Nucleon TMDPDFs from Lattice QCD

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Parton Model

Any hadron with large momentum can be considered as a composition of a number of point-like "partons". (By Richard Feynman in 1969)

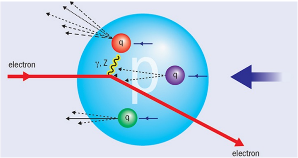


Figure 1: DIS with parton model, from CERN.

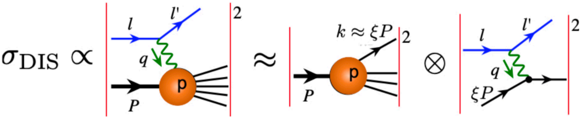


Figure 2: DIS factorization. [1]

What are TMDPDFs

Parton Distribution Functions: the probability to find a parton (quark or gluon) in a hadron as a function of momentum fraction x.

Transverse-Momentum-Dependent PDFs: the distribution densities to find a parton carrying a longitudinal momentum fraction x and transverse momentum k_T in a hadron.

TMDPDFs can be regarded as a tomography of the nucleon, which reveals the nucleons 3D internal structure.

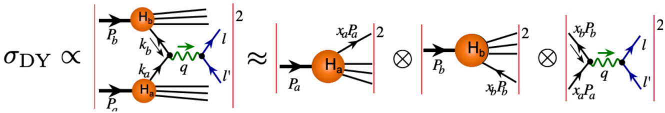


Figure 3: Drell Yan factorization.[1]

Why Care about TMDPDFs

- Understanding the inner structure of hadrons.
- TMD processes are the most important processes in high energy collisions, like SIDIS on EIC.
- TMDPDFs are important inputs for experiments on TMD processes.

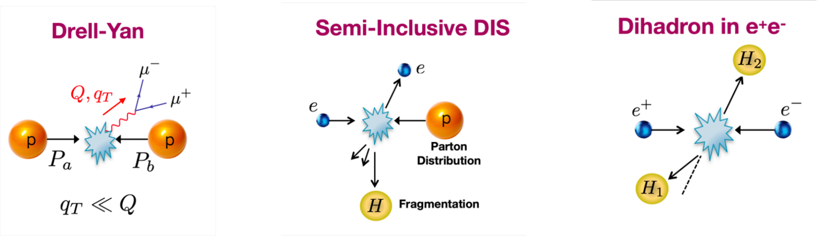


Figure 4: TMD processes. [1]

Lattice QCD

Path integral formalism

$$\langle \hat{O} \rangle = \int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \hat{O}\ e^{iS_{\mathrm{QCD}}(A,\psi,\bar{\psi})}$$

$$\langle \hat{O} \rangle = \int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \hat{O}\ e^{-S^E_{\mathrm{QCD}}(A,\psi,\bar{\psi})}$$

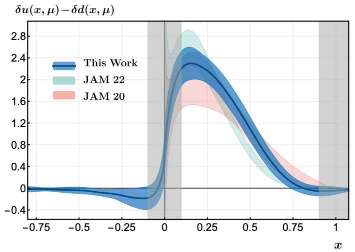
Monte Caro Sampling

$$P(A,\psi,\bar{\psi}) = \frac{e^{-S^E_{\mathrm{QCD}}(A,\psi,\bar{\psi})}}{\int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\ e^{-S^E_{\mathrm{QCD}}(A,\psi,\bar{\psi})}}$$

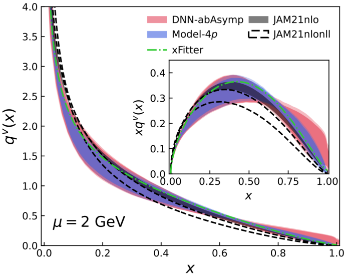
$$\langle \hat{O} \rangle = \langle \hat{O} \rangle_{\text{Sample average}}$$

Lattice QCD

Lattice QCD is a powerful non-perturbative method



(a) Proton transversity PDF. [2]



(b) Pion valance PDF. [3]

Figure 5: Achievements on Lattice QCD.

