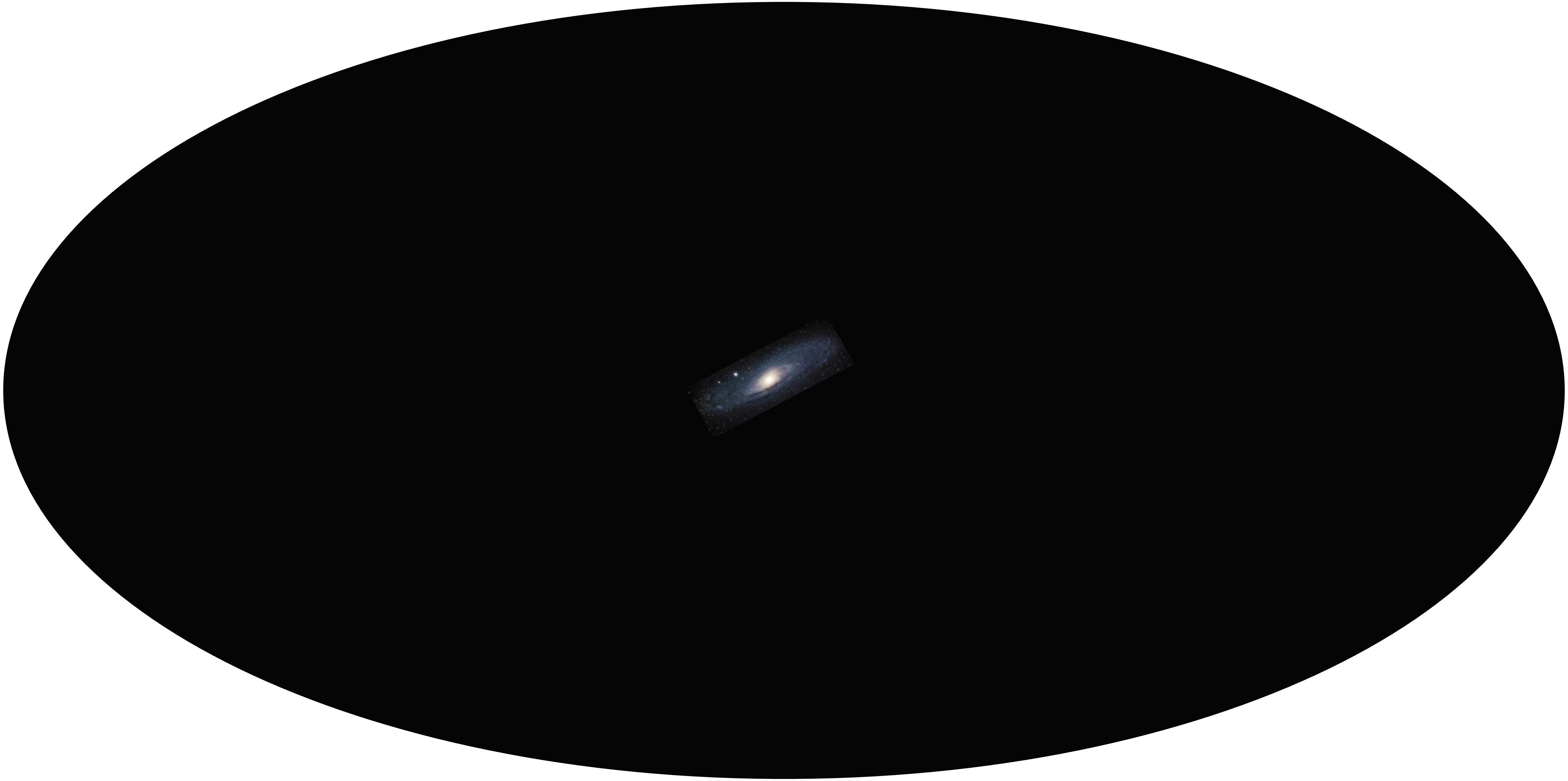
Constraining Dark Matter Substructure with Gaia Wide Binaries

Edward D. Ramirez
Rutgers University

arXiv:2209.08100 with Matthew R. Buckley

Dark Matter Substructure



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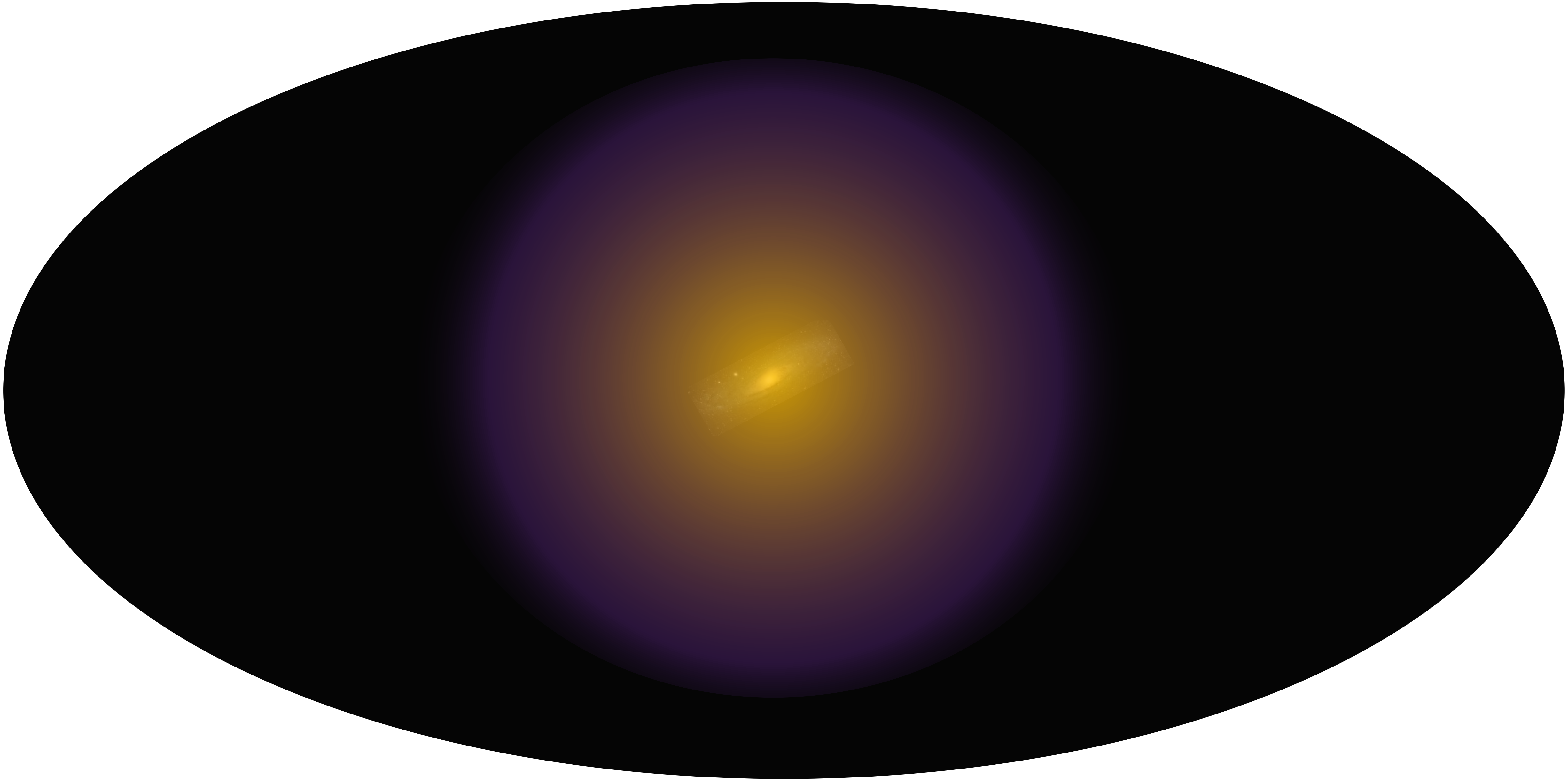
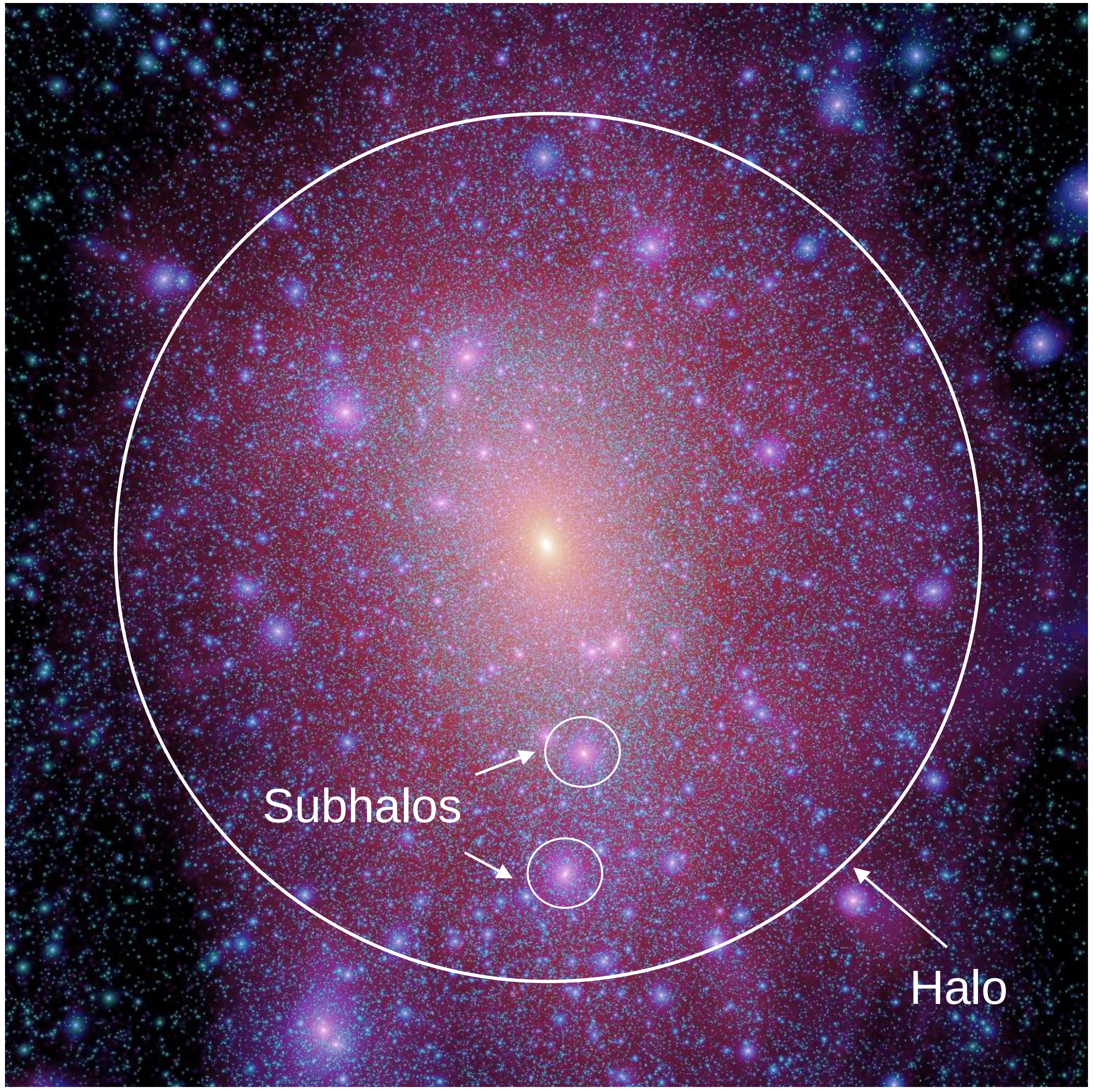


Dark Matter Substructure

Prediction: *The Milky Way hosts a population of dark matter subhalos*



Springel et al. [0809.0898]

Dark Matter Substructure

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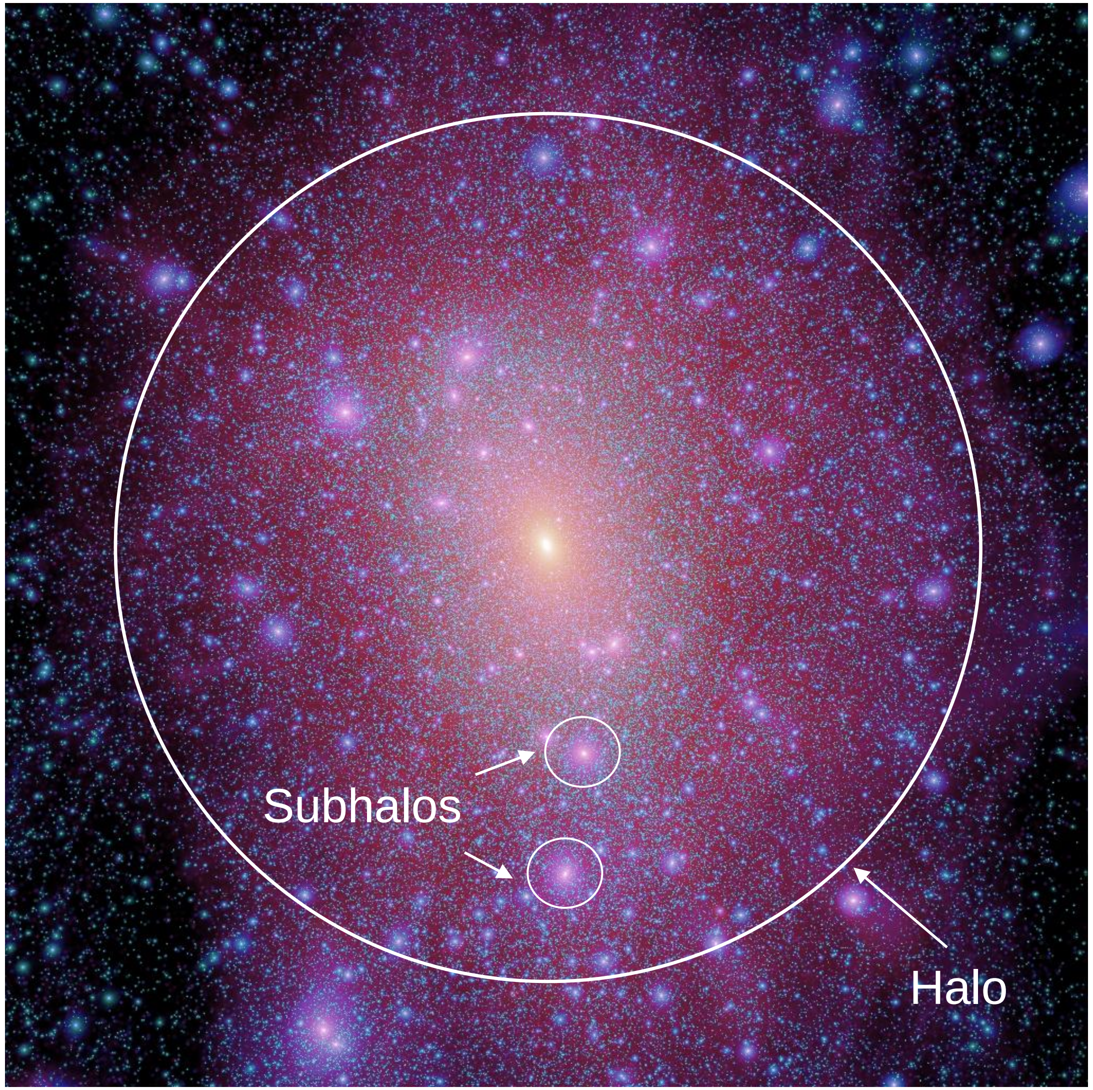
Why would this be interesting to a particle physicist?



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Dark Matter Substructure

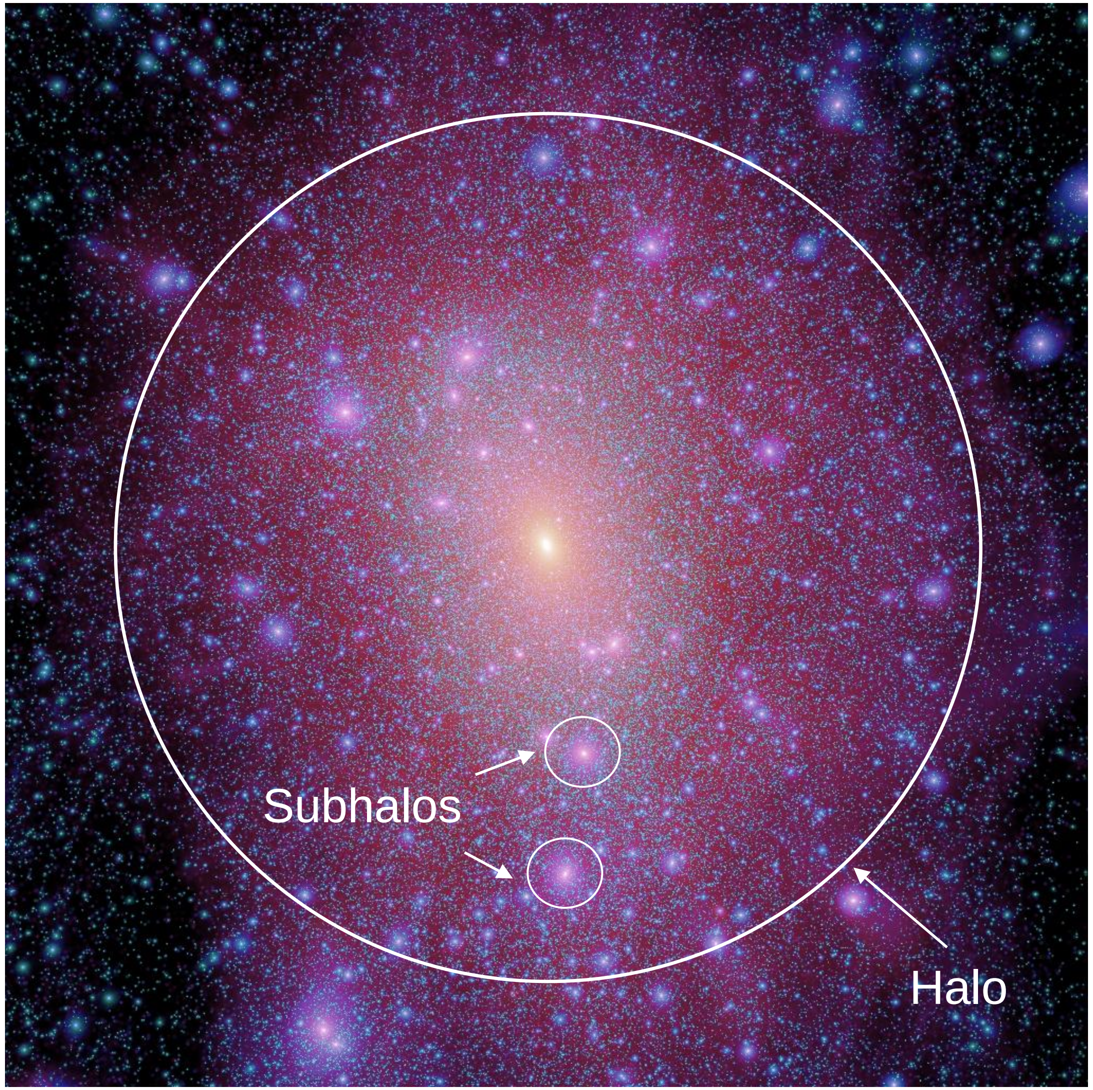
Prediction: *The Milky Way hosts a population of dark matter subhalos*

↓

Why would this be interesting to a particle physicist?

↓

Characteristics of subhalos depend on dark matter microphysics



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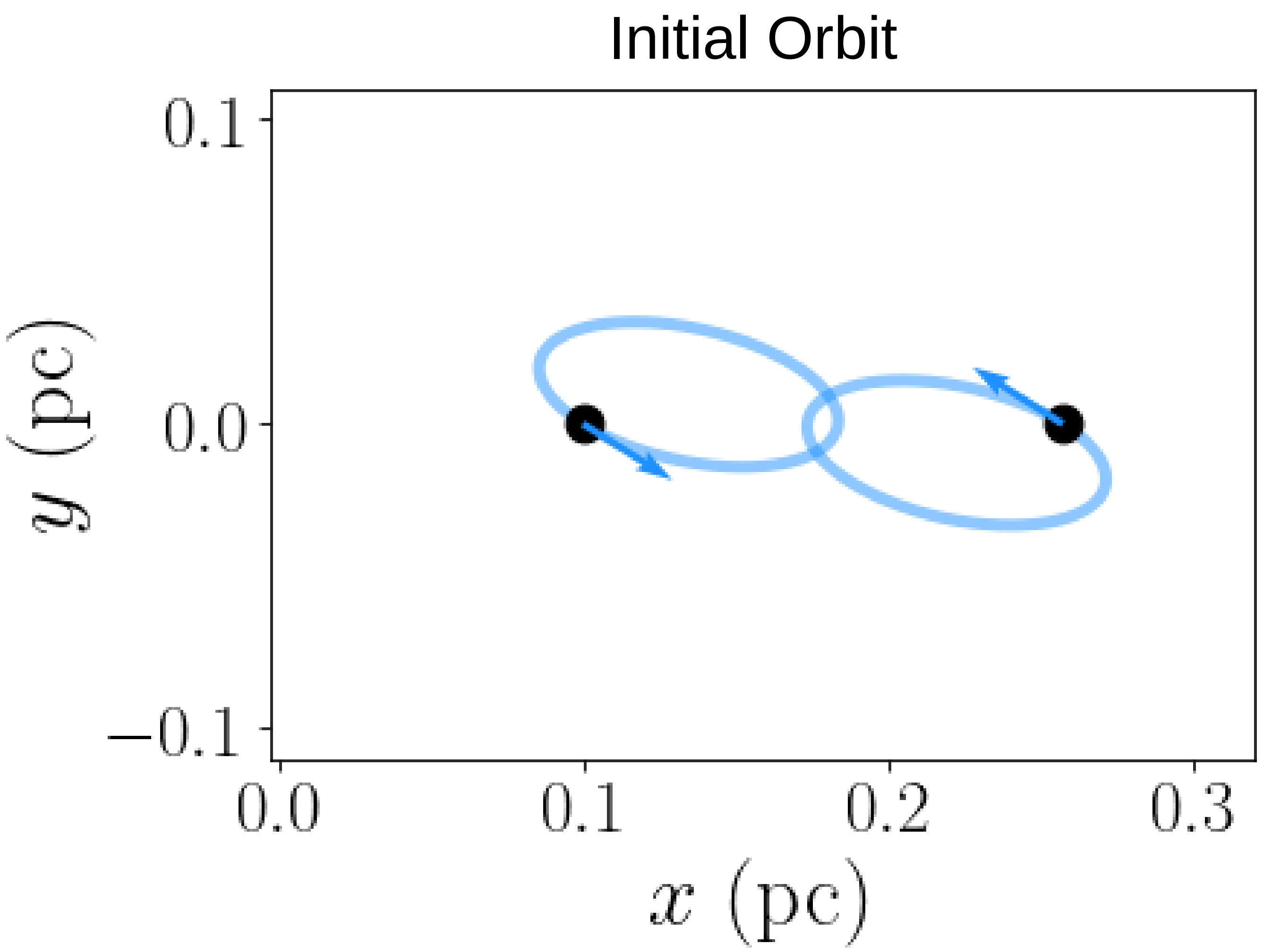
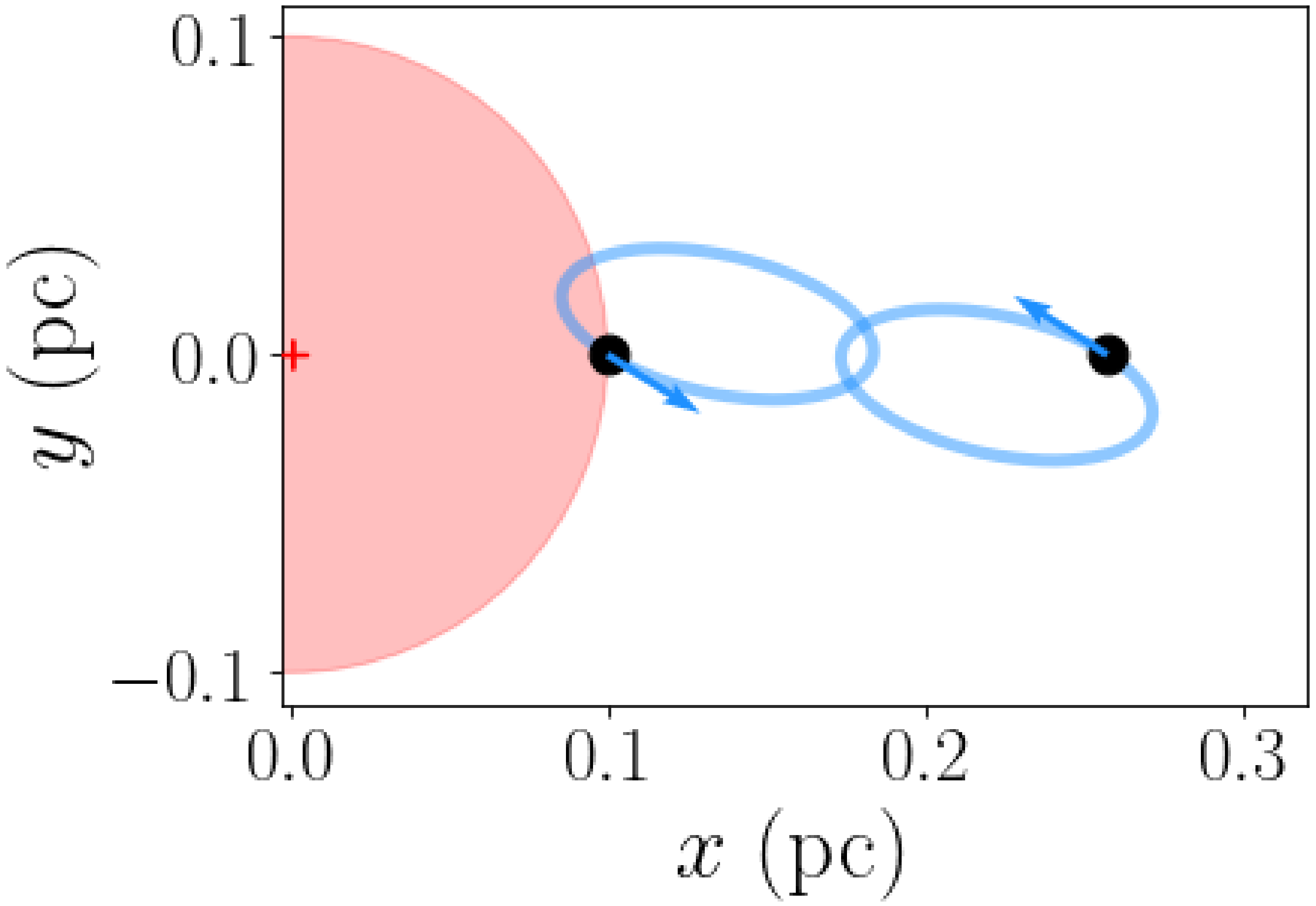
Wide Binaries as Probes of Dark Matter

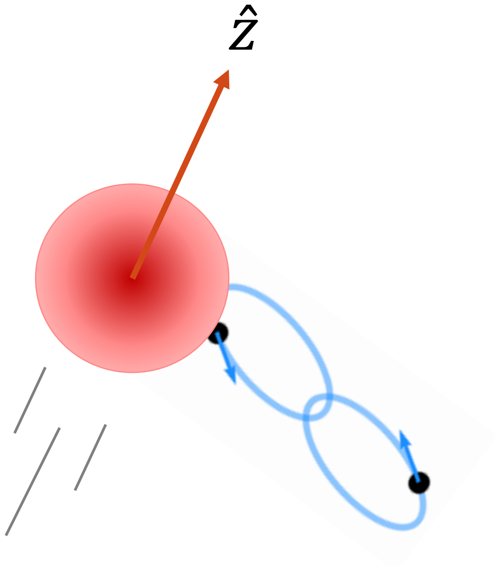


Ramirez et al. [2209.08100]

Wide Binaries as Probes of Dark Matter

Subhalo Encounter





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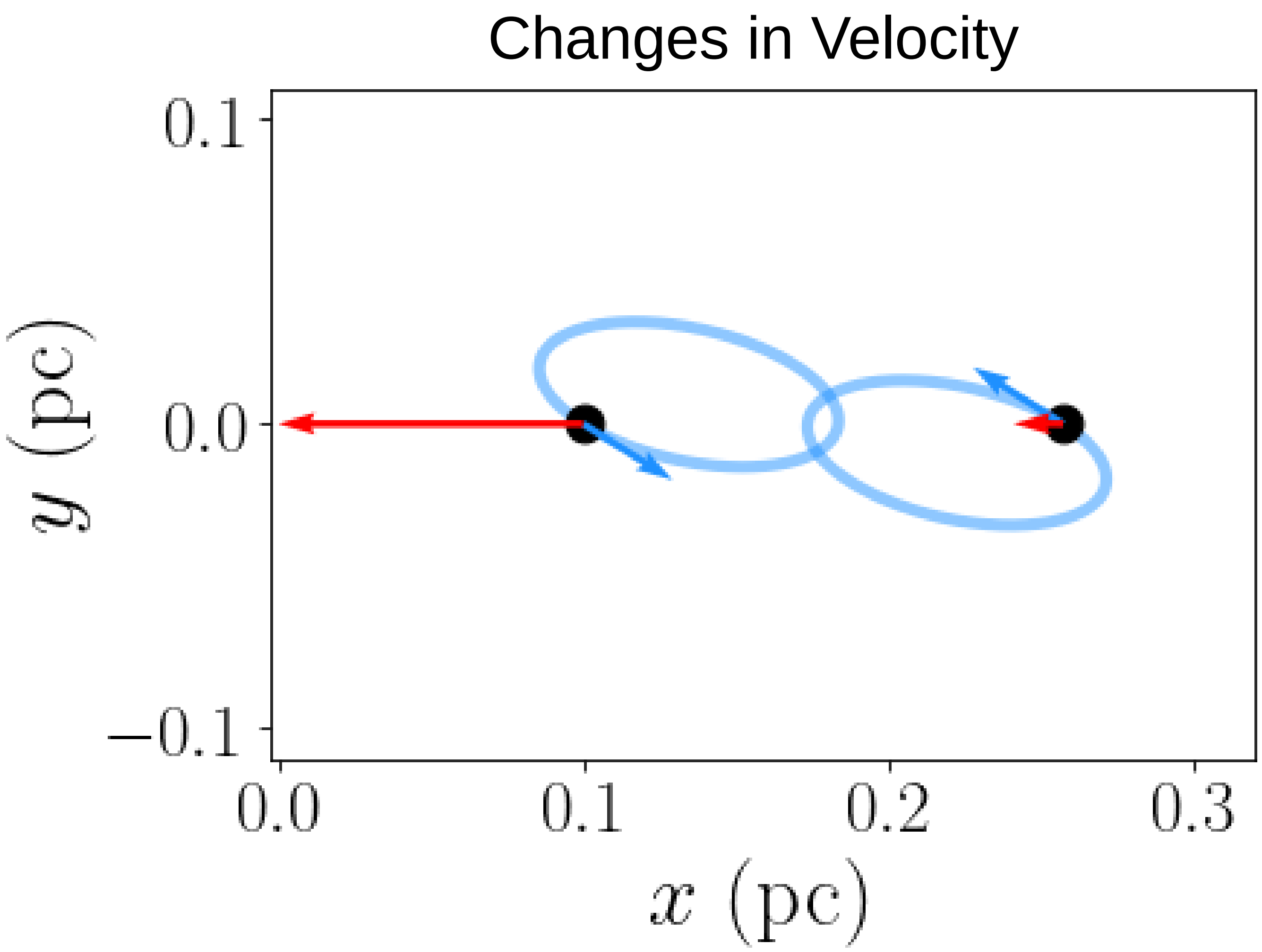

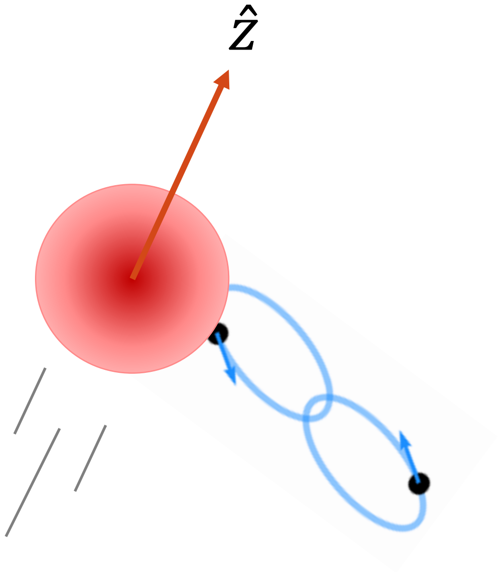

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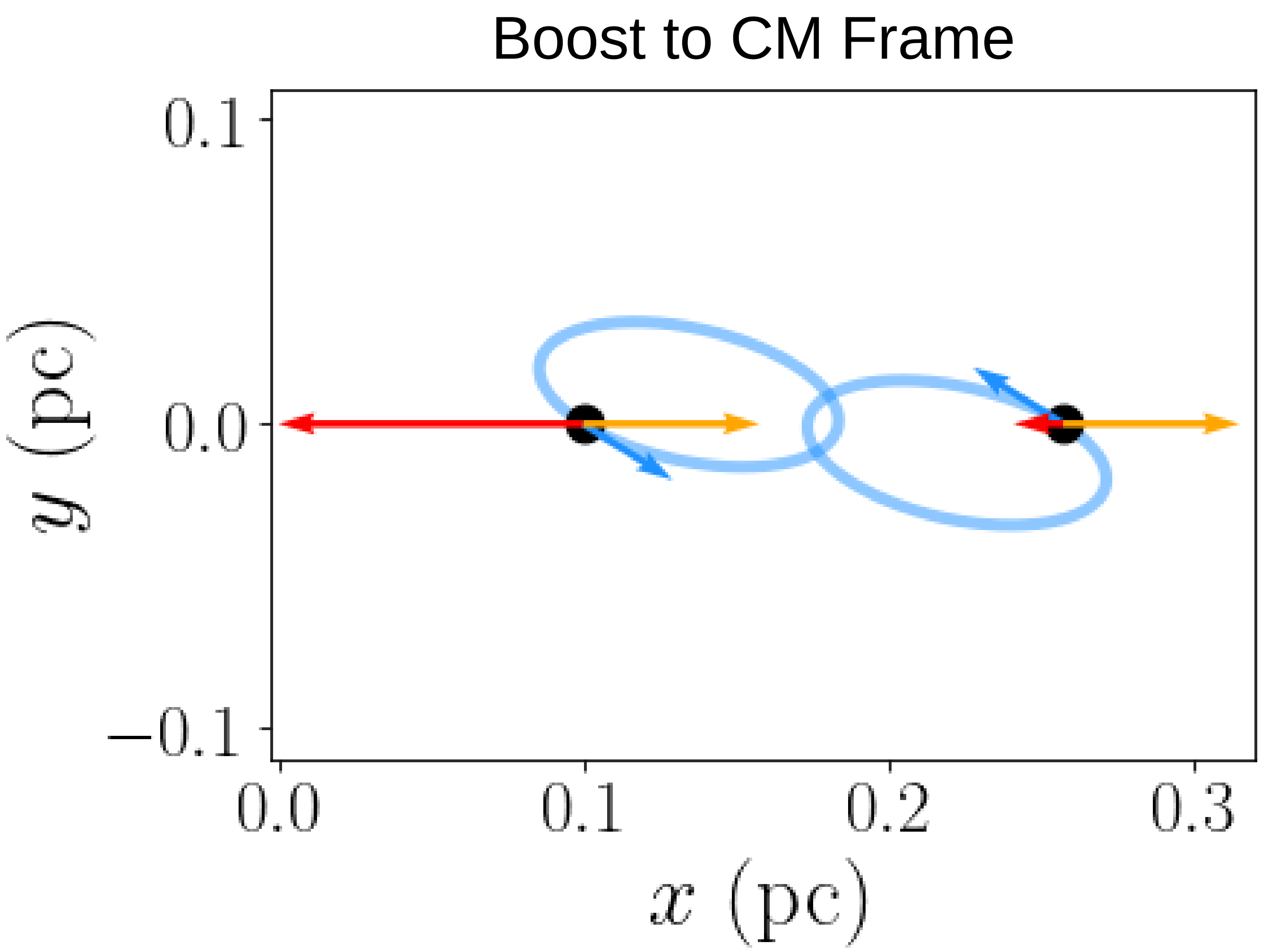
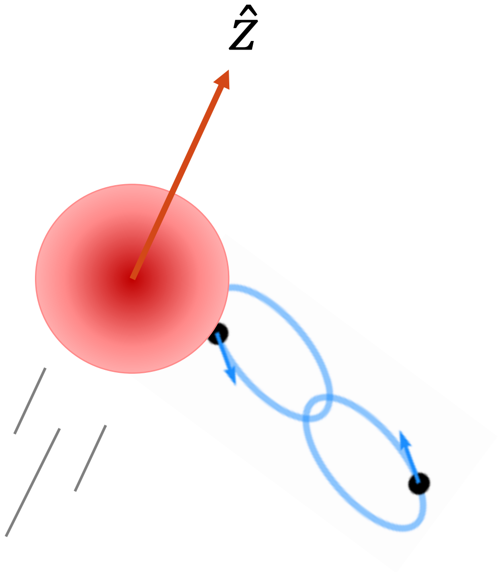
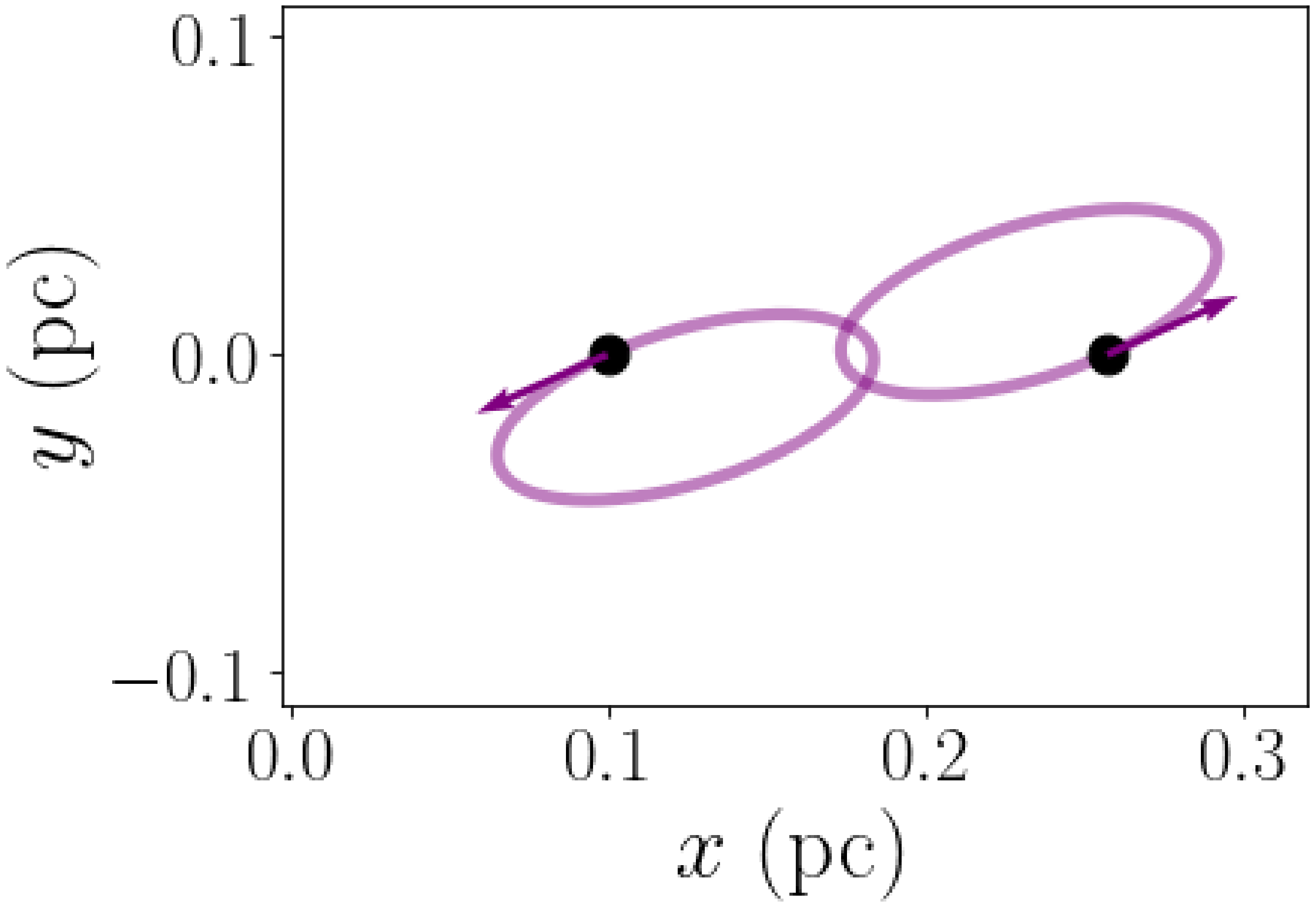


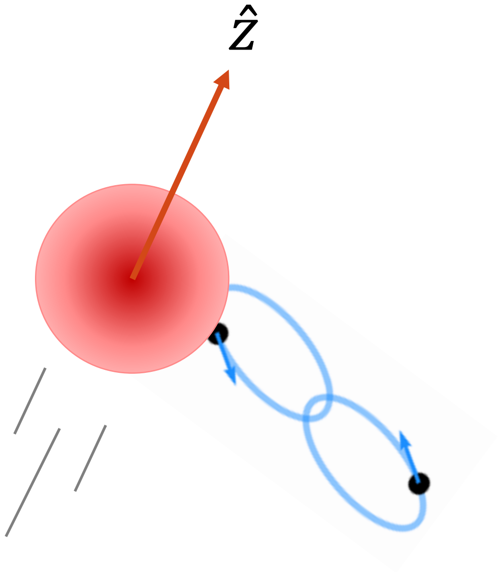


Ramirez et al. [2209.08100]

Wide Binaries as Probes of Dark Matter

New Orbit





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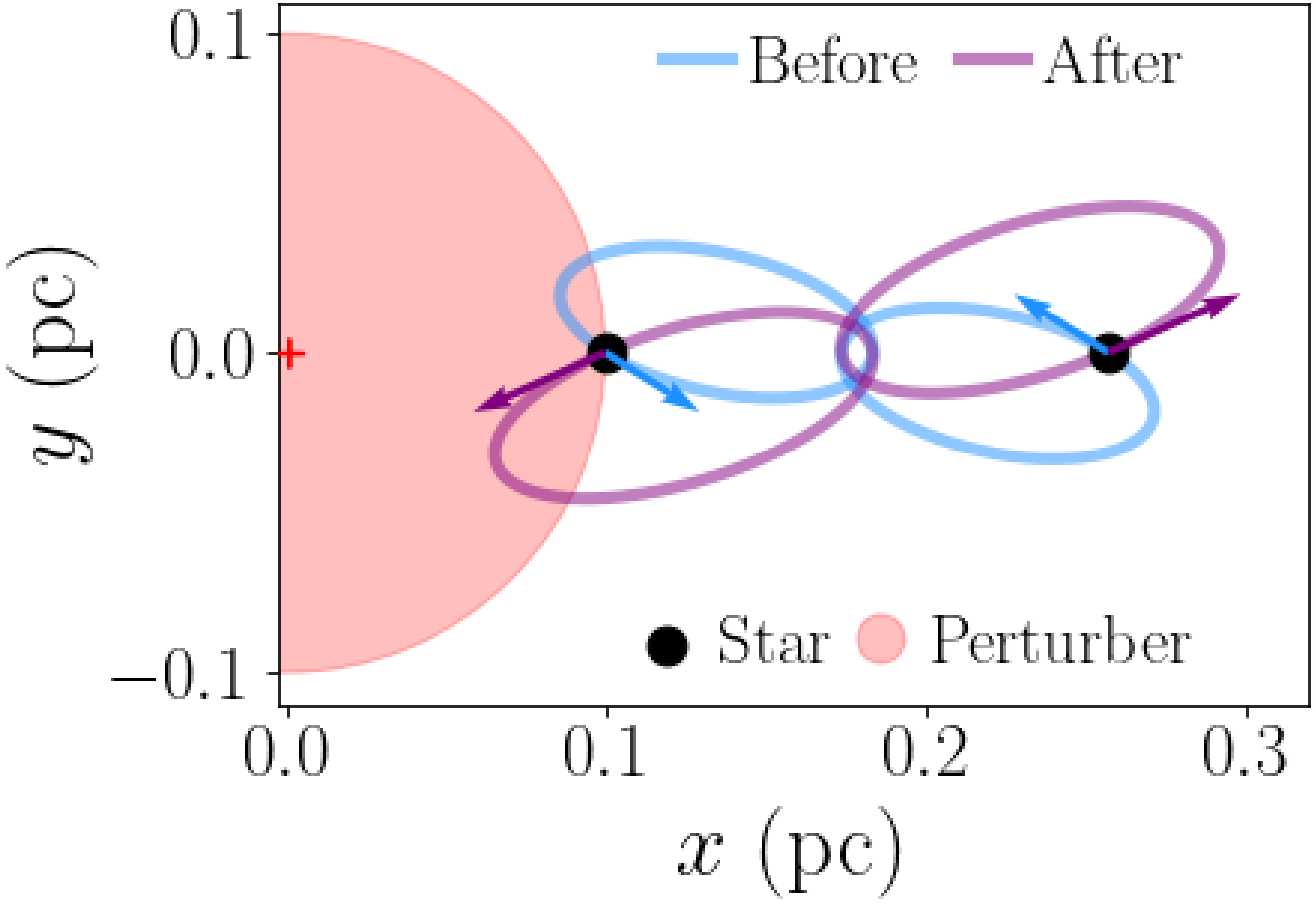

