Some applications of the eikonal model with Coulomb and curvature corrections in *pp* and $\bar{p}p$ scattering

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Outline

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III. Applications

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- The differential cross section in the dip region

IV. Conclusions

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I. Introduction

- Coulomb and nuclear interactions in the scattering of charged particles has been studied by many authors.
- Particular emphasis is placed on the use of Coulomb-nuclear interference (CNI) effects to determine the real part of the nuclear scattering amplitude in pp and $\bar{p}p$ scattering at high center-of-mass energies W and small squares of the momentum transfer $q^2 = |t|$.
- Cahn's work [Z. Phys. C 15, 253 (1982)] and Kundrát and Lokajiček's work [Z. Phys. C 63, 619 (1994)] seem to have become standard in the analysis of CNI effects at high energies.
- They separate the Coulomb and pure nuclear effects in a spin-independent scattering amplitude with its components expressed in terms of convolutions involving the nuclear and Coulomb amplitudes with the effects of the proton electromagnetic form factors included.
- Their result is rather cumbersome to use.

I. Introduction

- In a recent paper [PRD **102**, 036025 (2020)], we presented a very simple way of calculating Coulomb and form-factor corrections to the *pp* scattering amplitude in the context of an eikonal model.
- Our eikonal approach was based on a realistic model which fits the *pp* and $\bar{p}p$ data from 4.5 GeV to cosmic ray energies.
- Our approach was much simpler than that of Cahn and Kundrát & Lokajiček.

- ❑ Here, we modify the approach discussed in our recent paper and use it to evaluate the basic parameters *B*, ρ, and σ_{tot} at W= 53 GeV, 62.3 GeV, 8 TeV and 13 TeV. The results of the basic parameters calculated using our simple eikonal approach agree well with the values determined in other analyses.
- ❑ We also investigate the differential cross sections in the dip region for *pp* and $\bar{p}p$ elastic scattering at W = 53 GeV and 1.96 TeV. We find that Coulomb effects are significant there and must be taken into account in attempts to detect odderon effects from differences in the *pp* and $\bar{p}p$ cross sections.

II. Simple Eikonal approach for CNI effects

- In the absence of significant spin effects, the spin-averaged differential cross section for *pp* and $\bar{p}p$ scattering can be written in terms of a **single spin-independent amplitude**

$$f(s,q^2) = i\int_0^\infty db\, b\, \left(1 - e^{2i\left(\delta_c^{tot}(b,s)+\delta_N(b,s)\right)}\right) J_0(qb) \quad (1)$$

full Coulomb phase shift including the effects of the finite charge structure of the proton $\delta_c^{tot}(b,s) = \delta_c(b,s) + \delta_c^{FF}(b,s)$

The nuclear phase shift

- where $q^2 = -t$: the square of the invariant momentum transfer,

  b : the impact parameter.

- Eq. (1) can be rearranged in the form

$$f(s,q^2) = f_c(s,q^2) + f_c^{FF}(s,q^2) + f_{N,c}(s,q^2) \qquad (2)$$

Simple Eikonal approach for CNI effects

$$f(s,q^2) = f_c(s,q^2) + f_c^{FF}(s,q^2) + f_{N,c}(s,q^2) \qquad (2)$$

Coulomb amplitude without form factors (FFs)

Term accounts for the effects of the FFs on the Coulomb scattering

Term includes for the effects of the nuclear scattering as modified by the Coulomb and FFs effects

$$f_c(s,q^2) = i\int_0^\infty db\, b\,(1 - e^{2i\delta_c(s,b)})J_0(qb)$$

$$f_c^{FF}(s,q^2) = i\int_0^\infty db\, b e^{2i\delta_c(s,b)}(1 - e^{2i\delta_c^{FF}(s,b)})J_0(qb)$$

$$f_{N,c}(s,q^2) = i\int_0^\infty db\, b e^{2i\delta_c(s,b)+2i\delta_c^{FF}(s,b)}\left(1 - e^{2i\delta_N(s,b)}\right)J_0(qb)$$

The pure nuclear amplitude

$$f_N(s,q^2) = i\int_0^\infty db\, b\,(1 - e^{2i\delta_N(s,b)})J_0(qb)$$

In Eq. (2), we can divide out a common Coulomb phase from all terms; f_c is the real.

