

Pheno 2023, 8-10 May

Master Integrals for electroweak
corrections to $gg \rightarrow \gamma\gamma$

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Based on: 2305.xxxx with Amlan Chakraborty
and Ciaran Williams

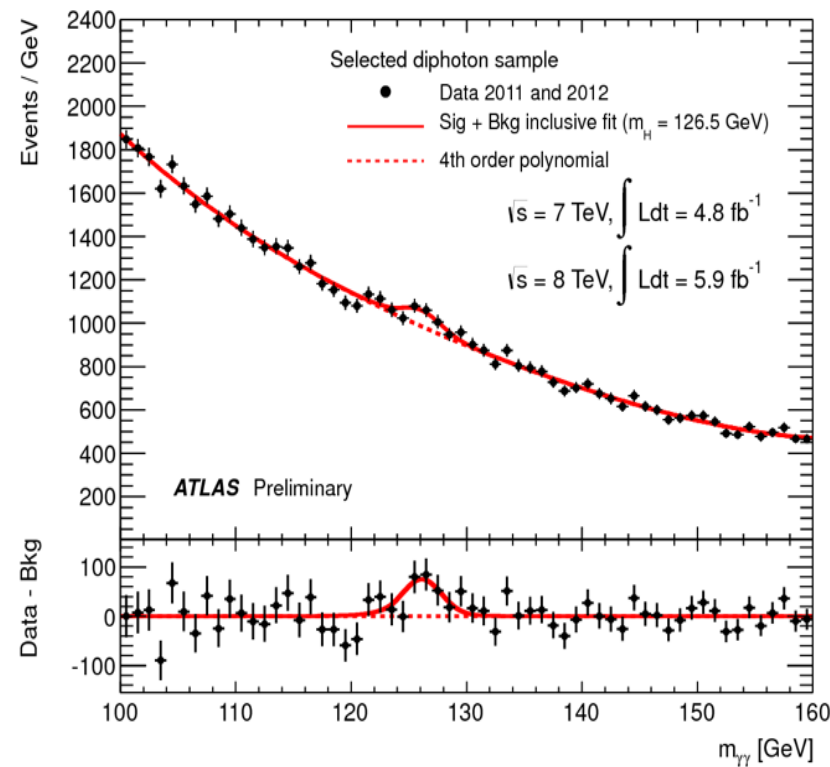


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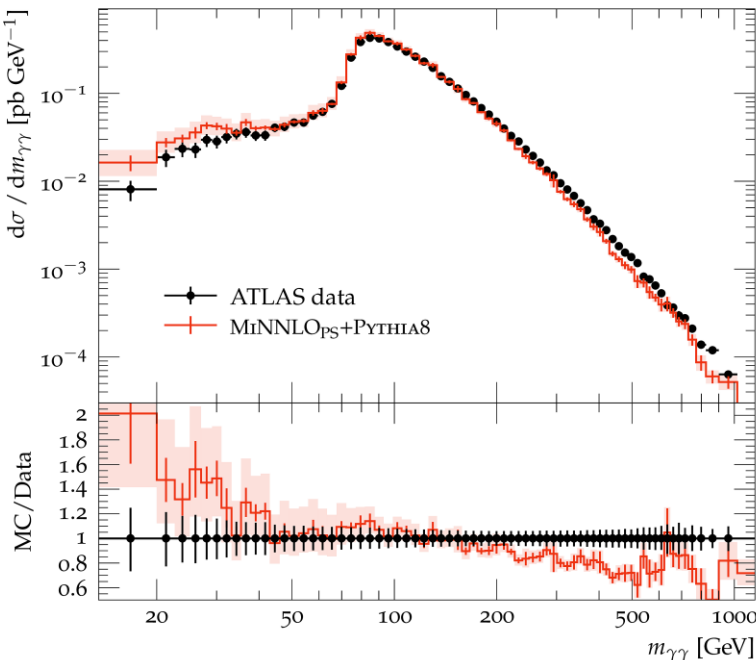
Department of Physics



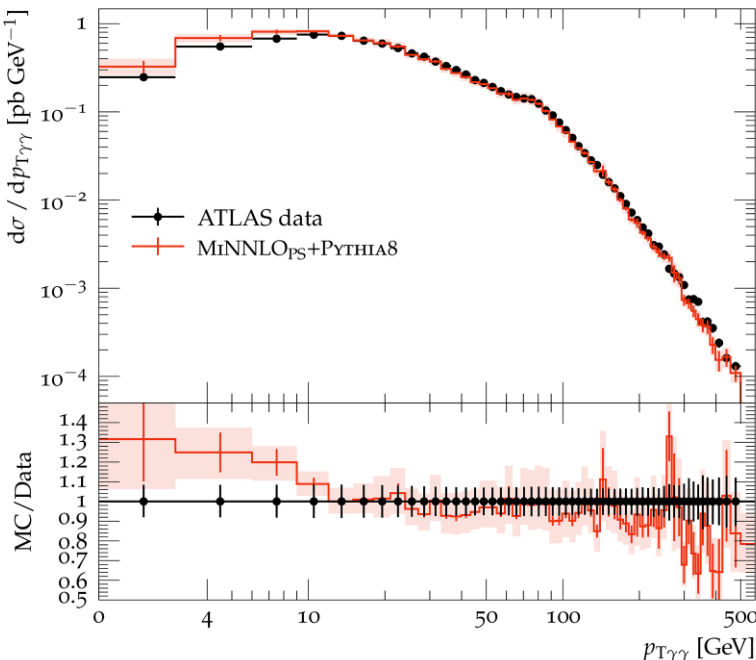
Diphoton channel has a crucial role in Higgs phenomenology. High precision data needs high theoretical predictions.



