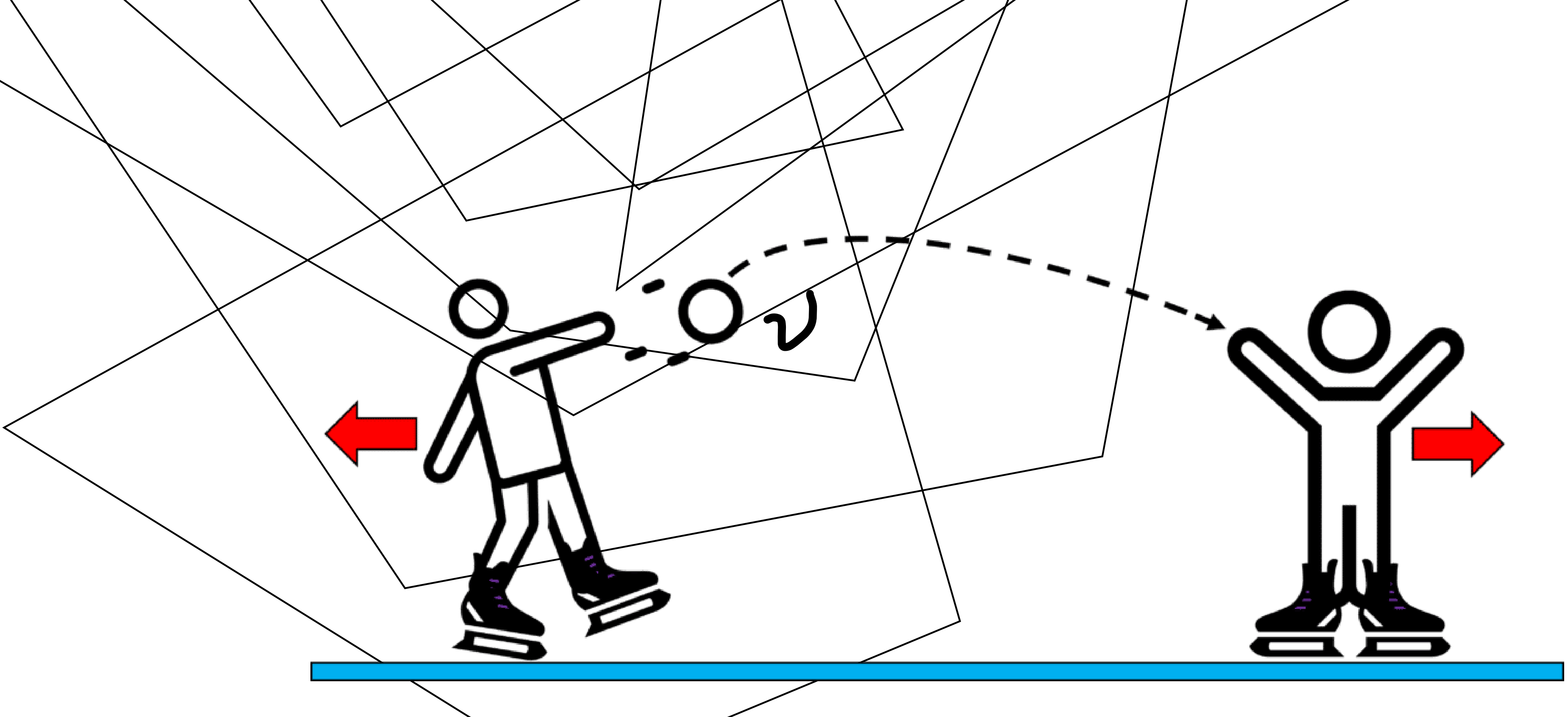
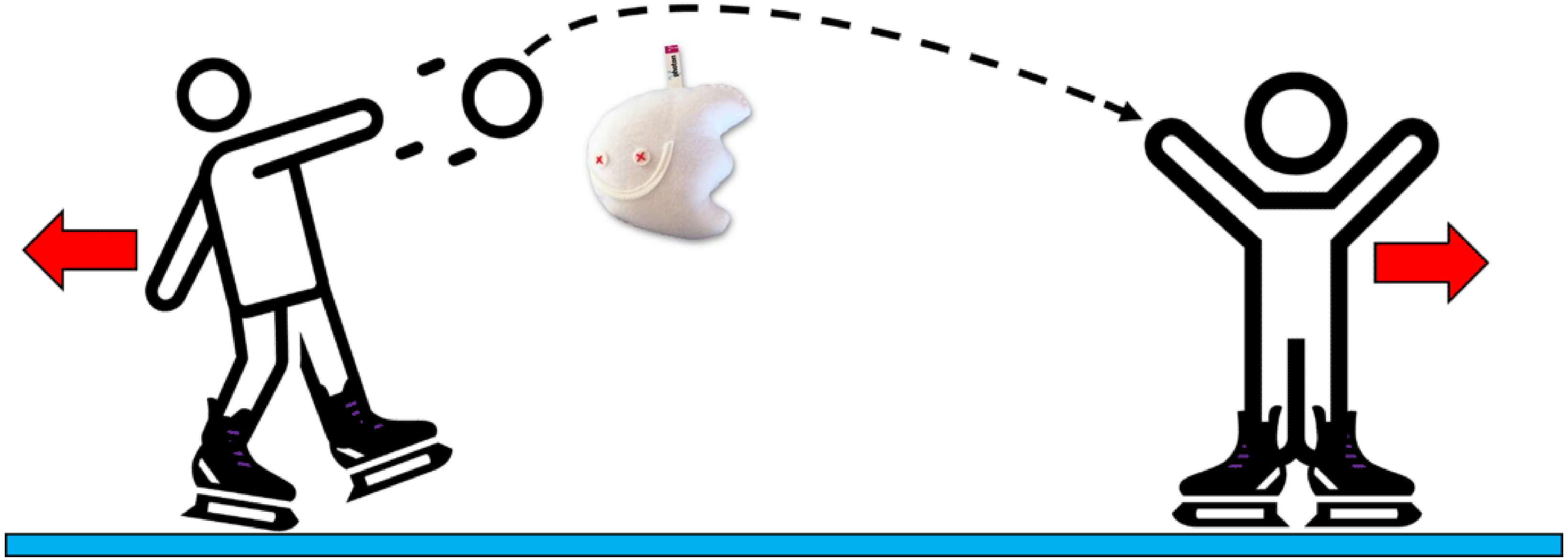


