_V} \frac{d\sigma(y)}{dy_V} \left[\frac{d\sigma(p_\perp^V, y_V)}{dp_\perp^V dy_V} \left(\frac{d\sigma(y)}{dy_V} \right)^{-1} \right] \left(1 + \cos^2\theta + \sum_{i=0}^7 A_i(p_\perp^V, y_V) P_i(\cos\theta, \phi) \right)$$

Pythia Parton Shower tuned to p_\perp^Z data
NNLO-QCD

- The tuning is done with one fixed choice of the pQCD scales
→ QCD scale uncertainties are estimated *a posteriori*, studying the ratio $(d\sigma^{th}/dp_\perp^W) / (d\sigma^{th}/dp_\perp^Z)$
are **not** evaluated directly on the main observables (p_\perp^ℓ, M_\perp)

Breakdown of modelling uncertainties

ATLAS-CONF-2023-004

Obs.	Mean [MeV]	Elec. Unc.	PDF Unc.	Muon Unc.	EW Unc.	PS & A_i Unc.	Bkg. Unc.	Γ_W Unc.	MC stat. Unc.	Lumi Unc.	Recoil Unc.	Total sys.	Data stat.	Total Unc.
p_T^ℓ	80360.1	8.0	7.7	7.0	6.0	4.7	2.4	2.0	1.9	1.2	0.6	15.5	4.9	16.3
m_T	80382.2	9.2	14.6	9.8	5.9	10.3	6.0	7.0	2.4	1.8	11.7	24.4	6.7	25.3

CDF, Science 376, 170-176 (2022)

Source of systematic uncertainty	m_T fit			p_T^ℓ fit			p_T^ν fit		
	Electrons	Muons	Common	Electrons	Muons	Common	Electrons	Muons	Common
Lepton energy scale	5.8	2.1	1.8	5.8	2.1	1.8	5.8	2.1	1.8
Lepton energy resolution	0.9	0.3	-0.3	0.9	0.3	-0.3	0.9	0.3	-0.3
Recoil energy scale	1.8	1.8	1.8	3.5	3.5	3.5	0.7	0.7	0.7
Recoil energy resolution	1.8	1.8	1.8	3.6	3.6	3.6	5.2	5.2	5.2
Lepton $u_{ }$ efficiency	0.5	0.5	0	1.3	1.0	0	2.6	2.1	0
Lepton removal	1.0	1.7	0	0	0	0	2.0	3.4	0
Backgrounds	2.6	3.9	0	6.6	6.4	0	6.4	6.8	0
p_T^Z model	0.7	0.7	0.7	2.3	2.3	2.3	0.9	0.9	0.9
p_T^W/p_T^Z model	0.8	0.8	0.8	2.3	2.3	2.3	0.9	0.9	0.9
Parton distributions	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9
QED radiation	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7
Statistical	10.3	9.2	0	10.7	9.6	0	14.5	13.1	0
Total	13.5	11.8	5.8	16.0	14.1	7.9	18.8	17.1	7.4

LHCb, JHEP 01 (2022) 036

Source	Size (MeV)
Parton distribution functions	9
Total theoretical syst. uncertainty (excluding PDFs)	17
Transverse momentum model	11
Angular coefficients	10
QED FSR model	7
Additional electroweak corrections	5
Total experimental syst. uncertainty	10
Momentum scale and resolution modelling	7
Muon ID, tracking and trigger efficiencies	6
Isolation efficiency	4
QCD background	2
Statistical	23
Total uncertainty	32

Each uncertainty item has been studied propagating the associated variations to the templates and in turn to m_W ;
important role of correlations (e.g. for PDFs)

Breakdown of modelling uncertainties

ATLAS-CONF-2023-004

Obs.	Mean [MeV]	Elec. Unc.	PDF Unc.	Muon Unc.	EW Unc.	PS & A_i Unc.	Bkg. Unc.	Γ_W Unc.	MC stat. Unc.	Lumi Unc.	Recoil Unc.	Total sys.	Data stat.	Total Unc.
p_T^ℓ	80360.1	8.0	7.7	7.0	6.0	4.7	2.4	2.0	1.9	1.2	0.6	15.5	4.9	16.3
m_T	80382.2	9.2	14.6	9.8	5.9	10.3	6.0	7.0	2.4	1.8	11.7	24.4	6.7	25.3

CDF, Science 376, 170-176 (2022)

Source of systematic uncertainty	m_T fit			p_T^ℓ fit			p_T^ν fit		
	Electrons	Muons	Common	Electrons	Muons	Common	Electrons	Muons	Common
Lepton energy scale	5.8	2.1	1.8	5.8	2.1	1.8	5.8	2.1	1.8
Lepton energy resolution	0.9	0.3	-0.3	0.9	0.3	-0.3	0.9	0.3	-0.3
Recoil energy scale	1.8	1.8	1.8	3.5	3.5	3.5	0.7	0.7	0.7
Recoil energy resolution	1.8	1.8	1.8	3.6	3.6	3.6	5.2	5.2	5.2
Lepton $u_{ }$ efficiency	0.5	0.5	0	1.3	1.0	0	2.6	2.1	0
Lepton removal	1.0	1.7	0	0	0	0	2.0	3.4	0
Backgrounds	2.6	3.9	0	6.6	6.4	0	6.4	6.8	0
p_T^Z model	0.7	0.7	0.7	2.3	2.3	2.3	0.9	0.9	0.9
p_T^W/p_T^Z model	0.8	0.8	0.8	2.3	2.3	2.3	0.9	0.9	0.9
Parton distributions	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9
QED radiation	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7
Statistical	10.3	9.2	0	10.7	9.6	0	14.5	13.1	0
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LHCb, JHEP 01 (2022) 036

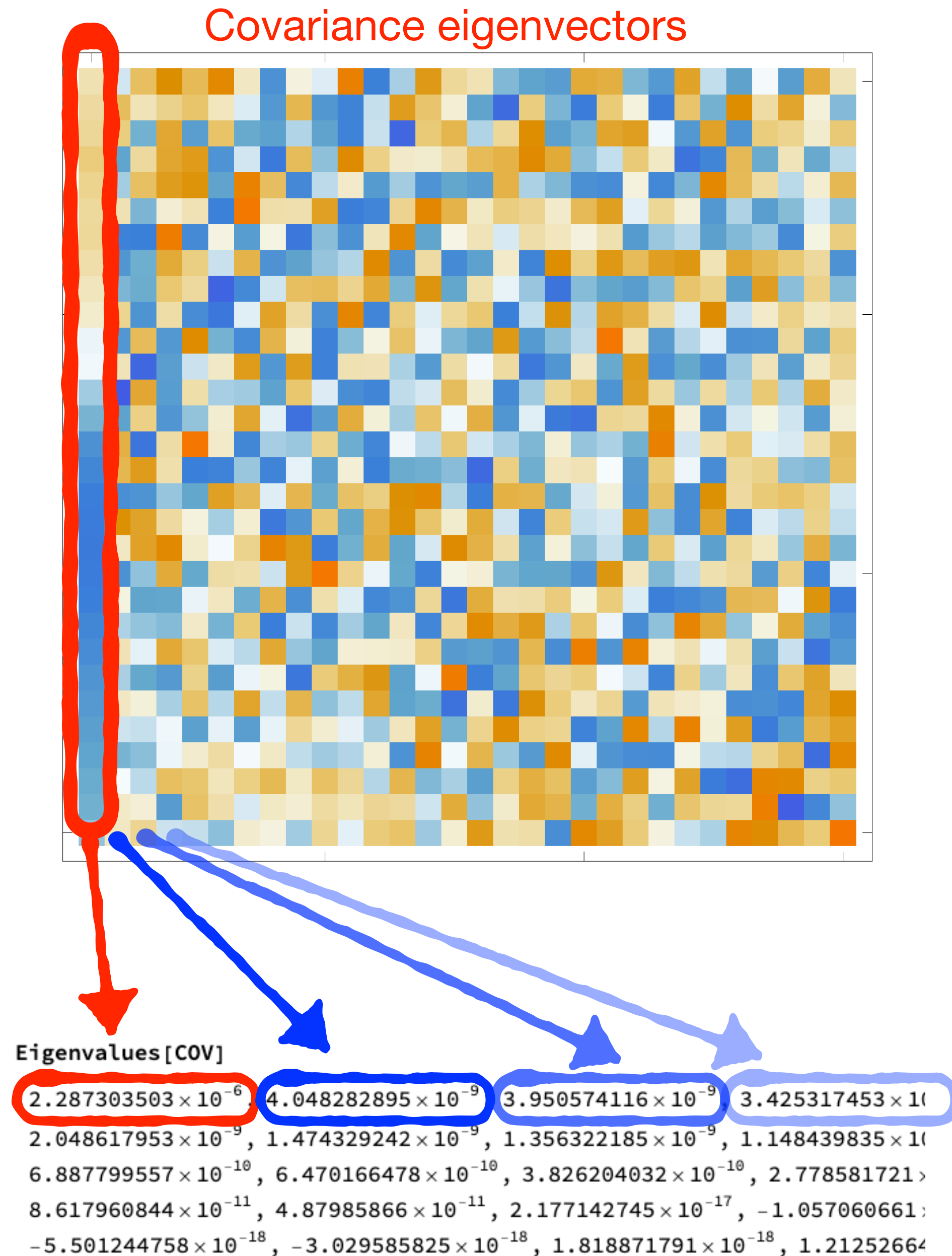
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Muon ID, tracking and trigger efficiencies	6
Isolation efficiency	4
QCD background	2
Statistical	23
Total uncertainty	32

Can we understand these values for the QCD uncertainties in a more general way ?
How much do they depend on the details of the data-driven approach ?

MW from a jacobian asymmetry

L.Rottoli, P.Torrielli, AV, [arXiv:2301.04059](https://arxiv.org/abs/2301.04059)

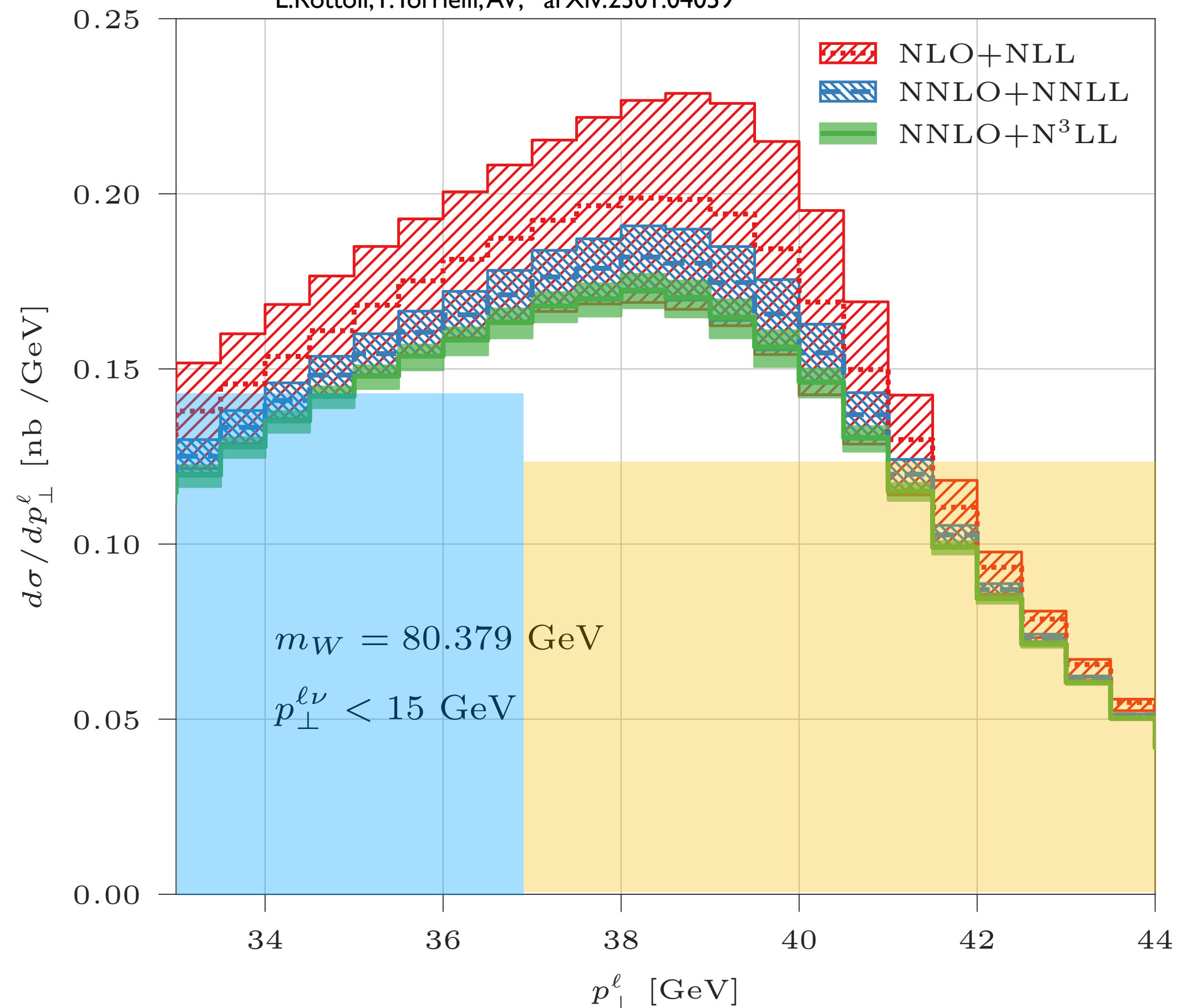
Sensitivity to the W boson mass: covariance with respect to m_W variations



- The p_{\perp}^{ℓ} spectrum includes N bins.
- After the rotation which diagonalises the m_W covariance, we have N linear combinations of the primary bins independent of each other under m_W variations.
- The combination associated to the (by far) largest eigenvalue i.e. the combination most sensitive to m_W exhibits a very clear and simple pattern:
two regions where the coefficients have similar sizes and constant sign
- The point where the coefficients change sign is very stable at different orders in QCD and with different bin ranges and it is found at $p_{\perp}^{\ell} \sim 37$ GeV

The jacobian asymmetry $\mathcal{A}_{p_\perp^\ell}$

L.Rottoli, P.Torrielli, AV; arXiv:2301.04059



$$L_{p_\perp^\ell} \equiv \int_{p_\perp^{\ell, \min}}^{p_\perp^{\ell, \text{mid}}} dp_\perp^\ell \frac{d\sigma}{dp_\perp^\ell},$$