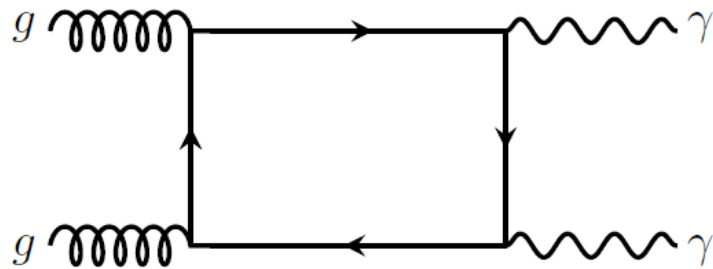
ATLAS Collaboration, 2012



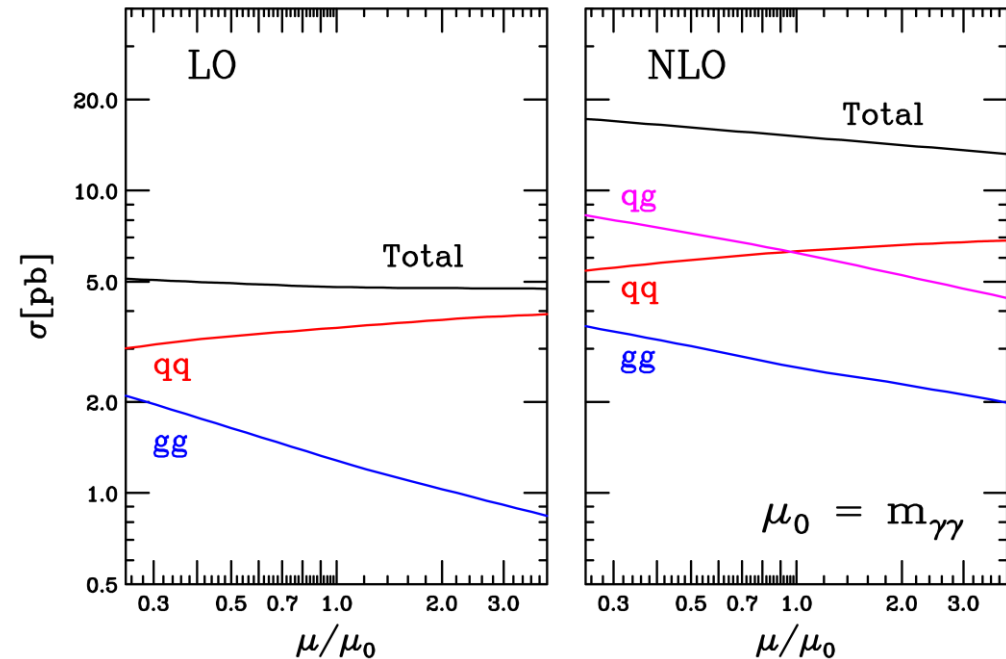
Gavardi, Oleari, Rea, 2022



- As part of the partonic process $pp \rightarrow \gamma\gamma$, the corrections enter at NNLO in QCD.



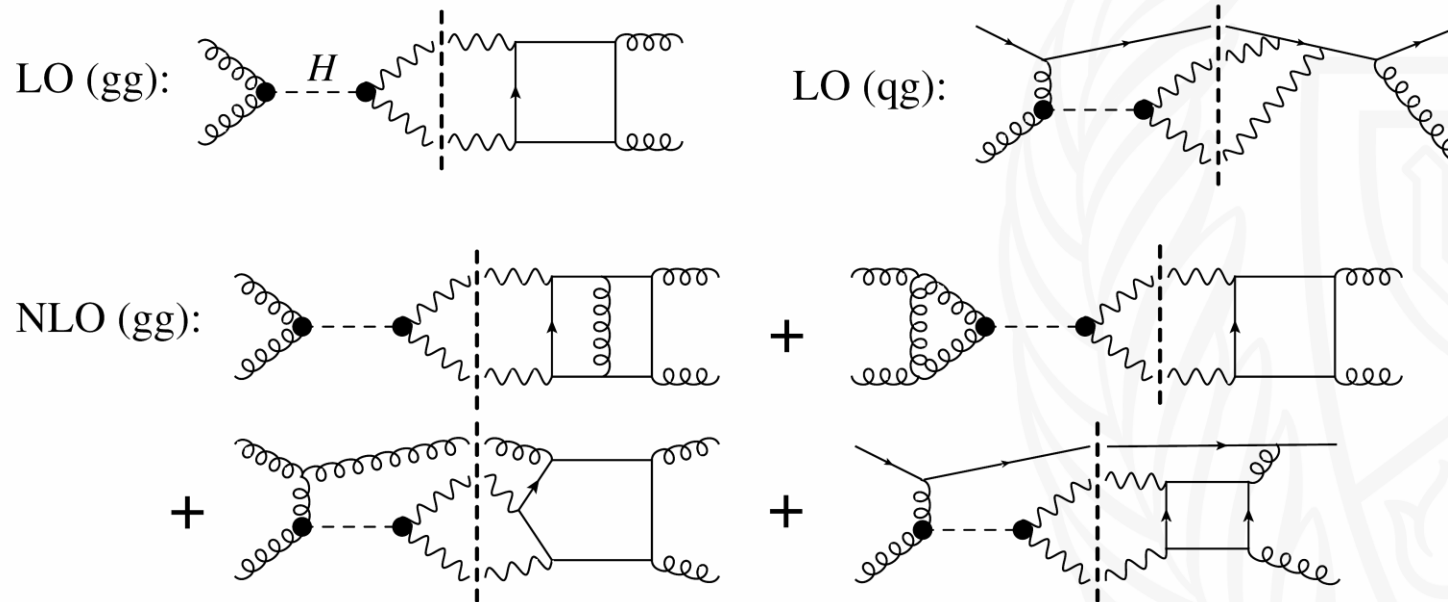
- Sizable corrections to the cross-section due to large initial-state gluon flux.
- Given the quality of recent LHC data, $\mathcal{O}(\alpha)$ corrections for the $gg \rightarrow \gamma\gamma$ channel are required to improve our theoretical predictions.



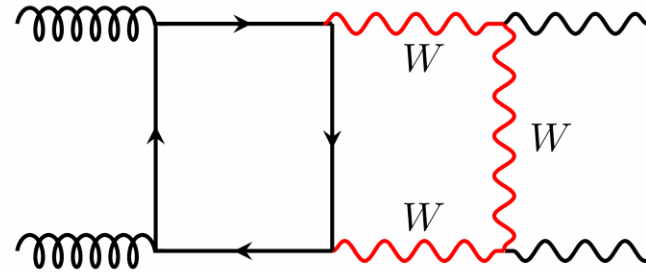
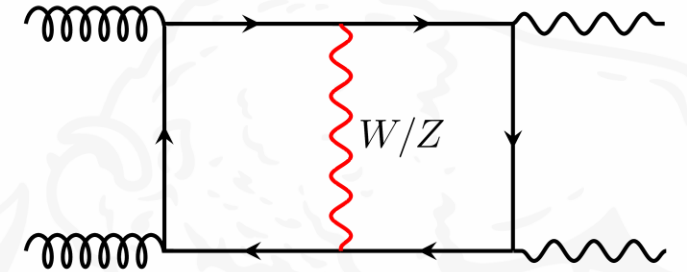
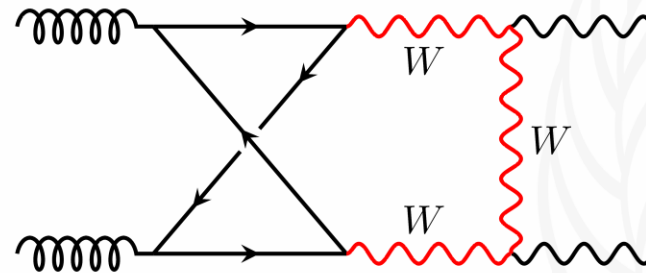
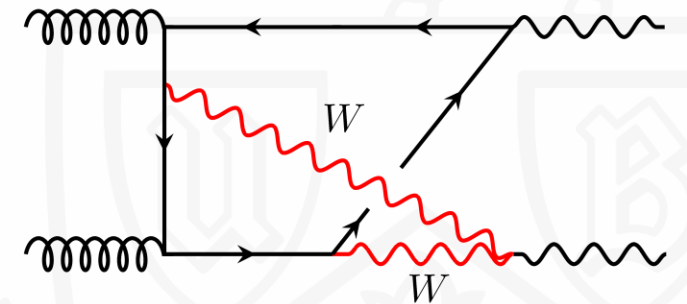
Campbell, Ellis, Williams, 2011

