

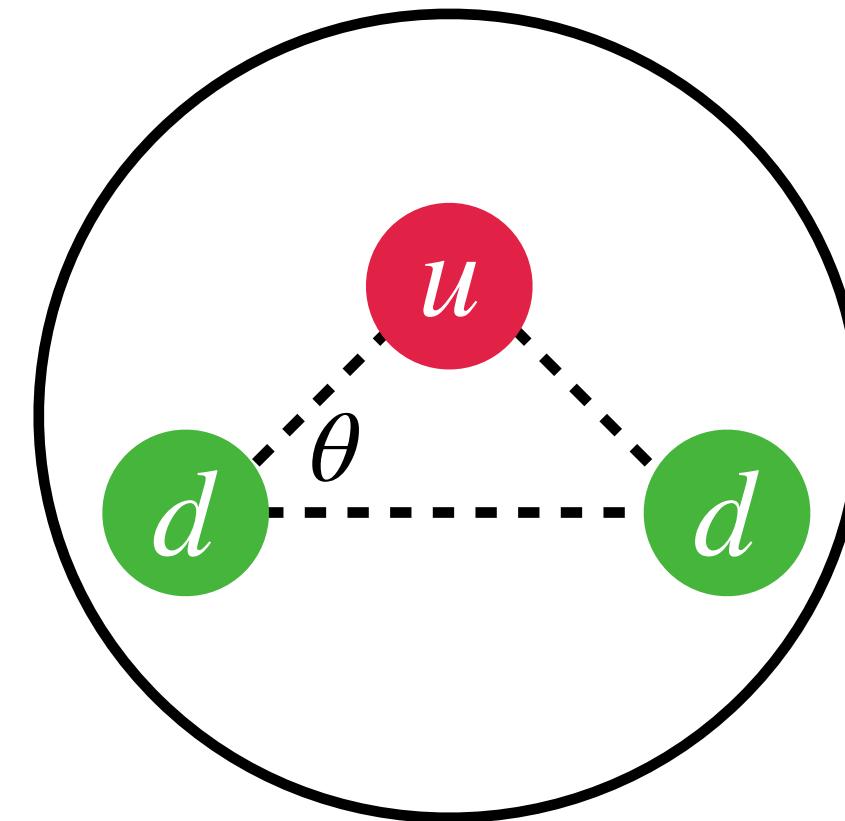
# The Irreducible Axion Background

**Kevin Langhoff - UC Berkeley  
Nadav Outmezguine (UCB) and Nick Rodd (CERN)  
[2208.07882]**

# Why Axions?

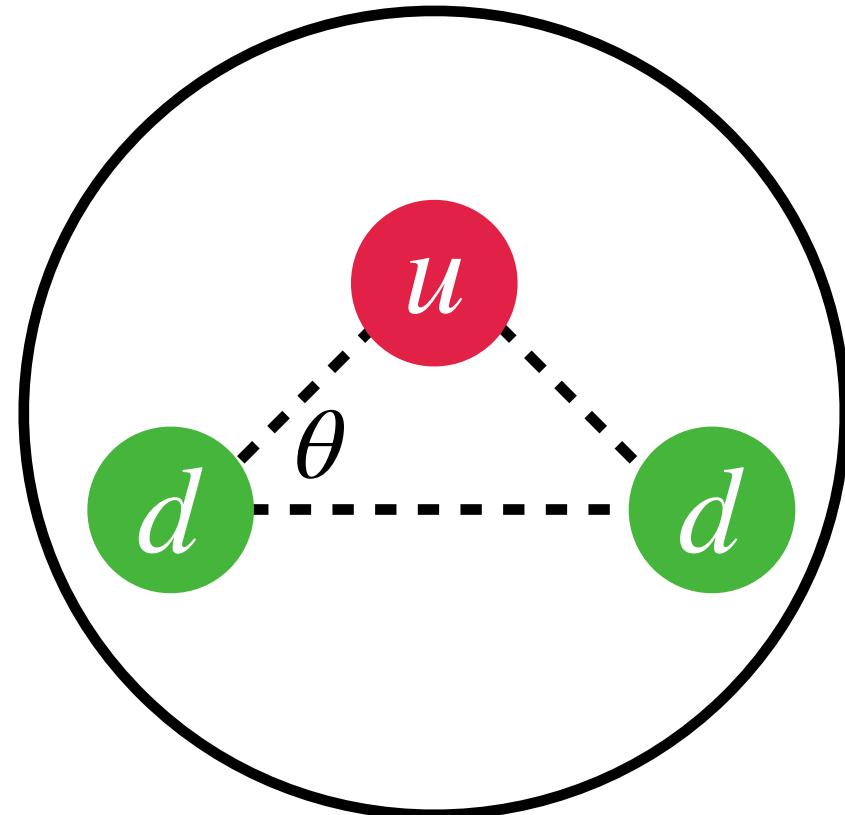
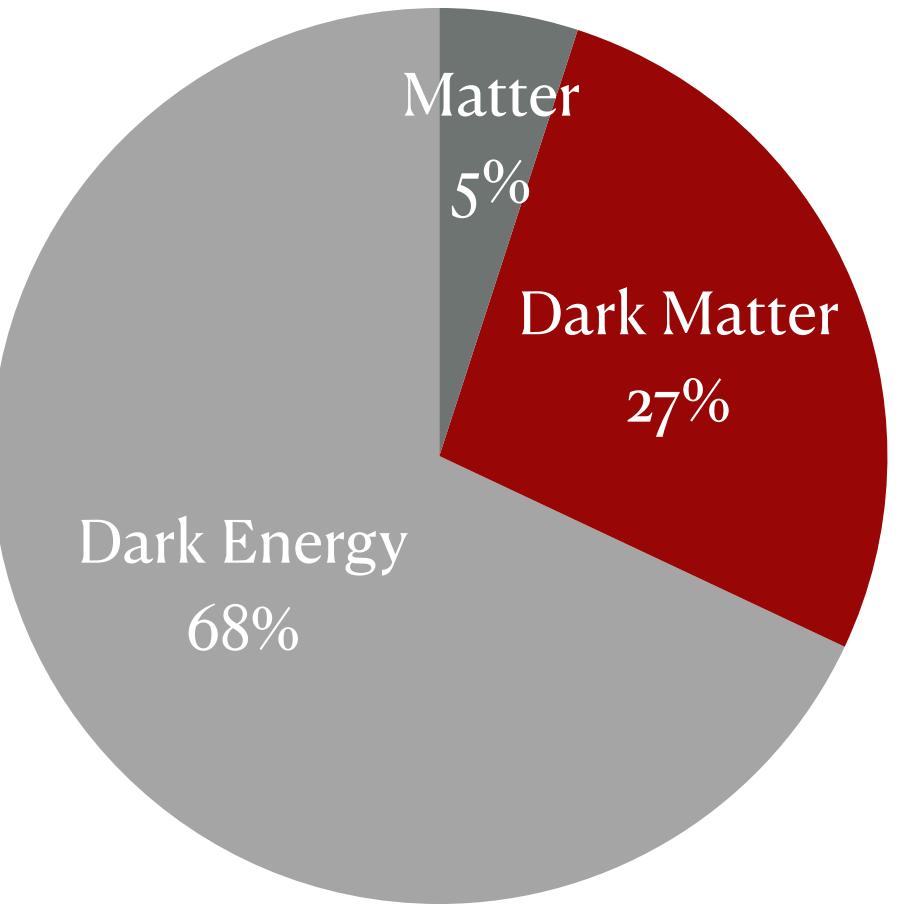
# Why Axions?

- Strong CP



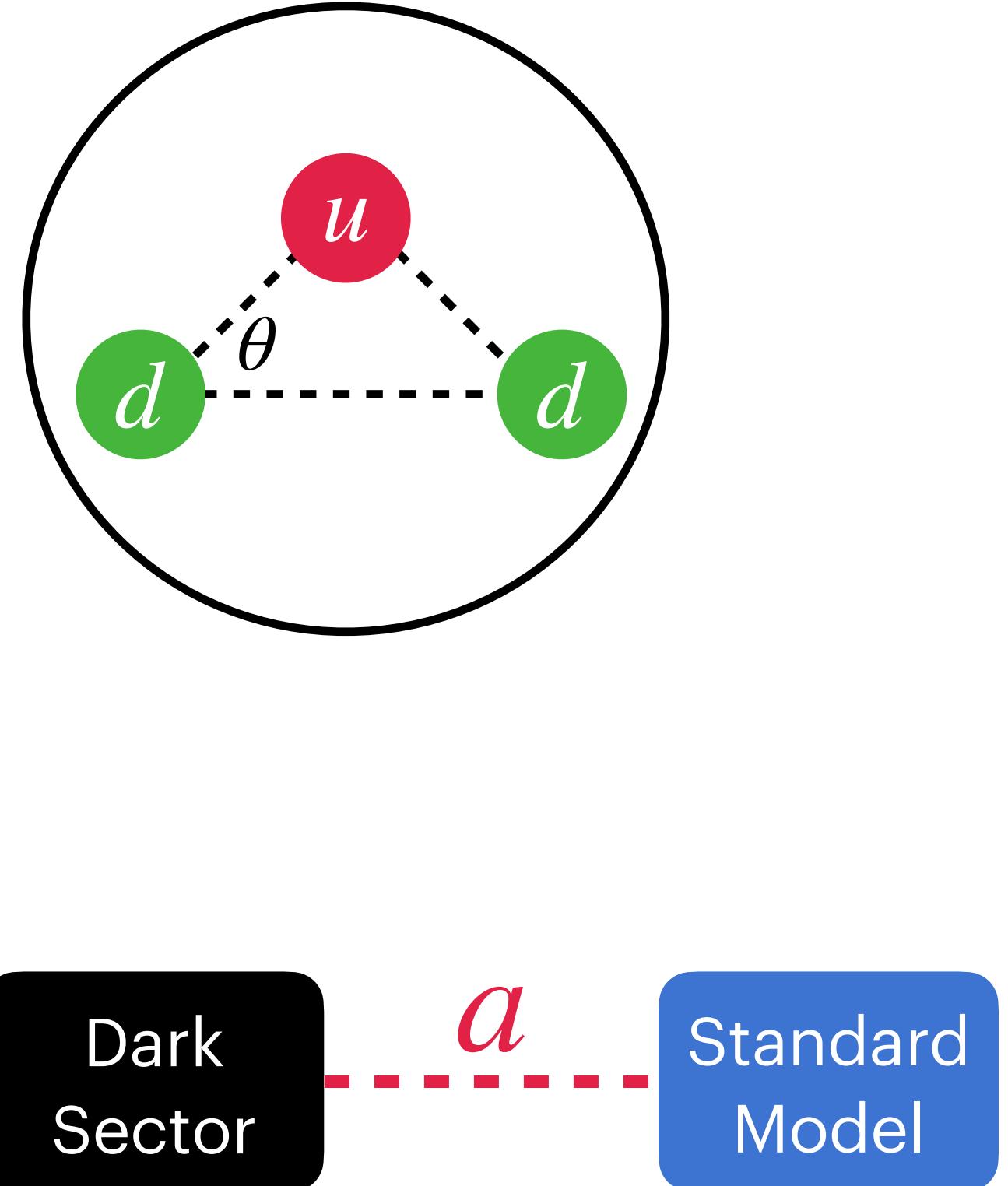
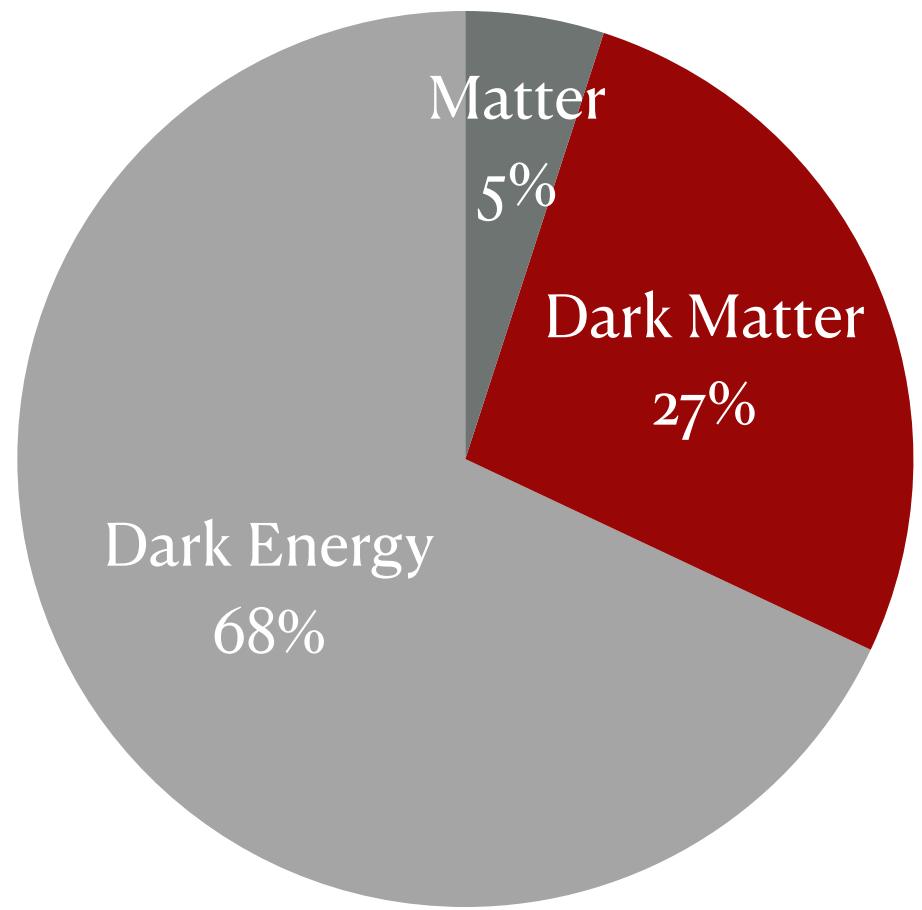
# Why Axions?

- Strong CP
- Dark matter candidate



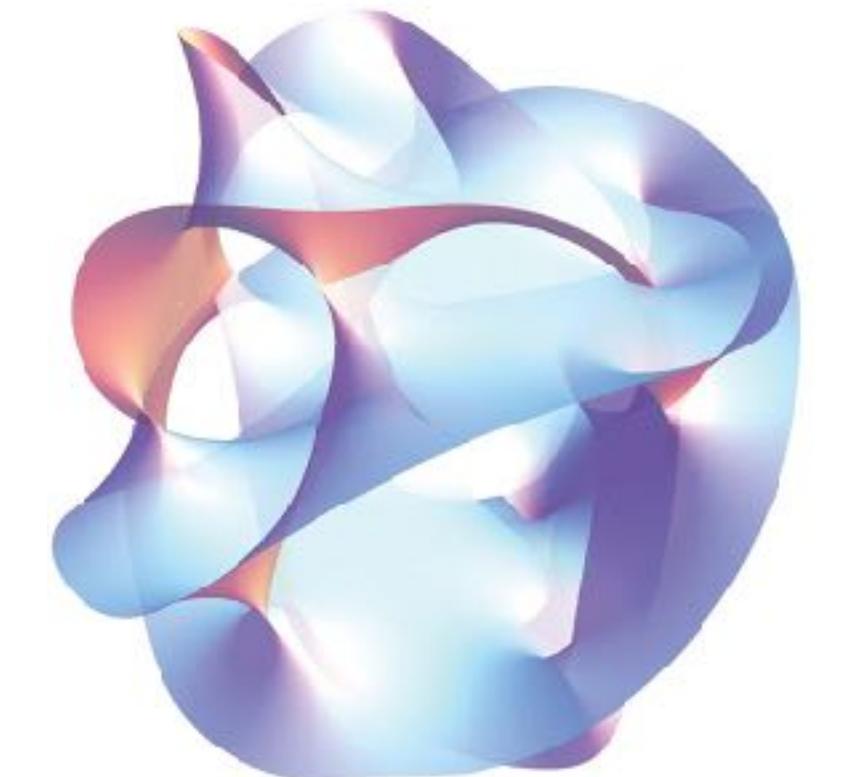
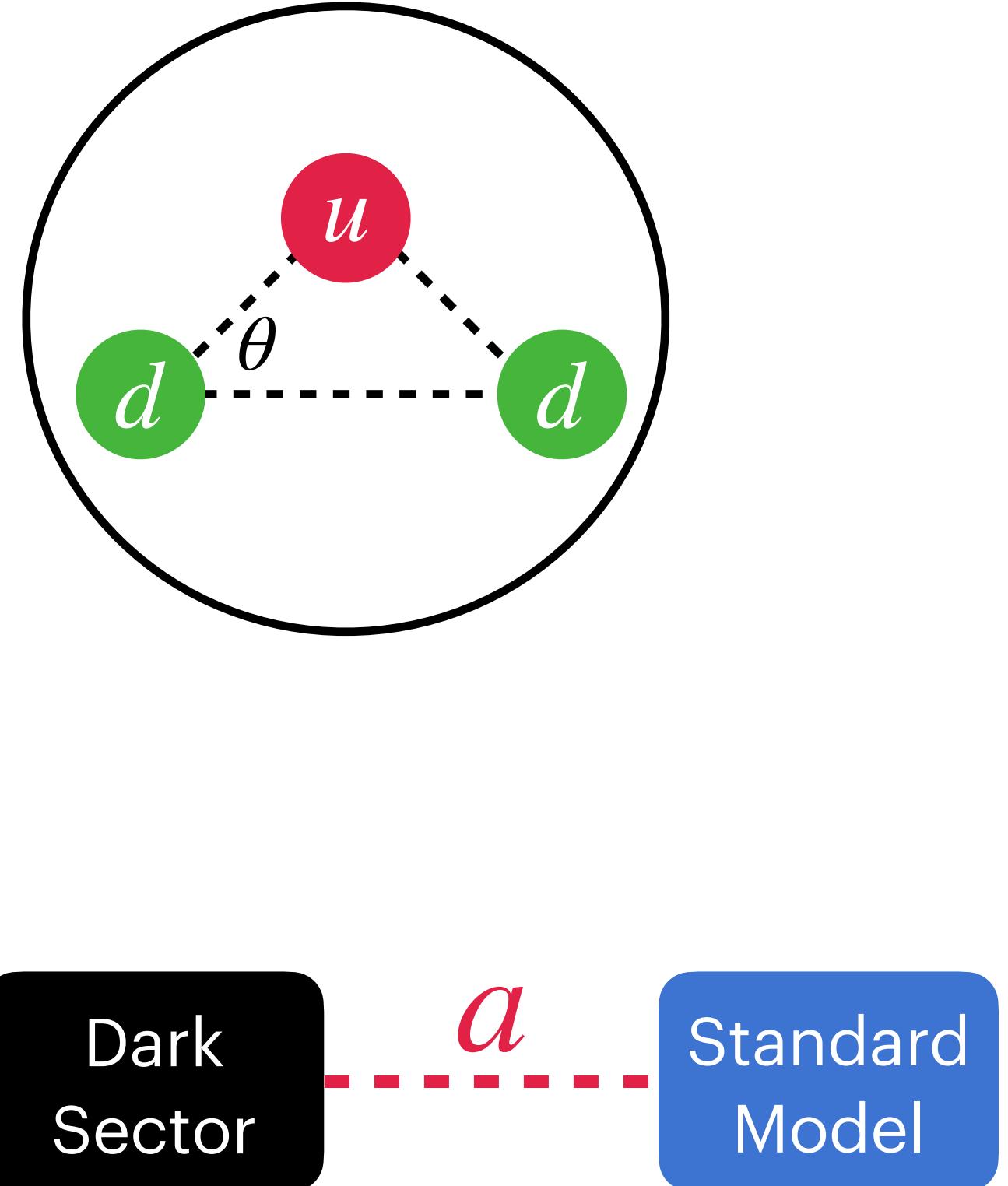
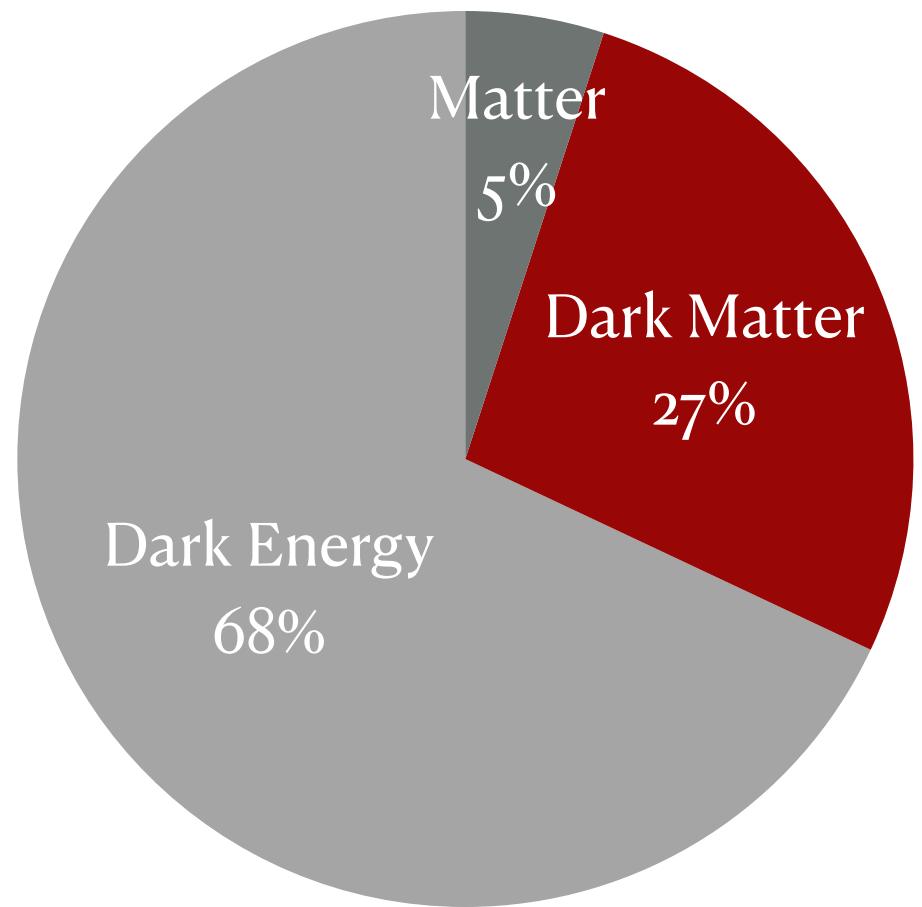
# Why Axions?

- Strong CP
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- Potential mediator to dark sector



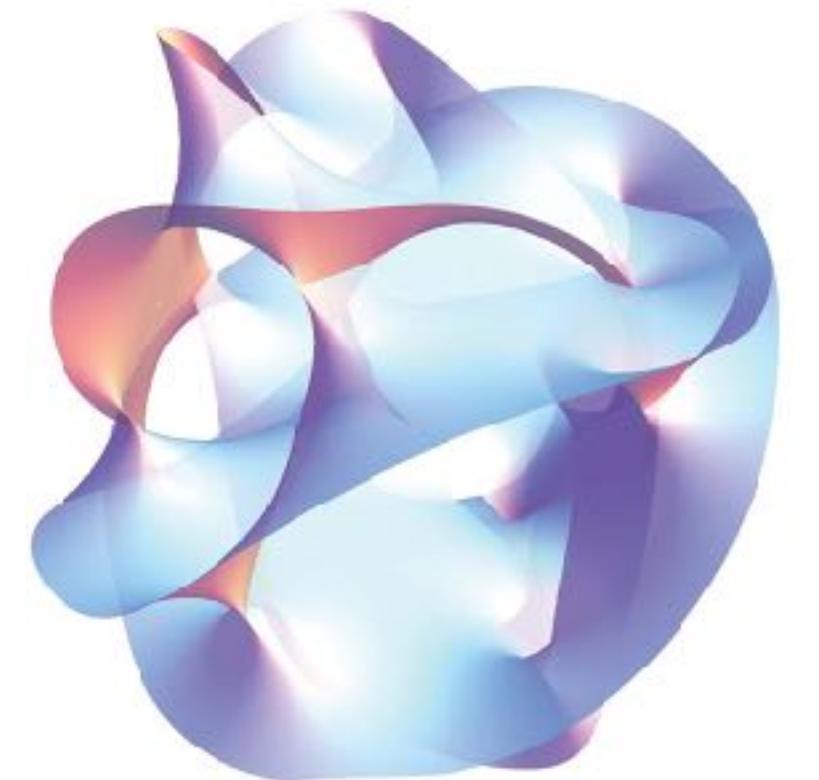
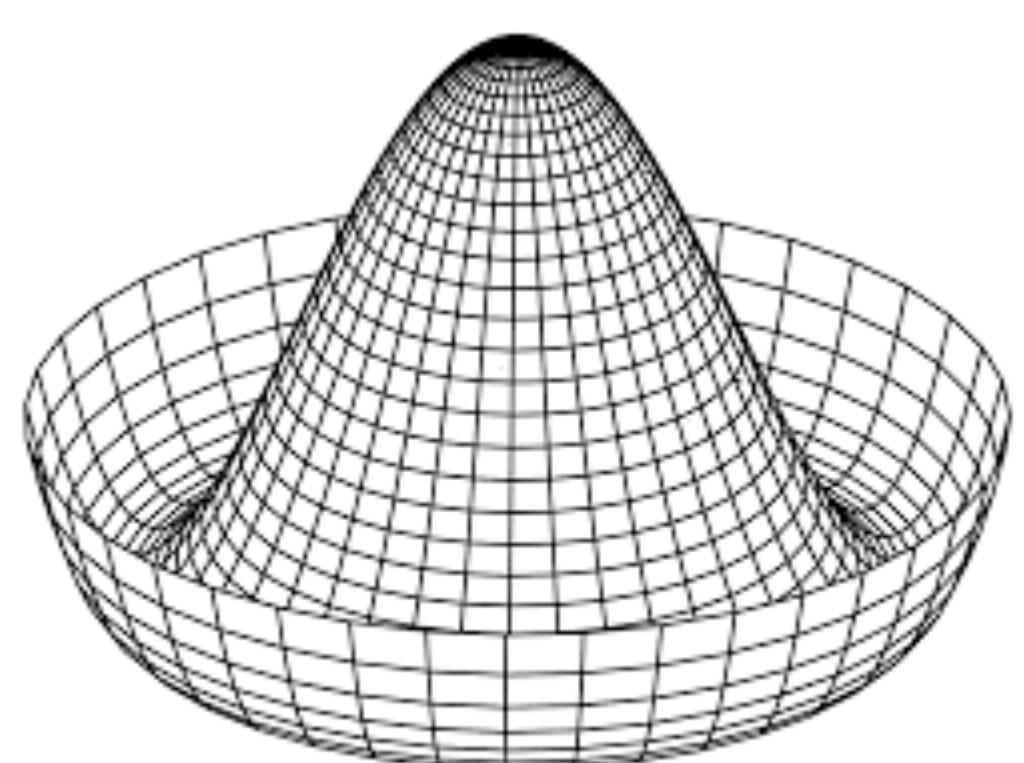
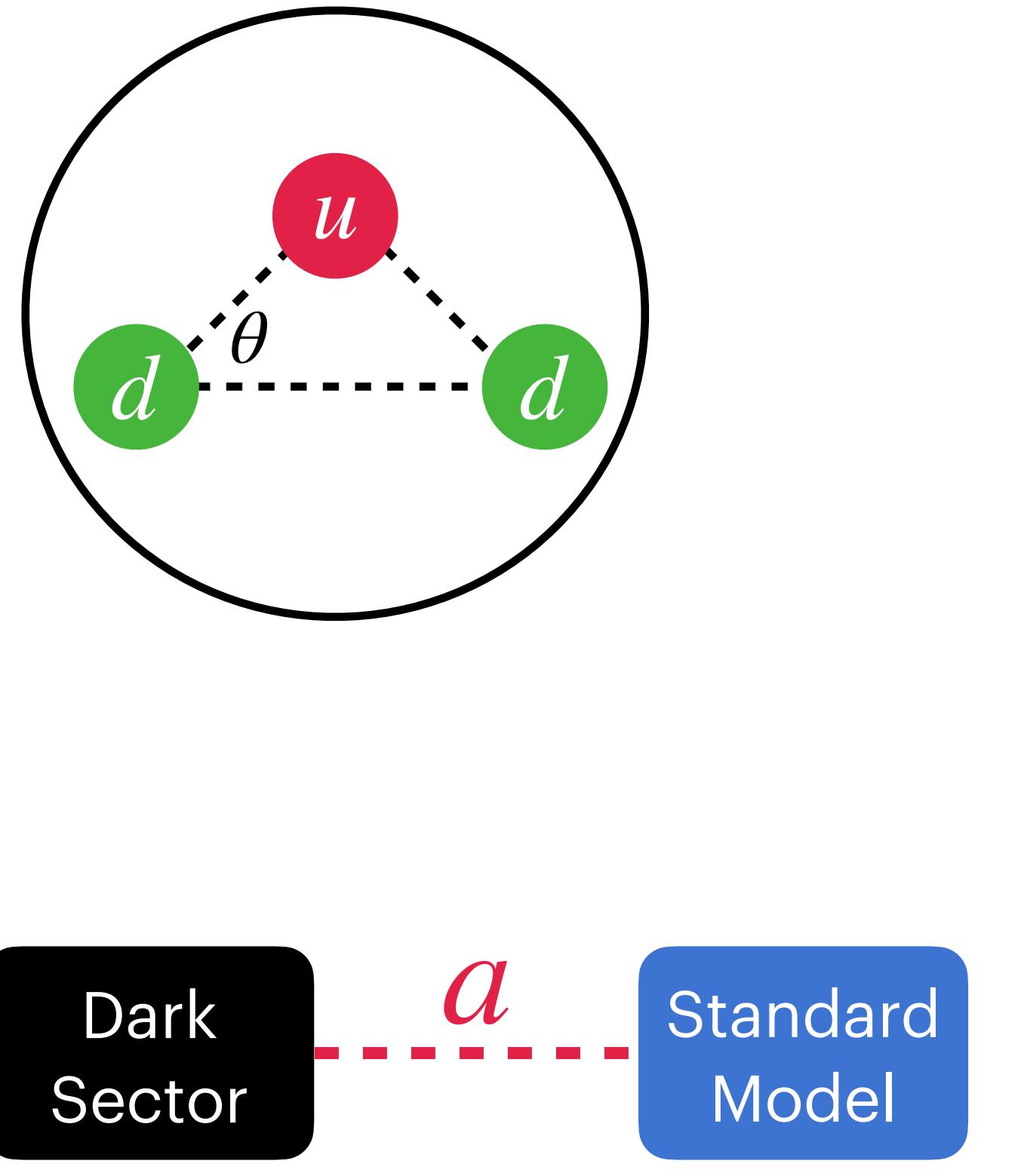
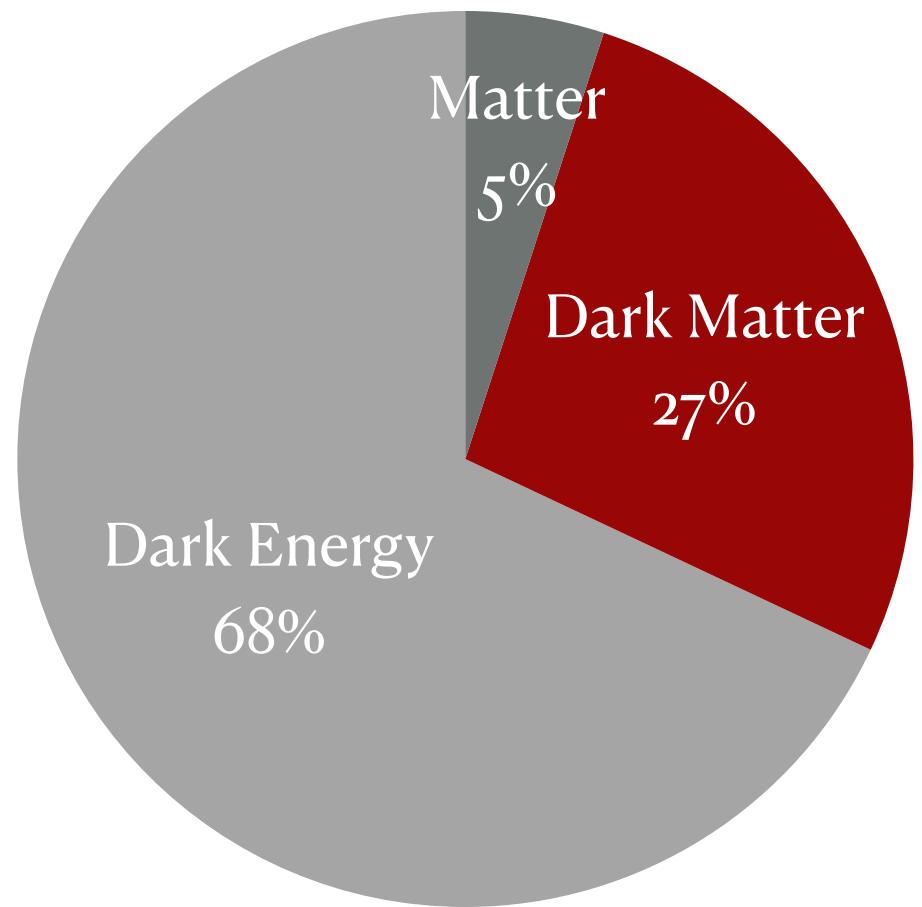
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- Strong CP
- Dark matter candidate
- Potential mediator to dark sector
- Prevalent in string theories.
- Goldstone bosons of global symmetries