Large Momentum Effective Theory(LaMET)

Quasi distribution: Equal time, P_z-dependent.
Light-cone distribution: Separate on time, universal.

$$(\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 = 0$$
$$(\Delta \tau)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 = 0$$

$$\boxed{\tilde{f}_\Gamma\left(x, b_\perp, \zeta_z, \mu\right)} \sqrt{S_I\left(b_\perp, \mu\right)} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right) K(b_\perp, \mu)} \boxed{f\left(x, b_\perp, \mu, \zeta\right)} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta_z}\right)$$

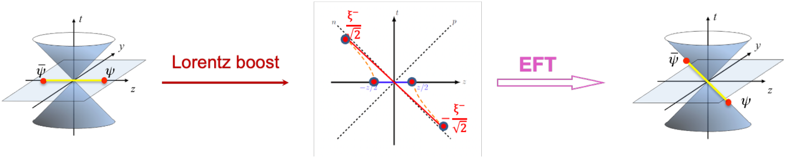


Figure 6: Large Momentum Effective Field Theory.

Recipe

Correlator on lattice.

Ground state fit.

Renormalization

x-distribution.

LaMET matching.

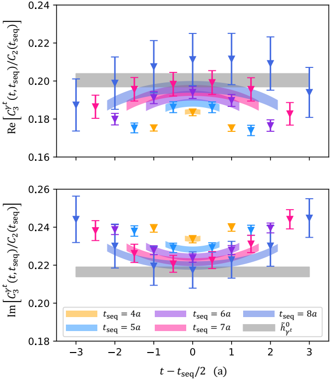


Figure 7: Ground state fit.[4]

Renormalization

$$\tilde{h}_\Gamma\left(z, b_\perp, P^z, a, \mu\right) = \lim_{L\to\infty} \frac{\tilde{h}^0_\Gamma\left(z, b_\perp, P^z, a, L\right)}{\sqrt{Z_E\left(2L+z, b_\perp, a\right)} Z_O(1/a, \mu, \Gamma)} \quad (1)$$

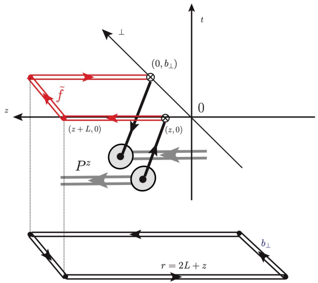


Figure 8: Non-perturbative renormalization.[5]

Z_E: divergence from Wilson links

Z_O: divergence from quark-gauge link vertices

Comparison with Phenomenological Results

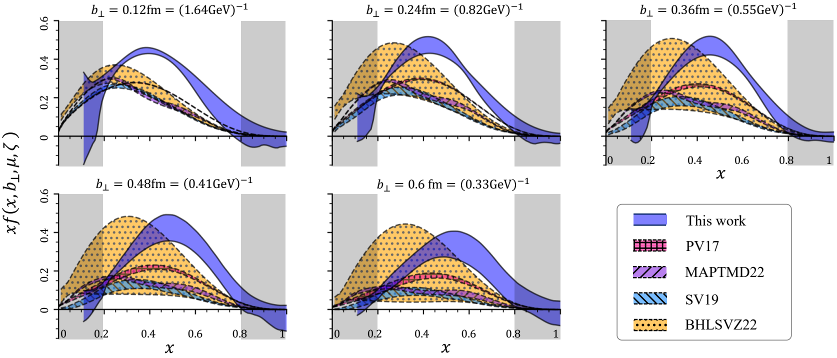


Figure 9: Final results on x-dependence. [4][6][7][8][9]

Comparison with Phenomenological Results

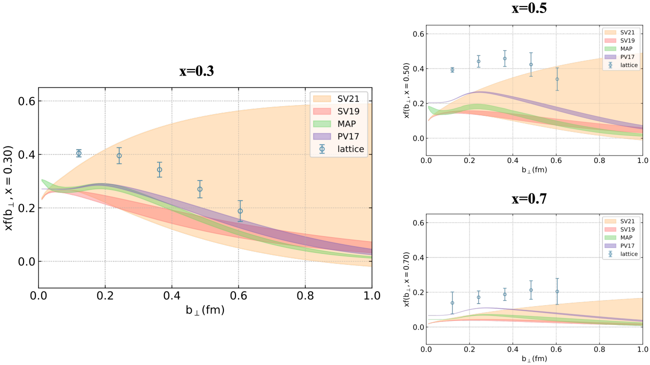


Figure 10: Final results on b-dependence. [4][6][7][8][9]