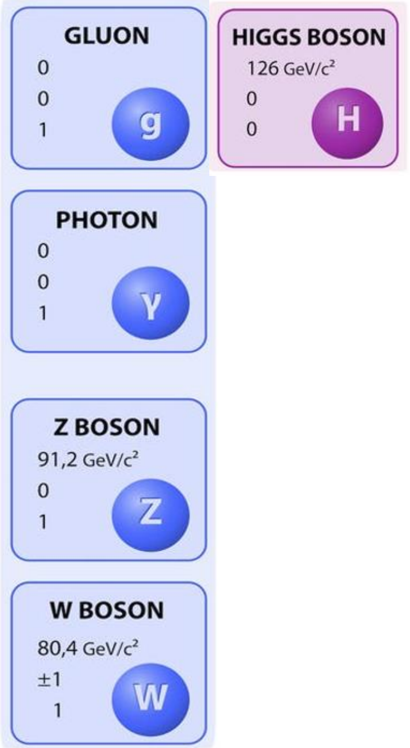
QUANTUM FORCES FROM VIRTUAL NEUTRINOS

Mitrajyoti Ghosh

Cornell University

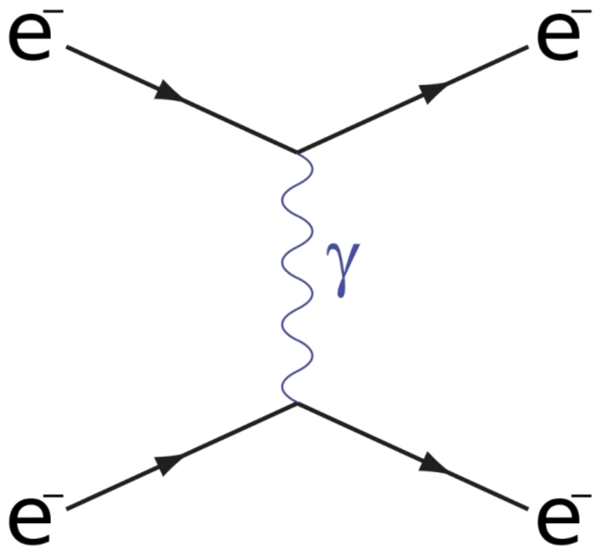
FORCES VIA VIRTUAL PARTICLE EXCHANGE





WARNING: BAD ANALOGY ONLY. **DO NOT TRY THIS AT HOME.**

THE COULOMB POTENTIAL



In the non-relativistic limit:

$$\mathcal{M} \sim \frac{1}{\mathbf{q}^2}$$

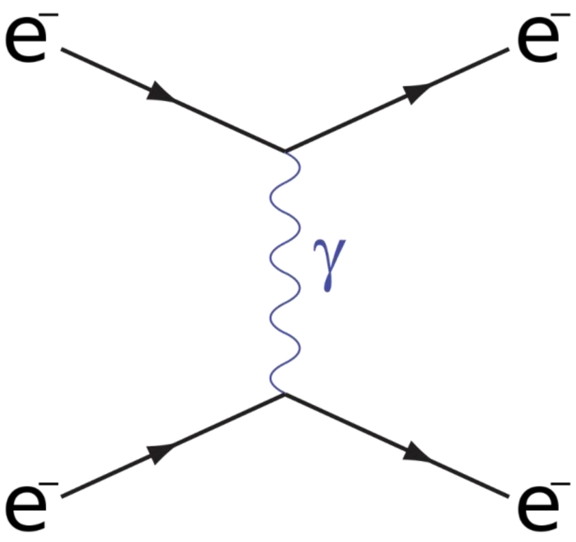
$$V(\mathbf{r}) \sim \int \mathrm{d}^3\mathbf{q} \;\; \frac{1}{\mathbf{q}^2}\, e^{-i\mathbf{q}\cdot\mathbf{r}} \sim \frac{1}{r}$$

FOR MASSIVE PARTICLE EXCHANGE

$$V(\mathbf{r}) \sim \int d^3\mathbf{q} \left(\frac{1}{\mathbf{q}^2 + m^2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}} \sim \frac{e^{-mr}}{r}$$

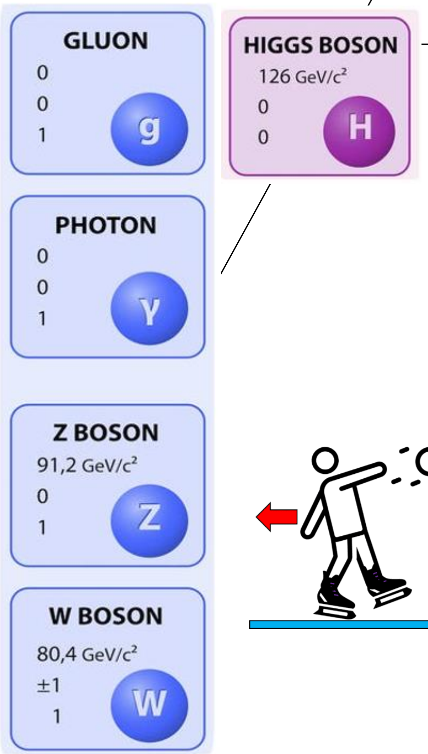
- Higher masses imply higher "virtuality" .
- Exponentially decaying potentials: The **probability of exchanging a virtual mass over larger distance falls exponentially with distance**. Falls faster when virtuality is high.

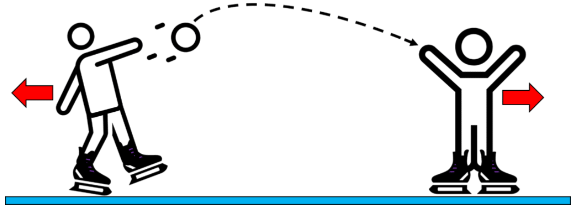
CLASSICAL FORCE MEDIATION AT TREE LEVEL



Who can do the job?