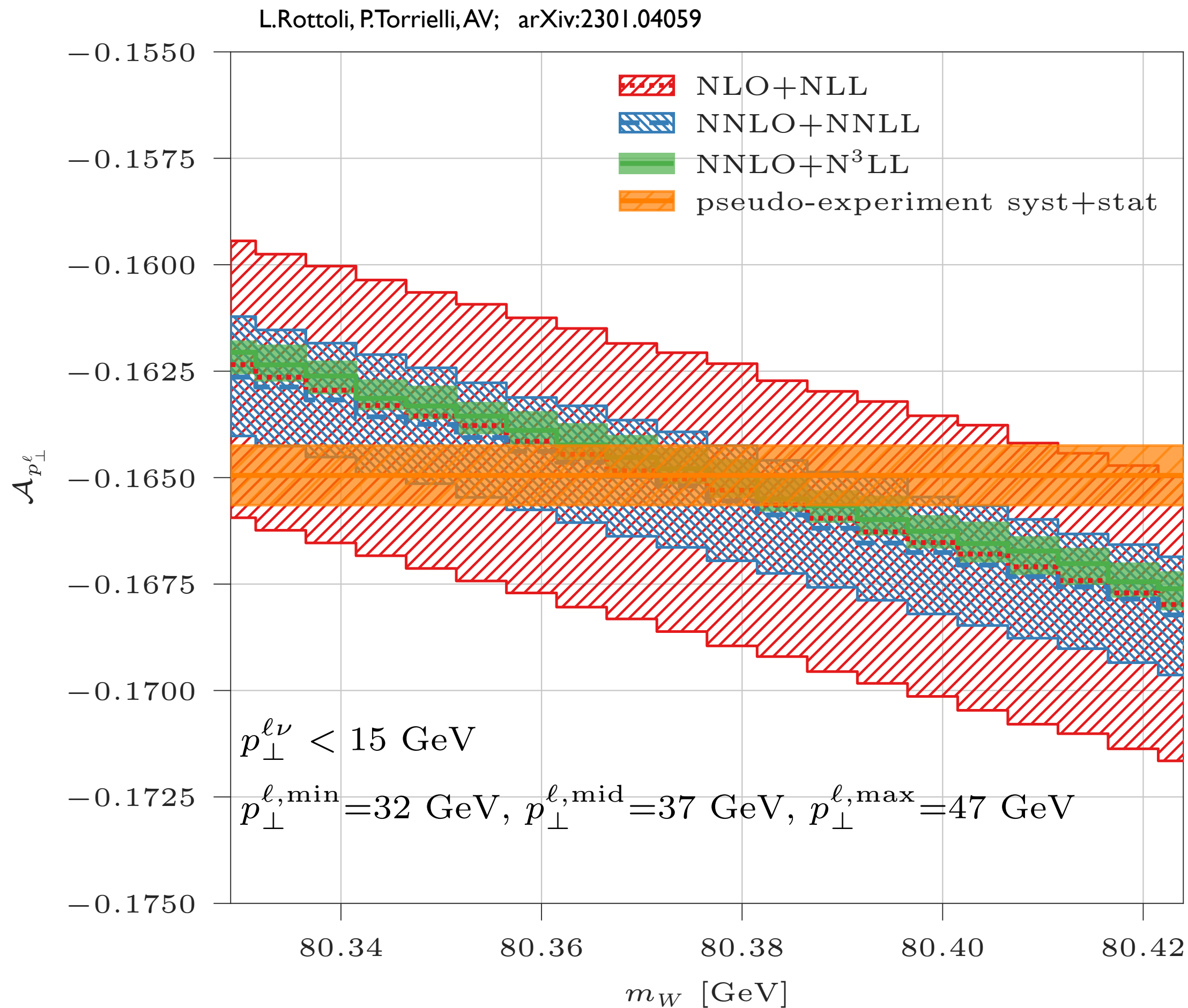
$$U_{p_\perp^\ell} \equiv \int_{p_\perp^{\ell, \text{mid}}}^{p_\perp^{\ell, \max}} dp_\perp^\ell \frac{d\sigma}{dp_\perp^\ell}$$

$$\mathcal{A}_{p_\perp^\ell}(p_\perp^{\ell, \min}, p_\perp^{\ell, \text{mid}}, p_\perp^{\ell, \max}) \equiv \frac{L_{p_\perp^\ell} - U_{p_\perp^\ell}}{L_{p_\perp^\ell} + U_{p_\perp^\ell}}$$

The asymmetry is an observable (i.e. it is measurable via counting): its value is one single scalar number
It depends only on the edges of the two defining bins

Increasing m_W shifts the position of the peak to the right → Events migrate from the blue to the orange bin
→ The asymmetry decreases

The jacobian asymmetry $\mathcal{A}_{p_\perp^\ell}$ as a function of m_W



The asymmetry \mathcal{A}_{p_\perp} has a linear dependence on m_W ,
stemming from the linear dependence on the end-point position

The slope of the asymmetry expresses the sensitivity to m_W ,
in a given setup $(p_\perp^{\ell,\min}, p_\perp^{\ell,\text{mid}}, p_\perp^{\ell,\max})$

The slope is the same with every QCD approximation
(factorization of QCD effects, perturbative and non-perturbative)

The “large” size of the two bins $\mathcal{O}(5 - 10)$ GeV leads to

- small statistical errors
- excellent stability of the QCD results (inclusive quantity)
- ease to unfold the data to particle level (m_W combination)

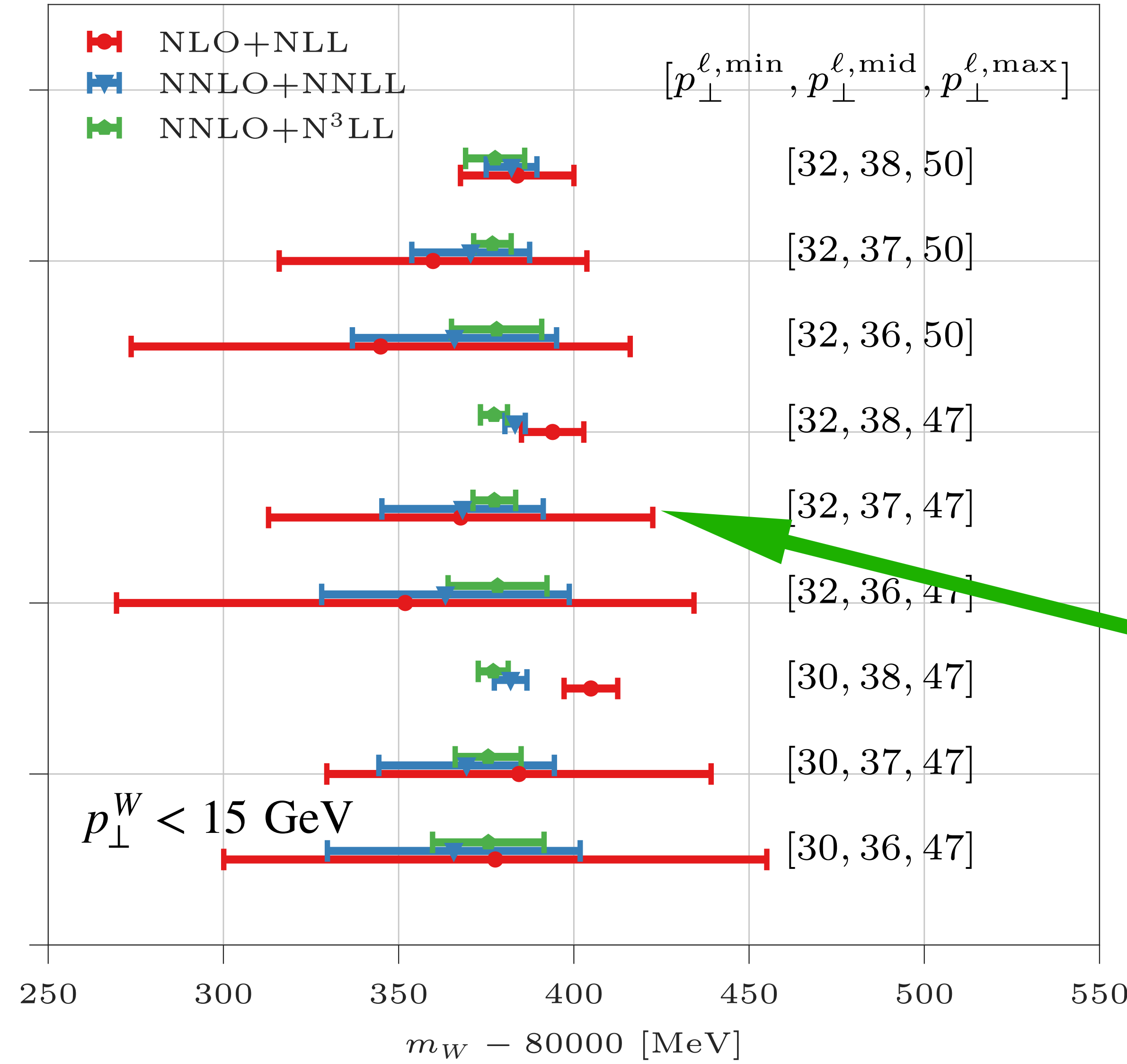
The experimental value and the theoretical predictions can be directly compared (m_W from the intersection of two lines)

The theoretical uncertainty can be directly read
from the two intersections of the experimental line (orange) with a theoretical uncertainty band

m_W determination as a function of the $\mathcal{A}_{p_\perp^\ell}$ parameters

as pseudo-experimental value we choose the NNLO+N3LL result with $m_W = 80.379$

L.Rottoli, P.Torrielli, AV; arXiv:2301.04059



Each QCD scale-variations band determines an m_W interval (intersection with the central experimental line)

We first check the convergence order-by-order.
If we observe it, then we take the size of the m_W interval as estimator of the residual pQCD uncertainty

We do not trust the scale variations alone
→ cfr the choice with $p_\perp^{\ell, \text{mid}} = 38$ GeV

A pQCD uncertainty at the ± 5 MeV level is achievable based on CCDY data alone

This uncertainties on m_W can be estimated in a purely pQCD framework
without the need of a data-driven approach

It is evident that at NNLL-QCD the typical size of the QCD uncertainties is $\mathcal{O}(\pm 20 \text{ MeV})$

Important role of the N3LL corrections

Information transfer from NCDY to CCDY : a validation exercise

- NNLO+N3LL with central scales $\mu_R = \mu_F = \mu_Q = 1$ is our MC truth = pseudodata both for NCDY and CCDY
- we take NNLO+NNLL as theory model
- for **different scale choices** we compute the reweighing functions **from** NNLO+NNLL **to** the p_\perp^Z pseudodata

$$\mathcal{R}(\mu_R, \mu_F, \mu_Q; p_\perp^Z) = \left(\frac{d\sigma^{\text{NNLO+N3LL}}(1,1,1)}{dp_\perp^Z} \right) \left(\frac{d\sigma^{\text{NNLO+NNLL}}(\mu_R, \mu_F, \mu_Q)}{dp_\perp^Z} \right)^{-1}$$

- we then use the appropriate reweighing function in CCDY at NNLO+NNLL for **each different scale choice**

$$\frac{d\sigma^{\text{NNLO+NNLL-rwg}}(\mu_R, \mu_F, \mu_Q)}{dp_\perp^W} = \mathcal{R}(\mu_R, \mu_F, \mu_Q; p_\perp^W) \frac{d\sigma^{\text{NNLO+NNLL}}(\mu_R, \mu_F, \mu_Q)}{dp_\perp^W}$$

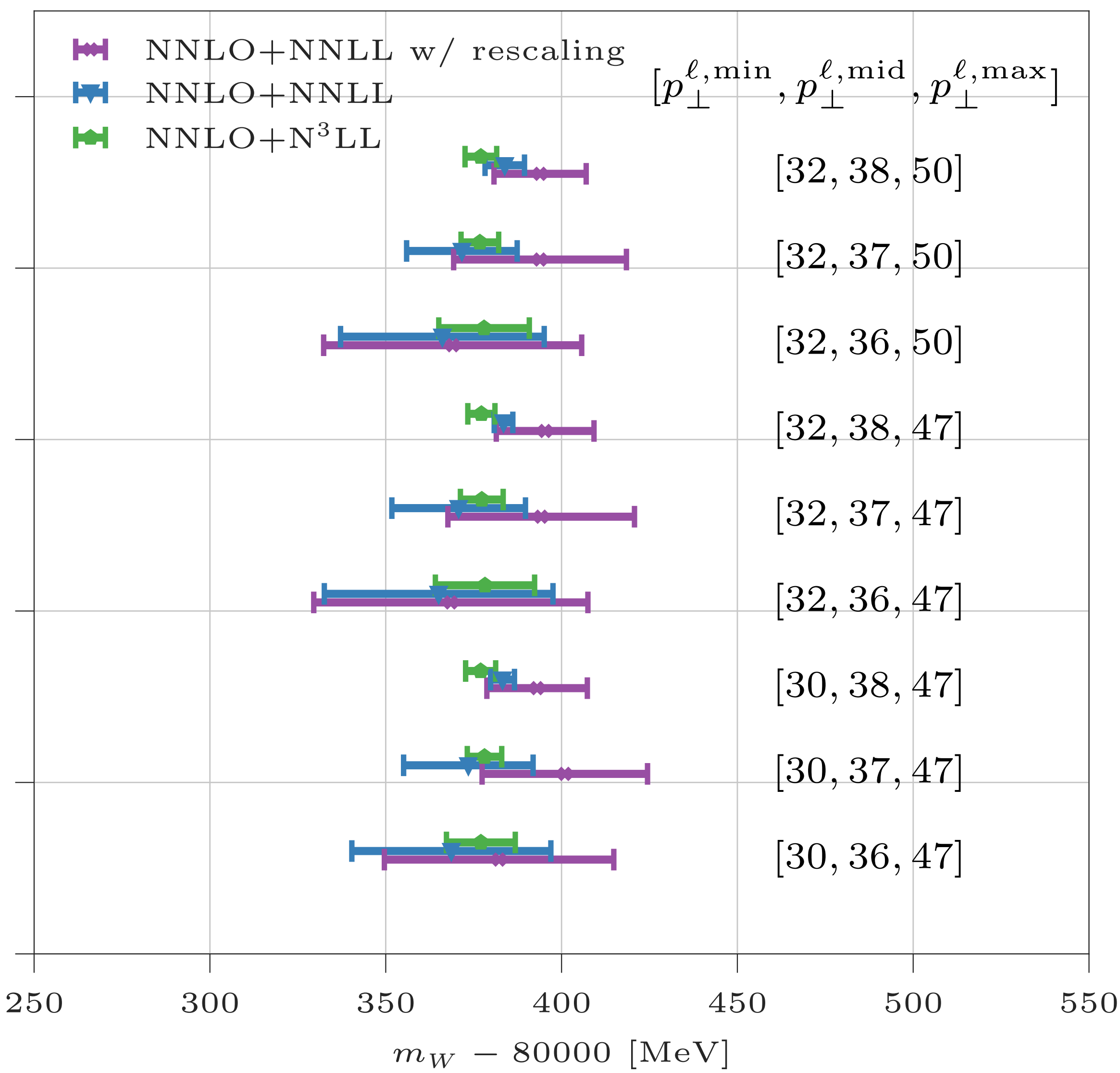
- we compare the reweighed results and the CCDY pseudodata and study the residual scale dependence

$$\frac{d\sigma^{\text{NNLO+NNLL-rwg}}(\mu_R, \mu_F, \mu_Q)}{dp_\perp^W} \leftrightarrow \frac{d\sigma^{\text{NNLO+N3LL}}(1,1,1)}{dp_\perp^W}$$

- naive expectation: since by construction all the scale choices match the p_\perp^Z pseudodata, then also in CC-DY we should find the same (i.e. no scale dependence)