- The interference between the diphoton channel $gg \rightarrow H \rightarrow \gamma\gamma$ and the QCD background $gg \rightarrow \gamma\gamma$ is crucial.

$$\hat{\delta}_{gg \rightarrow H \rightarrow \gamma\gamma} = -2 (\hat{s} - m_H^2) \frac{\text{Re}(\mathcal{A}_{gg \rightarrow H} \mathcal{A}_{H \rightarrow \gamma\gamma} \mathcal{A}_{\text{cont}}^*)}{(\hat{s} - m_H^2)^2 + m_H^2 \Gamma_H^2} - 2m_H \Gamma_H \frac{\text{Im}(\mathcal{A}_{gg \rightarrow H} \mathcal{A}_{H \rightarrow \gamma\gamma} \mathcal{A}_{\text{cont}}^*)}{(\hat{s} - m_H^2)^2 + m_H^2 \Gamma_H^2}$$

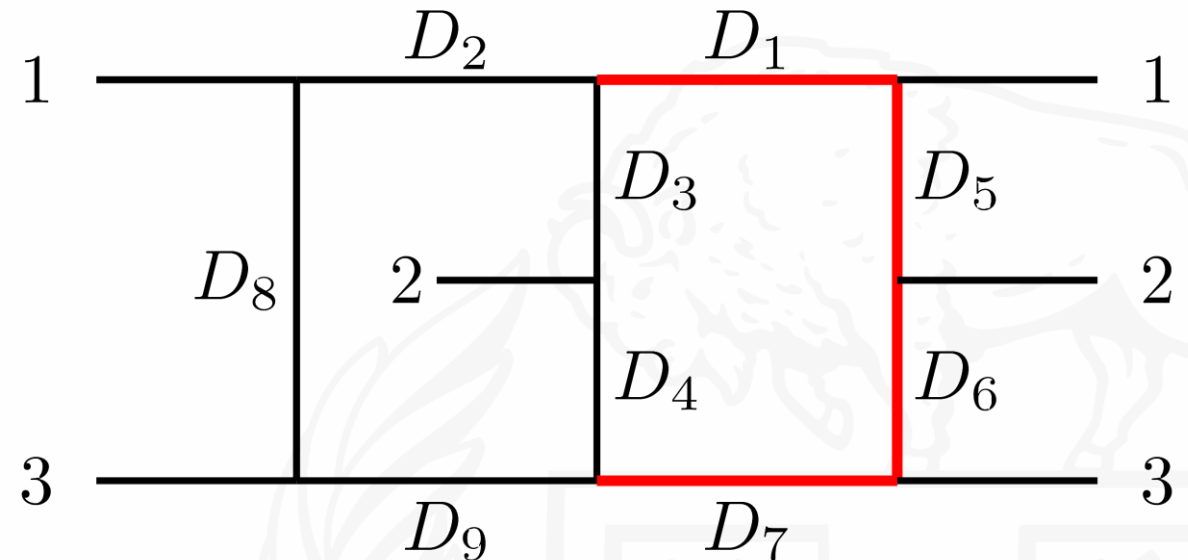


- We present the evaluation of the master integrals for massless quark loops.
- Different topologies depending on the vector boson flow. Here we focus on N_I .
- To extract the helicity amplitude, we introduce 8 projectors.
- Combination of thousands of integrals in the form factors \mathcal{F}_i .

(a) Topology P_I (b) Topology P_{II} (c) Topology N_I (d) Topology N_{II}

$$\mathcal{A}(s, t) = \sum_{i=1}^8 \mathcal{F}_i T_i$$

- We introduce an auxiliary topology to account for all the possible scalar products involving the loop momenta.
- All the scalar integrals can be written in terms of the auxiliary topology's propagators.
- These integrals are reduced to a minimal set of Master Integrals (MIs) by using Kira
(*Klappert, Lange, Maierhöfer, Usovitsch, 2023*).
- Laporta Basis for N_I : 32 MIs.



$$I_{a_1 \dots a_9}^{N_I} = \mathcal{C}(\epsilon) \int \frac{d^d \ell_1}{(2\pi)^d} \frac{d^d \ell_2}{(2\pi)^d} \frac{1}{D_1^{a_1} \dots D_9^{a_9}}$$

- General strategy to evaluate MIs: Differential Equation method (DEQ).

$$\text{Laporta Basis } \vec{\mathcal{I}}(\{x_i\}) \longrightarrow \frac{\partial}{\partial x_i} \vec{\mathcal{I}}(\{x_i\}) = \underbrace{\mathcal{A}(\{x_i\}, \epsilon)} \vec{\mathcal{I}}(\{x_i\})$$

- The complexity goes up pretty quickly for the Laporta Basis. Solution: Canonical Basis.

$$\text{Canonical Basis } \vec{\mathcal{G}}(\{x_i\}) \longrightarrow \frac{\partial}{\partial x_i} \vec{\mathcal{G}}(\{x_i\}) = \underbrace{\epsilon \mathcal{A}(\{x_i\})} \vec{\mathcal{G}}(\{x_i\})$$

- The factorization of the space-time parameter allows to write the solution in an iterative way:

$$\vec{\mathcal{G}}(\{x_i\}) = \left(\mathbb{1} + \epsilon \int_{\gamma} d\mathcal{A} + \epsilon^2 \int_{\gamma} d\mathcal{A} d\mathcal{A} + \dots \right) \vec{\mathcal{G}}(\{x_i^0\})$$

- The kinematic information is encoded in the alphabet $\{\eta_k\}$ of the differential equation:

$$d\mathcal{A} = \sum_k \mathcal{M}_k d \log \eta_k$$

- The solution for rational letters is expressed in terms of Goncharov Polylogarithms (GPLs).
- Efficient numerical evaluation of GPLs using GiNaC/HandyG (*Naterop, Signer, Ulrich, 2016*).
- Non-rational letters may or may not be represented in terms of GPLs. We implement a generalized approach based on Chen's Iterated Integrals.