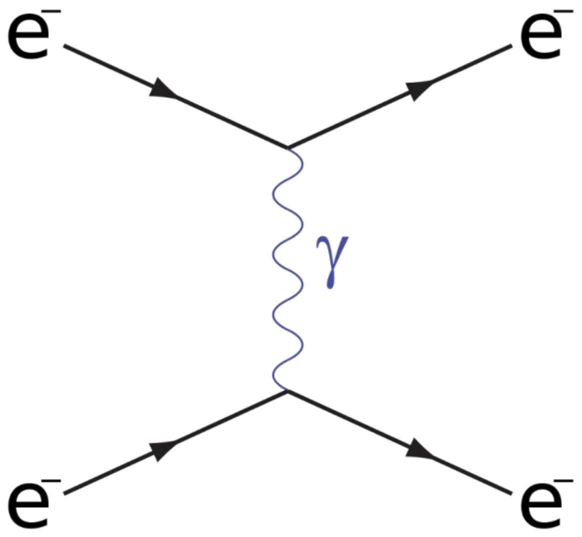
- A vector boson (spin 1)
- A scalar (spin 0)
- A fermion (spin ½)





A single fermion exchanged cannot give us a potential as it alters the angular momentum state of the species involved. Potentials are classical.

QUANTUM FORCES (LOOPS)



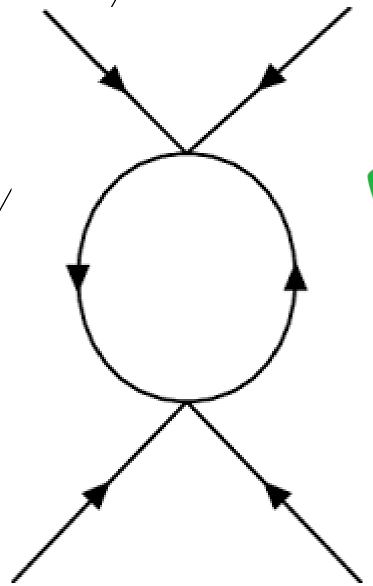
Who can do the job?

- A scalar ✓
- A vector boson ✓
- A fermion ✗

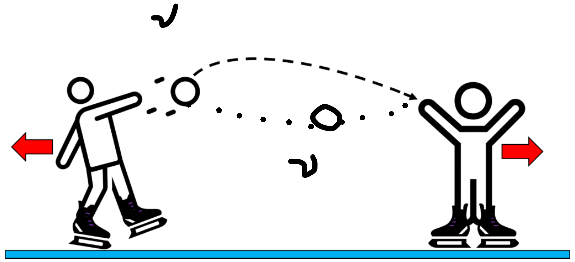
A single fermion exchanged cannot give us a potential as it alters the angular momentum state of the species involved. Potentials are classical.

Two Fermions?

Behave like bosons. Allowed!



✓



LONG RANGED FORCES! NICE!

Short ranged forces not preferred for the sake of experiment

$$V(\mathbf{r}) \sim \int \mathrm{d}^3\mathbf{q} \ \left(\frac{1}{\mathbf{q}^2 + m^2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}} \sim \frac{e^{-mr}}{r}$$

Medieval English Longbow, Range ~ 350 meters

Range of the weak force $\sim 10^{-18}$ meters

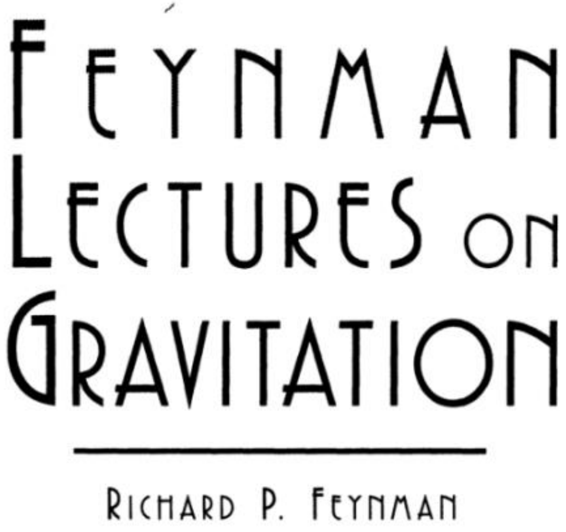
LONG RANGED FORCES! NICE!

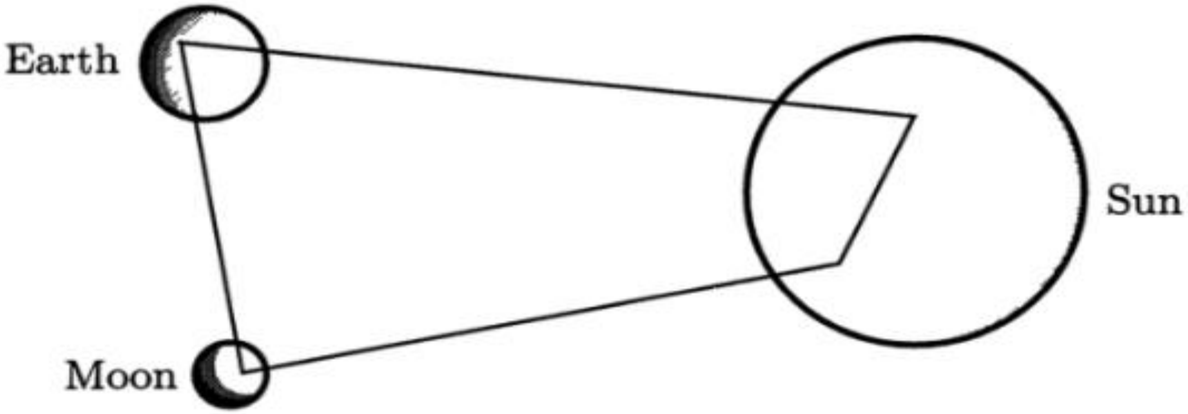
Short ranged forces not preferred for the sake of experiments

The Two-Neutrino Force!

Range $\sim 1\mu$m

The neutrino-force as an explanation for gravity??





