Entanglement and Bell's inequalities with boosted semi-leptonic top quarks at the LHC

Alberto Navarro

Oklahoma State University

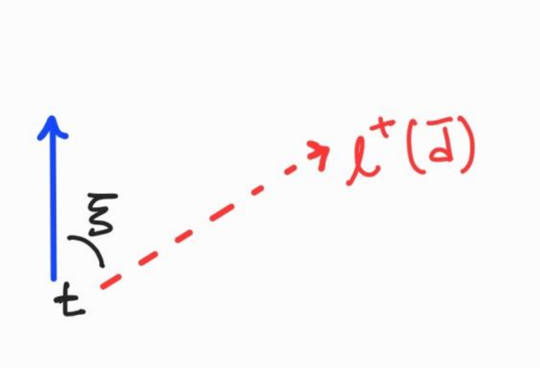
With Z.Dong, D.Gonçalves and K. Kong

Why top semi-leptonic quarks?

- Top quarks are good candidates because they decay before hadronization and spin decorrelation effects take place.

- This implies that the top quark final states correlate with the top quark polarization axis as

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\xi_k} = \frac{1}{2}\left(1+\beta_k p\cos\xi_k\right), \qquad \beta_k = \begin{cases} +1, & \text{for } l^+ \text{ or } \bar{d}\text{-quark.} \\ -0.31, & \text{for } \bar{\nu} \text{ or } u\text{-quark.} \\ -0.41, & \text{for } b\text{-quark.} \end{cases}$$



- We obtain the top spin information from angular distributions involving its decay products. Charged leptons and down-type quarks are the best polarimeters.

The semi-leptonic final state has a larger rate, so larger statistics.

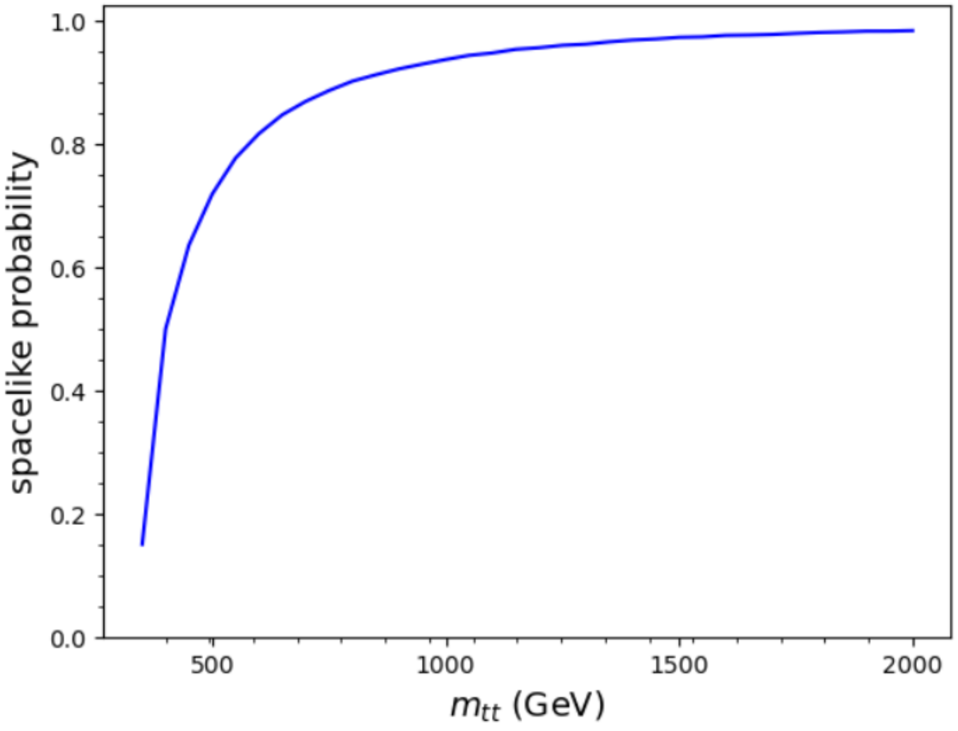
Space-like separation

In the top pair CM frame, their distance between their decay location is given by $(t_1 + t_2)v$.