The solution can be evaluated (up to boundary constants) at any fixed order:



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$$\vec{\mathcal{G}}^{(1)}(\{x_i\}) = \int_{\gamma} d\mathcal{A} \vec{\mathcal{G}}^{(0)}(\{x_i^0\}) + \vec{\mathcal{G}}^{(1)}(\{x_i^0\})$$



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$$\vec{\mathcal{G}}^{(2)}(\{x_i\}) = \int_{\gamma} d\mathcal{A} d\mathcal{A} \vec{\mathcal{G}}^{(0)}(\{x_i^0\}) + \int_{\gamma} d\mathcal{A} \vec{\mathcal{G}}^{(1)}(\{x_i^0\}) + \vec{\mathcal{G}}^{(2)}(\{x_i^0\})$$

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$$\vec{\mathcal{G}}^{(3)}(\{x_i\}) = \int_{\gamma} d\mathcal{A} d\mathcal{A} d\mathcal{A} \vec{\mathcal{G}}^{(0)}(\{x_i^0\}) + \int_{\gamma} d\mathcal{A} d\mathcal{A} \vec{\mathcal{G}}^{(1)}(\{x_i^0\}) + \int_{\gamma} d\mathcal{A} \vec{\mathcal{G}}^{(2)}(\{x_i^0\}) + \vec{\mathcal{G}}^{(3)}(\{x_i^0\})$$

The solution can be evaluated (up to boundary constants) at any fixed order:

$$\vec{\mathcal{G}}^{(1)}(\{x_i\}) = \int_{\gamma} d\mathcal{A} \vec{\mathcal{G}}^{(0)}(\{x_i^0\}) + \vec{\mathcal{G}}^{(1)}(\{x_i^0\})$$

$$\vec{\mathcal{G}}^{(2)}(\{x_i\}) = \int_{\gamma} d\mathcal{A} d\mathcal{A} \vec{\mathcal{G}}^{(0)}(\{x_i^0\}) + \int_{\gamma} d\mathcal{A} \vec{\mathcal{G}}^{(1)}(\{x_i^0\}) + \vec{\mathcal{G}}^{(2)}(\{x_i^0\})$$

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$$\begin{aligned} \vec{\mathcal{G}}^{(4)}(\{x_i\}) = & \int_{\gamma} d\mathcal{A} d\mathcal{A} d\mathcal{A} d\mathcal{A} \vec{\mathcal{G}}^{(0)}(\{x_i^0\}) + \int_{\gamma} d\mathcal{A} d\mathcal{A} d\mathcal{A} \vec{\mathcal{G}}^{(1)}(\{x_i^0\}) + \int_{\gamma} d\mathcal{A} d\mathcal{A} \vec{\mathcal{G}}^{(2)}(\{x_i^0\}) \\ & + \int_{\gamma} d\mathcal{A} \vec{\mathcal{G}}^{(3)}(\{x_i^0\}) + \vec{\mathcal{G}}^{(4)}(\{x_i^0\}). \end{aligned}$$

The solution can be evaluated (up to boundary constants) at any fixed order:

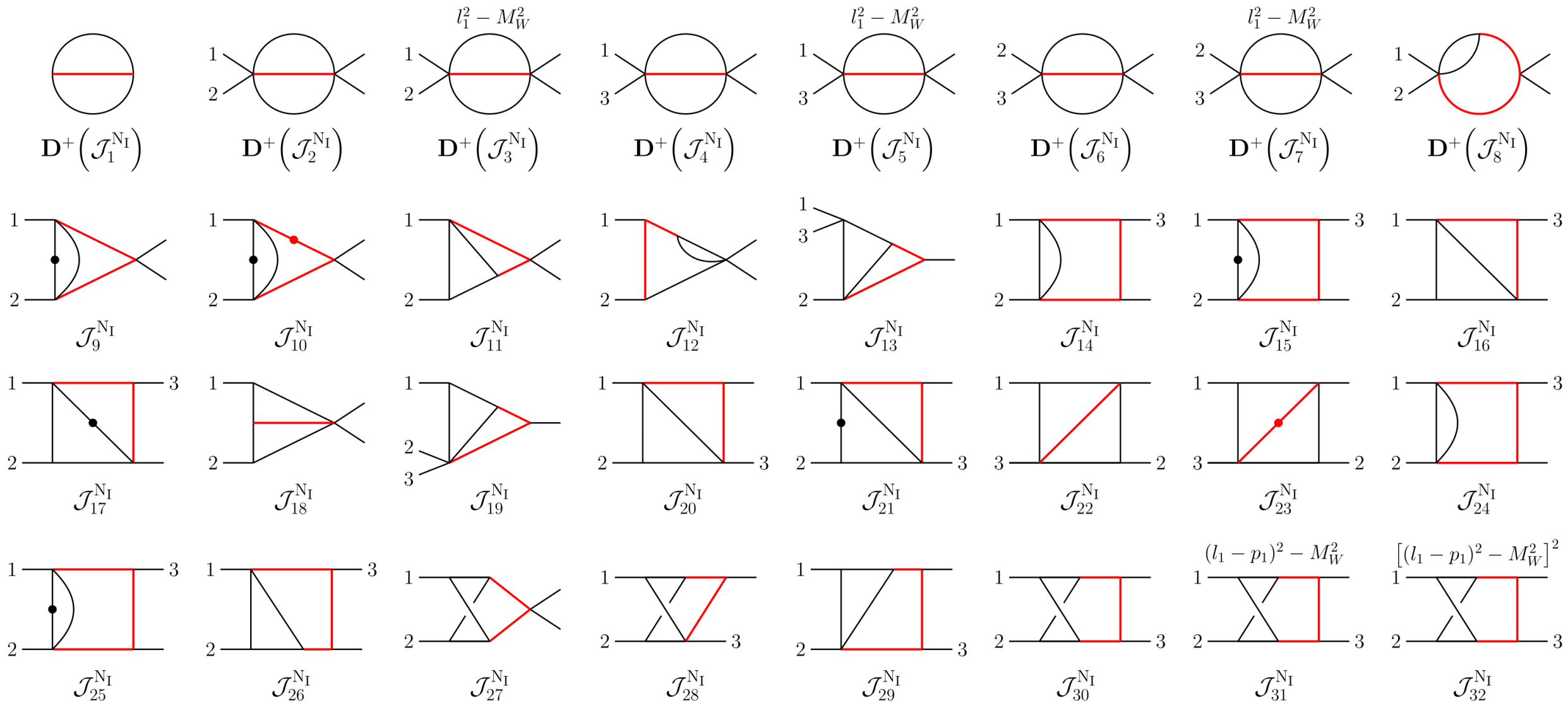
$$\vec{\mathcal{G}}^{(1)}(\{x_i\}) = \int_{\gamma} d\mathcal{A} \vec{\mathcal{G}}^{(0)}(\{x_i^0\}) + \vec{\mathcal{G}}^{(1)}(\{x_i^0\})$$