Simple Eikonal approach for CNI effects

For $\bar{p}p$ scattering, $\eta \rightarrow -\eta$

- For *pp* scattering, we find (for $\eta / F_Q^2(q^2) << 1$)

$$f_c(s,q^2) + f_c^{FF}(s,q^2) = -\frac{2\eta}{q^2} F_Q^2(q^2) e^{i\Phi_{c,FF}}$$

$$F_Q(q^2) = \frac{\mu^4}{(q^2+\mu^2)^2} \qquad \Phi_{c,FF} \sim -\eta \left(\frac{(q^2+\mu^2)^4}{\mu^8} - 1\right) \ln \frac{q^2}{q^2+\mu^2}$$

- We can separate $f_{N,c}(s,q^2)$ into two terms

$$f_{N,c}(s,q^2) = f_{N,c}^{Corr}(s,q^2) + f_N(s,q^2)$$

The pure nuclear amplitude

A single small term isolates the pieces of the full amplitude which involve both Coulomb-plus-form-factor and nuclear term

$\eta \sim \alpha = \frac{1}{137}; \mu^2 = 0.71\text{GeV}^2$

Simple Eikonal approach for CNI effects

- For very small q^2 ($q^2 \leq 0.2$ GeV^2), the real and imaginary parts of $f_{N,c}^{Corr}(s, q^2)$ can be fitted using the following parametrization.
- For example, the real part of $f_{N,c}^{Corr}$ is

$$Ref_{N,c}^{Corr}(s,t) = -(a_0 + b_0 \ln p) \ln t + (a_1 + b_1 \ln p) + (a_2 + b_2 \ln p)\, t + (a_3 + b_3 \ln p)\, t^2$$

where a_i , b_i (i=0,1,2, 3) are the parameters, $p = \sqrt{W^2/4 - m^2}$ is the proton momentum, and m is the proton mass.

Simple Eikonal approach for CNI effects

- We can now write the full amplitude in the form

$$f(s,q^2) = f_1(s,q^2) + f_N(s,q^2)$$

The pure nuclear amplitude

Coulomb amplitude and the mixed Coulomb-nuclear corrections

$$f_1(s,q^2) = f_c(s,q^2) + f_c^{FF}(s,q^2) + f_{N,c}^{Corr}(s,q^2)$$

- With our normalization, the different elastic scattering amplitude is

$$\frac{d\sigma}{dq^2}(s,q^2) = \pi|f(s,q^2)|^2$$

$$= \pi\left(|f_1|^2 + \frac{2|f_1||f_N|}{(1+\rho^2)^{1/2}}(sin\Phi_1 + \rho cos\Phi_1) + |f_N|^2\right)$$

$\rho(s,q^2) = \mathrm{Re}\, f_N(s,q^2)/\ \mathrm{Im}\, f_N(s,q^2)$

Φ_1 is the phase of $f_1(s,q^2)$

III. Applications

Fits to the differential cross sections:

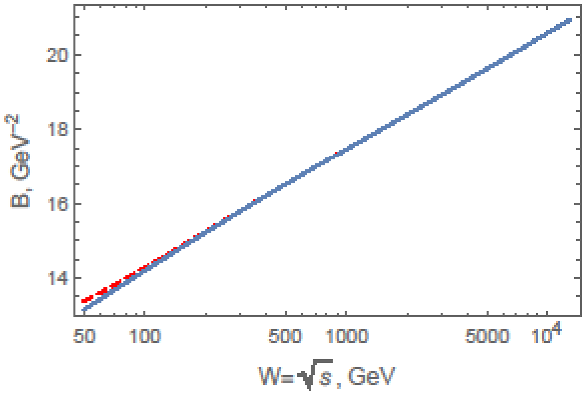
- Consider a model used frequently in the analysis of experimental data in which the purely nuclear part of the differential cross section is approximated as