Information transfer from NCDY to CCDY : a validation exercise

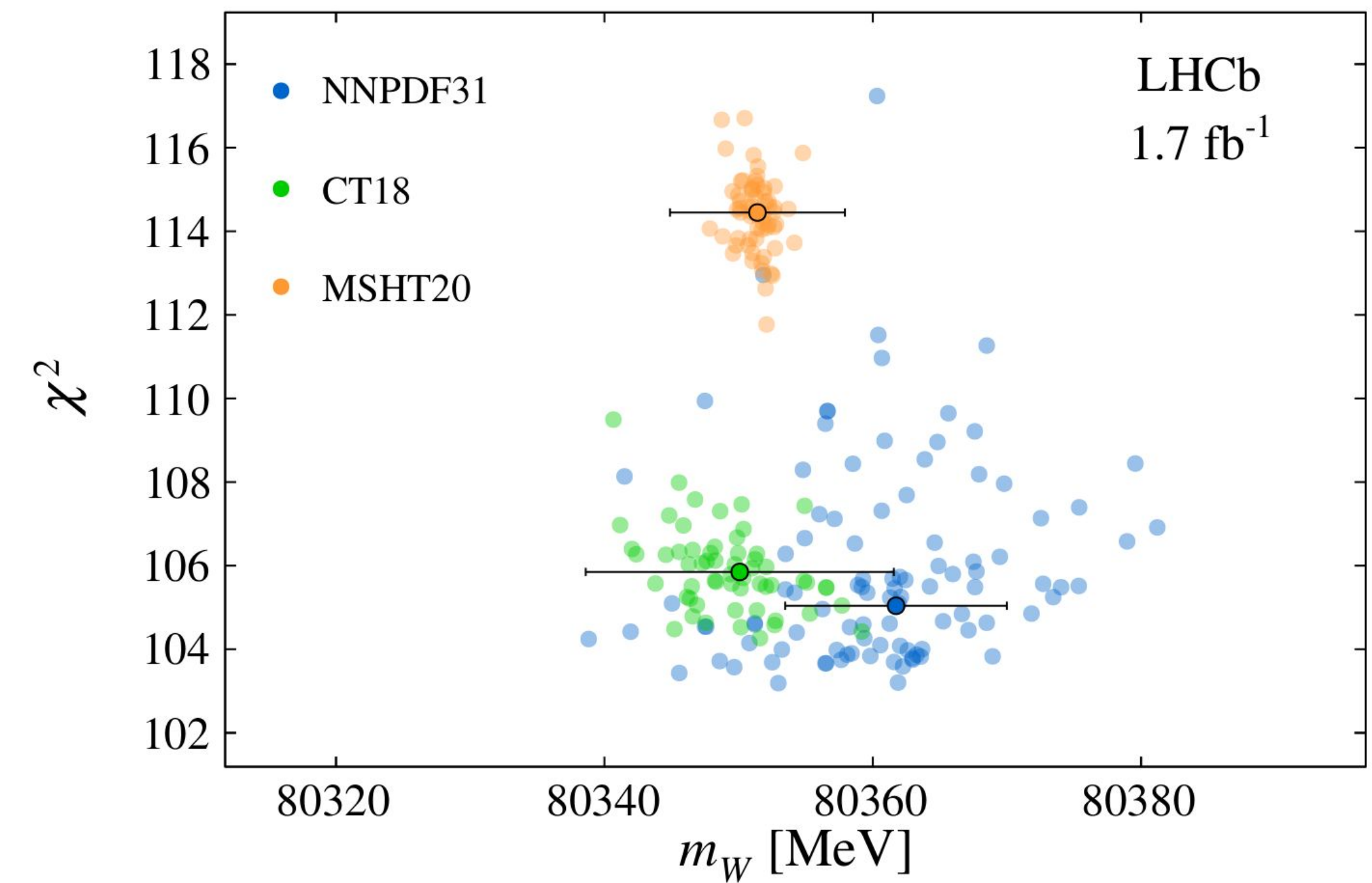
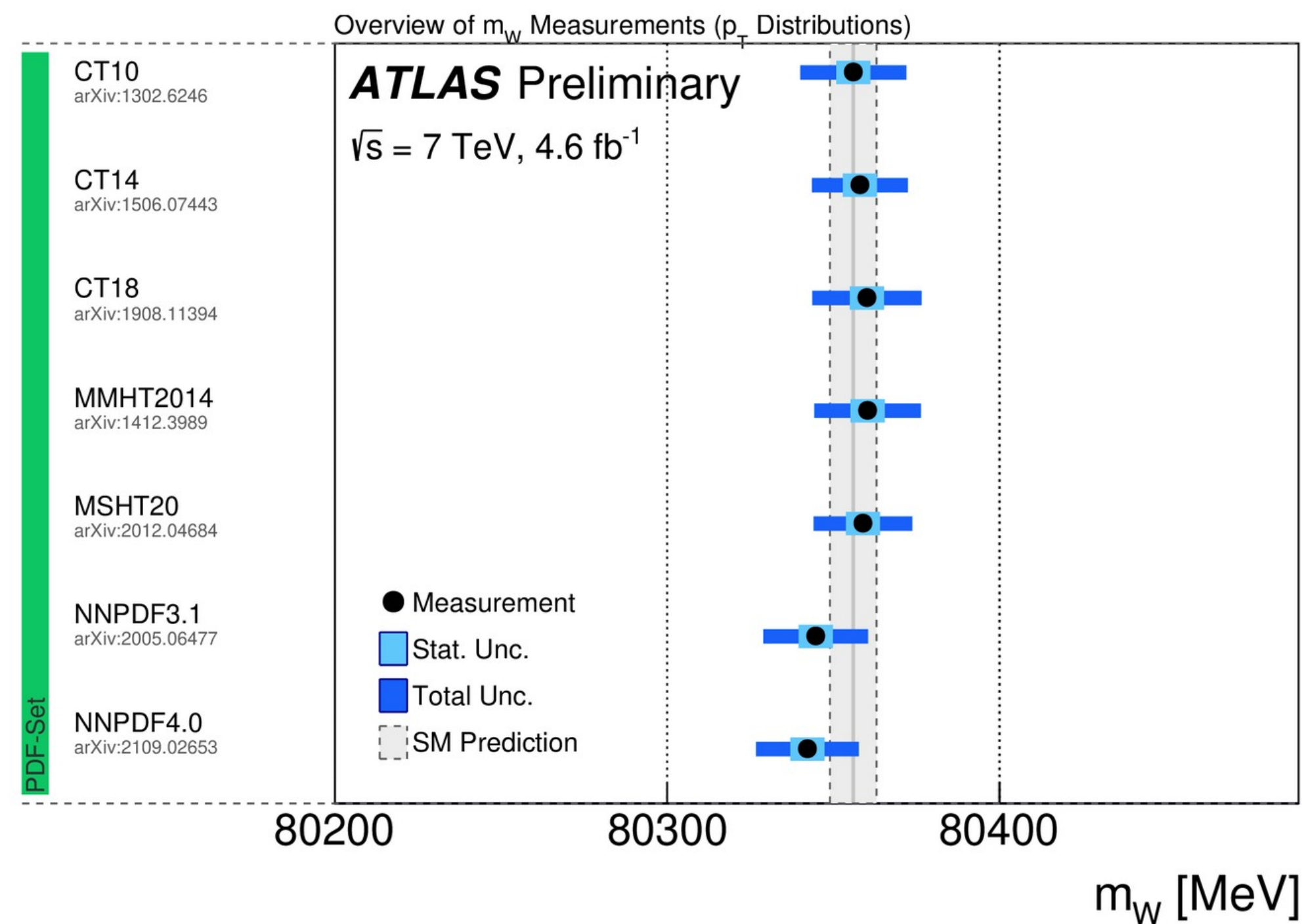
L.Rottoli, P.Torrielli,AV; arXiv:2301.04059



- we determine m_W using the three sets of distributions:
 - plain NNLO+NNLL
 - reweighed NNLO+NNLL
 - NNLO+N³LL
- the pQCD uncertainty on m_W estimated **with** or **without** reweighing is of similar size (in our case the **NNLO+NNLL QCD** uncertainty)
- the usage of the p_{\perp}^Z information improves the **accuracy** of the data description does **not** improve the **precision** of the fitting model
- usage of the **highest available perturbative order** is recommended to minimise the pQCD systematics in the transfer from Z to W

PDF uncertainty on MW: realistic estimates

[JHEP 01 (2022) 036], [LHCB-PAPER-2021-024]



ATLAS has chosen to use a profile likelihood analysis (ATLAS-CONF-2023-004)

Correlation effects within one PDF set are automatically included and profiled (cfr. E.Bagnaschi, AV, Phys.Rev.Lett. 126 (2021) 4, 041801)

LHCb is considering in a more conservative way the impact of different replica choices

The combination of ATLAS and LHCb results can lead to a reduction of the total PDF uncertainty (anti correlations)

PDF uncertainty on MW: exploiting the theoretical constraints

E.Bagnaschi, AV, Phys.Rev.Lett. 126 (2021) 4, 041801

all PDF replicas are correlated because the parton densities are developed in the same QCD framework

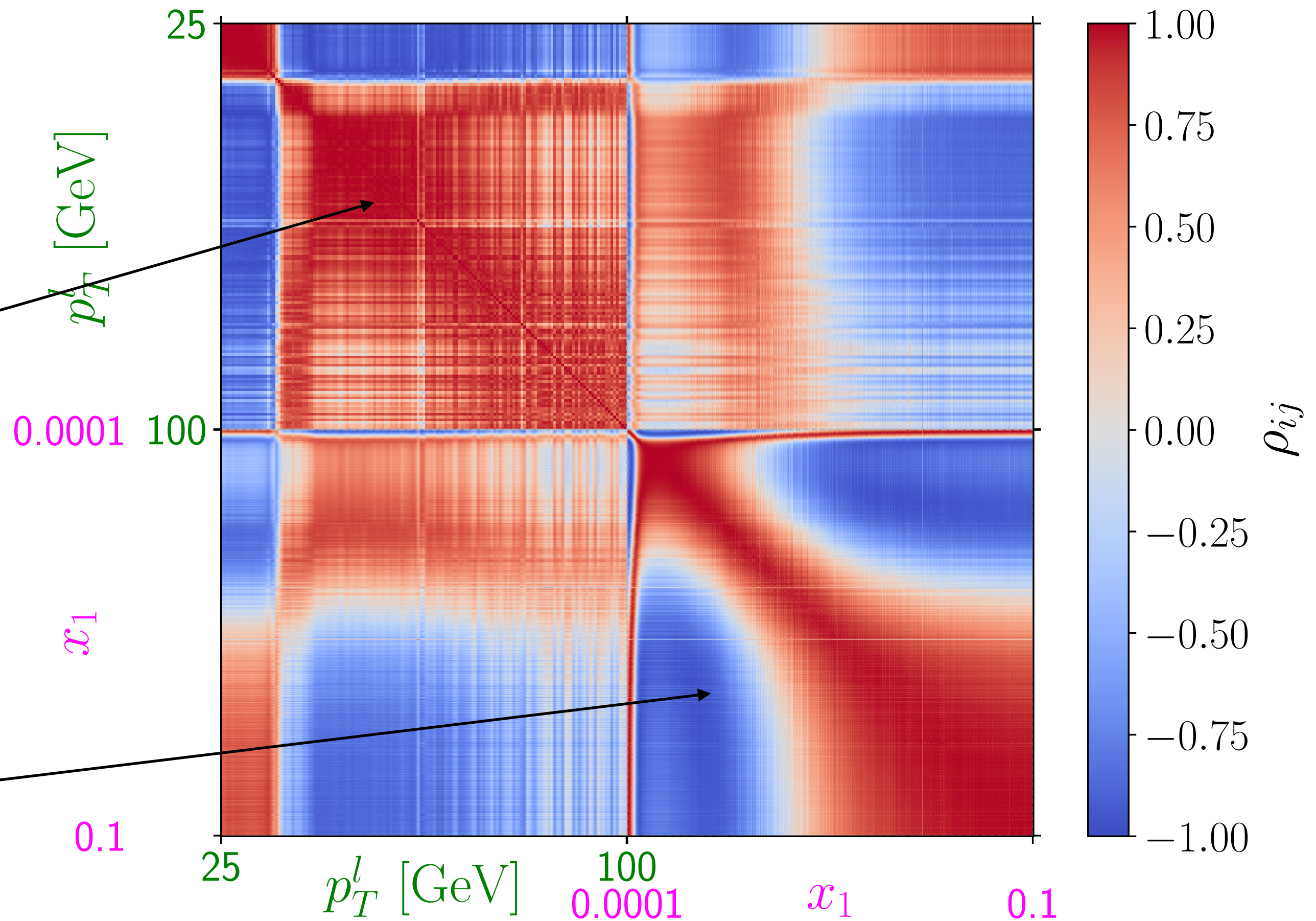
1) obey sum rules, 2) satisfy DGLAP equations, 3) are based on the same data set

the “unitarity constraint” of each parton density affects the parton-parton luminosities, which, convoluted with the partonic xsec, in turn affect the hadron-level xsec

$$\rho_{ij} = \frac{\langle (\mathcal{O}_i - \langle \mathcal{O}_i \rangle_{PDF}) (\mathcal{O}_j - \langle \mathcal{O}_j \rangle_{PDF}) \rangle_{PDF}}{\sigma_i \sigma_j}$$

the tails of the $\frac{d\sigma}{dp_{\perp}^{\ell}}$ distribution
are strongly (anti)-correlated w.r.t. PDF variations

the tails of the $\frac{d\sigma}{dx}$ distribution
are strongly (anti)-correlated w.r.t. PDF variations