The maximum distance that information can travel between their decay time is given by $|t_1 - t_2|c$

Thus, spacelike separation requires

$$\left|\frac{t_1 - t_2}{t_1 + t_2}\right| < v = \sqrt{1 - \frac{4m_t^2}{m_{t\bar{t}}^2}}$$



C. Severi et al 2022

Top quark production as a two-qubit system

For a system composed of two spin-1/2 particles the density matrix can be written as

$$\rho = \frac{I_4 + B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i + C_{ij} \sigma^i \otimes \sigma^j}{4}$$

Where $B_i^+ = \langle \sigma^i \otimes I_2 \rangle$ and $B_i^- = \langle I_2 \otimes \sigma^i \rangle$ are the polarizations of the two particles and $C_{ij} = \langle \sigma^i \otimes \sigma^j \rangle$ represents their spin correlations.

In general, for a system composed of two subsystems A and B, a quantum state is said to be separable if the mixed system is described by a density matrix

$$\rho = \sum_n p_n \rho_n^A \otimes \rho_n^B$$

An entangled state is defined as a non-separable state.

Entanglement criterion

The Peres-Horodecki criterion provides a necessary and sufficient condition for entanglement:

$$\rho^{T_2} = \sum_n p_n \rho_n^A \otimes (\rho_n^B)^T$$

If ρ^{T_2} has at least one negative eigenvalue, then ρ represents an entangled state.

For a system composed of two spin-1/2 particles, a sufficient condition for entanglement is

$$|C_{11} + C_{22}| - C_{33} > 1$$

Y. Afik, J. R. M. de Nova 2020

Only the diagonal elements of the correlation matrix are needed to test entanglement.

Bell's inequalities

The Clauser, Horne, Shimony, and Holt (CHSH) inequality is a realization of the Bell-type inequalities for bipartite system

$$\left|\vec{a}\cdot C(\vec{b}-\vec{b'}) \;+\; \vec{a'}\cdot C(\vec{b}+\vec{b'})\right| \leq 2$$

We may choose $\vec{a}, \vec{a}', \vec{b}$ and $\vec{b'}$ so that the l.h.s of the inequality is maximal and possibly larger than the r.h.s. It's been shown that the following choice

$$a_k = \delta_{ki}, \qquad a'_k = \delta_{kj},$$

$$b_i = -b'_i = \pm\frac{1}{\sqrt{2}}, \qquad b_j = b'_j = \pm\frac{1}{\sqrt{2}}, \qquad b_{k\neq i,j} = 0$$

Yields the set of inequalities $$|C_{ii} \pm C_{jj}| > \sqrt{2}$$

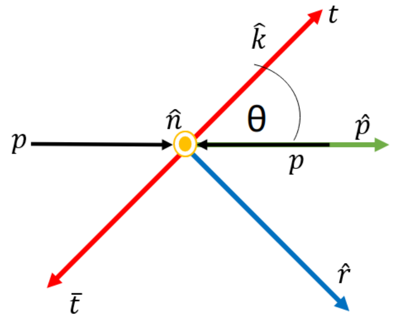
J.A. Aguilar, J.A. Casas 2022

Only two diagonal element of the correlation matrix are needed to test the Bell's inequality.

Reconstruction of the correlation matrix

The elements of the correlation matrix are extracted from forward-backward asymmetries

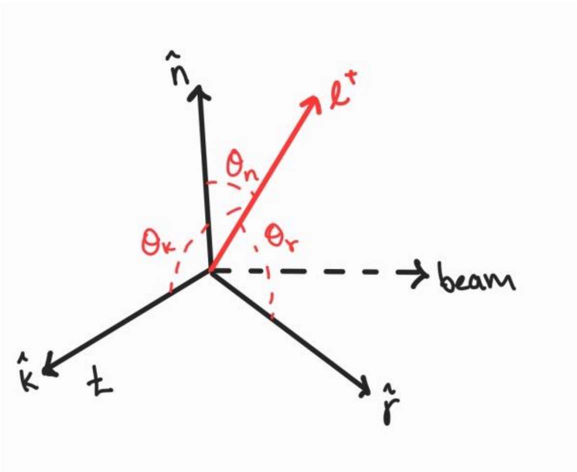
$$C_{ij} = \frac{4}{\beta_a \beta_b} \frac{N(c_{\theta_a^i} c_{\theta_b^j} > 0) - N(c_{\theta_a^i} c_{\theta_b^j} < 0)}{N(c_{\theta_a^i} c_{\theta_b^j} > 0) + N(c_{\theta_a^i} c_{\theta_b^j} < 0)}$$