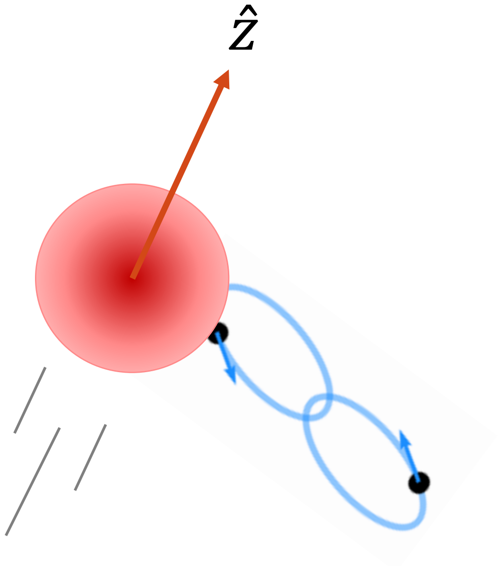

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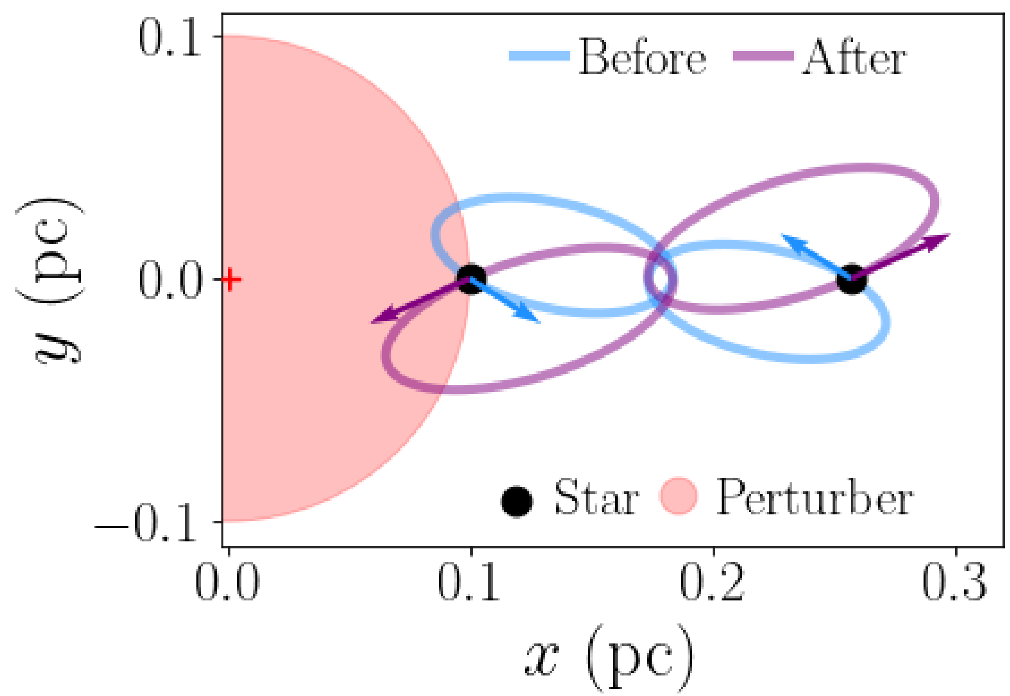
Wide Binaries as Probes of Dark Matter

Tidal Perturbation



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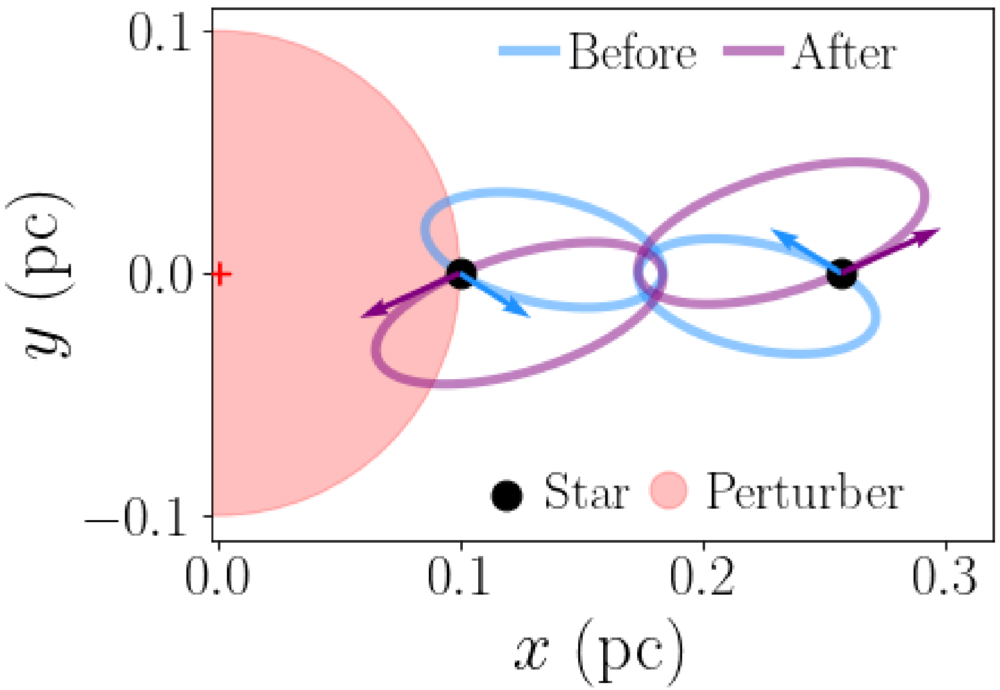

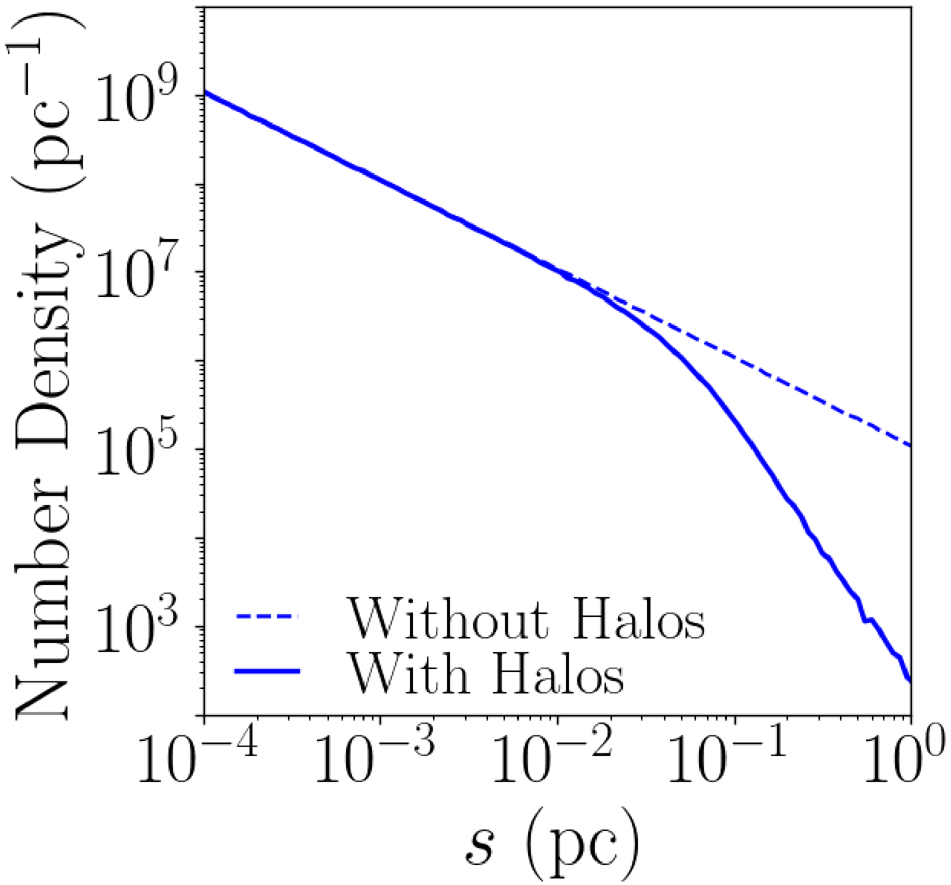

Ramirez et al. [2209.08100]

Wide Binaries as Probes of Dark Matter

Key: *Limits set on* ***extended*** *dark matter substructure*

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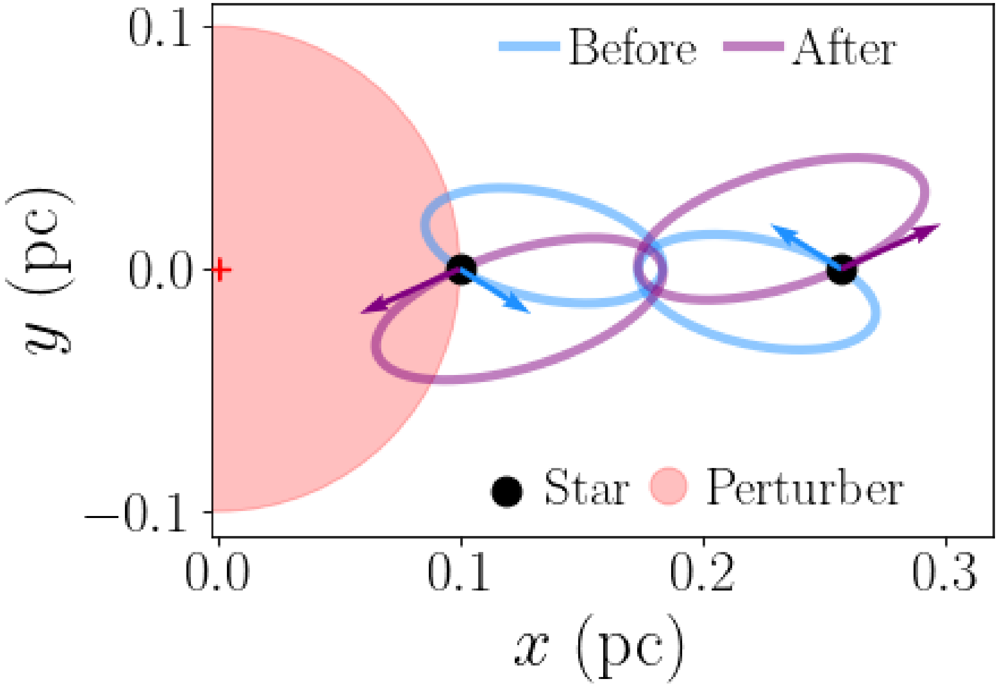

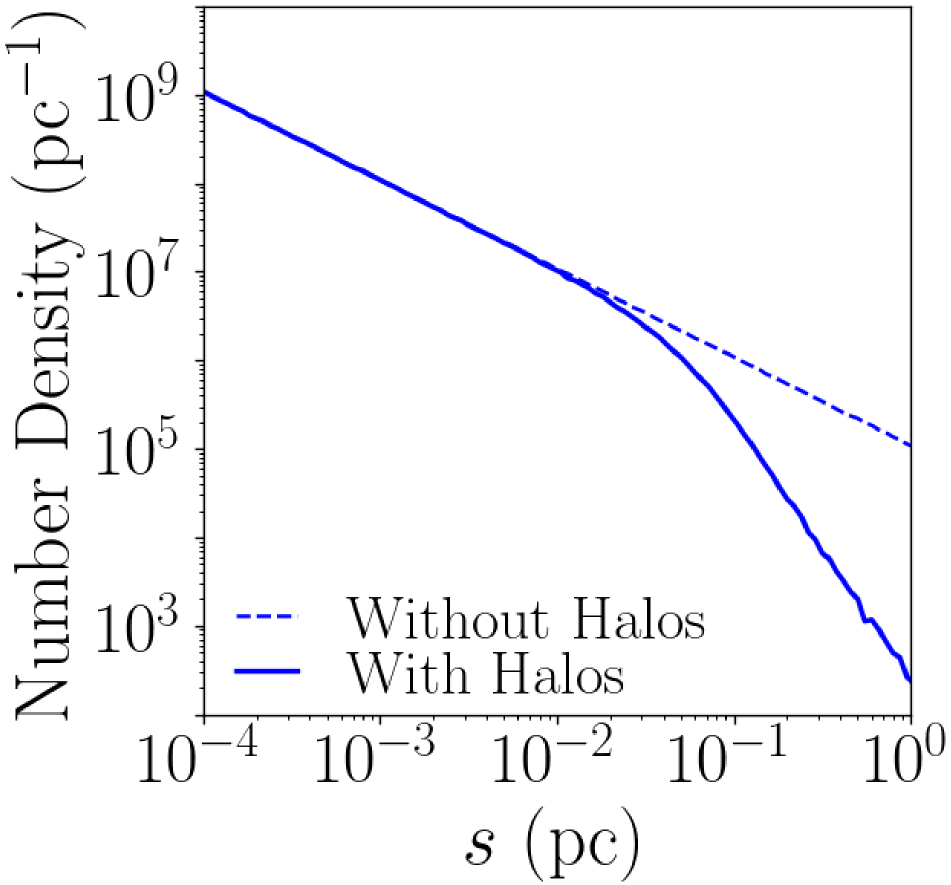

Ramirez et al. [2209.08100]

Extracting Wide Binaries from *Gaia*

- Initial Dataset

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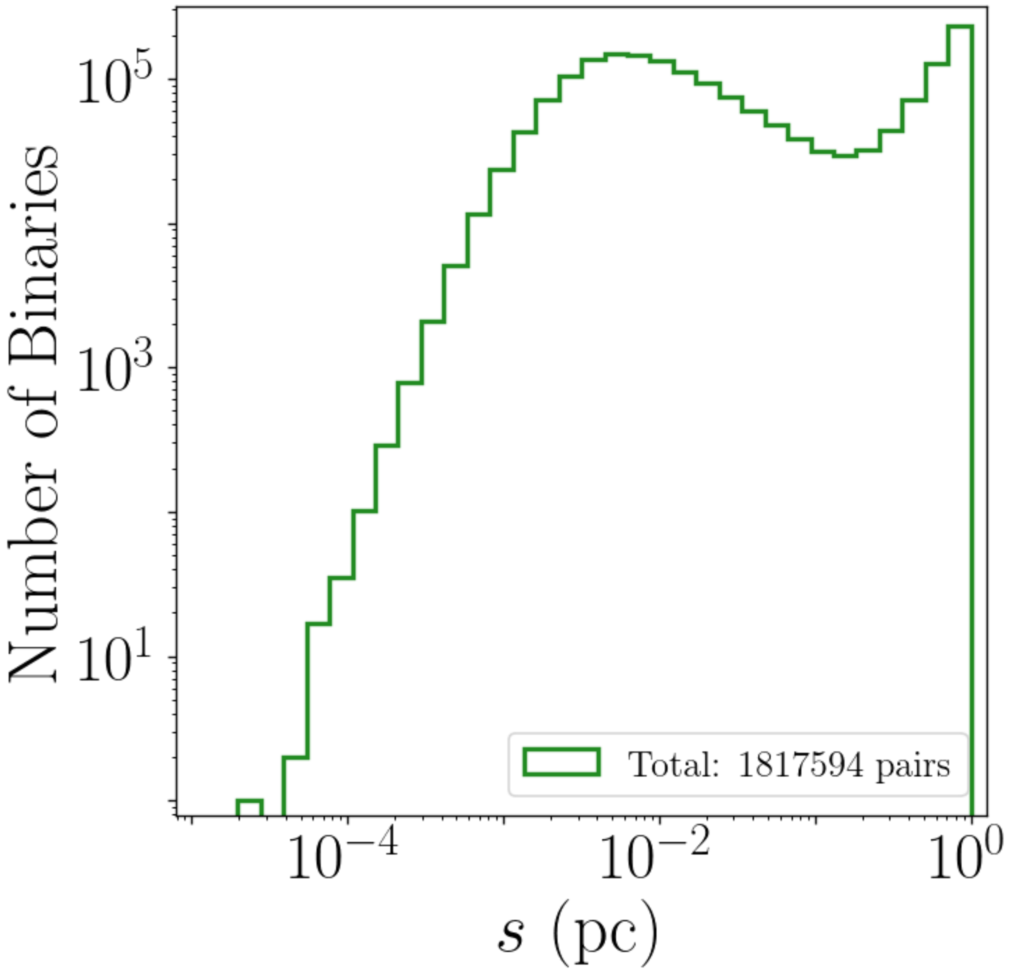
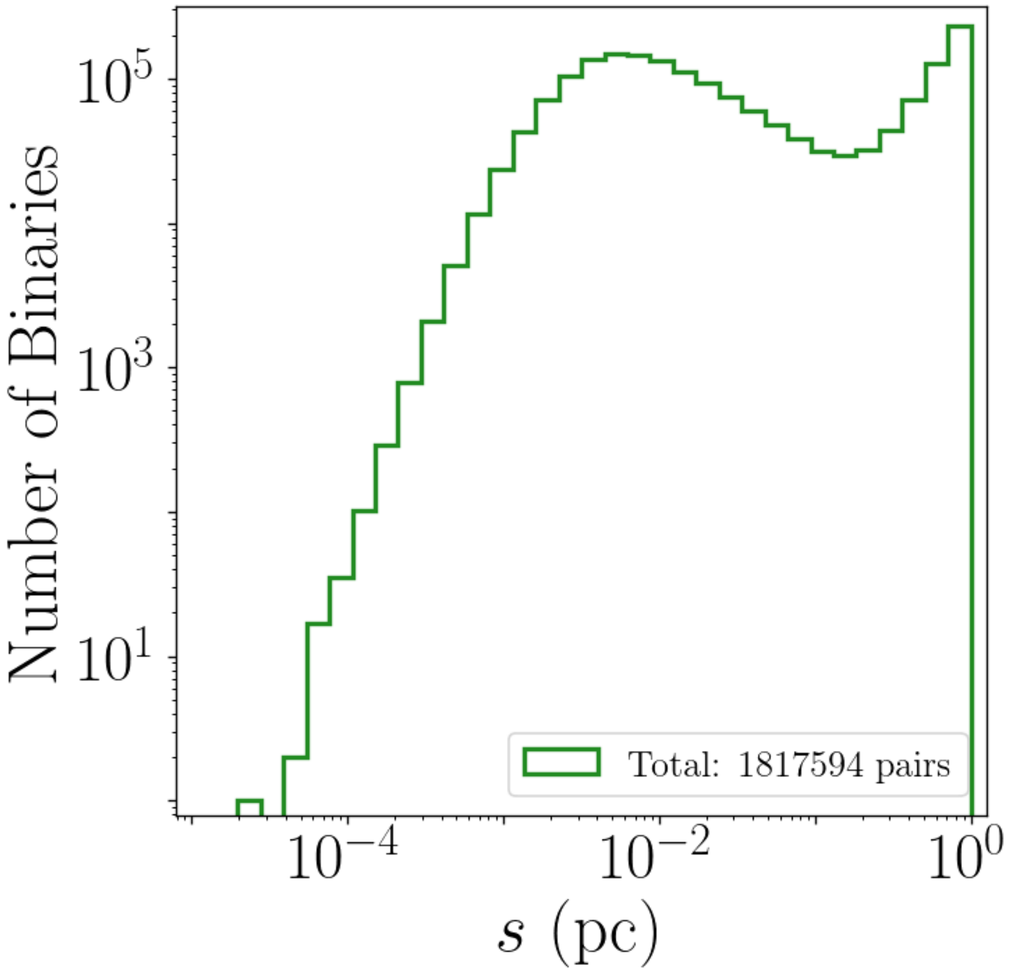
  - *Gaia* eDR3 Catalog*



*El-Badry et al. [2101.05282]

Extracting Wide Binaries from *Gaia*

- Initial Dataset
  - *Gaia* eDR3 Catalog*
- Main Steps for Building Catalog
  - Select stars with precise and complete set of measurements
  - Select pairs on Keplerian orbits



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Extracting Wide Binaries from *Gaia*

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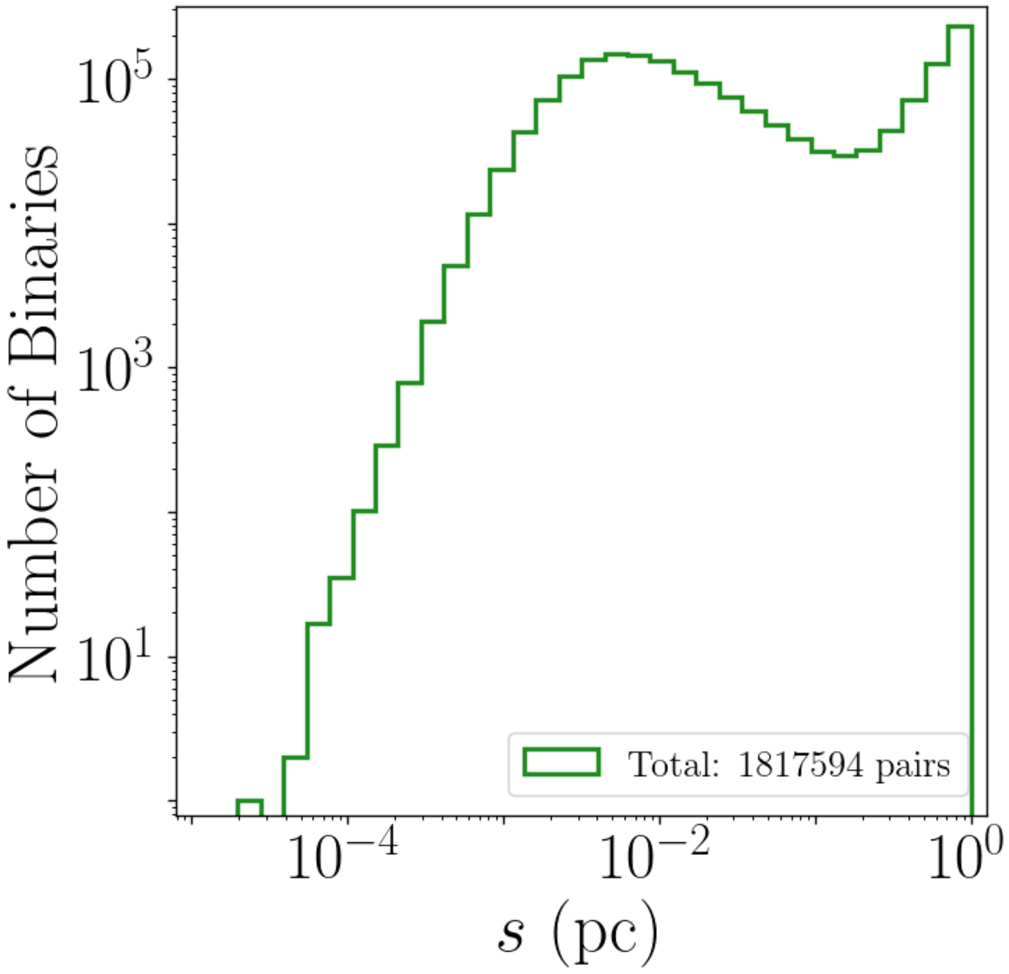
- Goals for Processed Data
  - Complete
  - Pure
  - Sensitive to Subhalos



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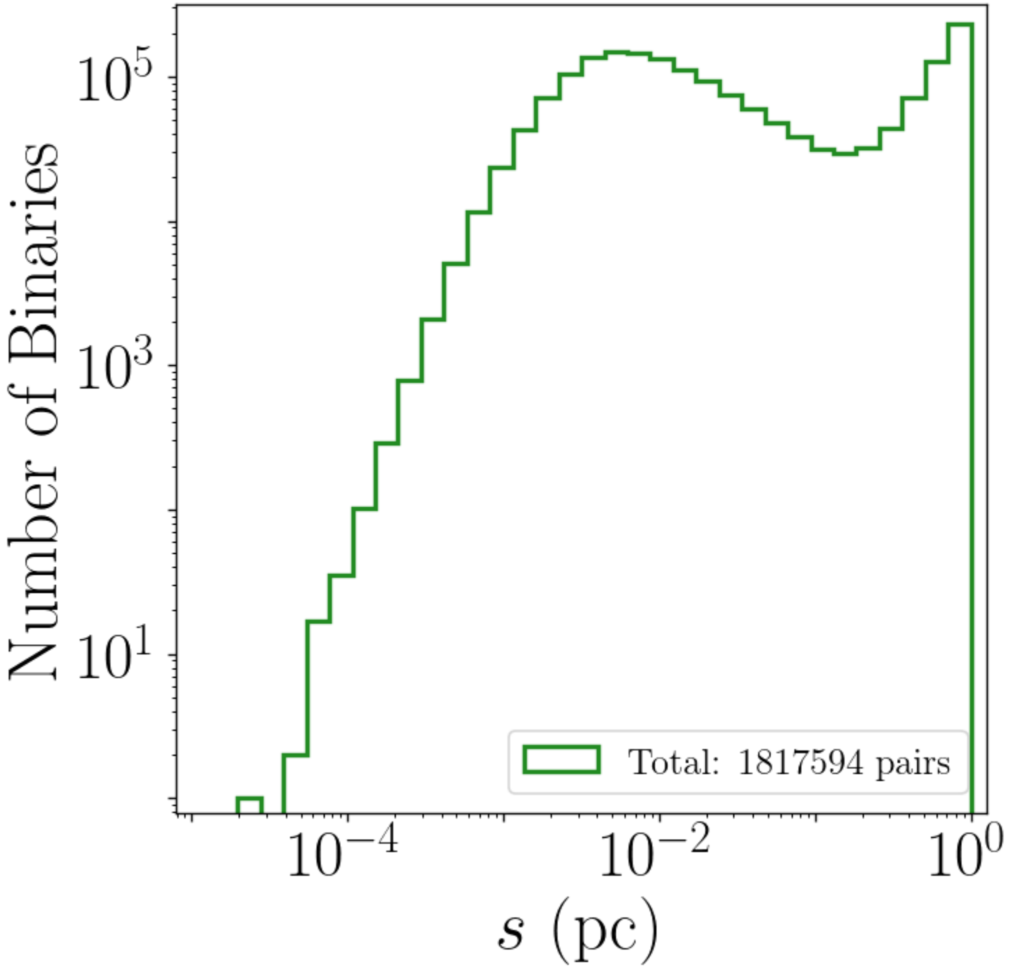
Completeness and Purity Cuts



Data taken from
El-Badry et al. [2101.05282]

Completeness and Purity Cuts

Incompleteness:

- Gaia limited angular resolution

$$\theta < 1.2 \text{ arcsec}$$

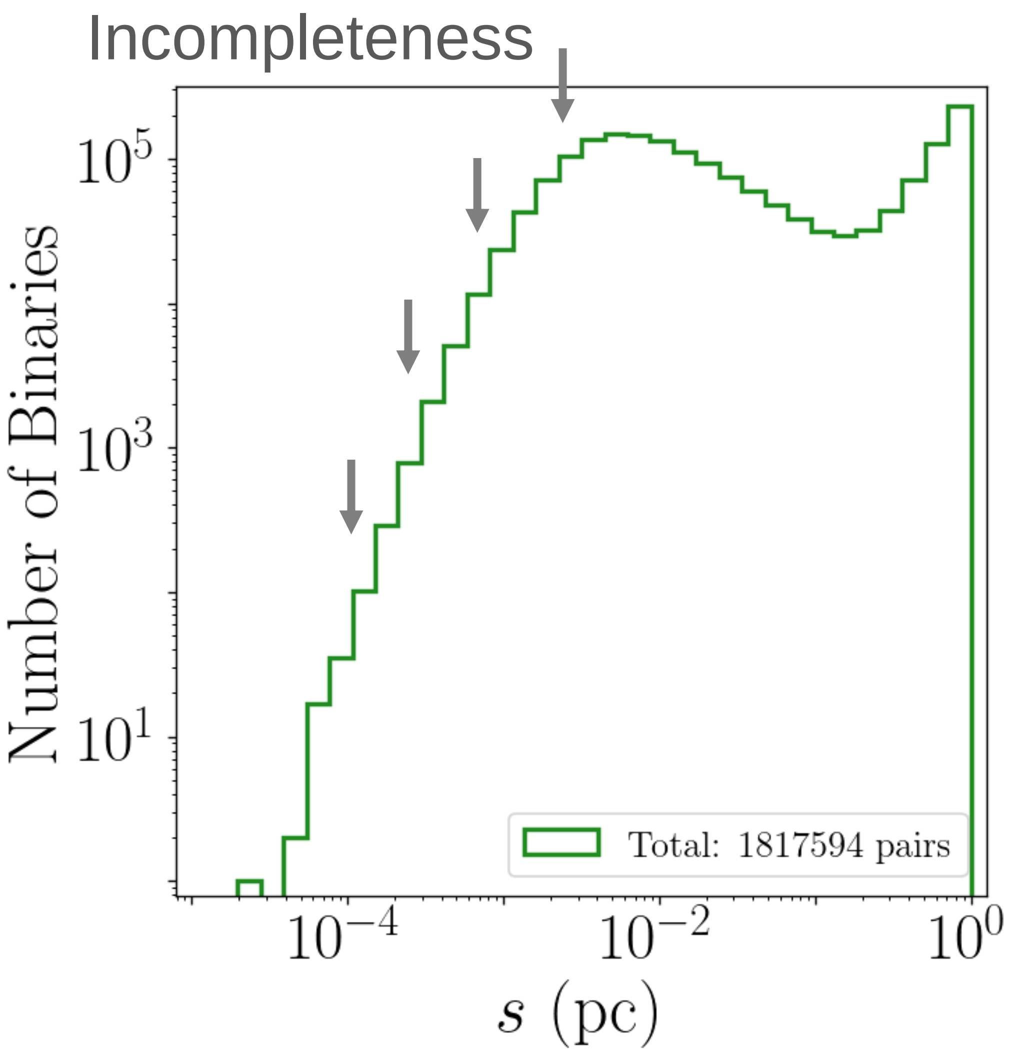
- Difficulty resolving nearby stars with similar magnitudes

$$\Delta G = |G_1 - G_2| \gg 0$$

Solution:

- Select binaries with high detection probability (> 0.999)

➡ Cutoff angle: $\theta_{\Delta G} \sim 3\ as$



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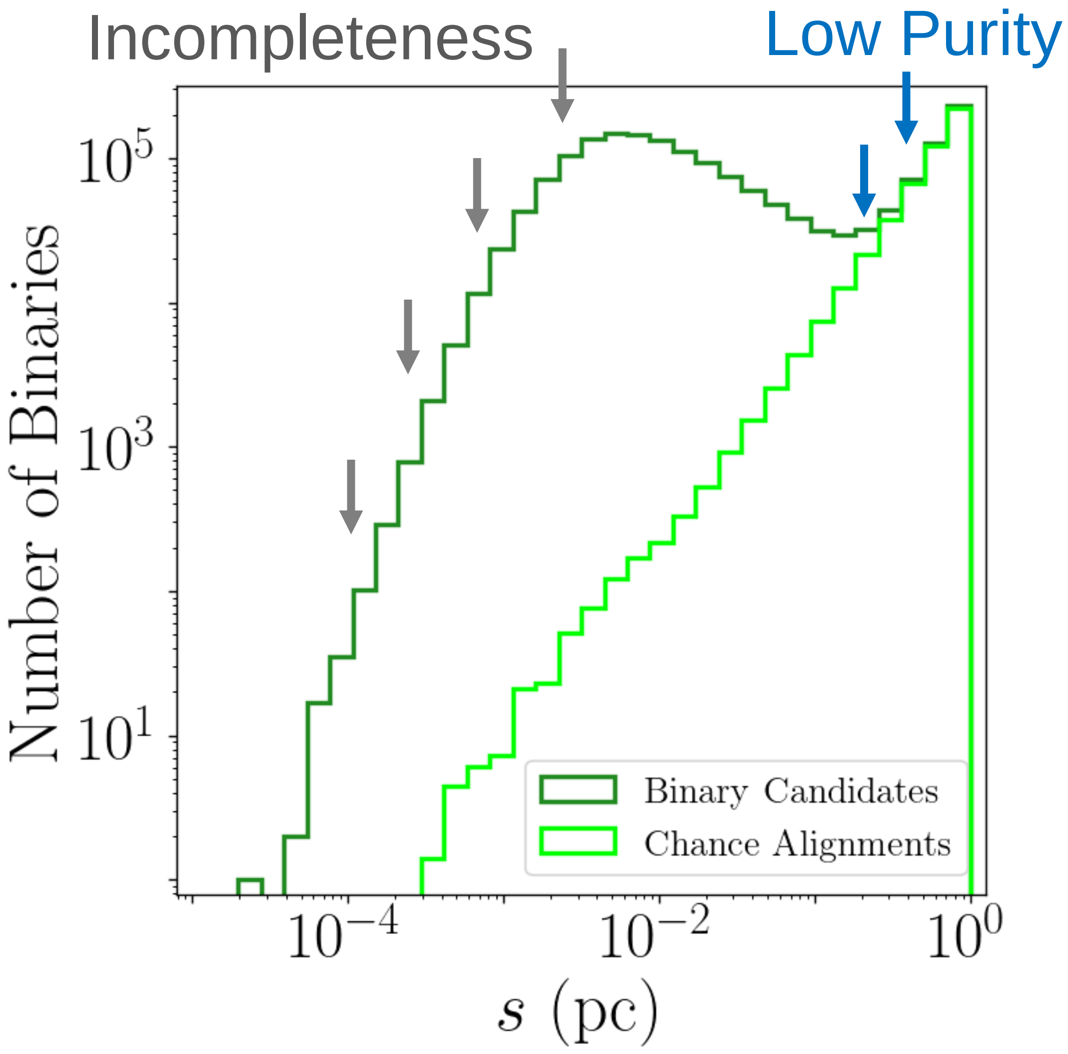

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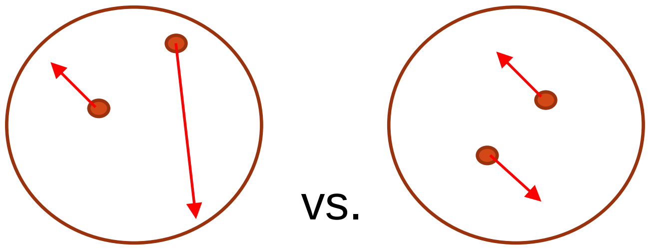
Low Purity:

- Binary candidates may not be true binaries, but are chance alignments



Solution:

- Filter out by imposing more stringent Keplerian condition

Binaries Sensitive to Subhalos

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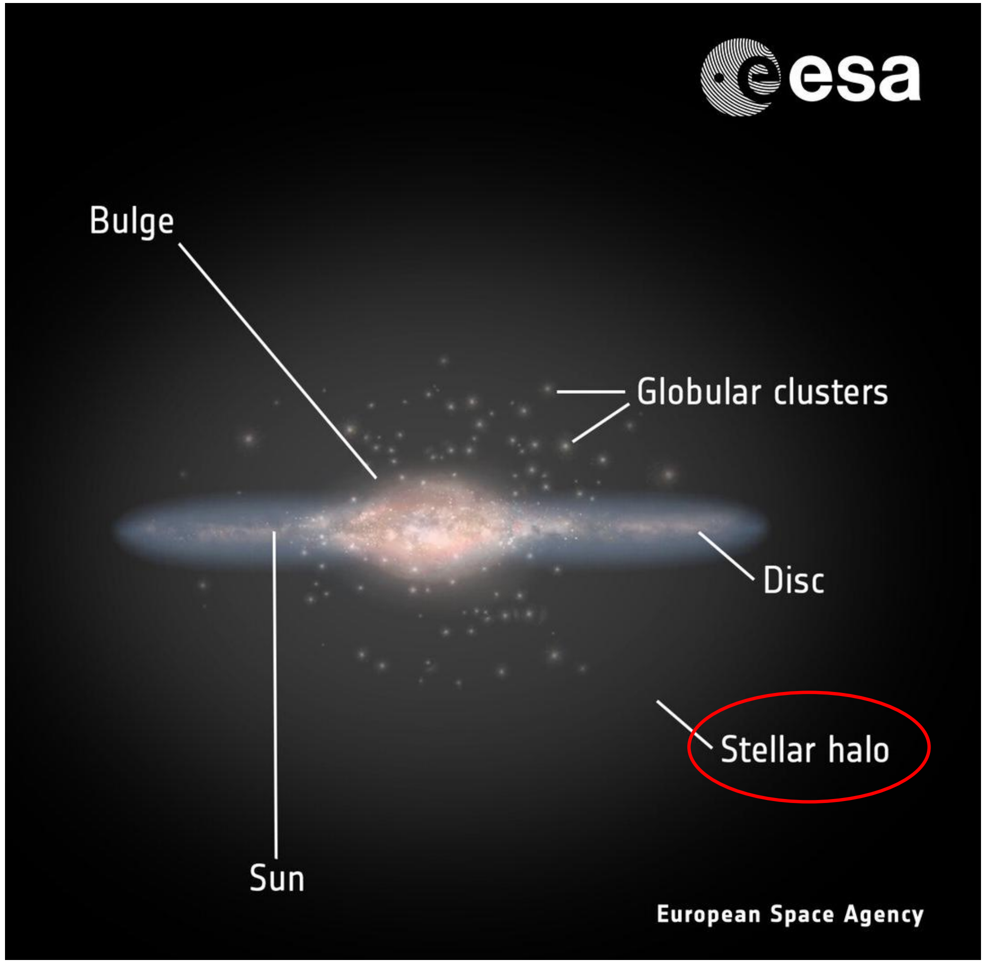
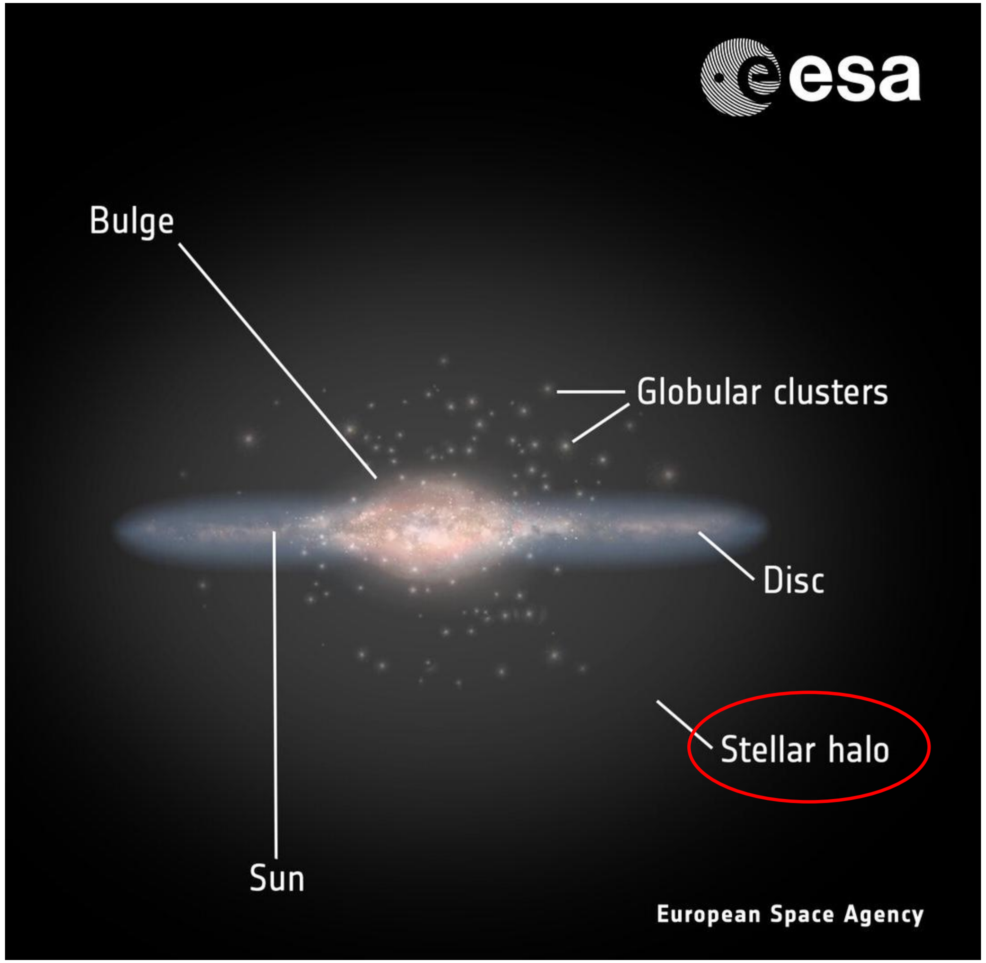


- Stellar Halo
  - Age $\gtrsim$ 10 Gyr
  - Sparse baryonic matter

Binaries Sensitive to Subhalos



- Stellar Halo
  - Age $\gtrsim$ 10 Gyr
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  - Interact with subhalos for the highest amount of time
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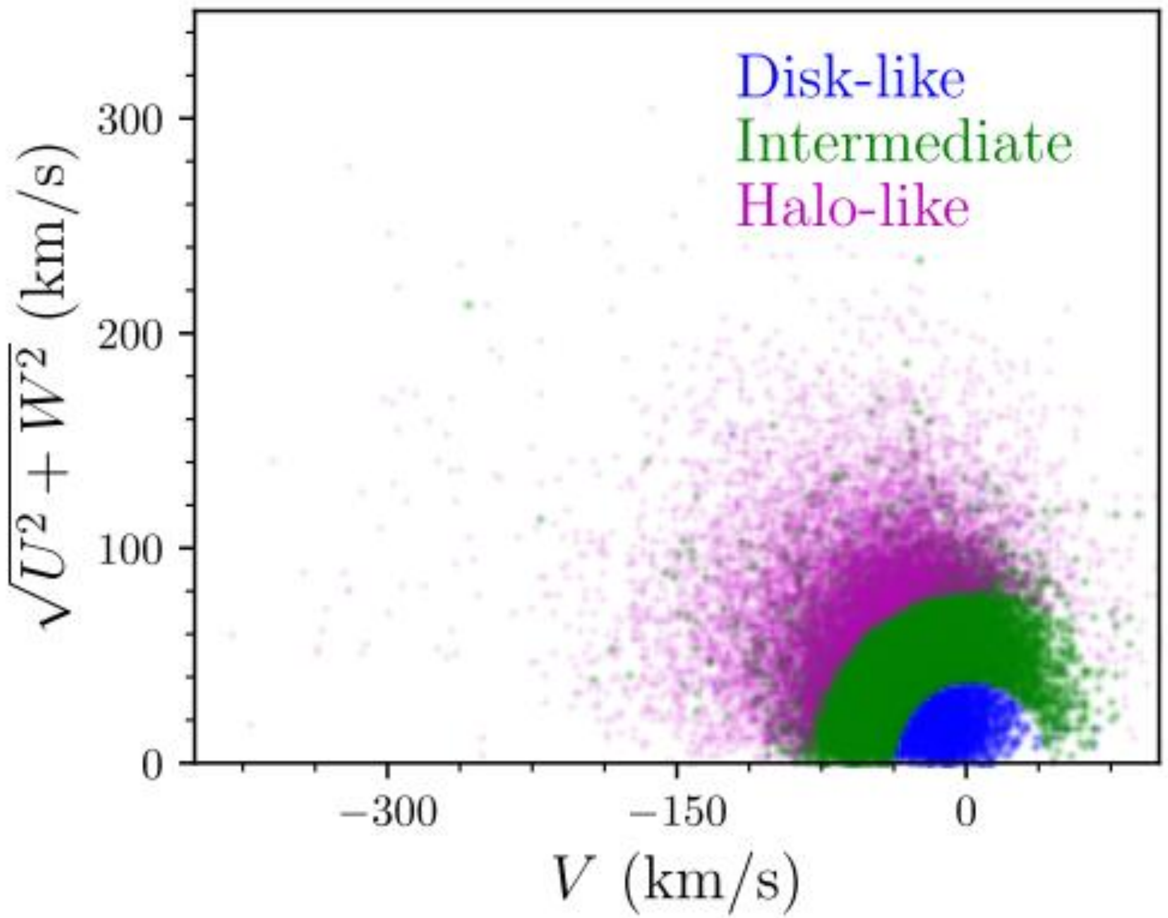

Tian [1909.04765]
(*Gaia* DR2)