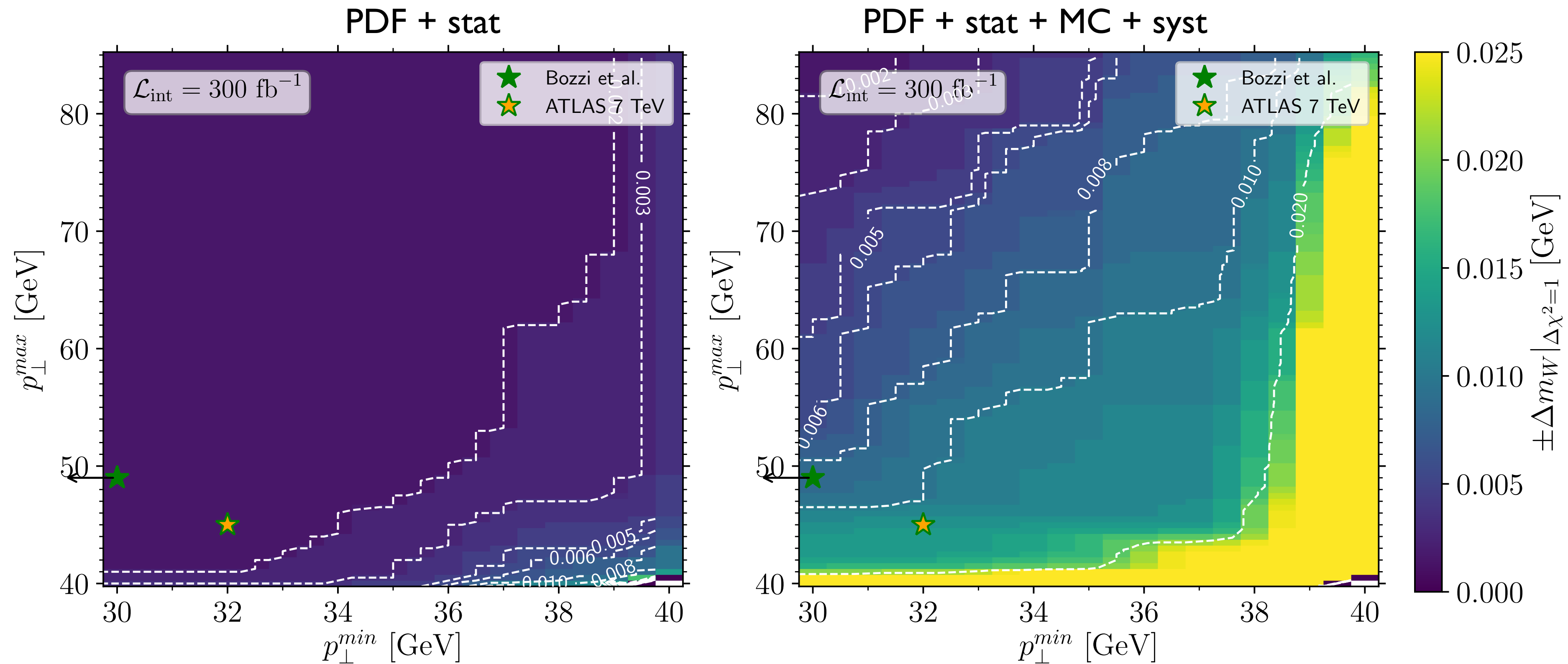


The uncertainty of PDF origin can be reduced to the few MeV level

PDF uncertainty on MW: exploiting the theoretical constraints

E.Bagnaschi, AV, Phys.Rev.Lett. 126 (2021) 4, 041801

scan over fitting windows for normalised distributions



$$\chi_{k,min}^2 = \sum_{r,s \in \text{bins}} \left(\mathcal{T}_{0,k} - \mathcal{D}^{\text{exp}} \right)_r C_{rs}^{-1} \left(\mathcal{T}_{0,k} - \mathcal{D}^{\text{exp}} \right)_s$$

$$C = \Sigma_{PDF} + \Sigma_{stat} + \Sigma_{MC} + \Sigma_{exp \text{ syst}}$$

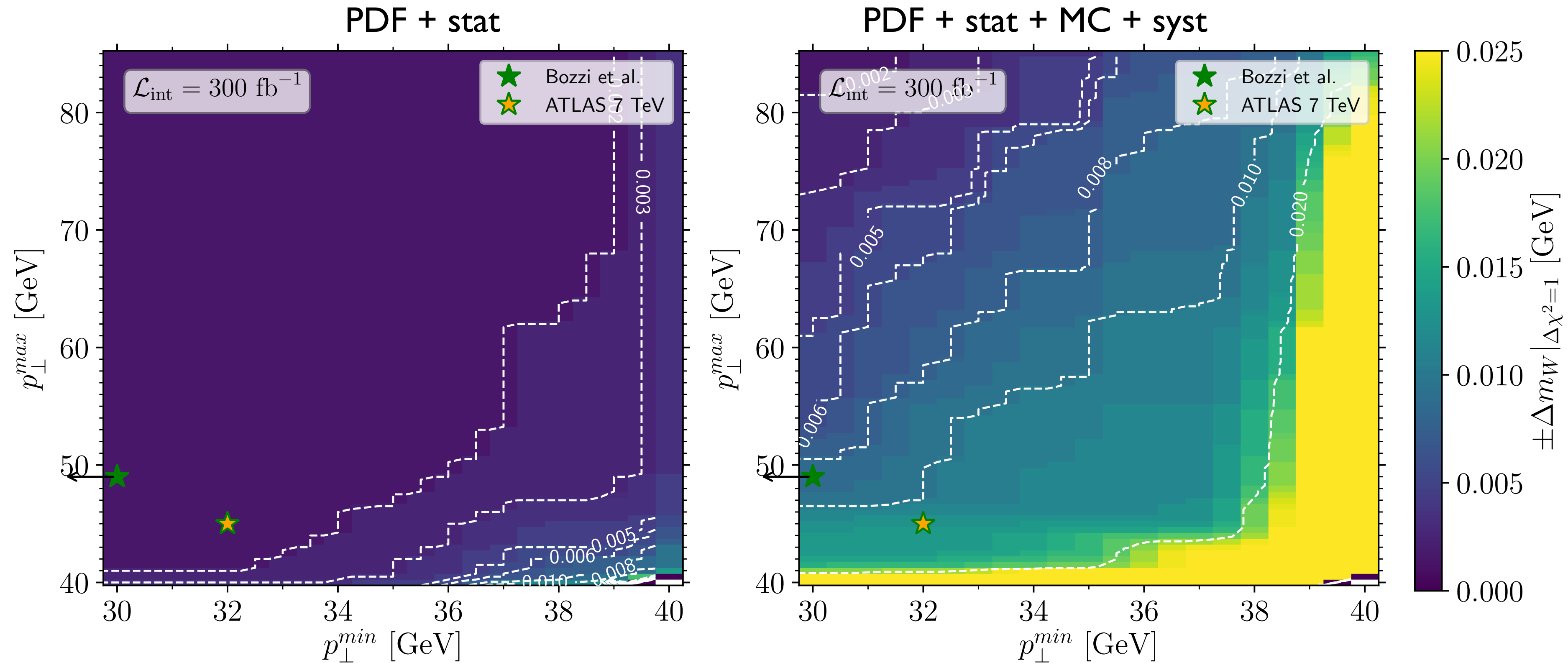
total covariance

total uncertainty determined
with $\Delta \chi^2 = 1$ rule

PDF uncertainty on MW: exploiting the theoretical constraints

E.Bagnaschi, AV, Phys.Rev.Lett. 126 (2021) 4, 041801

scan over fitting windows for normalised distributions



$$\chi_{k,min}^2 = \sum_{r,s \in bins} \left(\mathcal{T}_{0,k} - \mathcal{D}^{exp} \right)_r C_{rs}^{-1} \left(\mathcal{T}_{0,k} - \mathcal{D}^{exp} \right)_s$$

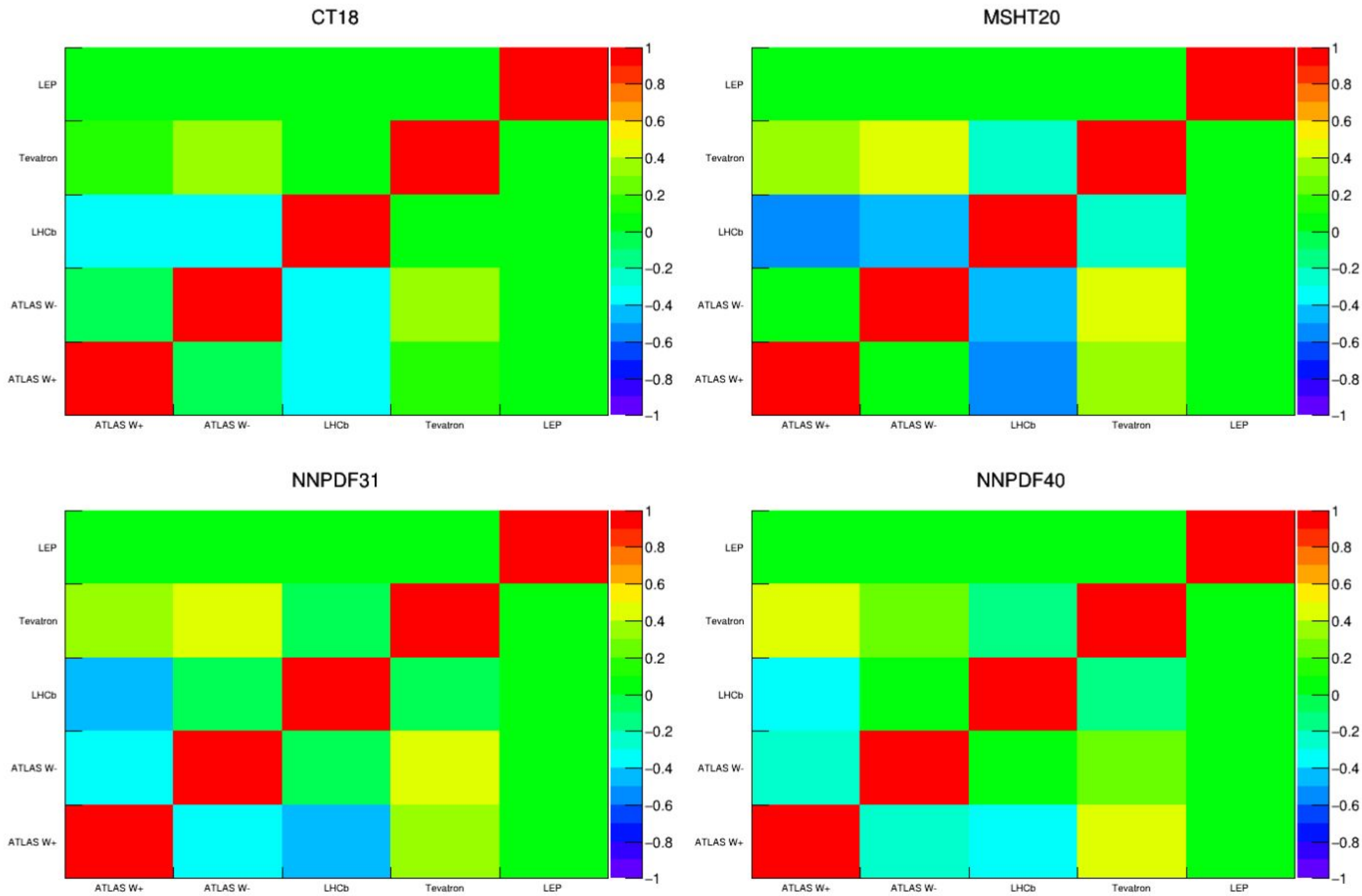
$$C = \Sigma_{PDF} + \Sigma_{stat} + \Sigma_{MC} + \Sigma_{exp syst} \quad \text{total covariance}$$

total uncertainty determined
with $\Delta\chi^2 = 1$ rule

- The PDF uncertainty is **not** a limiting factor for MW with high luminosity and a “perfect” detector
- The MC statistics needed is of at least $O(100B)$ of simulated events (several weeks on 1000 cores cluster)

PDF rapidity correlations

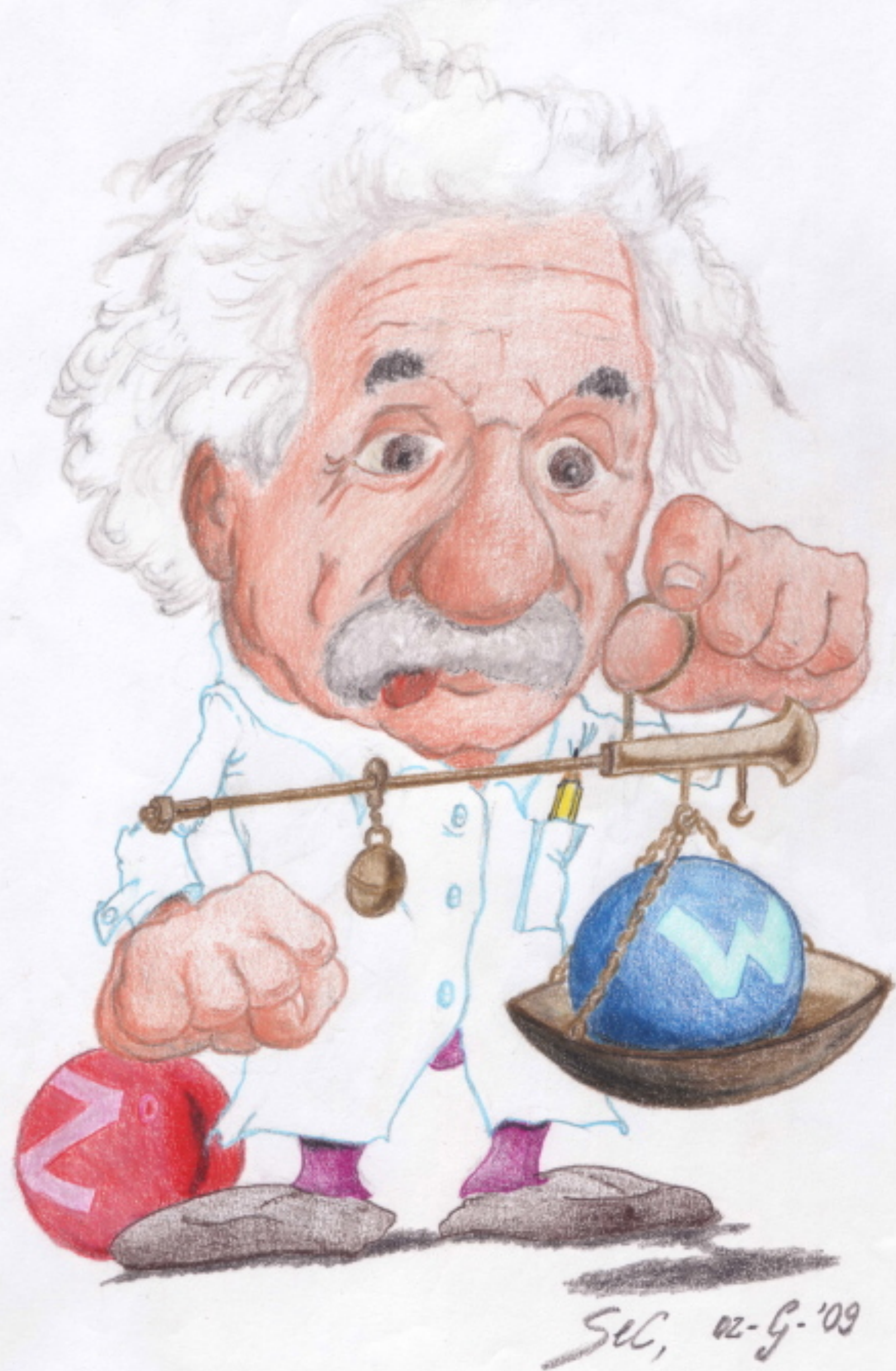
The anticorrelation of the LHCb results helps reducing the total PDF uncertainty



plot from Jan Kretschmar's talk at the EW WG general meeting (November 16th 2022)

Conclusions

- The determination of the W boson mass requires a detailed understanding of all the systematics of experimental and theoretical origin
- The QCD corrections, together with QED FSR, determine the baseline of the template shapes all the other effects have a smaller impact on the central m_W value
- The significance of the result, in the searches for BSM signals, depends on the size of the total error
- pQCD uncertainties have been studied so far relying on the availability of excellent p_{\perp}^Z data; the latter improve the accuracy but not the precision of the result → possible underestimate of these uncertainties
- The $\mathcal{A}_{p_{\perp}^{\ell}}$ asymmetry allows a transparent discussion of the propagation of the pQCD uncertainties to m_W , without the need of a data driven approach, with a clear understanding of the convergence of the perturbative series
- A robust reduction of the pQCD errors can be achieved exploiting the excellent features of the public tools implementing higher-order QCD corrections to DY, up to NNLO+N3LL-QCD accuracy, which have become available in the last few years
- The role of PDF correlations and uncertainties has been carefully discussed in the MW combination WG, the role of EW and mixed QCD-EW effects deserves a renovated scrutiny. (more material in the backup slides)