The axes are chosen in the so-called helicity basis

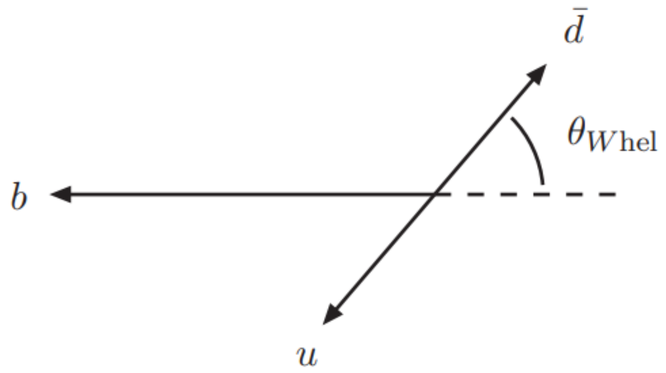
$$\hat{k} = \text{top direction}, \quad \hat{r} = \text{sign}(\cos\theta)\frac{\hat{p} - \hat{k}\cos\theta}{\sin\theta}, \quad \hat{n} = \hat{k} \times \hat{r}, \quad \hat{p} = (0, 0, 1).$$

We choose the charged lepton from the leptonic top and an optimal direction for the hadronic top to compute the C_{ij} coefficients.



Optimal hadronic direction

Consider the top three-body decay from the W rest frame.



The W polarization causes the cosine of this angle to be distributed as

$$\rho\left(c_{W\text{hel}}\right) \equiv \frac{3}{8} f_R \left(1 + c_{W\text{hel}}\right)^2 + \frac{3}{4} f_0 \left(1 - c_{W\text{hel}}^2\right) + \frac{3}{8} f_L \left(1 - c_{W\text{hel}}\right)^2$$

The soft-quark and hard-quark each have some probability of really being the d-quark

$$p(d \to q_{\text{soft}}) = \frac{\rho\left(-|c_{W\text{hel}}|\right)}{\rho\left(|c_{W\text{hel}}|\right) + \rho\left(-|c_{W\text{hel}}|\right)}$$

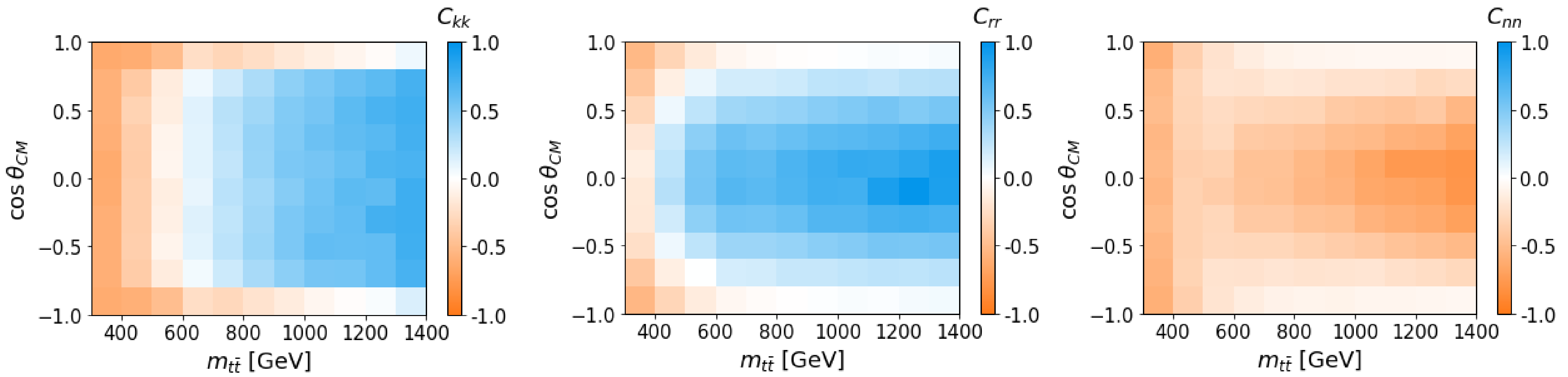
$$p(d \to q_{\text{hard}}) = \frac{\rho\left(|c_{W\text{hel}}|\right)}{\rho\left(|c_{W\text{hel}}|\right) + \rho\left(-|c_{W\text{hel}}|\right)} .$$