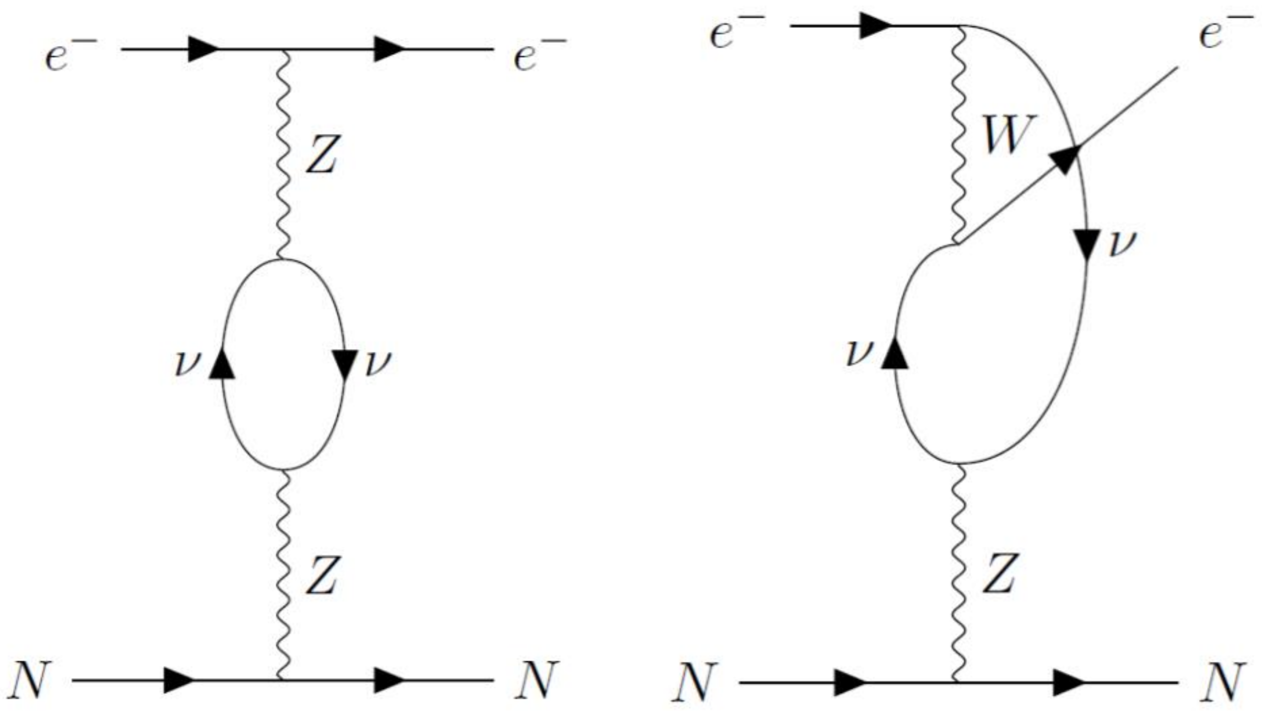
$$E = -G'^3 m_1 m_2 m_3 \pi^2 \frac{1}{(r_{12} + r_{23} + r_{13}) r_{12} r_{23} r_{13}}. \quad (2.4.4)$$

If one of the masses, say mass 3, is far away so that r_{13} is much larger than r_{12}, we do get that the interaction between masses 1 and 2 is inversely proportional to r_{12}.

What is this mass m_3? It evidently will be some effective average over all other masses in the universe. The effect of faraway masses spherically distributed about masses 1 and 2 would appear as an integral over an average density; we would have

$$E = -\frac{G'^3 m_1 m_2 \pi^2}{r_{12}} \int \frac{4\pi \rho(R) R^2 dR}{2R^3}, \quad (2.4.5)$$

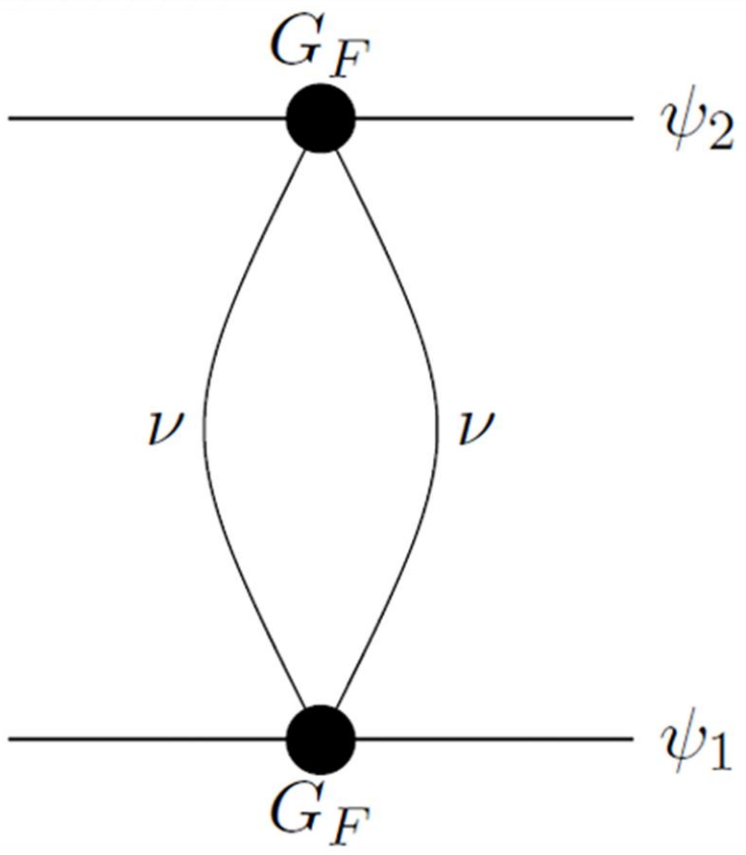
THE 2-NEUTRINO FORCE IN THE SM



We expect a force that is long ranged

Effects of this force would be sensitive to the mass of the neutrinos

THE NEUTRINO FORCE IN THE 4-FERMI APPROXIMATION



At energy scales relevant to us, we can integrate out the weak bosons.