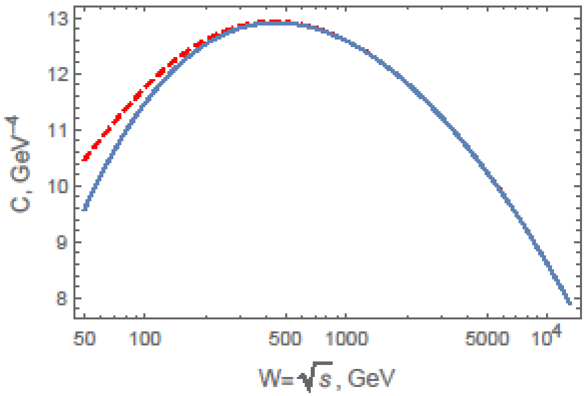
$$\frac{d\sigma}{dq^2}(s,q^2) \approx Ae^{-Bq^2+Cq^4-Dq^6+\cdots}$$

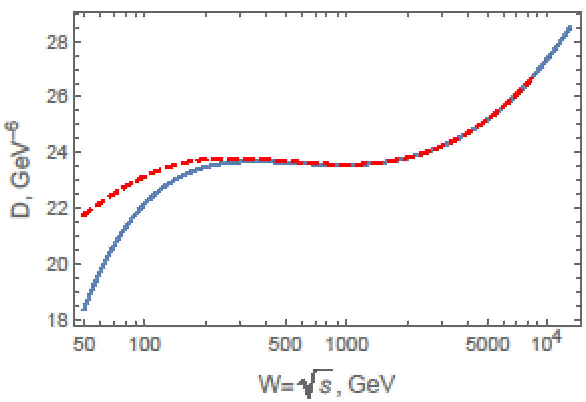
- Here B is the usual slope parameter and the parameters C, D, … which introduce curvature in $\frac{d\sigma}{dq^2}$.

- Taking the square root of $\frac{d\sigma}{dq^2}$ and introducing the phase of nuclear amplitude $\Phi_N = \frac{\pi}{2} - \arctan \rho(s,q^2)$, we have

$$\sqrt{\pi}\, f_N(s,q^2) = \sqrt{A}\, e^{-\frac{1}{2}(Bq^2-Cq^4+Dq^6+\cdots)}.$$







Applications

- We have calculated the curvature parameters *C* and *D* using our eikonal model [PRD **93**, 114009 (2016)].
- This reduces the number of free parameters by two relative to those used in other analysis of this type.
- We also plot the values of *B*, *C*, and *D*, calculated using our eikonal approach, versus *W* for the local momentum transfer $q_0^2 = 10^{-6}\,\mathrm{GeV}^2$ for *pp* (solid blue curves) and $\bar{p}p$ – (dashed red curves).
- The behavior at the lower energies is largely the results of the important of the Regge-like terms in the Eikonal functions at lower energies.

Applications

- Our eikonal results give

$$C = 9.779\ \text{GeV}^{-4},\ D = 18.83\ \text{GeV}^{-6}\ \text{at W} = 53\ \text{GeV}$$
$$C = 10.29\ \text{GeV}^{-4},\ D = 19.98\ \text{GeV}^{-6}\ \text{at W} = 62.3\ \text{GeV}$$
$$C = 9.176\ \text{GeV}^{-4},\ D = 26.53\ \text{GeV}^{-6}\ \text{at W} = 8\ \text{TeV}$$
$$C = 7.896\ \text{GeV}^{-4},\ D = 28.50\ \text{GeV}^{-6}\ \text{at W} = 13\ \text{TeV.}$$

- For $\rho(s, q^2)$ constant, we perform the least squares fits to the data for the differential cross sections (using data up to a maximum value q^2_{max}).
- The total cross section was derived via the optical theorem $\sigma^2_{tot} = \frac{16\pi A}{1+\rho^2}$.
- Table I shows the results of our fits to the ISR data at 53 GeV and 62.3 GeV and to the TOTEM data at 8 TeV and 13 TeV.