B. Tweedie 2014

The optimal hadronic direction is defined as the weighted average of the soft and hard-quark directions

$$\vec{q}_{\text{opt}}\left(|c_{W\text{hel}}|\right) \equiv p(d \to q_{\text{soft}})\hat{q}_{\text{soft}} + p(d \to q_{\text{hard}})\hat{q}_{\text{hard}}$$

Parton-level distributions



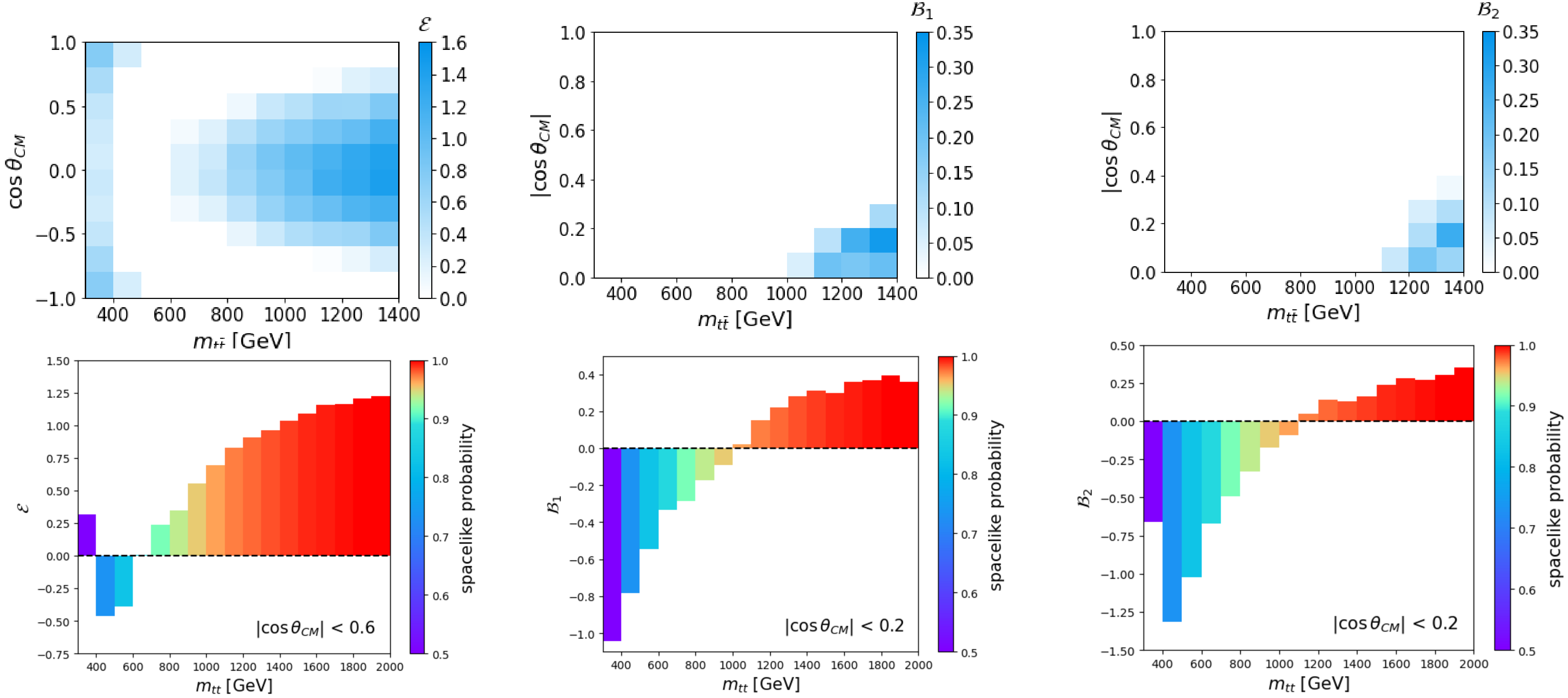
Given the relative signs of the C_{ii} coefficients, we introduce the entanglement and Bell's indicators

$$\mathcal{E} \equiv |C_{kk} + C_{rr}| - C_{nn} - 1 > 0.$$

$$\mathcal{B}_1 \equiv |C_{rr} - C_{nn}| - \sqrt{2} > 0,$$
$$\mathcal{B}_2 \equiv |C_{kk} + C_{rr}| - \sqrt{2} > 0.$$