# Today's Definition of Axions

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2}m_a^2 a^2 - \frac{1}{4}g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{g_{aee}}{2m_e} (\partial_\mu a) \bar{e} \gamma^\mu \gamma_5 e$$

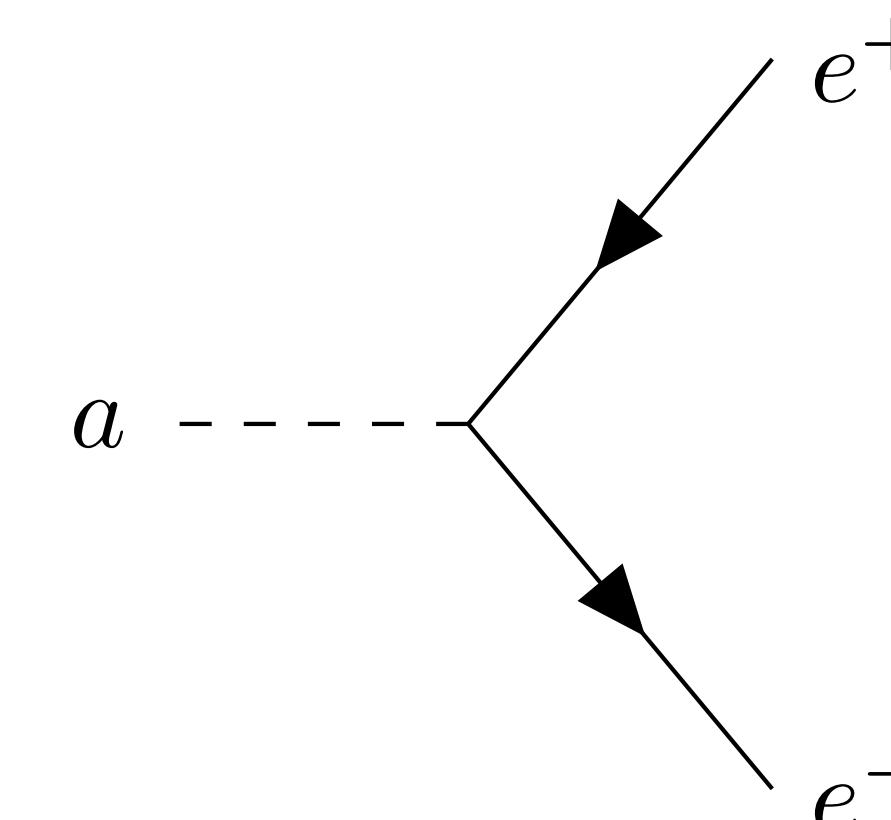
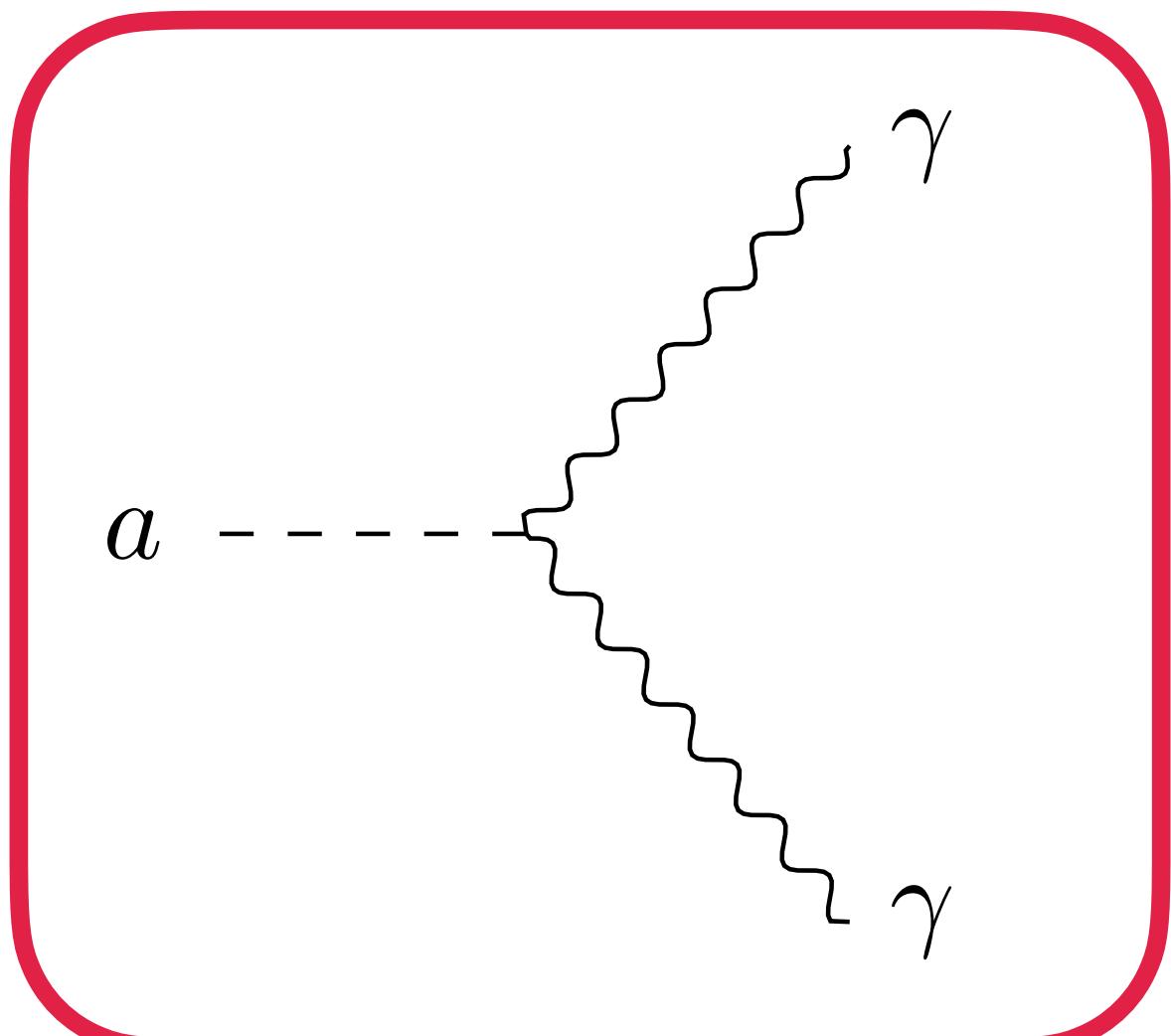
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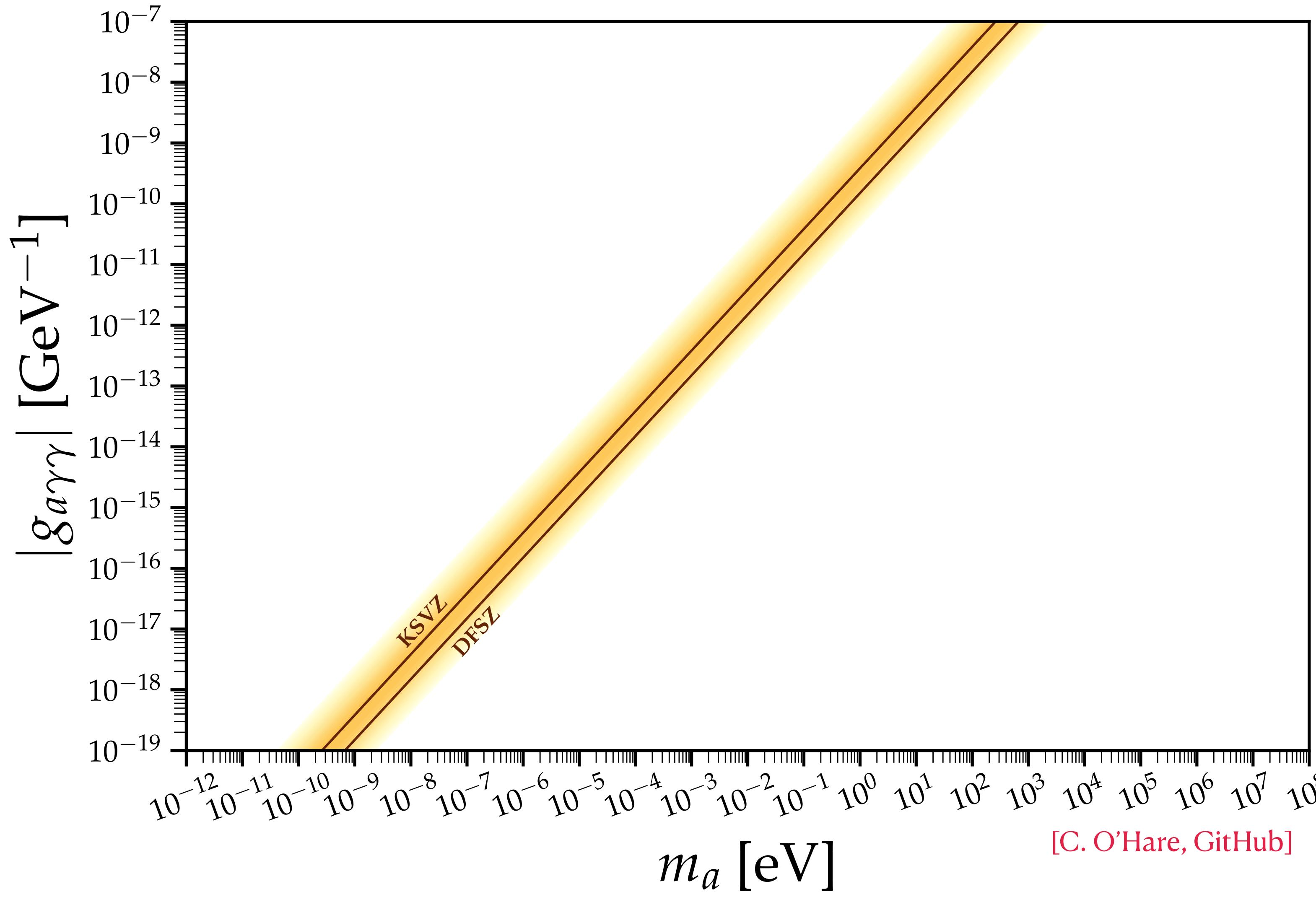
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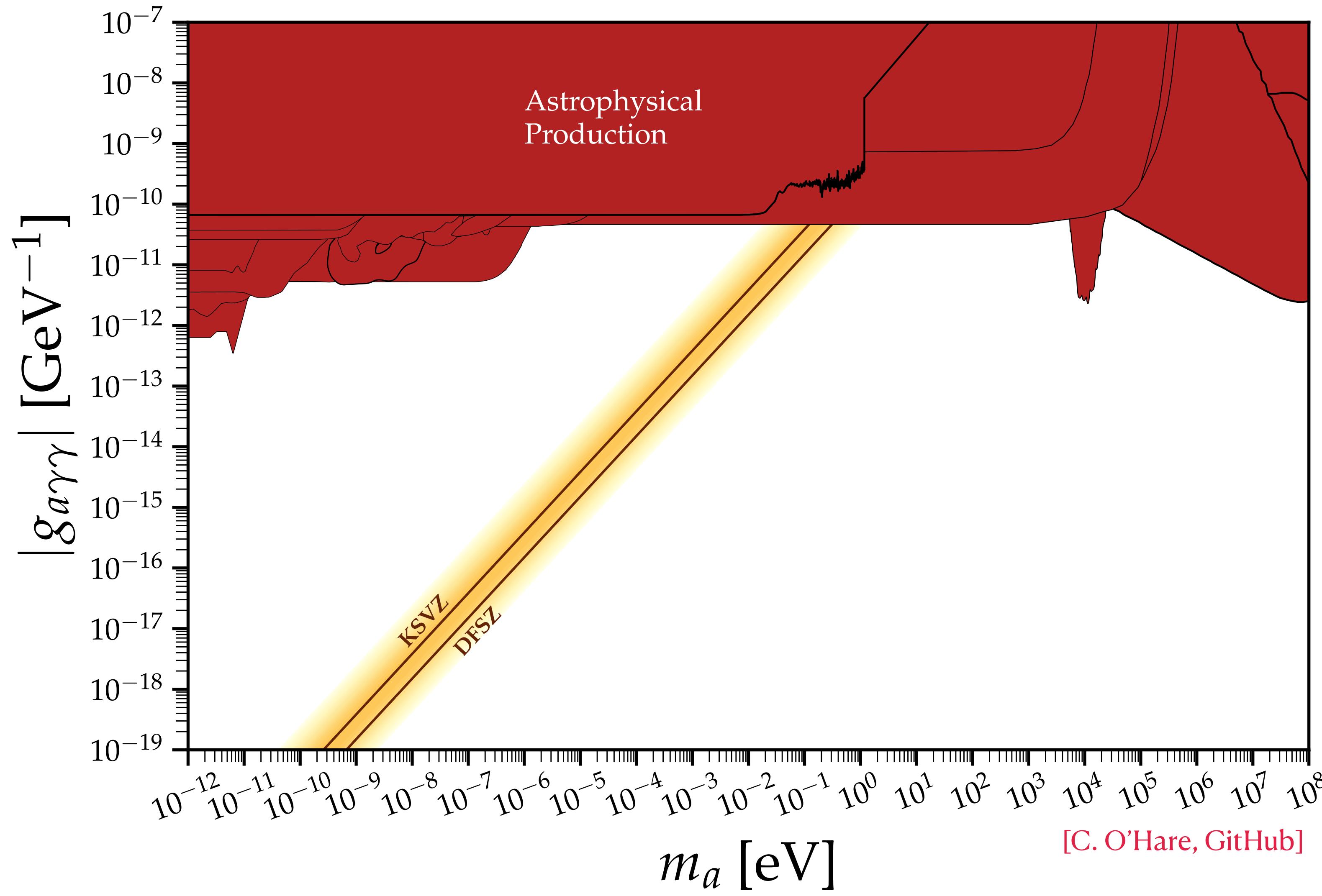
# Axion Parameter Space

If it solves strong CP (Canonically)



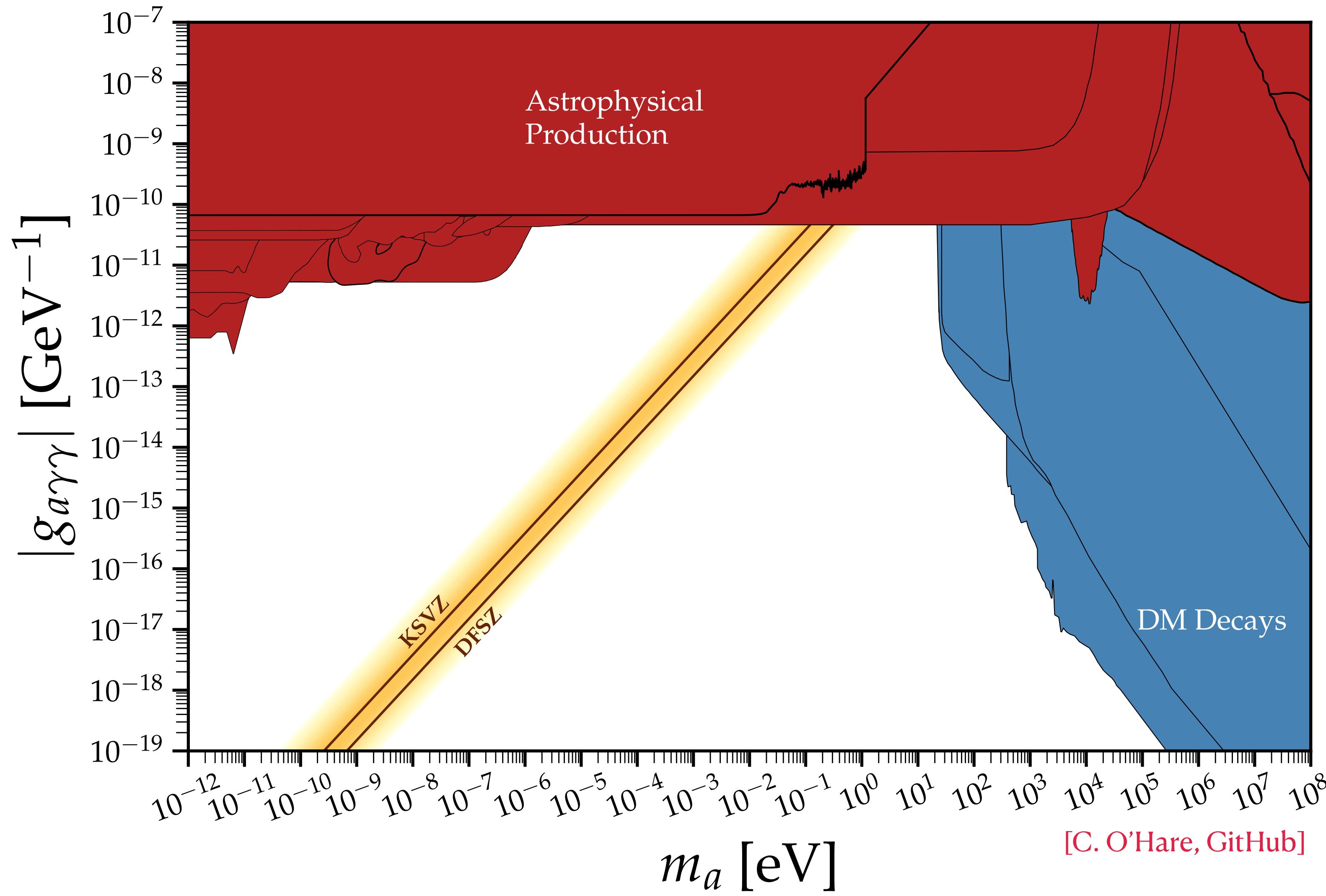
# Axion Parameter Space

If it exists



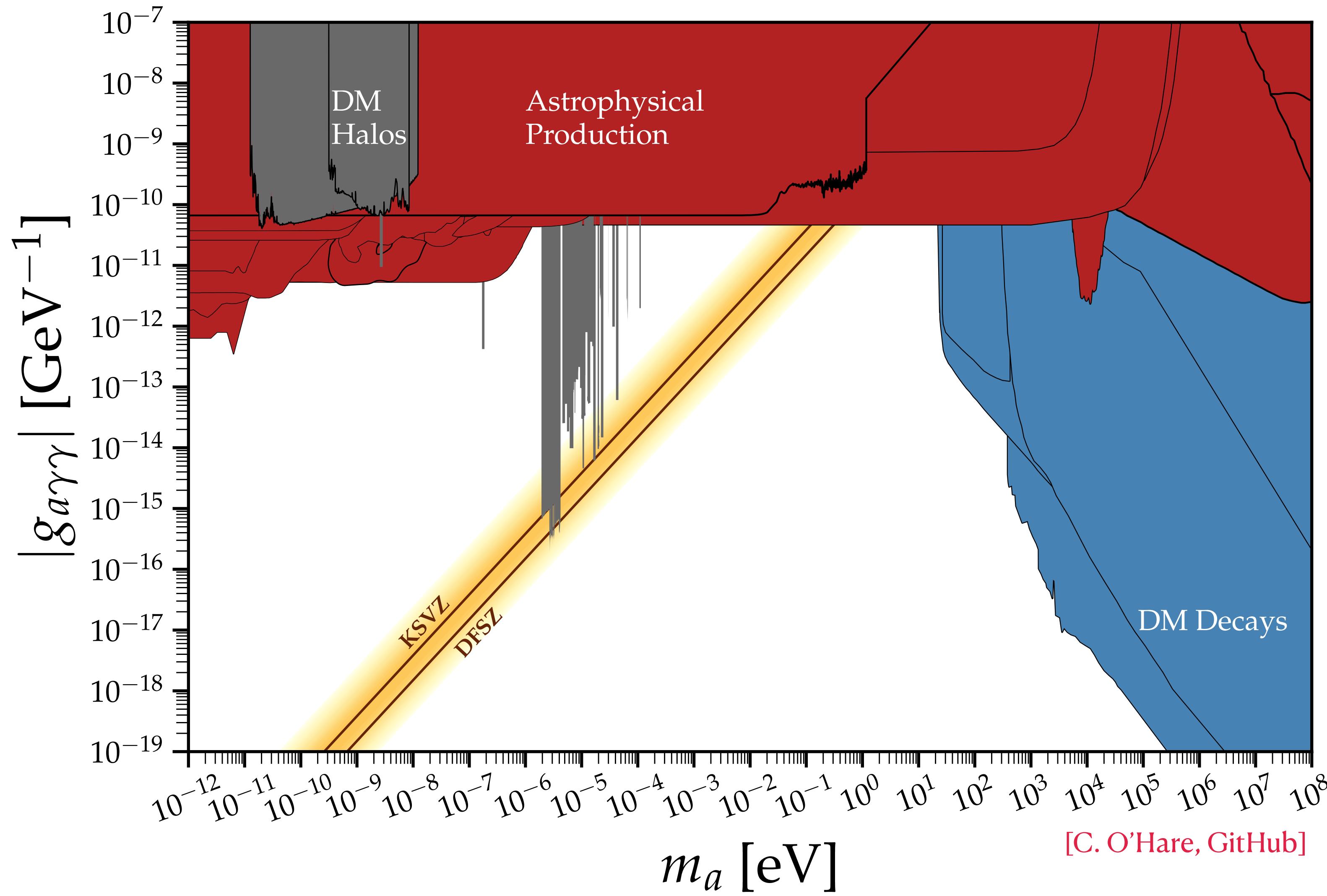
# Axion Parameter Space

If it is all of DM



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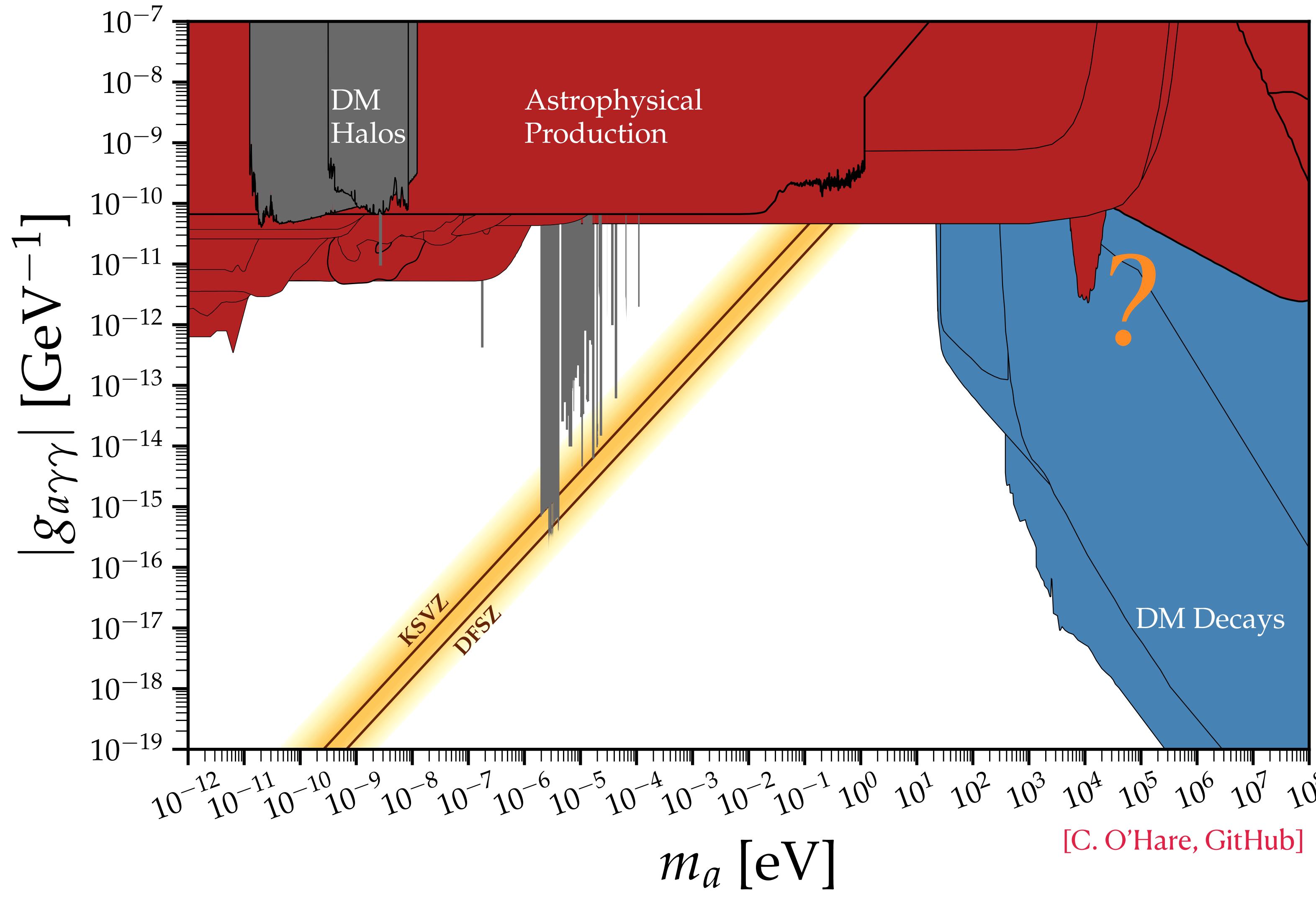
If it is all of DM



[C. O'Hare, GitHub]

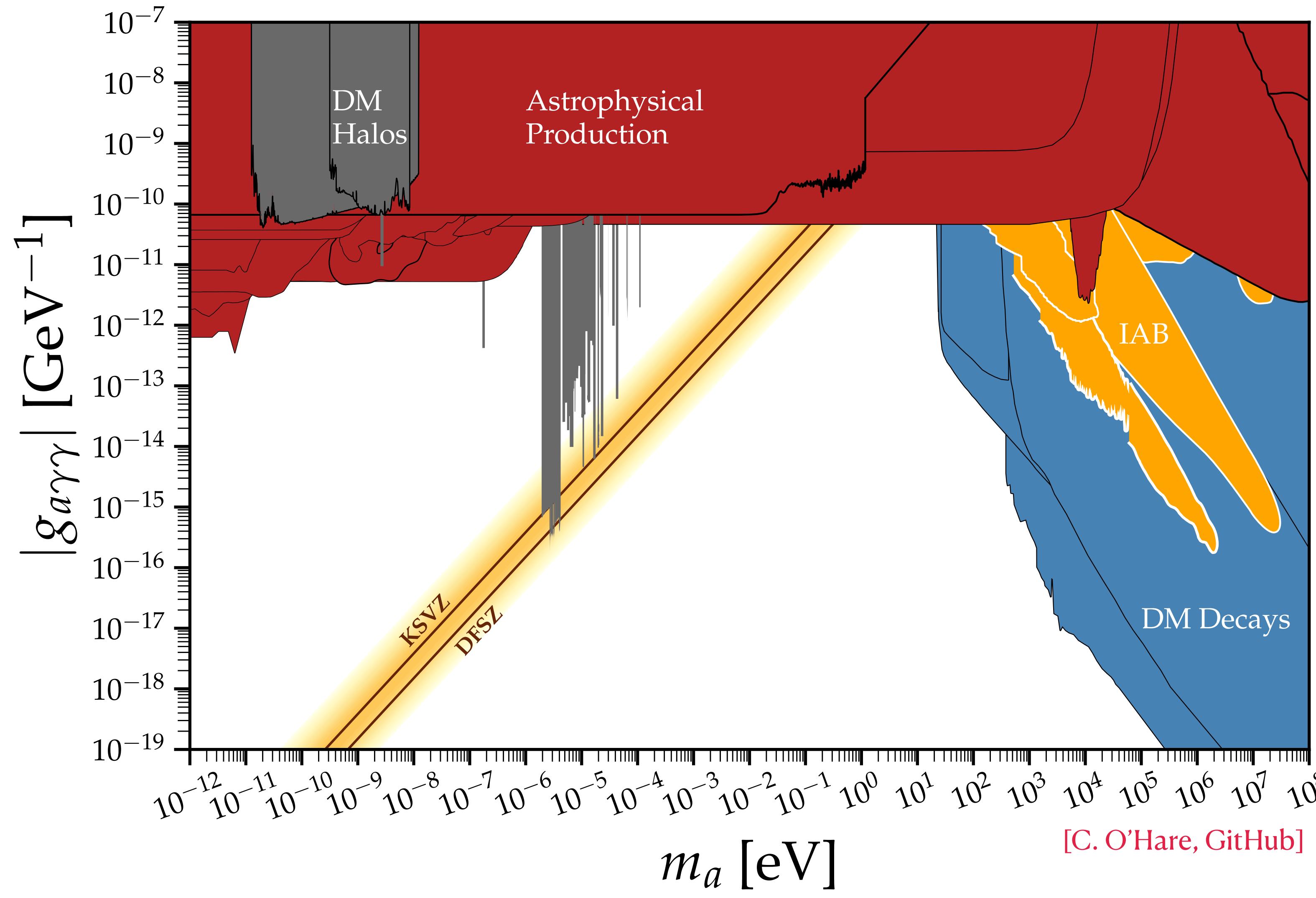
# Axion Parameter Space

What if it is not ALL of DM?