Applications

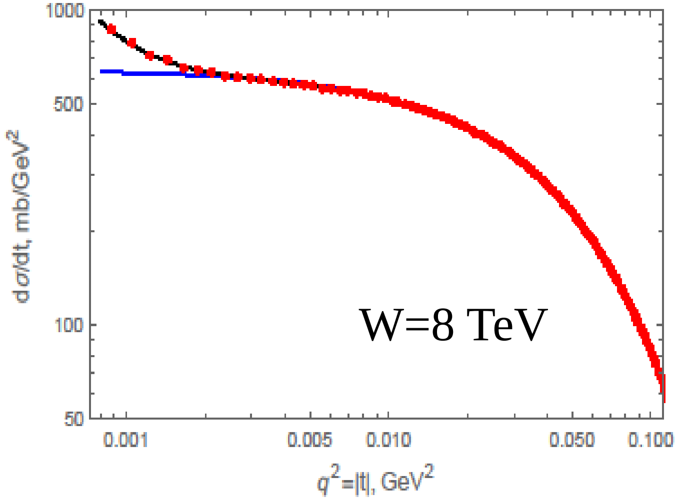
Results of the fits to the differential cross sections

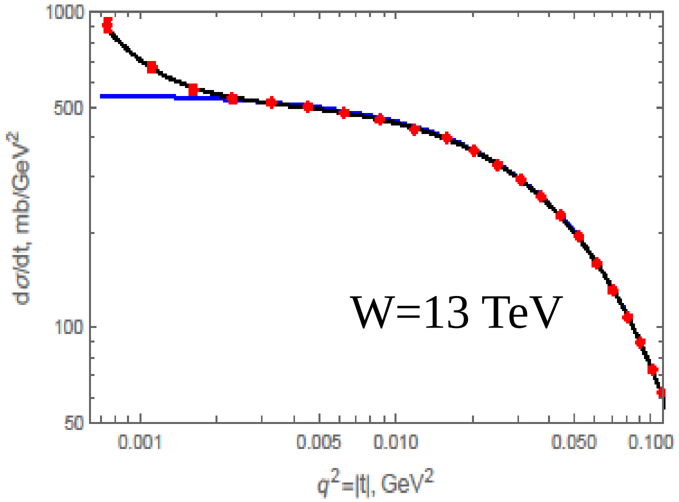
TABLE I. The results of our fits to the ISR data at 53 GeV and 62.3 GeV and to the TOTEM data at 8 TeV and 13 TeV. The Coulomb and Coulomb-hadronic interference contributions to the scattering were included in the fit. A, B, and ρ are the corresponding parameters in fits which included the curvature parameters C and D, with $(d\sigma/dq^2)_N \approx A\exp(-Bq^2 + Cq^4 - Dq^6)$. The parameters C and D were calculated using the comprehensive eikonal fit to the high energy pp and $\bar{p}p$ data.

(GeV^2)	W (GeV)	d.o.f	χ^2/d.o.f.	A (mb/GeV^2)	B (GeV^{-2})	ρ	σ_{tot}(mb)
1) q^2_{max} =0.07	13000	76	0.869	647.2 ± 0.7	21.23 ± 0.03	0.095 ± 0.004	112.0 ± 0.1
	8000	15	0.775	552.3 ± 2.9	20.68 ± 0.12	0.105 ± 0.020	103.4 ± 0.3
2) q^2_{max} =0.10	13000	93	0.956	645.8 ± 0.6	21.16 ± 0.02	0.091 ± 0.004	112.0 ± 0.1
	8000	18	0.710	551.5 ± 2.5	20.63 ± 0.08	0.102 ± 0.019	103.4 ± 0.3
	62.3	19	1.448	97.56 ± 0.72	13.40 ± 0.18	0.071 ± 0.018	43.59 ± 0.17
	53	18	2.048	92.98 ± 0.21	13.40 ± 0.07	0.082 ± 0.002	42.52 ± 0.05
3) q^2_{max} =0.15	13000	116	1.290	644.1 ± 0.5	21.09 ± 0.01	0.085 ± 0.004	111.9 ± 0.1
	8000	23	1.330	547.3 ± 2.1	20.43 ± 0.06	0.086 ± 0.018	103.1 ± 0.3

The results of the basic parameters from the fits are consistent with the values determined in other analyses.