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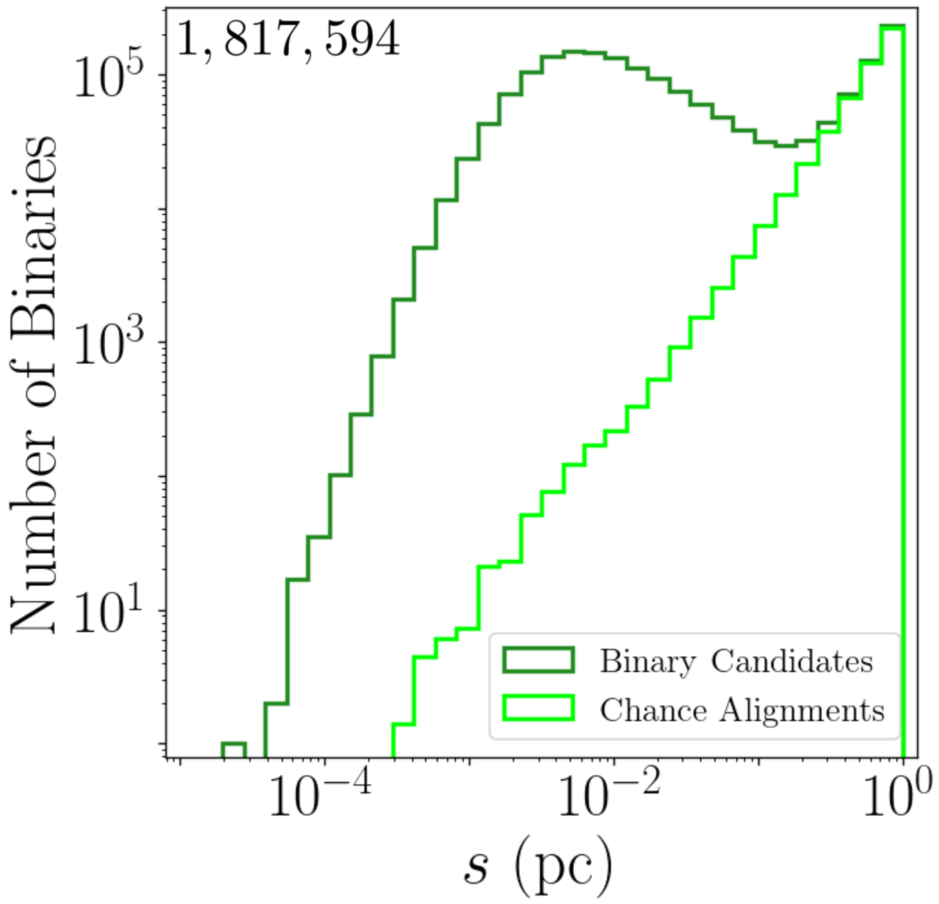
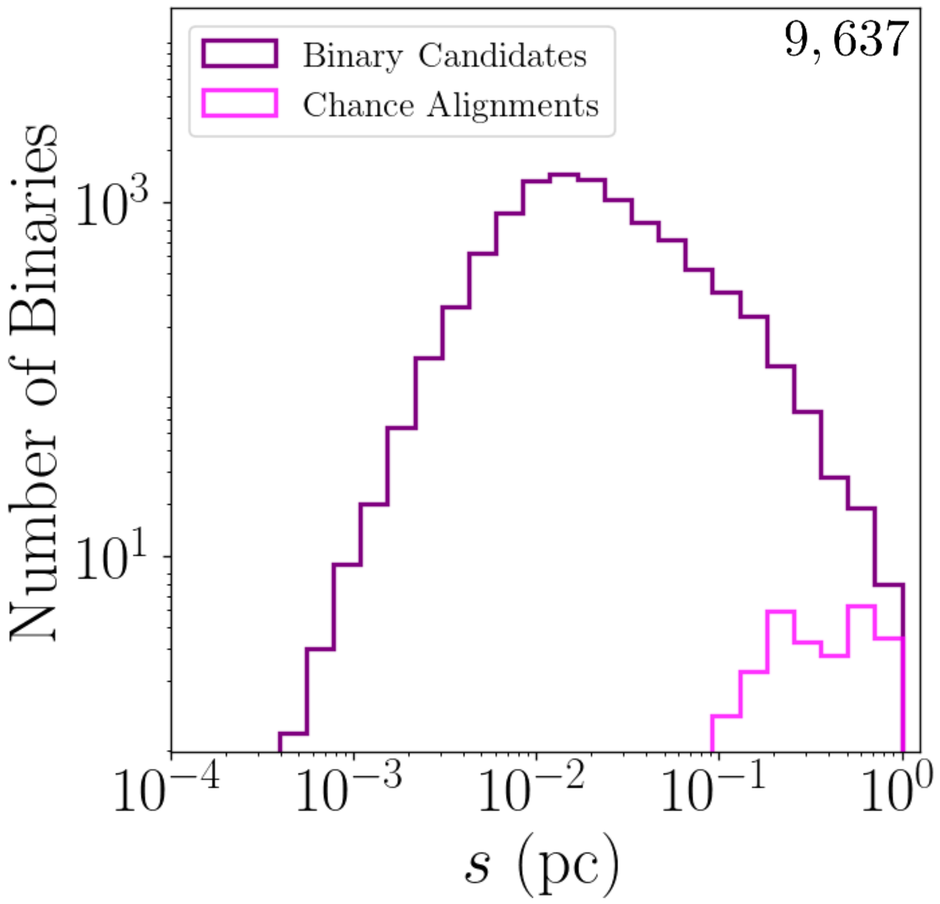
Selection Cut:

$$v_\perp > 85 \text{ km/s}$$
$$d < 700 \text{ pc}$$

Result of Cuts





Modelling Binary Evolution

- Single Binary, Single Subhalo

Ramirez et al. [2209.08100]

Modelling Binary Evolution

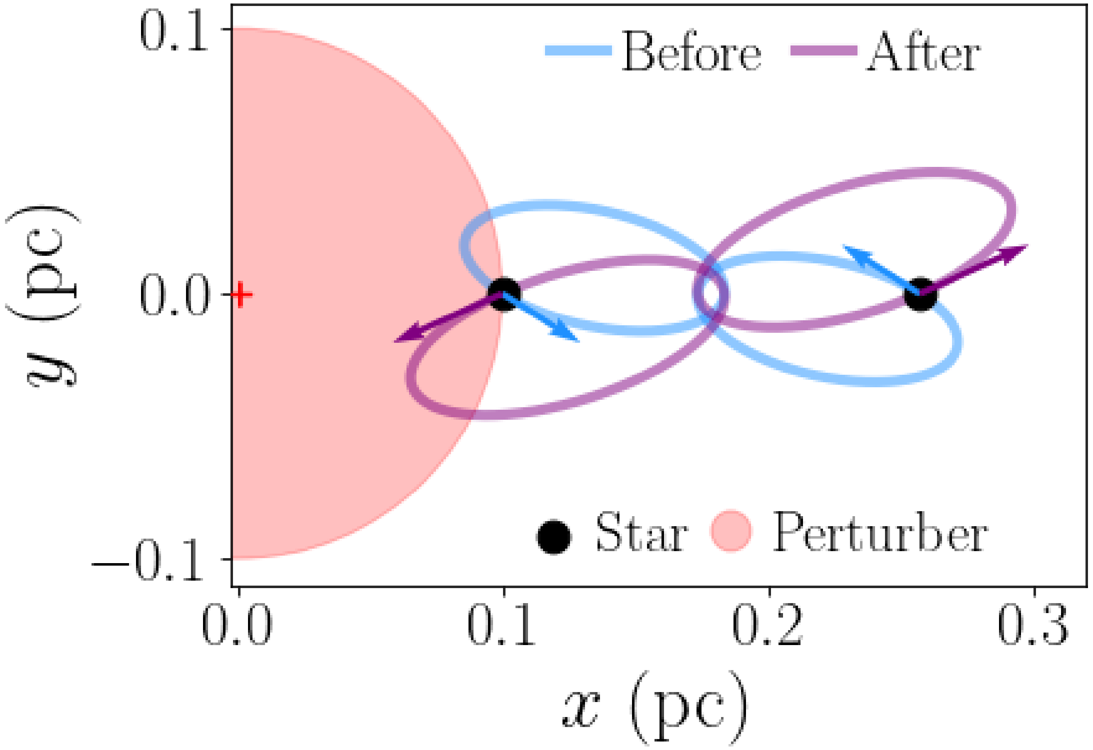
- Single Binary, Single Subhalo

Effect:

Change in Orbit

$$(a_0, e_0, \psi_0) \xrightarrow[\Delta\vec{v}]{} (a, e, \psi)$$

Analytic



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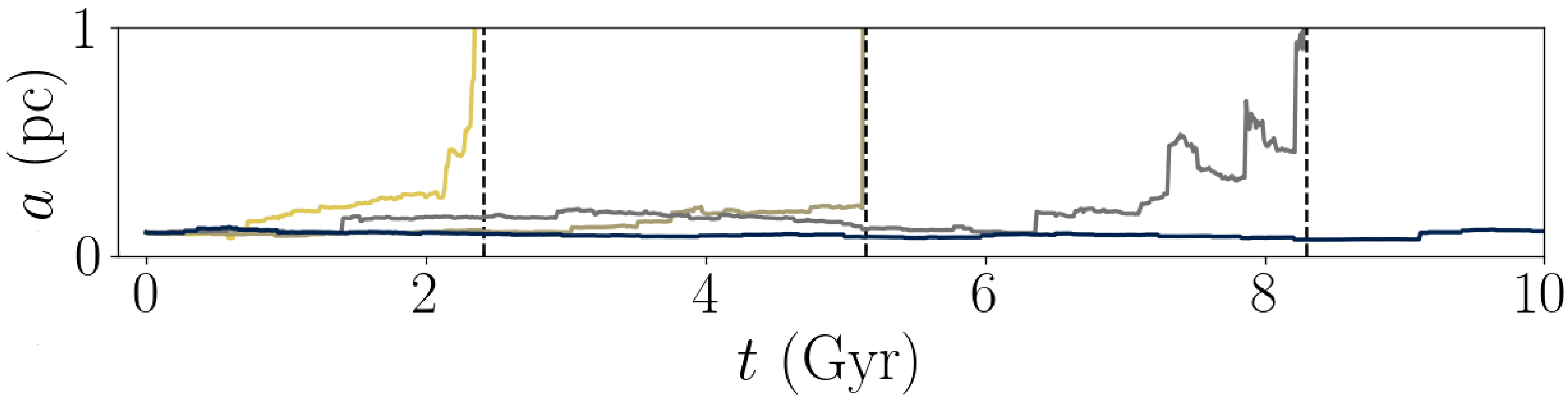
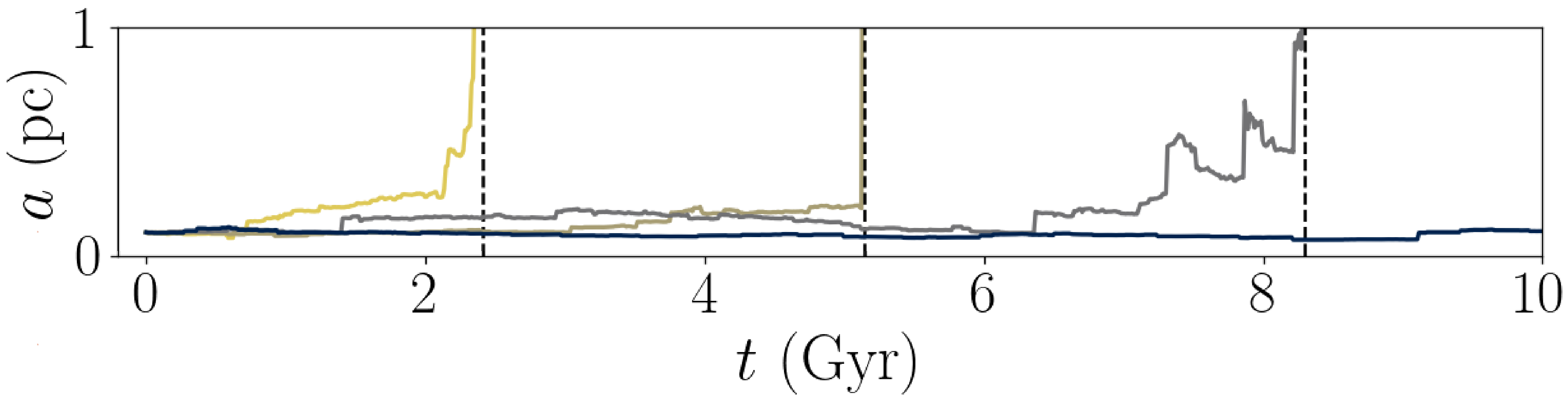


Ramirez et al. [2209.08100]

Modelling Binary Evolution

- Single Binary, Single Subhalo
- Single Binary, Many Subhalos
  - Random encounters lead to random evolution
  - Generally, binaries widen with time and may eventually be destroyed



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- Single Binary, Single Subhalo
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- Many Binaries, Many Subhalos
  - Evolve each individual binary as in previous case

Ramirez et al. [2209.08100]

Modelling Binary Evolution

- Single Binary, Single Subhalo
- Single Binary, Many Subhalos
- Many Binaries, Many Subhalos
  - Evolve each individual binary as in previous case
- Monte Carlo Simulation
  1) Sample binaries from some initial distribution representative of our dataset 10 Gyr ago
  2) Evolve binaries for 10 Gyr under repeated subhalo encounters
  3) Save present-day distribution of separations

Ramirez et al. [2209.08100]

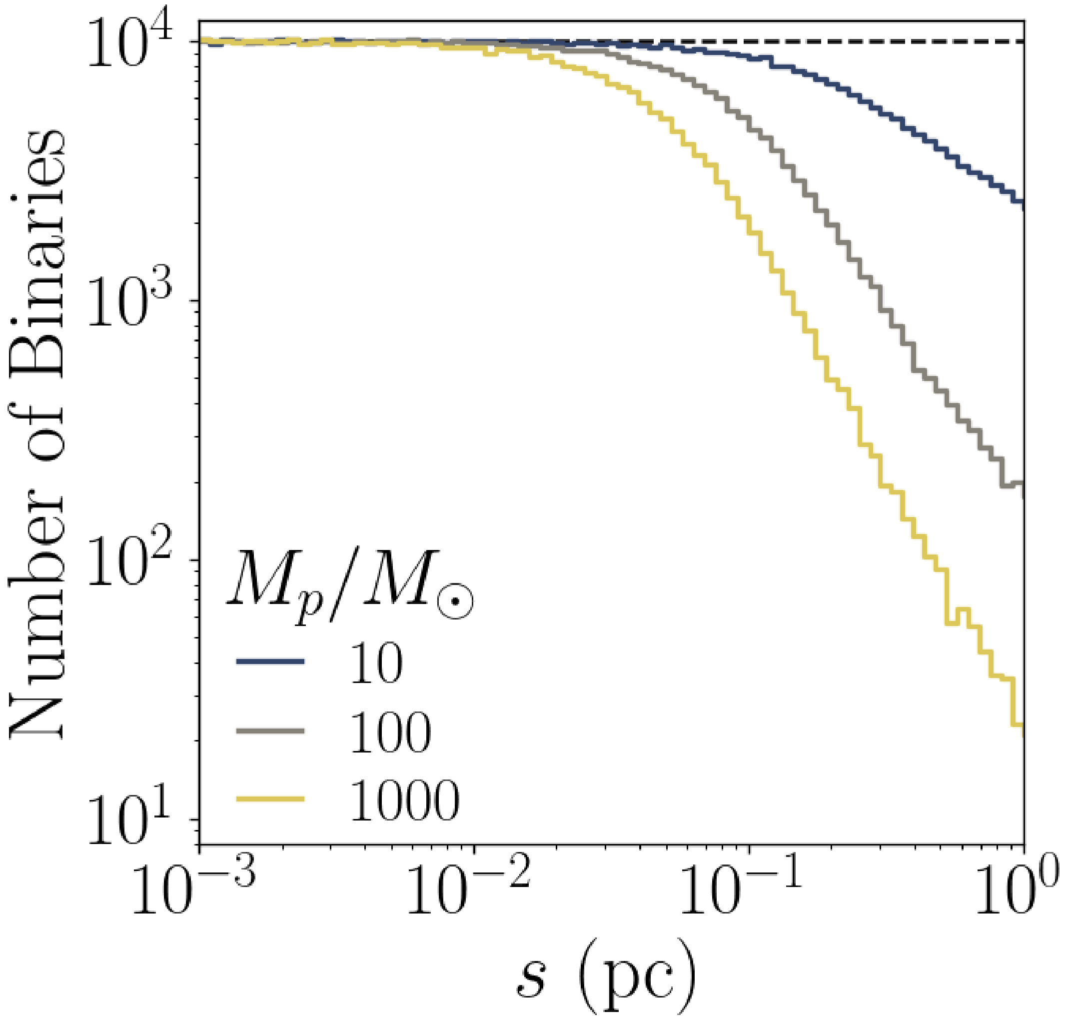
Simple Example

- Perturber Population

$$\begin{cases} M_p = \text{free} \\ R_p = 0.1 \text{ pc} \\ \rho(r) = \text{constant} \\ \rho_p(R_\odot) = \rho_{DM}(R_\odot) \end{cases}$$

- Initial Binary Population

  Log-flat separation distribution



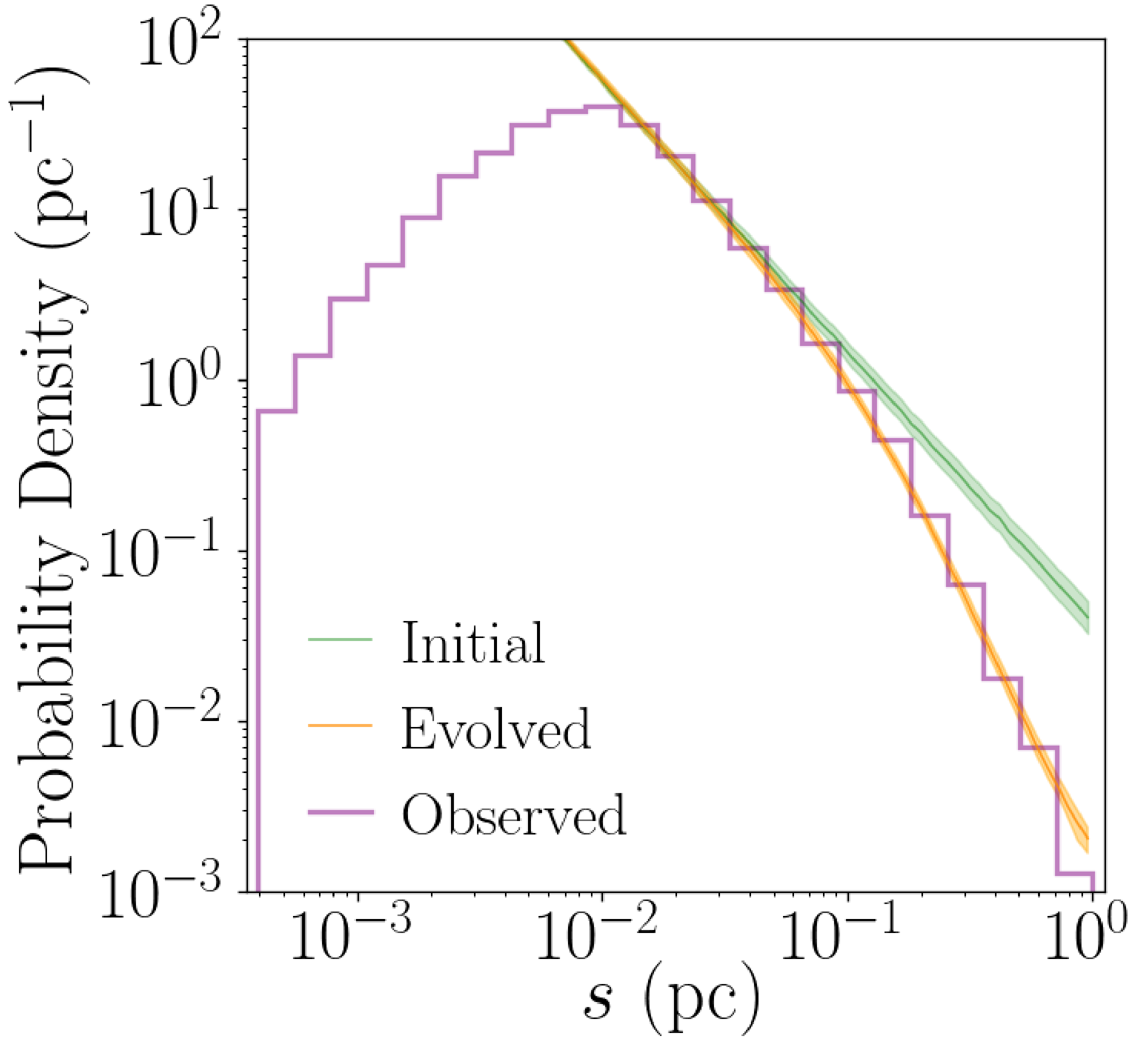
Ramirez et al. [2209.08100]

Setting Limits

Ramirez et al. [2209.08100]

Setting Limits

- From data and prediction,
  - Likelihood Function
  - Posterior Distribution



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Setting Limits

- From data and prediction,
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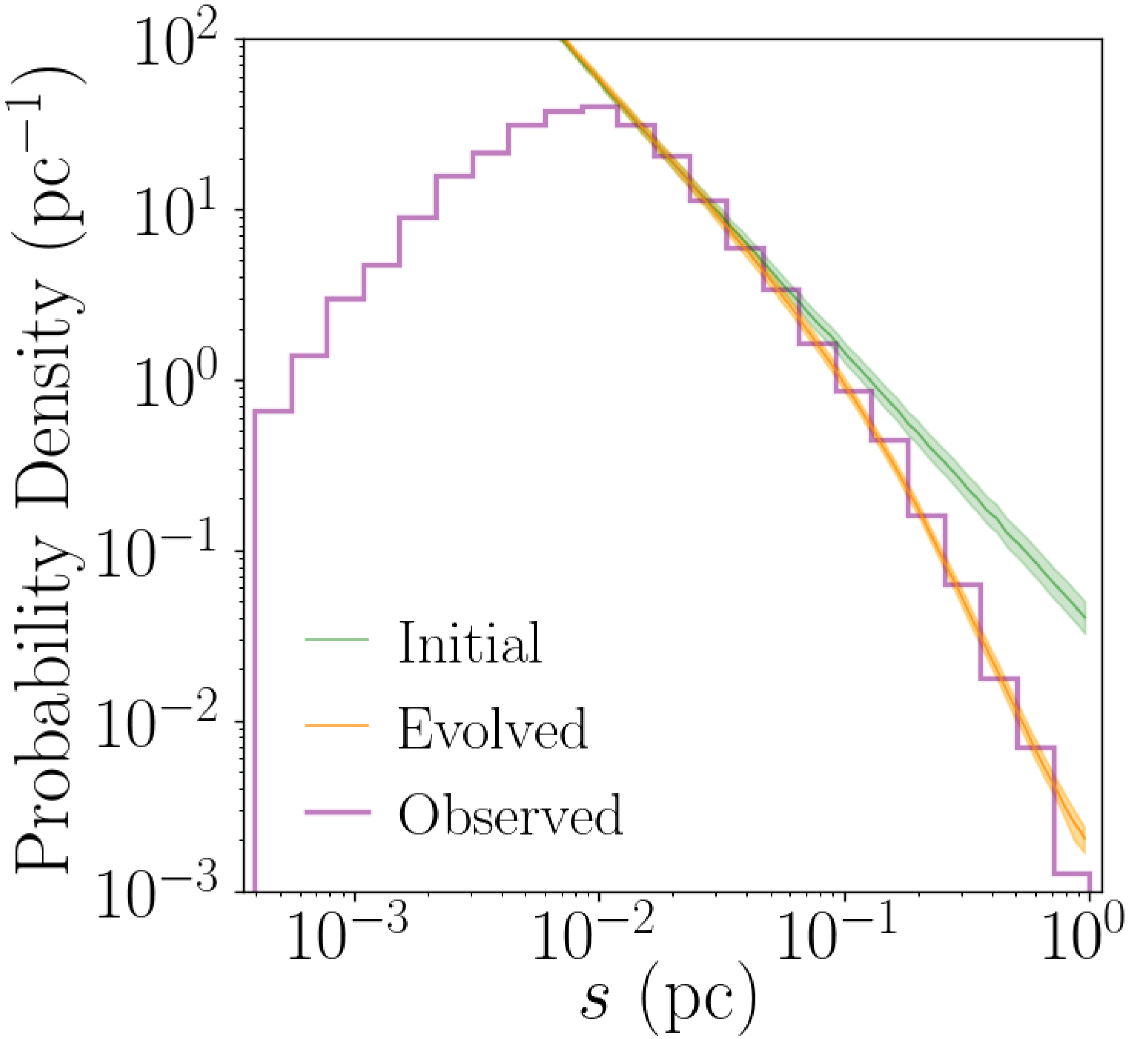
- Limits
  - 95% probability bound on

$$f_p = \frac{\rho_p(R_\odot)}{\rho_{DM}(R_\odot)}$$



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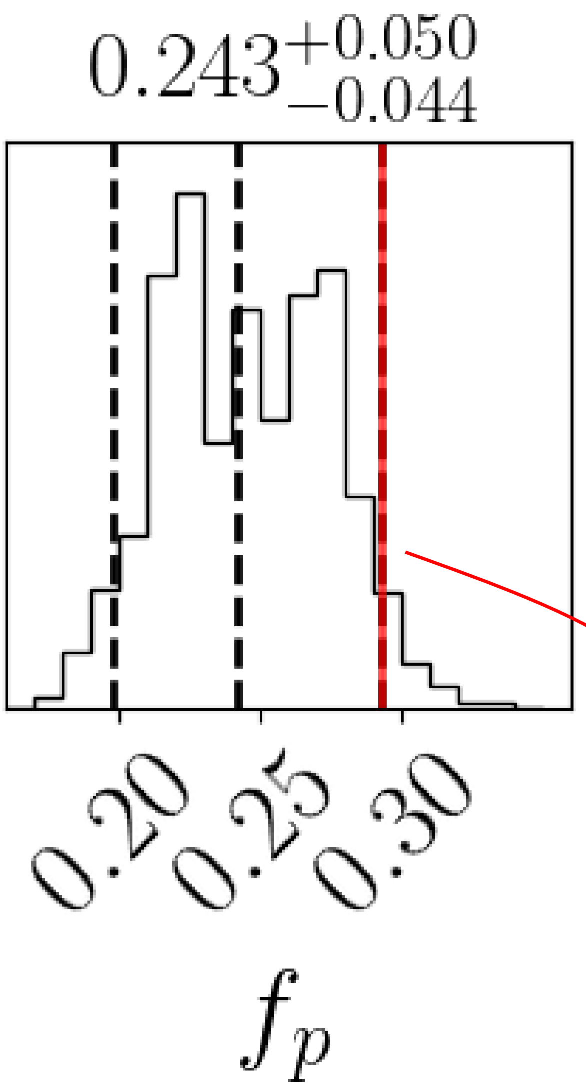
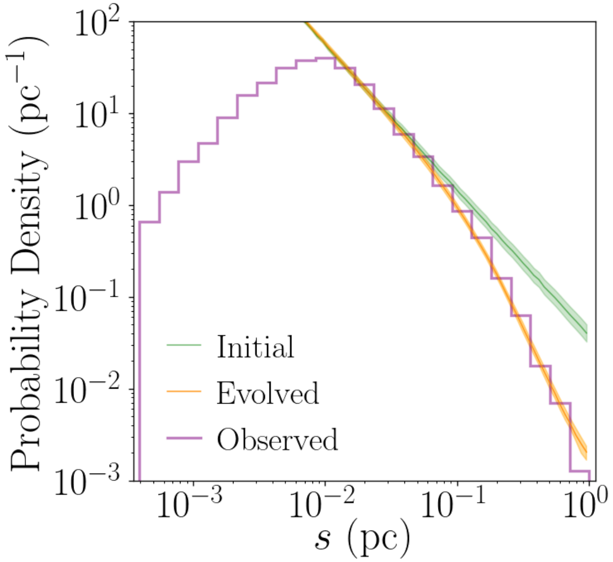




Subhalos:
$M_p = 10^3\ M_\odot$
$R_p = 0.1$ pc
$\rho(r) = \text{constant}$

$$\boxed{f_p < 0.293}$$

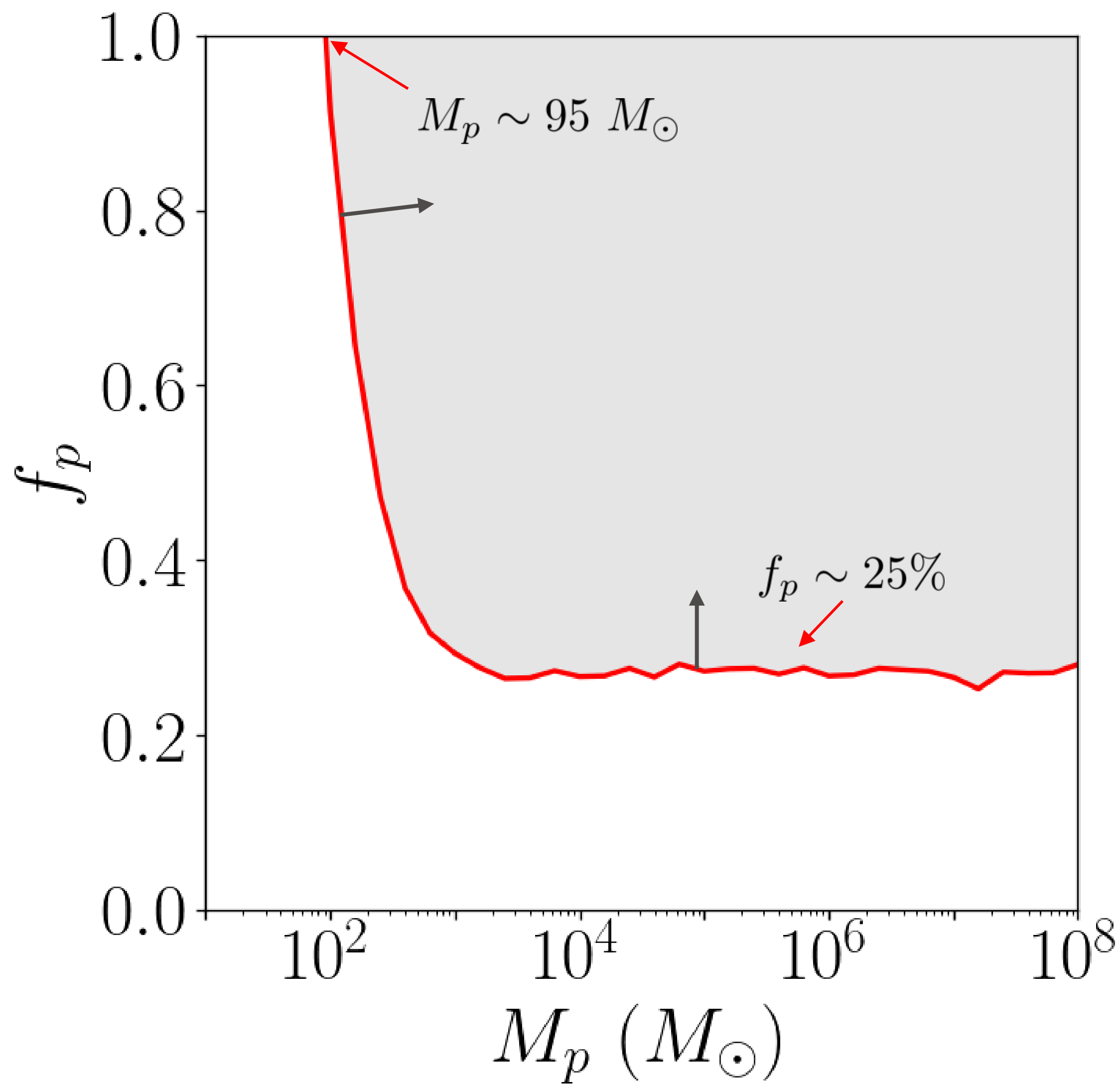
Ramirez et al. [2209.08100]

Limits on Uniform-Density Subhalos

- Perturber Population

$$\begin{cases} M_p = \text{free} \\ R_p = 0.1 \text{ pc} \\ \rho(r) = \text{constant} \end{cases}$$

- Key Points
  - $M_p > 95\ M_\odot$ cannot make up all the dark matter (at 95% level)
  - Can make up at most 25% of dark matter



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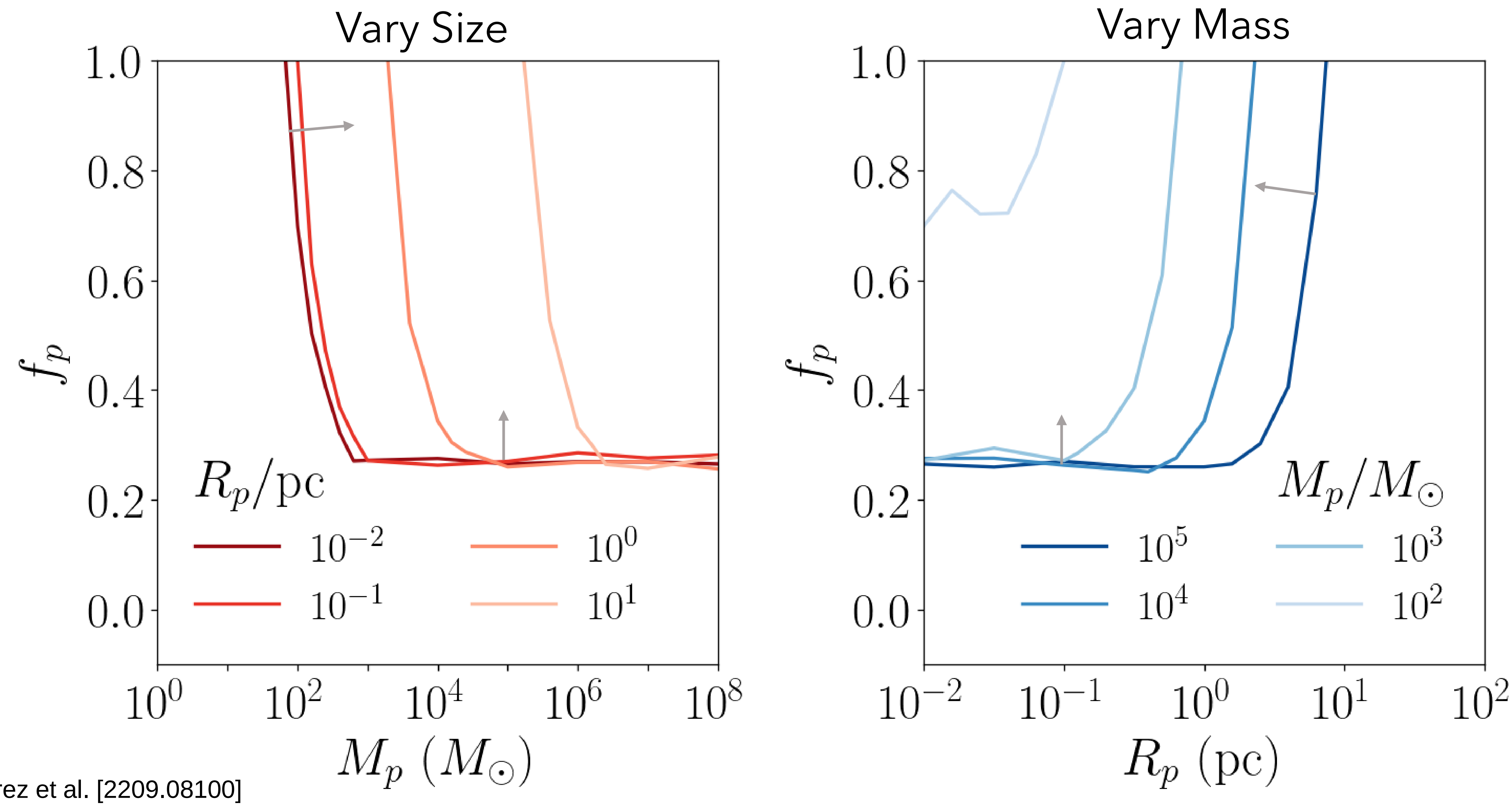

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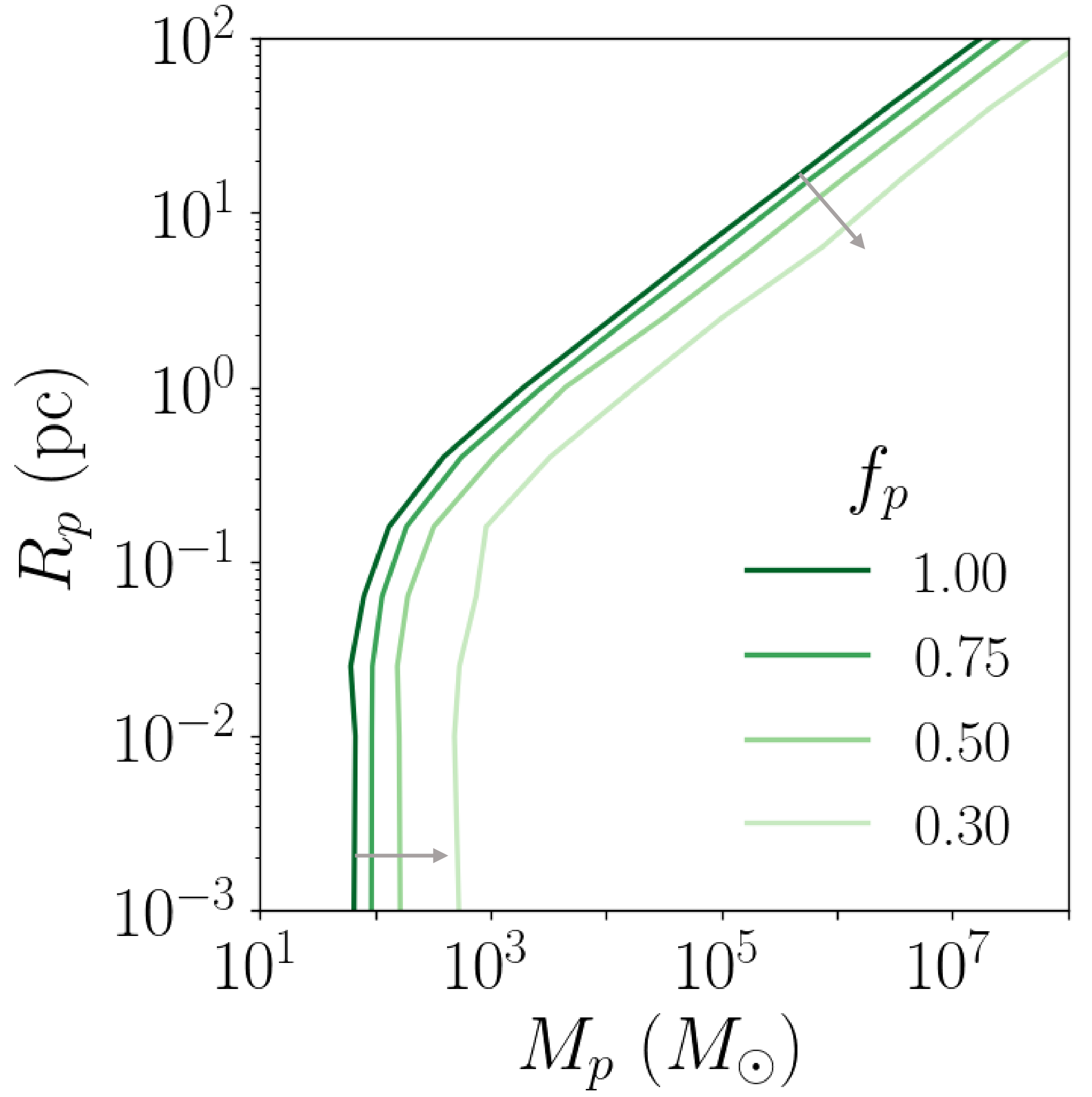

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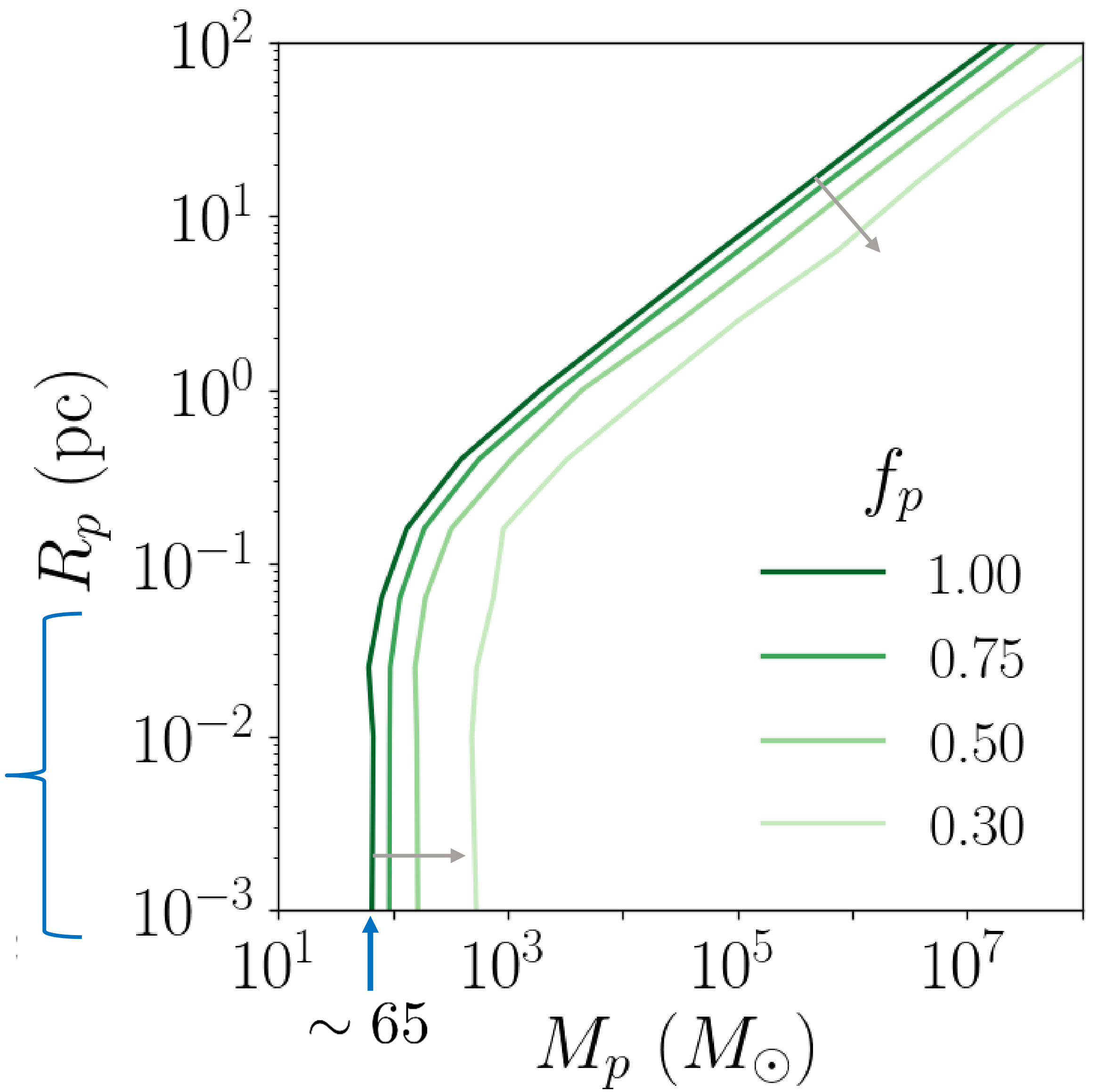