# Axion Parameter Space

What if it is not ALL of DM?



Irreducible Axion  
Background  
(Freeze-in relics)

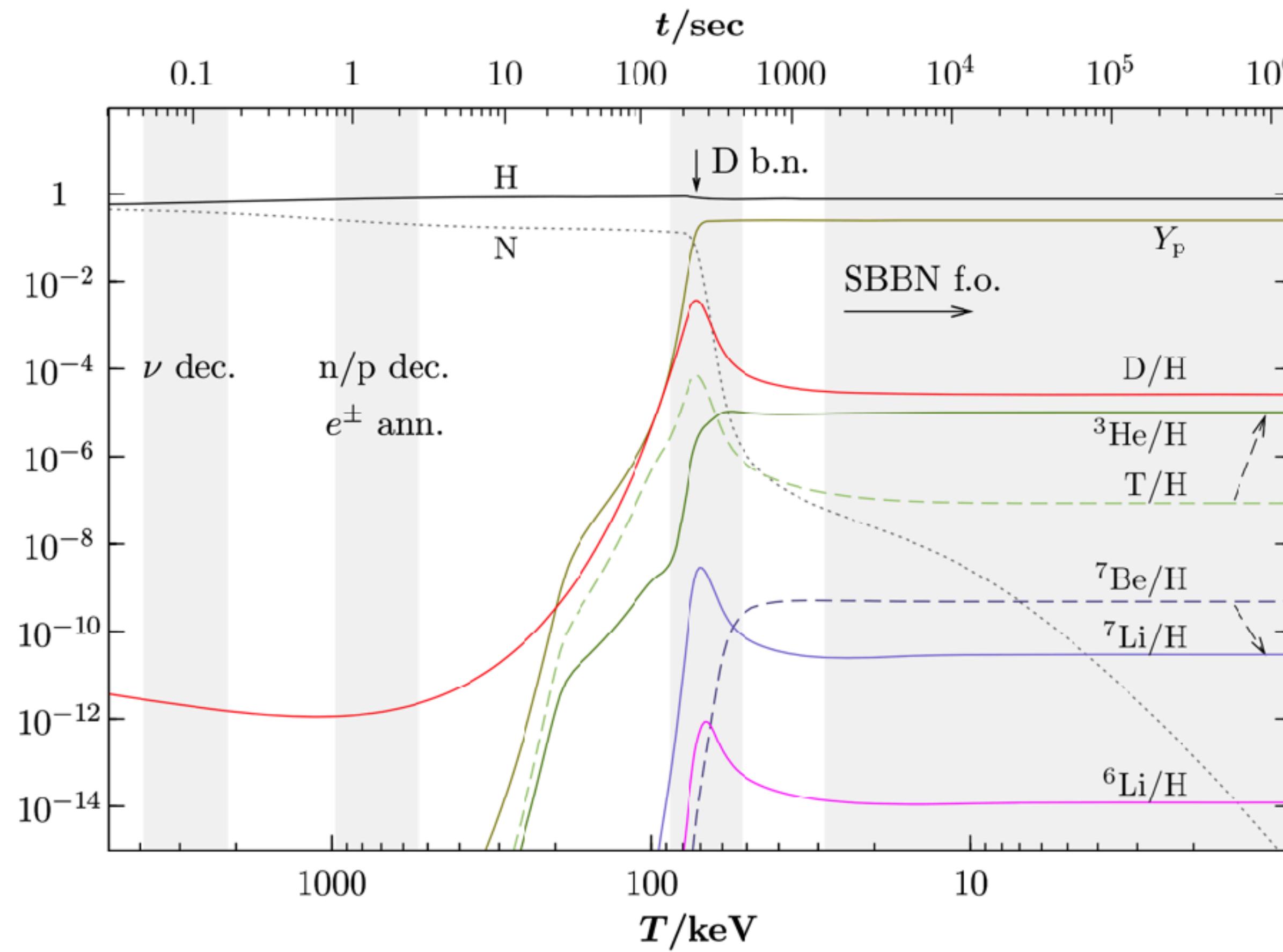
# **Irreducible Cosmic Abundance & Constraints**

# The General Picture

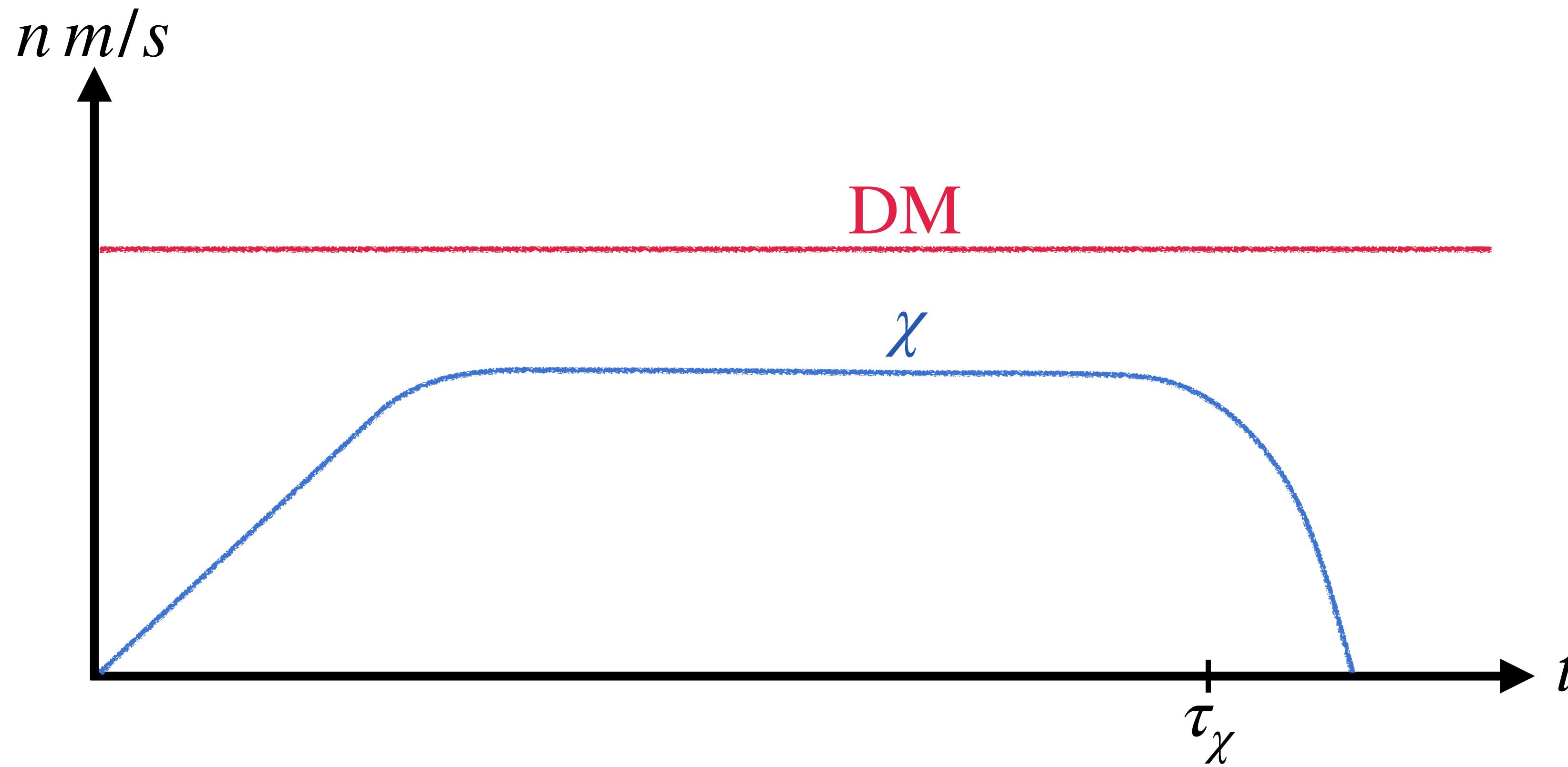
Dark matter may consist of **more than one species**.

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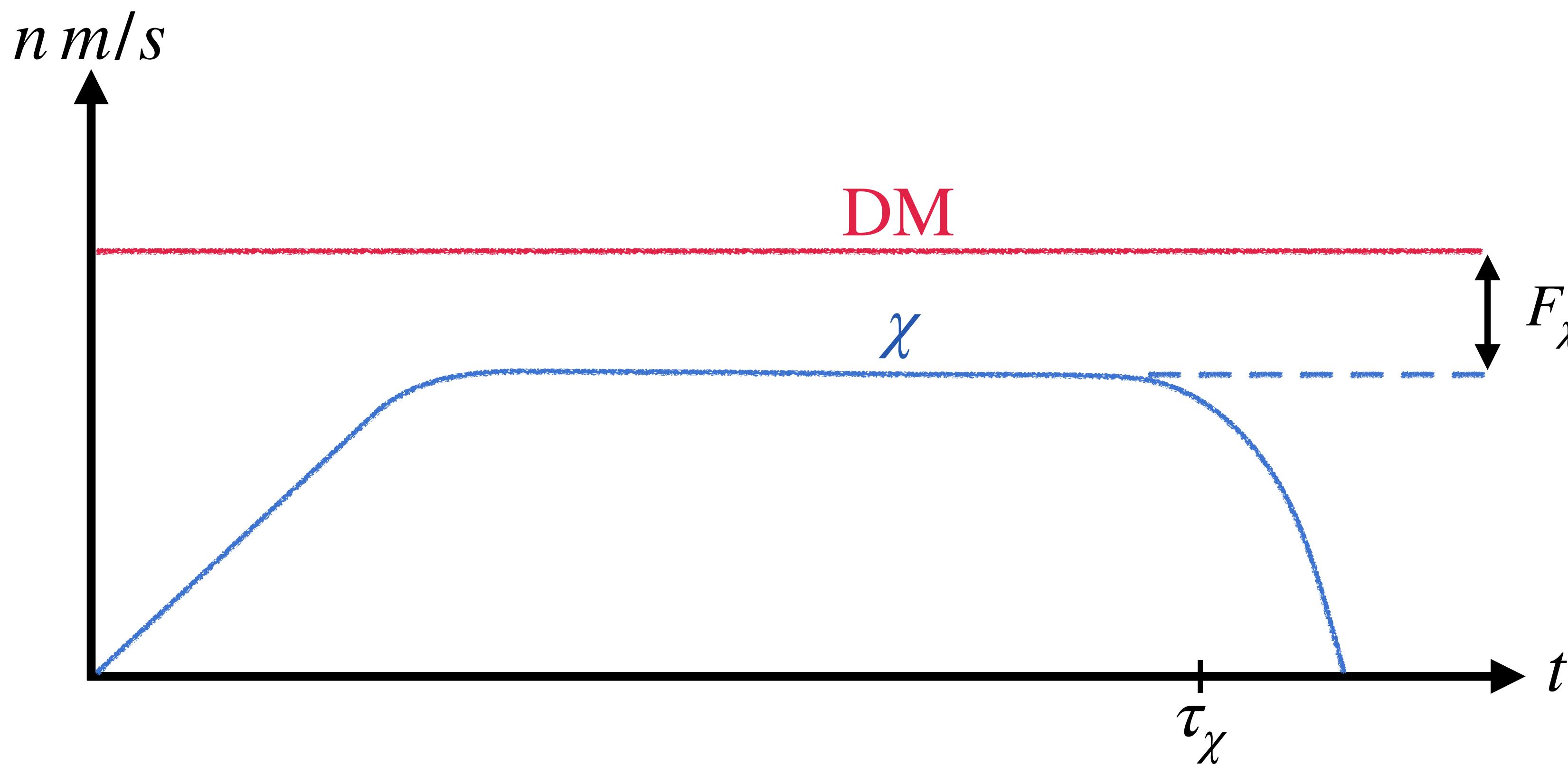
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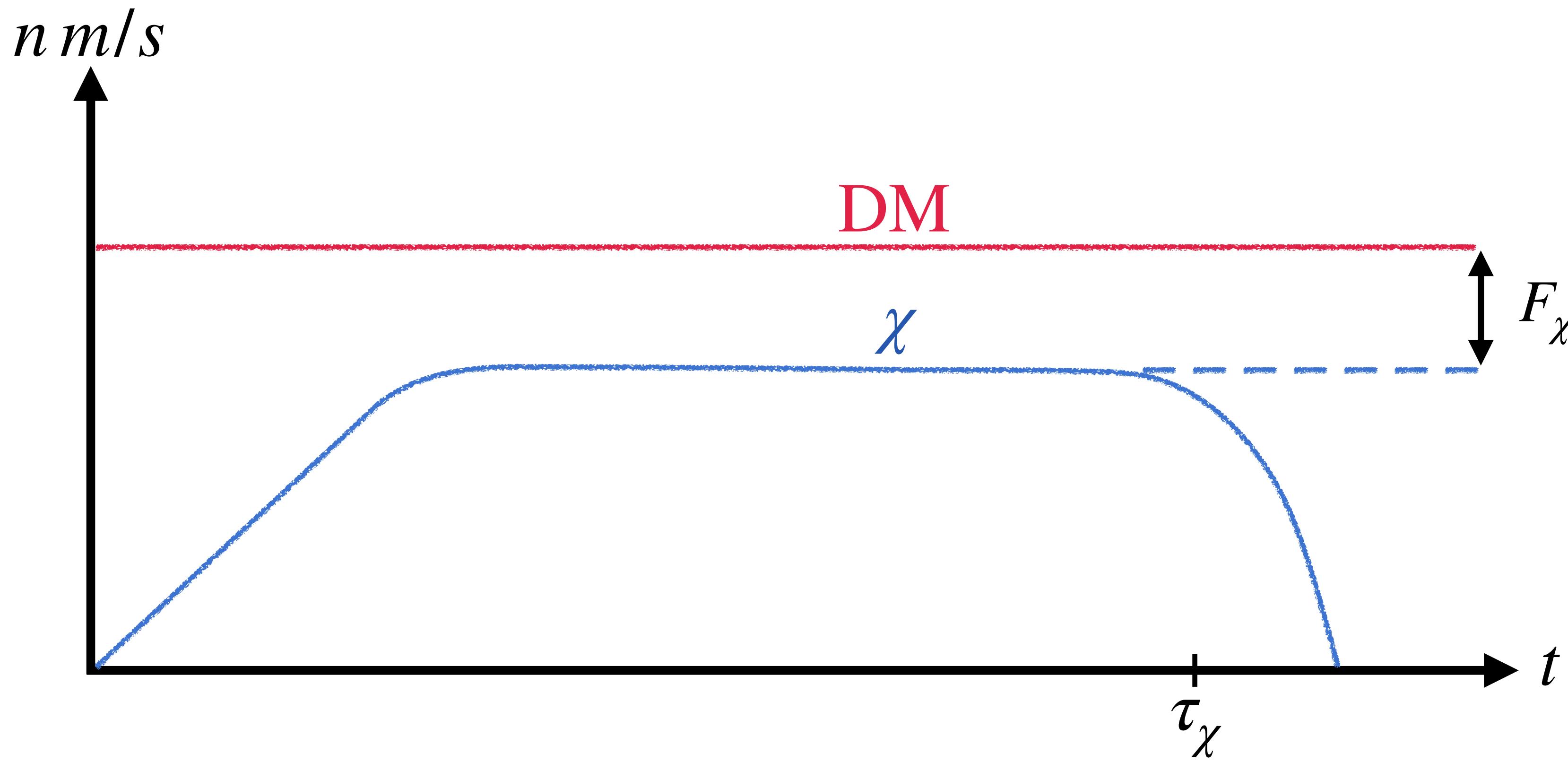
# Definition of $F_\chi$



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$$\rho_\chi \approx F_\chi \rho_{\text{DM}} e^{-t/\tau_\chi} \quad (\text{For non-relativistic } \chi \text{ after freeze-in})$$

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Constraints depending on  $C$  are not robust.

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Constraints obtained using  $F_{\chi,\text{irr}}(m_\chi, g_\chi)$  are **robust** under two mild assumptions:

1.  $\chi$  does not decay/annihilate to a dark sector.
2. Standard cosmology holds from BBN on.

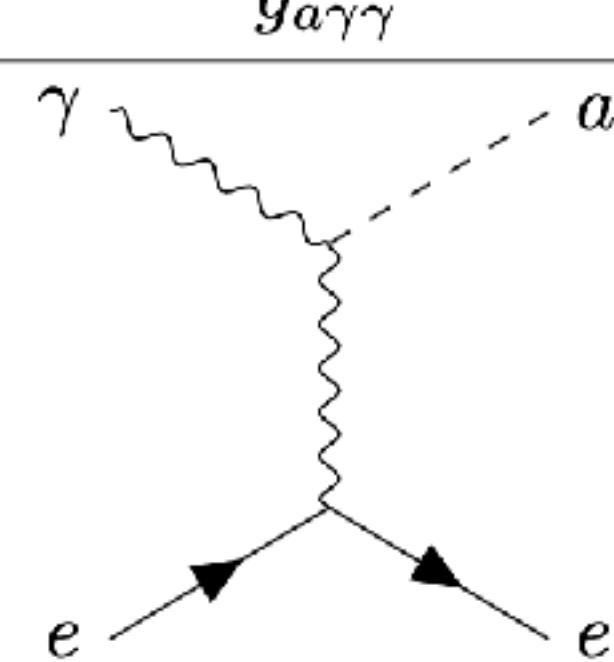
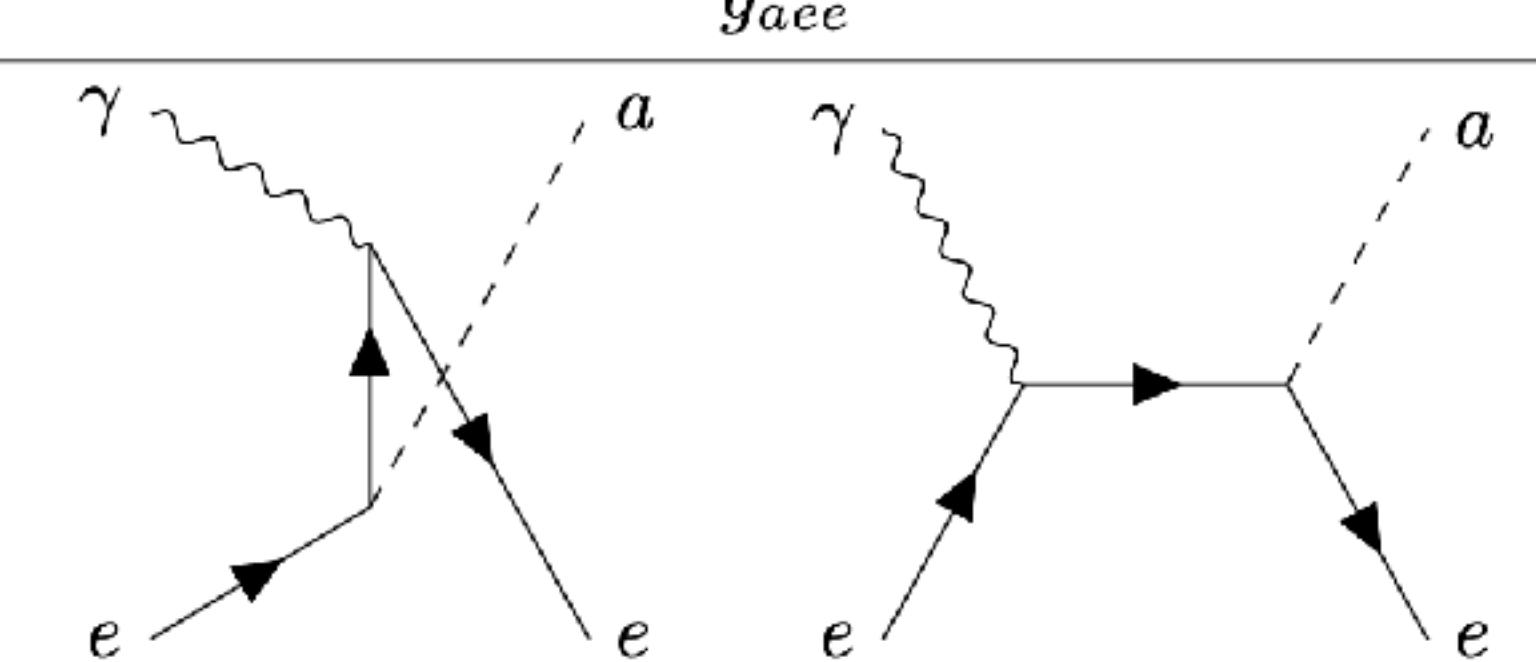
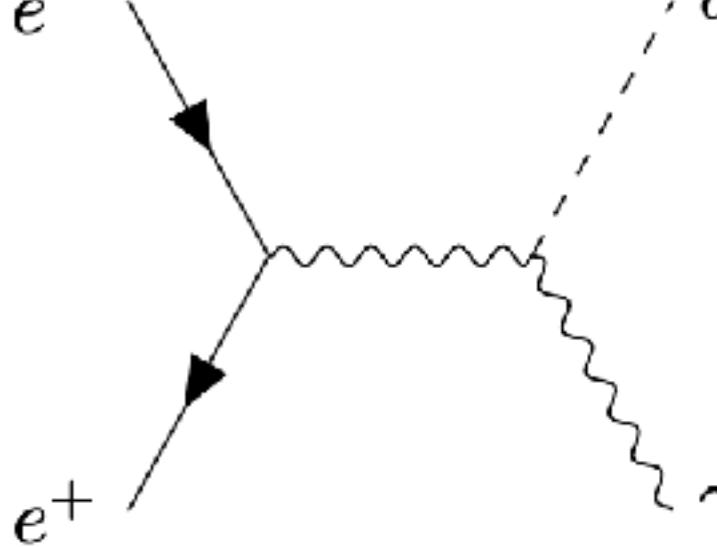
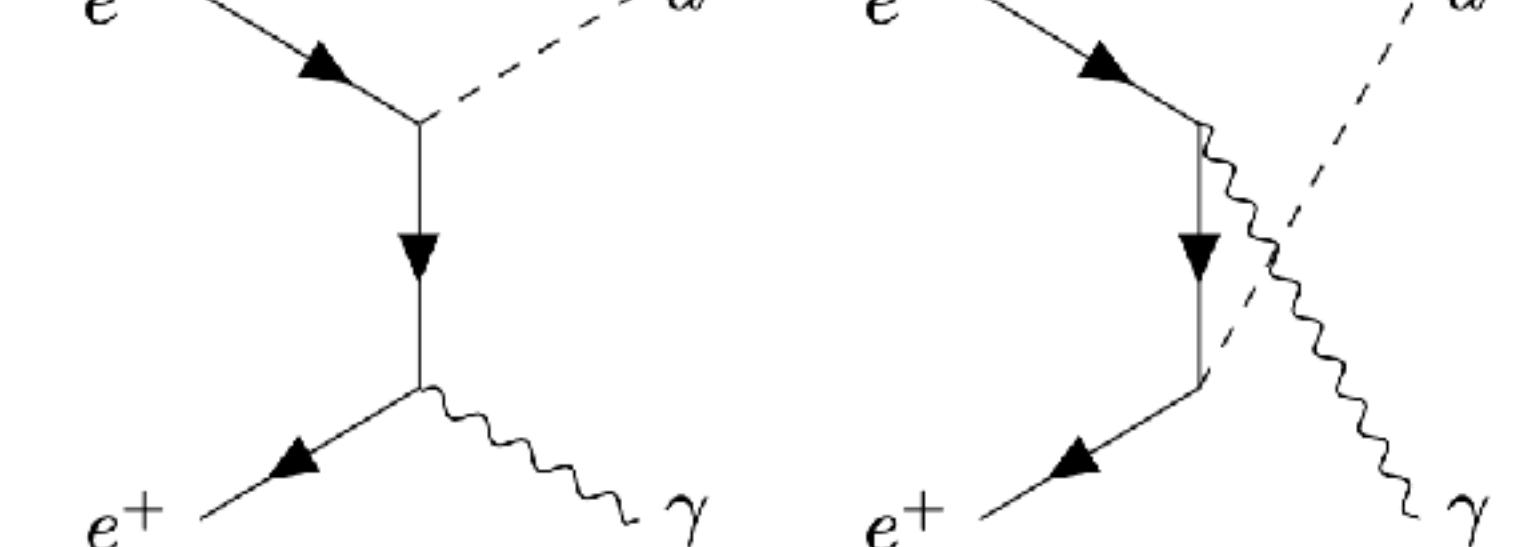
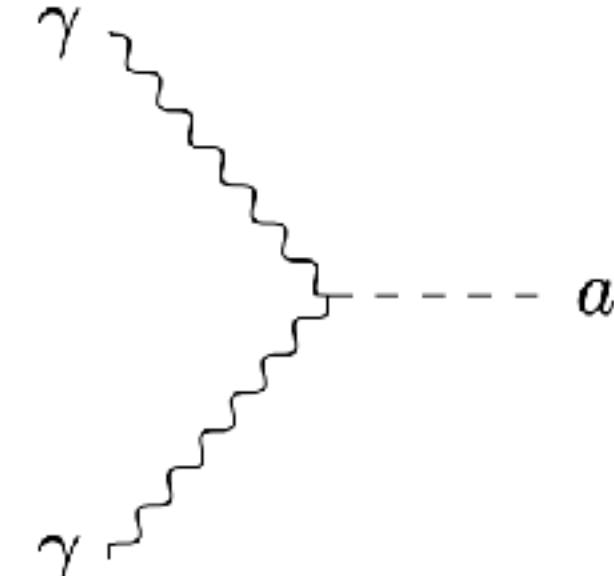
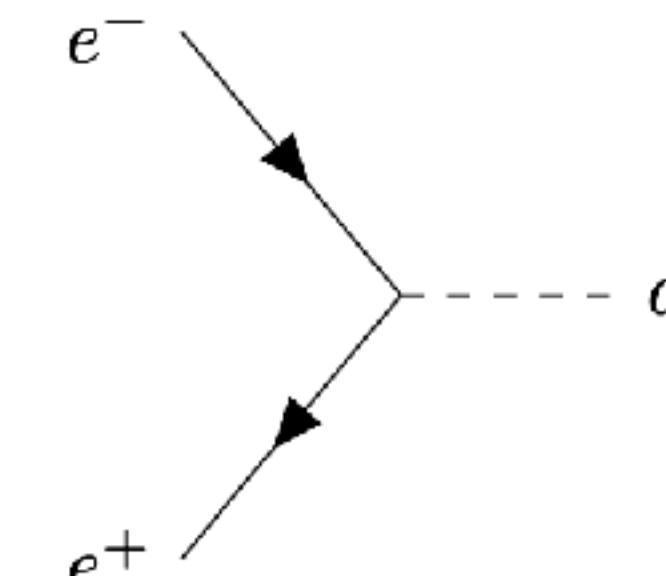
# **What is Freeze-In**

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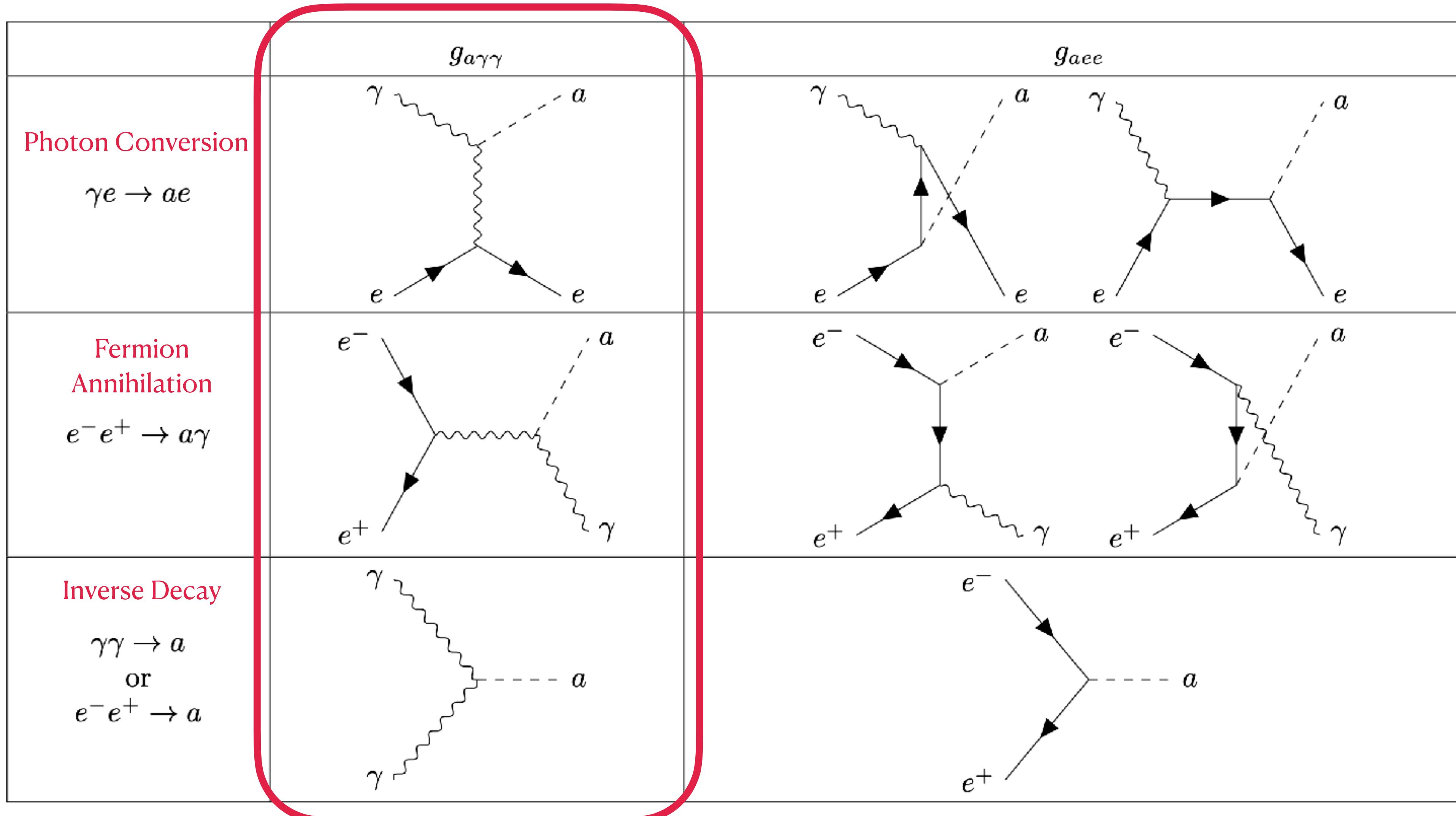
- Freeze-in is the process where particles are created from the primordial plasma of the universe without ever being in a state of thermal equilibrium with it.