Applications





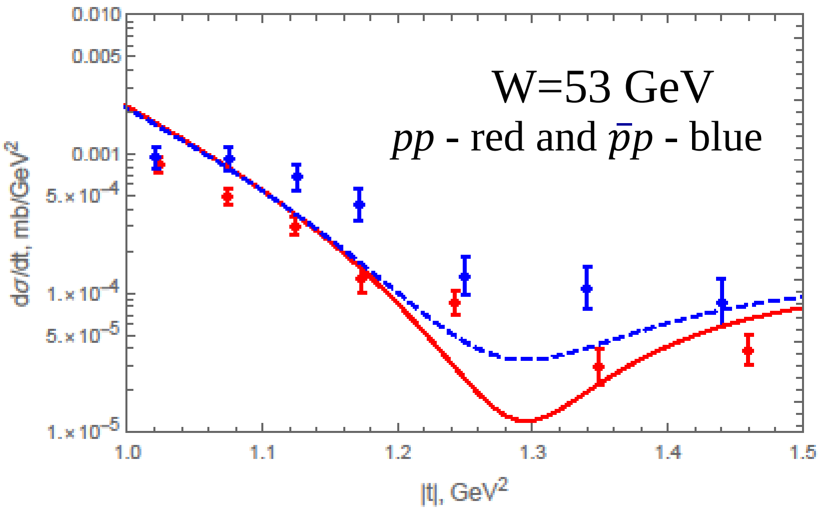
- $\frac{d\sigma}{dq^2}$ for the TOTEM data at 8 TeV and 13 TeV.
- $\frac{d\sigma}{dq^2}$ from the fit (black) and the purely nuclear result of the fit (blue).
- Data with their statistical errors are red.

Applications

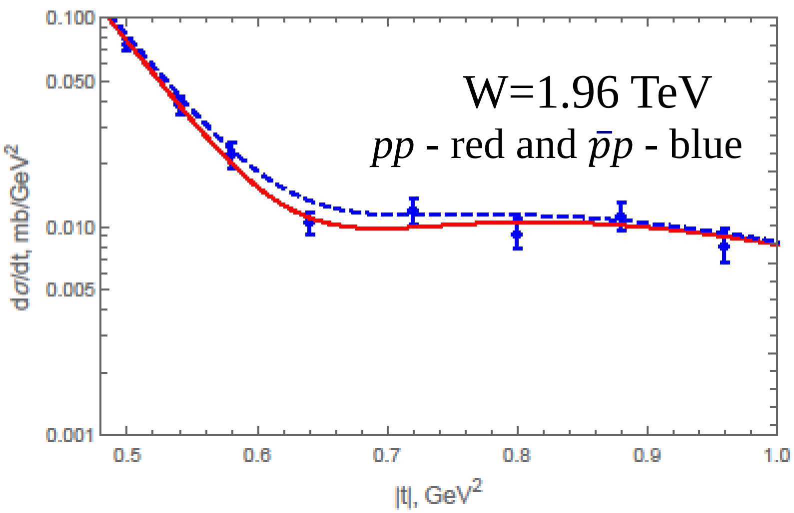
- Doing fits with our variable $\rho(s,q^2) = \rho(s)\,\frac{1-q^2/q_R^2}{1-q^2/q_I^2}$ (where q_R and q_I are the locations of zeros in the real and imaginary parts of $f(s,q^2)$) for $q^2 < 0.1$ GeV^2 , we find that the fit results and the fitting parameters do not change noticeably relative to the present results.
- This is because the sensitivity to the Coulomb-nuclear interference in the fitting is only at very small q^2.

III. Applications

The differential cross section $\frac{d\sigma}{dq^2}$ in the dip region



- At W = 53 GeV, the theoretical curves show a dip from the first diffraction zero in the dominant imaginary part of the nuclear scattering amplitude. This is predicted to be at $|t|=1.295\ \mathrm{GeV}^2$.



- At W = 1.96 TeV, the theoretical curves do not clearly show a dip. The predicted location of the zero in the imaginary part of the nuclear scattering amplitude is $|t|=0.683\ \mathrm{GeV}^2$.
- The are currently no *pp* data.
- We find that the Coulomb effects in the dip region are still significant on the scale given in The D0 and TOTEM collaboration paper [PRL 127, 062003 (2021)].

IV. Conclusions

- The results of the basic parameters calculated using our simple eikonal approach agree well with the values determined in other analyses.

- We find that Coulomb effects are significant in the dip region and must be taken into account in attempts to detect odderon effects from differences in the *pp* and $\bar{p}p$ cross sections.