Thank you

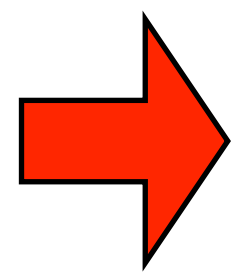
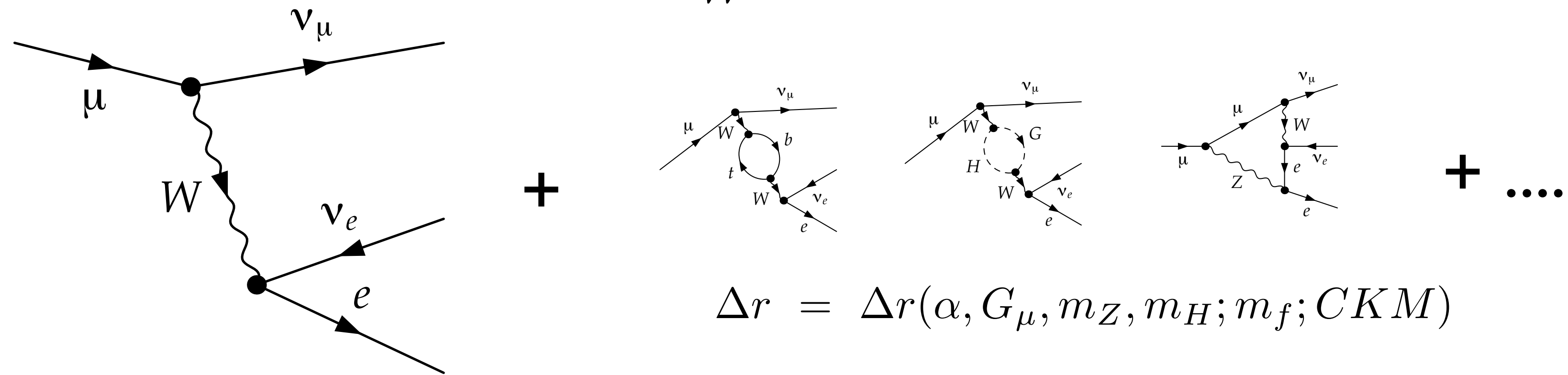
Backup

The W boson mass: theoretical prediction

$$\mathcal{L}_{SM} = \mathcal{L}_{SM}(\alpha, G_\mu, m_Z; m_H; m_f; CKM)$$

→ we can compute m_W

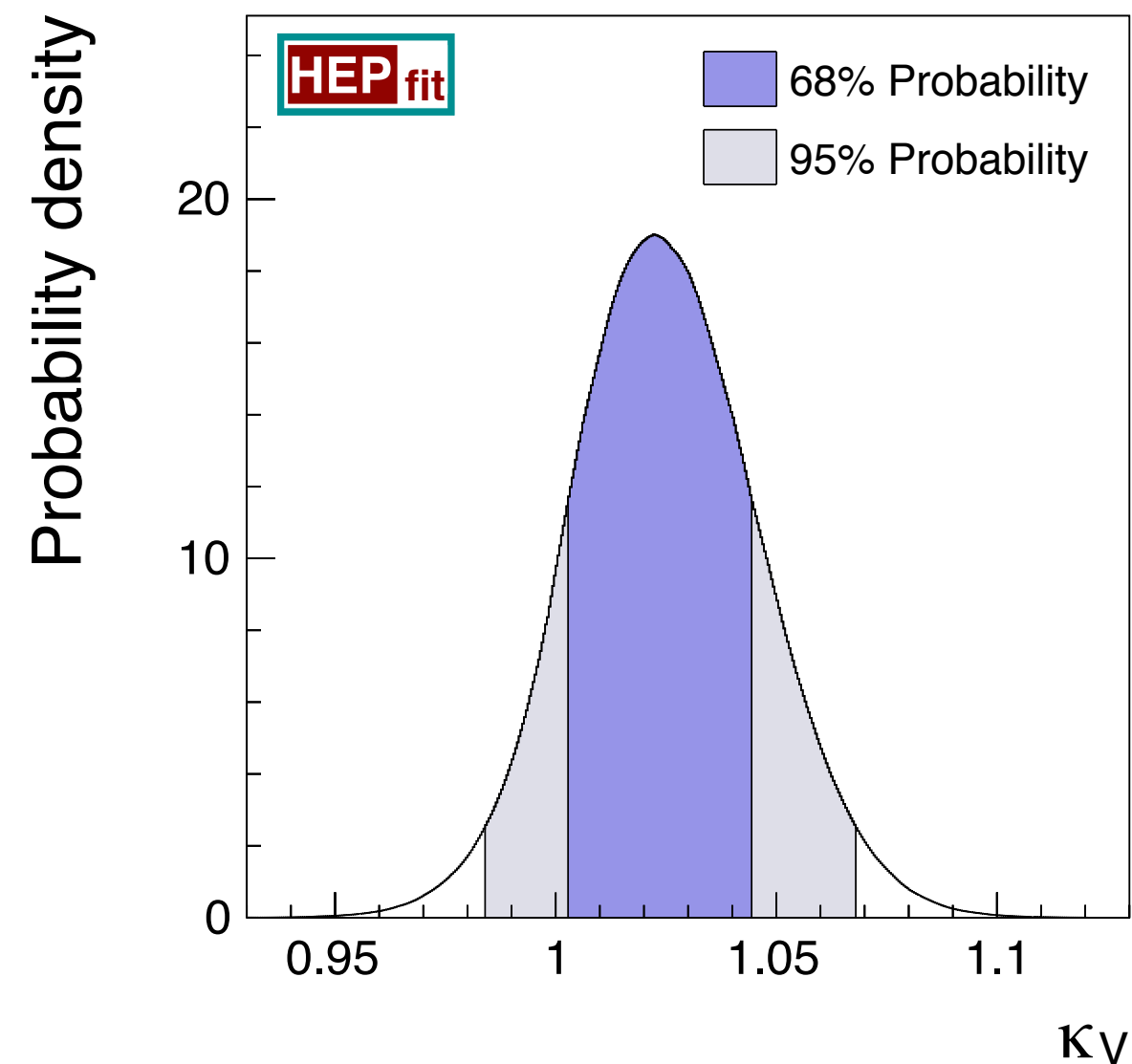
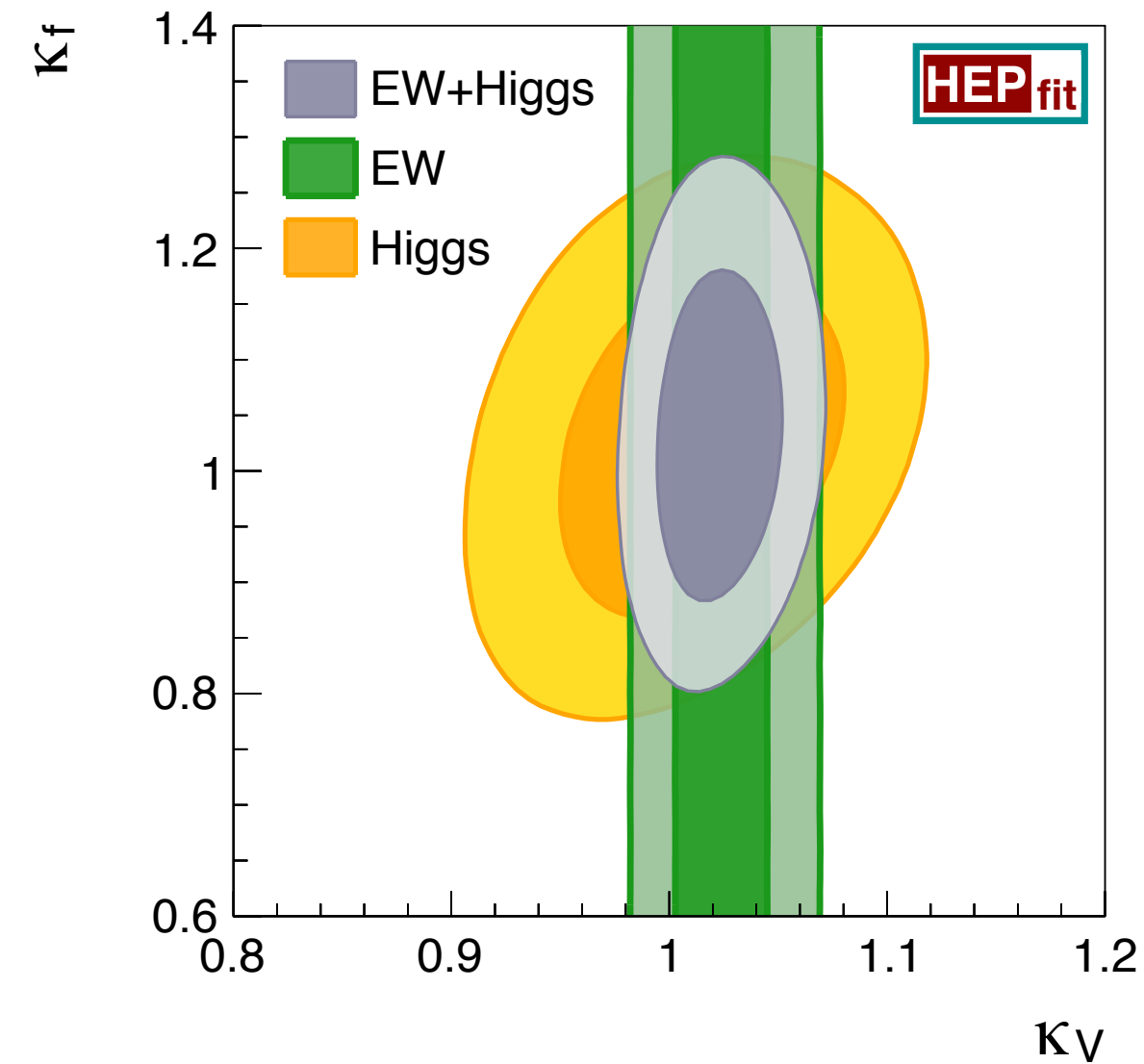
$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8m_W^2} (1 + \Delta r)$$



$$m_W^2 = \frac{m_Z^2}{2} \left(1 + \sqrt{1 - \frac{4\pi\alpha}{G_\mu \sqrt{2} m_Z^2} (1 + \Delta r)} \right)$$

Relevance of new high-precision measurement of EW parameters

de Blas et al, arXiv:1608.01509



$$\mathcal{L}_{\text{Eff}} = \sum_{d=4}^{\infty} \frac{1}{\Lambda^{d-4}} \mathcal{L}_d = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} \mathcal{L}_5 + \frac{1}{\Lambda^2} \mathcal{L}_6 + \dots$$

$$\mathcal{L}_d = \sum_i C_i^d \mathcal{O}_i \quad [\mathcal{O}_i] = d \xrightarrow[\text{Effects suppressed by } q=v, E < \Lambda]{\left(\frac{q}{\Lambda}\right)^{d-4}}$$

Λ : Cut-off of the EFT

$$\mathcal{O}_{\phi WB} = \phi^\dagger \sigma_a \phi B^{\mu\nu} W_{\mu\nu}^a \xrightarrow{\text{EWSB}} \begin{cases} v^2 B^{\mu\nu} W_{\mu\nu}^3 & \text{gauge boson masses} \\ v h B^{\mu\nu} W_{\mu\nu}^3 & h \rightarrow ZZ, \gamma\gamma \end{cases}$$

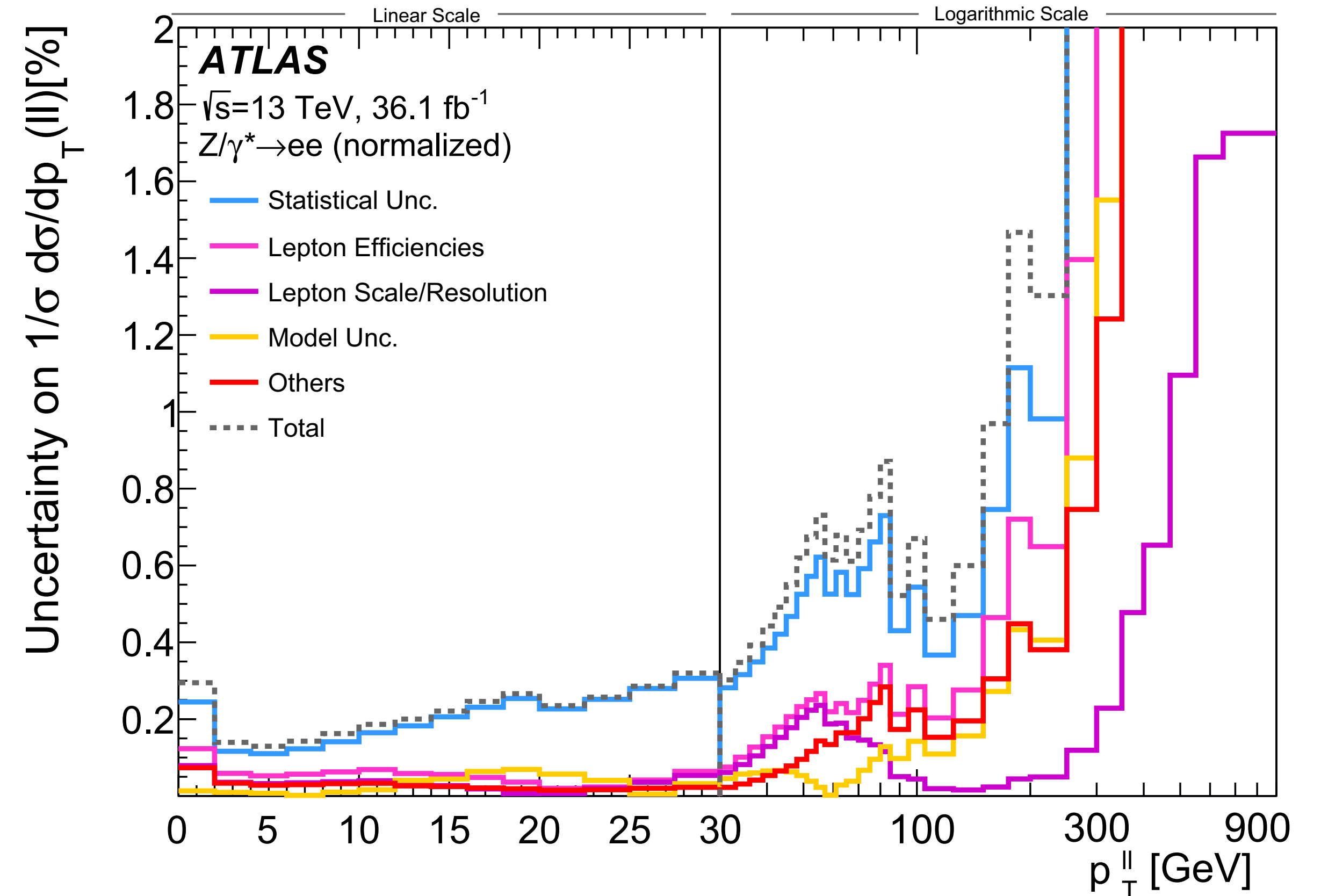
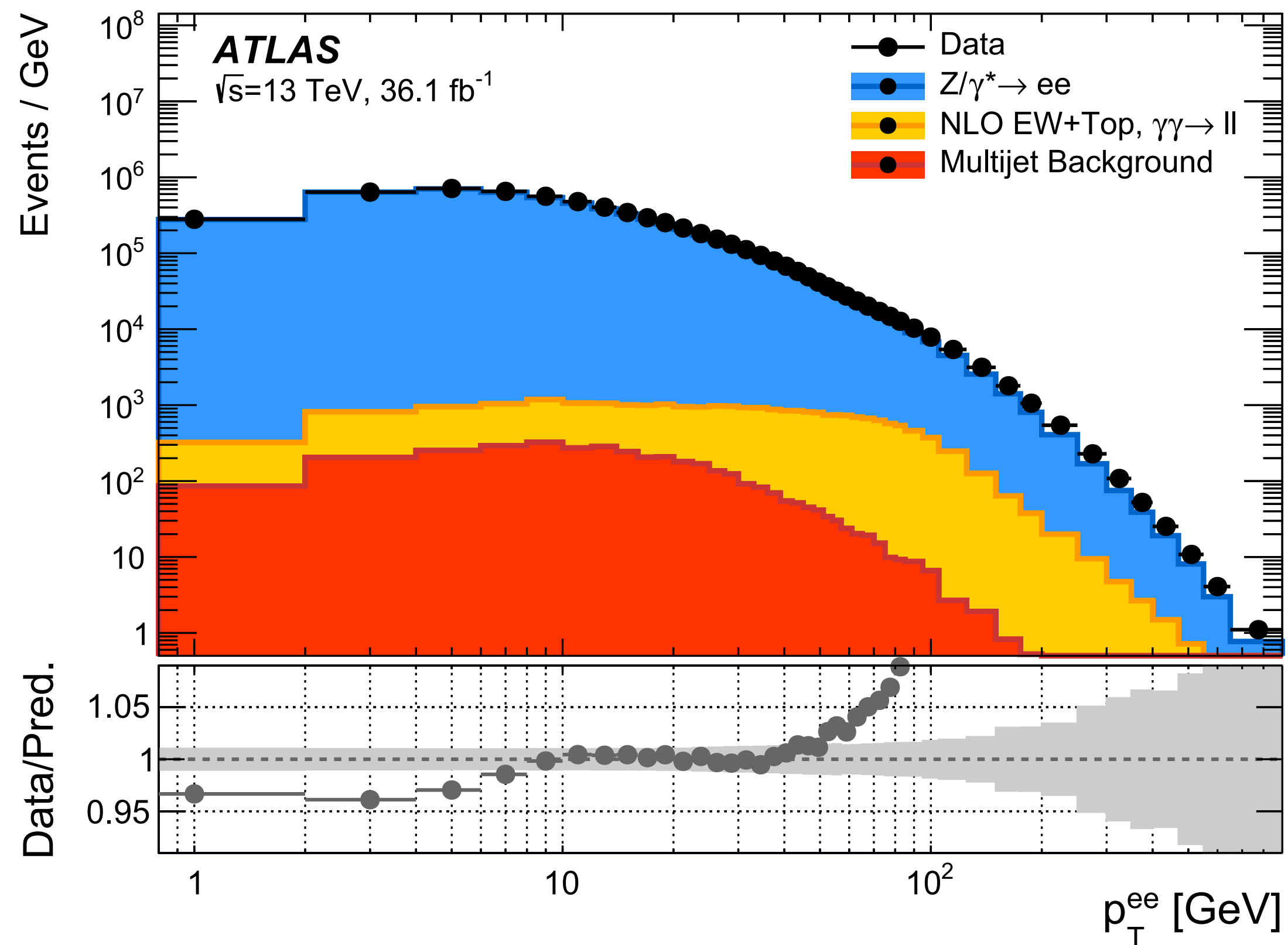
$$M_W^2 = M_Z^2 c^2 \left[1 - \frac{c^2}{c^2 - s^2} \left(\frac{1}{2} C_{\phi D} + 2 \frac{s}{c} C_{\phi WB} + \frac{s^2}{c^2} \Delta_{G_\mu} \right) \frac{v^2}{\Lambda^2} \right]$$