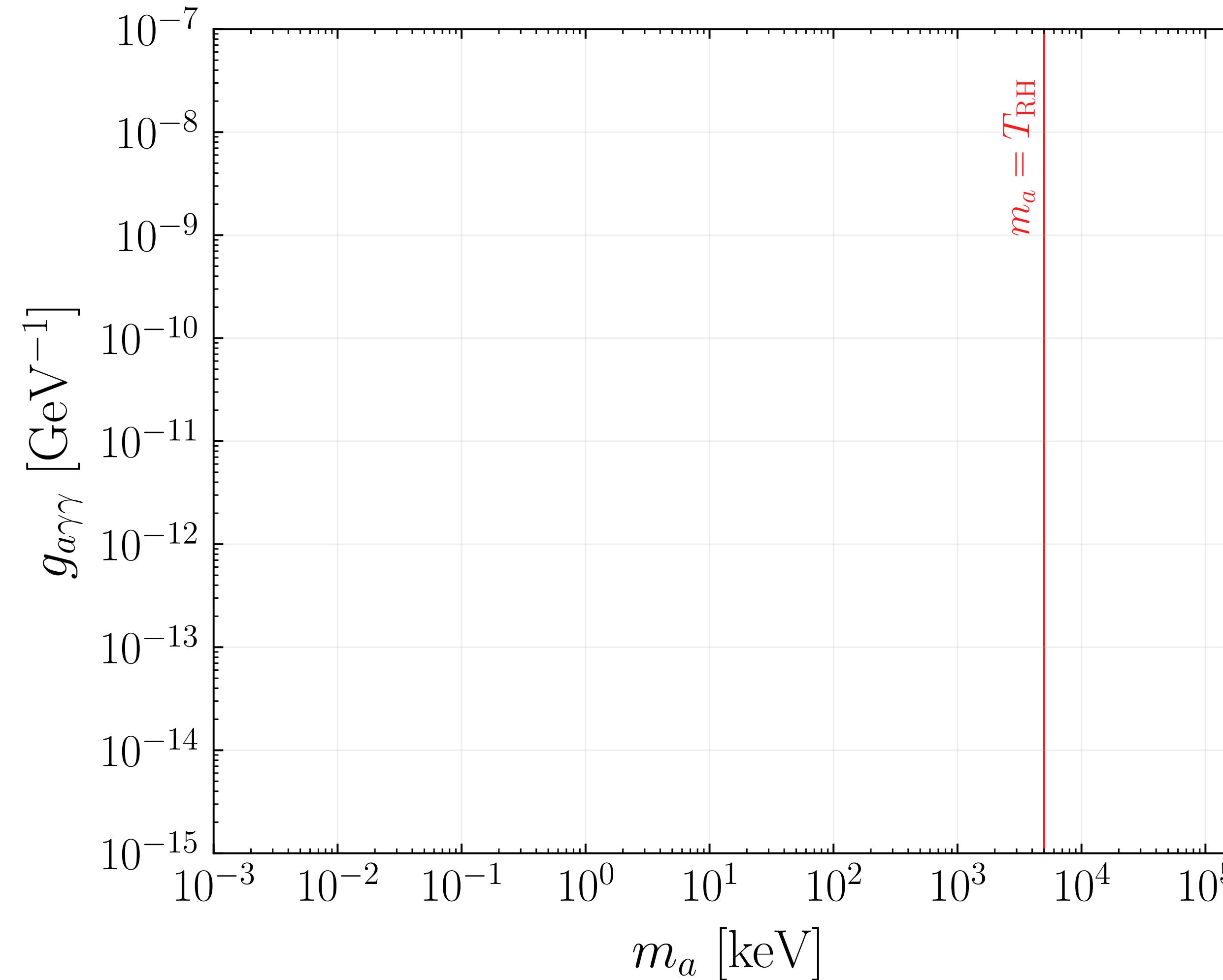
# General Production of Axions

	$g_{a\gamma\gamma}$	$g_{ae e}$
Photon Conversion $\gamma e \rightarrow ae$		
Fermion Annihilation $e^- e^+ \rightarrow a \gamma$		
Inverse Decay $\gamma\gamma \rightarrow a$ or $e^- e^+ \rightarrow a$		

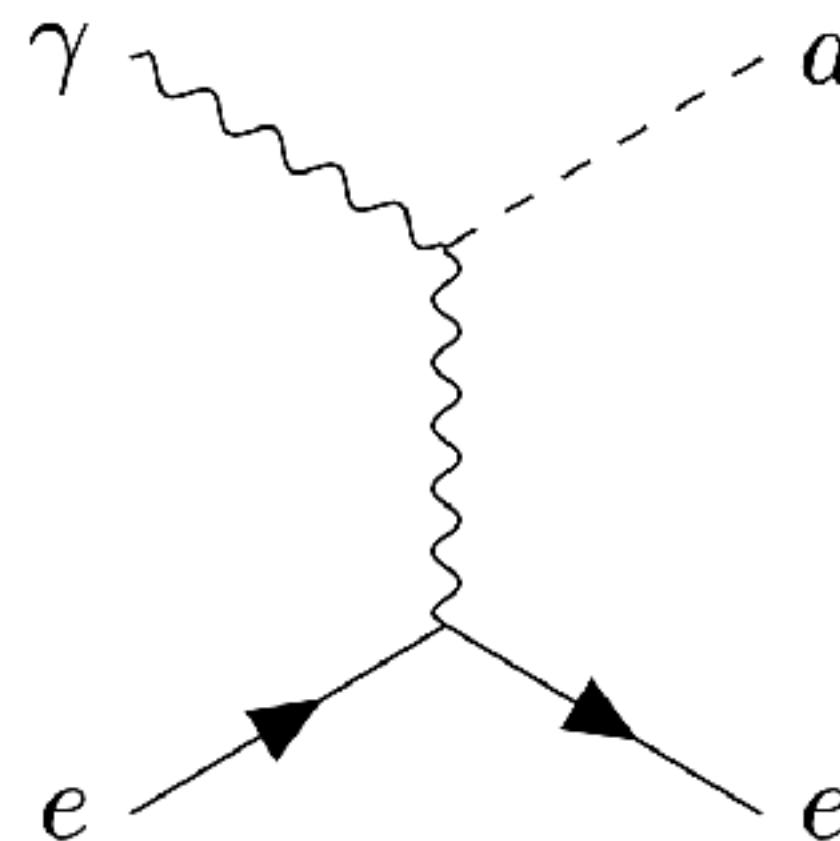
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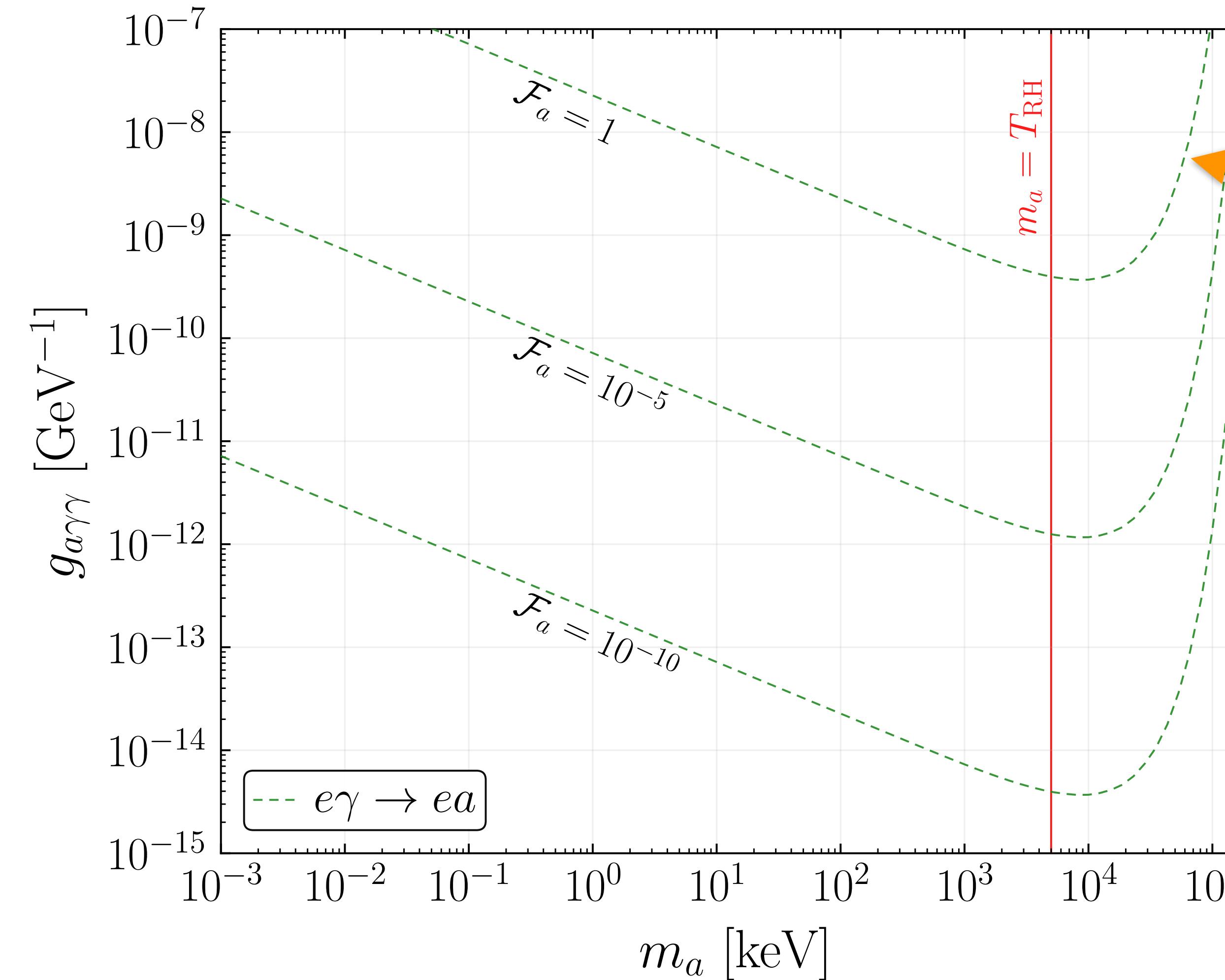
# Irreducible Freeze-In Background



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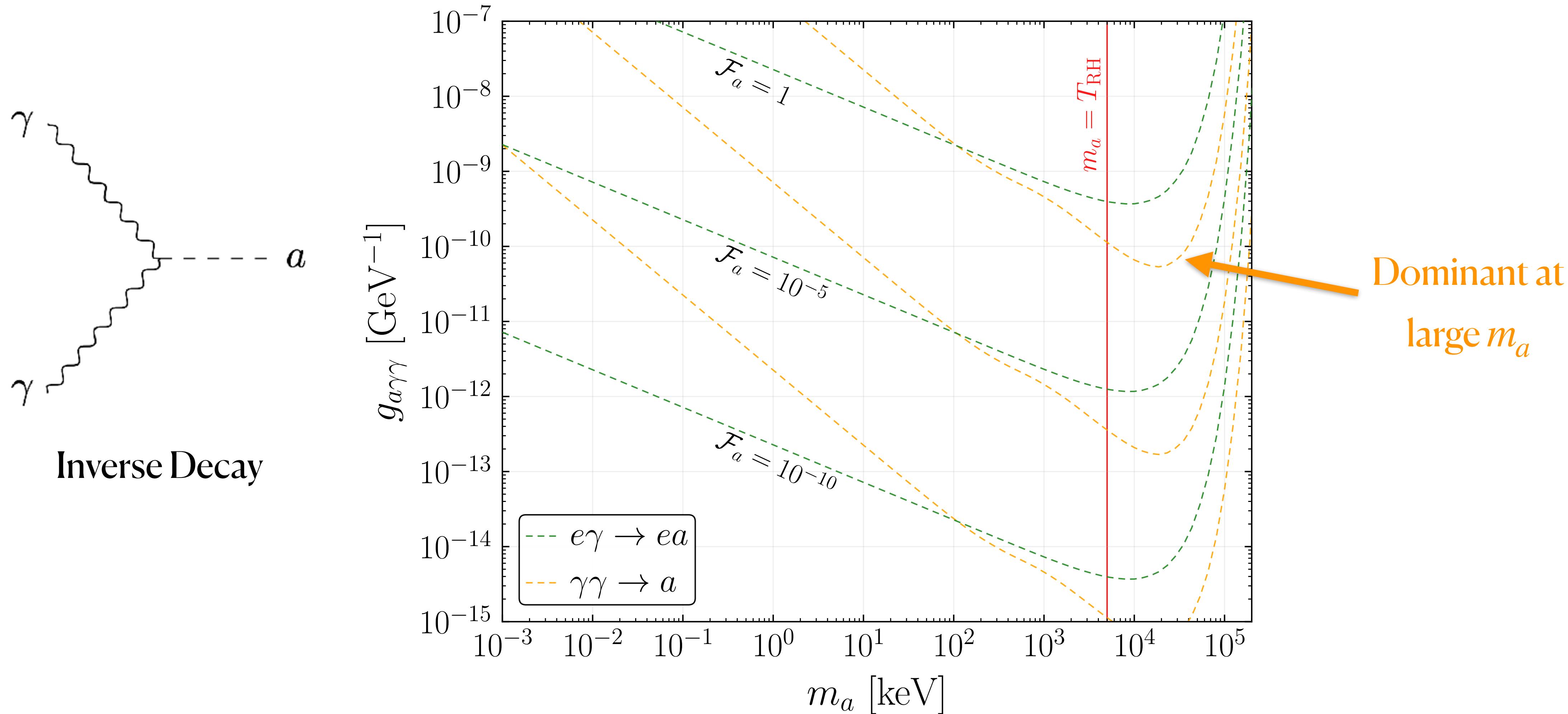


Photon Conversion

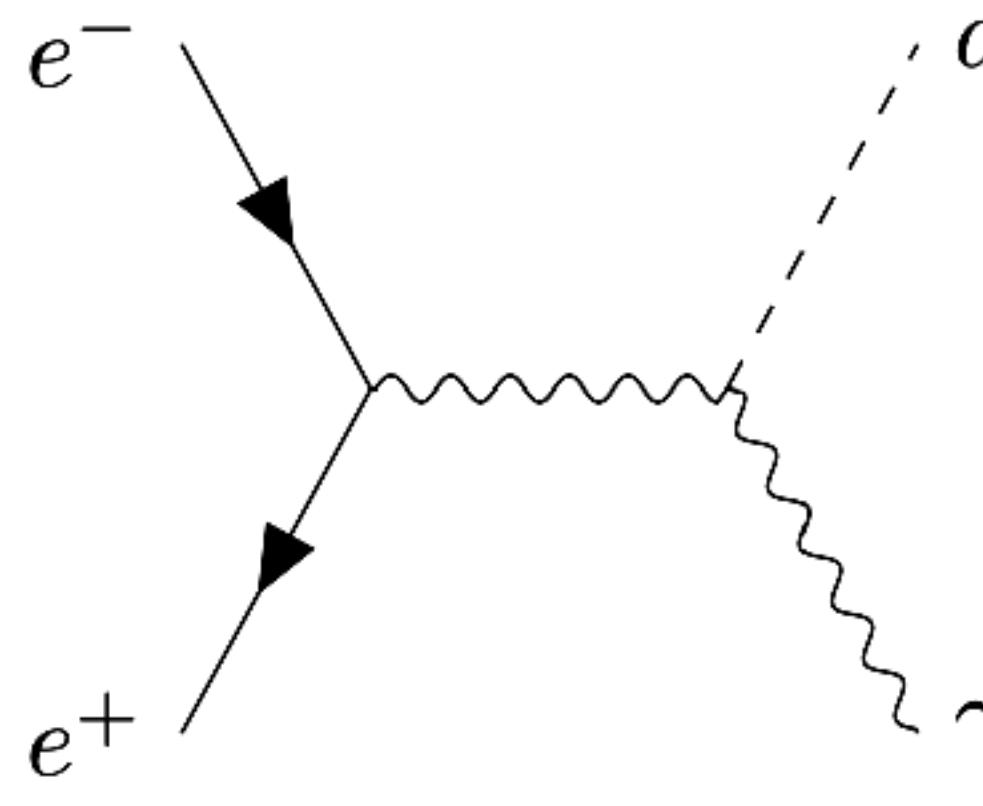


Exponentially suppressed for  
 $m_a \gg T_{\text{RH}}$

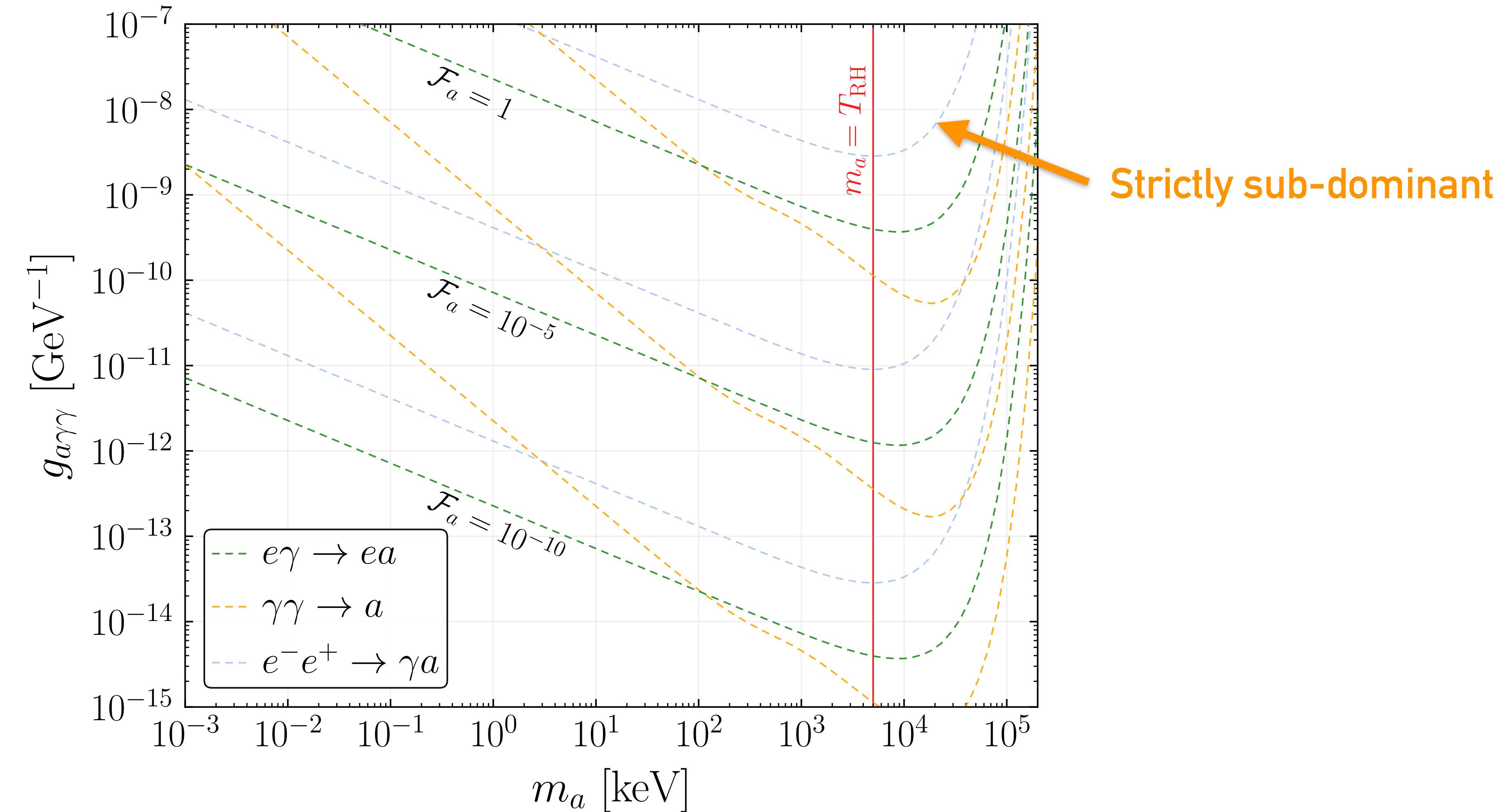
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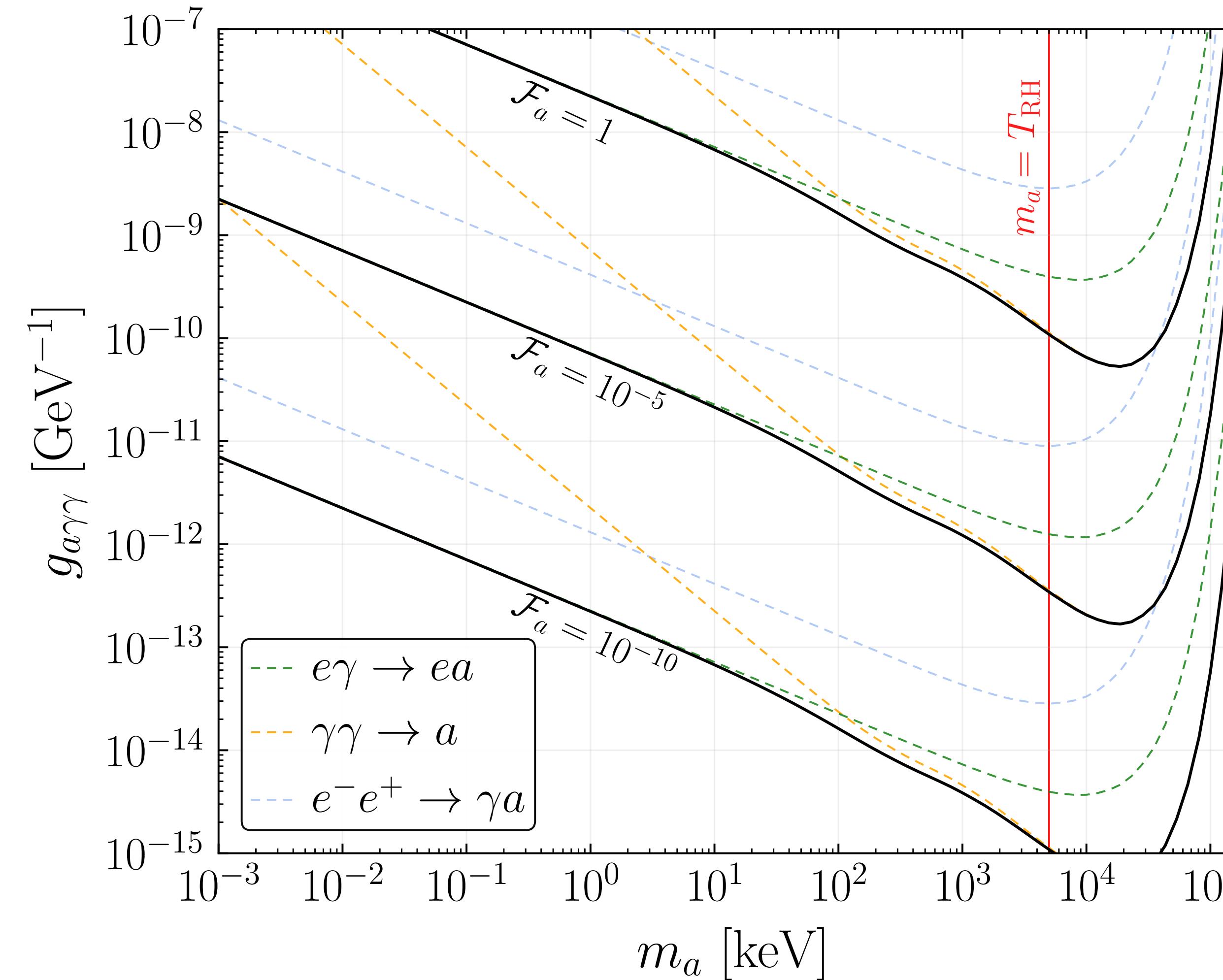
# Irreducible Freeze-In Background



Fermion Annihilation



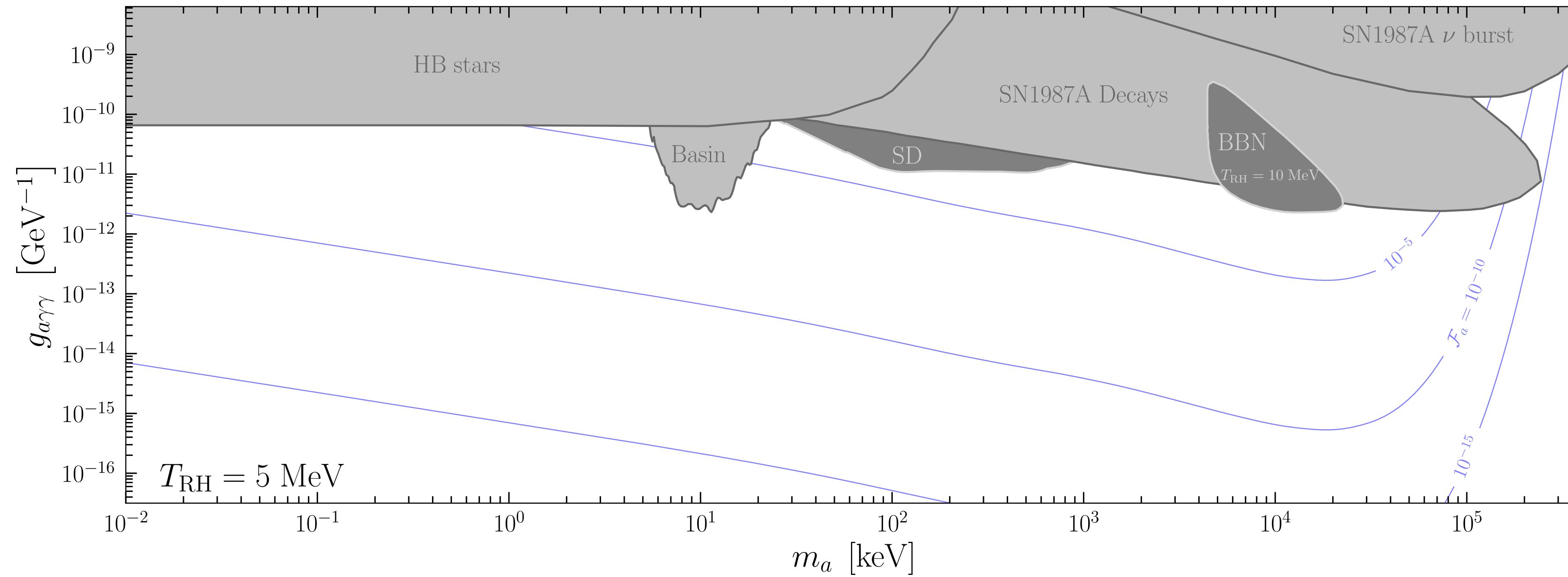
# Irreducible Freeze-In Background



# Astrophysical and Cosmological Constraints

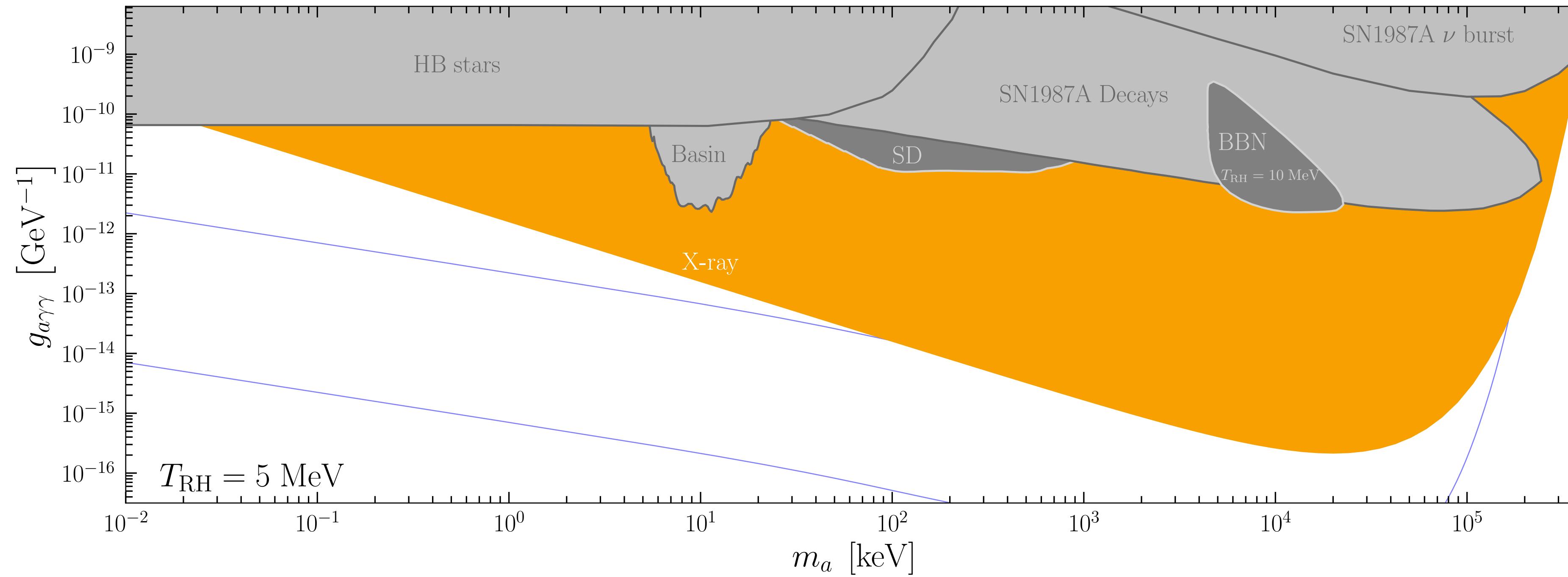
# Intuition: X-rays

- Consider the benchmark constraint  $\tau_{\text{DM}} > 10^{28}\text{s} \implies \tau_a > F_a \tau_{\text{DM}}$ .