Limits are independent of R_p for $R_p \lesssim 0.1$ pc

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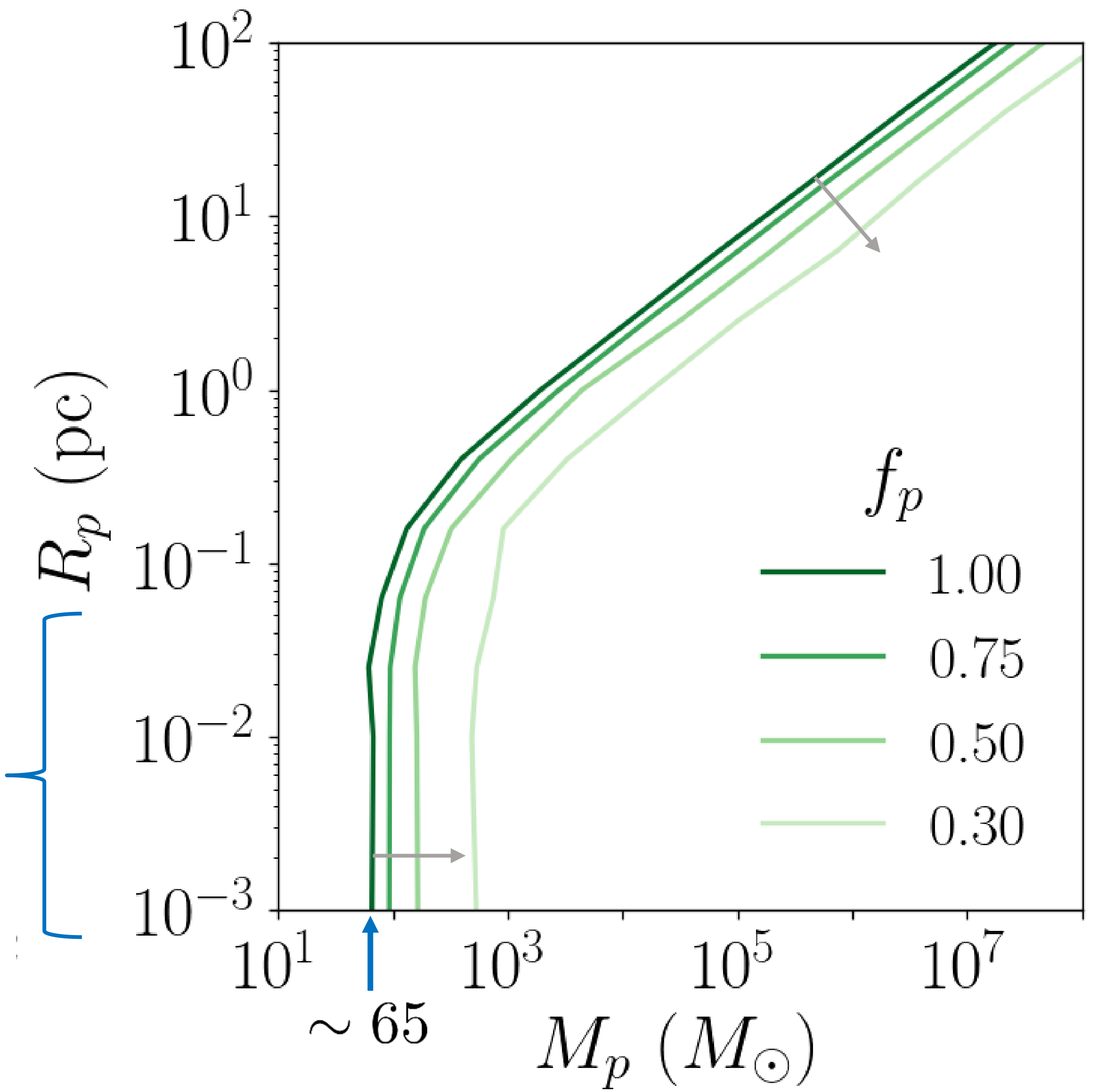
Limits on Uniform-Density Subhalos



Limits are independent of R_p for $R_p \lesssim 0.1$ pc

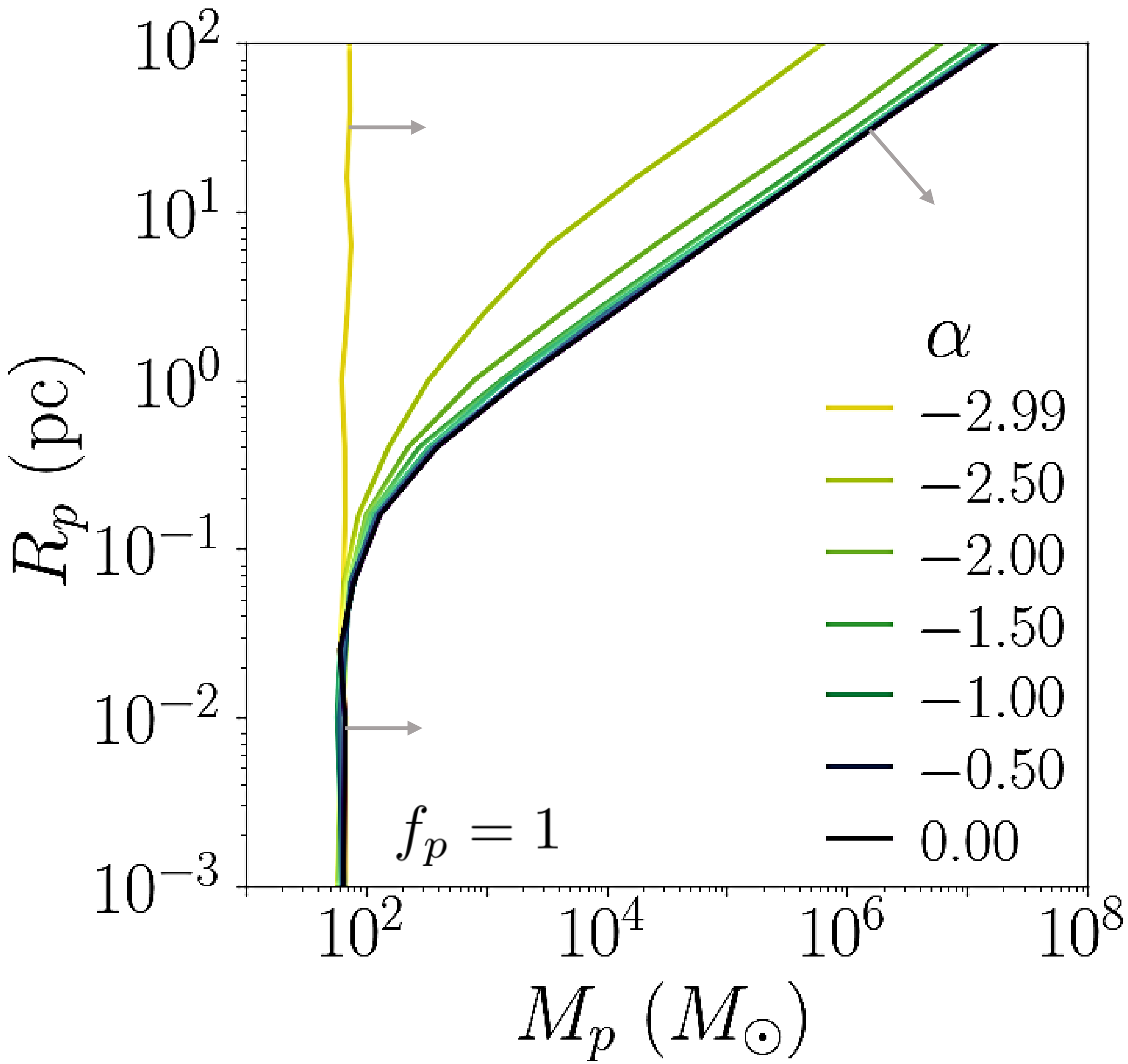
Limits weaken for $R_p \gtrsim 0.1$ pc

Ramirez et al. [2209.08100]

Effects of the Density Profile

- How do limits change with density profile?
  - Consider power-law density profiles:

$$\rho(r;\alpha) = \begin{cases} \rho_0 \left(\frac{r}{R_p}\right)^{\alpha} & , r \leq R_p \\ 0 & , r > R_p \end{cases}$$



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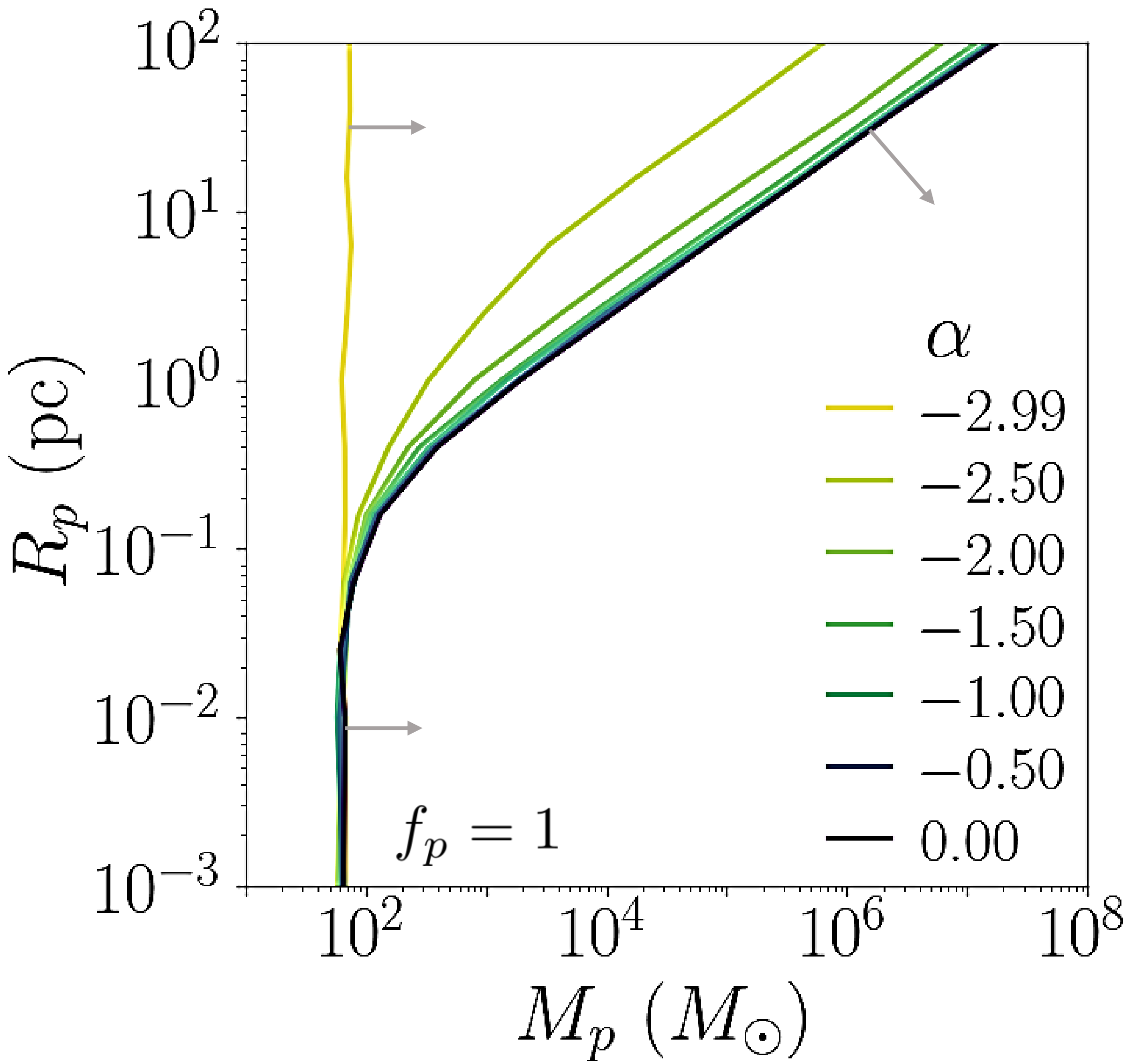
Higher central densities lead to stronger constraints



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Limits on Gravity-only DM Subhalos

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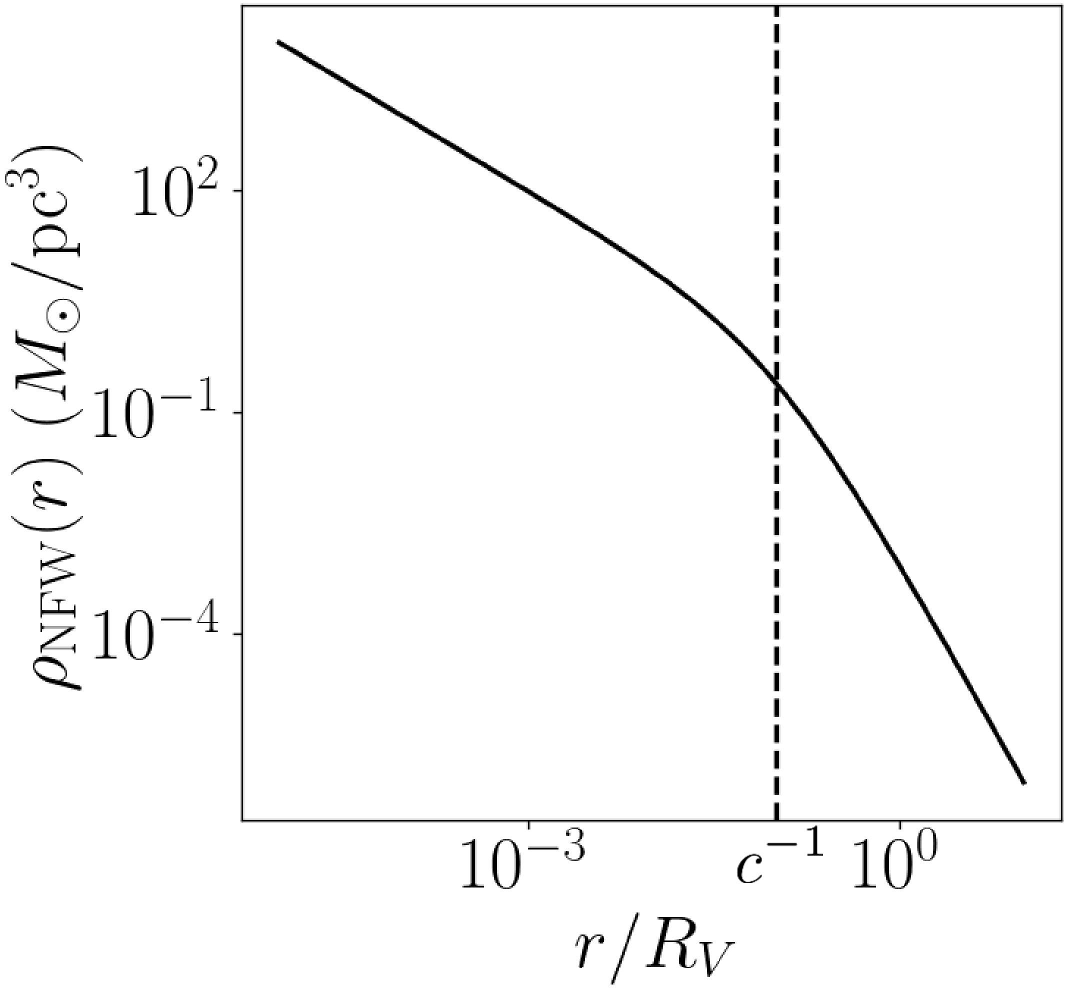
- NFW density profile

$$\rho_{\rm NFW}(r) = c^{-3}\rho_0 \left(\frac{r}{R_V}\right)^{-1} \left(c^{-1} + \frac{r}{R_V}\right)^{-2}$$

  - Free parameters: (c, M_V, R_V)



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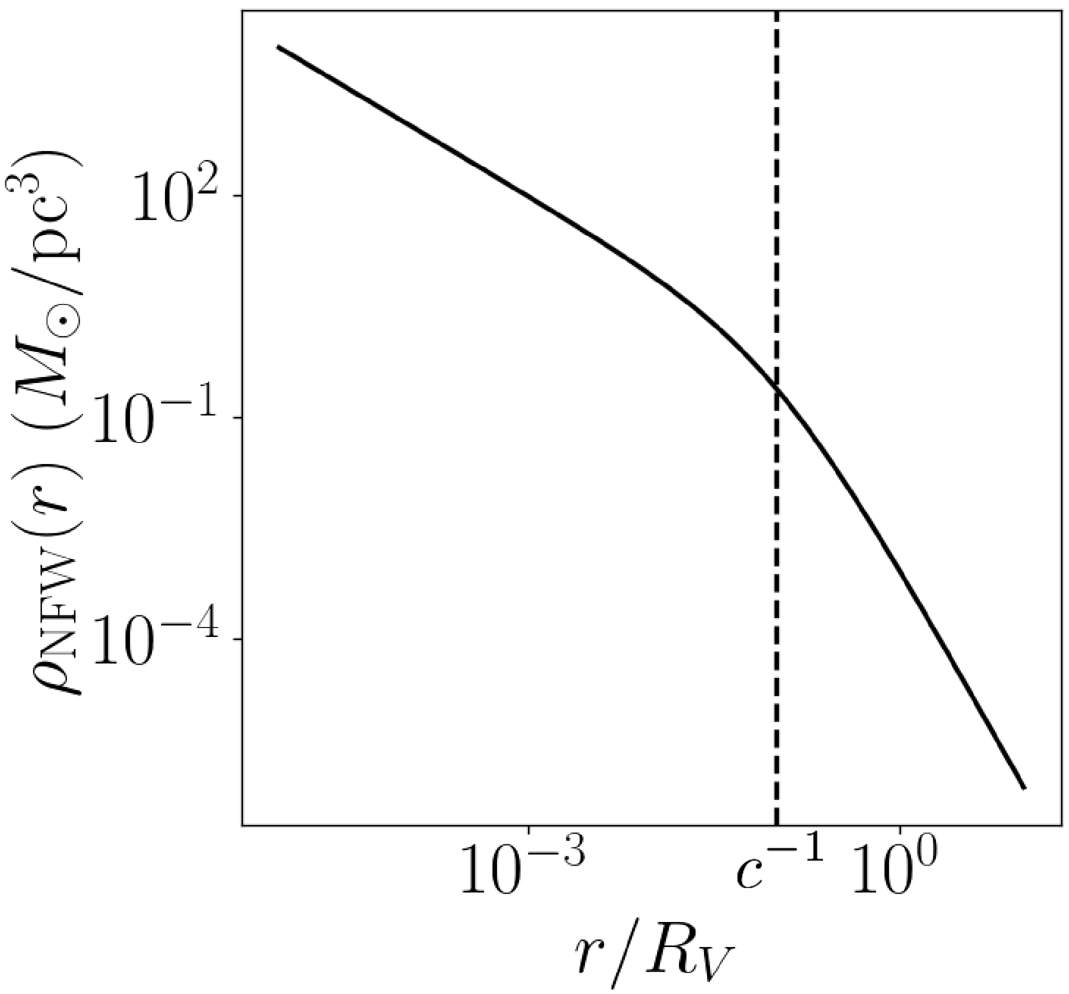
  - Free parameters: (c, M_V, R_V)
  - Two relations

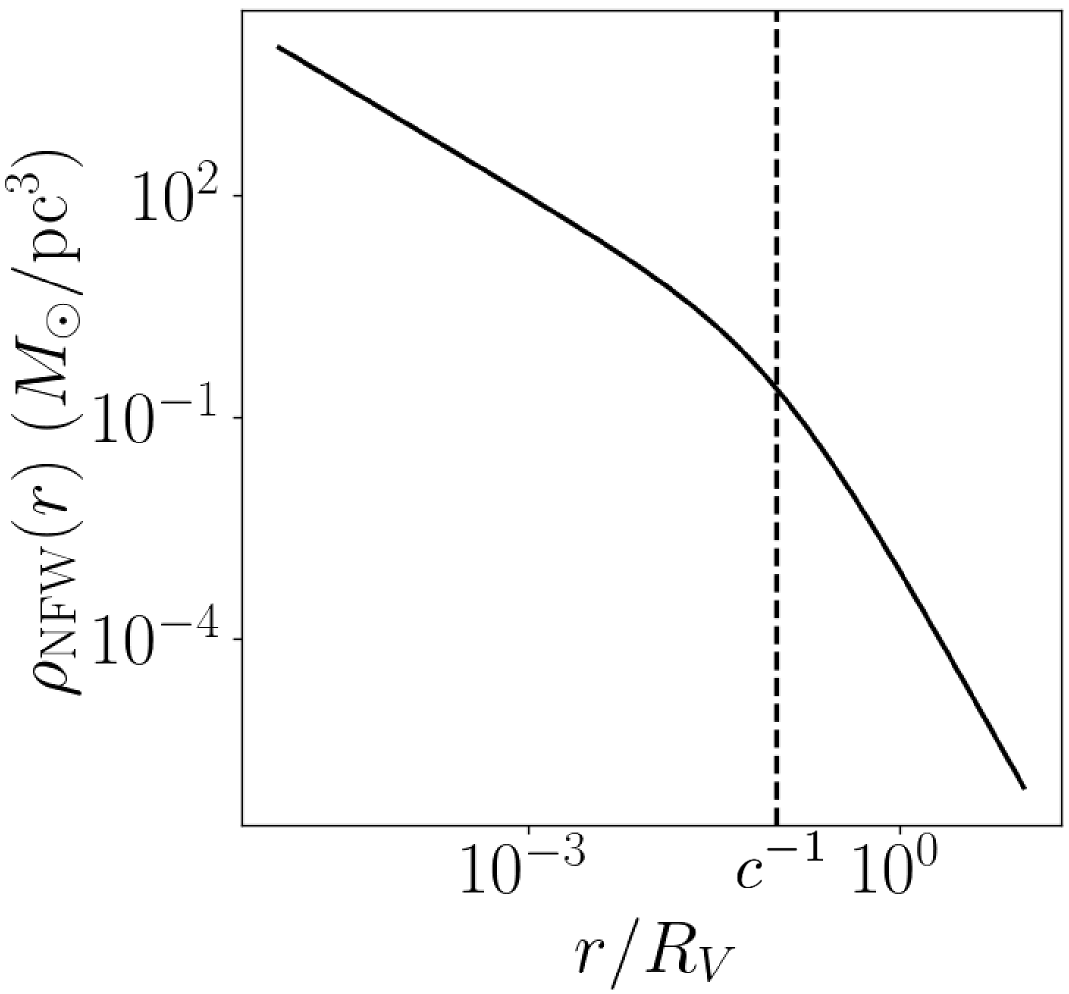


Limits on Gravity-only DM Subhalos

- NFW density profile

$$\rho_{\mathrm{NFW}}(r) = c^{-3}\rho_0 \left(\frac{r}{R_V}\right)^{-1}\left(c^{-1} + \frac{r}{R_V}\right)^{-2}$$

  - Free parameters: (c, M_V, R_V)
  - Two relations
- (c, M_V) Relation:



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Diemand et al. [0805.1244]

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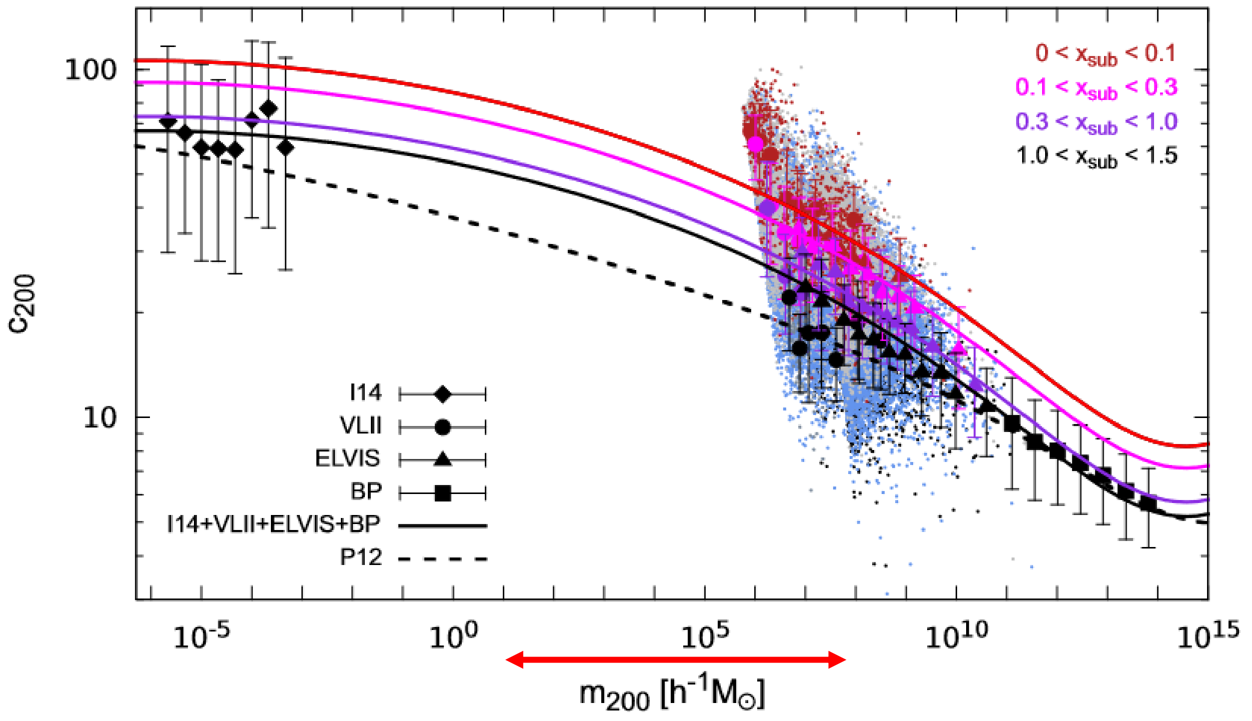
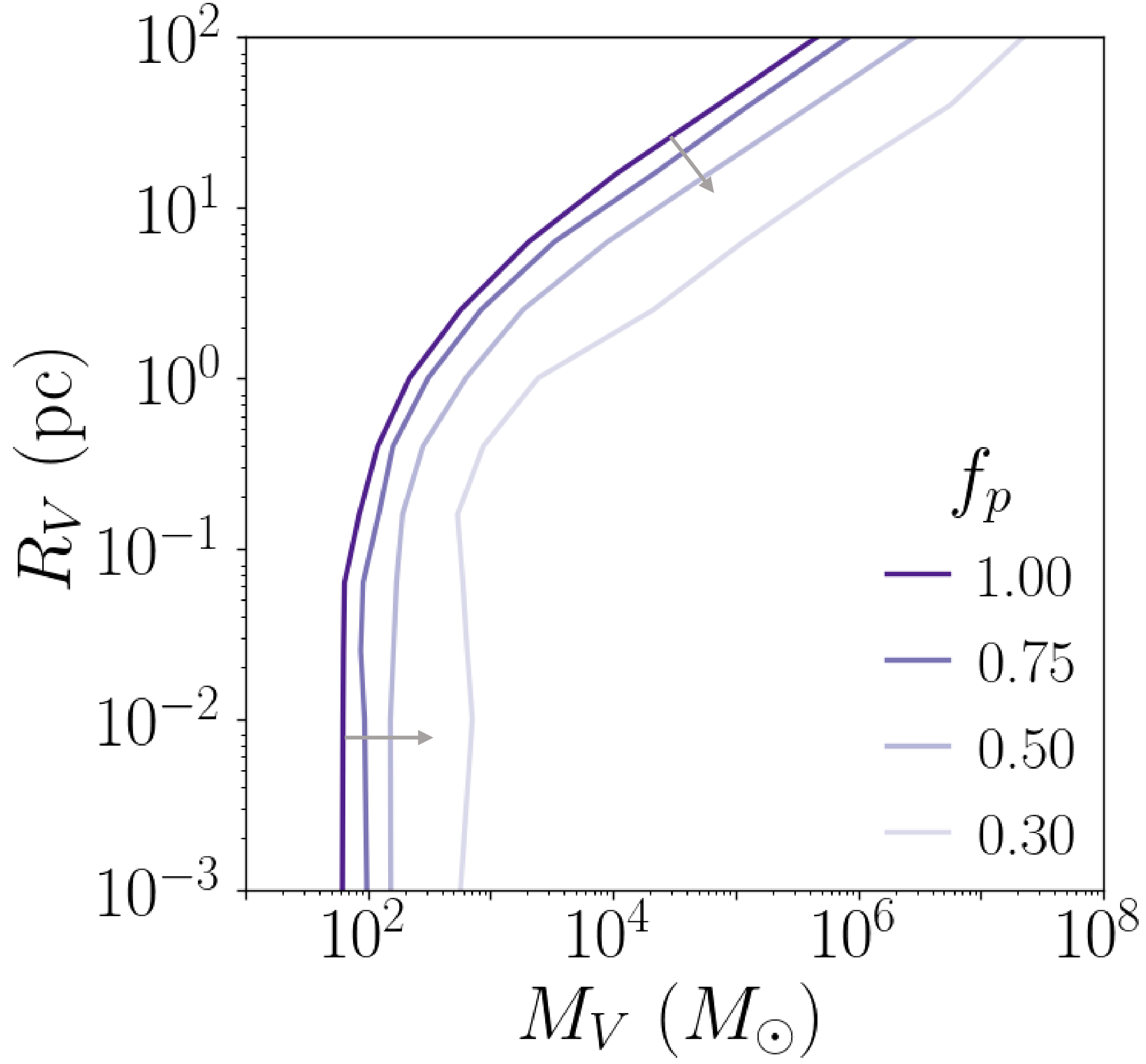


Moliné et al. [1603.04057]

Limits on Gravity-only DM Subhalos



Ramirez et al. [2209.08100]

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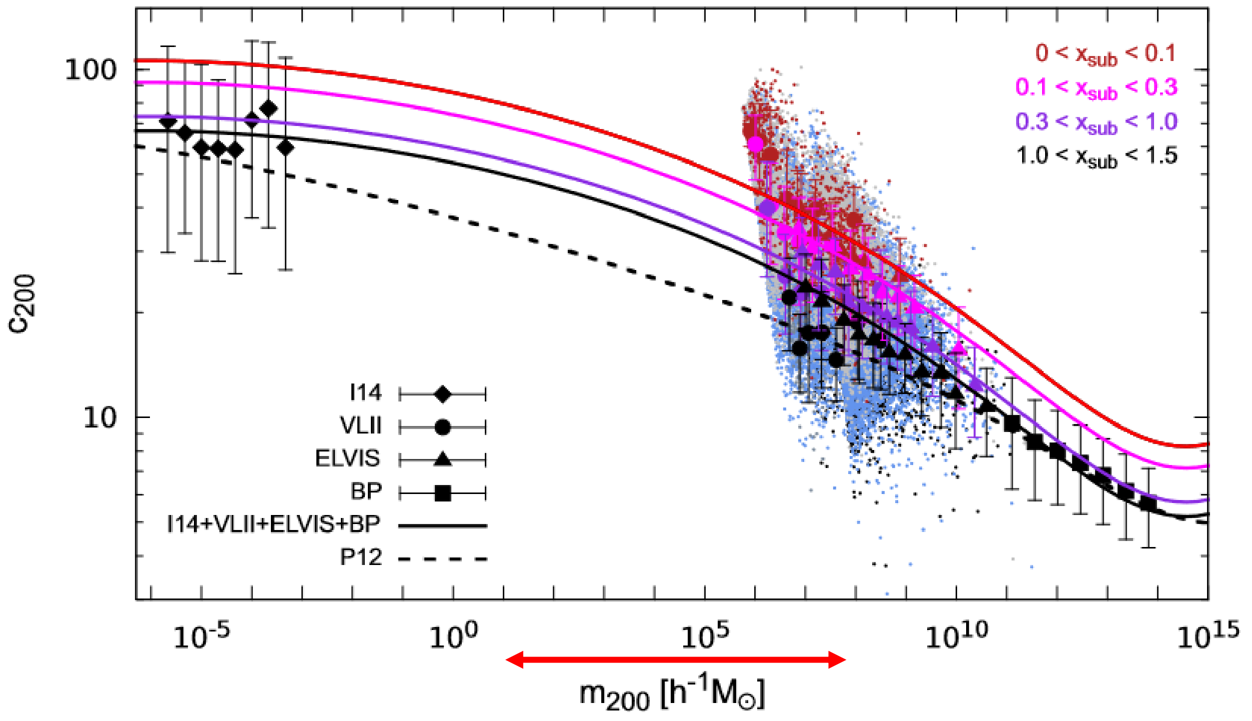
- (c, M_V) Relation:
  - VL-2 / ELVIS / BolshoiP simulations
  - $c \sim 100$
- Canonical NFW mass

$$M_V^* = \left(\frac{4\pi R_V^3}{3}\right)\rho_c \Delta$$



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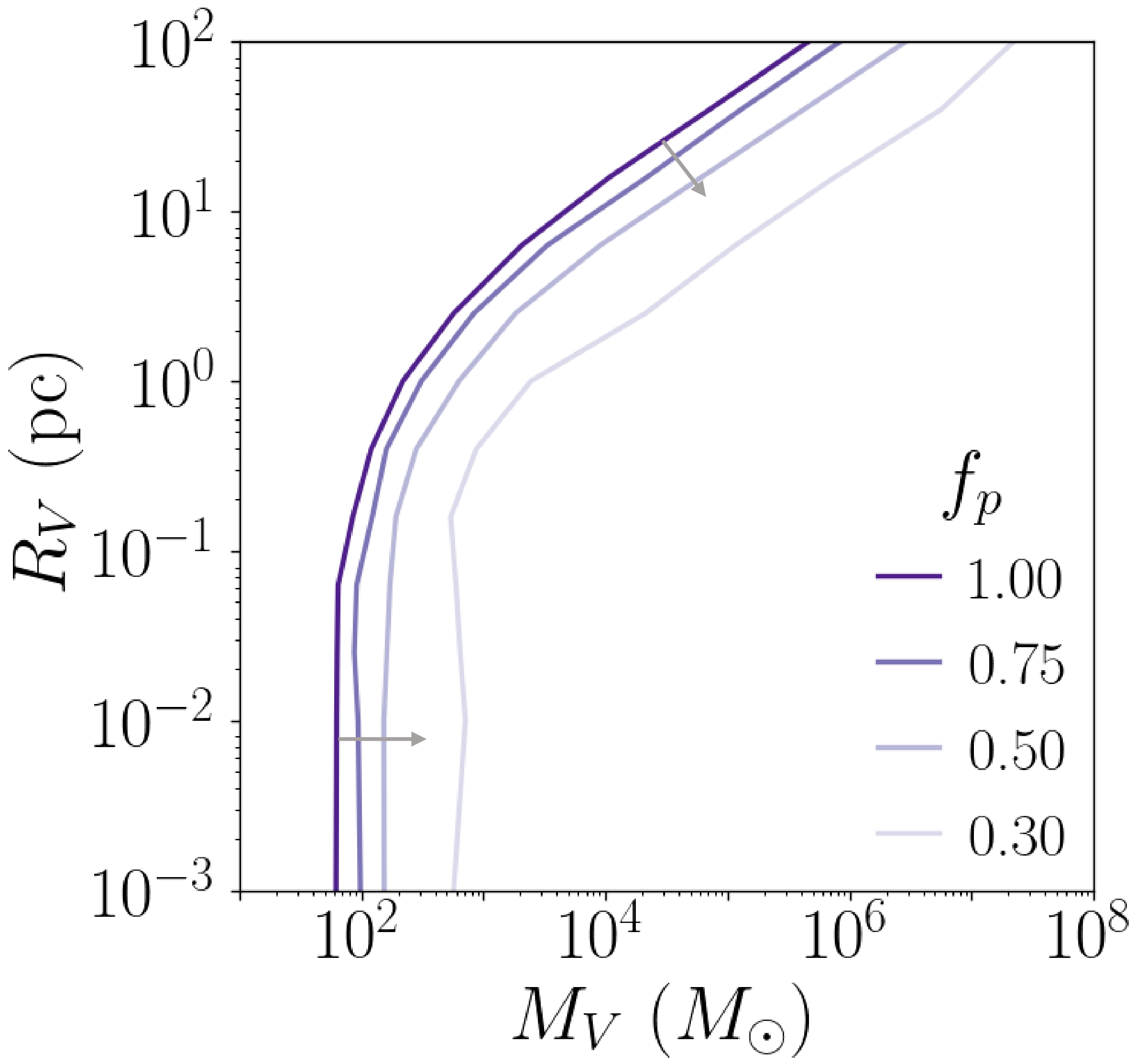

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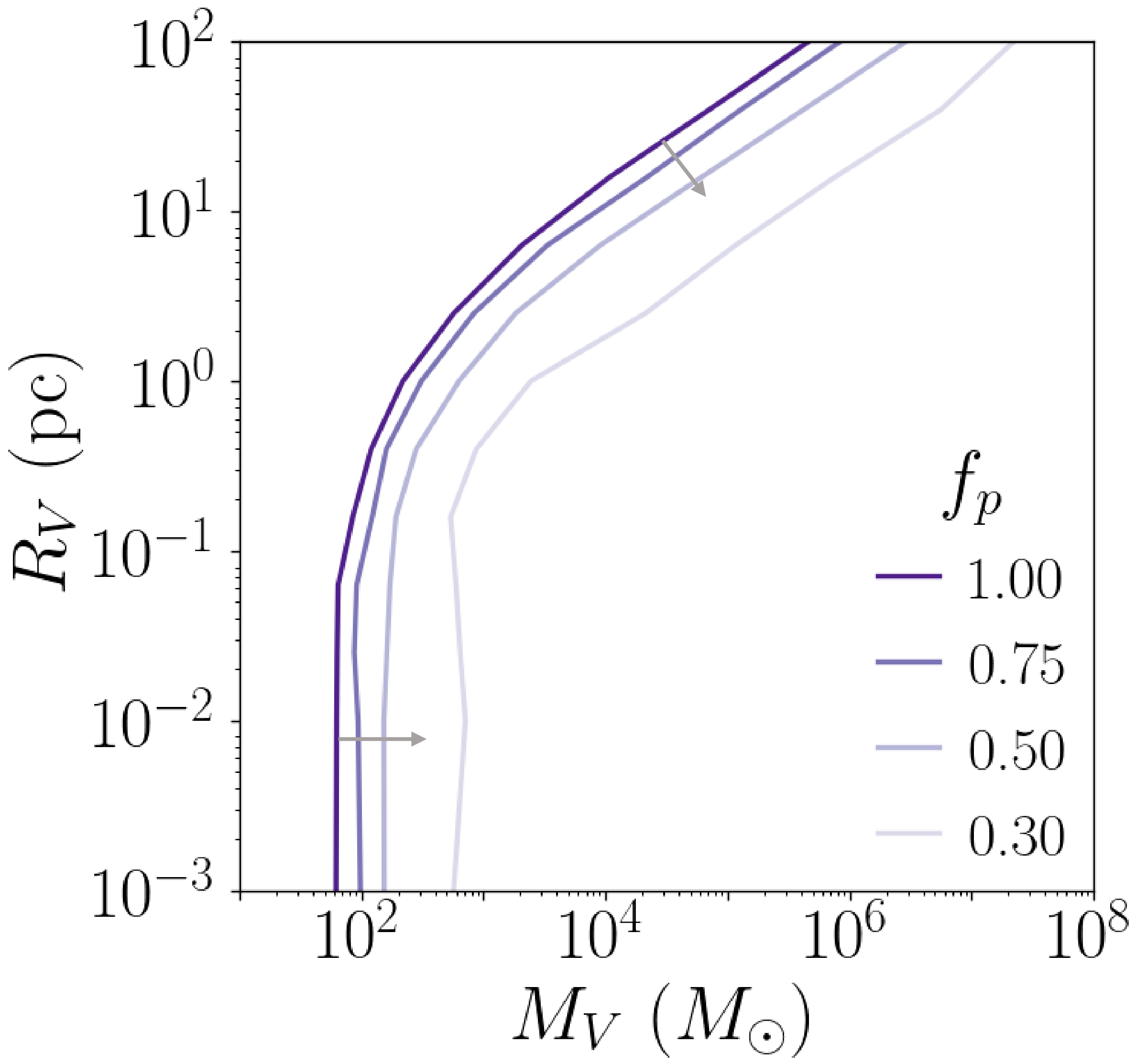

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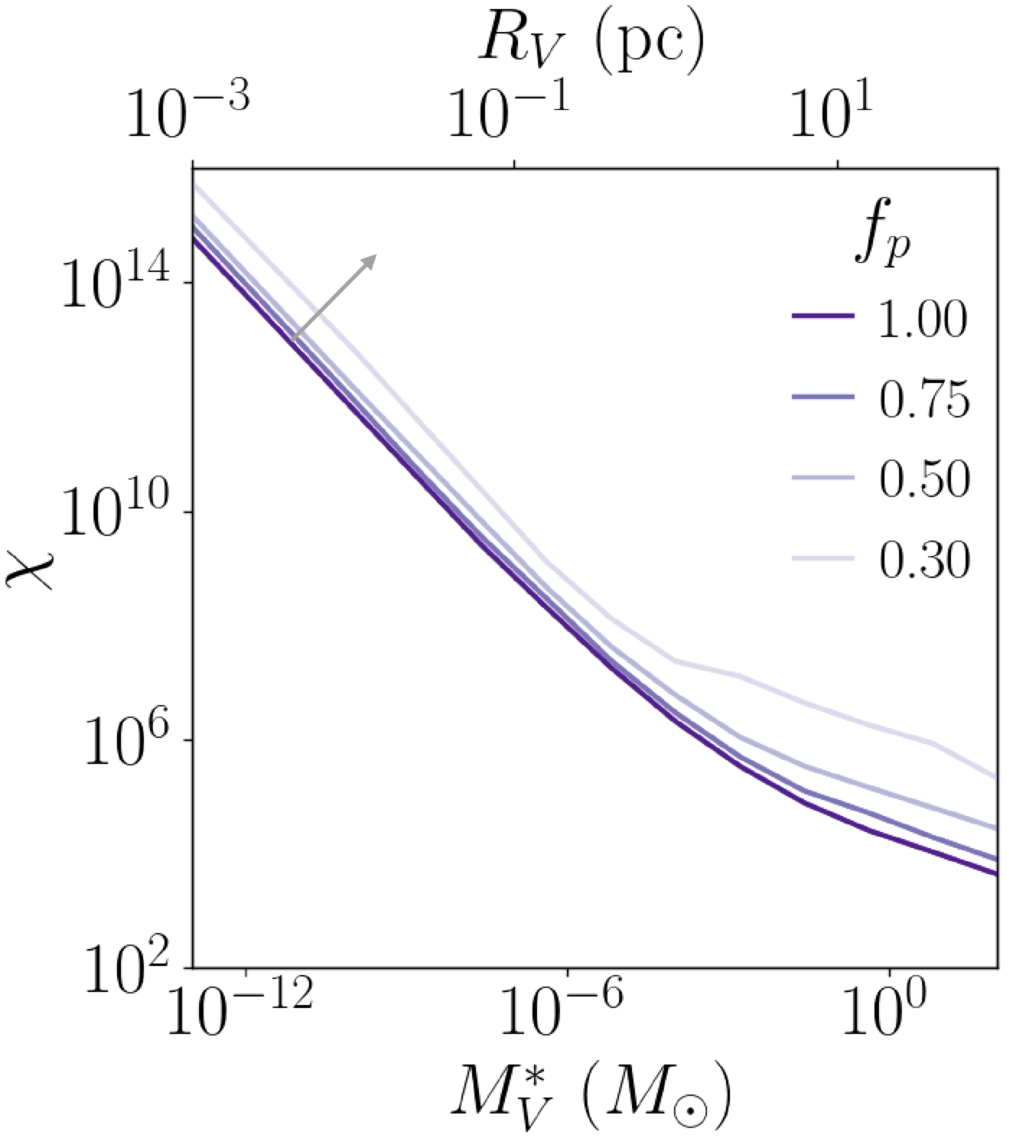

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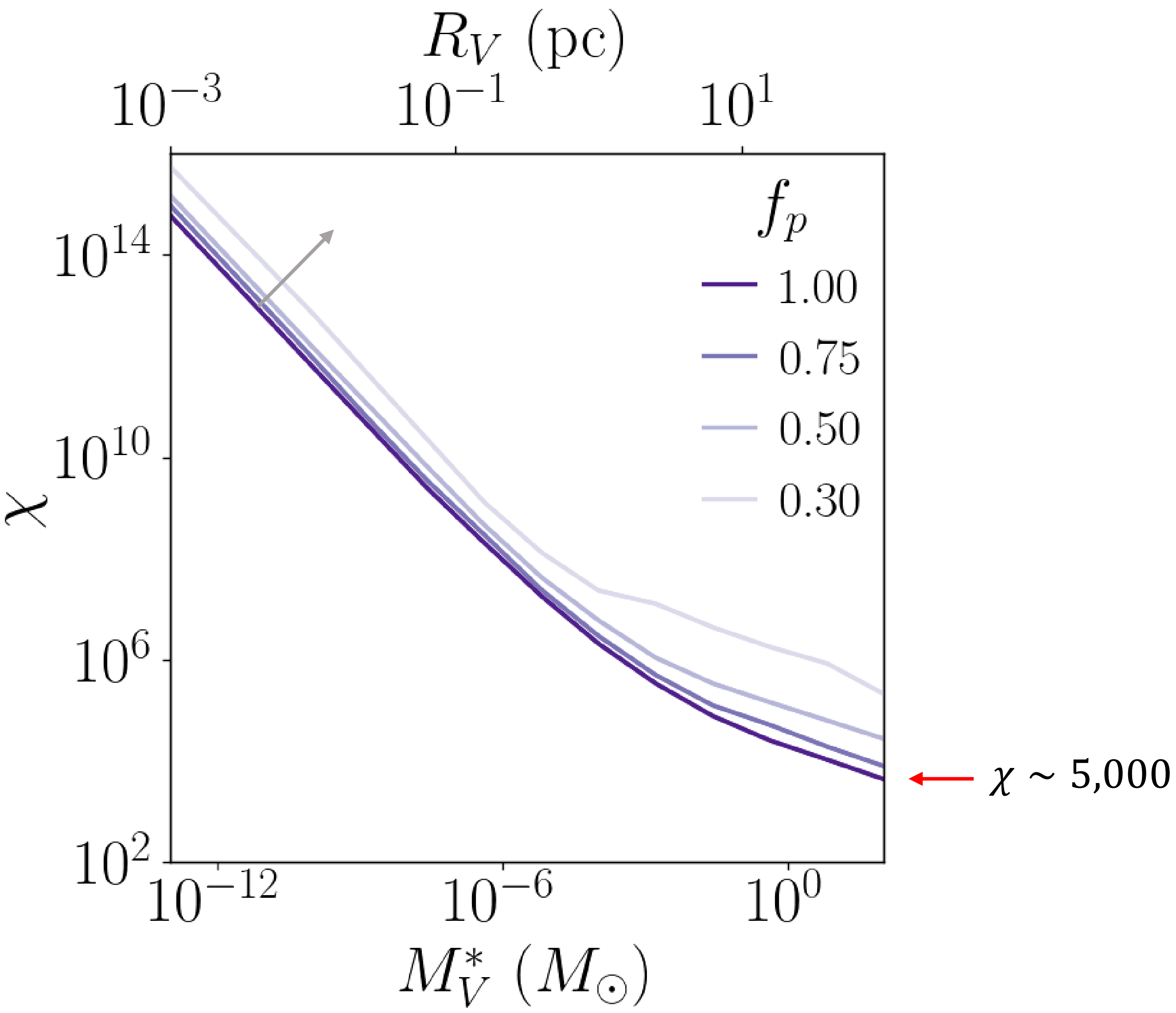


Ramirez et al. [2209.08100]

Conclusions

- Wide binaries can set limits on a wide variety of subhalos
- General results:
  - Subhalos smaller than 0.1 pc cannot make up 100% of the local dark matter density if $M_p \gtrsim 65\ M_\odot$
  - Limits on subhalos larger than 0.1 pc depend on their density profiles
  - Higher central densities lead to stronger constraints
- NFW result:
  - NFW subhalos must be at least ~5,000 more massive than predicted by gravity-only dark matter simulations to be constrained by binaries
- First limits on O(1 pc) halos

Email: edwardramirez@physics.rutgers.edu

Backup Slides

Why Substructure?