A precise measurement of m_W and $\sin^2 \theta_{\text{eff}}$ constrains several dim-6 operators contributing to Higgs and gauge interaction vertices.

Today still one of the strongest constraints

Lepton-pair transverse momentum distribution

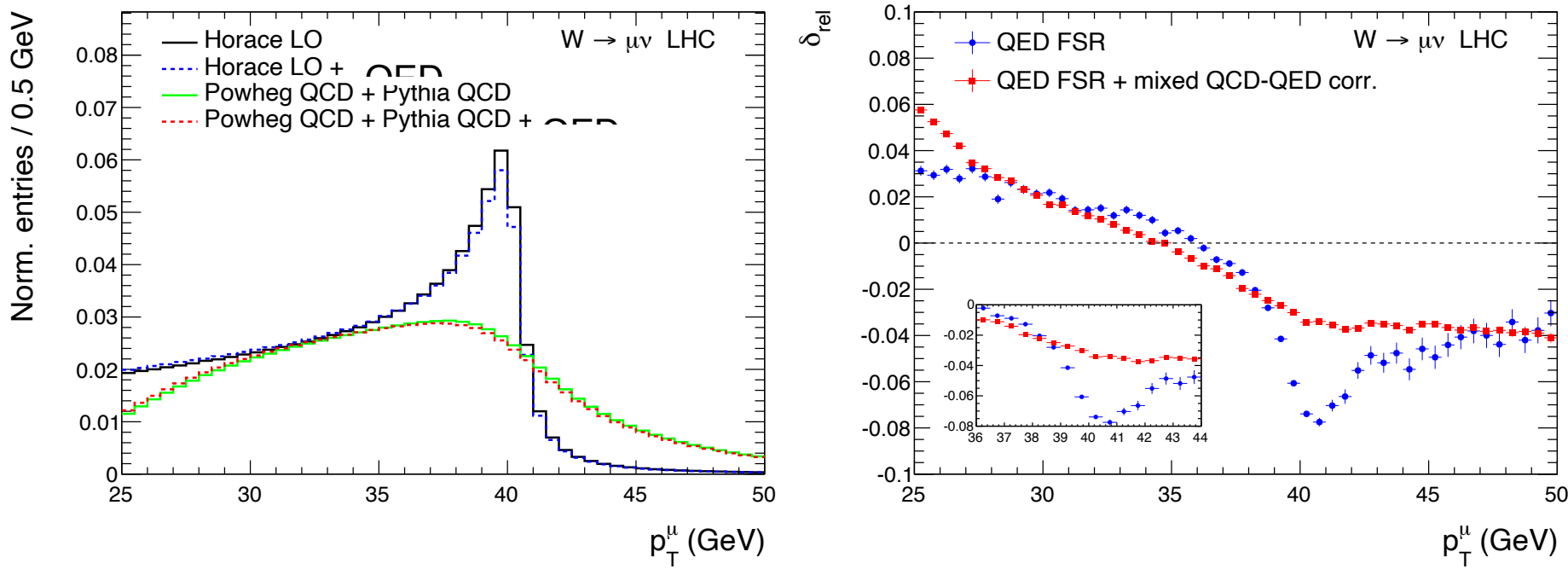
- A crucial role in precision EW measurements (m_W in particular) is played by the p_\perp^Z distribution
 - ▷ m_W is extracted from the fit to the p_\perp^l, M_\perp and E_\perp^{miss} distributions
 - ▷ the p_\perp^l and p_\perp^ν simulation strongly depends on a precise knowledge of the p_\perp^W distribution
 - ▷ a precise p_\perp^W measurement is not yet available → we rely on p_\perp^Z and extrapolate from it
 - ▷ p_\perp^Z is used to calibrate Monte Carlo tools (Parton Shower at low- p_\perp^Z)



Impact of EW and mixed QCDxEW corrections on MW

C.Carloni Calame, M.Chiesa, H.Martinez, G.Montagna, O.Nicrosini, F.Piccinini, AV, arXiv:1612.02841

$pp \rightarrow W^+, \sqrt{s} = 14 \text{ TeV}$		M_W shifts (MeV)			
Templates accuracy: LO		$W^+ \rightarrow \mu^+ \nu$		$W^+ \rightarrow e^+ \nu$	
Pseudo-data accuracy		M_T	p_T^ℓ	M_T	p_T^ℓ
1	HORACE only FSR-LL at $\mathcal{O}(\alpha)$	-94±1	-104±1	-204±1	-230±2
2	HORACE FSR-LL	-89±1	-97±1	-179±1	-195±1
3	HORACE NLO-EW with QED shower	-90±1	-94±1	-177±1	-190±2
4	HORACE FSR-LL + Pairs	-94±1	-102±1	-182±2	-199±1
5	PHOTOS FSR-LL	-92±1	-100±2	-182±1	-199±2

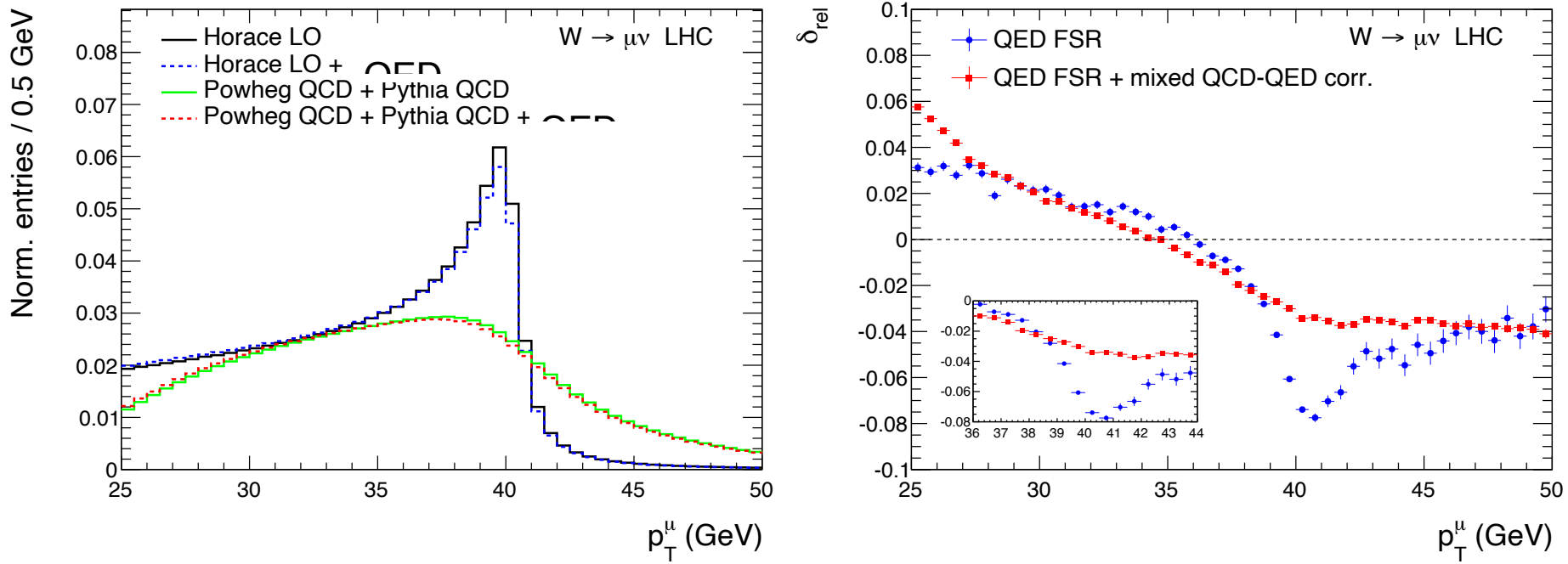


- QED FSR plays the major role
- subleading QED and weak induce further O(4 MeV) shifts

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- QED FSR plays the major role
- subleading QED and weak induce further O(4 MeV) shifts

the impact on MW of the mixed QCD QED-FSR corrections strongly depends on the underlying QCD shape/model

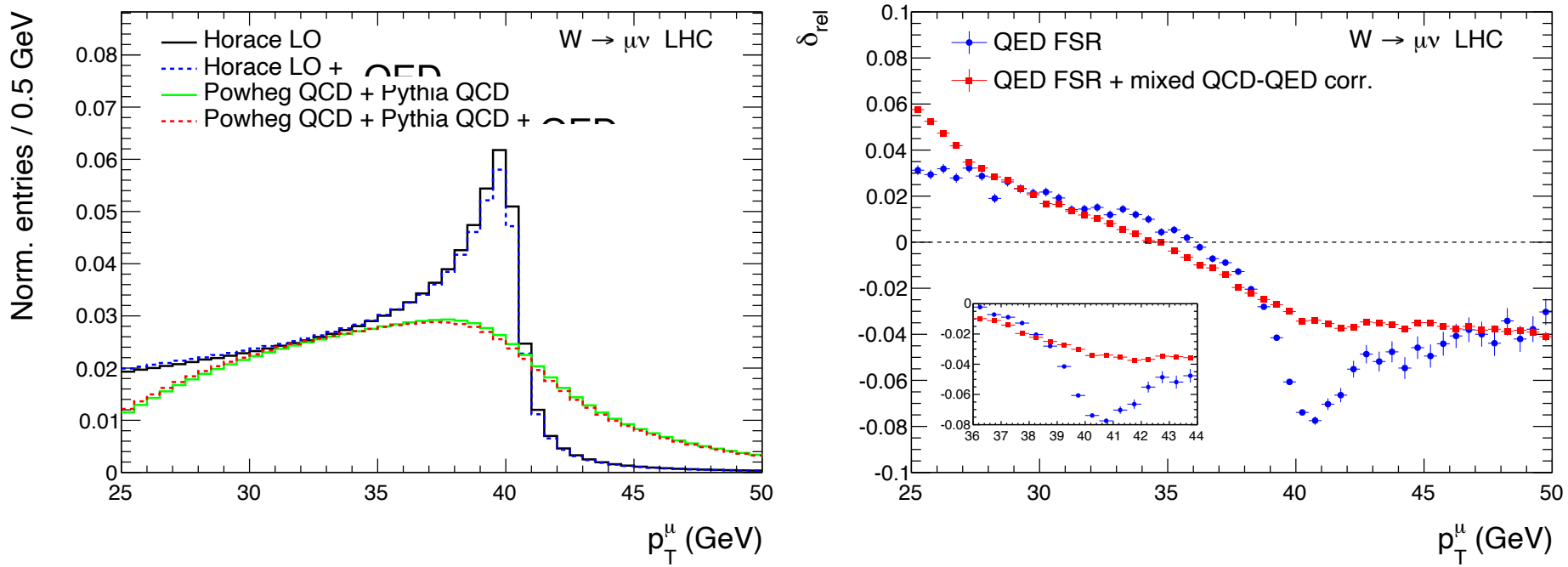
$pp \rightarrow W^+, \sqrt{s} = 14 \text{ TeV}$			M_W shifts (MeV)			
Templates accuracy: NLO-QCD+QCD _{PS}			$W^+ \rightarrow \mu^+ \nu$		$W^+ \rightarrow e^+ \nu(\text{dres})$	
Pseudodata accuracy		QED FSR	M_T	p_T^ℓ	M_T	p_T^ℓ
1	NLO-QCD+(QCD+QED) _{PS}	PYTHIA	-95.2±0.6	-400±3	-38.0±0.6	-149±2
2	NLO-QCD+(QCD+QED) _{PS}	PHOTOS	-88.0±0.6	-368±2	-38.4±0.6	-150±3
3	NLO-(QCD+EW)+(QCD+QED) _{PS} two-rad	PYTHIA	-89.0±0.6	-371±3	-38.8±0.6	-157±3
4	NLO-(QCD+EW)+(QCD+QED) _{PS} two-rad	PHOTOS	-88.6±0.6	-370±3	-39.2±0.6	-159±2

- the bulk of the corrections is included in the analyses
- what is the associated uncertainty ?
 - what happens if we change the underlying QCD model ?