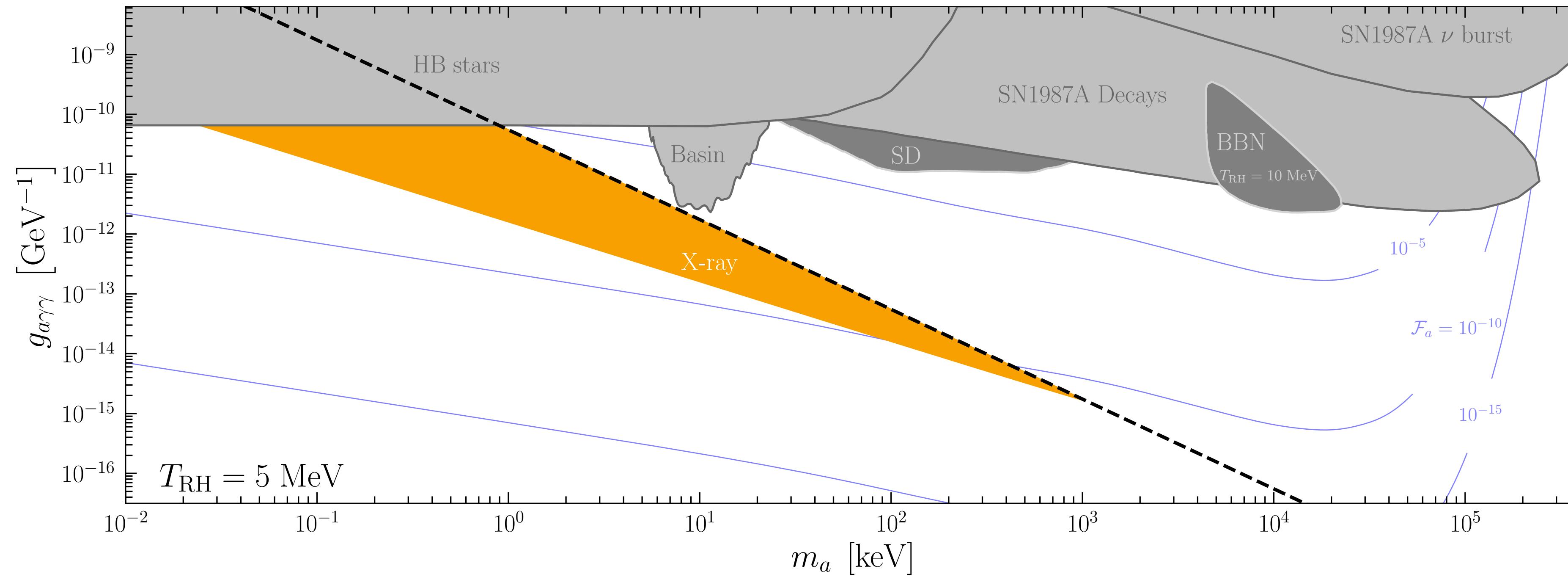
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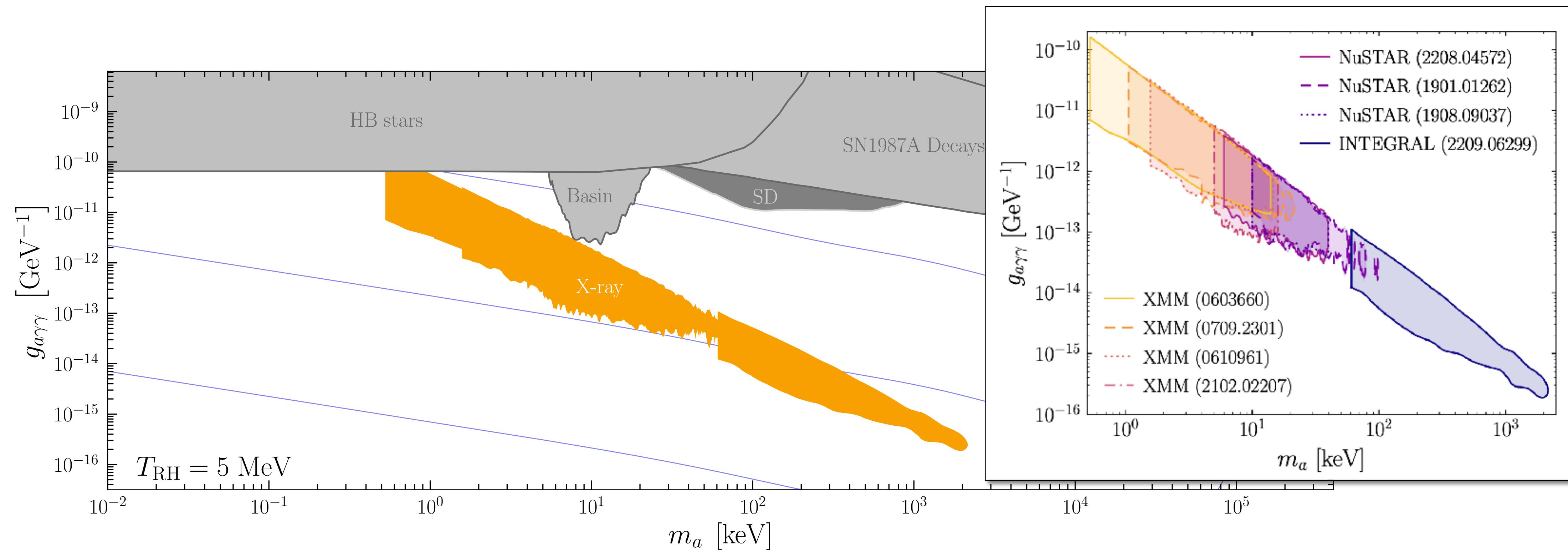
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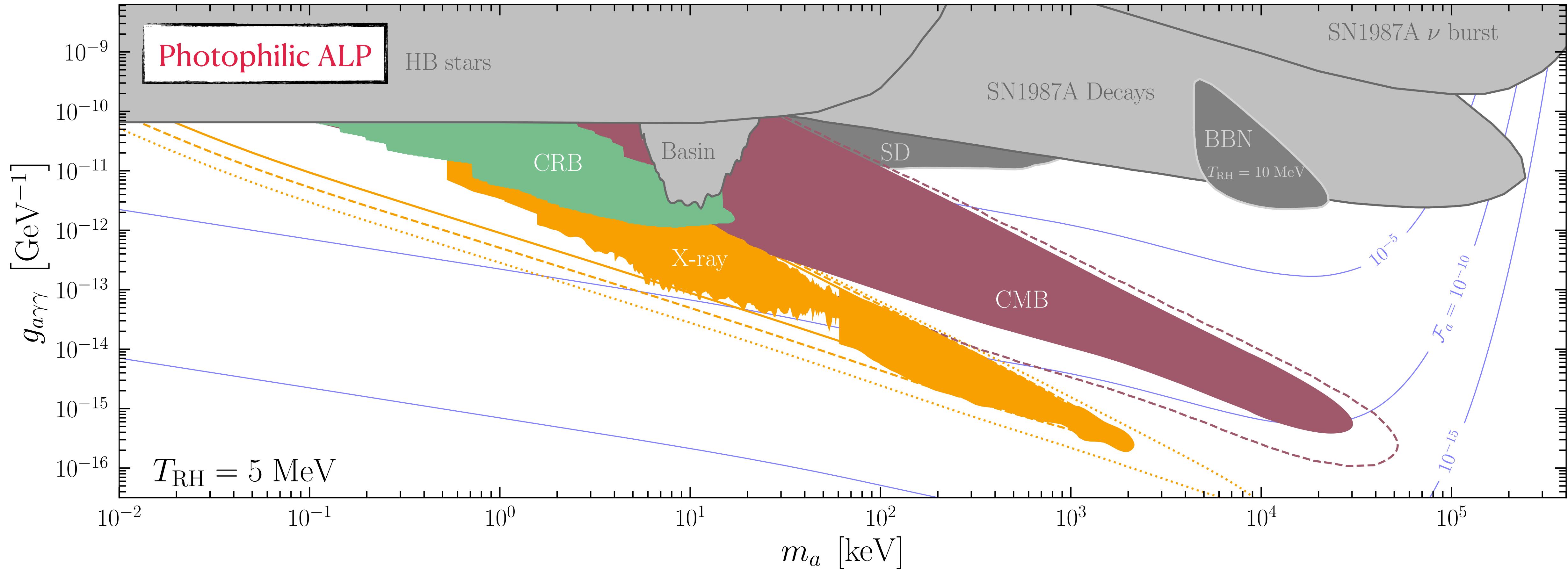


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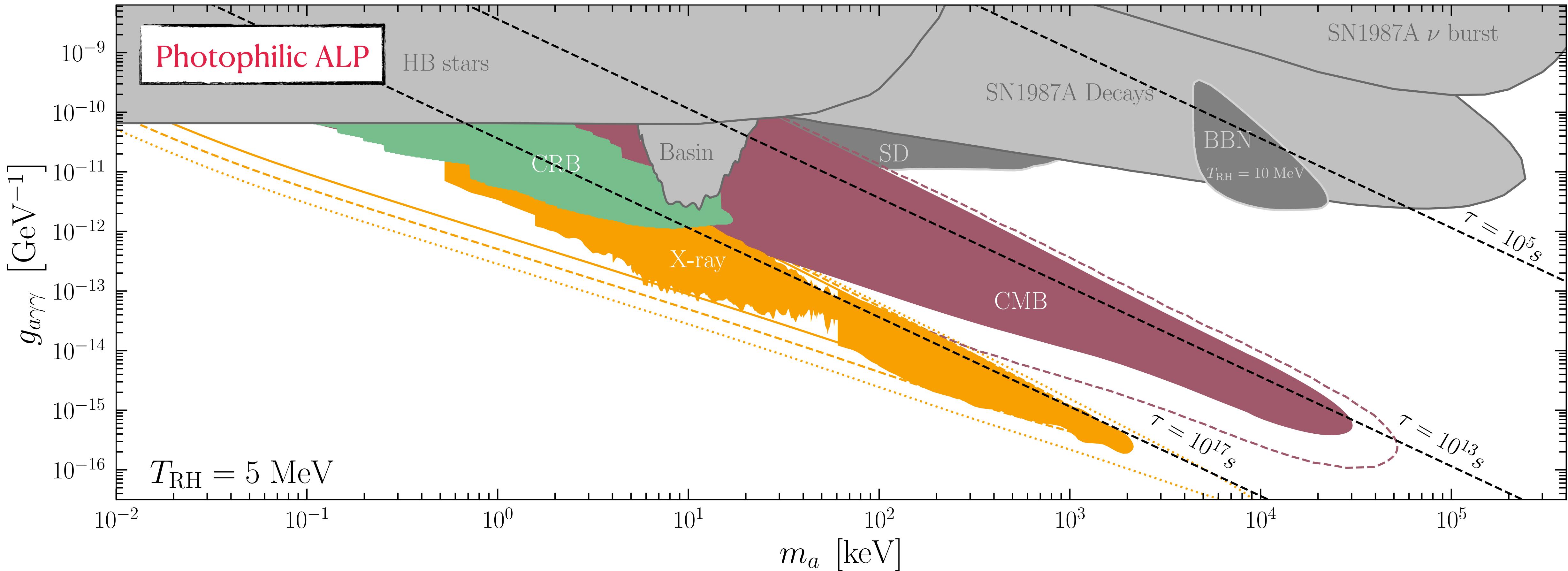
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# Photophilic Axion Bounds



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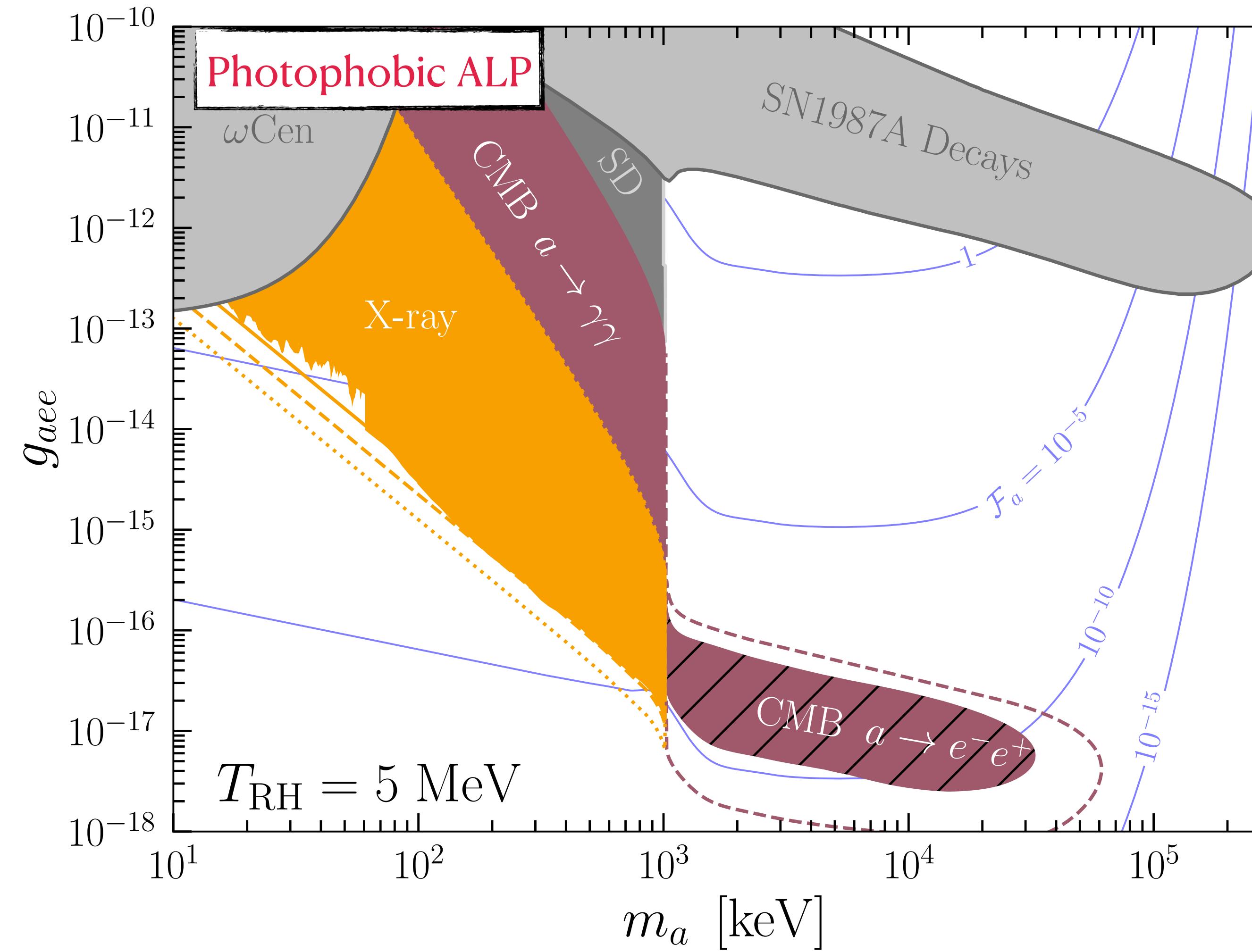


# Generalizations

# Photophobic Axion Constraints

<p>Photon Conversion <math>\gamma e \rightarrow ae</math></p>	<p><math>g_{a\gamma\gamma}</math></p>	<p><math>g_{ae e}</math></p>
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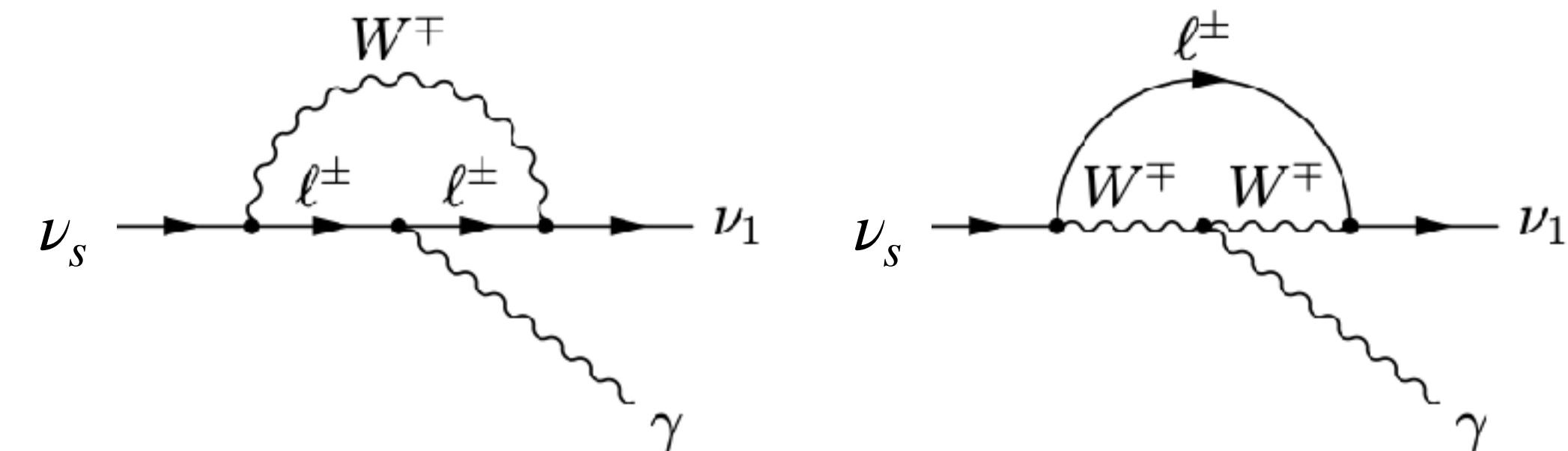


# Production of Sterile Neutrinos

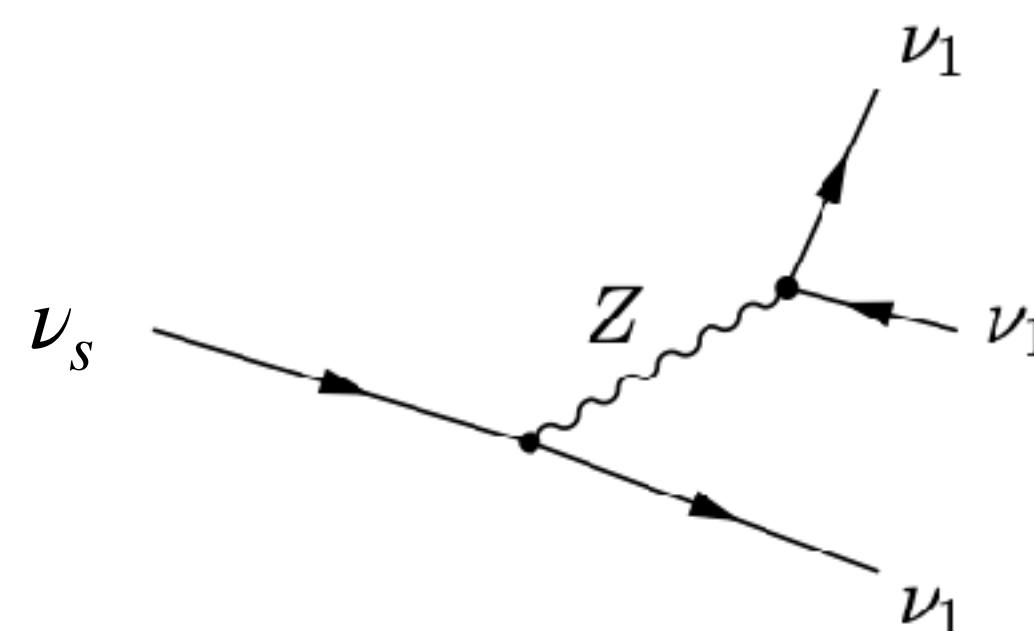
- For simplicity, assume sterile neutrino mixes only with  $\nu_e$ .

$$\begin{pmatrix} \nu_e \\ \nu_s \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

- Observed through radiative decays.

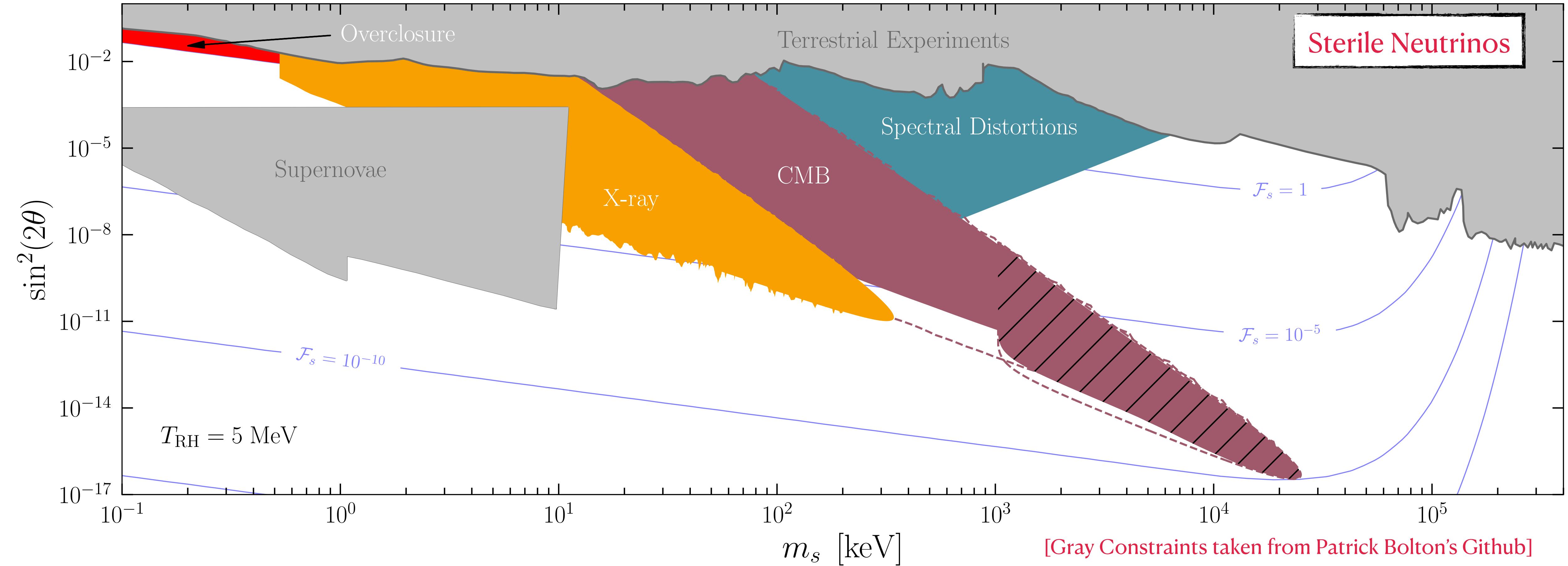


- However lifetime determined by different process.



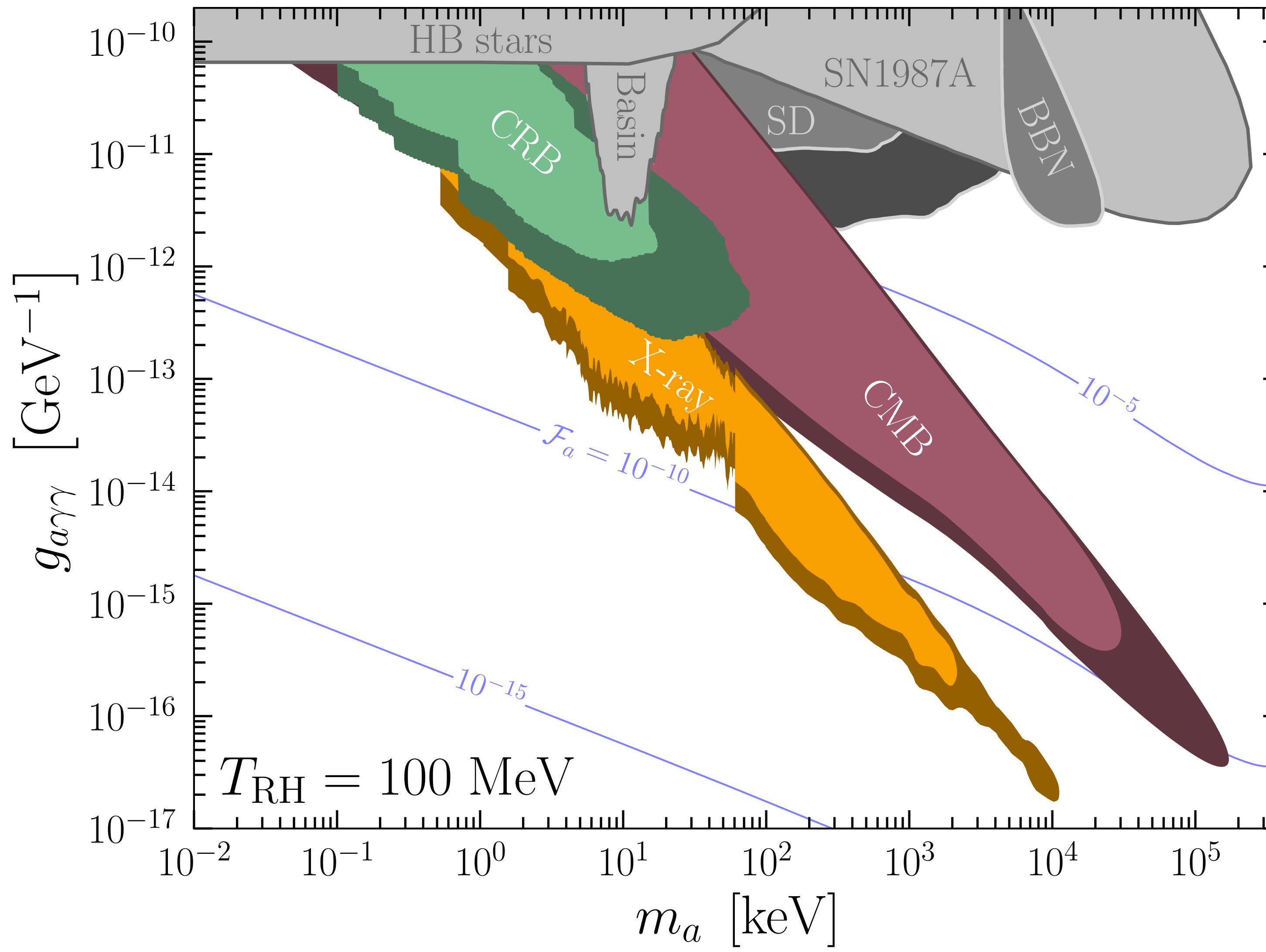
(Also to  $e^+e^-$  when  $m_s > 2m_e$ )