Connections Between Microphysics and Structure

- Dark matter particle physics affects
  1. Halo abundance
  2. Halo density profiles

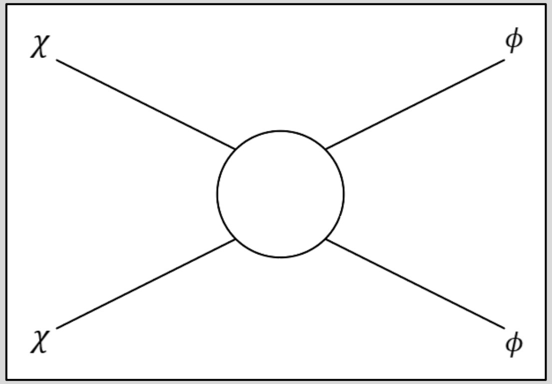
- Example (Abundance):
  - Warm dark matter:

    Same as cold dark matter, but has high thermal velocities

➡ Removes fluctuations at length scales smaller than

$$\lambda_{fs} \sim \sigma t$$

- Example (Density Profiles):
  - Self-interacting dark matter:



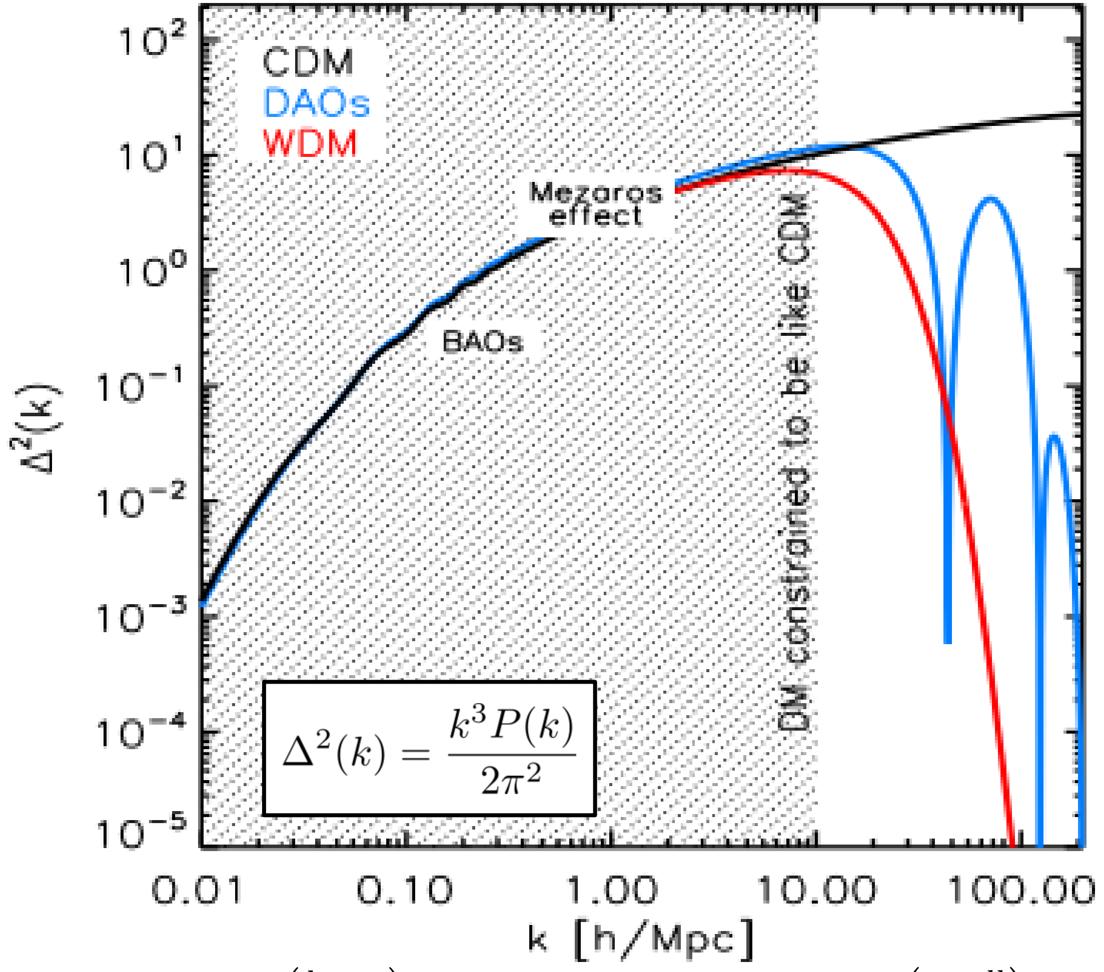
➡ Dark matter interacts more frequently in higher-density regions

$$\Gamma = \int d^3x \, \frac{\rho(\vec{x})^2}{2m_\chi^2} \langle \sigma_T v \rangle$$

➡ Dark matter may diffuse out of higher-density regions

The Effects of Dark Matter Microphysics on Structure Formation

Abundance

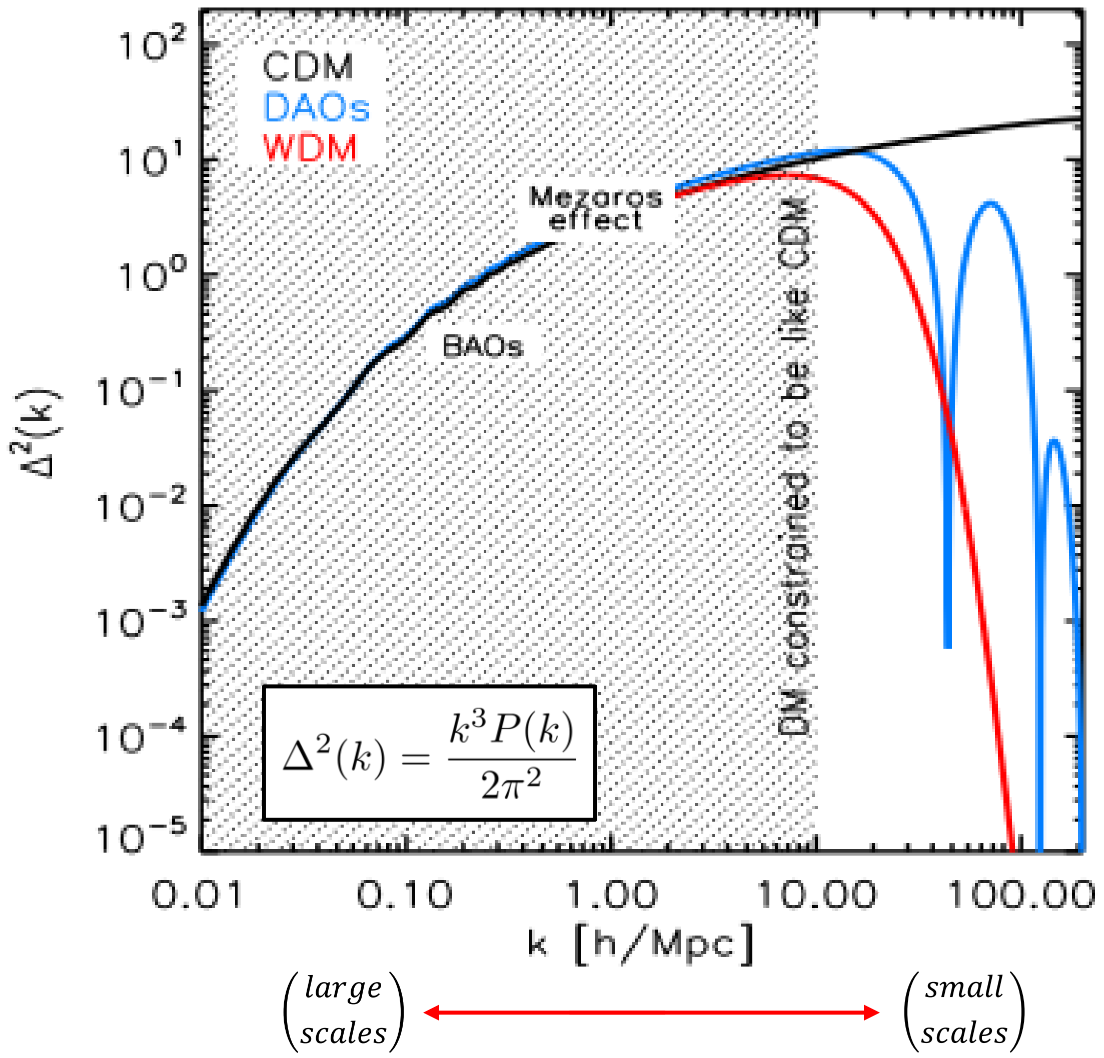


$\begin{pmatrix} large \\ scales \end{pmatrix} \longleftrightarrow \begin{pmatrix} small \\ scales \end{pmatrix}$

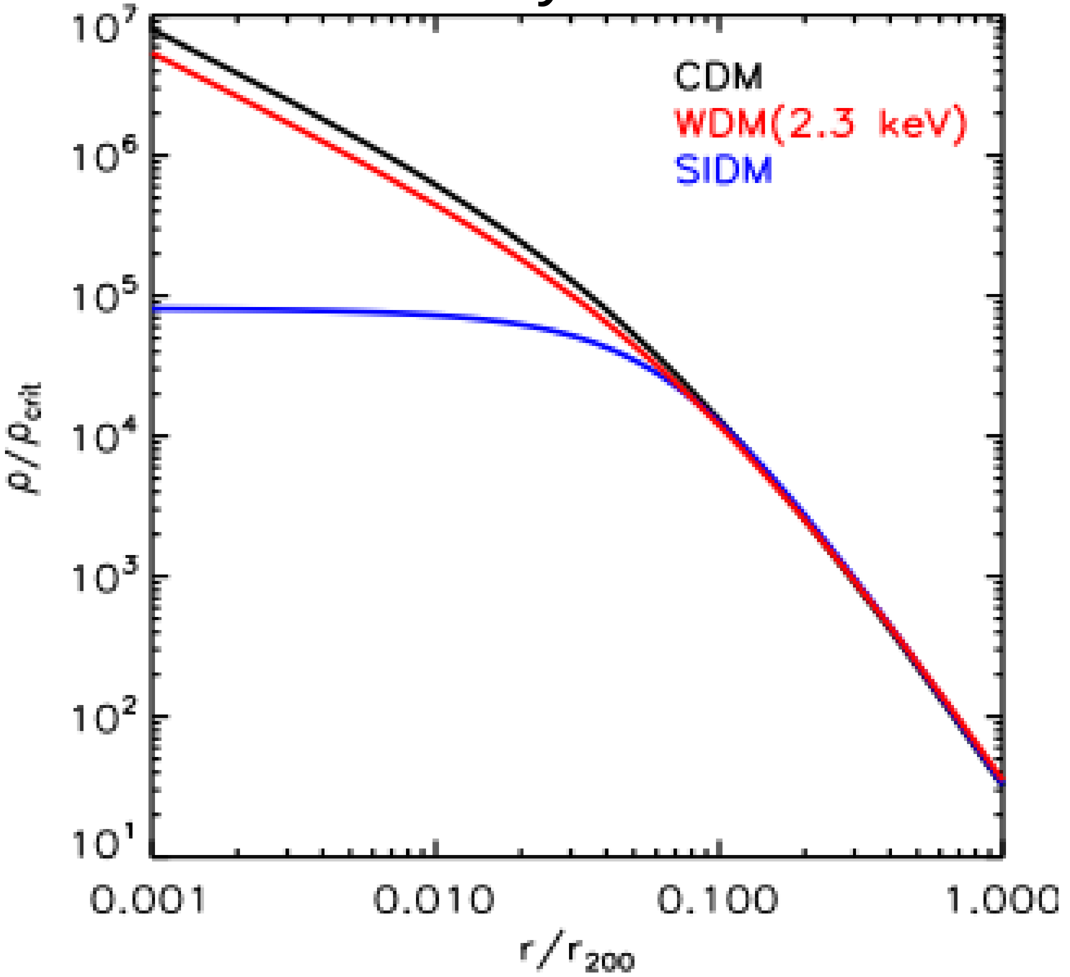
Zavala et al. [1907.11775]

The Effects of Dark Matter Microphysics on Structure Formation

Abundance



Density Profiles



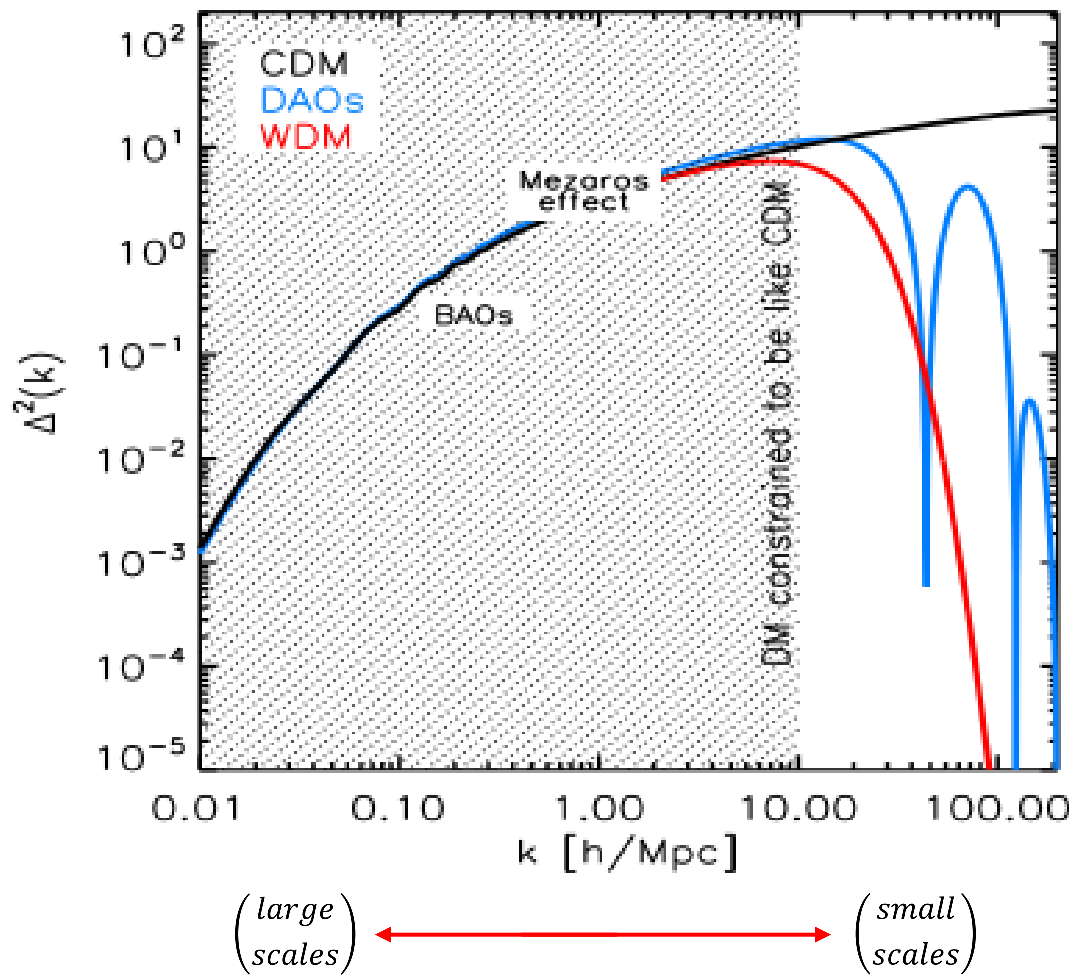
Zavala et al. [1907.11775]

Why Analyze Dark Matter Structures?

- Pros:
  - Model-independent probes of dark matter
  - Connected to cosmology and galaxy evolution
- Cons:
  - Difficult to observe
  - Difficult to model
  - High systematic uncertainty

Why Subhalos?

- Below the scale of dwarf galaxies
  - Not understood
  - Inside Milky Way
  - Abundance
- Small-scale halos may be more sensitive to microphysics
  - Age
  - Density



Zavala et al. [1907.11775]

Data

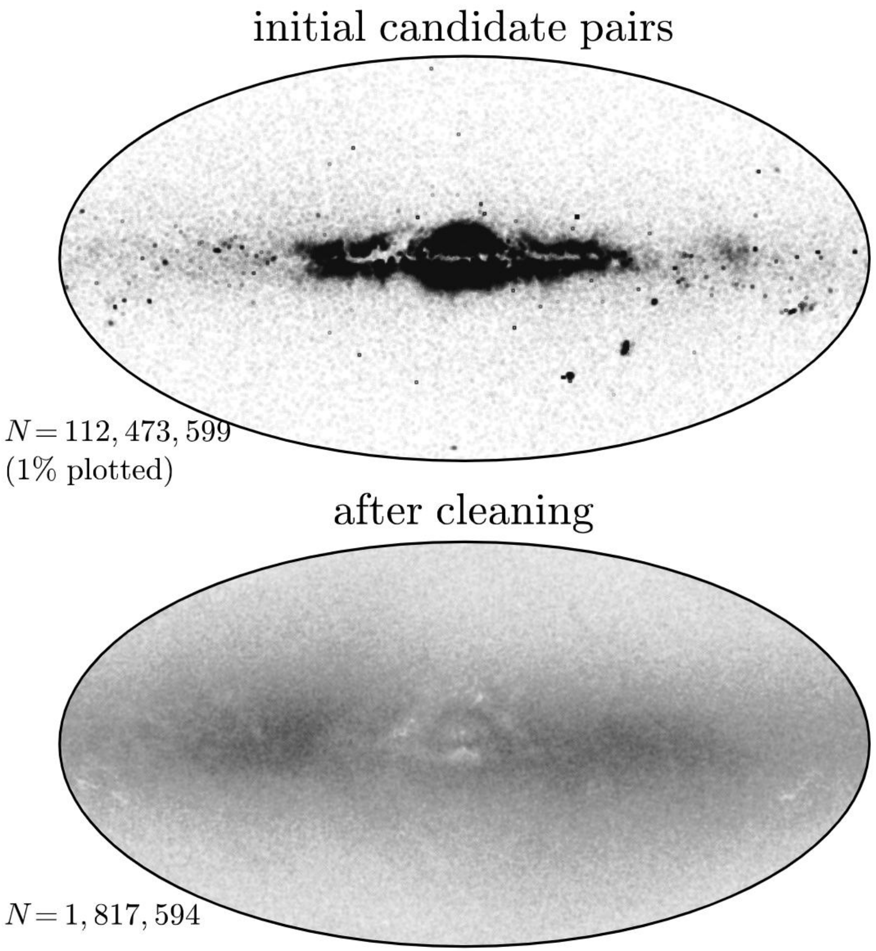
Extracting Binaries from *Gaia* eDR3

Steps to Creating Catalog:

1. Select well-measured stars
   a) High precision
   b) Complete astrometric and photometric measurements

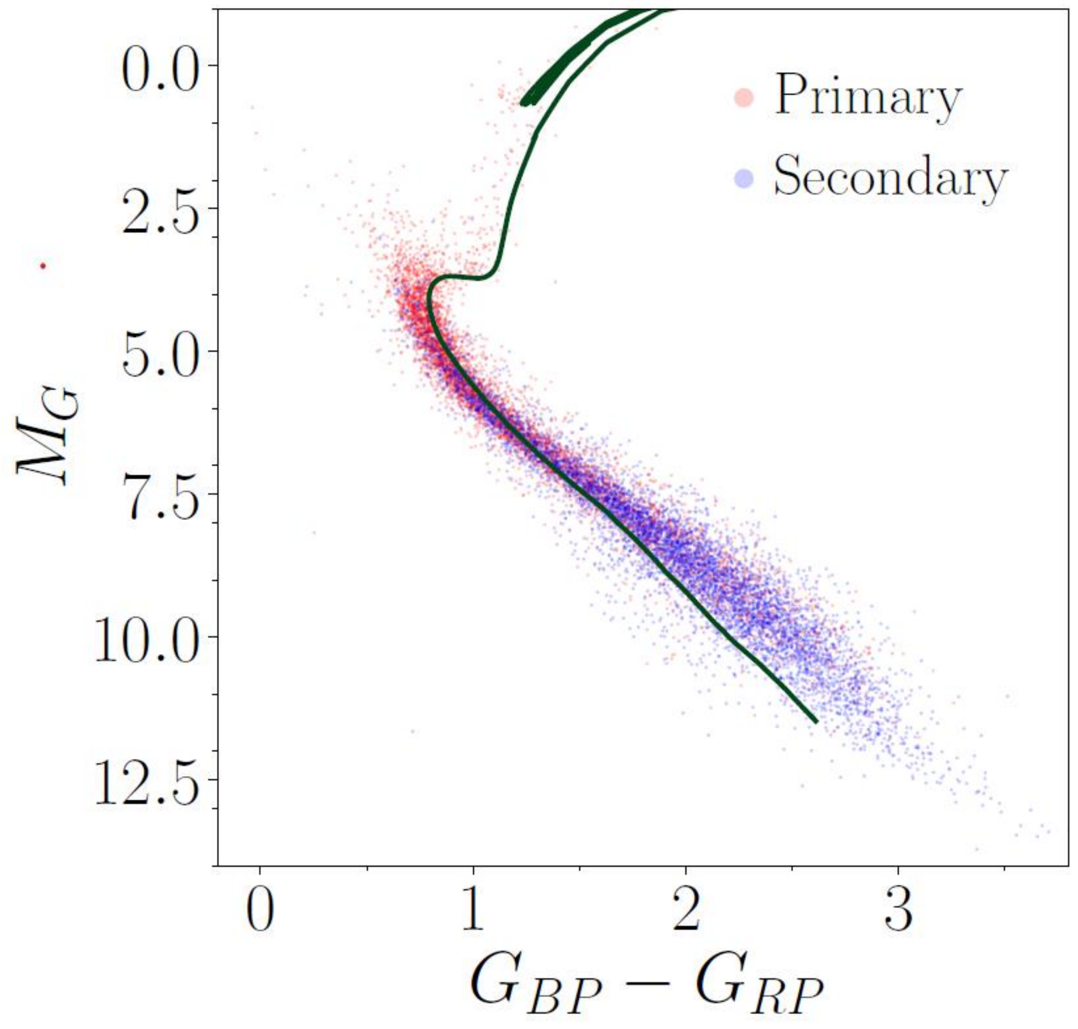
2. Select stellar pairs consistent with Keplerian orbits

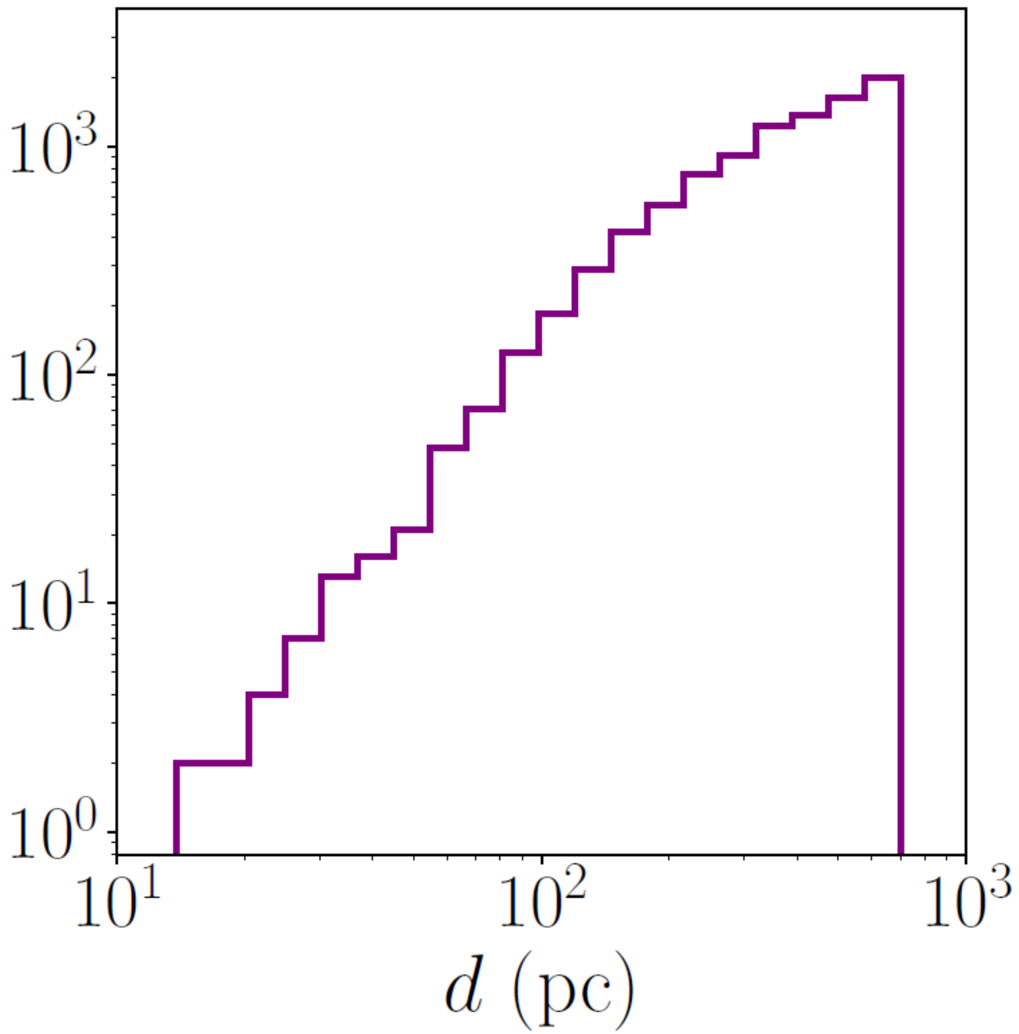
3. Filter out bound systems of three or more stars



El-Badry [2101.05282]

Additional Useful Data





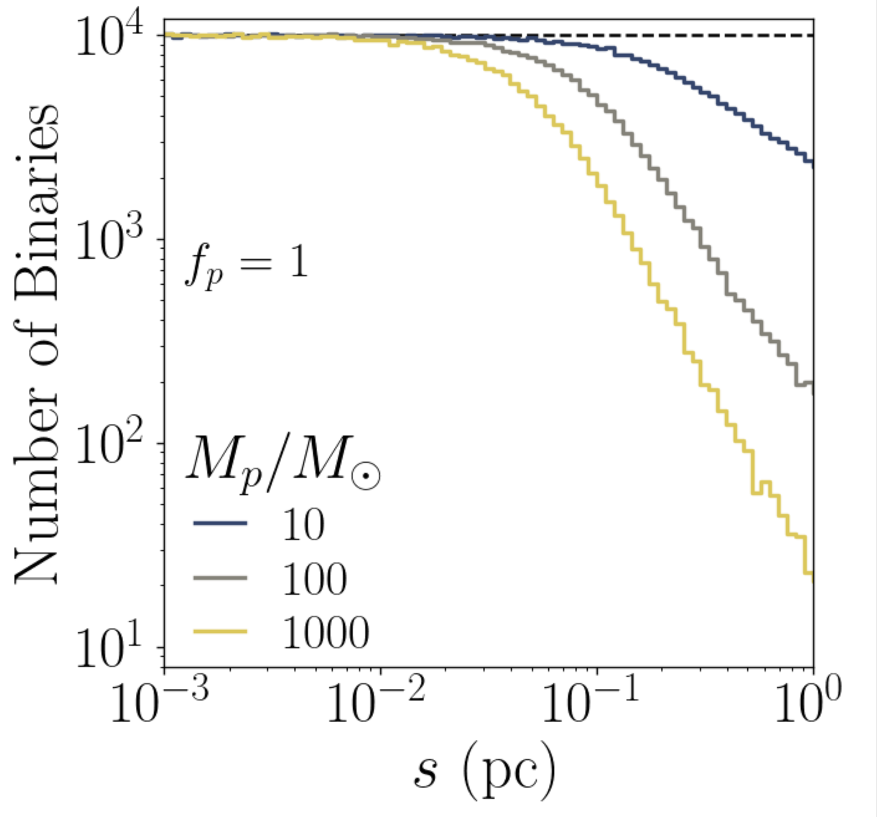
Binary Evolution Model

Binary Evolution Modelling Strategy

- **Goal:**
  - Data-driven model of binary evolution under the influence of subhalos

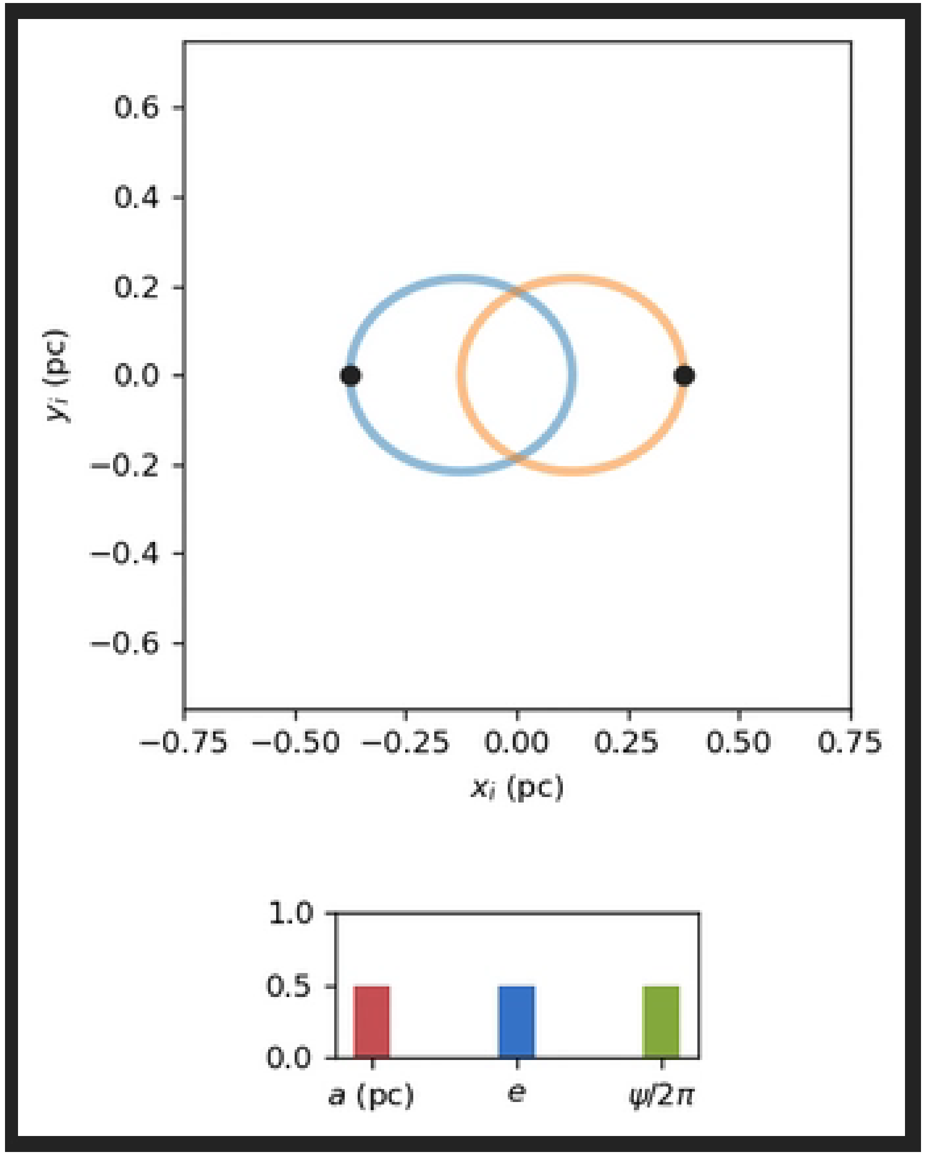
1. Single binary, single perturber
   - ➡ Describe the effect of a passing subhalo on a binary's orbit
2. Single binary, multiple perturbers
   - ➡ Scattering matrix formalism of binary evolution interacting with perturbers
3. Multiple binaries, multiple perturbers
   - ➡ Infer the present-day separation distribution from the scattering matrix

- **Principle Object:**
  - The distribution of projected separations



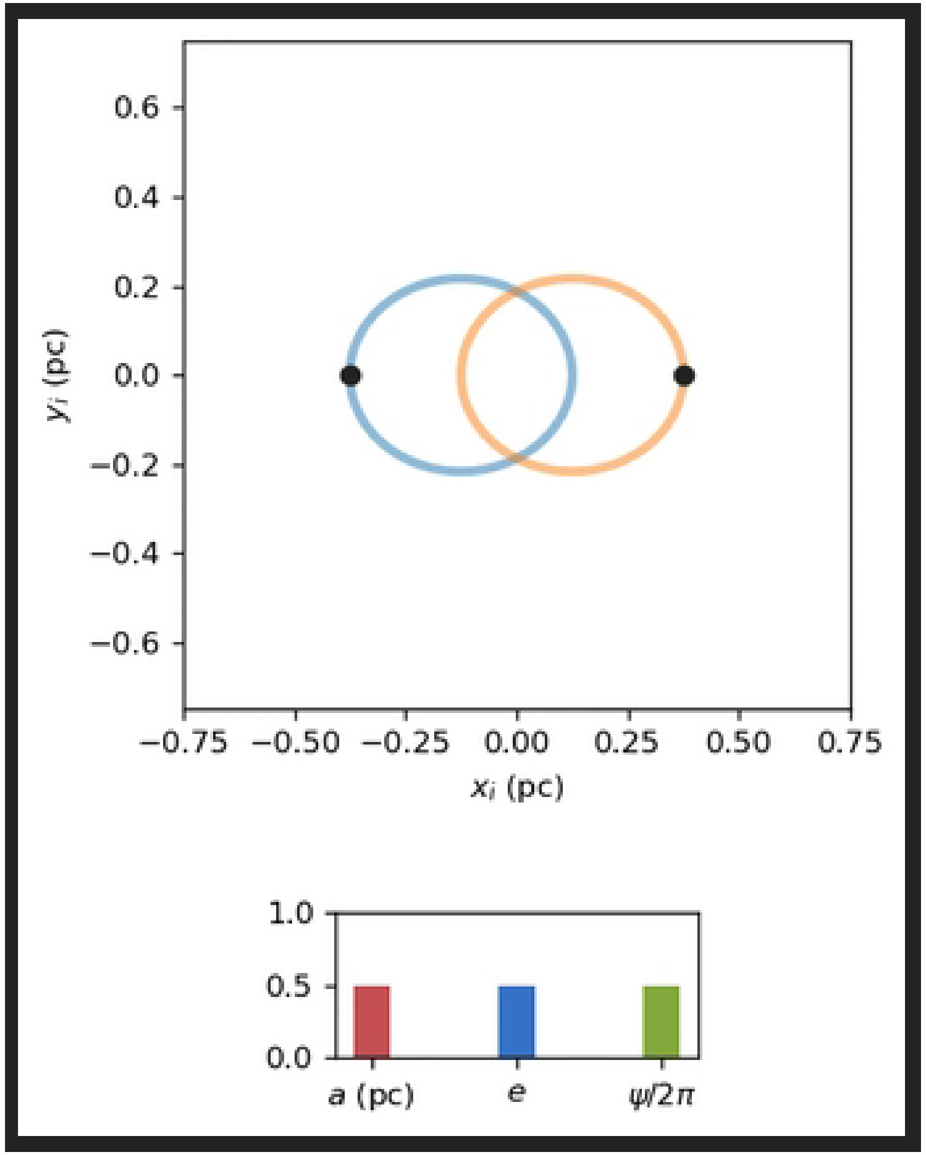
Unperturbed Binary Orbits

- Binary Orbital Parameters



Unperturbed Binary Orbits

- Binary Orbital Parameters
  - a: Semimajor Axis



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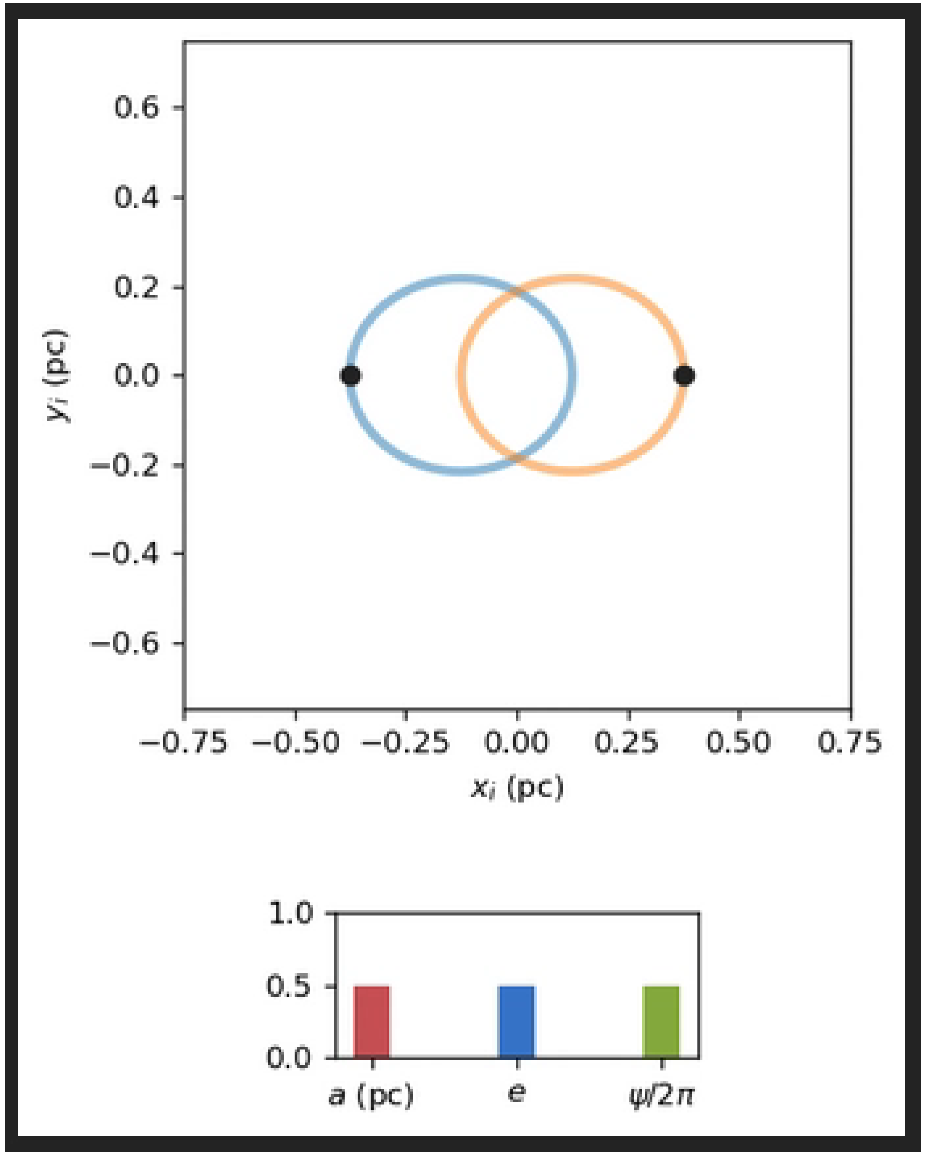
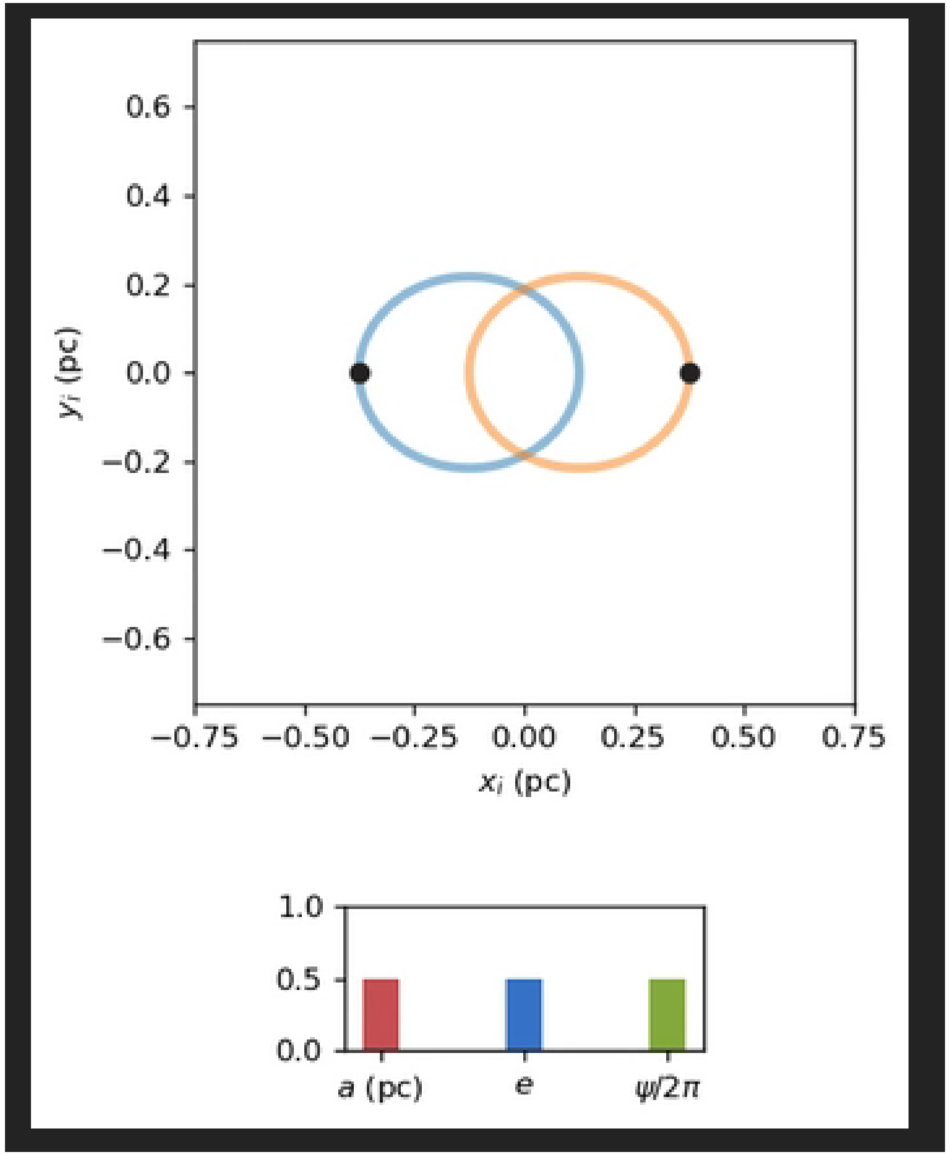