$$\vec{\mathcal{G}}^{(2)}(\{x_i\}) = \int_{\gamma} d\mathcal{A} d\mathcal{A} \vec{\mathcal{G}}^{(0)}(\{x_i^0\}) + \int_{\gamma} d\mathcal{A} \vec{\mathcal{G}}^{(1)}(\{x_i^0\}) + \vec{\mathcal{G}}^{(2)}(\{x_i^0\})$$

$$\vec{\mathcal{G}}^{(3)}(\{x_i\}) = \int_{\gamma} d\mathcal{A} d\mathcal{A} d\mathcal{A} \vec{\mathcal{G}}^{(0)}(\{x_i^0\}) + \int_{\gamma} d\mathcal{A} d\mathcal{A} \vec{\mathcal{G}}^{(1)}(\{x_i^0\}) + \int_{\gamma} d\mathcal{A} \vec{\mathcal{G}}^{(2)}(\{x_i^0\}) + \vec{\mathcal{G}}^{(3)}(\{x_i^0\})$$

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Efficient semi-numerical implementation thanks to the recursive nature of the iterated integrals.



The Canonical base is obtained via Magnus Series Expansion (*Blanes, Casas, Oteo, Ros, 2008*). Here we show the 7-propagators family:

$$\mathcal{G}_{30}^{\text{NI}} = -2\epsilon^2 \left(2(x-1)\mathcal{F}_{28} + (1-x-2y+4xy)\mathcal{F}_{30} + \mathcal{F}_{31} \right),$$

$$\mathcal{G}_{31}^{\text{NI}} = -\epsilon^2 \frac{\sqrt{r_1}}{2} \left(2(x-1)\mathcal{F}_{28} + (1-x)\mathcal{F}_{30} + (1-2x)\mathcal{F}_{31} \right),$$

$$\begin{aligned} \mathcal{G}_{32}^{\text{NI}} = & \epsilon^2 (y\mathcal{F}_{30} + \mathcal{F}_{31} + \mathcal{F}_{32}) + \frac{1}{8x} (15\mathcal{G}_1^{\text{NI}} + 8\mathcal{G}_2^{\text{NI}} - 8\mathcal{G}_3^{\text{NI}} + 10\mathcal{G}_4^{\text{NI}} - 17\mathcal{G}_5^{\text{NI}} \\ & + 4\mathcal{G}_6^{\text{NI}} - 4\mathcal{G}_7^{\text{NI}} - 24\mathcal{G}_{13}^{\text{NI}} - 12\mathcal{G}_{16}^{\text{NI}} - 2\mathcal{G}_{17}^{\text{NI}} + 8\mathcal{G}_{18}^{\text{NI}} - 8\mathcal{G}_{19}^{\text{NI}} + 20\mathcal{G}_{20}^{\text{NI}} \\ & + 2\mathcal{G}_{21}^{\text{NI}} - 24\mathcal{G}_{22}^{\text{NI}} - 8\mathcal{G}_{23}^{\text{NI}}). \end{aligned}$$

The differential equation is expressed in terms of the variables:

$$x = -\frac{p_1 \cdot p_3}{p_1 \cdot p_2} \quad y = \frac{M_W^2}{2p_1 \cdot p_2}$$

➤ Non-Rational letters:

$$r_1 = 1 - 4y$$

$$r_2 = x^2(1 - 4y) + 2xy + y^2$$

$$r_3 = x^2(1 - 4y) + x(6y - 2) + (y - 1)^2$$

$$r_4 = -x(4xy - x - 4y)$$

$$r_5 = xy(x - 1)$$

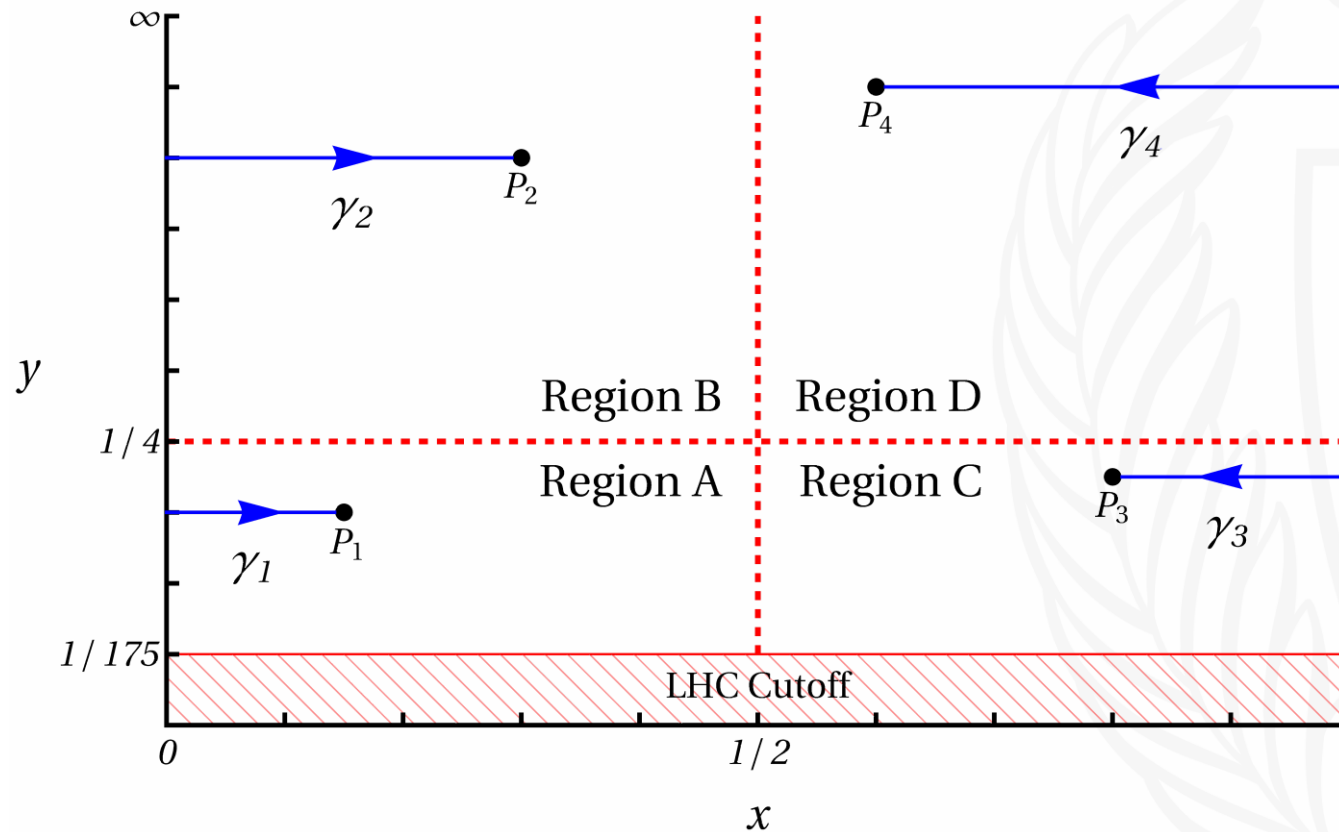
$$r_6 = -(x - 1)(x(4y - 1) + 1)$$

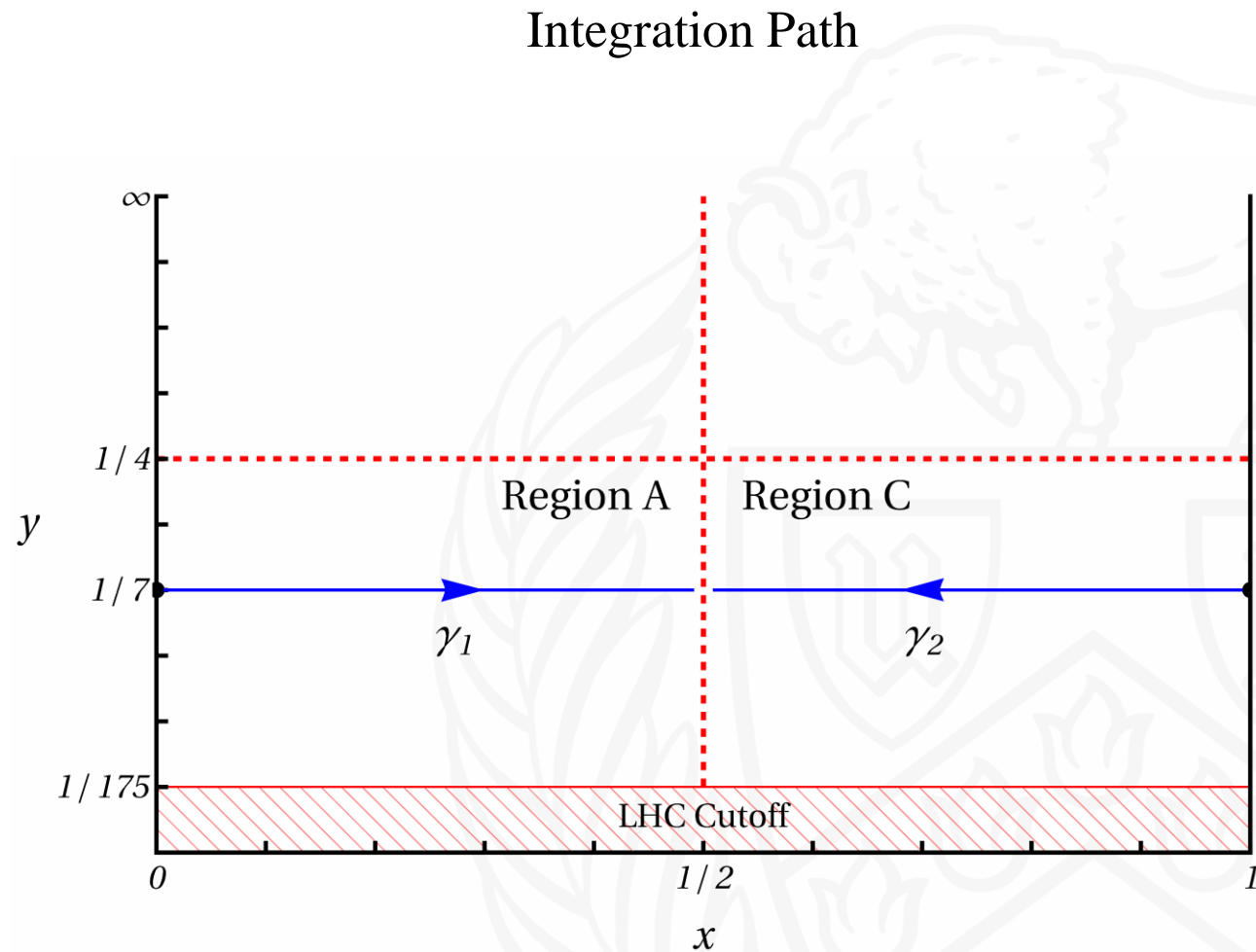
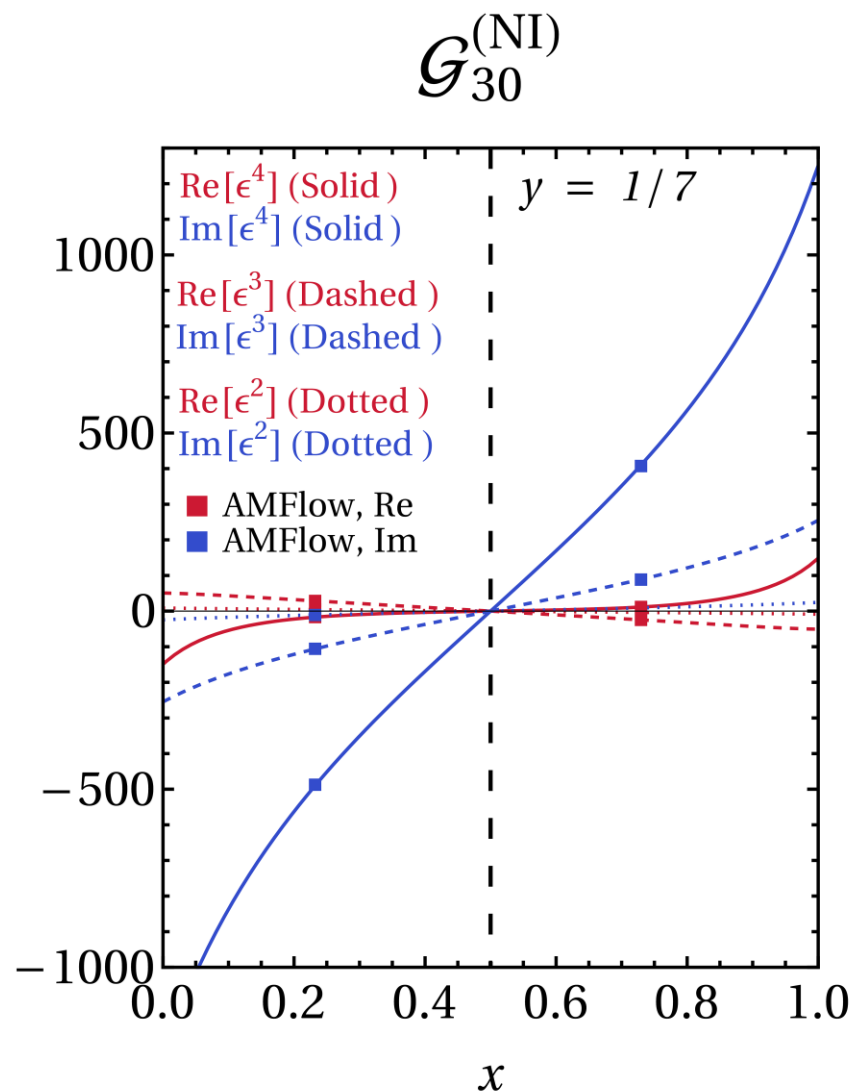
➤ For efficiency reasons, we only rationalize r_1 .