- For a long time, neutrinos were thought to be massless

$$\boxed{V(r) = \frac{G_F^2}{4\pi^3 r^5}}$$

Feinberg, Sucher, Au (1989)
Sikivie, Hsu. (1994)

- Then mass was included, but three neutrino families and flavor mixing not accounted for   Grifols, Masso, Toldra (1996)

$$\boxed{V_{\nu\nu}^{\mathrm{Dirac}}(r) = \frac{G_F^2 m_\nu^3}{4\pi^3}\frac{K_3(2m_\nu r)}{r^2}, \qquad V_{\nu\nu}^{\mathrm{Majorana}}(r) = \frac{G_F^2 m_\nu^2}{2\pi^3}\frac{K_2(2m_\nu r)}{r^3}}$$

- Much weaker than gravity or electromagnetic forces at large distances.

In 4-Fermi theory, this force is suppressed by **two powers of G_F.**

NEUTRINO FORCES IN NEUTRINO BACKGROUNDS

Can we do better than $1/r^5$?

MODIFIED FERMION PROPAGATOR IN A BACKGROUND

In vacuum

$$\frac{i(\not{p}+m)}{p^2-m^2+i\epsilon}$$

In a background

I NEED SOME SPACE!

MODIFIED FERMION PROPAGATOR IN A BACKGROUND

$$(\not{p}+m)\left\{\frac{i}{p^2-m^2+i\epsilon}-(2\pi)\delta\left(p^2-m^2\right)\left[\Theta\left(p^0\right)n_+(\mathbf{p})+\Theta\left(-p^0\right)n_-(\mathbf{p})\right]\right\}$$

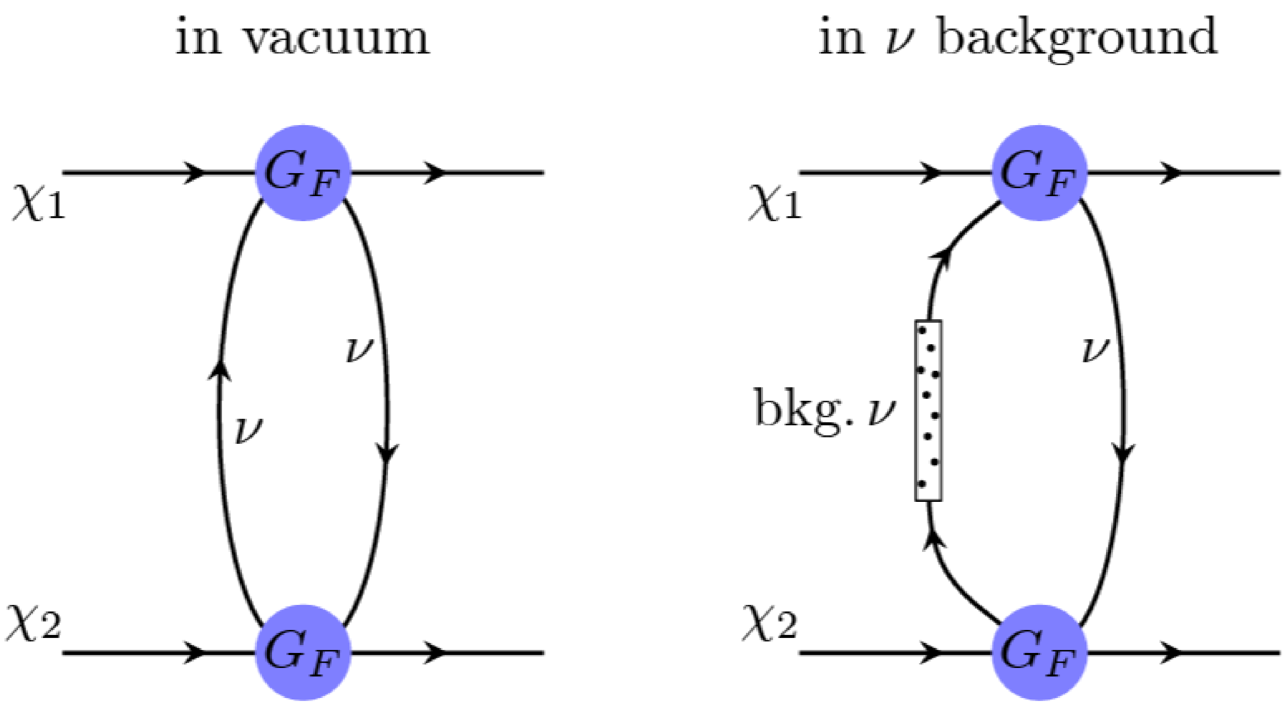
Vacuum propagator

Background correction

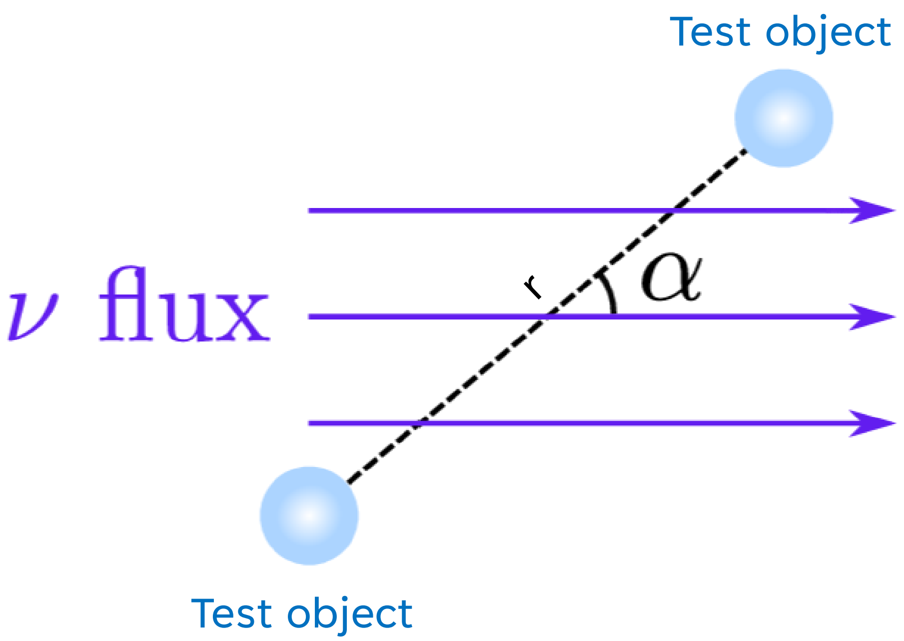
- In the background potential, the delta function puts the fermion on shell!