Unperturbed Binary Orbits

- Binary Orbital Parameters
  - a: Semimajor Axis
  - e: Eccentricity

Specify Orbit



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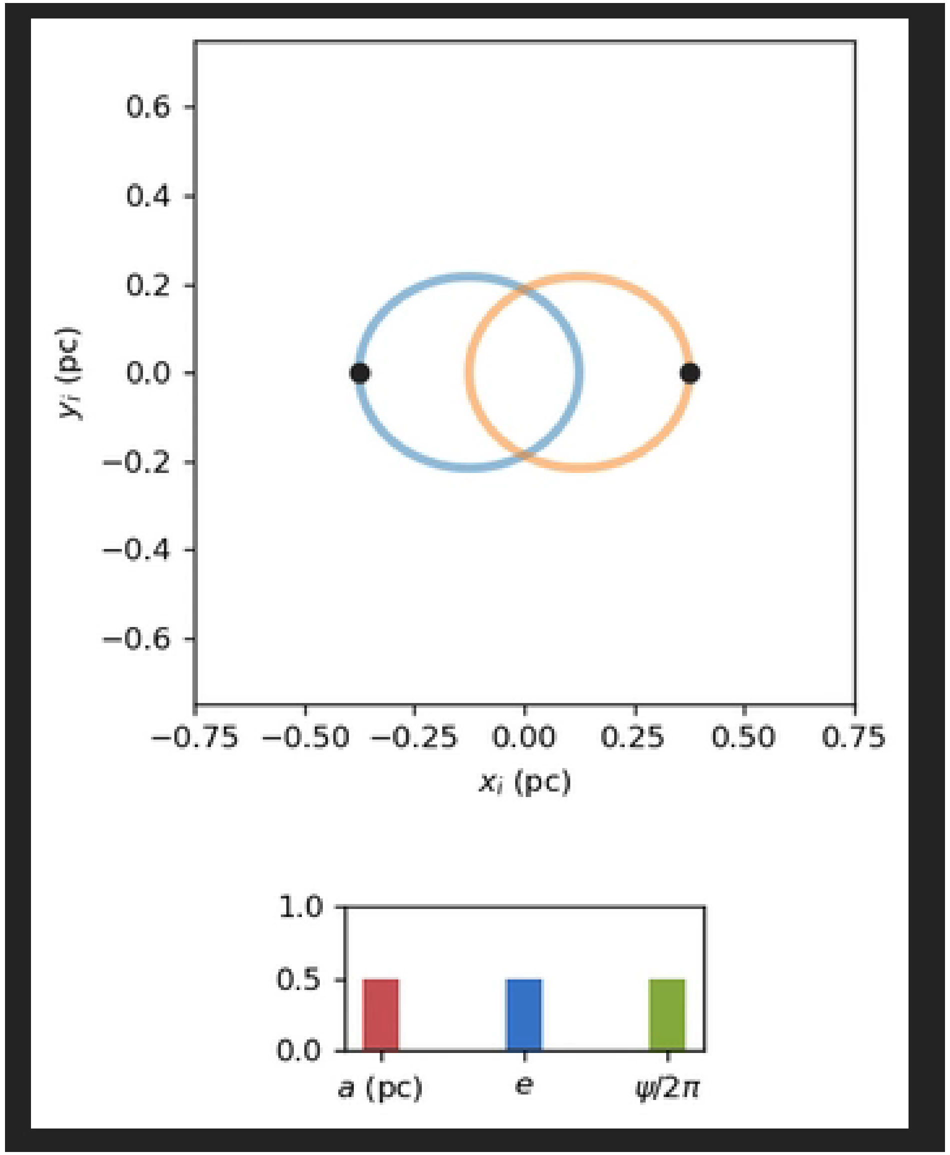
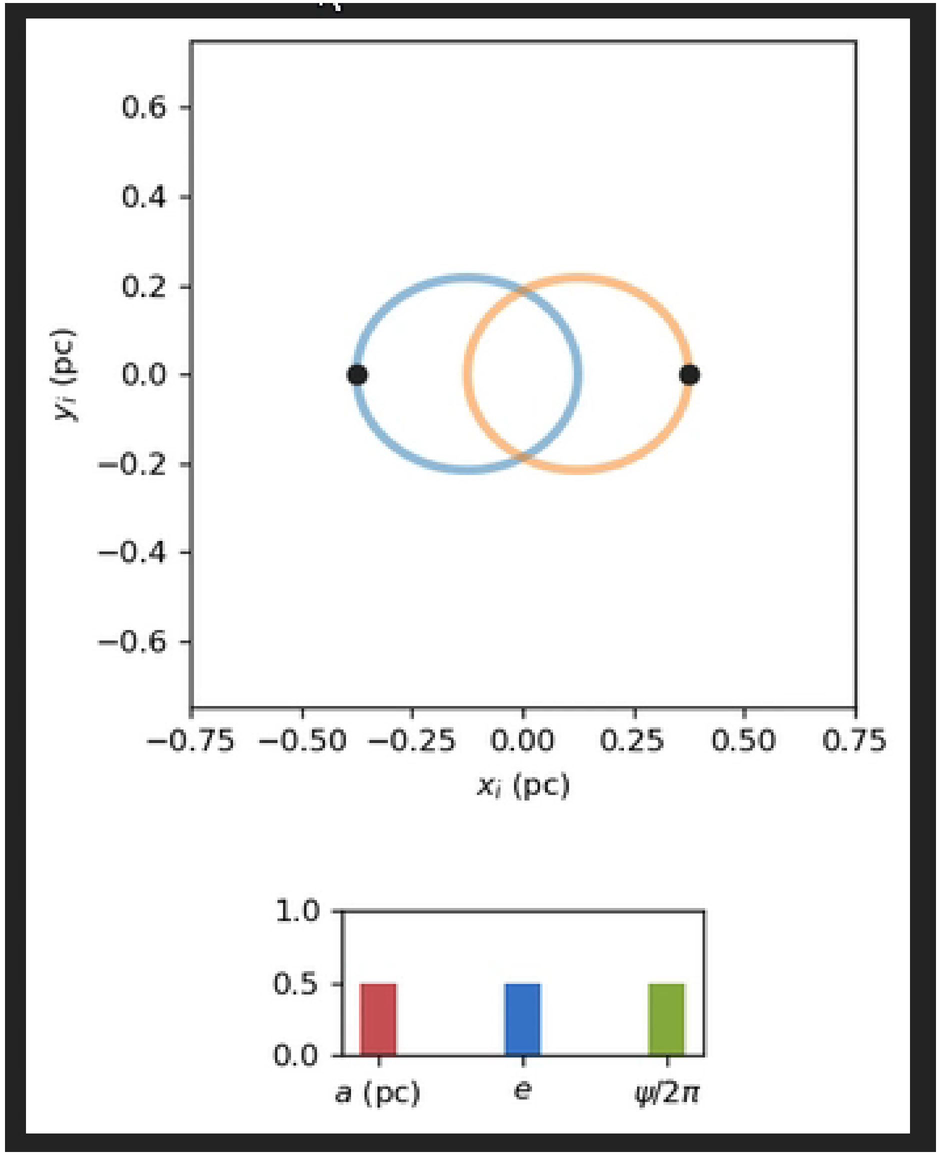


Unperturbed Binary Orbits

- Binary Orbital Parameters
  - a: Semimajor Axis
  - e: Eccentricity
  - ψ: Eccentric Anomaly

Specify Orbit (a, e)

Specify Phase (ψ)



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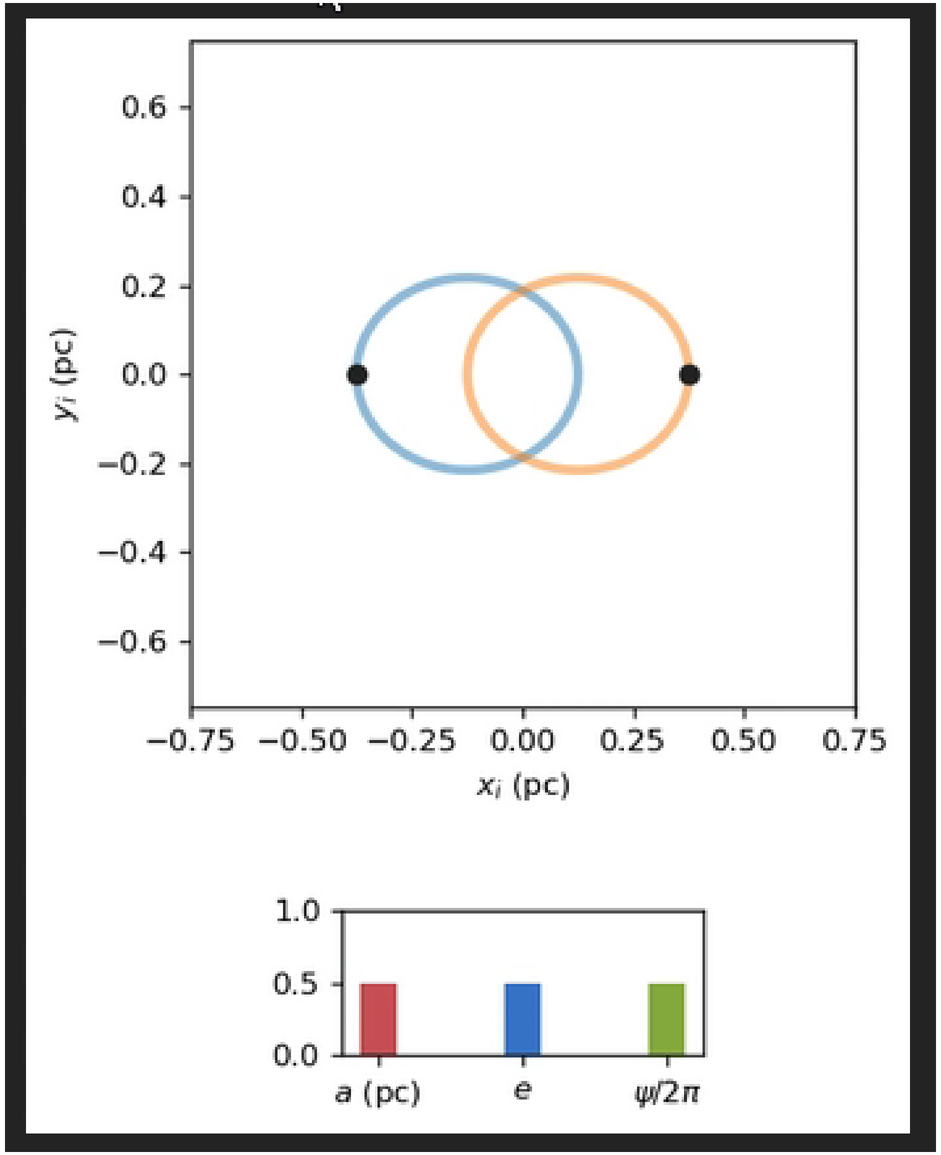
$\left.\begin{array}{l} a \\ e \end{array}\right\}$ Specify Orbit

$\left.\psi\right\}$ Specify Phase



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  - a: Semimajor Axis
  - e: Eccentricity
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a, e: Specify Orbit

ψ: Specify Phase

- Equations of Motion

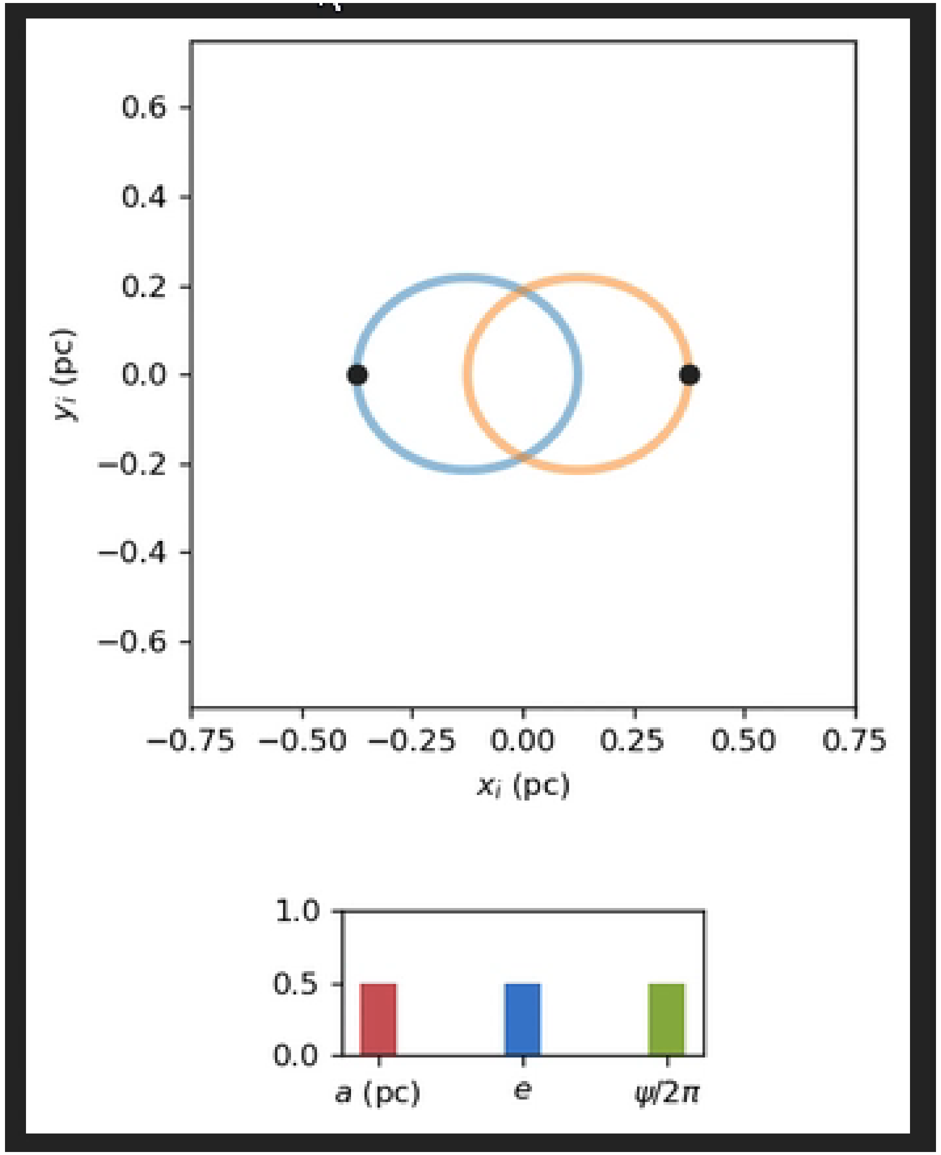
$$\begin{cases} r = a(1 - e\cos\psi) \\ t = \frac{P}{2\pi}(\psi - e\sin\psi) \end{cases}$$

Evolution of Physical Separation

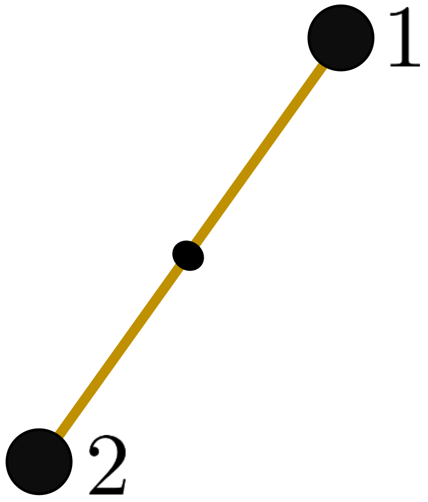
Evolution of Eccentric Anomaly

$$P = a^{3/2}\sqrt{4\pi^2/GM}$$

Orbital Period

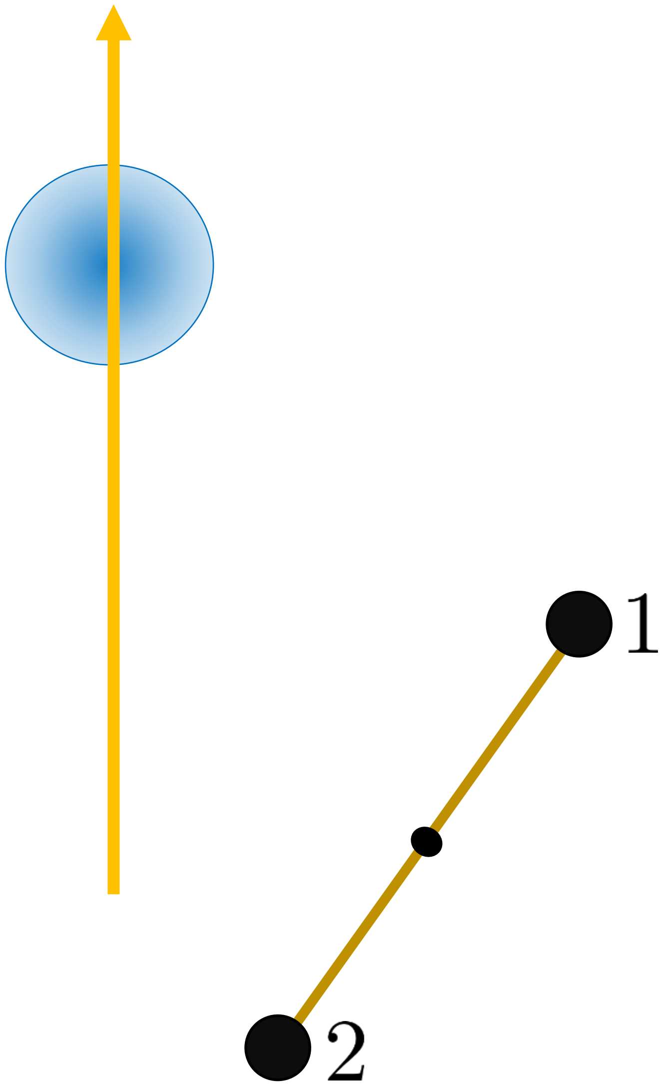


A Single Binary-Perturber Encounter



A Single Binary-Perturber Encounter

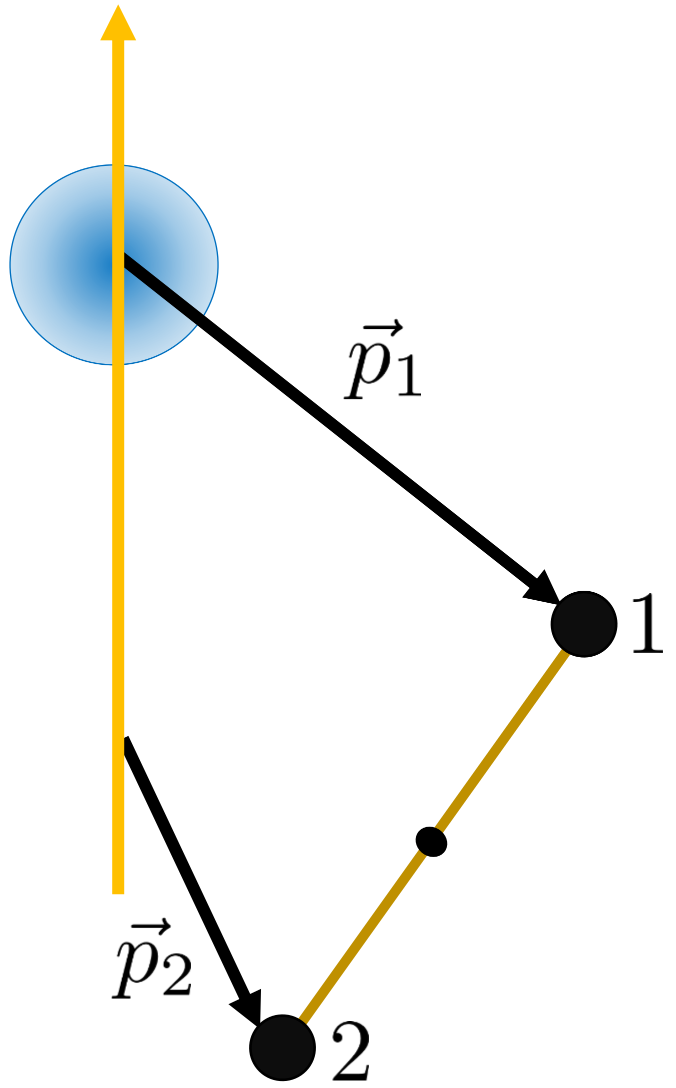
- The Impulse Approximation
  - Binary positions fixed during encounter



A Single Binary-Perturber Encounter

- The Impulse Approximation
  - Binary positions fixed during encounter
  - Encounter results in velocity kicks on the stellar components

$$\Delta\vec{v}_i = -\frac{2GM_p}{v_p}U(p_i)\frac{\vec{p_i}}{p_i^2}$$



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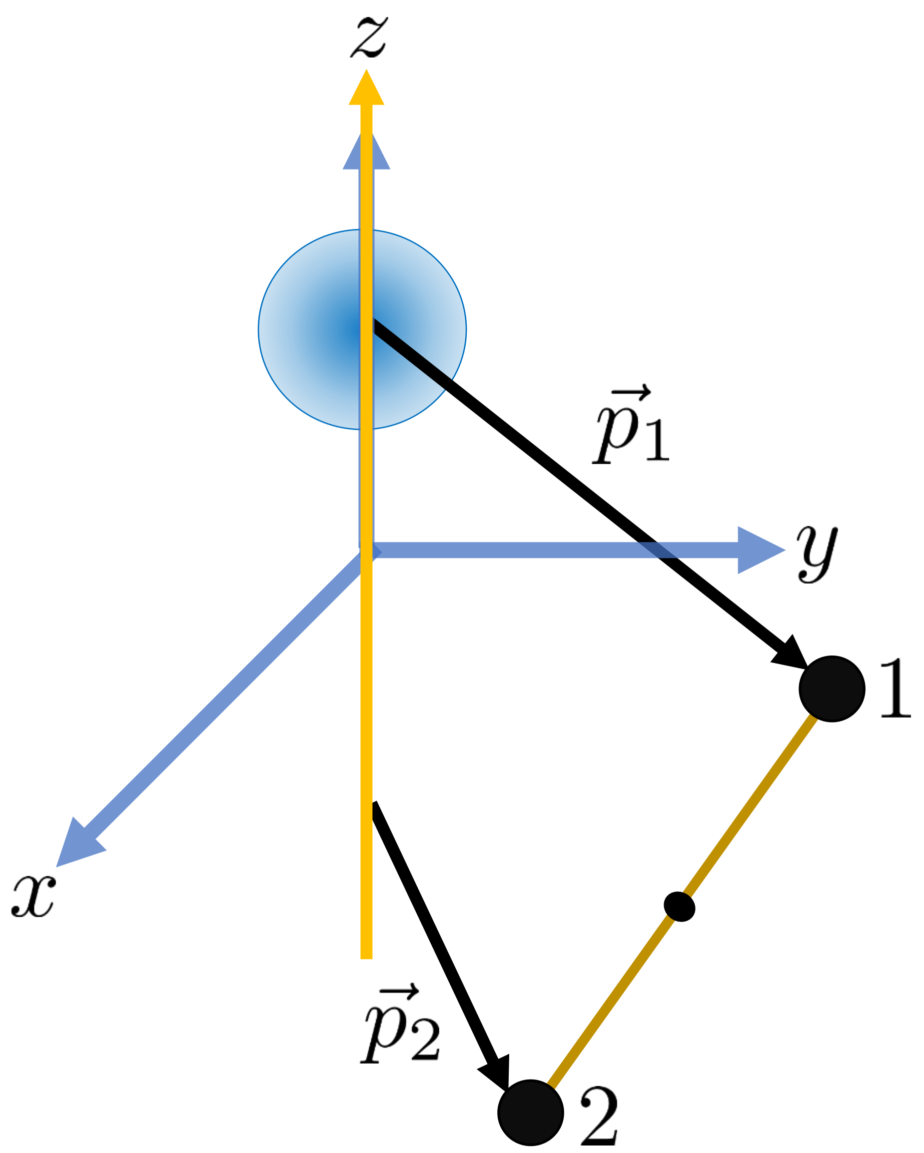
- Encounter Geometry



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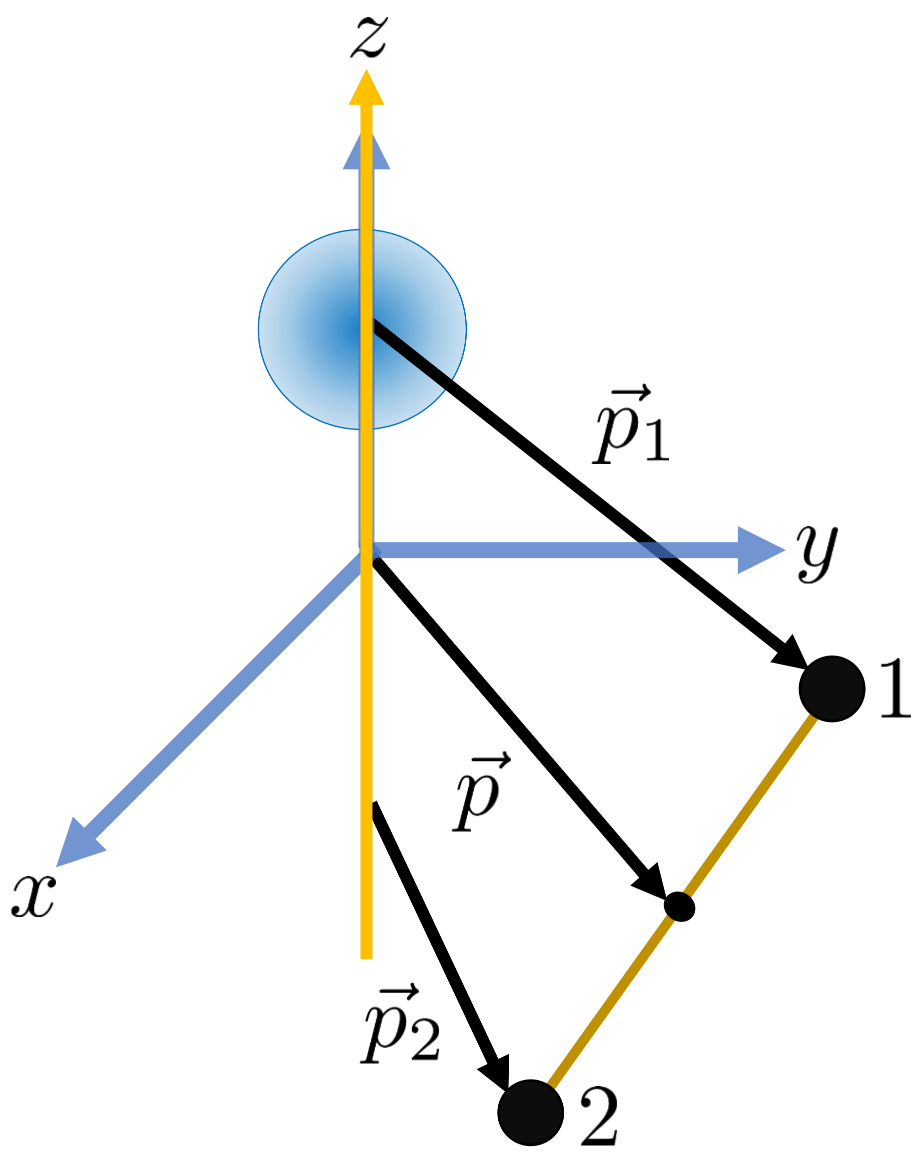
  - p: Impact Parameter



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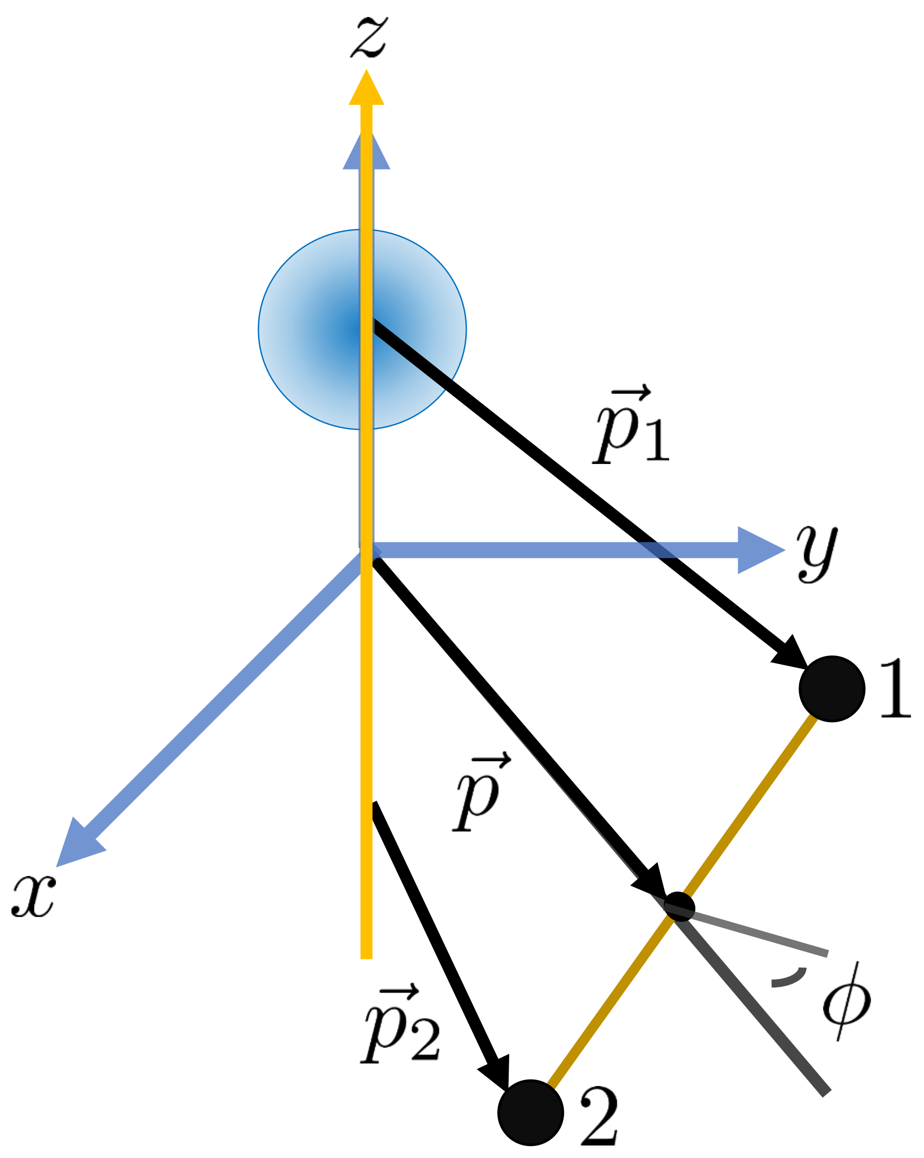

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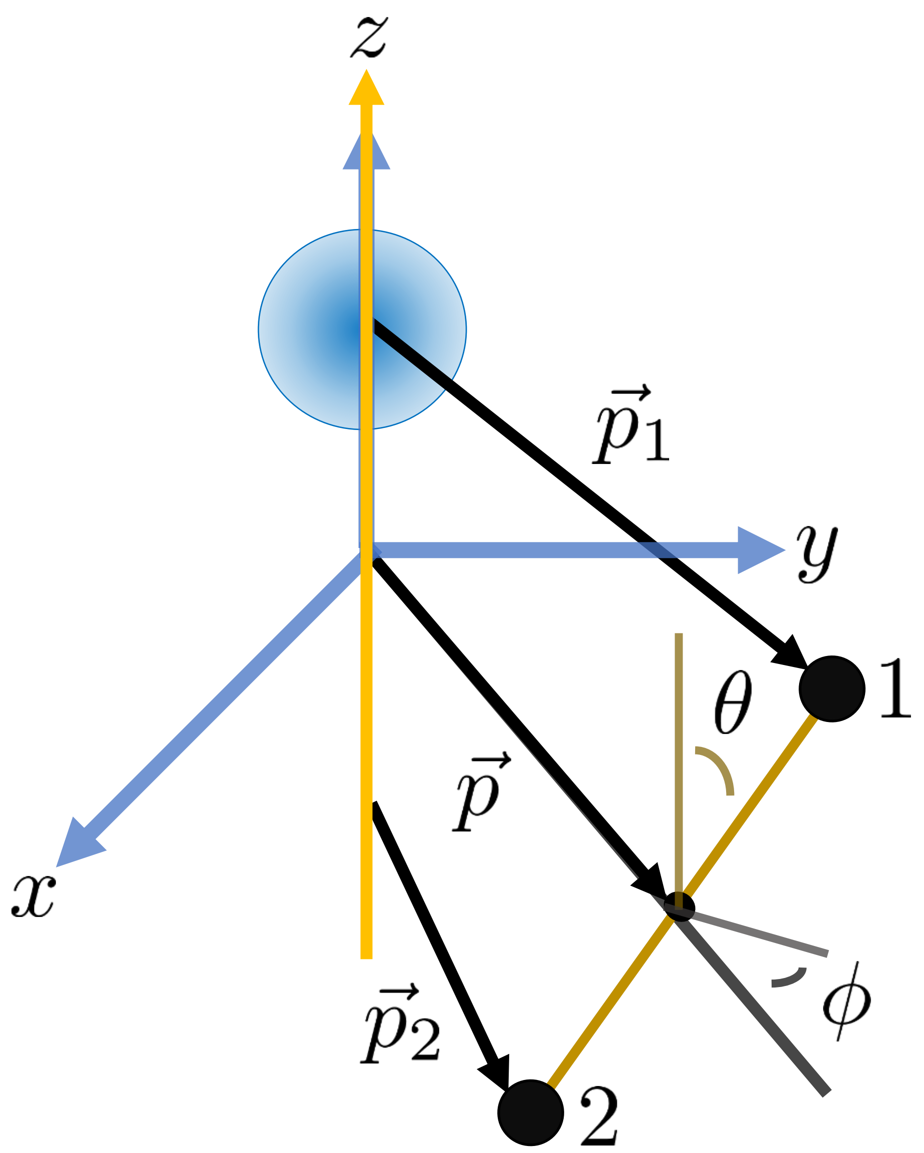

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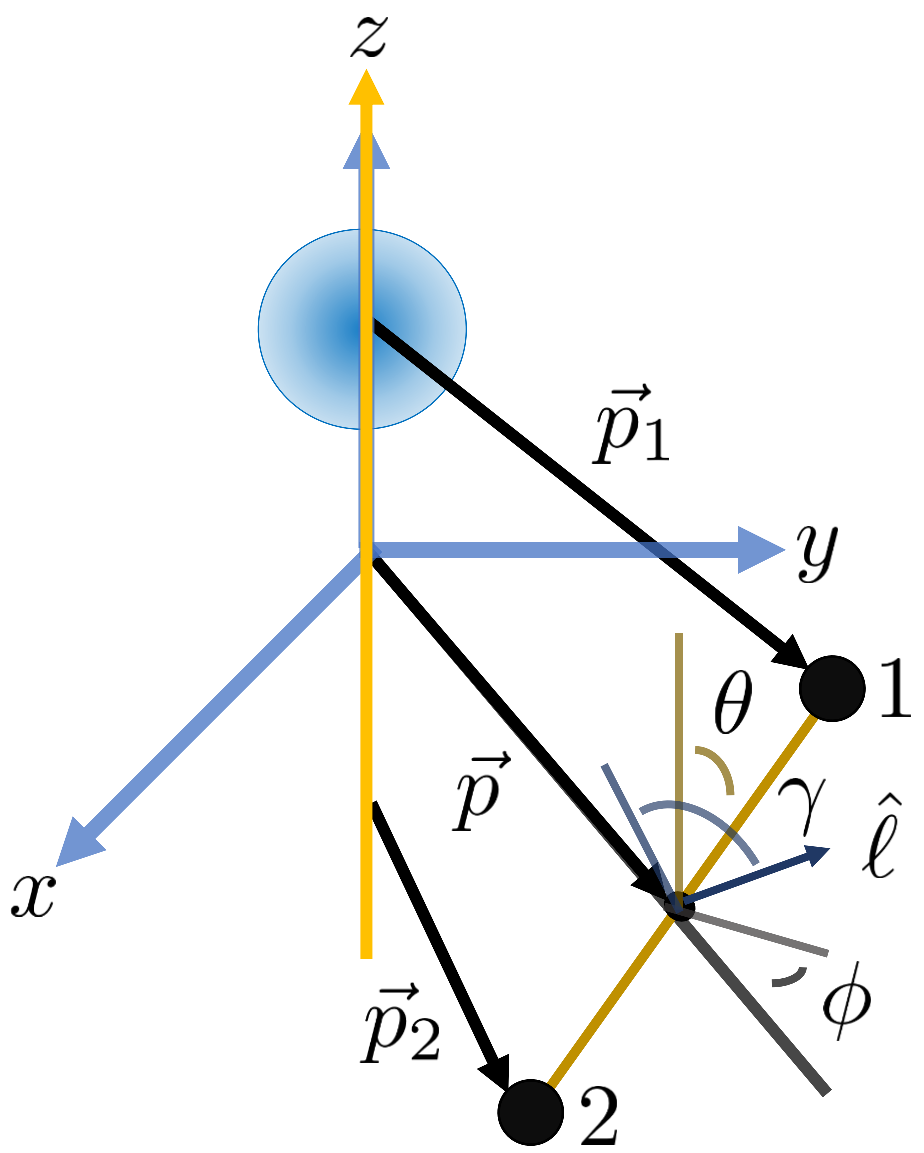
  - θ: Polar Angle
  - γ: Angle for orbital plane

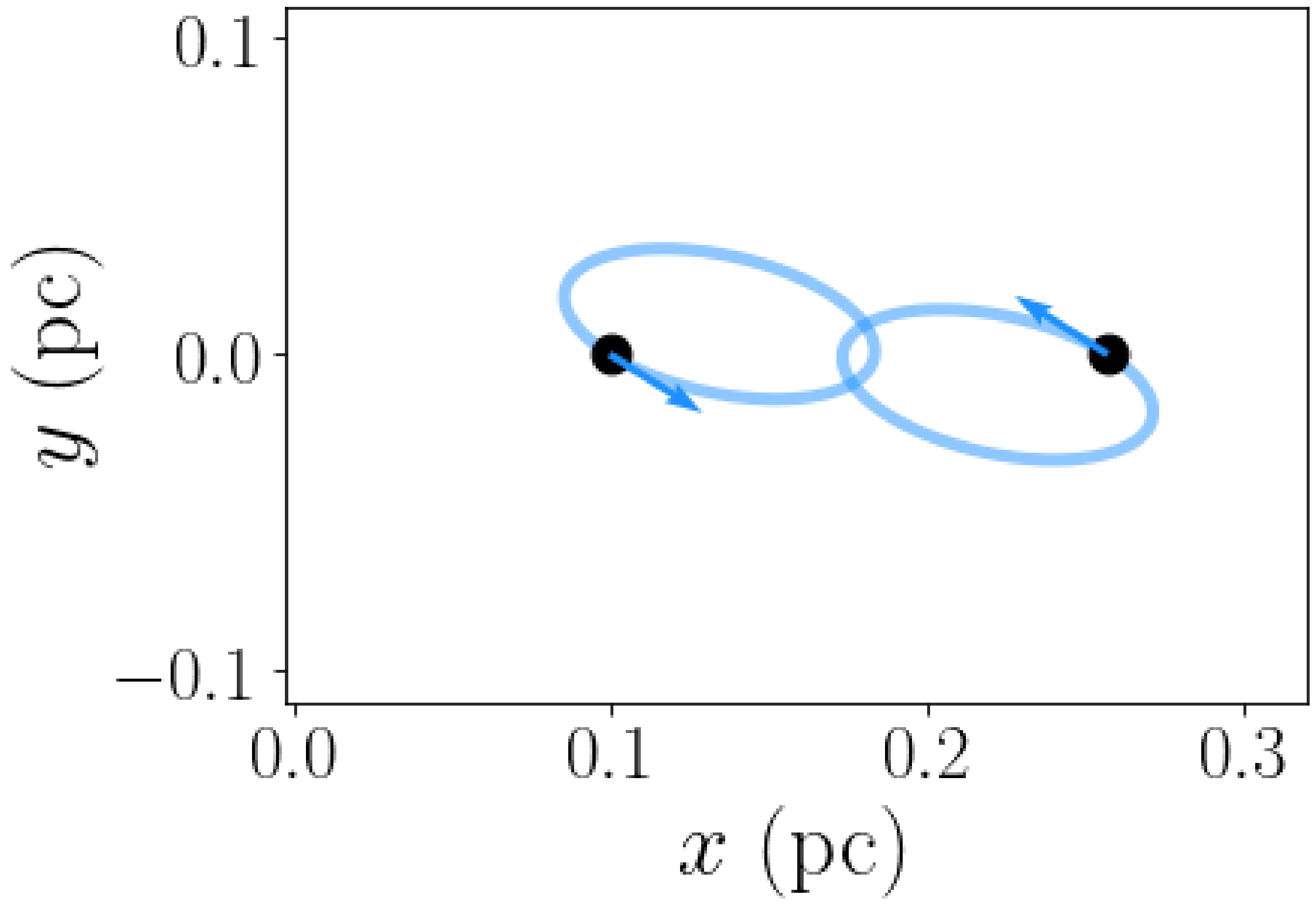


The Effect of the Encounter

The Effect of the Encounter

- Example:
  - Two equal mass binaries
  - Uniform-density perturber

Initial Orbit



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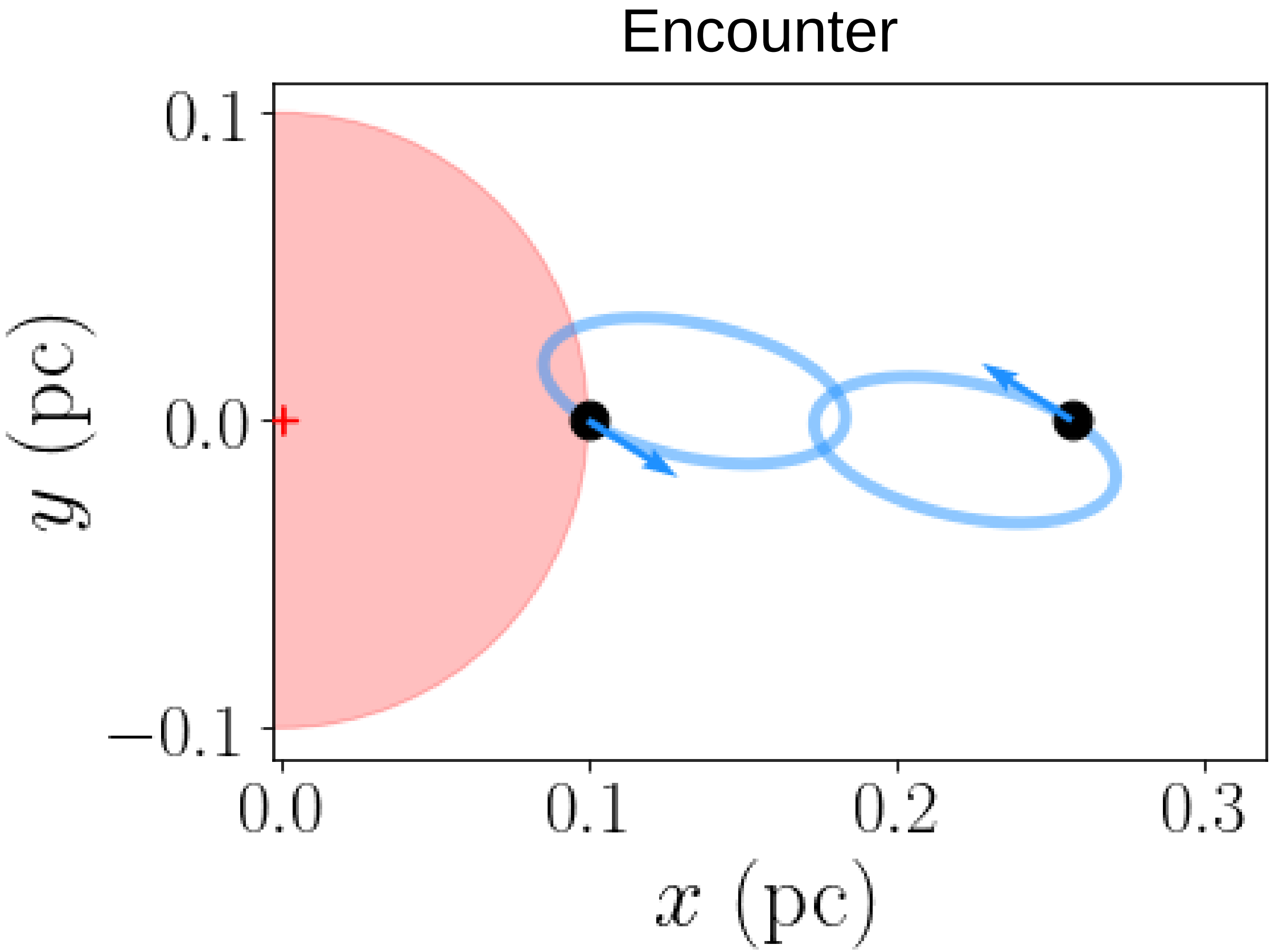