➤ Mixed Chen/GPL representation.

Region A : $[s > 4M_W^2, u < t], \quad y = \frac{z_A}{2} \left(1 - \frac{z_A}{2}\right)$ Region B : $[s \leq 4M_W^2, u < t], \quad y = \frac{1 - 2z_B + z_B^2}{4z_B^2}$

Region C : $[s > 4M_W^2, u > t], \quad y = \frac{z_A}{2} \left(1 - \frac{z_A}{2}\right)$ Region D : $[s \leq 4M_W^2, u > t], \quad y = \frac{1 - 2z_B + z_B^2}{4z_B^2}$





Summary:

- Evaluation of the MIs for the electroweak corrections to $gg \rightarrow \gamma\gamma$ for massless quark loops.
- Efficient calculation at any kinematic point.
- Versatility on the alphabet: rational vs non-rational letters.
- Solution available for every topology.

Future Directions:

- Evaluation of the amplitude and differential distributions (pheno analysis).
- Inclusion of the top-quark mediated diagrams.
- Complete analysis of the interference order $\mathcal{O}(\alpha)$.

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Grazie!



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Backup Slides



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- We define the color-stripped amplitude at 2 loops.
- The number of independent tensor structures corresponds to the physical helicity configurations.
- The projectors are built from the tensor structures.
- The integral occurrences define the Laporta basis for each topology.

$$\mathcal{A}(s, t) = \sum_{i=1}^8 \mathcal{F}_i T_i$$

$$T_1 = \epsilon_1 \cdot p_3 \ \epsilon_2 \cdot p_1 \ \epsilon_3 \cdot p_1 \ \epsilon_4 \cdot p_2 ,$$

$$T_2 = \epsilon_1 \cdot p_3 \ \epsilon_2 \cdot p_1 \ \epsilon_3 \cdot \epsilon_4, \quad T_3 = \epsilon_1 \cdot p_3 \ \epsilon_3 \cdot p_1 \ \epsilon_2 \cdot \epsilon_4,$$

$$T_4 = \epsilon_1 \cdot p_3 \ \epsilon_4 \cdot p_2 \ \epsilon_2 \cdot \epsilon_3, \quad T_5 = \epsilon_2 \cdot p_1 \ \epsilon_3 \cdot p_1 \ \epsilon_1 \cdot \epsilon_4,$$

$$T_6 = \epsilon_2 \cdot p_1 \ \epsilon_4 \cdot p_2 \ \epsilon_1 \cdot \epsilon_3, \quad T_7 = \epsilon_3 \cdot p_1 \ \epsilon_4 \cdot p_2 \ \epsilon_1 \cdot \epsilon_2,$$

$$T_8 = \epsilon_1 \cdot \epsilon_2 \ \epsilon_3 \cdot \epsilon_4 + \epsilon_1 \cdot \epsilon_4 \ \epsilon_2 \cdot \epsilon_3 + \epsilon_1 \cdot \epsilon_3 \ \epsilon_2 \cdot \epsilon_4$$

- Issues when trying to generalize γ_5 to d-dimensions:

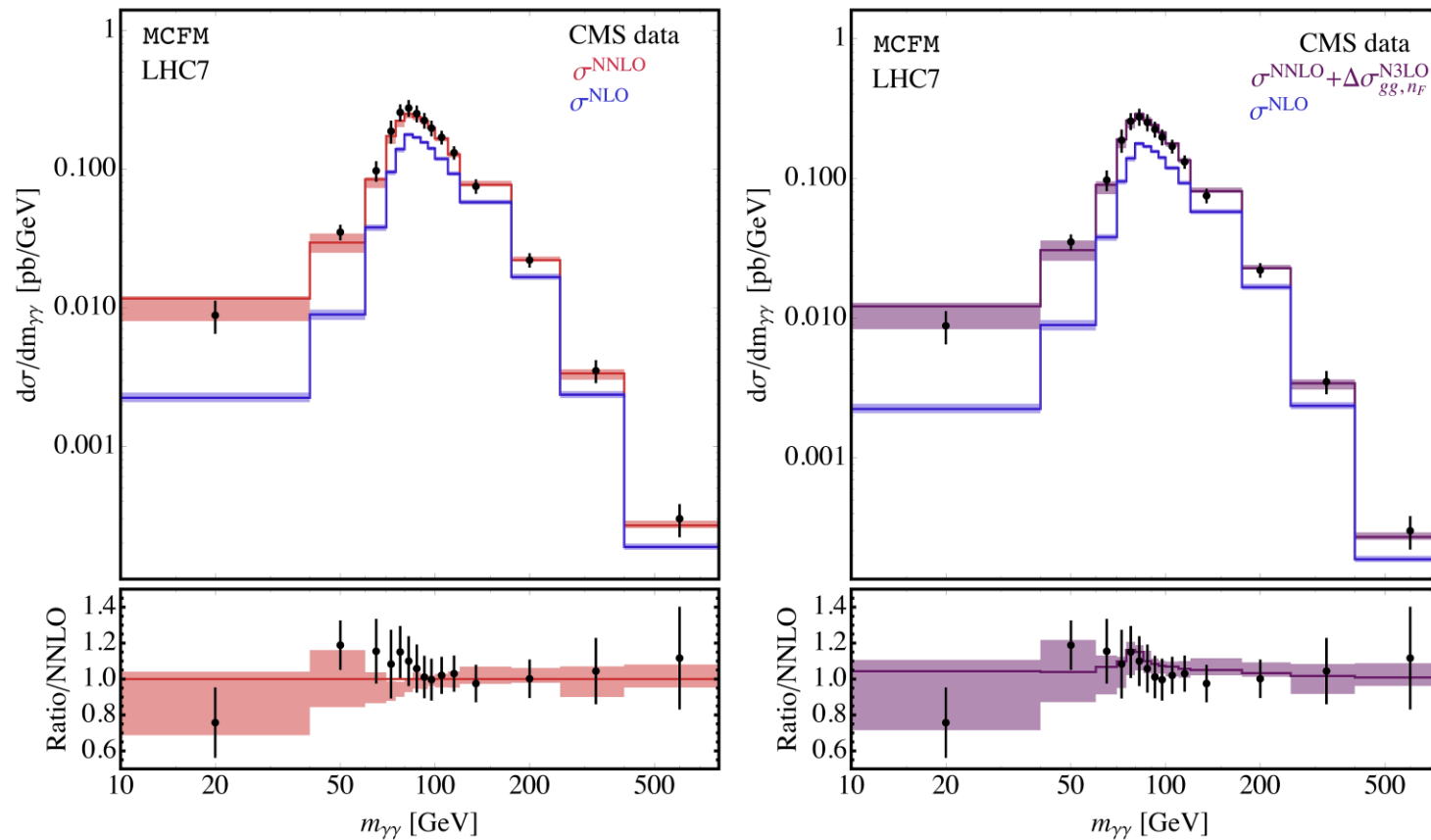
$$\begin{aligned}\{\gamma_d^\mu, \gamma_5\} &= 0 & 0 \leq \mu \leq d-1 \\ \text{Tr}(\gamma_d^\mu \gamma_d^\nu \gamma_d^\rho \gamma_d^\sigma \gamma_5) &= 4i\epsilon^{\mu\nu\rho\sigma} & \mu, \nu, \rho, \sigma \in \{0, 1, 2, 3\}\end{aligned}$$

- Anti-commutation relation and cyclicity of the trace can't be satisfied at the same time.
- Solution: 't Hooft-Veltman prescription (*'t Hooft, Veltman, 1972*).

$$\gamma_5 = -\frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma$$

- No new Mis are introduced due to the Levi-Civita tensor (only amplitude issue).
- No new Tensor structure either, the 4-contractions vanish, and the others are linearly dependent.

Seizable corrections coming from the gg channel, even at NNLO.



Campbell, Ellis, Williams, 2016

The kinematic information is encoded in the alphabet of the differential equation:

$$d\mathcal{A} = \sum_k \mathcal{M}_k d \log \eta_k$$

The solution for rational letters is expressed in terms of Goncharov Polylogarithms (GPLs):

$$\begin{aligned} G(a; x_0) &= \int_0^{x_0} \frac{dt}{t - a}, \\ G(a_n, \dots, a_1; x_0) &= \int_0^{x_0} G(a_{n-1}, \dots, a_1; x_0) \frac{dt}{t - a_n}, \\ G(\vec{0}_n; x_0) &= \frac{1}{n!} \log(x_0)^n \end{aligned}$$

$$\begin{aligned}
 \mathcal{J}_1 &= \mathbf{D}^-(I_{111000000}^{N_I}), & \mathcal{J}_2 &= \mathbf{D}^-(I_{011001000}^{N_I}), & \mathcal{J}_3 &= \mathbf{D}^-(I_{(-1)11001000}^{N_I}), & \mathcal{J}_4 &= \mathbf{D}^-(I_{010100100}^{N_I}), \\
 \mathcal{J}_5 &= \mathbf{D}^-(I_{(-1)10100100}^{N_I}), & \mathcal{J}_6 &= \mathbf{D}^-(I_{001000110}^{N_I}), & \mathcal{J}_7 &= \mathbf{D}^-(I_{(-1)01000110}^{N_I}), & \mathcal{J}_8 &= \mathbf{D}^-(I_{111001000}^{N_I}), \\
 \mathcal{J}_9 &= I_{110201000}^{N_I}, & \mathcal{J}_{10} &= I_{210201000}^{N_I}, & \mathcal{J}_{11} &= I_{111101000}^{N_I}, & \mathcal{J}_{12} &= I_{111011000}^{N_I}, \\
 \mathcal{J}_{13} &= I_{111100100}^{N_I}, & \mathcal{J}_{14} &= I_{110101100}^{N_I}, & \mathcal{J}_{15} &= I_{110201100}^{N_I}, & \mathcal{J}_{16} &= I_{011101100}^{N_I}, \\
 \mathcal{J}_{17} &= I_{012101100}^{N_I}, & \mathcal{J}_{18} &= I_{111100010}^{N_I}, & \mathcal{J}_{19} &= I_{111000110}^{N_I}, & \mathcal{J}_{20} &= I_{101100110}^{N_I}, \\
 \mathcal{J}_{21} &= I_{102100110}^{N_I}, & \mathcal{J}_{22} &= I_{011100110}^{N_I}, & \mathcal{J}_{23} &= I_{011100210}^{N_I}, & \mathcal{J}_{24} &= I_{101001110}^{N_I}, \\
 \mathcal{J}_{25} &= I_{102001110}^{N_I}, & \mathcal{J}_{26} &= I_{111101100}^{N_I}, & \mathcal{J}_{27} &= I_{111101010}^{N_I}, & \mathcal{J}_{28} &= I_{111100110}^{N_I}, \\
 \mathcal{J}_{29} &= I_{111001110}^{N_I}, & \mathcal{J}_{30} &= I_{111101110}^{N_I}, & \mathcal{J}_{31} &= I_{1111(-1)1110}^{N_I}, & \mathcal{J}_{32} &= I_{1111(-2)1110}^{N_I}
 \end{aligned}$$

Pre-canonical basis satisfy a linear DEQ in ϵ . The canonical basis is extracted via subsequent application of the Magnus Exponent method.