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Extra slides. Background

- Consider p-p and pbar-p scattering at high energies. Neglecting the small effects of the nucleon spins, we describe the scattering amplitude and cross sections in an impact parameter.
- The spin-independent eikonal scattering amplitude and differential elastic scattering amplitude are

$$f(s,t) = i\int_0^\infty dbb\,(1-e^{i\chi(b,s)})J_0(b\sqrt{-t}), \qquad (1)$$

$$\frac{d\sigma}{dt}(s,t) = \pi\left|f(s,t)\right|^2. \qquad (2)$$

$s = W^2 = 4(p^2 + m^2)$ - square of total energy in the C.M. system,
p – the C.M. momentum of either incident particle,
b = j/p , j is the partial wave angular momentum,
$t = -2p^2 (1 - \cos\theta)$ – the invariant 4 - momentum transfer.

Extra slides. Eikonal Model

$\chi(b,s) = \chi_R + i\,\chi_I$ - the eikonal function

$$\sigma_{elas}(s) = 2\pi\int_0^\infty dbb\left|1-e^{i\chi}\right|^2 \quad , \qquad (3)$$

$$\sigma_{tot}(s) = 4\pi\,\mathrm{Im}\,f(s,0) = 4\pi\int_0^\infty dbb\,(1-\cos\chi_R e^{-\chi_I}), \quad (4)$$

$$\sigma_{inelas}(s) = \sigma_{tot} - \sigma_{elas} = 2\pi\int_0^\infty dbb\,(1-e^{-2\chi_I}), \qquad (5)$$

$$\rho = \mathrm{Re}\,f(s,0)\,/\,\mathrm{Im}\,f(s,0)$$

$$= -\int_0^\infty dbb\,e^{-\chi_I}\sin\chi_R \,/ \int_0^\infty dbb\,(1-\cos\chi_R e^{-\chi_I}), \qquad (6)$$

$$\mathrm{B} = \frac{d}{dt}\left[\ln\frac{d\sigma}{dt}(s,t)\right]$$

$$\approx \frac{1}{2}\int_0^\infty dbb^3\,(1-e^{-\chi_I})\,/\int_0^\infty dbb(1-e^{-\chi_I}). \qquad (7)$$

Extra slides. Eikonal Fit: An update

$$\chi_{p\bar{p}}(b,W) = (\chi_E(b,W) + \chi_O(b,W))/2\ , \quad (8)$$

$$\chi_{pp}(b,W) = (\chi_E(b,W) - \chi_O(b,W))/2\ . \quad (9)$$

where

$$\chi_E(b,W) = i\left[\sigma_{qq}(\tilde{W})A(b,\mu_{qq}) + \sigma_{qg}(\tilde{W})A(b,\mu_{qg}) + \sigma_{gg}(\tilde{W})A(b,\mu_{gg})\right],\ (10)$$

$$\chi_O(b,W) = -C_5\Sigma_{gg}\left(\frac{m_0}{\tilde{W}}\right)^{2-2\alpha_1} A(b,\mu_{odd})\ . \quad (11)$$

$$\tilde{W} = We^{-i\pi/4}$$

A(b, μ) – overlap functions, σ_{ij} – describing interactions between components i and j.

- We fit data on total cross sections for W ≥ 5.3 GeV and the elastic cross sections, ρ and B for energies W ≥ 10 GeV.
- Fix σ_{tot} at W = 4 GeV to match the results obtained from low-energy data.