2022, J.A. Aguilar, J.A. Casas

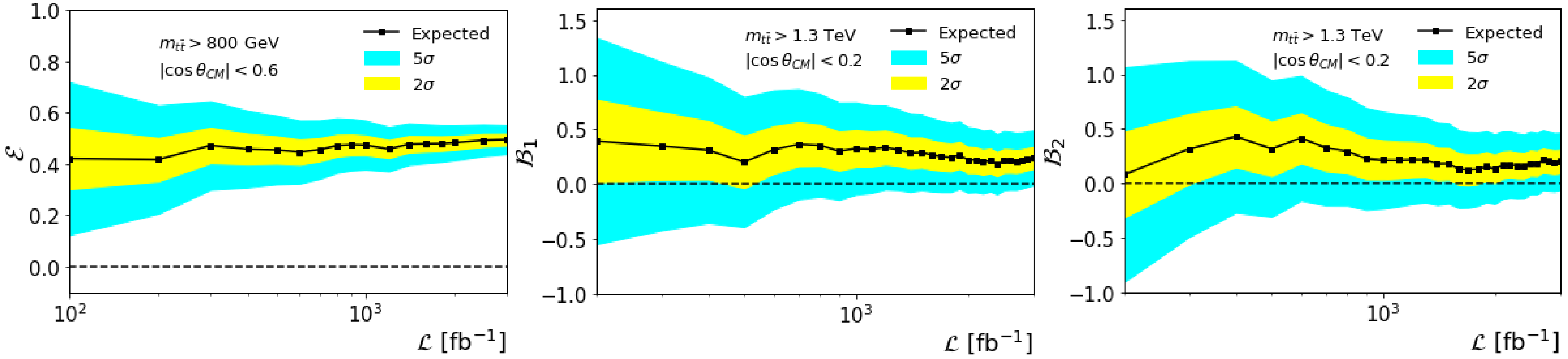
Entanglement and Bell's indicators



Our analysis focuses on the high invariant mass region.

Sensitivity

- We simulated $pp \to t\bar{t} \to l^{\pm}\nu 2b2j$ events at the LHC.
- The hadronic top reconstruction is done with HepTopTagger.
- We use LBN for the leptonic top reconstruction.
- The angular distributions are unfolded with the TSVDUnfold package.



Summary

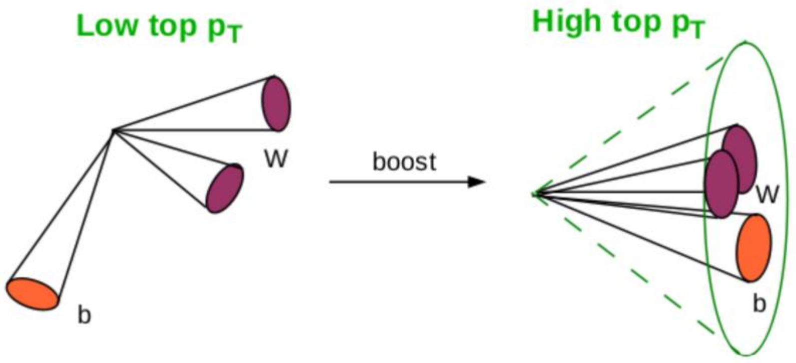
- Top quark pair-production at the LHC provides a window to study the foundations of QM in the high-energy regime.
- It is possible to use boosted semi-leptonic tops to study entanglement and violation of the Bell's inequality.
- The optimal hadronic direction allows us to recover the hadronic top spin information quite well.
- Entanglement is present at threshold and high invariant mass regions.
- The violation of the Bell’s inequality may be addressed in the high invariant mass region, where the tops are spacelike separated.