- Proportional to the density of fermions and anti-fermions in the background.

THE MODIFIED NEUTRINO FORCE



MONOCHROMATIC DIRECTIONAL BACKGROUNDS



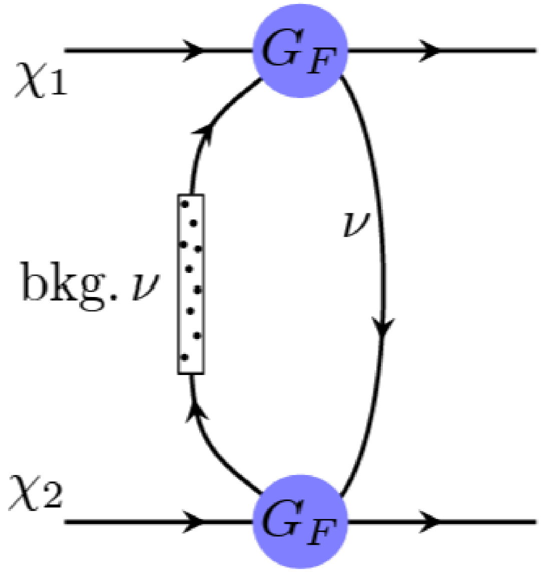
Consider a beam with the following properties:

1. Monochromatic beam with all neutrinos travelling in the same direction
2. Beam energy $E_\nu \gg m_\nu$, neutrino mass being ignored in this talk.
3. Beam flux density ϕ_0

NAÏVE EXPECTATIONS

Oscillations

Exchange of a "real" neutrino, as opposed to a virtual one gives oscillations



Radial dependence

From "geometry" of one virtual particle exchange, expect $1/r$

NAÏVE EXPECTATIONS

Oscillations

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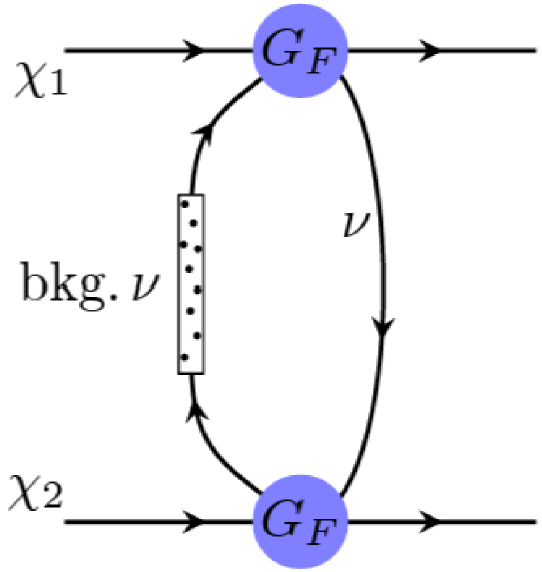
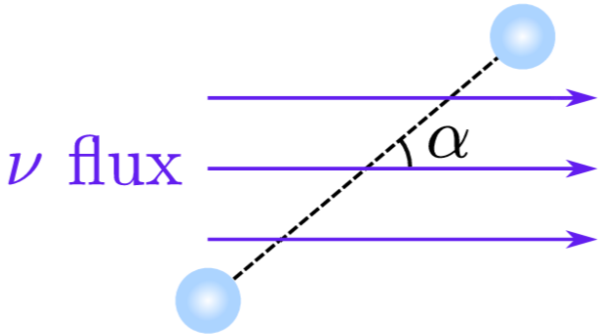


Radial dependence

From "geometry" of one virtual particle exchange, expect $1/r$

$$V(r) \sim \frac{e^{-2iE_\nu r f(\alpha)}}{r} \sim \frac{1}{r}\cos(2E_\nu r f(\alpha))$$