Impact of EW and mixed QCDxEW corrections on MW

C.Carloni Calame, M.Chiesa, H.Martinez, G.Montagna, O.Nicrosini, F.Piccinini, AV, arXiv:1612.02841

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5	PHOTOS FSR-LL	-92±1	-100±2	-182±1	-199±2



- QED FSR plays the major role
- subleading QED and weak induce further O(4 MeV) shifts

the impact on MW of the mixed QCD QED-FSR corrections strongly depends on the underlying QCD shape/model

$pp \rightarrow W^+, \sqrt{s} = 14 \text{ TeV}$			M_W shifts (MeV)			
Templates accuracy: NLO-QCD+QCD _{PS}			$W^+ \rightarrow \mu^+ \nu$		$W^+ \rightarrow e^+ \nu(\text{dres})$	
Pseudodata accuracy		QED FSR	M_T	p_T^ℓ	M_T	p_T^ℓ
1	NLO-QCD+(QCD+QED) _{PS}	PYTHIA	-95.2±0.6	-400±3	-38.0±0.6	-149±2
2	NLO-QCD+(QCD+QED) _{PS}	PHOTOS	-88.0±0.6	-368±2	-38.4±0.6	-150±3
3	NLO-(QCD+EW)+(QCD+QED) _{PS} two-rad	PYTHIA	-89.0±0.6	-371±3	-38.8±0.6	-157±3
4	NLO-(QCD+EW)+(QCD+QED) _{PS} two-rad	PHOTOS	-88.6±0.6	-370±3	-39.2±0.6	-159±2

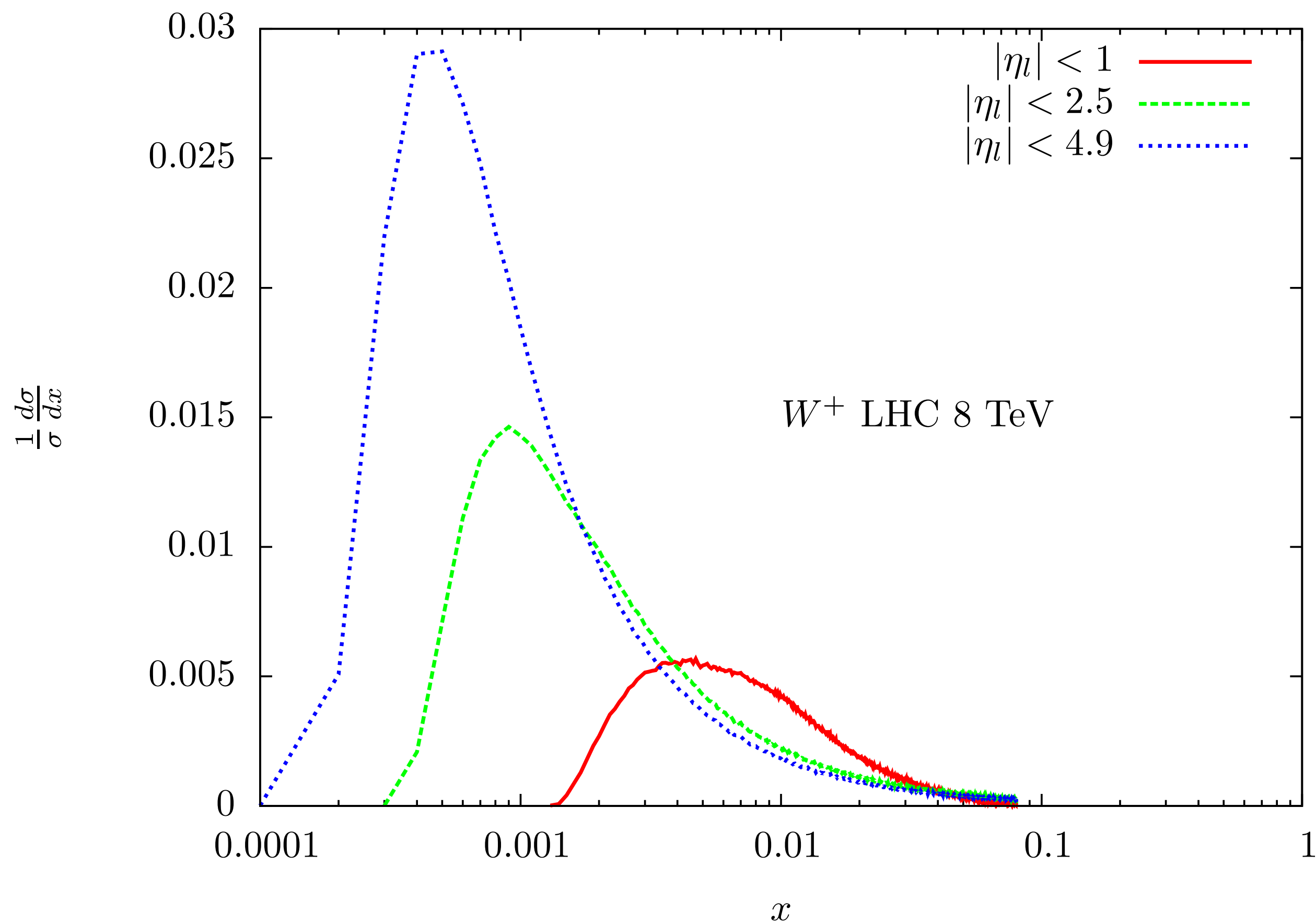
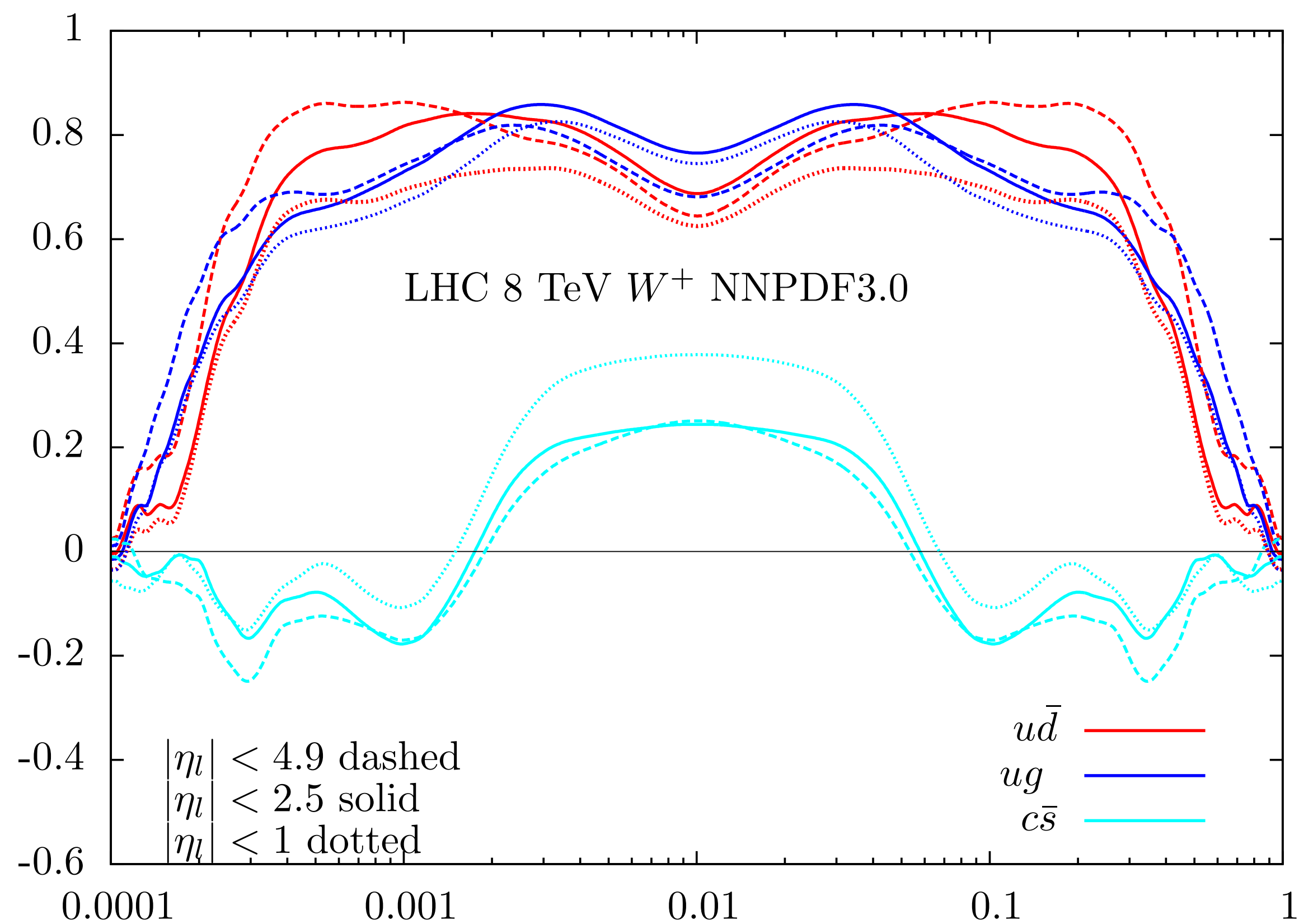
- the bulk of the corrections is included in the analyses
- what is the associated uncertainty ?
 - what happens if we change the underlying QCD model ?

can we constrain the formulation, for the $\alpha\alpha_s$ contribution ?

very stable behaviour of the M_\perp distribution in contrast to the p_\perp^l case

Rapidity acceptance and the relevant partonic-x range

G.Bozzi, L.Citelli, AV, arXiv:1501.05587

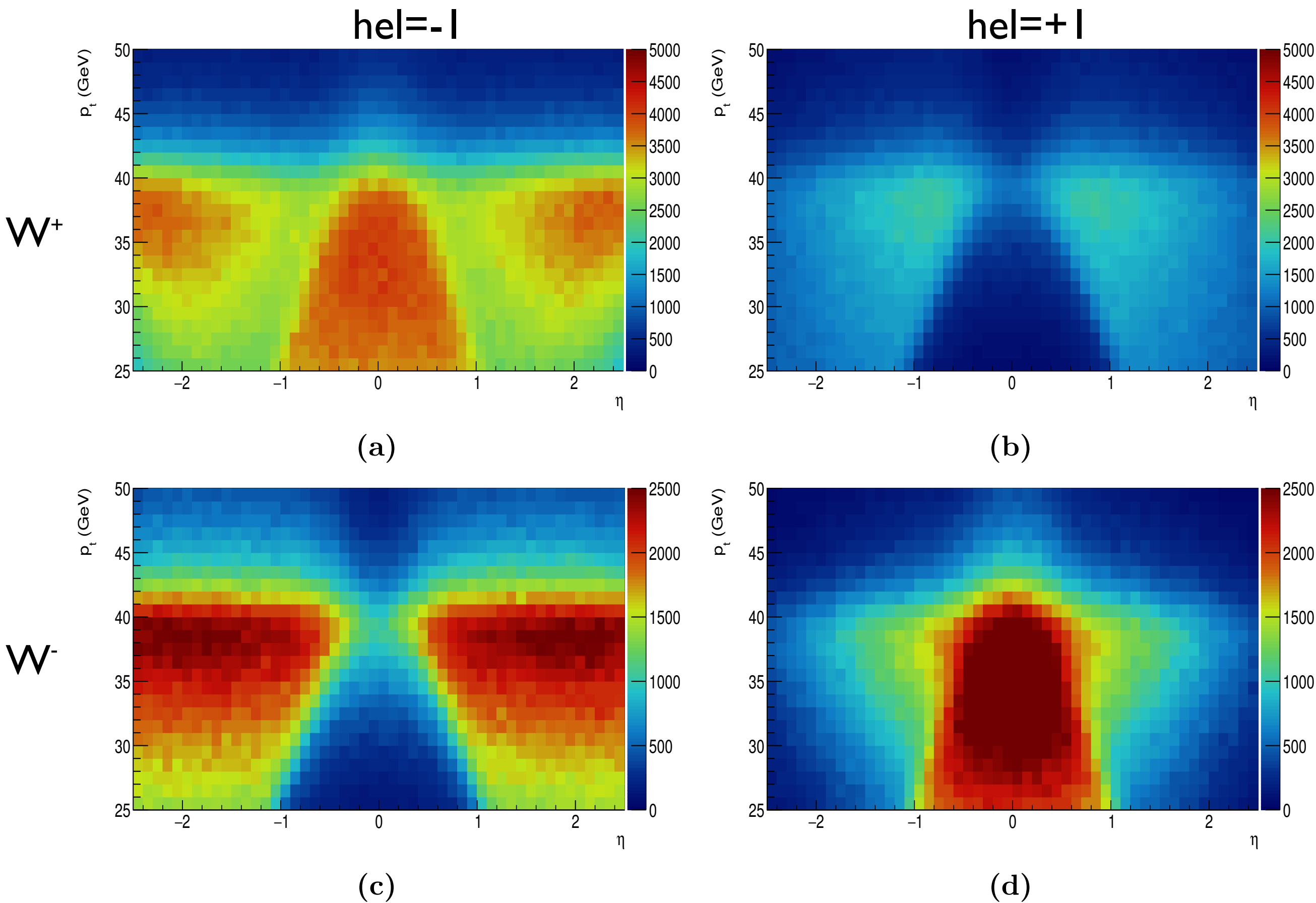


$$\rho(x, \tau) = \frac{\langle \mathcal{P}_{ij}(x, \tau) \frac{d\sigma}{dp_{\perp}^l} \rangle - \langle \mathcal{P}_{ij}(x, \tau) \rangle \langle \frac{d\sigma}{dp_{\perp}^l} \rangle}{\sigma_{\mathcal{P}_{ij}}^{PDF} \sigma_{d\sigma/dp_{\perp}^l}^{PDF}}$$

$$\frac{1}{\sigma} \frac{d\sigma}{dx}$$

PDF uncertainty on MW: exploiting the power of data

E.Manca, O.Cerri, N.Foppiani, G.Rolandi, JHEP12 (2017) 130



The fingerprint of W^\pm helicity states, in the double differential $p_{t\text{lep}}\text{-}\eta_{\text{lep}}$ distribution in CC-DY offers a very strong constraint

- to determine the PDFs

and/or

- to determine MW with an effective profiling of the PDFs

In the second case, the resulting uncertainty estimate should be reduced compared to a naive χ^2 analysis thanks to the information stored in the data that “effectively discards the least probable replicas”

Comments on the χ^2 minimisation in the template fit

$$\chi^2 = (\vec{d} - \vec{t})^T \cdot C^{-1} \cdot (\vec{d} - \vec{t})$$

$$C = \Sigma_{stat} + \Sigma_{syst,exp} + \Sigma_{MC} + \Sigma_{PDF} + \Sigma_{syst,th}$$

The $\Sigma_{syst,th}$ contribution to the covariance matrix is never included, because of the non-statistical nature of theory uncertainties

The χ^2 minimisation leads to sensible and stable results when the deviation of the data from the templates is comparable to the size of the eigenvalues of the covariance matrix