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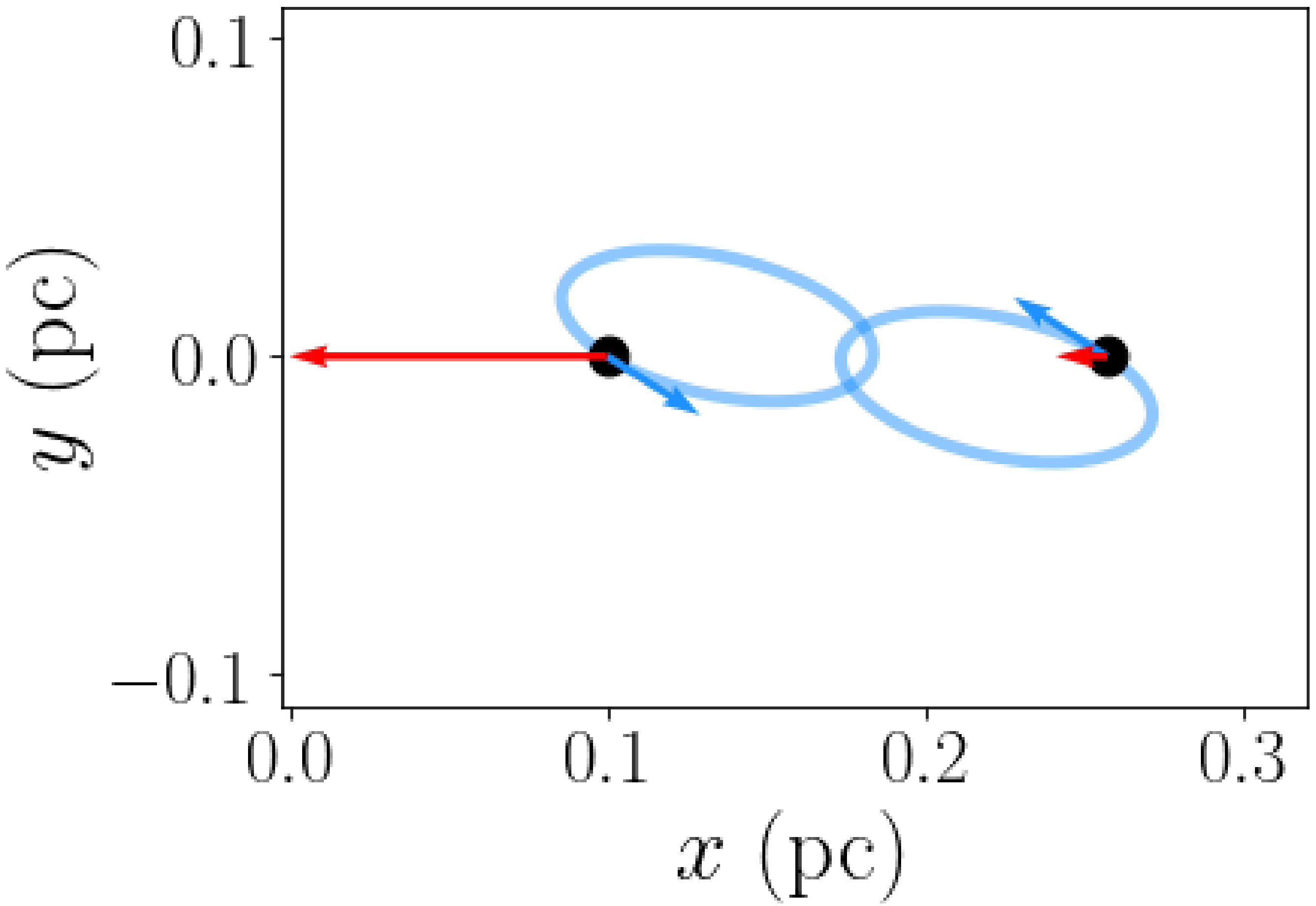
Velocity Kicks



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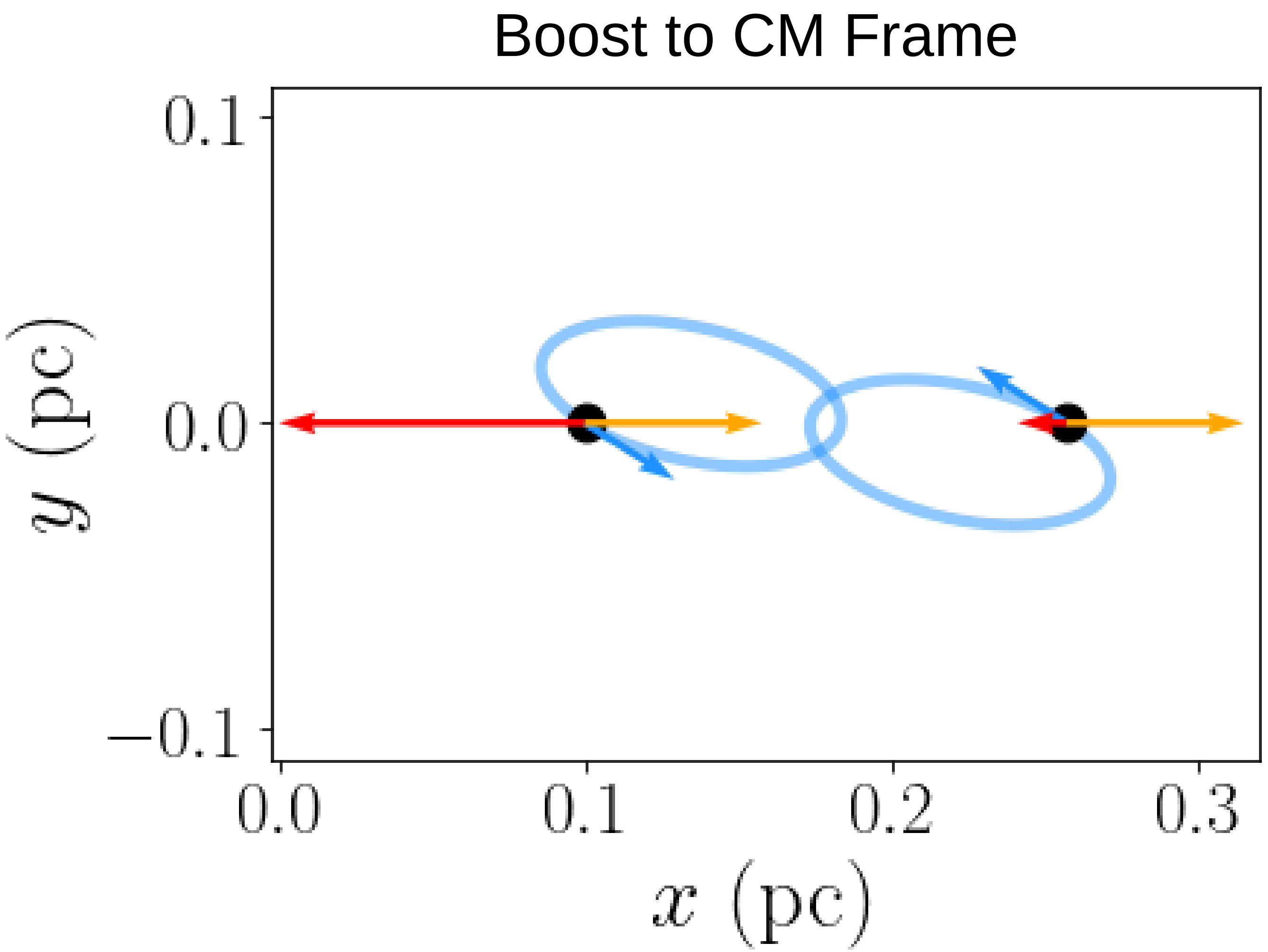

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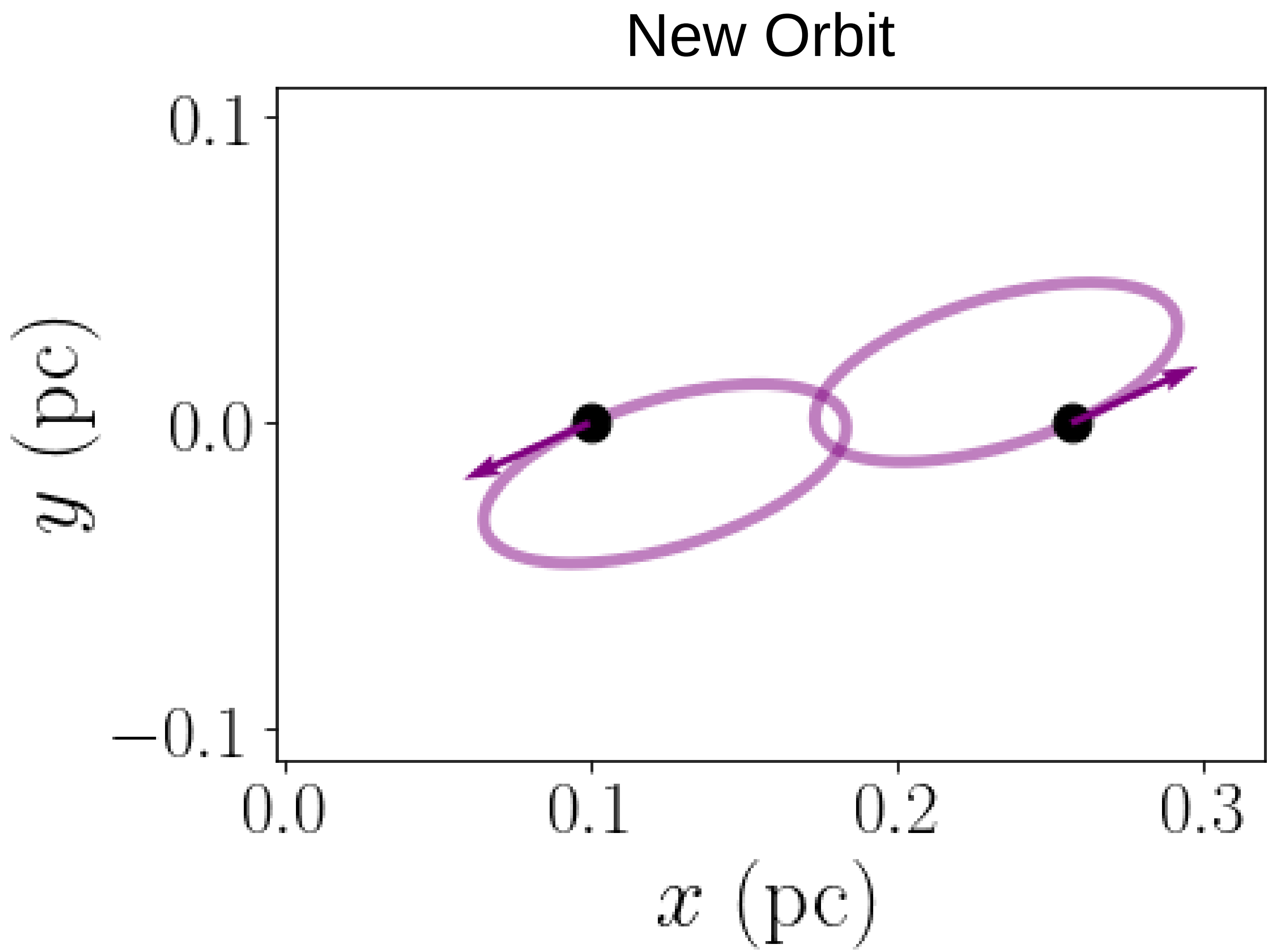

The Effect of the Encounter

- Example:
  - Two equal mass binaries
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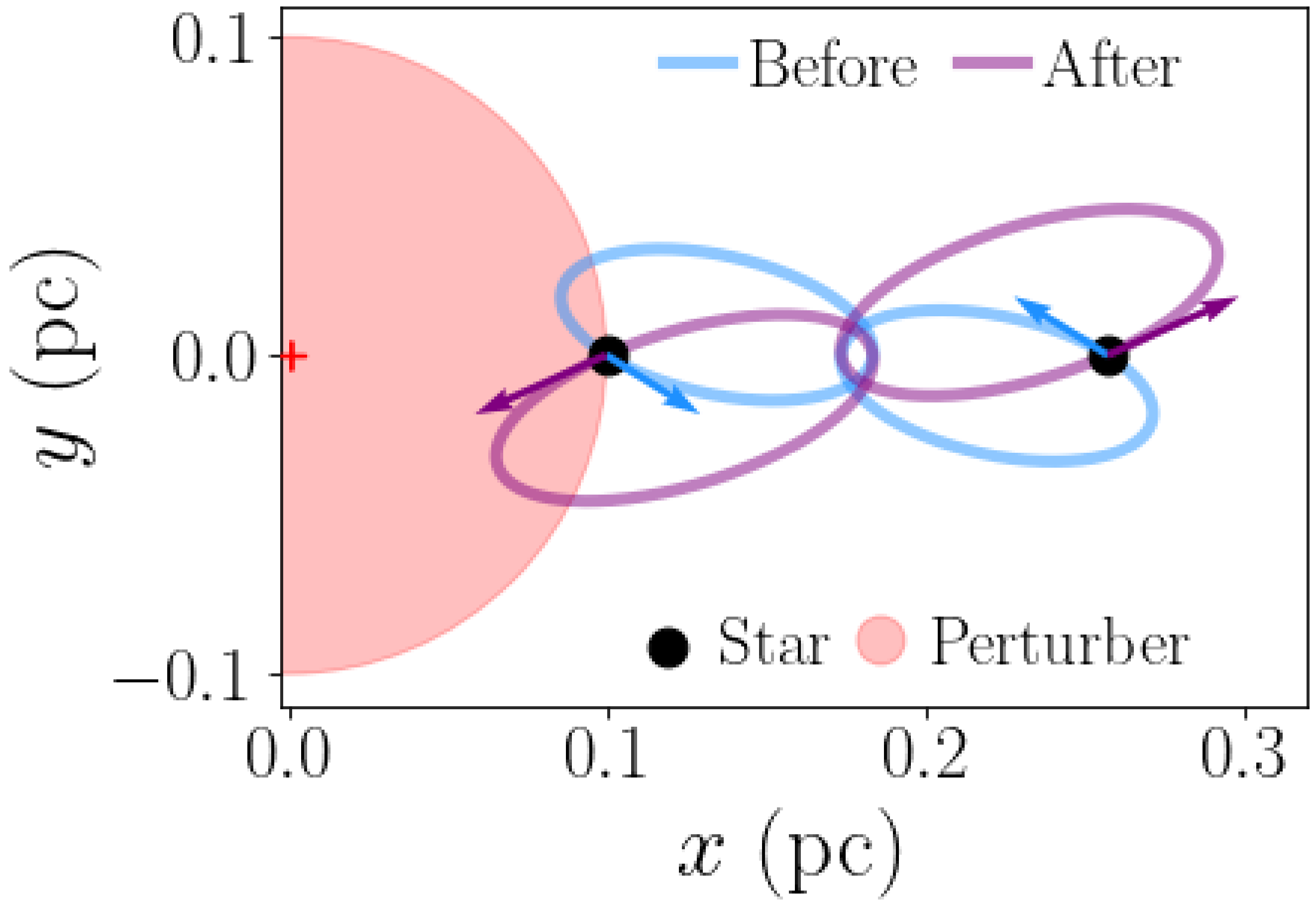
- **Effect:**

  Change in Orbit

  $$(a_0, e_0, \psi_0) \xrightarrow{\Delta\vec{v}} (a, e, \psi)$$

  Analytic

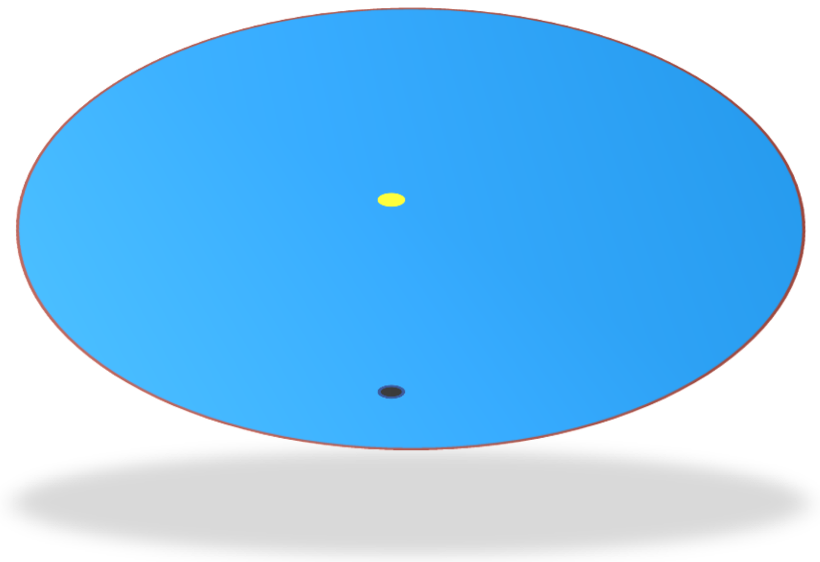


Multiple Encounters on a Single Binary

- The effect of an encounter is deterministic

$$\Delta\vec{v}_i = -\frac{2GM_p}{v_p}U(p_i)\frac{\vec{p_i}}{p_i^2}$$

- Encounters are random



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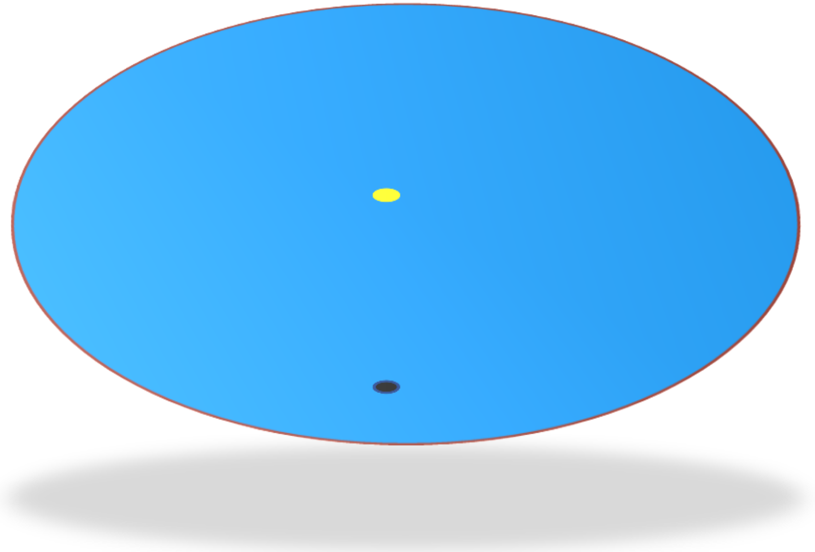

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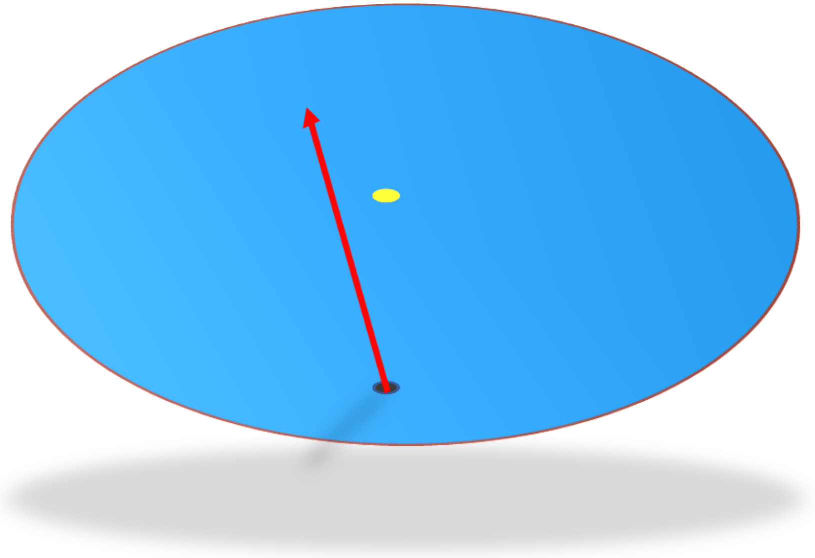

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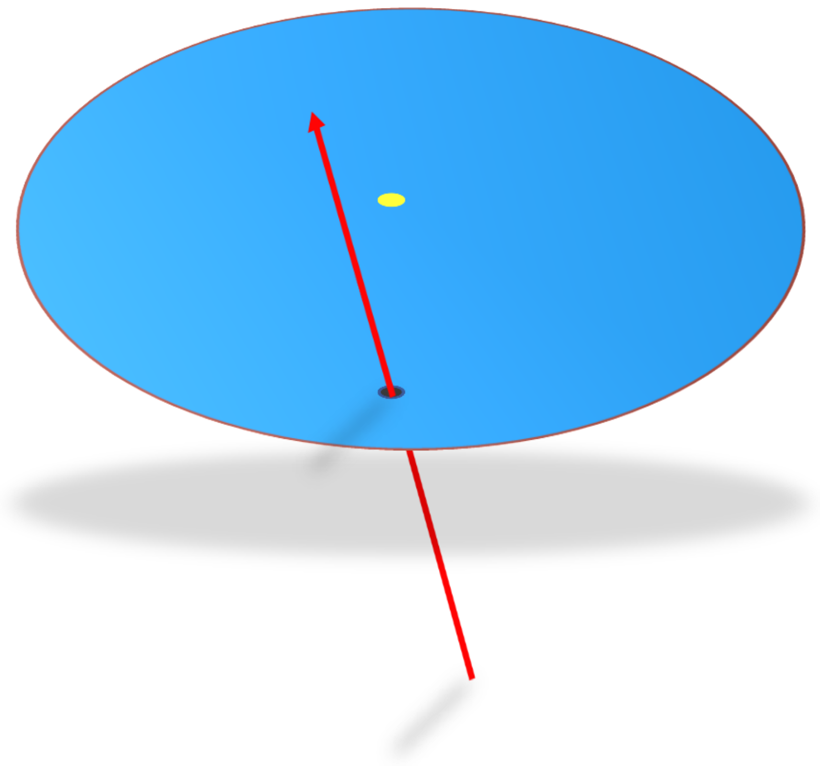

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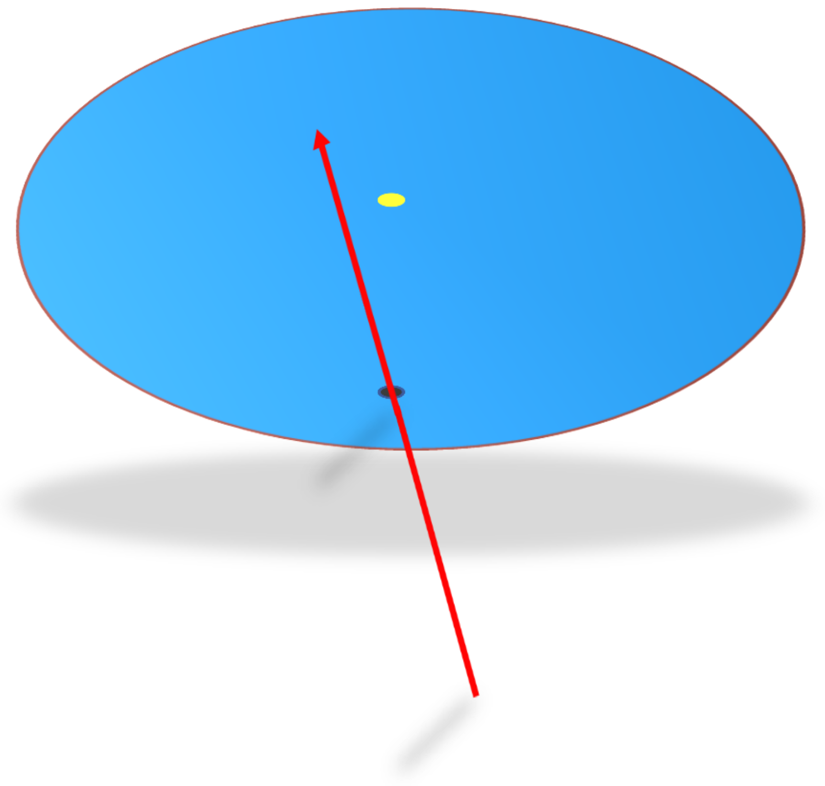
- Encounters are random
  - p: Uniform in disk
  - ϕ: Uniform



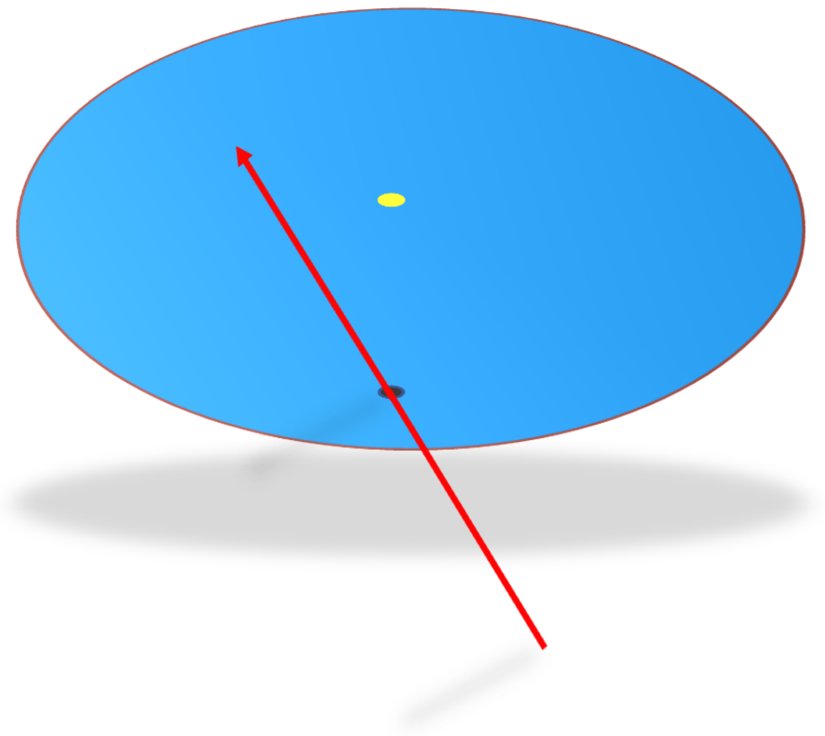
Multiple Encounters on a Single Binary

- The effect of an encounter is deterministic

$$\Delta\vec{v}_i = -\frac{2GM_p}{v_p}U(p_i)\frac{\vec{p_i}}{p_i^2}$$

- Encounters are random
  - p: Uniform in disk
  - ϕ: Uniform
  - $\sin\theta$: Uniform

$\left.\begin{matrix}\phi \\ \sin\theta\end{matrix}\right\}$ Ω uniform



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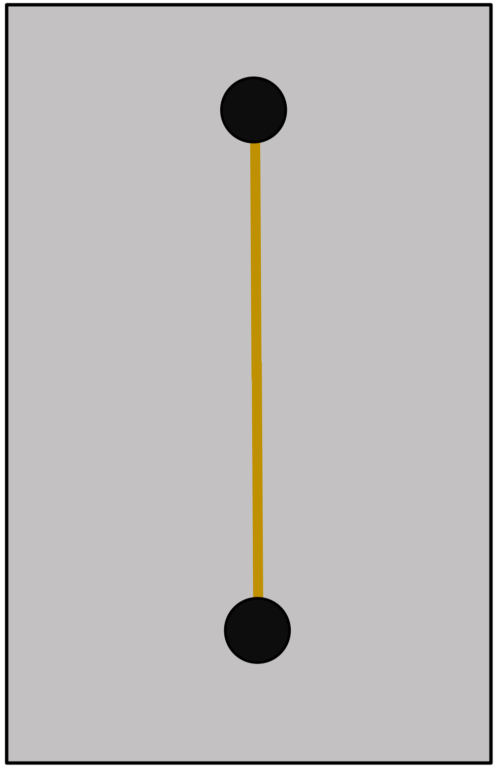

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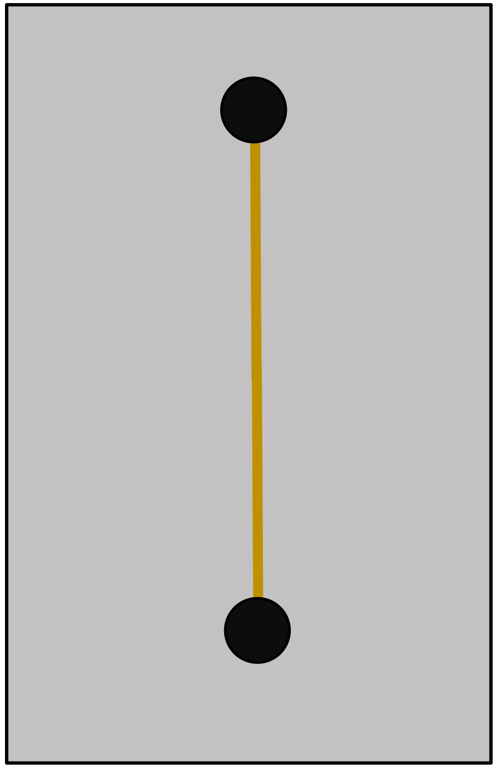

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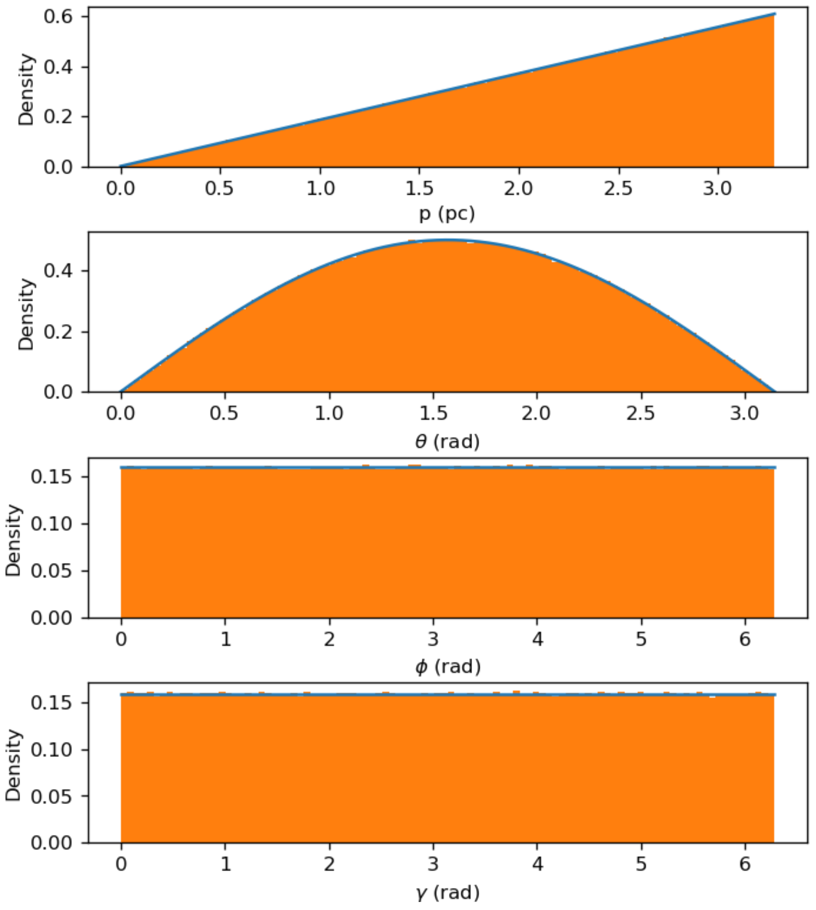

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➡ Evolution is random

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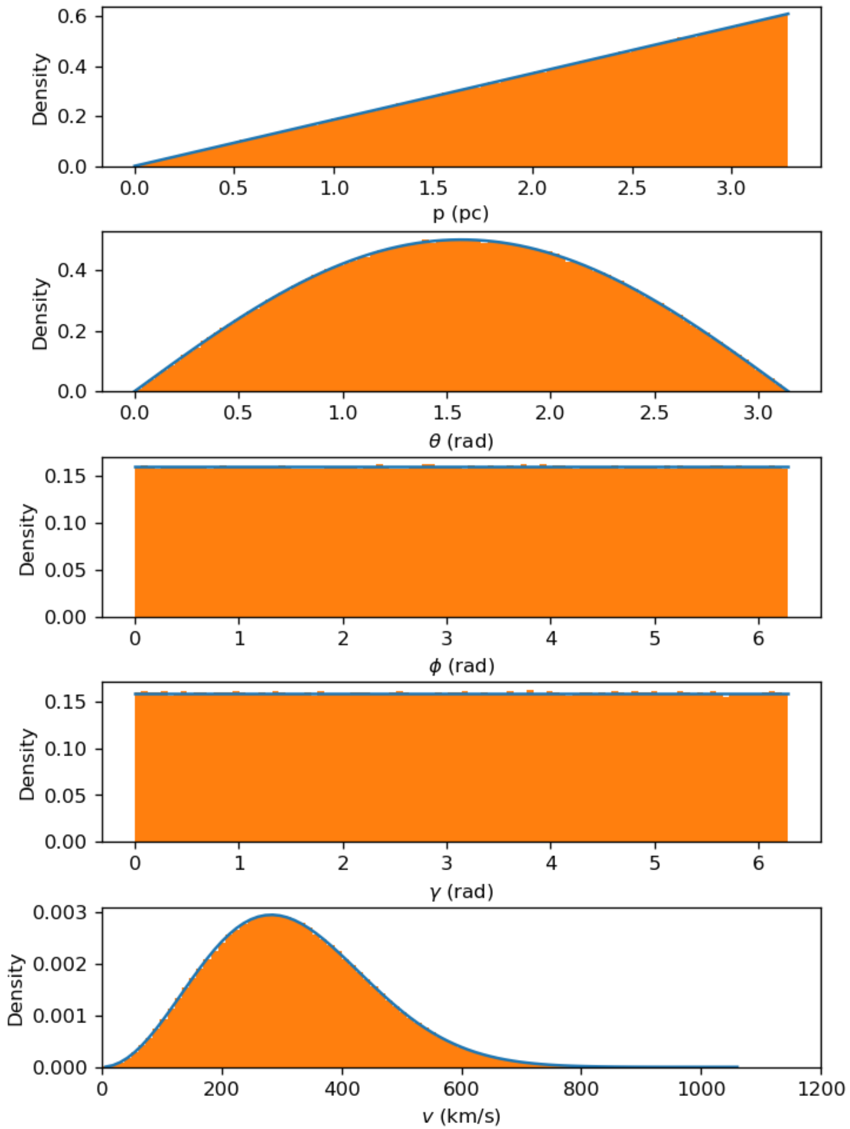
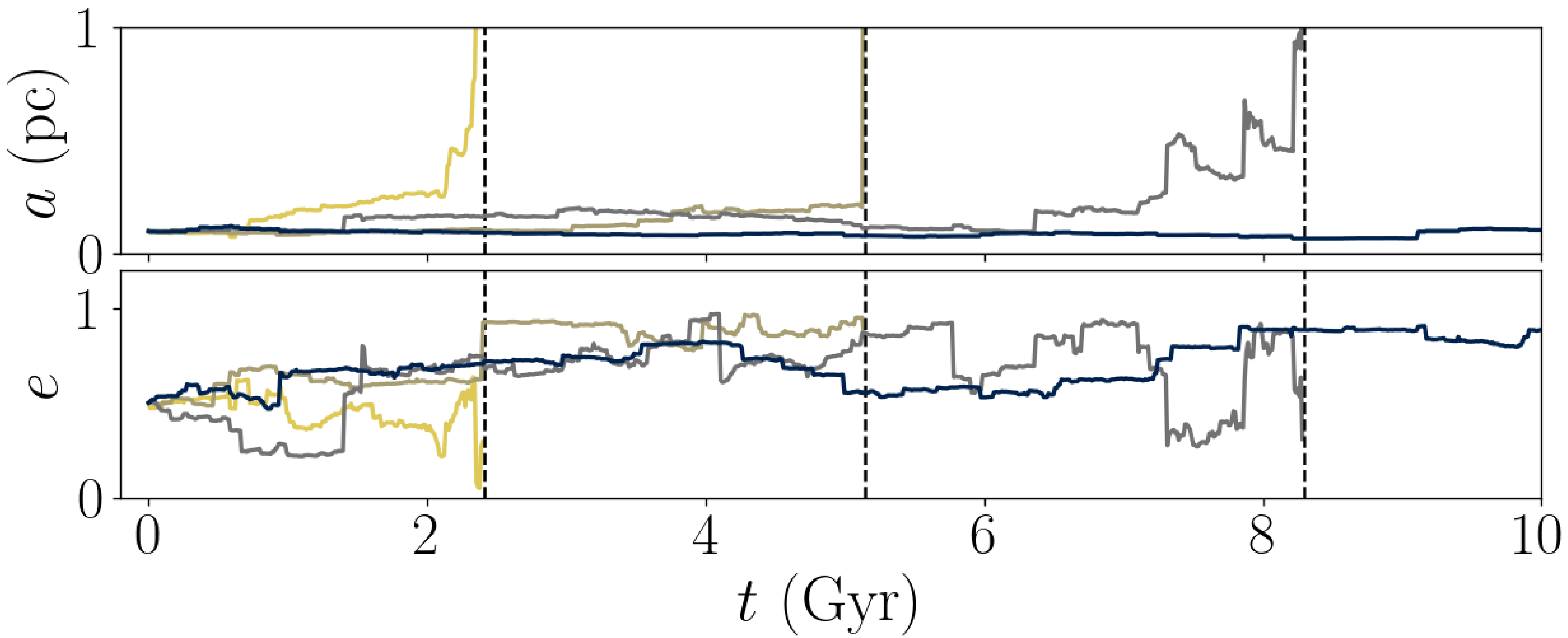


Random Evolution

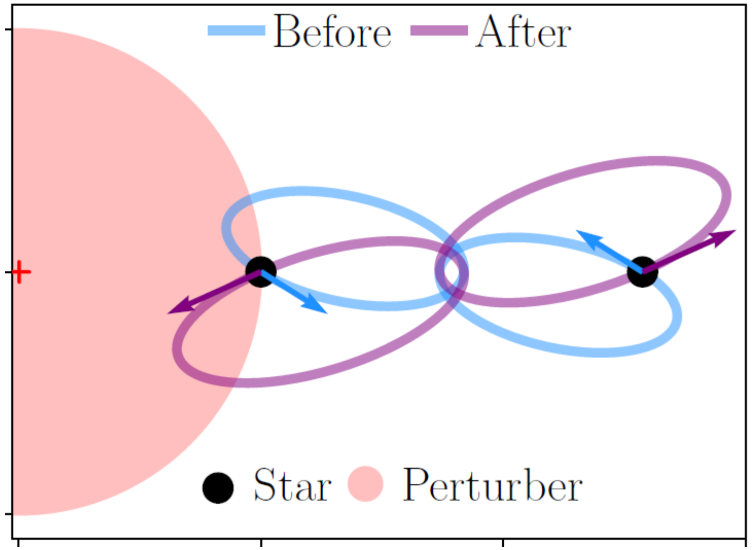


Scattering Matrix: $Pr(\vec{q}_0 \to \vec{q})$

The Effect of Single Random Encounter

$$Pr(\vec{q}_0 \to \vec{q}_1) = f_1(\vec{q}_1|\vec{q}_0)$$

$$\vec{q}_0 \to \vec{q}_1$$



- Binary orbital state

$$\vec{q} = (a, e, \psi)$$

- Single random encounter

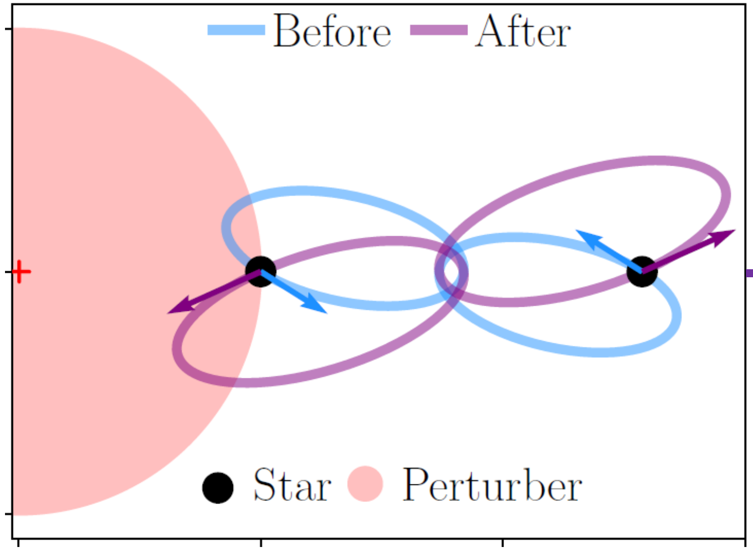
$$\vec{q}_0 \to \vec{q}_1$$

Two Random Encounters

What if we don't know $\vec{q}_1$?