Extra slides. Eikonal Fit: An update

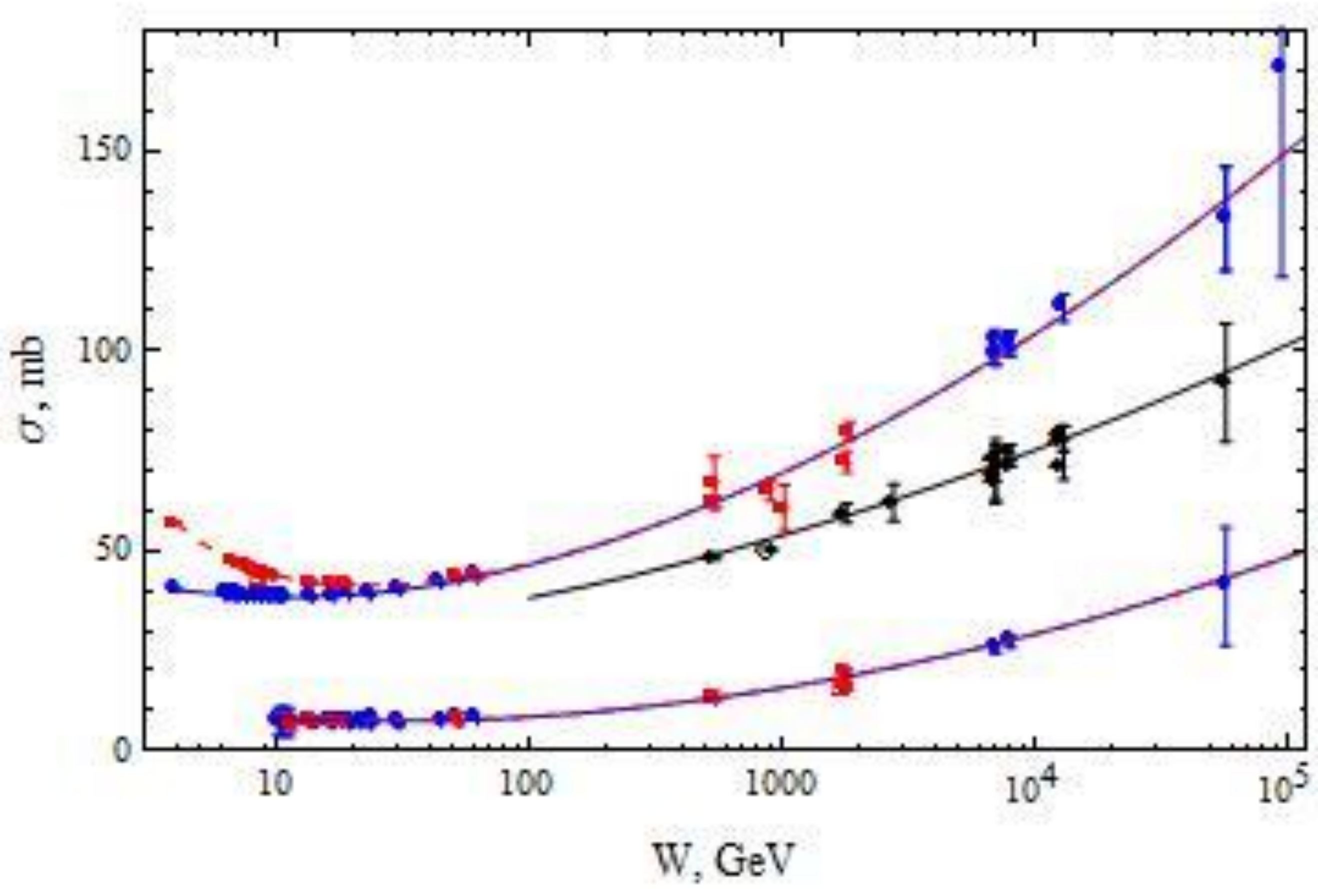
- 9 parameter fit
- 189 datum points
- Seive algorithm eliminates 14 outliers
- dof = 166; χ^2 /dof = 0.985
- $\mathcal{R}$ χ^2 /dof =1.08
- Fixed parameters

$$m_0 = 0.6 \text{ GeV}, W_0 = 4 \text{ GeV},$$
$$\mu_{\text{gg}} = 0.705 \text{ GeV}, \mu_{\text{qq}} = 0.89 \text{ GeV},$$
$$\mu_{\text{odd}} = 0.6 \text{ GeV}, \alpha_{\text{S}} = 0.5,$$
$$\Sigma_{gg} = 19.635 \text{ GeV}^{-2}.$$

- The fitted parameters

$$C_0 = 6.790 \pm 0.07$$
$$C_1 = 26.80 \pm 0.02$$
$$C_2 = -0.187 \pm 0.0004$$
$$C_3 = -2.480 \pm 0.004$$
$$C_4 = 13.75 \pm 0.013$$
$$C_5 = -26.13 \pm 0.02$$
$$\alpha_1 = 0.3188 \pm 0.0003$$
$$\alpha_2 = 0.4866 \pm 0.0001$$
$$\beta_1 = 0.1474 \pm 0.0002$$

Extra slides. Comprehensive Fit: An update



Fits, top to bottom, to the total, inelastic, and elastic scattering cross sections