# Sterile Neutrino Constraints

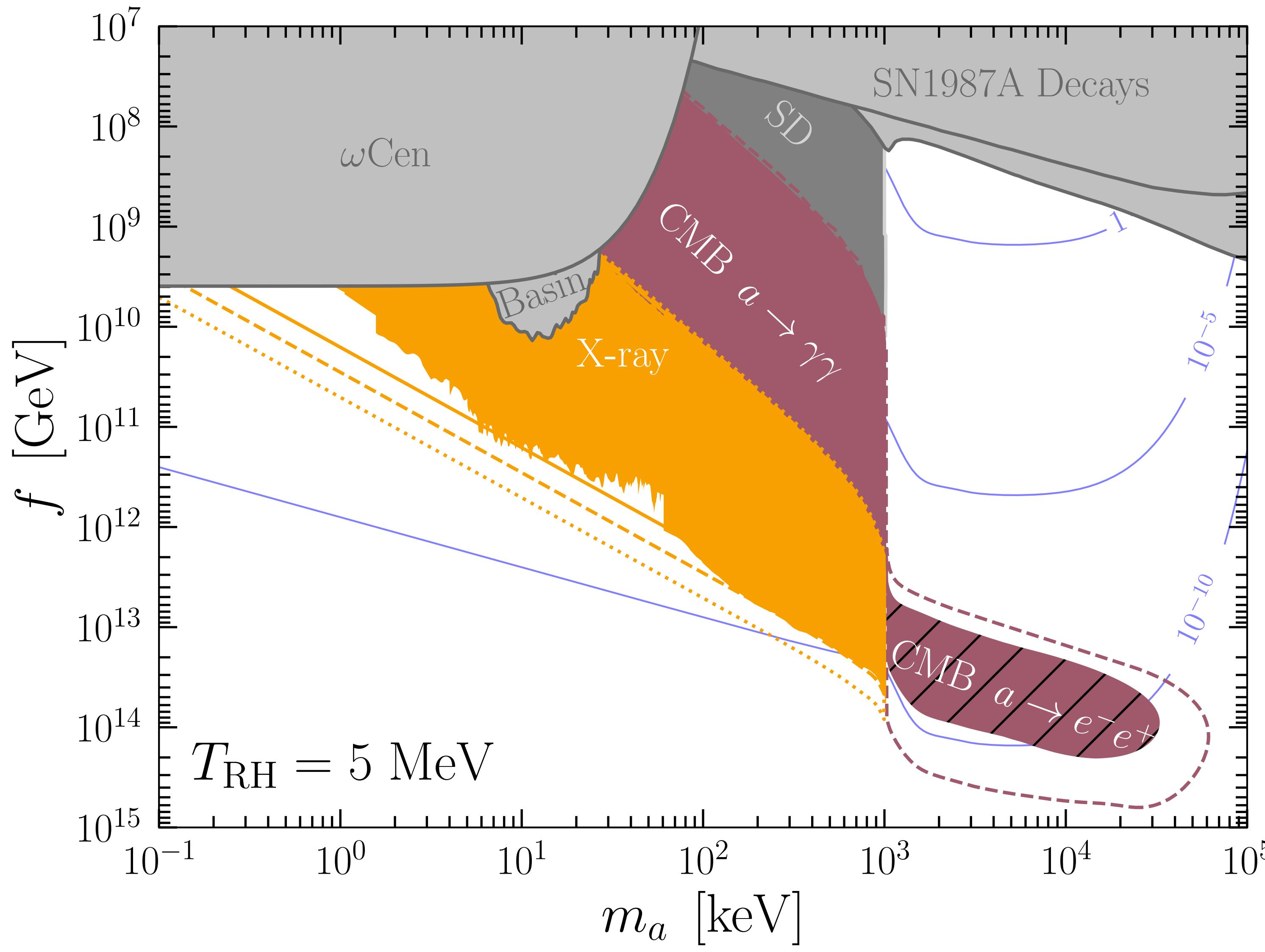


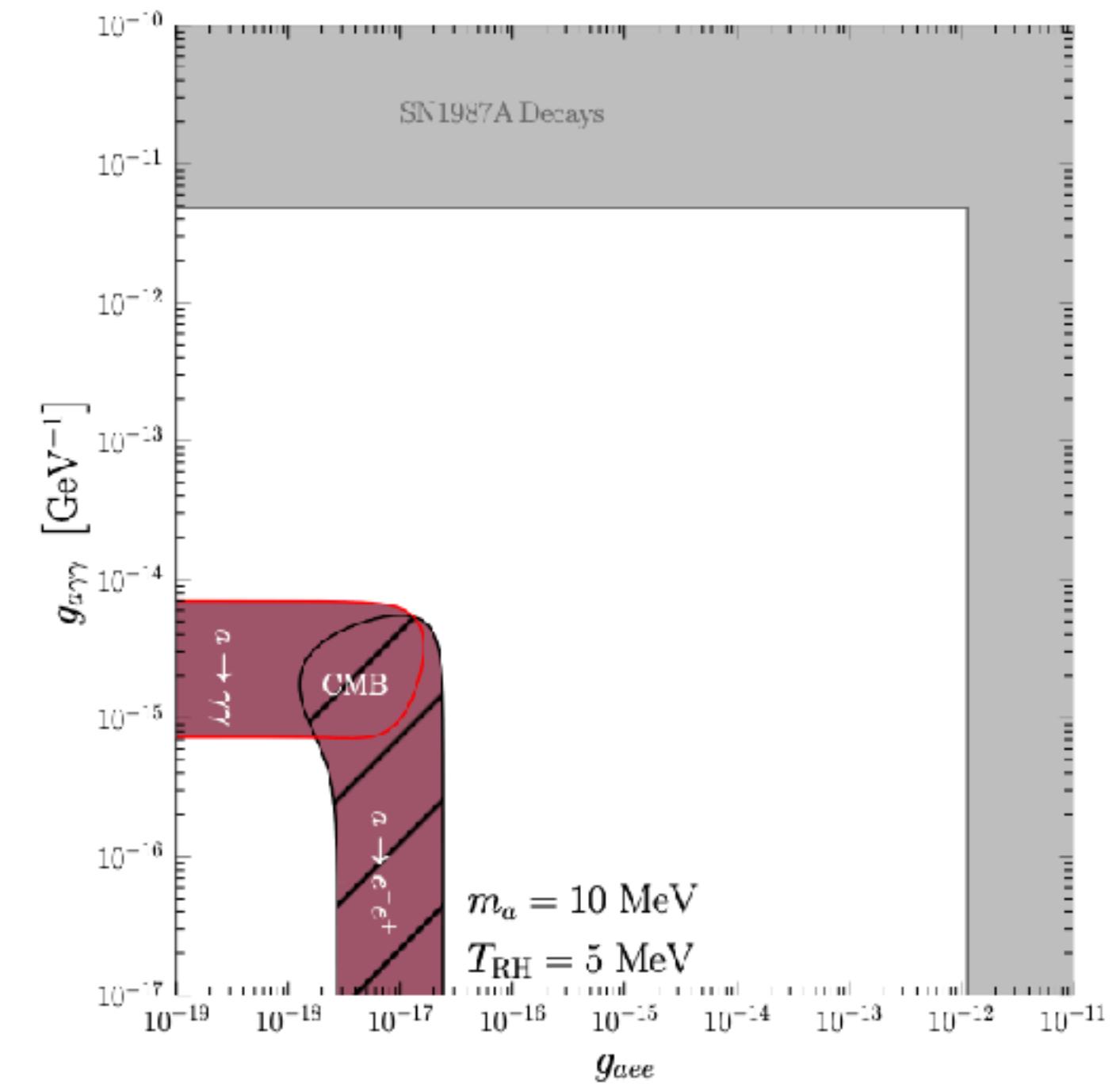
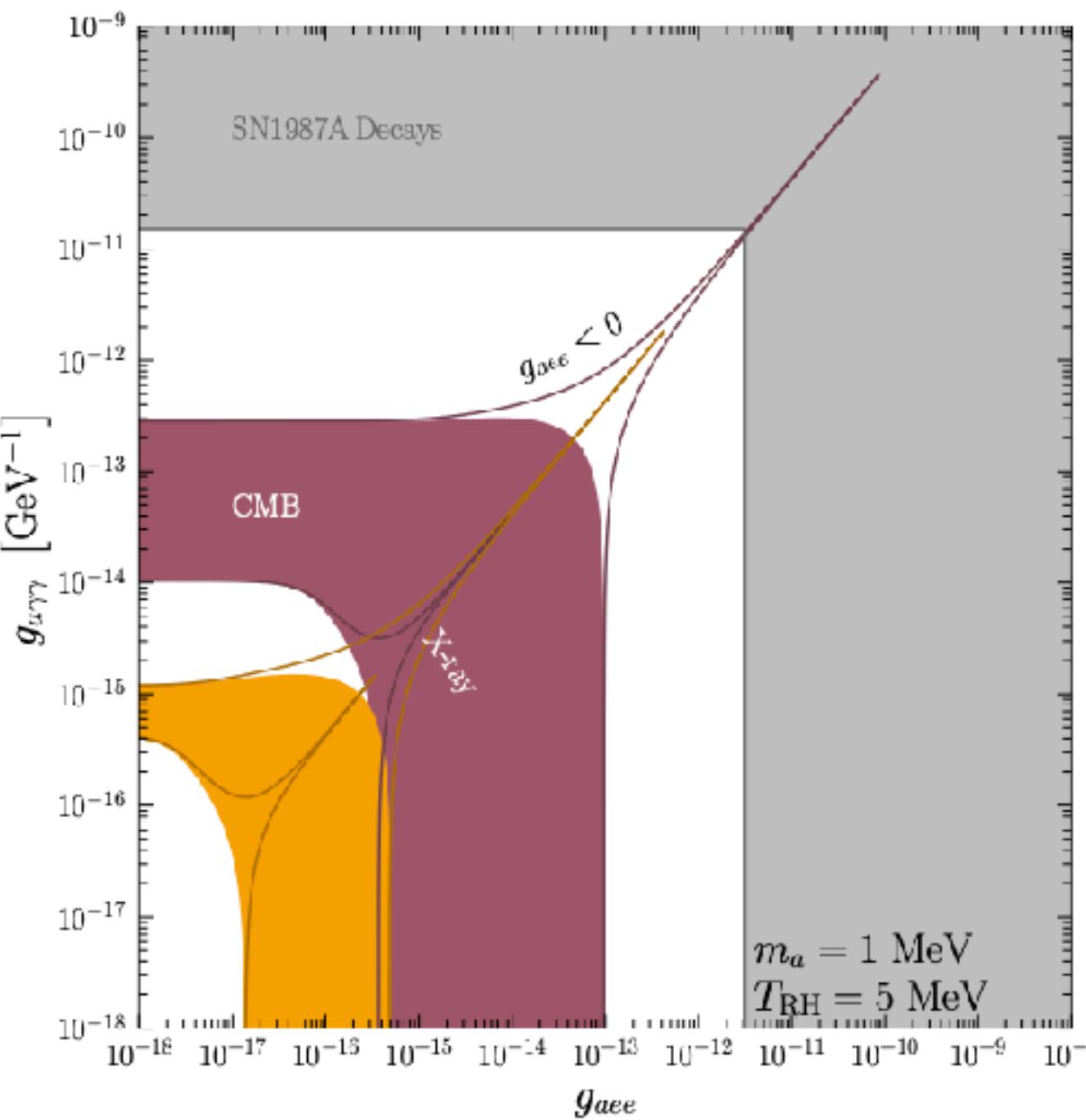
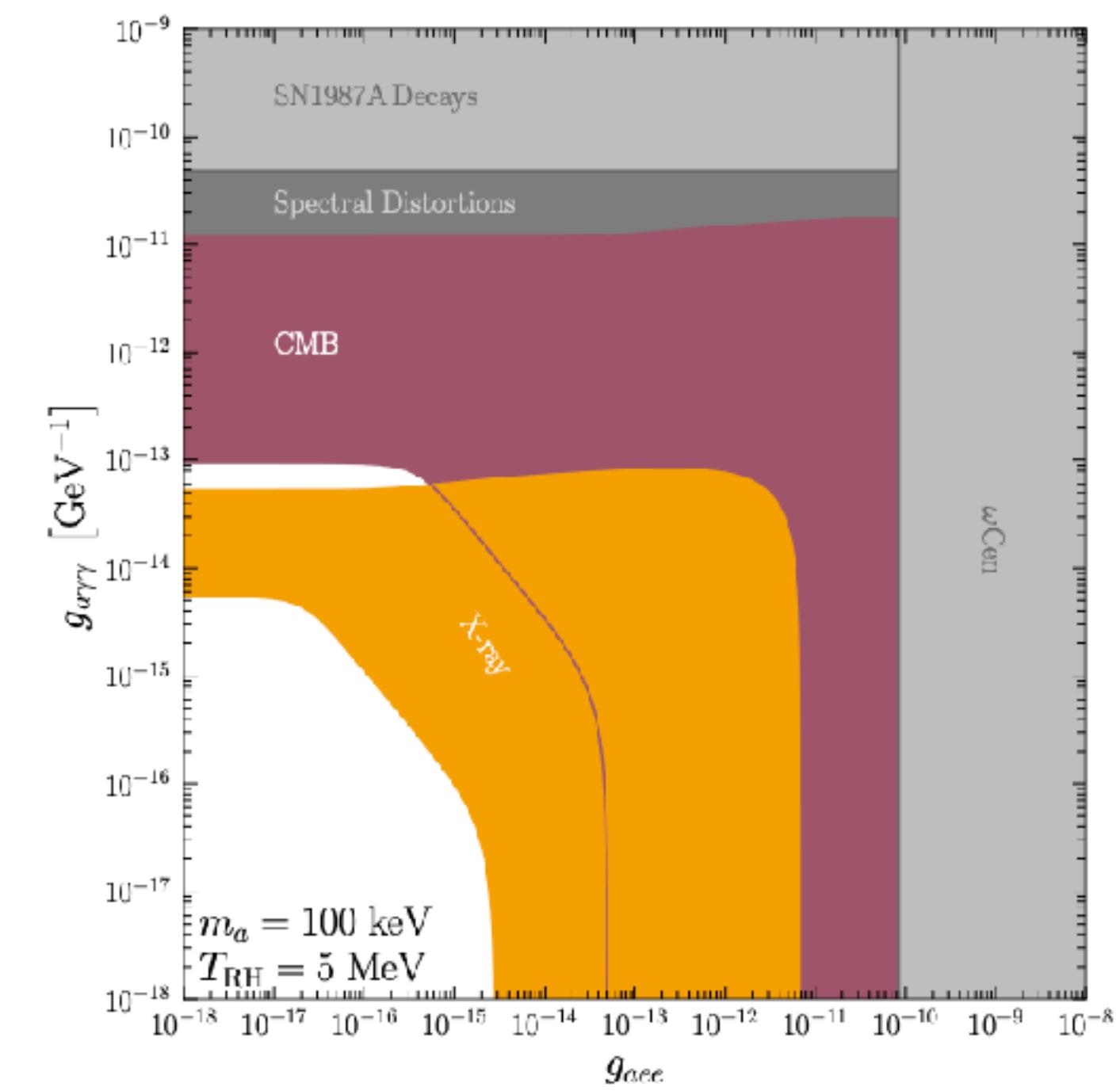
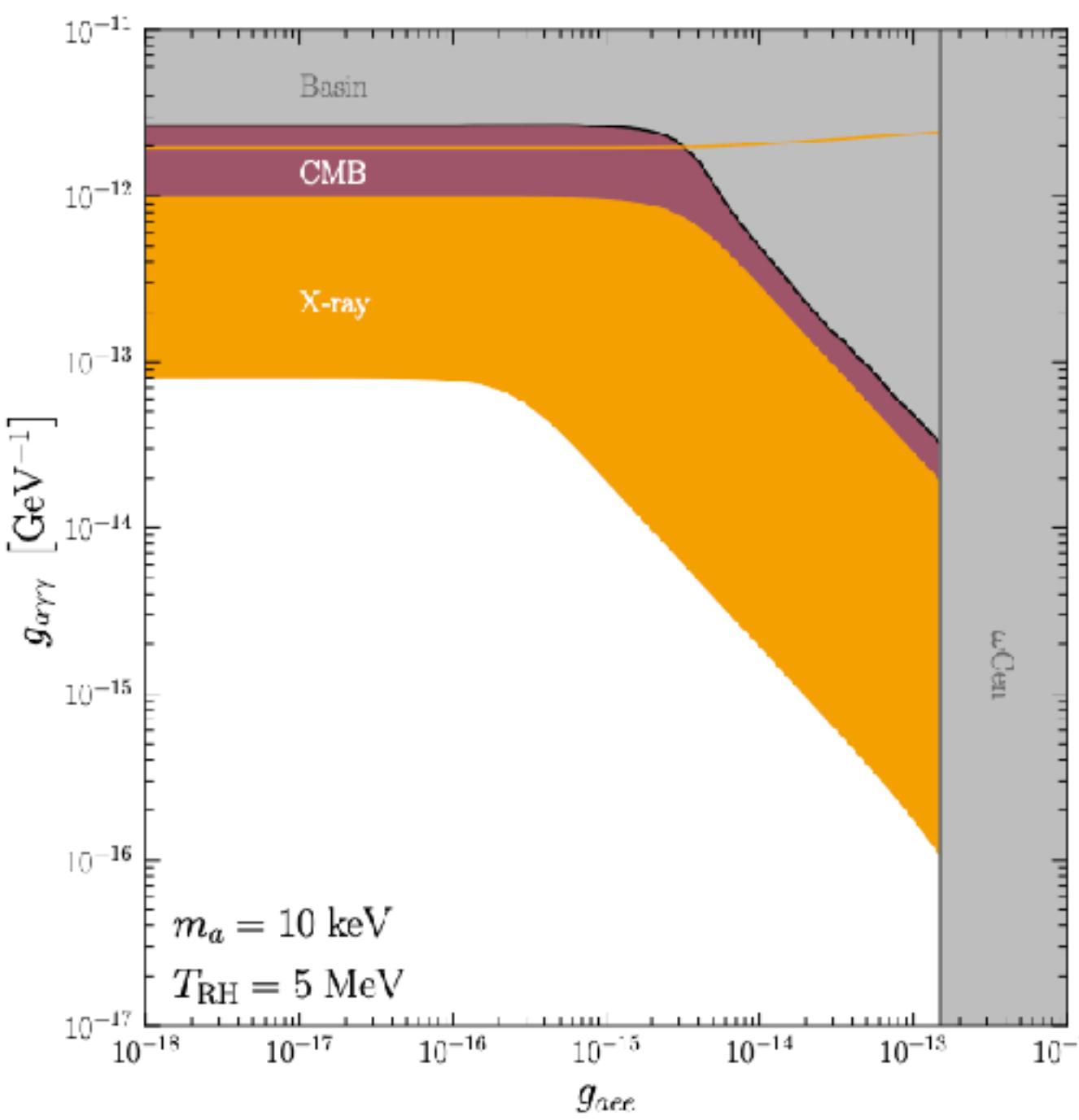
# Thank You

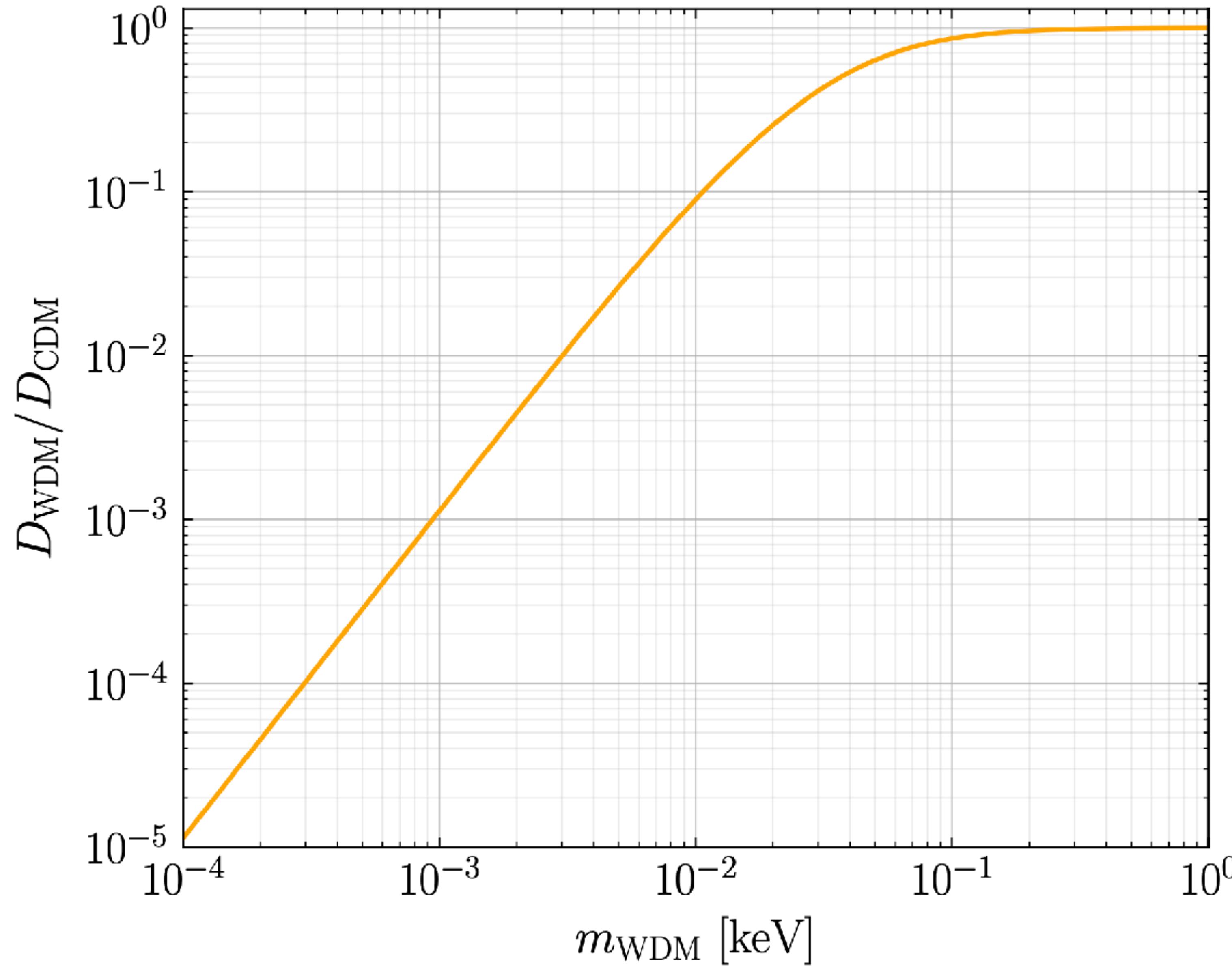
# Photophilic Axions with $T_{RH} = 100$ MeV



# Axions with “Universal” Couplings







# Production of Axions

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$$R(t) = \sum_{\text{process}} \int \left( \prod_i d\Pi_i \right) \left( \prod_f d\Pi_f \right) (2\pi)^4 \delta^4 \left( \sum_i p_i - \sum_f p_f \right) |\mathcal{M}_{i \rightarrow f}|^2 \times \Phi$$

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$$\Phi = (f_a^{\text{eq}} - f_a) \times \left[ \prod_i \left( 1 \pm f_i^{\text{eq}} \right) \prod_{f \neq a} f_f^{\text{eq}} - \prod_{f \neq a} \left( 1 \pm f_f^{\text{eq}} \right) \prod_i f_i^{\text{eq}} \right]$$

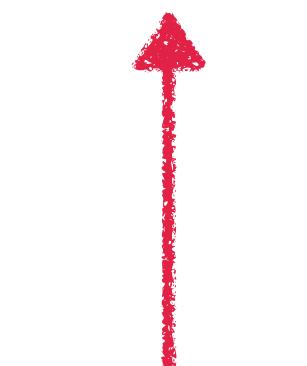
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$$R(t) = \sum_{\text{process}} \int \left( \prod_i d\Pi_i \right) \left( \prod_f d\Pi_f \right) (2\pi)^4 \delta^4 \left( \sum_i p_i - \sum_f p_f \right) |\mathcal{M}_{i \rightarrow f}|^2 \times \Phi$$

$$\Phi = (f_a^{\text{eq}} - f_a) \times \left[ \prod_i (1 \pm f_i^{\text{eq}}) \prod_{f \neq a} f_f^{\text{eq}} - \prod_{f \neq a} (1 \pm f_f^{\text{eq}}) \prod_i f_i^{\text{eq}} \right]$$



$\ll f_a^{\text{eq}}$

# Production of Axions

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$\uparrow$   
 $\ll f_a^{\text{eq}}$

Note dropping  $f_a$  decouples the Boltzmann equation!  
(Production by different processes are independent)

# Production of Axions

Make the following definitions:

$$1. \quad x = m_a/T$$

$$2. \quad Y_a = n_a/s$$

$$3. \quad \tilde{g}(x) = 1 - \frac{1}{3} \frac{d \log g_{\star,s}}{d \log x}$$

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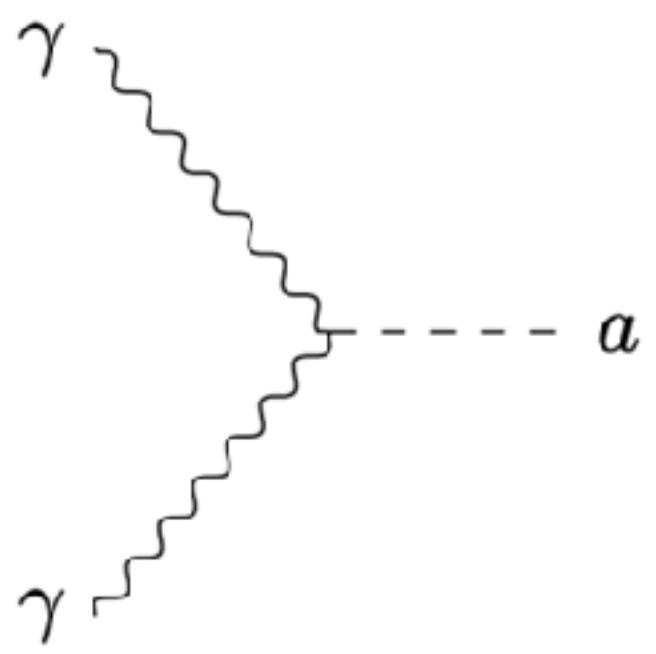
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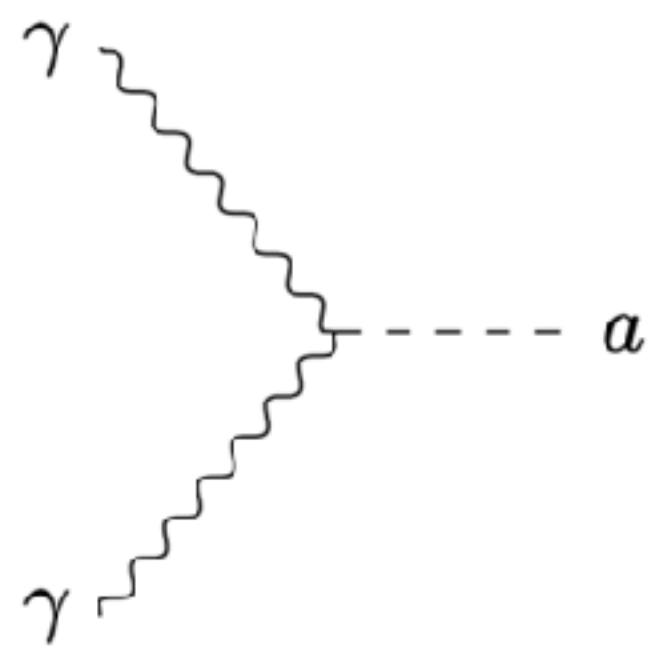
$$\frac{dY_a}{dx} = \frac{\tilde{g}(x)}{xH(x)s(x)} R(x) \implies \mathcal{F}_a \simeq \frac{m_a s_0}{\rho_{\text{DM},0}} Y_a(\infty)$$

(Ignoring Axion Decay)

# Production of Axions (Inverse Decay)

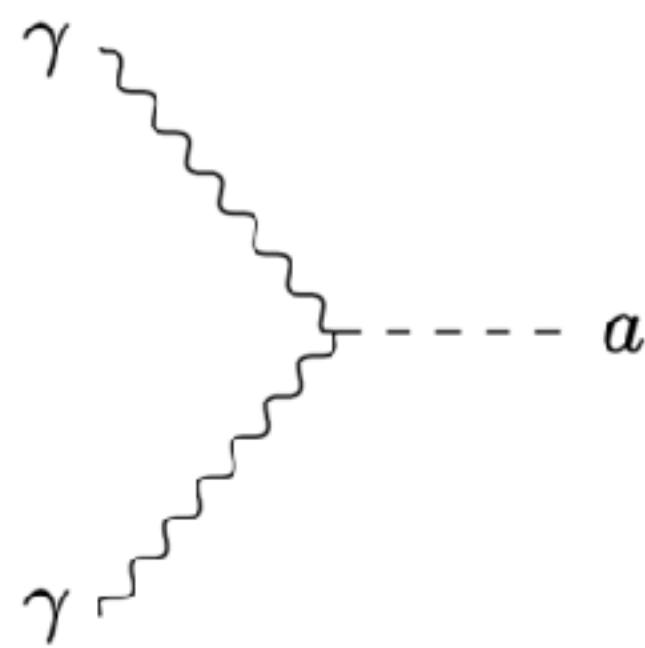


# Production of Axions (Inverse Decay)



$$\sum |\mathcal{M}_{\gamma\gamma \rightarrow a}|^2 = \frac{1}{2} g_{a\gamma\gamma}^2 m_a^2 (m_a^2 - 4m_\gamma^2)$$

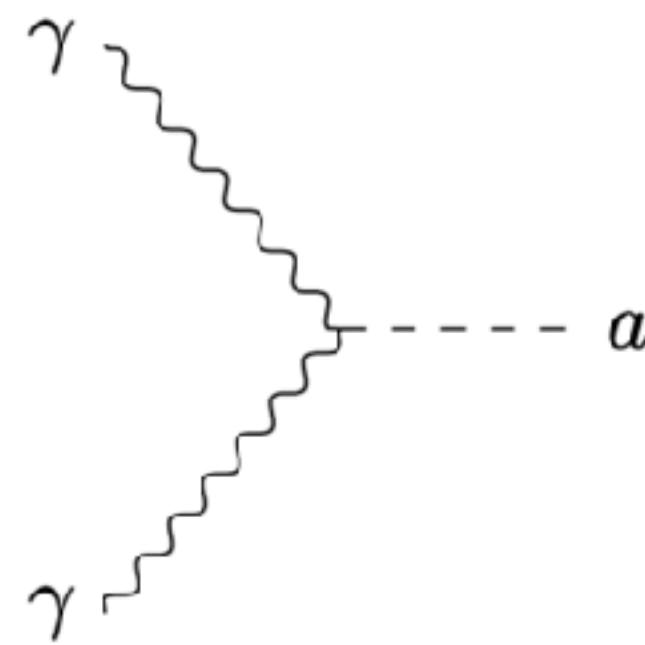
# Production of Axions (Inverse Decay)



$$\sum |\mathcal{M}_{\gamma\gamma \rightarrow a}|^2 = \frac{1}{2} g_{a\gamma\gamma}^2 m_a^2 (m_a^2 - 4m_\gamma^2)$$

$$R_{\text{ID}}(T) = \sum |\mathcal{M}_{\gamma\gamma \rightarrow a}|^2 \int d\Pi_1 d\Pi_2 d\Pi_a (2\pi)^4 \delta^4(p_1 + p_2 - p_a) \times f_a^{\text{eq}} [1 + (f_1^{\text{eq}} + f_2^{\text{eq}})]$$

# Production of Axions (Inverse Decay)

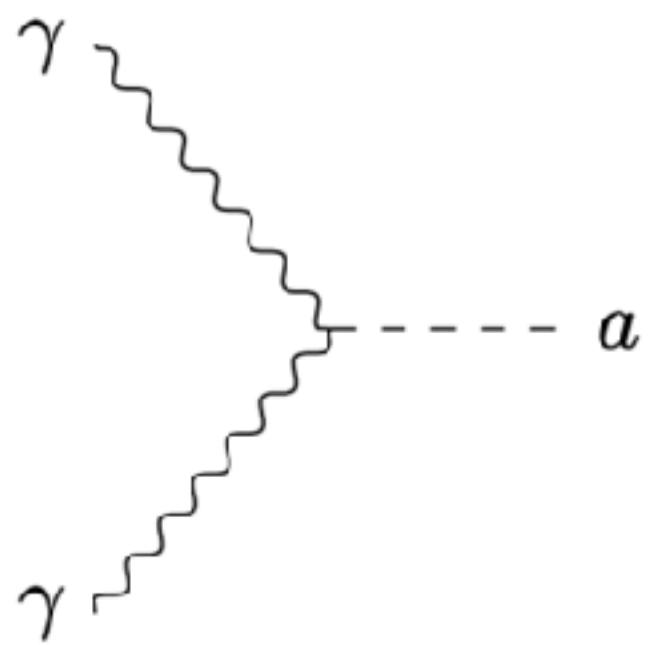


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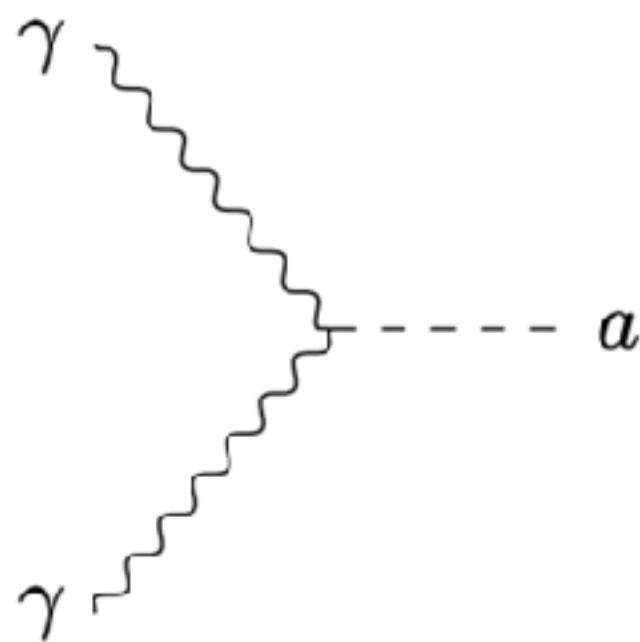
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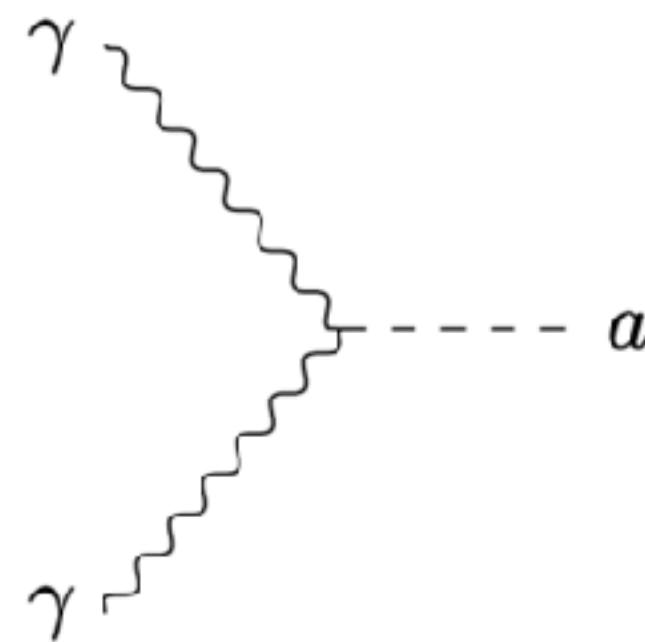
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- $R_{\text{ID}}(T) = 0$  for  $2m_\gamma(T) > m_a$  where  $m_\gamma(T) \approx eT/3 \sim T/10$

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- Similar calculation for electrons.