Improvement

Direction	Status	Prospect
Lattice spacing	a $= 0.12$ fm	Continuum limit
Hadron momentum	$P_z = 2.58$ GeV	Larger γ factor
Transverse behavior	b $= 0.6$ fm	QCD Confinement
More		Higher power correction

Other TMDPDFs

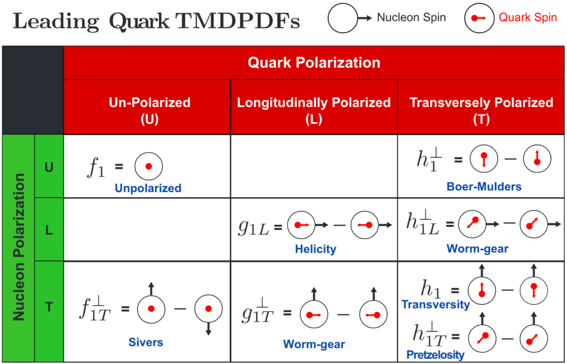


Figure 11: Leading quark TMDPDFs. [1]

[1] R. Boussarie *et al.*, “TMD Handbook,” 4 2023.

[2] F. Yao *et al.*, “Nucleon Transversity Distribution in the Continuum and Physical Mass Limit from Lattice QCD,” 8 2022.

[3] X. Gao, A. D. Hanlon, N. Karthik, S. Mukherjee, P. Petreczky, P. Scior, S. Shi, S. Syritsyn, Y. Zhao, and K. Zhou, “Continuum-extrapolated NNLO valence PDF of the pion at the physical point,” *Phys. Rev. D*, vol. 106, no. 11, p. 114510, 2022.

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Thank You

Unpolarized Transverse-Momentum-Dependent Parton Distributions of the Nucleon from Lattice QCD

(Lattice Parton Collaboration (LPC))

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Figure 12: e-Print: 2211.02340. [4]

Back Up: Lattice Setup

2+1+1 flavors of HISQ action by MILC (a=0.12 fm);

Valance pion mass: 310 MeV, 220 MeV;

Gamma structure: γ^t and γ^z;

Hadron momentum: 1.72 GeV, 2.15 GeV, 2.58 GeV.

Back Up: Uncertainties

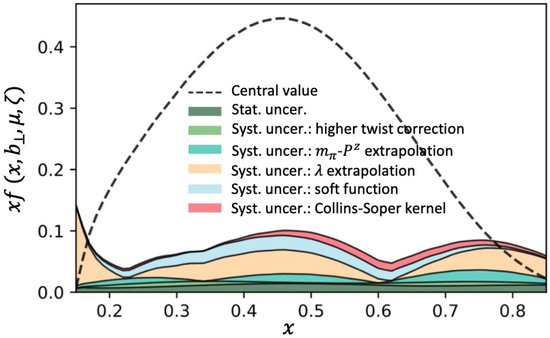

Figure 13: Comparison between different uncertainties. [4]

Back Up: Definition

PDF

$$q(x,\epsilon) \equiv \int \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \left\langle P \left| \bar{\psi}\left(\xi^-\right) \gamma^+ U\left(\xi^-, 0\right) \psi(0) \right| P \right\rangle,$$

$$U\left(\xi^-, 0\right) = P \exp\left(-ig \int_0^{\xi^-} d\eta^- A^+\left(\eta^-\right)\right).$$

TMDPDF

$$\tilde{f}_{i/p}^{0(\mathrm{u})}\left(x, \mathbf{b}_T, \epsilon, \tau, xP^+\right)$$
$$\equiv \int \frac{\mathrm{d}b^-}{2\pi} e^{-ib^-\left(xP^+\right)} \left\langle p(P) \left| \left[\bar{\psi}_i^0\left(b^\mu\right) W_{\sqsubset}\left(b^\mu, 0\right) \frac{\gamma^+}{2} \psi_i^0(0)\right]_\tau \right| p(P) \right\rangle,$$

$$\tilde{S}_{n_a n_b}^0\left(b_T, \epsilon, \tau\right) = \frac{1}{N_c} \left\langle 0 \left| \mathrm{Tr}\left[W\left(b_T\right)\right]_\tau \right| 0 \right\rangle.$$