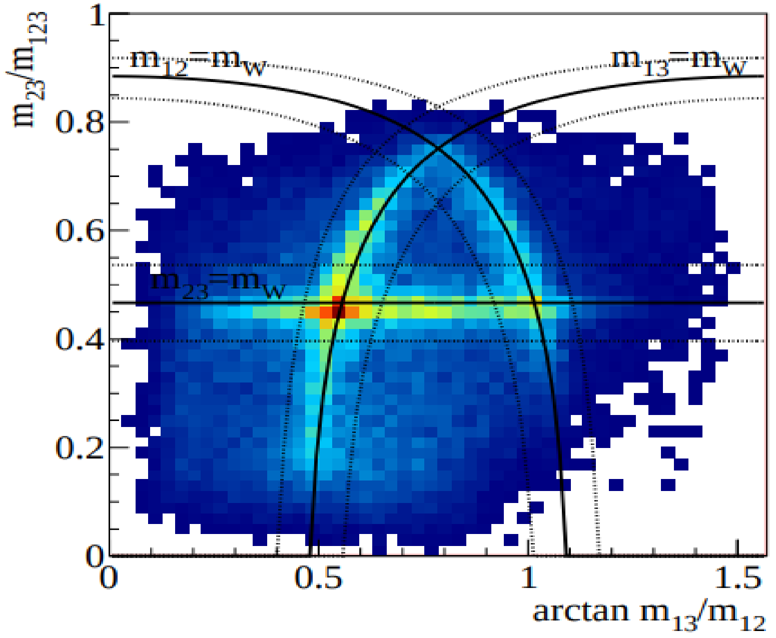
HepTopTagger

Top decay products are collimated for boosted tops.

HepTopTagger identifies the best top candidate and the three subjets within it.

We use machine learning (LBN) to determine the b-jet.



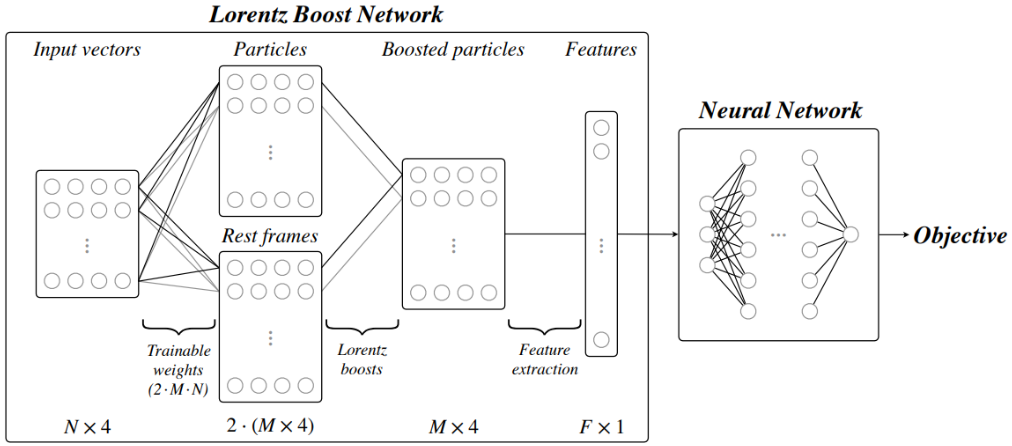


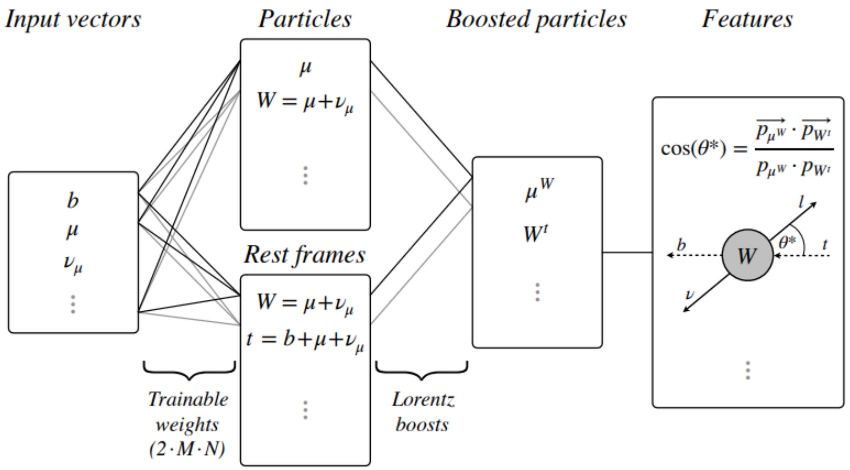
2010, Plehn, Spannowsky, Takeuchi, Zerwas

LBN

The Lorentz Boost Network (LBN) is composed of two stages of architectures.

Physical features such as mass and angles between those vectors are then extracted and input into another neural network.





2018, Erdmann, Geiser, Rath, Rieger