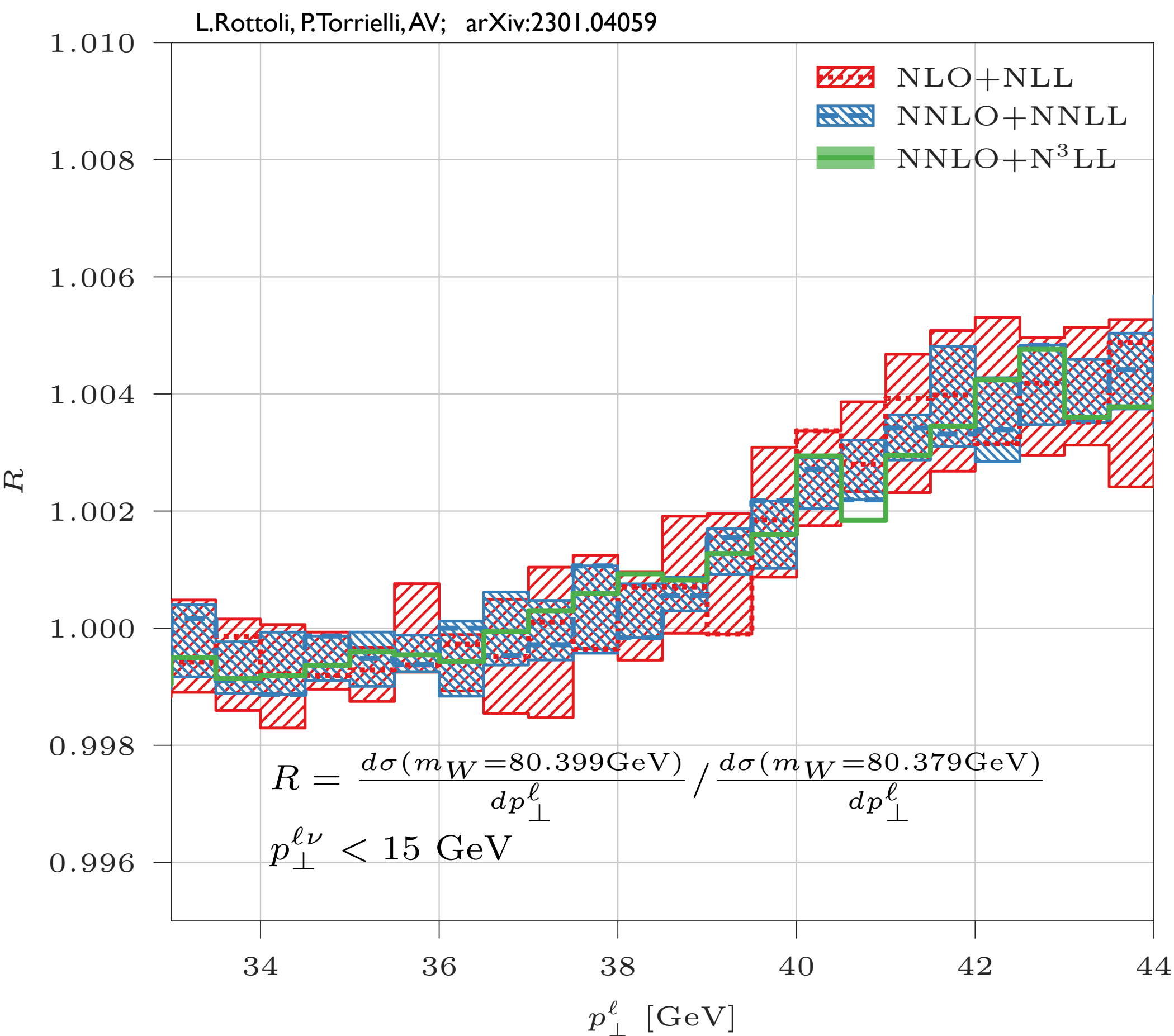
but

the lepton transverse momentum distribution receives very large corrections in QCD, much larger than 0.1% ;

the absence of $\Sigma_{syst,th}$ makes it impossible to assign a “sensible” contribution to the χ^2 , e.g. when applying scale variations (instability of the χ^2 minimisation)

→ the data driven approach remains the only way to pursue a template fit approach
at the price of losing the possibility to study the theoretical uncertainties on the modelling

Sensitivity to the W boson mass: independence from QCD approximation



The determination of m_W requires the possibility to appreciate the distortion of the distribution induced by 2 different mass hypotheses

A shift by $\Delta m_W = 20 \text{ MeV}$ distorts the distribution at few per mille level

In pure QCD,
the distortion is **independent of the QCD approximation or scale choice**

The process can be **factorized** in production (with QCD effects) times propagation and decay of the W boson.

The sensitivity to m_W stems from the propagation and decay part

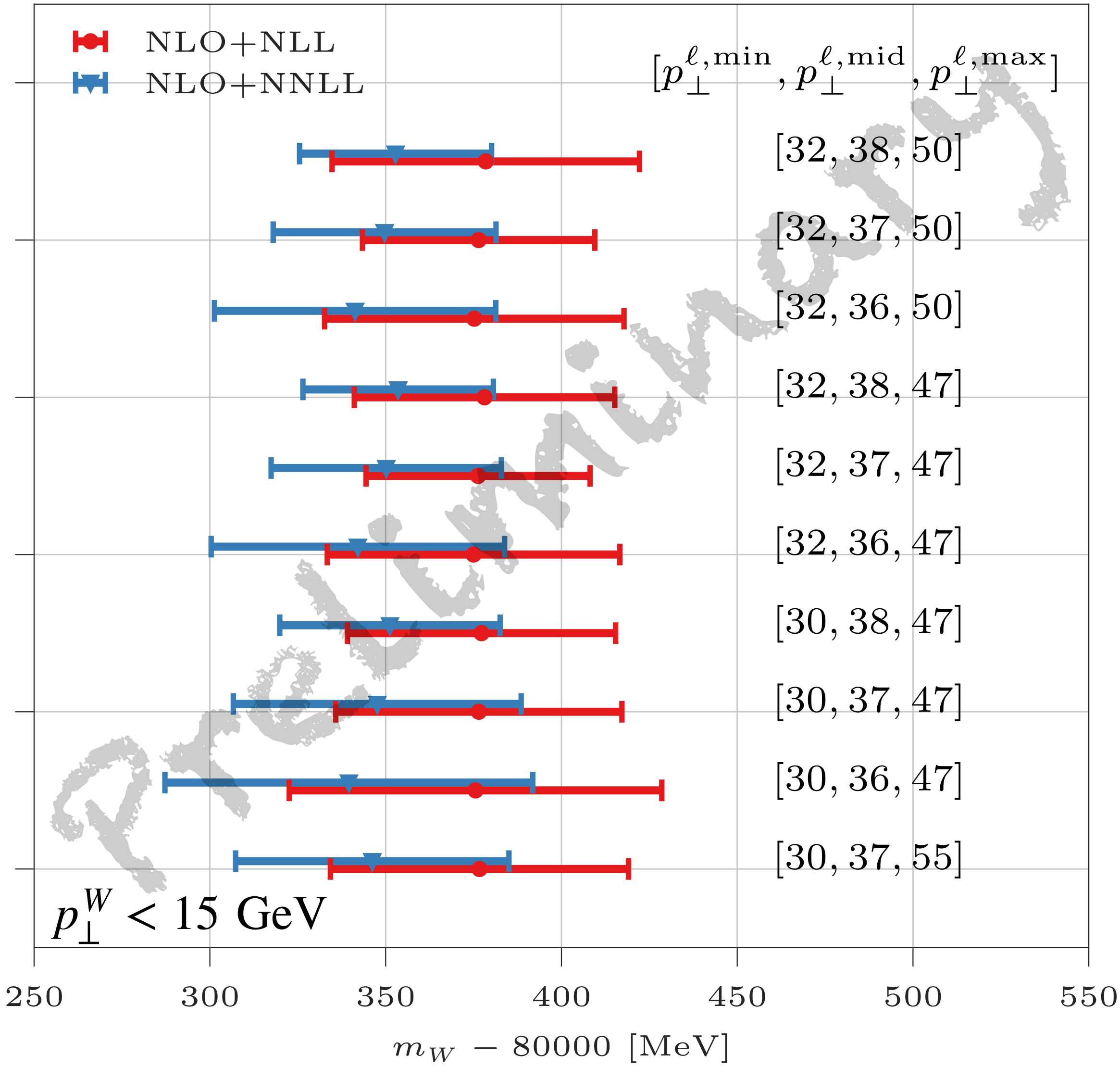
The sensitivity to m_W is independent of the QCD approximation
The central value and the uncertainty on m_W instead do depend on the QCD approximation

The study of the covariance matrix for m_W variations shows that **one specific combination** of bins **carries the bulk of the sensitivity** to m_W → **following this indication, we design a new observable**

m_W determination at the Tevatron as a function of the $\mathcal{A}_{p_\perp^\ell}$ parameters (no p_\perp^Z reweighting)

as pseudo-experimental value we choose the NNLO+N3LL result with $m_W = 80.379$

L.Rottoli, P.Torrielli, AV; arXiv:2301.04059

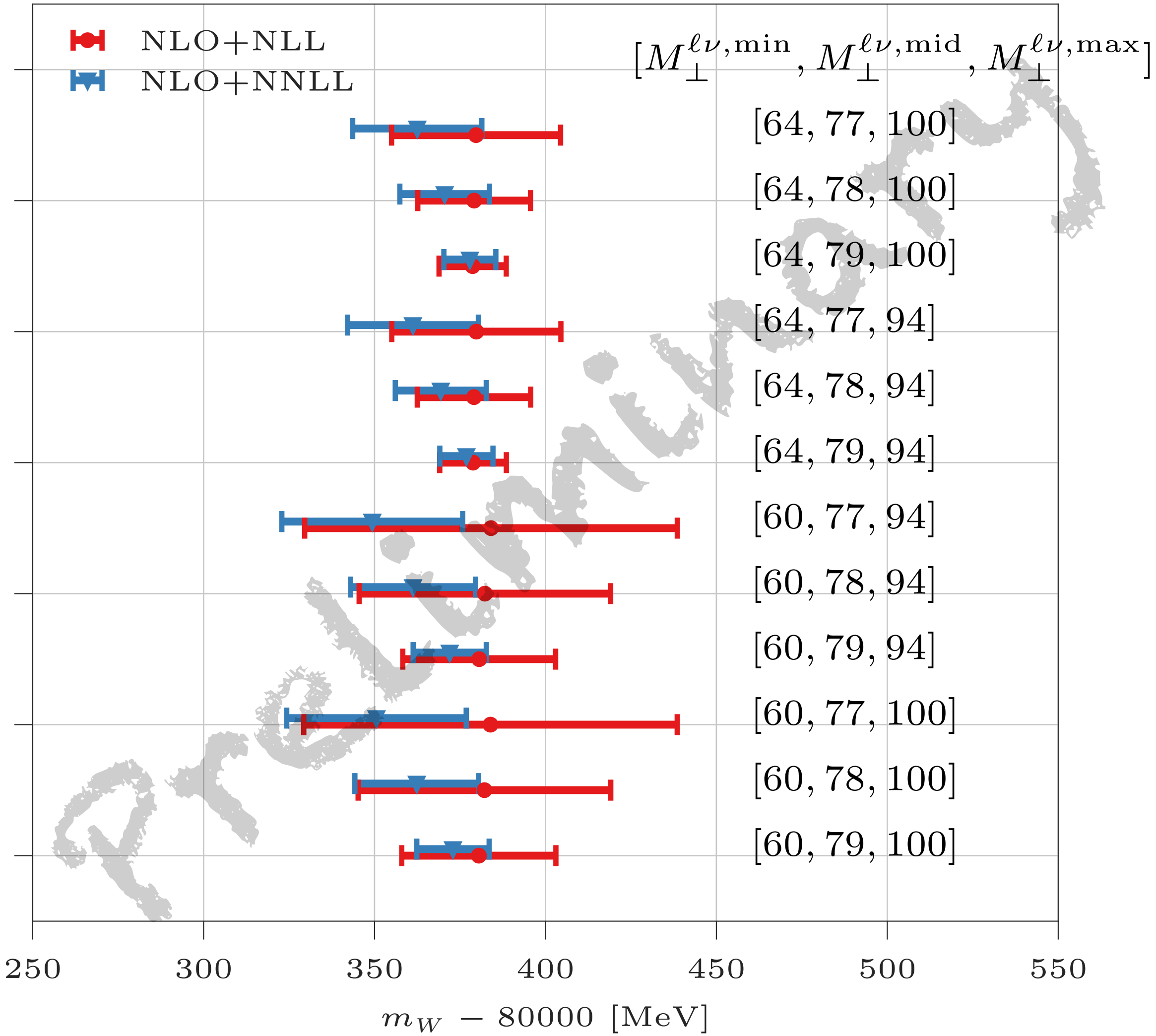


- we compute $\mathcal{A}_{p_\perp^\ell}$ at the Tevatron, from CC-DY, as a function of m_W
we vary the QCD scales in the canonical ranges
- in the most optimistic configuration, at NLO+NNLL, a range of values $\Delta m_W \sim \pm 30 \text{ MeV}$ is found
- NLO+NNLL is the same perturbative accuracy available in ResBos
- it is difficult to expect a very significant uncertainty reduction thanks to the p_\perp^Z data information only (cfr. previous slides)
- usage of the **highest available perturbative order** is recommended to minimize the pQCD systematics in the transfer from Z to W

m_W determination at the Tevatron as a function of the $\mathcal{A}_{M_{\perp}^{\ell\nu}}$ parameters (no p_{\perp}^Z reweighting)

as pseudo-experimental value we choose the NNLO+N3LL result with $m_W = 80.379$

L.Rottoli, P.Torrielli, AV; arXiv:2301.04059



- we compute $\mathcal{A}_{M_{\perp}^{\ell\nu}}$ at the Tevatron, from CC-DY, as a function of m_W
we vary the QCD scales in the canonical ranges
- NLO+NNLL is the same perturbative accuracy available in ResBos
- we neglect important detector simulation effects
→ optimistic estimates for the uncertainty
- in the most optimistic configuration, at NLO+NNLL, a range of values $\Delta m_W \sim \pm 10$ MeV is found

Sensitivity to the W boson mass: covariance w.r.t. M_W variations

The sensitivity to m_W can be quantified by means of a matrix of covariance w.r.t. m_W variations

$$\mathcal{C}_{ij} \equiv \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle \quad \text{with} \quad \langle \sigma \rangle \equiv \frac{1}{N_W} \sum_{k=1}^{N_W} \sigma(m_W = m_W^{(k)})$$

and σ_i represents the i -th bin of the p_{\perp}^{ℓ} distribution

The diagonalization of the covariance matrix yields N_{bins} linear combinations of the σ_i transforming independently of each other under m_W variations

The eigenvalues express the sensitivity for a given Δm_W shift, and help classifying the different combinations

The first eigenvalue is 564 times the second one (in size)

The associated linear combination has a peculiar structure:

all coefficients are positive (negative) for $p_{\perp}^{\ell} < 37$ ($p_{\perp}^{\ell} > 37$) GeV

Explicit check that the value $p_{\perp}^{\ell} \sim 37$ is very stable changing QCD approximation or bin range

This value can be appreciated also in the plot of the ratio \rightarrow indication for the definition of a new observable