THE LEADING LONG-DISTANCE BEHAVIOR



Vectorial Couplings

Beam parameters

Radial dependence

Oscillations

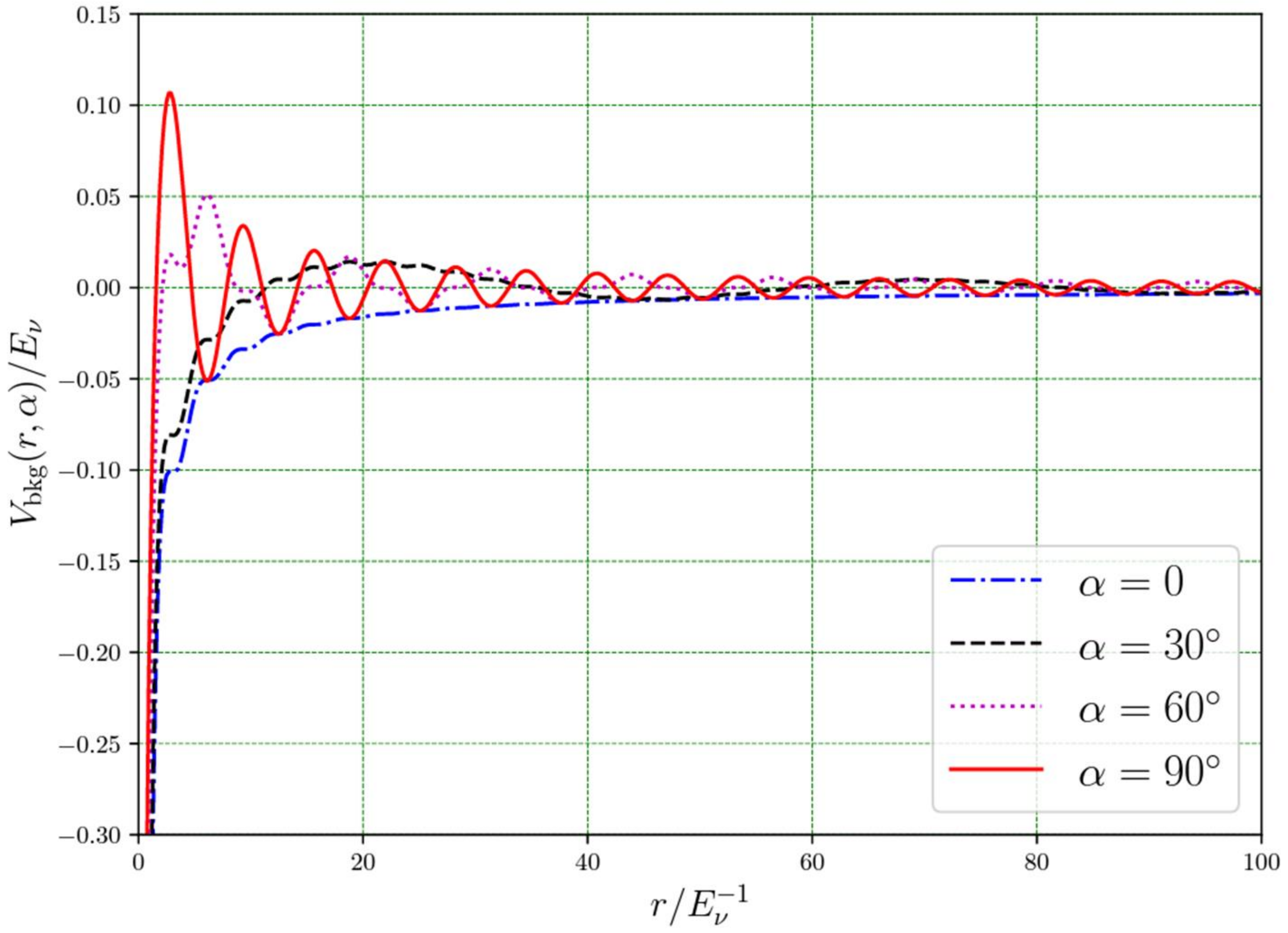
$$V_{\text{bkg}}\left(r \gg E_\nu^{-1}, \alpha \ll 1\right) = -\frac{g_V^1 g_V^2}{\pi} G_F^2 \times \Phi_0 E_\nu \times \frac{1}{r} \times \cos\left(\frac{\alpha^2 E_\nu r}{2}\right)$$

Non-leading terms that go as $1/r^2$ etc. exist, but are negligible at large distances

INTUITION WAS:

$$V(r) \sim \frac{e^{-2iE_\nu r f(\alpha)}}{r} \sim \frac{1}{r}\cos(2E_\nu r f(\alpha))$$

THE GENERAL FORM OF $V_{bkg}(r)$



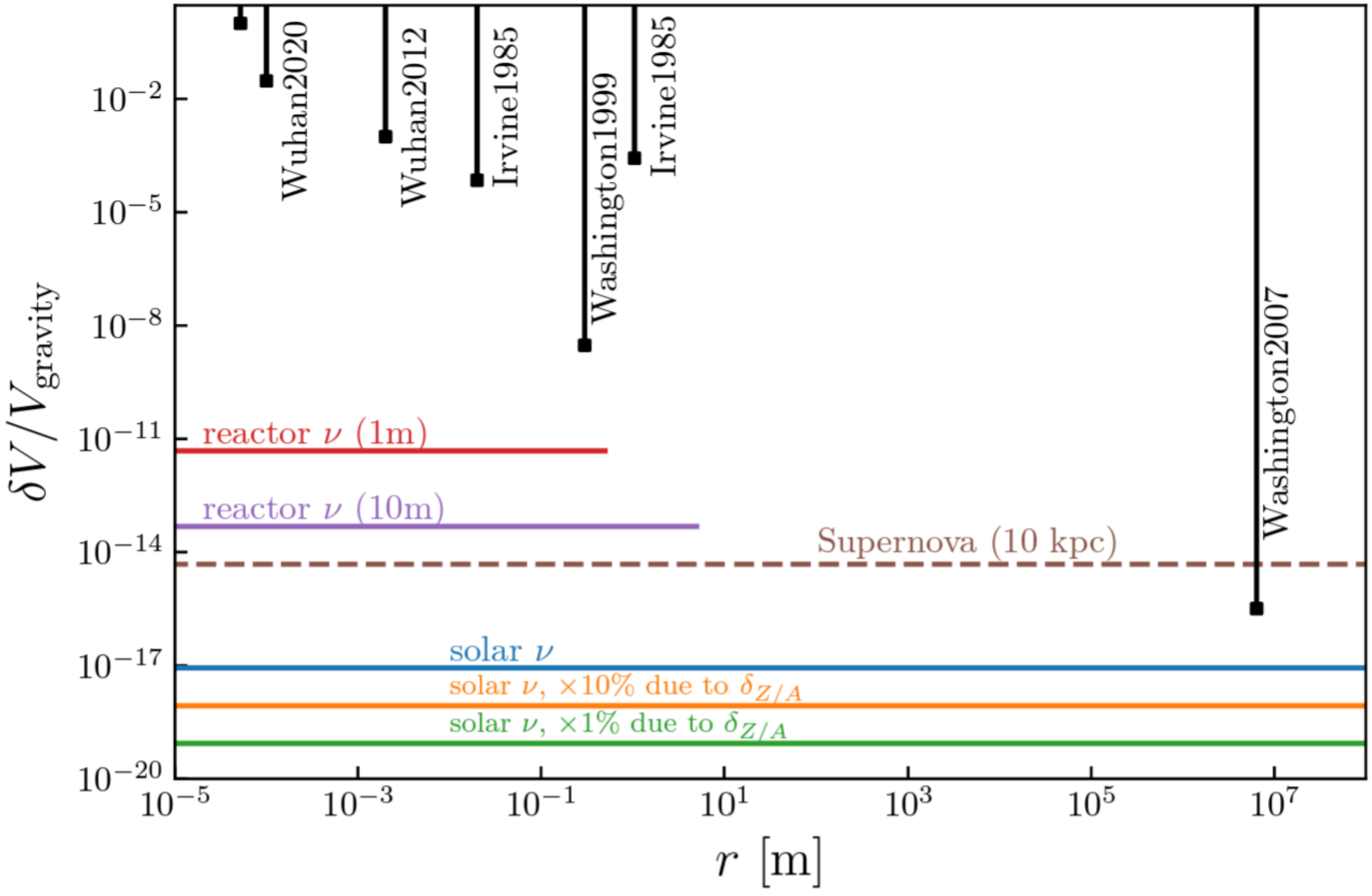
COMPARING WITH GRAVITY AT $\alpha = 0$

$$\frac{V_{\mathrm{bkg}}(r)}{V_{\mathrm{grav}}(r)} \sim 10^{-13}$$

Can fifth-force Experiments achieve
this sensitivity in the future?