# Production of Axions (2→2)

$$R_{2\rightarrow 2}(T) \approx \frac{g_1 g_2 T}{32\pi^4} \int_{s_{\min}}^{\infty} ds \lambda(s, m_1^2, m_2^2) \frac{K_1(\sqrt{s}/T)}{\sqrt{s}} \sigma_{12\rightarrow 3a}(s) \quad [\text{D'Eramo et al, 2017}]$$

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$$\begin{aligned} \sigma_{\text{FA}}(s) = & \frac{\alpha g_{a\gamma\gamma}^2}{24\beta} \left(1 - \frac{m_a^2}{s}\right)^3 \left(1 + \frac{2m_e^2}{s}\right) + \frac{\alpha g_{aee}^2}{2s^2(s - m_a^2)\beta^2} \left[ (s^2 - 4m_e^2 m_a^2 + m_a^4) \ln\left(\frac{1 + \beta}{1 - \beta}\right) - 2\beta m_a^2 s \right] \\ & - \frac{\alpha g_{a\gamma\gamma} g_{aee} m_e}{2s\beta^2} \left(1 - \frac{m_a^2}{s}\right)^2 \ln\left(\frac{1 + \beta}{1 - \beta}\right) \end{aligned}$$

$$\begin{aligned} \sigma_{\text{PC}}(s) = & \frac{\alpha g_{a\gamma\gamma}^2}{32s^2} \left[ 2(2s^2 - 2m_a^2 s + m_a^4) \ln\left(\frac{s - m_a^2}{m_\gamma^2}\right) - 7s^2 + 10m_a^2 s - 5m_a^4 \right] \\ & + \frac{\alpha g_{aee}^2}{8s^3} \left[ 2(2s^2 - 2m_a^2 s + m_a^4) \ln\left(\frac{s}{m_e^2}\right) - 3s^2 + 10m_a^2 s - 7m_a^4 \right] \\ & - \frac{\alpha g_{a\gamma\gamma} g_{aee} m_e}{8s^3(s - m_a^2 + m_e^2)} \left[ 2(s^3 + m_a^6) \ln\left(\frac{(s - m_a^2)^2}{(s + m_a^2)m_e^2}\right) - 3(s + m_a^2)(s - m_a^2)^2 \right] \end{aligned}$$

# **Types of Freeze-In (Rough Idea)**

There exist many types of freeze-in, but can generally be classified into two groups.

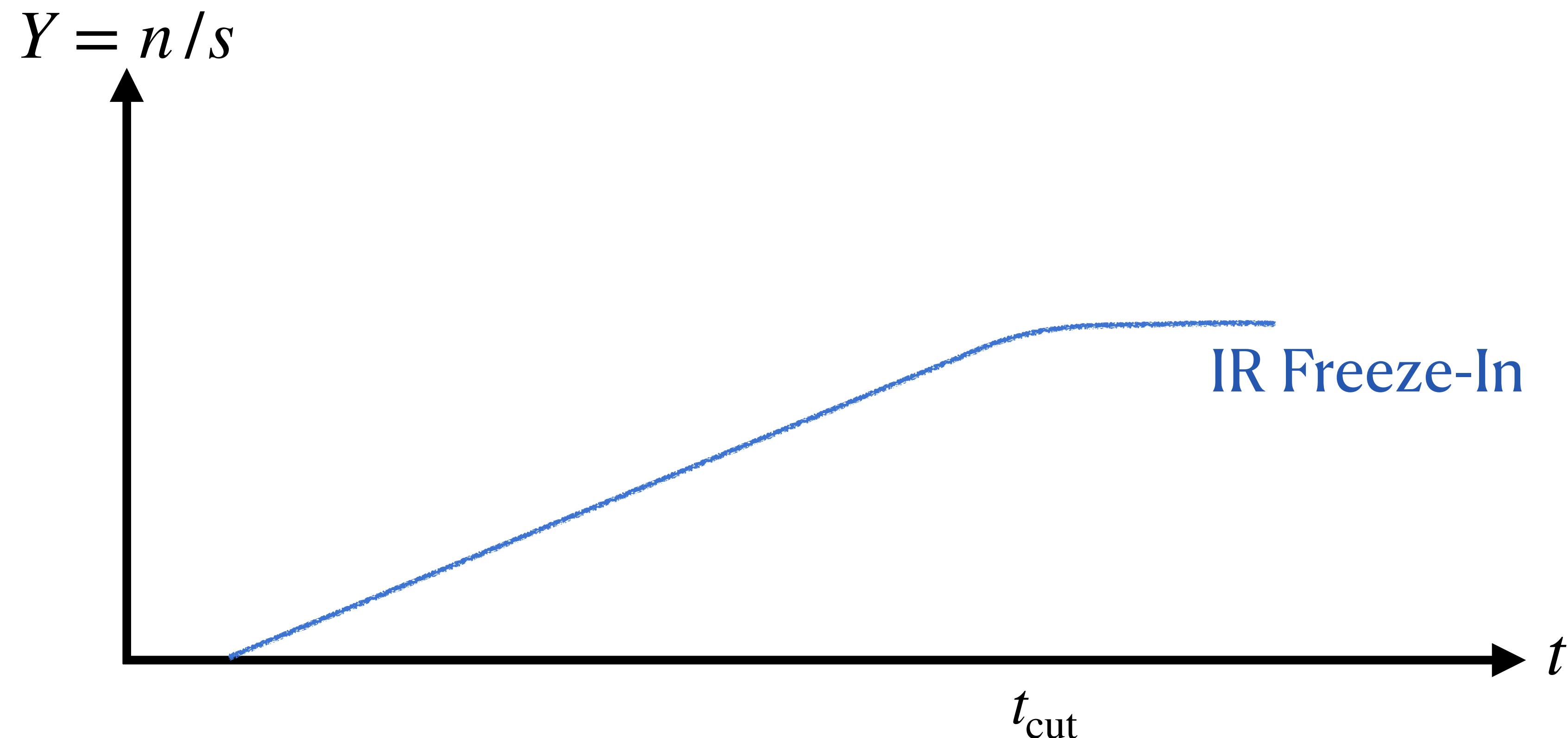
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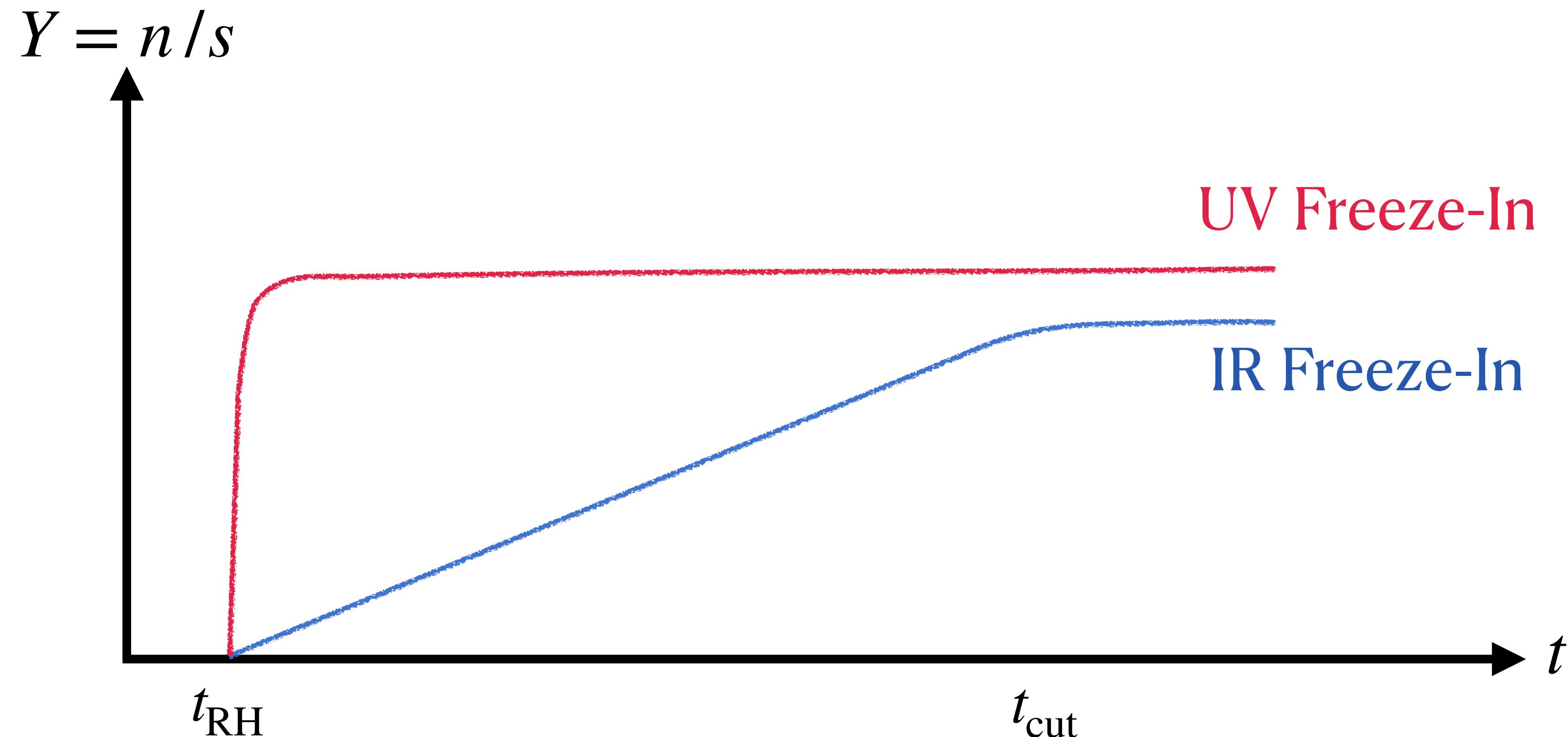
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# Logic of Freeze-In (Simplified)

$$\frac{dn_a}{dt} + 3Hn_a = n_{\text{SM}}\Gamma_{\text{SM}\rightarrow a} - n_a\Gamma_{a\rightarrow\text{SM}}$$

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Axion Production                                    Axion Destruction

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$$\frac{dn_a}{dt} + 3Hn_a = \boxed{n_{\text{SM}} \Gamma_{\text{SM} \rightarrow a} - n_a \Gamma_{a \rightarrow \text{SM}}} \approx 0$$

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Define:  $Y_a = n_a/s \sim n_a R^{-3}$

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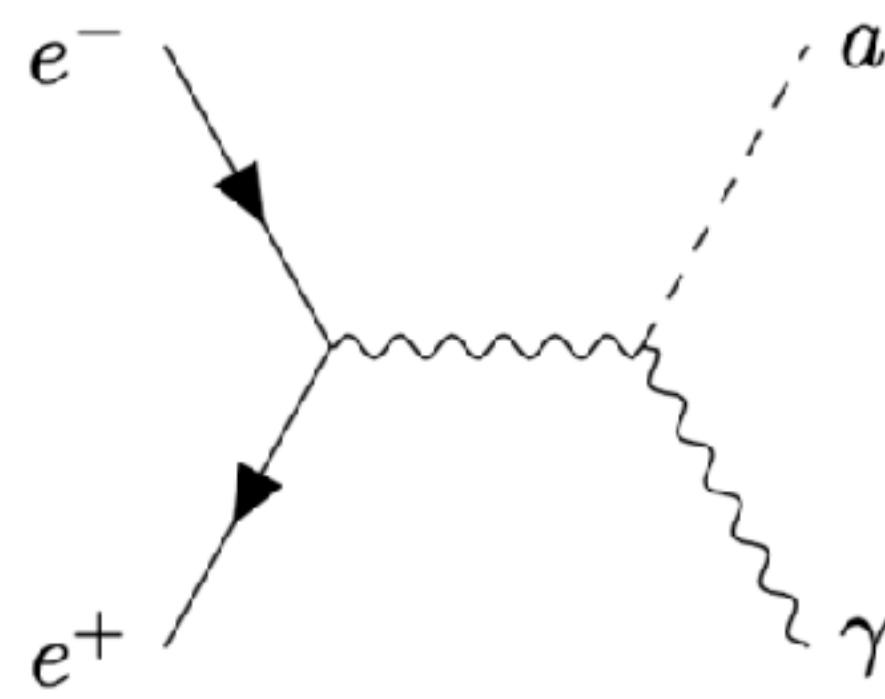
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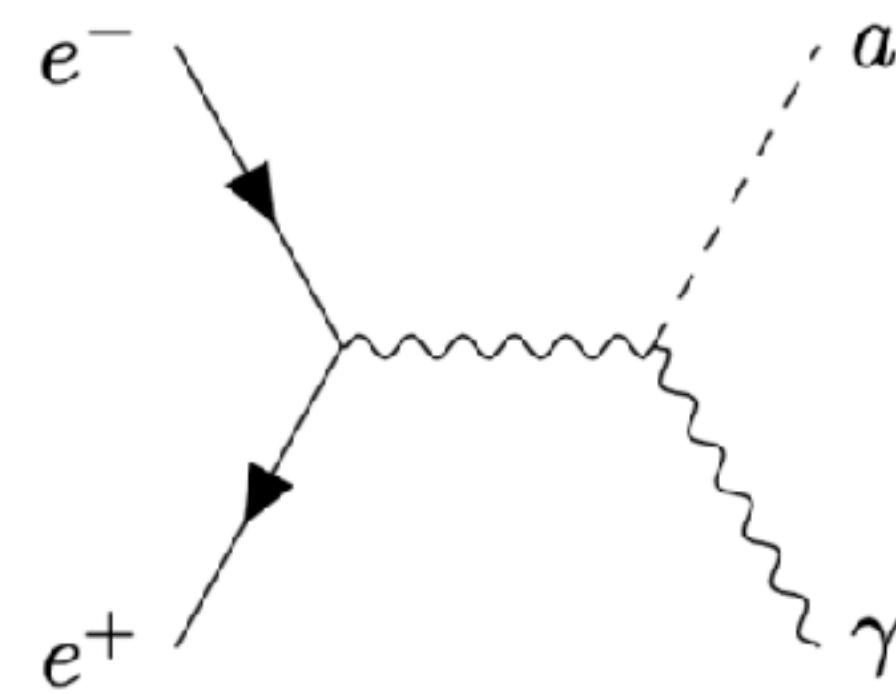
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# Freeze-In Example

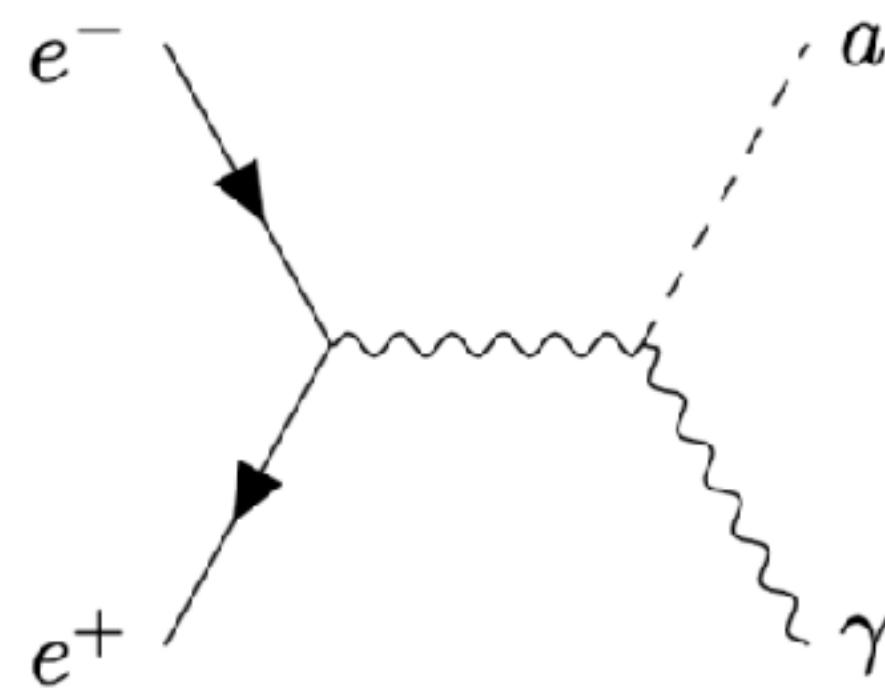


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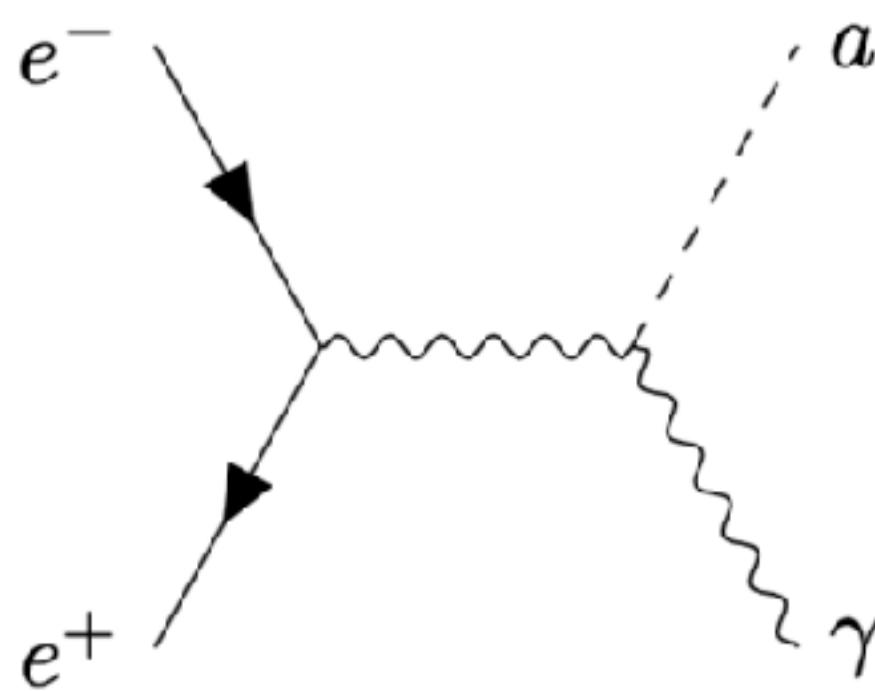
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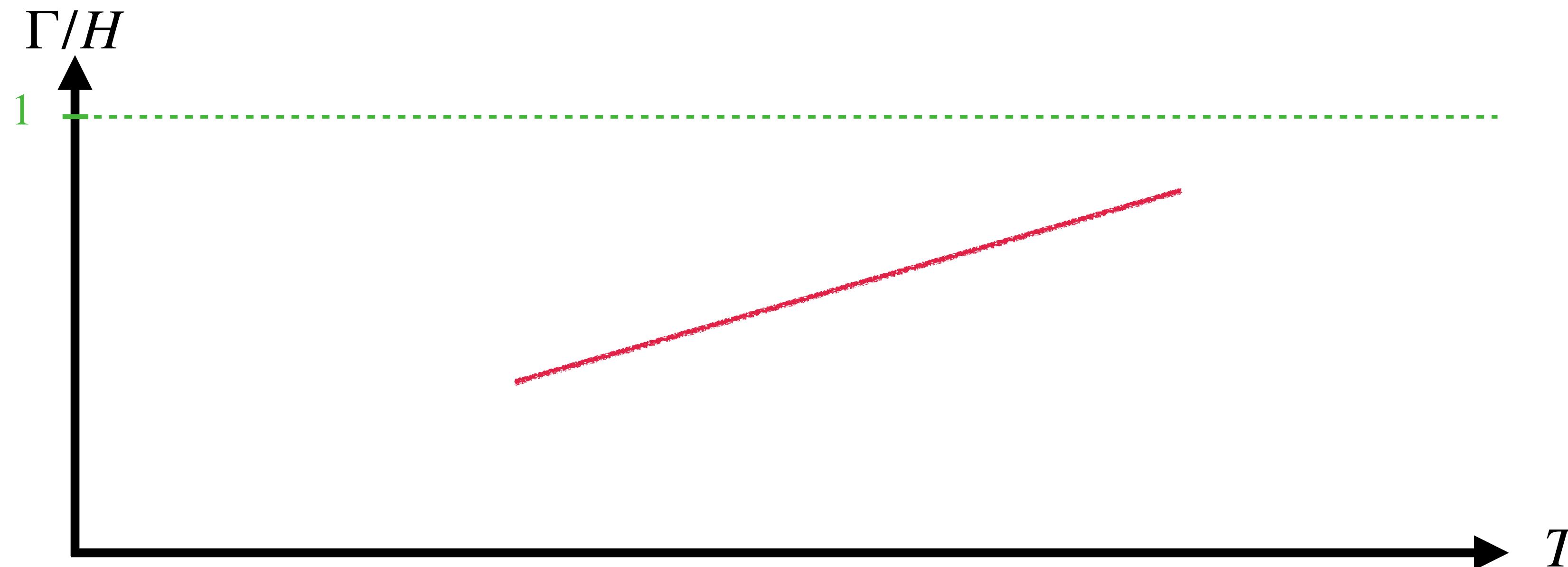