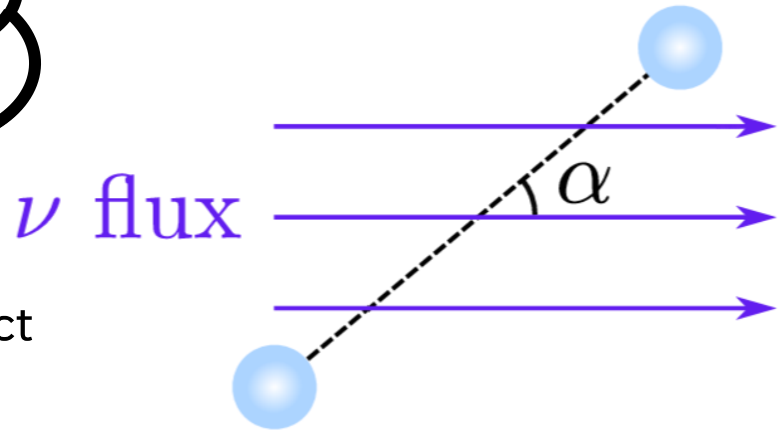
**Currently have a sensitivity of 10^{-9},
just 4 orders of magnitude above.**

Where are our experiments?



HOWEVER..

$$\cos\left(\frac{\alpha^2 E_\nu r}{2}\right)$$



- Considering macroscopic objects: the **finite size** of the object means that the net force between them will require an integration over the angle α, which can kill the leading $1/r$ dependence.

$$\Delta(\alpha^2) \lesssim (E_\nu r)^{-1}$$

- In realistic scenarios there will be a certain energy distribution in the neutrino beam, which can also smear out the force and kill the $1/r$ dependence.

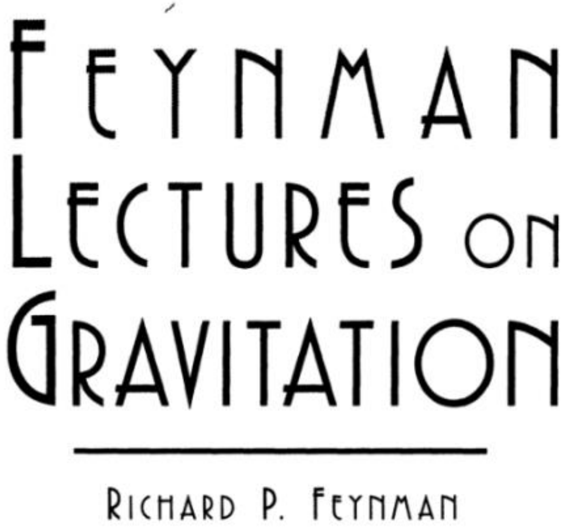
Assuming that the beam is truly monochromatic, we find, for test objects of size R:

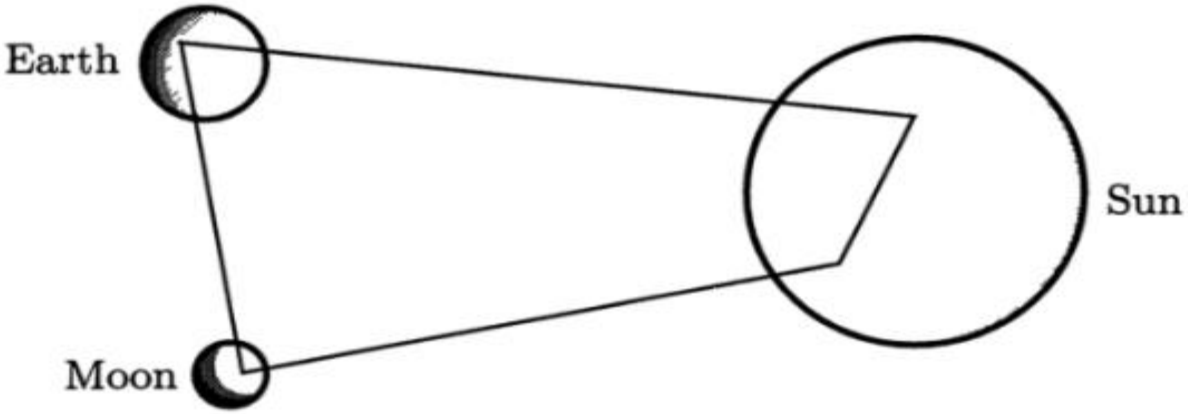
$$\alpha \lesssim (E_\nu R)^{-1}$$

If hydrogen atoms are used as test objects, we need $\alpha \ll 10^{-2}$.

NEED LOWER ENERGY BACKGROUNDS!

The uncanny similarity of this work and Feynman's





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Background!

FEYNMAN: $r^{-3} \to 1/r$ **US:** $r^{-5} \to 1/r$

CONCLUSIONS

1. Background effects can **greatly enhance static forces** obtained from particle exchange.

2. In the context of the two-neutrino force, the presence of a directional background takes the radial dependence **from a $\mathbf{1/r^5}$ to a $\mathbf{1/r}$**!

3. The $1/r$ dependence, however, is fragile and is easily killed by the smearing effects of finite size of the objects and/or the energy spread in the background neutrino beam.

4. Still, seems exciting as a way to finally probe the neutrino-force that has eluded us so far!

WHAT NEXT?

1. Computation of the force outside 4-Fermi regime, incorporate neutrino mass and mixing in background

2. **Separate calculations for Dirac and Majorana neutrinos.** Maybe possible to distinguish them this way.

3. Most importantly, try to design some experiments to probe the **longest range 2-fermion mediated force** in the Standard Model.

Thanks!