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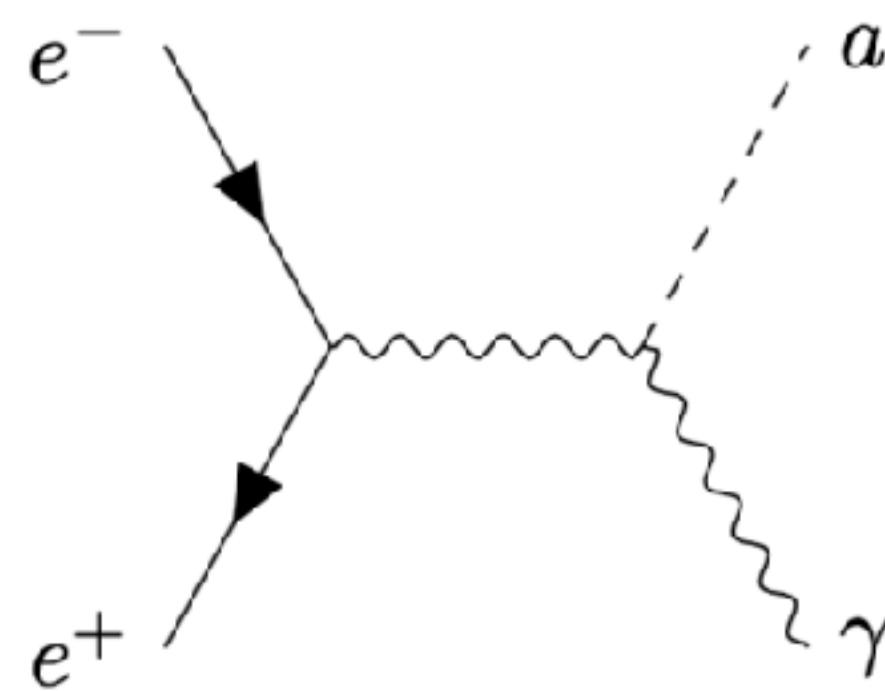
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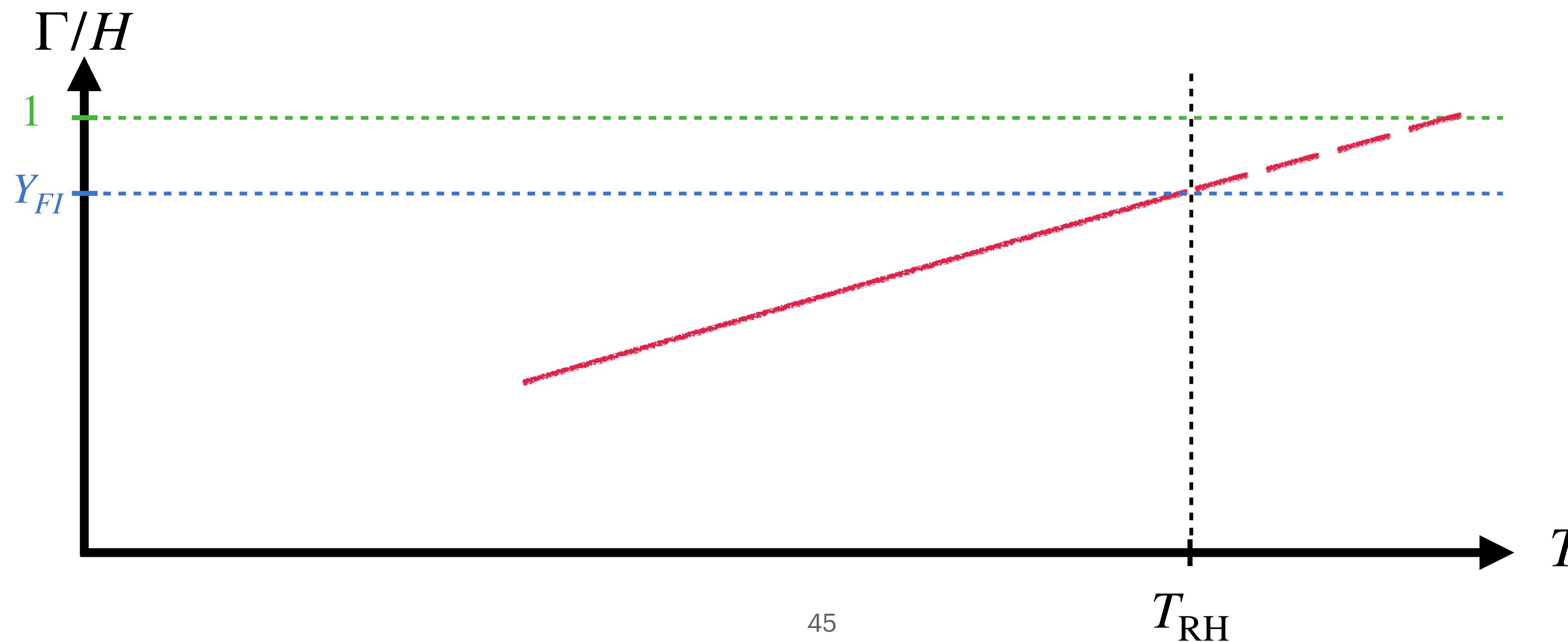
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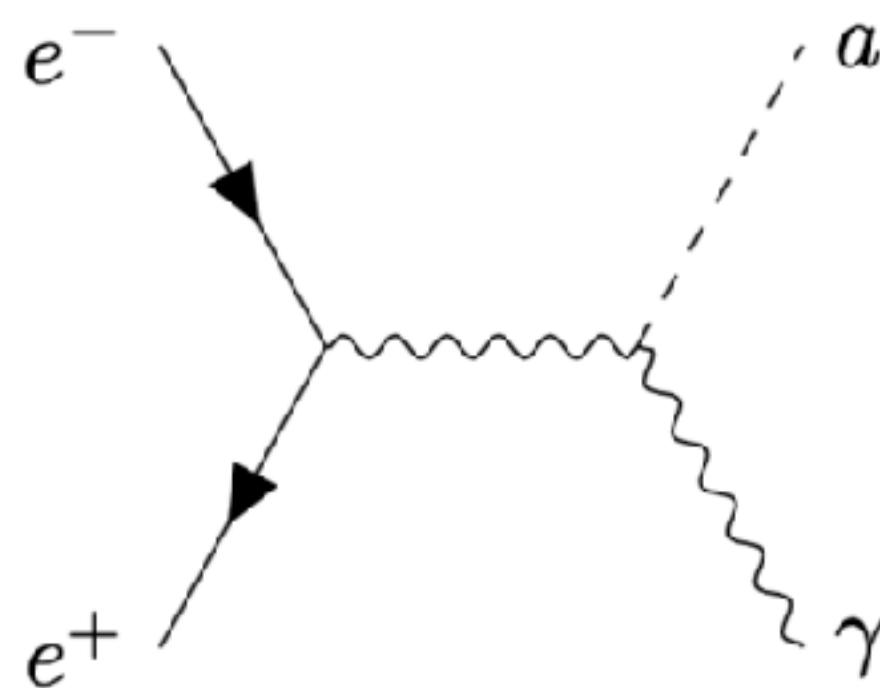
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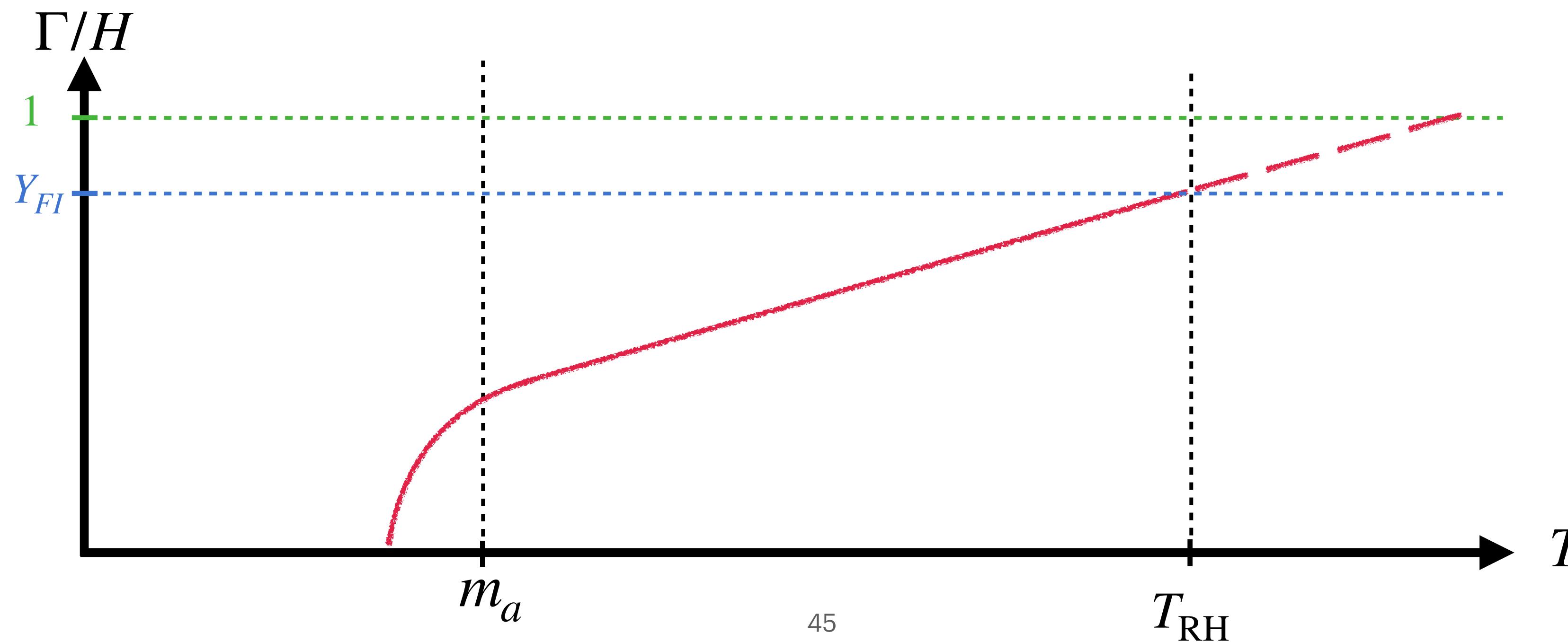
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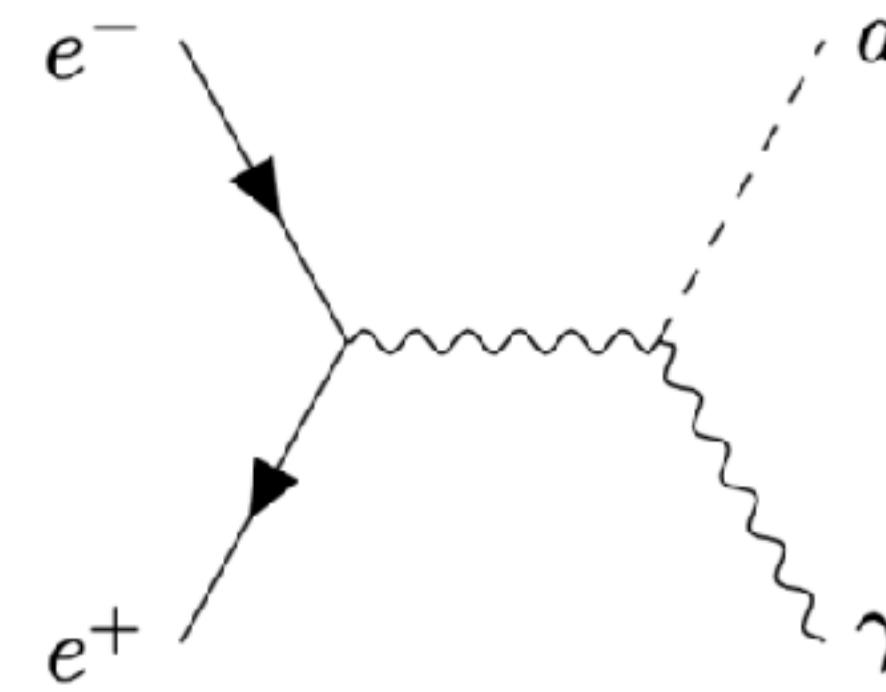
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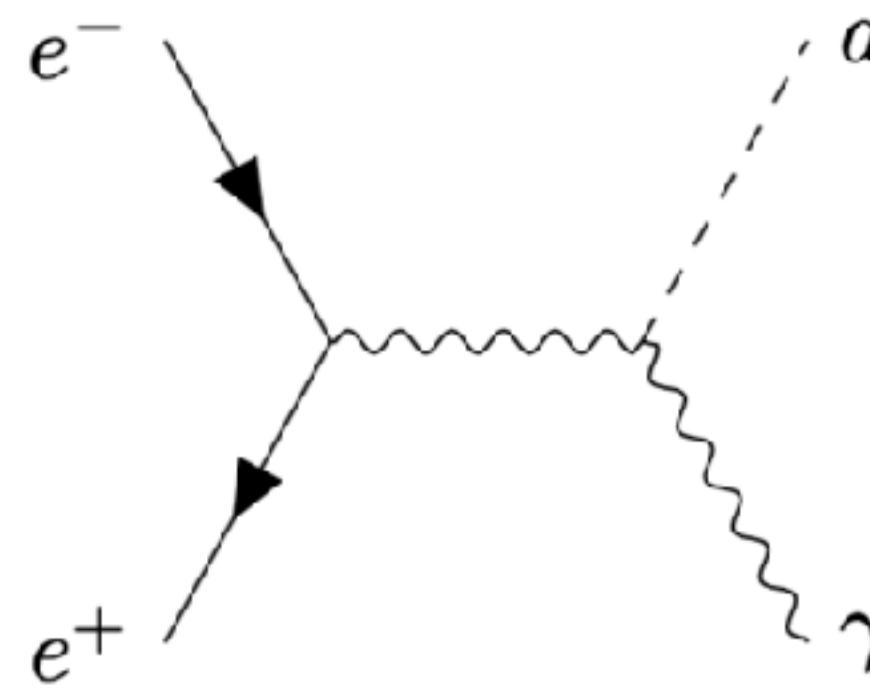
Define:  $Y_a = n_a/s \sim n_a R^{-3}$

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# UV Freeze-In Example

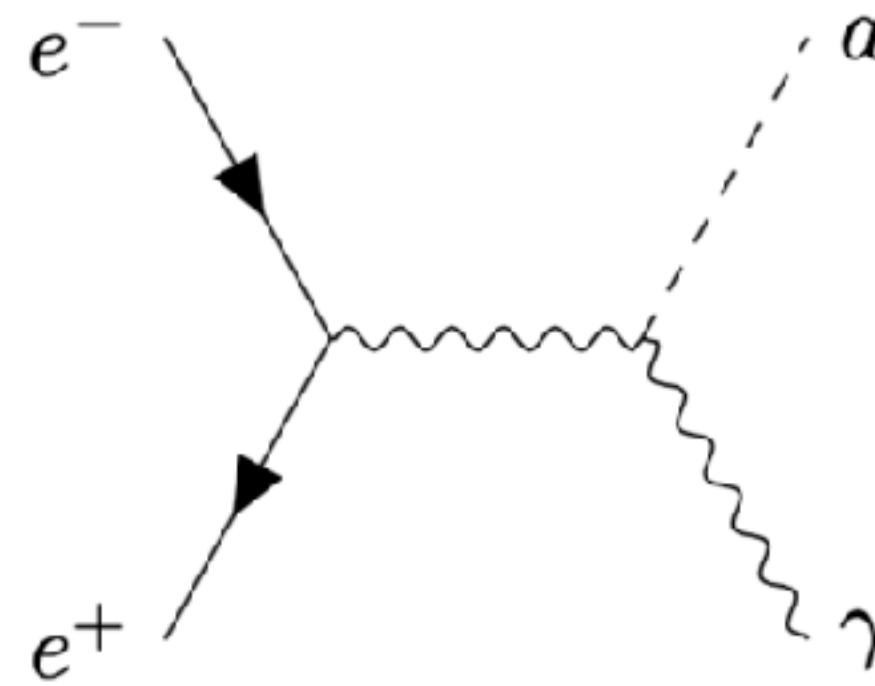


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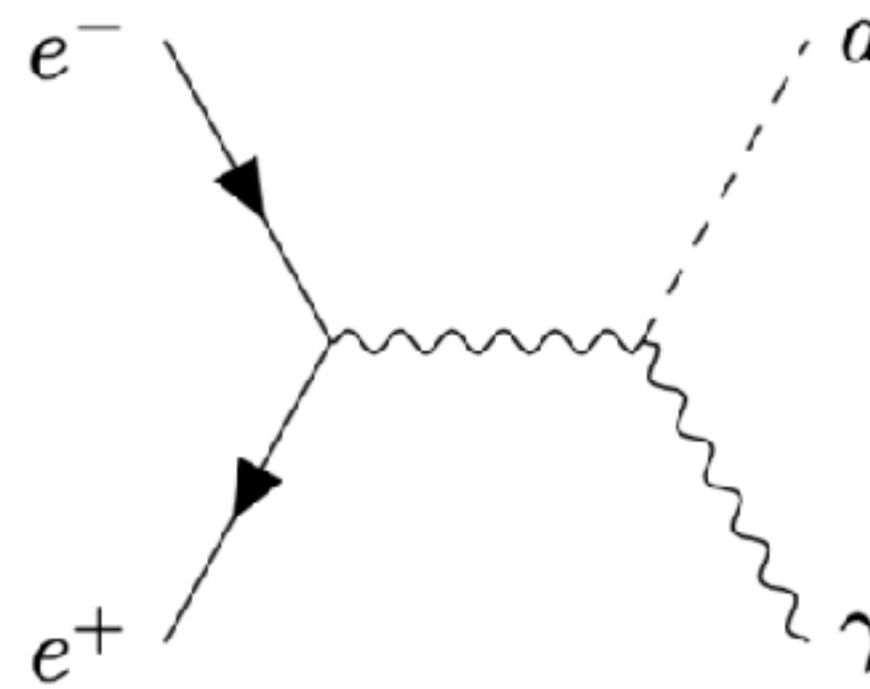
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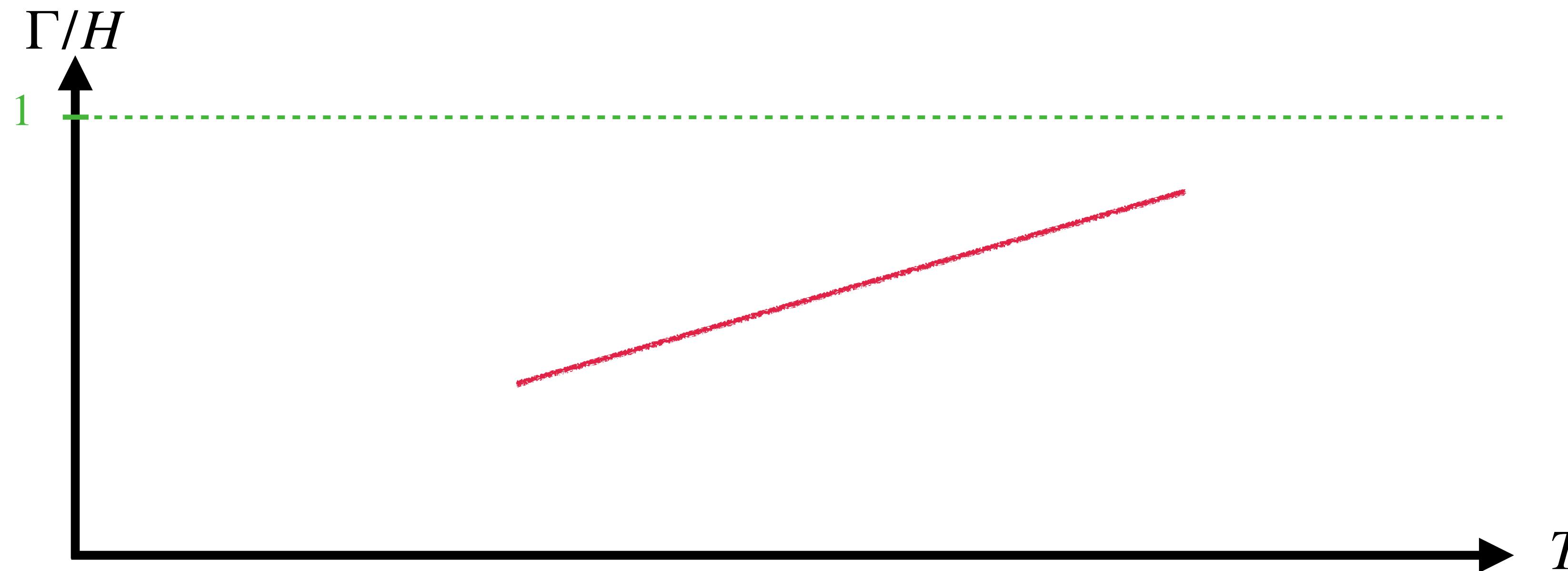


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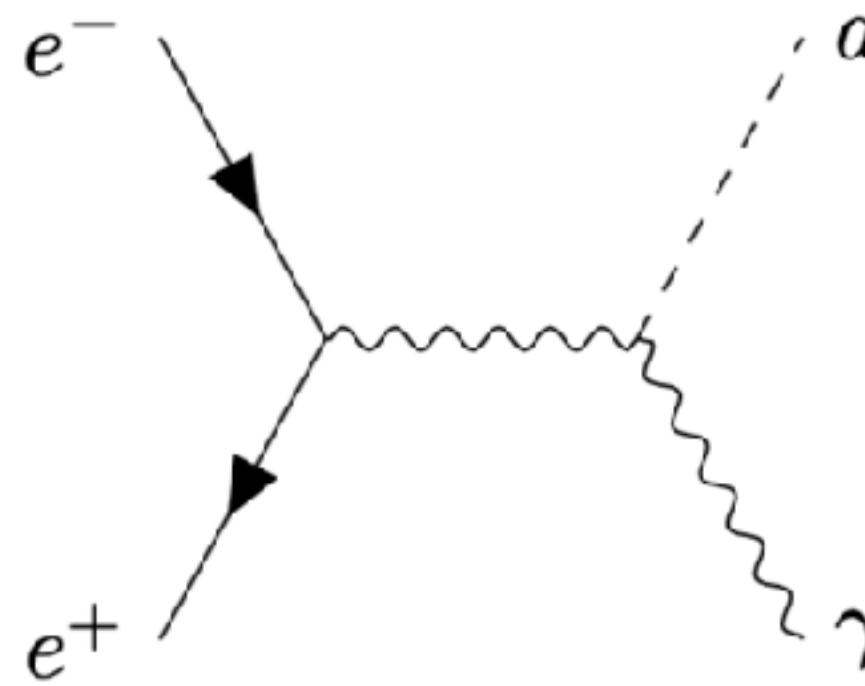
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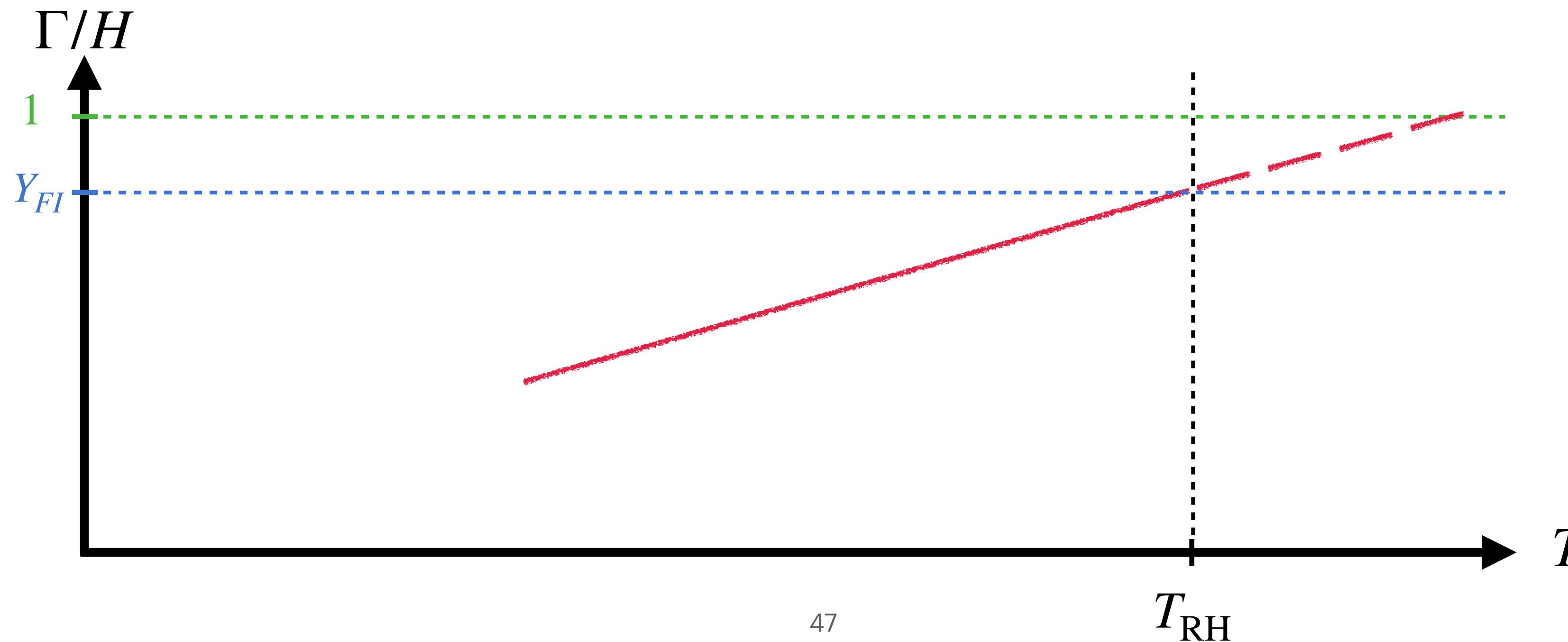
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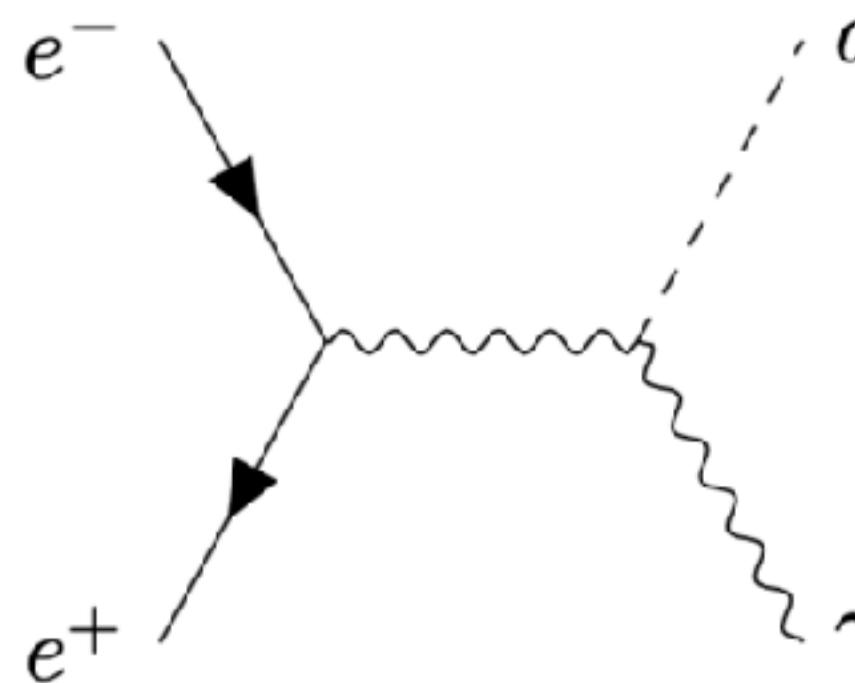
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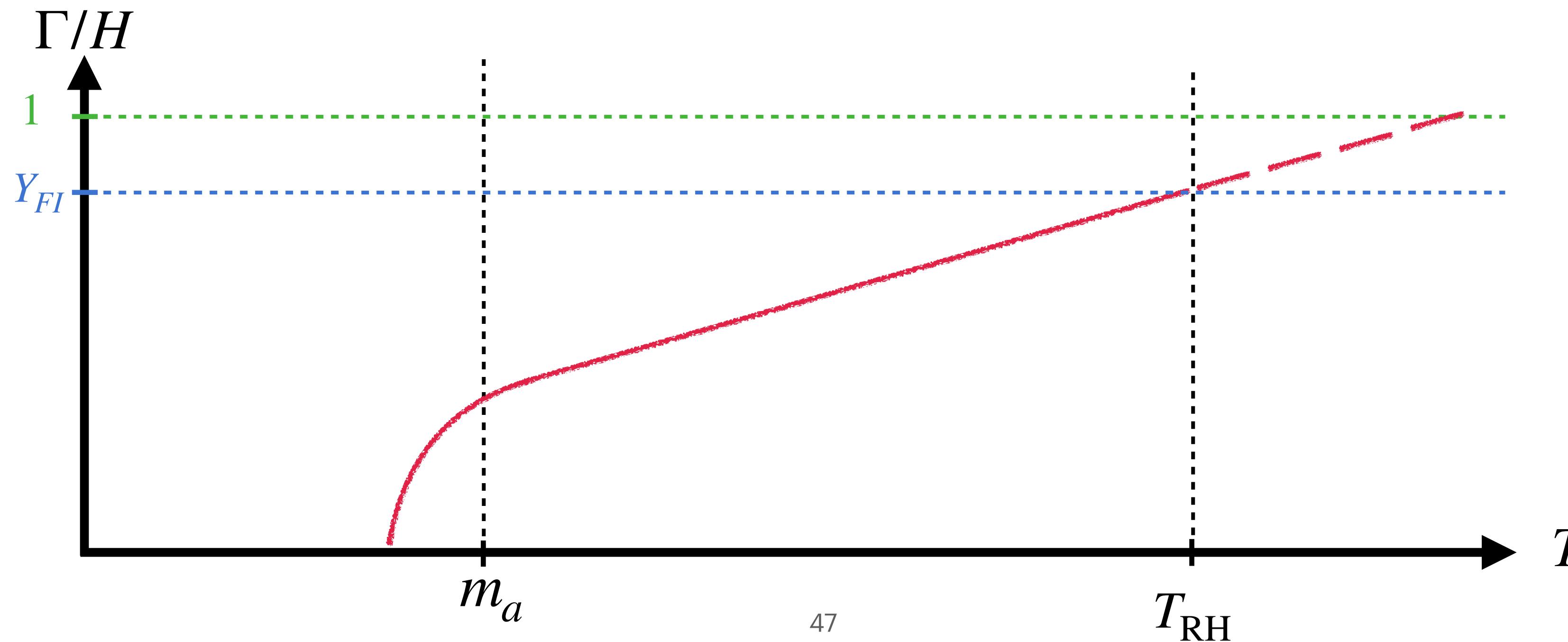
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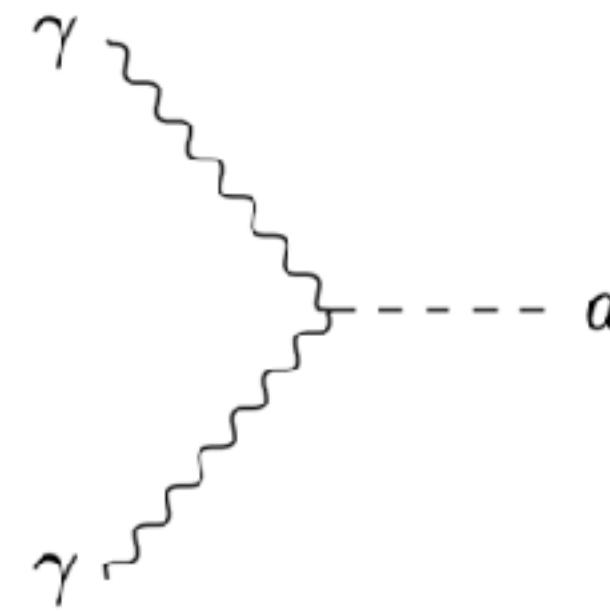
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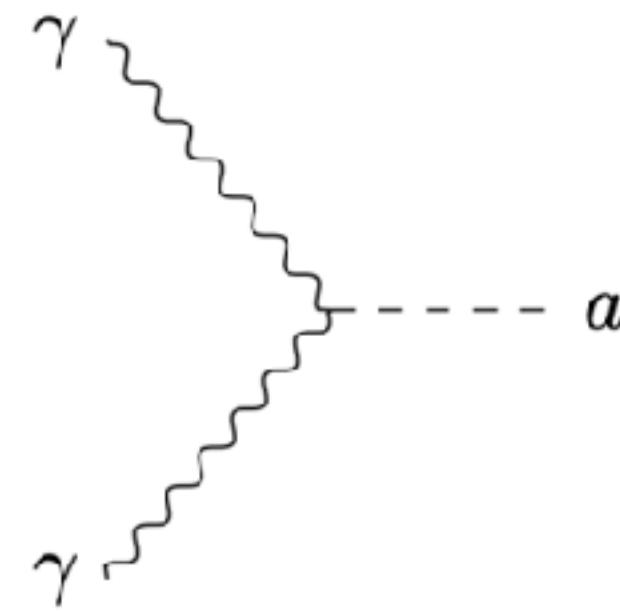


# IR Freeze-In Example



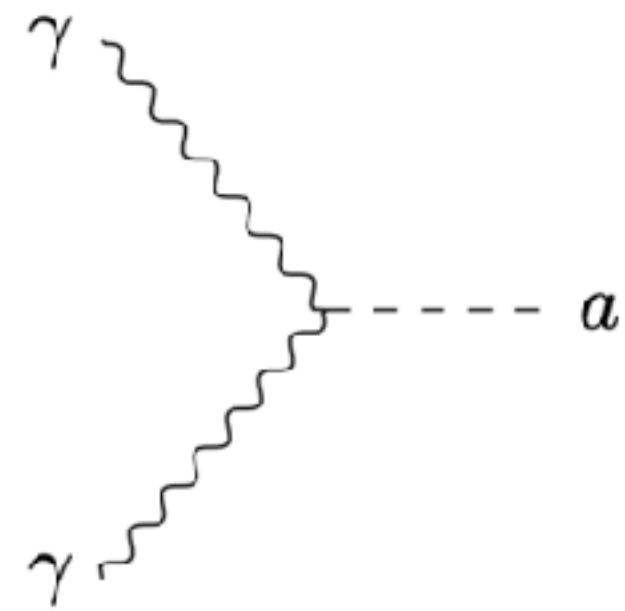
$$\Gamma \sim g_{a\gamma\gamma}^2 T^3 \text{ (Naively)}$$

# IR Freeze-In Example



$$\Gamma \sim \begin{cases} 0, & m_\gamma(T) > m_a/2 \\ g_{a\gamma\gamma}^2 m_a T^2, & m_\gamma(T) < m_a/2 \end{cases} \quad \text{where } m_\gamma(T) \approx eT/3$$

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