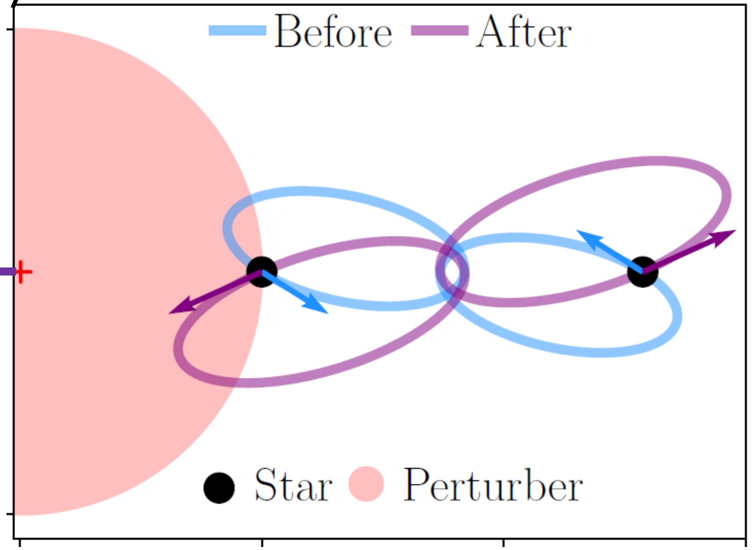
$$Pr(\vec{q}_0 \to \vec{q}_1 \to \vec{q}_2) \propto \underline{f_2(\vec{q}_2|\vec{q}_1)} f_1(\vec{q}_1|\vec{q}_0)$$

$\vec{q}_0 \to \vec{q}_1$



Evolve Freely to Next Encounter

$\vec{q}_1 \to \vec{q}_2$

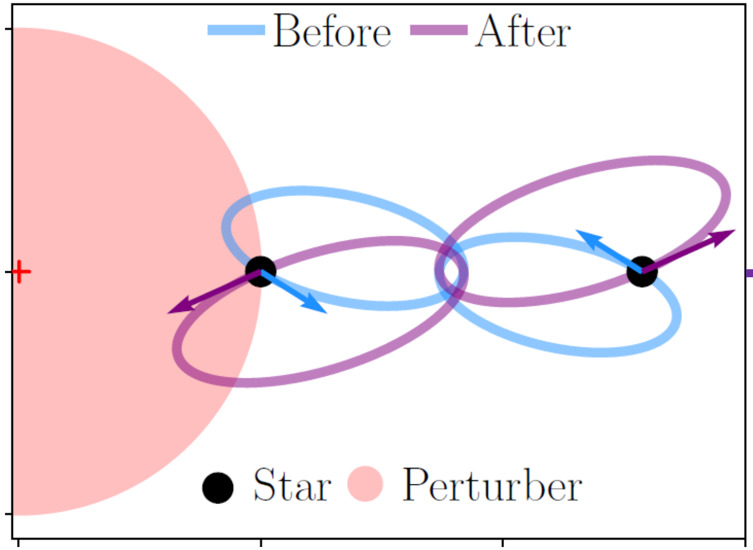


Two Random Encounters

Need to integrate over intermediate state

$$Pr(\vec{q}_0 \to \vec{q}_2) = \underline{\int d\vec{q}_1}\ f_2(\vec{q}_2|\vec{q}_1) f_1(\vec{q}_1|\vec{q}_0)$$

$\vec{q}_0 \to \vec{q}_1$



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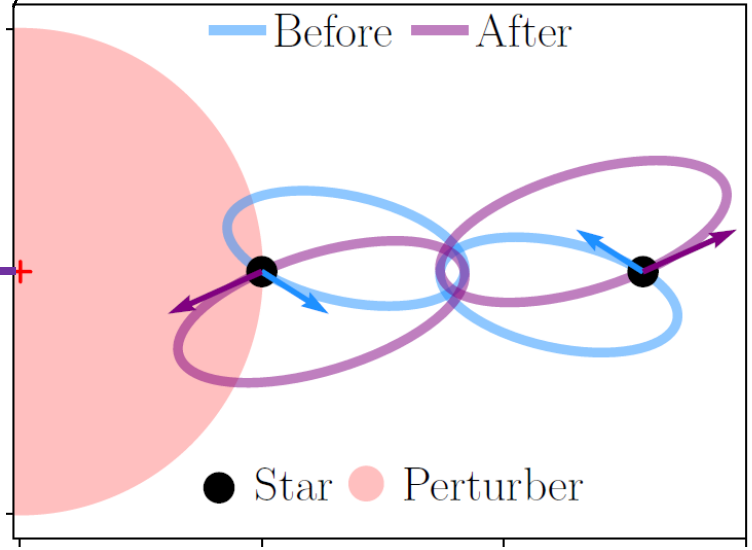
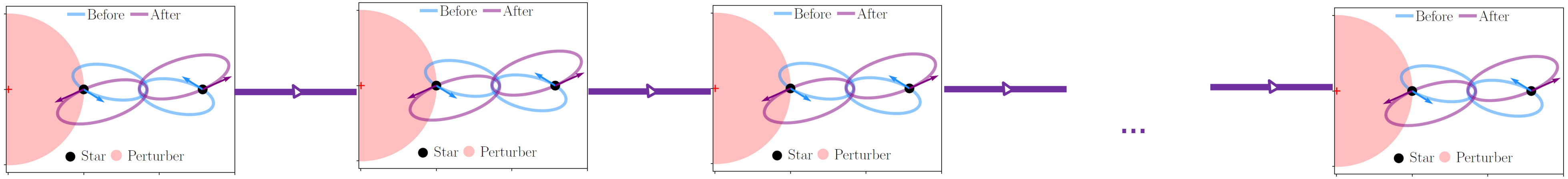


N Random Encounters

Need to integrate over intermediate states

$$Pr(\vec{q}_0 \to \vec{q}_N) = \int \prod_{i=1}^{N-1} \left[\, d\vec{q}_i \;\; f_{i+1}(\vec{q}_{i+1}|\vec{q}_i)\right] \;\; f_1(\vec{q}_1|\vec{q}_0)$$

$\vec{q}_0 \to \vec{q}_1$ $\qquad$ $\vec{q}_1 \to \vec{q}_2$ $\qquad$ $\vec{q}_2 \to \vec{q}_3$ $\qquad$ ... $\qquad$ $\vec{q}_{N-1} \to \vec{q}_N$



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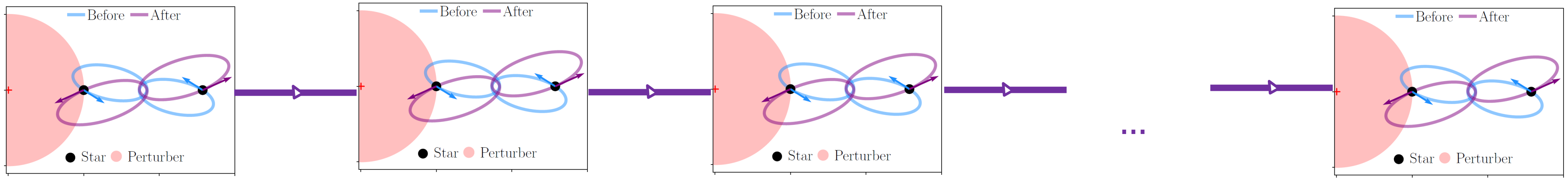


Monte Carlo Simulation

Scattering Matrix Estimate:
Evolve a high number of synthetic binaries representative of the observed dataset and obtain the frequency distribution in $\vec{q}$

N Random Encounters

Uniformly Spaced Encounters

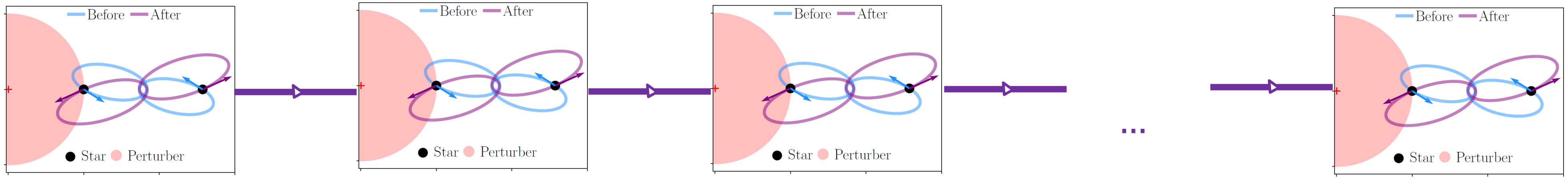
$$N = \mathrm{int}\left[\frac{T}{\langle \delta t \rangle}\right] \propto f_p$$

Need to integrate over intermediate states

$$Pr(\vec{q}_0 \to \vec{q}_N) = \int \prod_{i=1}^{N-1} \left[\, d\vec{q}_i \;\, f_{i+1}(\vec{q}_{i+1}|\vec{q}_i)\right] \;\, f_1(\vec{q}_1|\vec{q}_0)$$

$f_p \equiv \rho_{subhalo}/\rho_{DM}$

$\vec{q}_0 \to \vec{q}_1$ $\quad$ $\vec{q}_1 \to \vec{q}_2$ $\quad$ $\vec{q}_2 \to \vec{q}_3$ $\quad$ $\vec{q}_{N-1} \to \vec{q}_N$



Monte Carlo Simulation

Scattering Matrix Estimate:
Evolve a high number of synthetic binaries representative of the observed dataset and obtain the frequency distribution in $\vec{q}$

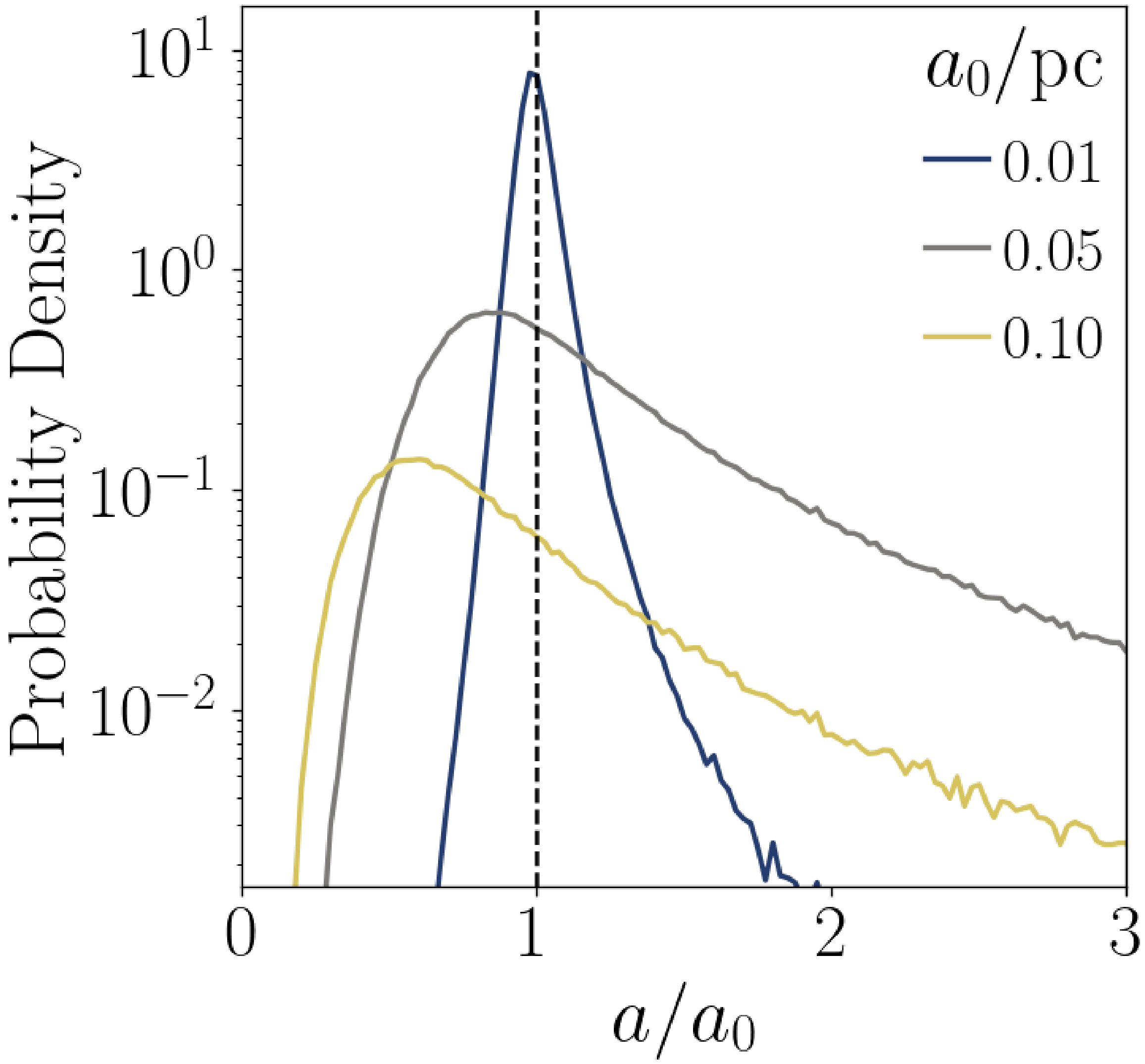
Scattering Matrix of Three Types of Binaries

- Simulation

Binaries:
$a_0 = 0.01, 0.05, 0.1$ pc
$e_0 = 0.5$
$\frac{\psi_0}{2\pi} = 0$
$M = 1\ M_\odot$

Perturbers:
$M_p = 10^3\ M_\odot$
$R_p = 0.1$ pc
$\rho(r) = constant$
$f_p = 1$

- Steps:
  1. Generate 10^6 identical binaries
  2. Evolve each binary for $T = 10$ Gyr:
     $$\vec{q}_0 \to \cdots \to \vec{q}_N \equiv \vec{q}$$
  3. Generate histogram of the semimajor axis a



Multiple Encounters on a Multiple Binaries

- Two processes determine the fate of the binary population
  - Assembly process of initial population of binaries
  - Subsequent evolution of the initial binary population

$$\underbrace{\phi(\vec{q})}_{\text{Present-Day Distribution of Binaries}} = \int d\vec{q}_0 \; \underbrace{Pr(\vec{q}_0 \rightarrow \vec{q})}_{\text{Scattering Matrix}} \; \underbrace{\phi_0(\vec{q}_0)}_{\text{Initial Distribution of Binaries}}$$

Initial Distribution of Binaries

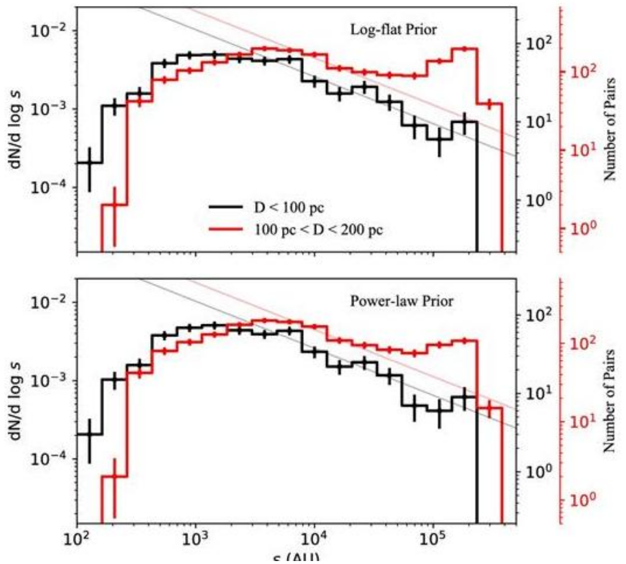
$$\phi_0(\vec{q}_0) : \vec{q}_0 = (a_0, e_0, \psi_0)$$

$\phi_0(a_0)$

- Unknown. Observations suggest it is given by a power law

$$\phi_0(a_0|\lambda) \propto a_0^\lambda,$$

where λ is an unknown parameter we float when setting limits on subhalos.



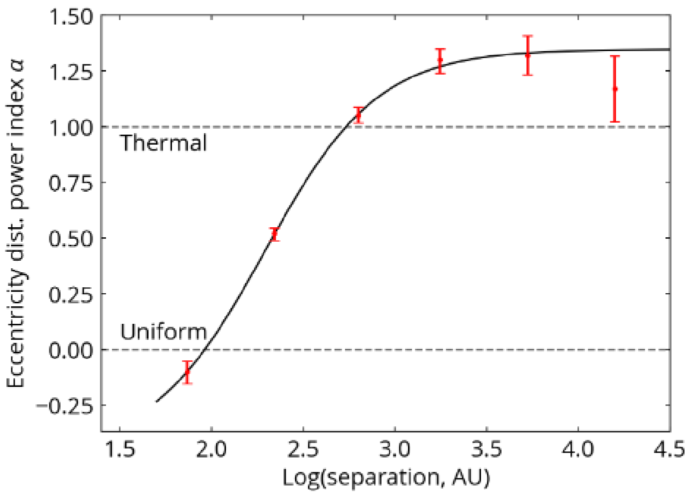
Andrews et al. [1704.07829]

$\phi_0(e_0)$

- Widest binaries obey a super-thermal distribution

$$\phi(e_0) \propto e_0^\kappa \ (\kappa > 1)$$

- As a conservative assumption, we take $\kappa = 1$ (thermal)

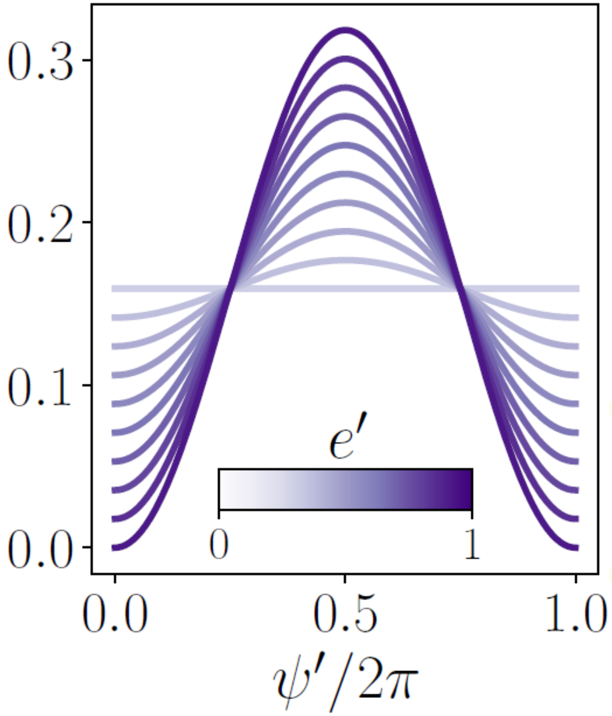


Hwang et al. [2111.01789]

$\phi_0(\psi_0)$

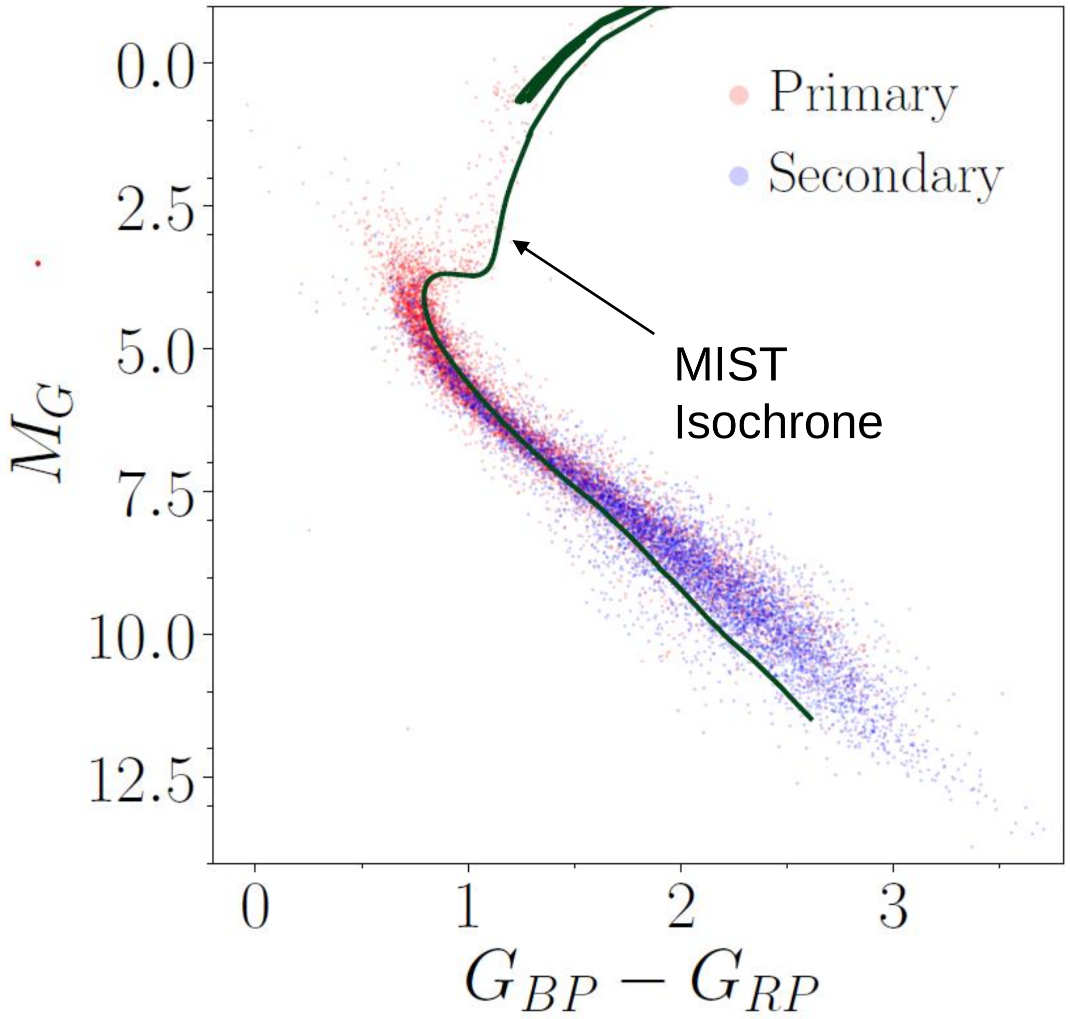
- Distributed in dynamical time t with uniform probability

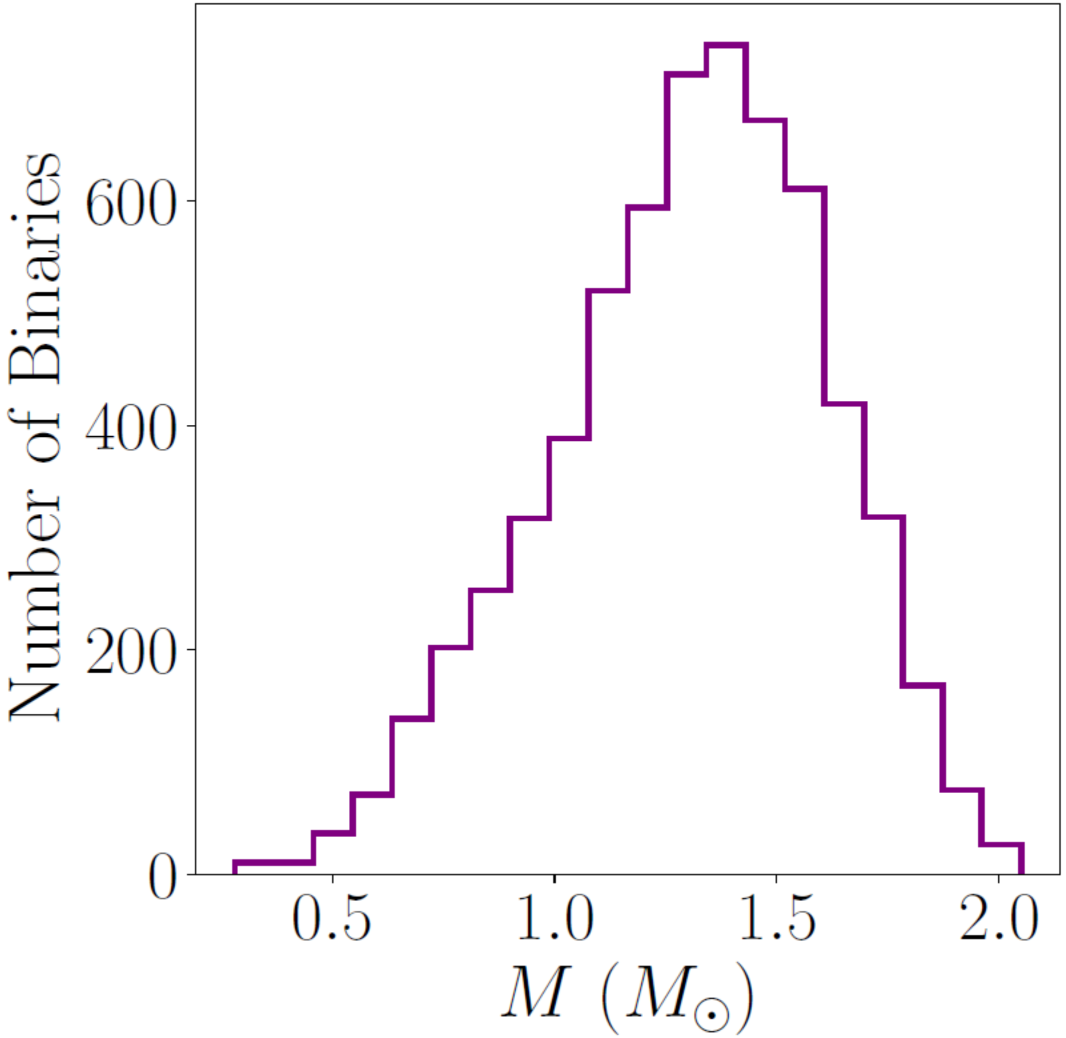
$$\phi_0(\psi_0|e_0) = \frac{1}{2\pi}(1 - e_0 \cos\psi_0)$$



Distribution of Binary Masses

Extinction-corrected using Bayestar 2015



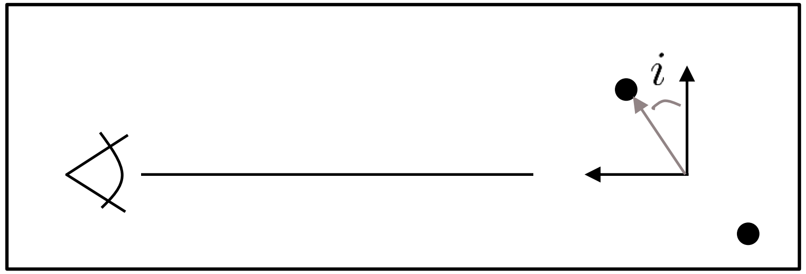


From $\vec{q}$ to s

$$\phi(\vec{q}) = \int d\vec{q}_0 \; Pr(\vec{q}_0 \to \vec{q}) \; \phi_0(\vec{q}_0)$$

- Projected physical separation

$$s = r \sin i$$



- Random orientations

$$p(i) = \cos i$$

- Distribution of Projected Separations

$$\phi(s) = \underbrace{\int d\sin i \int d\vec{q} \; \delta(s - r\cos i)}_{\text{Projects } \vec{q} \to s} \; \phi(\vec{q})$$

Calculating the Separation Distribution

Simulation:

1. Generate 10^4 binaries uniformly in bins of a_0
2. For each bin, evolve each binary for $T = 10$ Gyr:

$$\vec{q}_0 \to \cdots \to \vec{q}_N \equiv \vec{q}$$

3. Convert $\vec{q} \to s$
4. Generate histogram of s

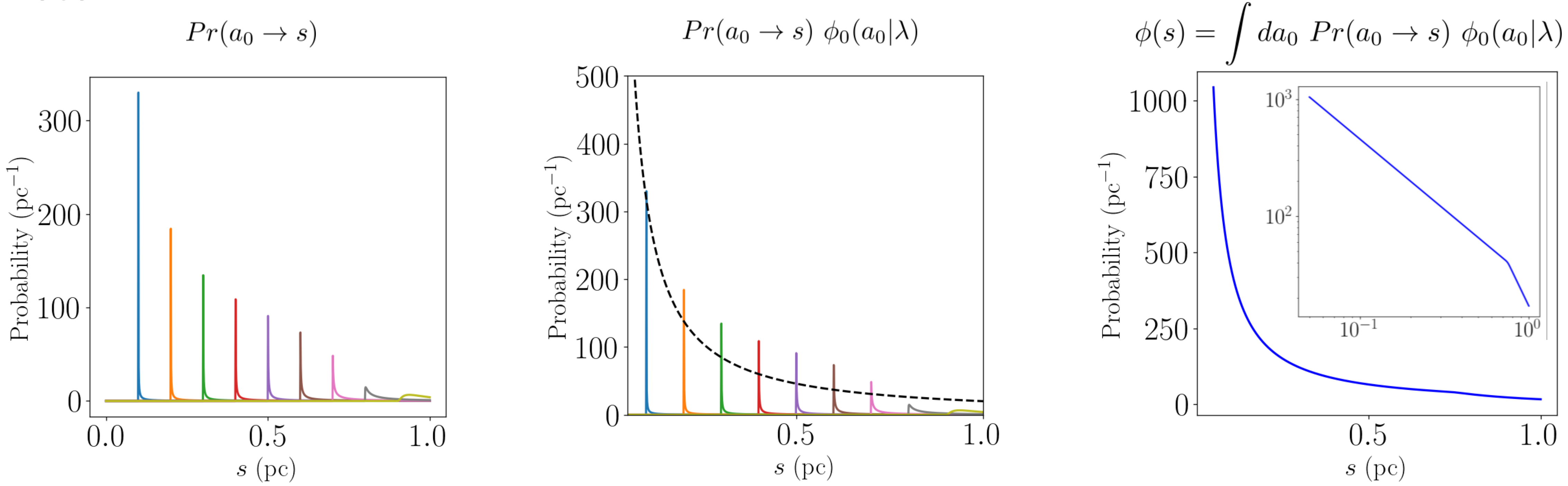
$$Pr(a_0 \to s)$$

Integration:

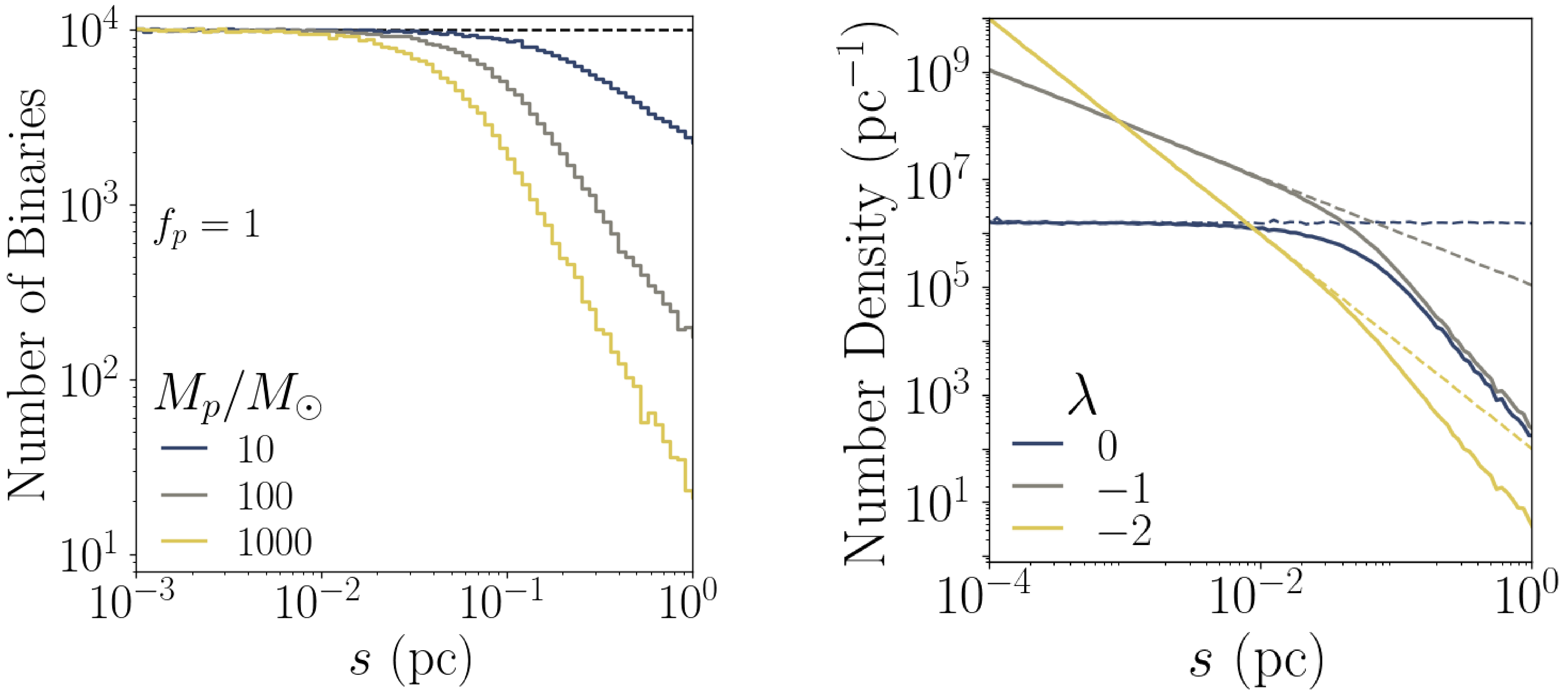
5. Specify power law index λ and integrate for $\phi(s)$

$$\phi(s) = \int da_0 \; \underbrace{Pr(a_0 \to s)}_{\text{Simulation}} \; \underbrace{\phi_0(a_0|\lambda)}_{\text{Free}}$$

Sketch:



Calculating the Separation Distribution



Statistical Methods

Statistical Methods

- **Summary:**
  - Dataset:

    Separations $\{s_i\} \to \vec{s}$
  - Model:

    Distribution of binary projected separations: $\phi(s|\vec{m})$ $[\vec{m} = (\lambda, f_p)]$

- **Goal:**
  - Set limits on dark matter substructure via the model parameter f_p

- **Idea:**
  - Given $\phi(s|\vec{m})$,

  ➡ Probability of obtaining the data given the model:

  $\mathcal{L}(\vec{s}|\vec{m})$ (Likelihood Function)

  - Bayes' Theorem,

  ➡ Probability of what the true model is given the data:

  $$\mathcal{L}(\vec{m}|\vec{s}) = \frac{\mathcal{L}(\vec{s}|\vec{m})\ \pi(\vec{m})}{\int d\vec{m}'\ \mathcal{L}(\vec{s}|\vec{m}')\ \pi(\vec{m}')}$$ (Posterior Distribution)

  - Upper Limit on Parameter $\vec{m}$,

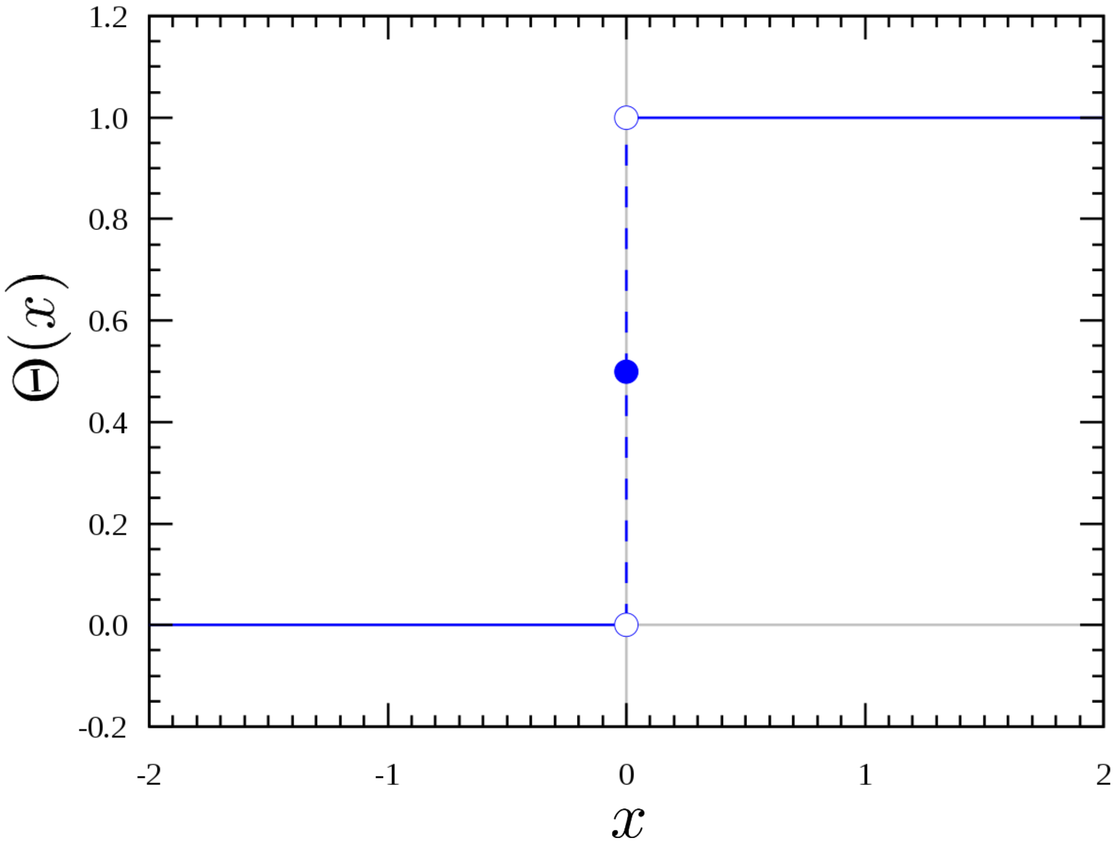
  ➡ 95% probability bound on $\vec{m}$

Additional Modelling: Detection

- So far, $\phi(s|\vec{m})$ gives only the probability of **existence**
- Recall:
  - Dataset roughly complete, but we select binaries with angular separations $\theta > \theta_{\Delta G}$
- Probability of **Detection:**

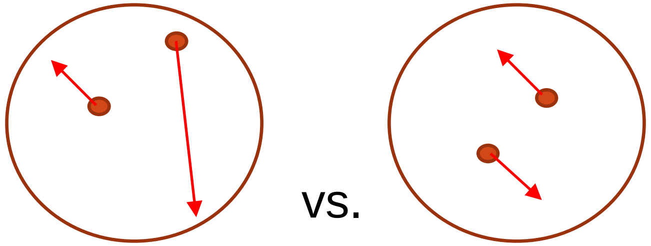
$$p(s|d, \Delta G; \vec{m}) \propto \phi(s|\vec{m})\ \Theta(s/d - \theta_{\Delta G})$$

Performs completeness selection cut



Additional Modelling: Contamination

- Recall:
  - Dataset may be slightly contaminated by chance alignments

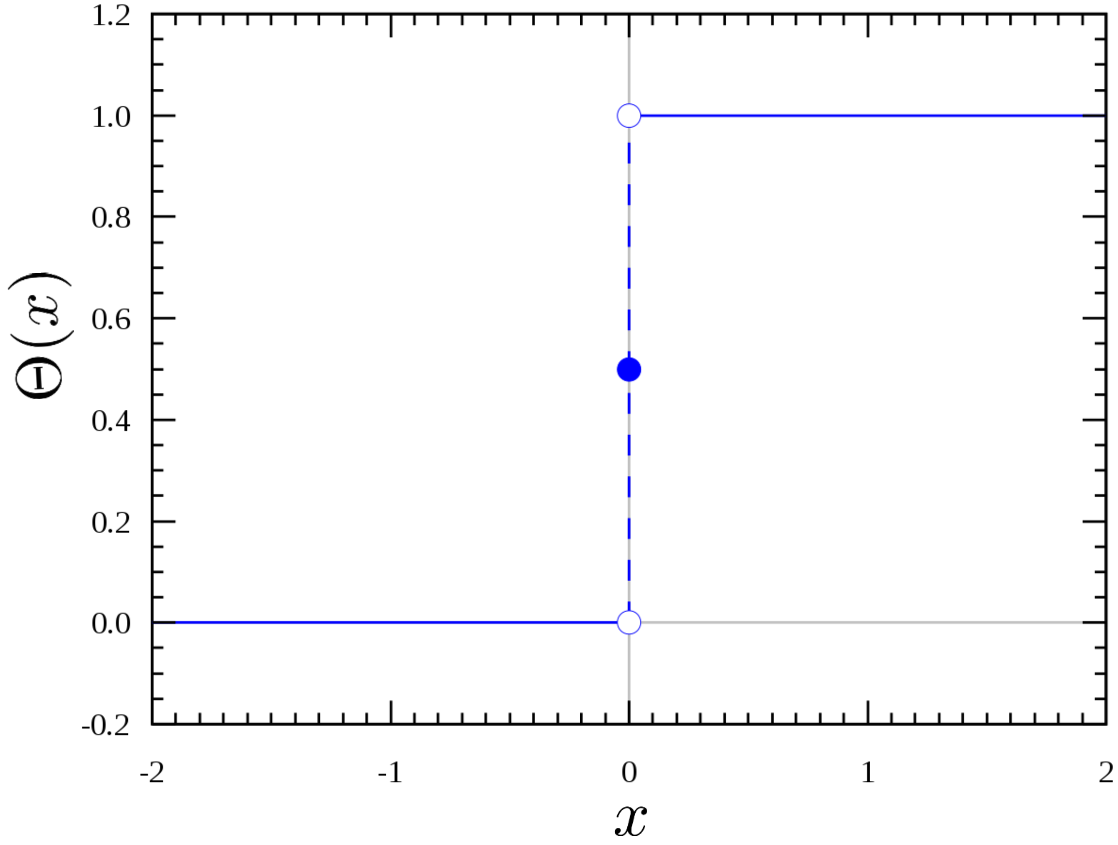


- Chance alignment model:
  - Obey a power-law separation distribution:

$$\phi_c(s|\lambda_c) \propto s^{\lambda_c}$$

- Detection model:

$$p_c(s|d, \Delta G; \lambda_c) \propto \phi_c(s|\lambda_c)\ \Theta(s/d - \theta_{\Delta G})$$



Likelihood Function

- Probability of Detecting a Binary OR Chance Alignment

$$\mathcal{P}(s|d,\Delta G,\mathcal{R};\vec{m},\lambda_c) = \underbrace{(1-\mathcal{R})\ p(s|d,\Delta G;\vec{m})}_{\text{Binary Detection Probability}} + \underbrace{\mathcal{R}\ p_c(s|d,\Delta G;\lambda_c)}_{\text{Chance Alignment Detection Probability}}$$

  - $\mathcal{R}$: Chance alignment probability

- Likelihood Function

$$\mathcal{L} = \prod_i \mathcal{P}(\underbrace{s_i|d_i,\Delta G_i,\mathcal{R}_i}_{\text{Data}},\underbrace{\vec{m},\lambda_c}_{\text{Model Parameters}})$$

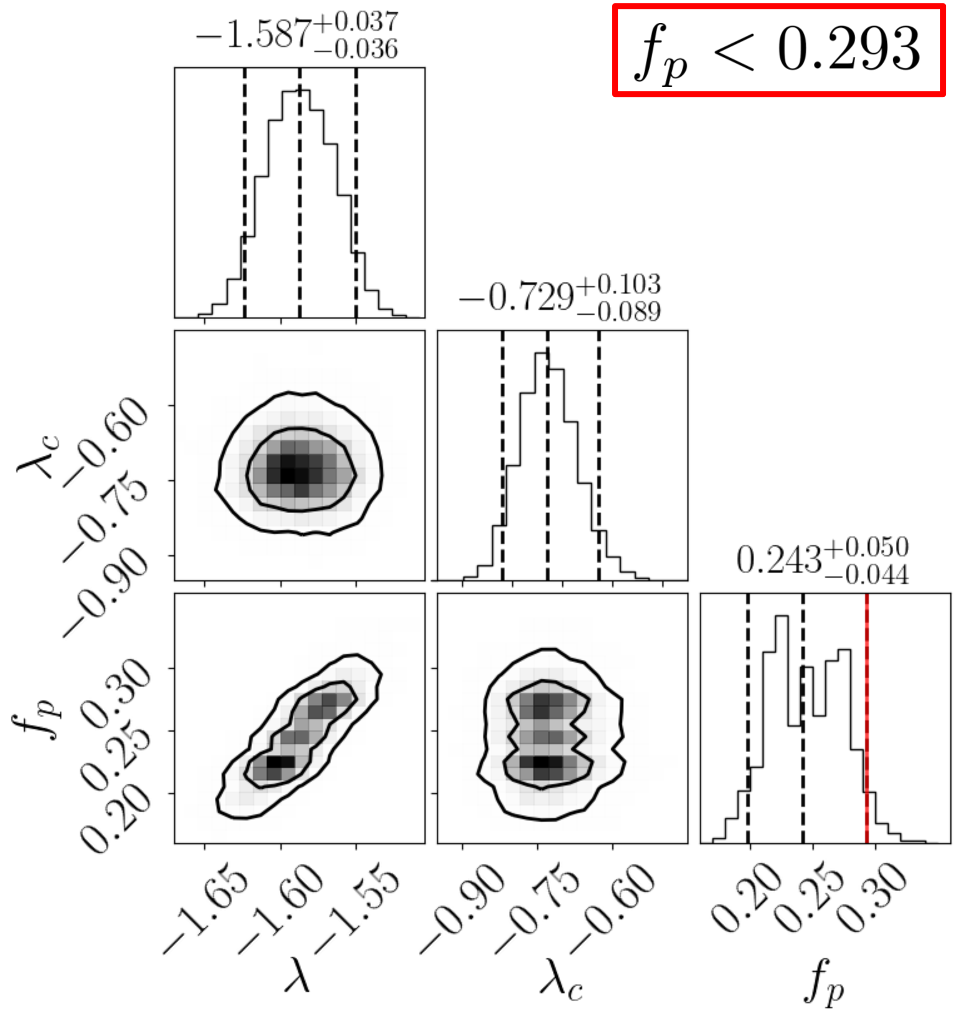
- Posterior estimated by numerical (MCMC) sampling
  - Limits on model parameters are reported as 95% probability bounds

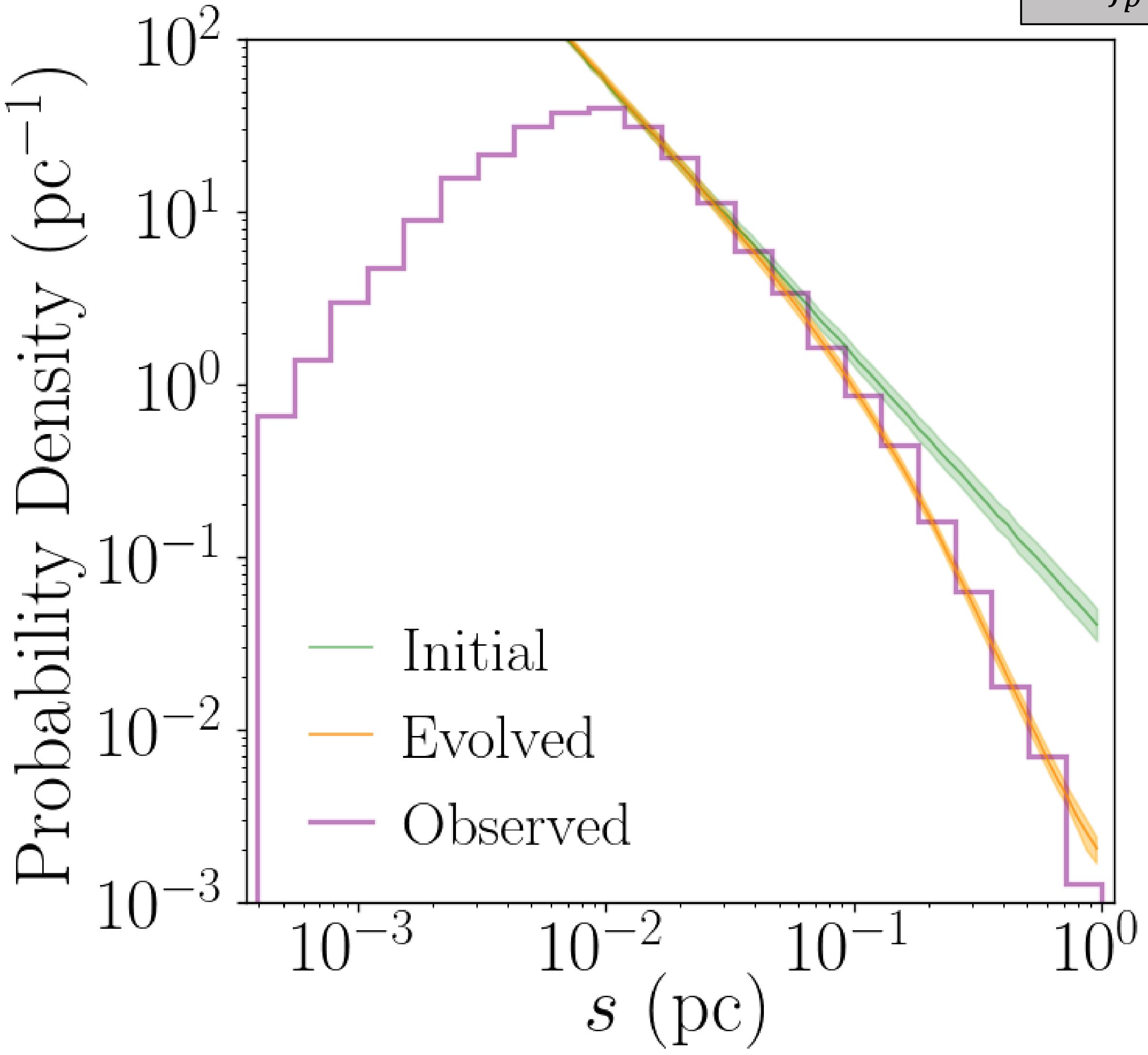
Limits

Example: Limits

Perturbers:
$M_p = 10^3\ M_\odot$
$R_p = 0.1$ pc
$\rho(r) = constant$
$f_p = Free$

$\boxed{f_p < 0.293}$





Limits on Uniform-Density Subhalos

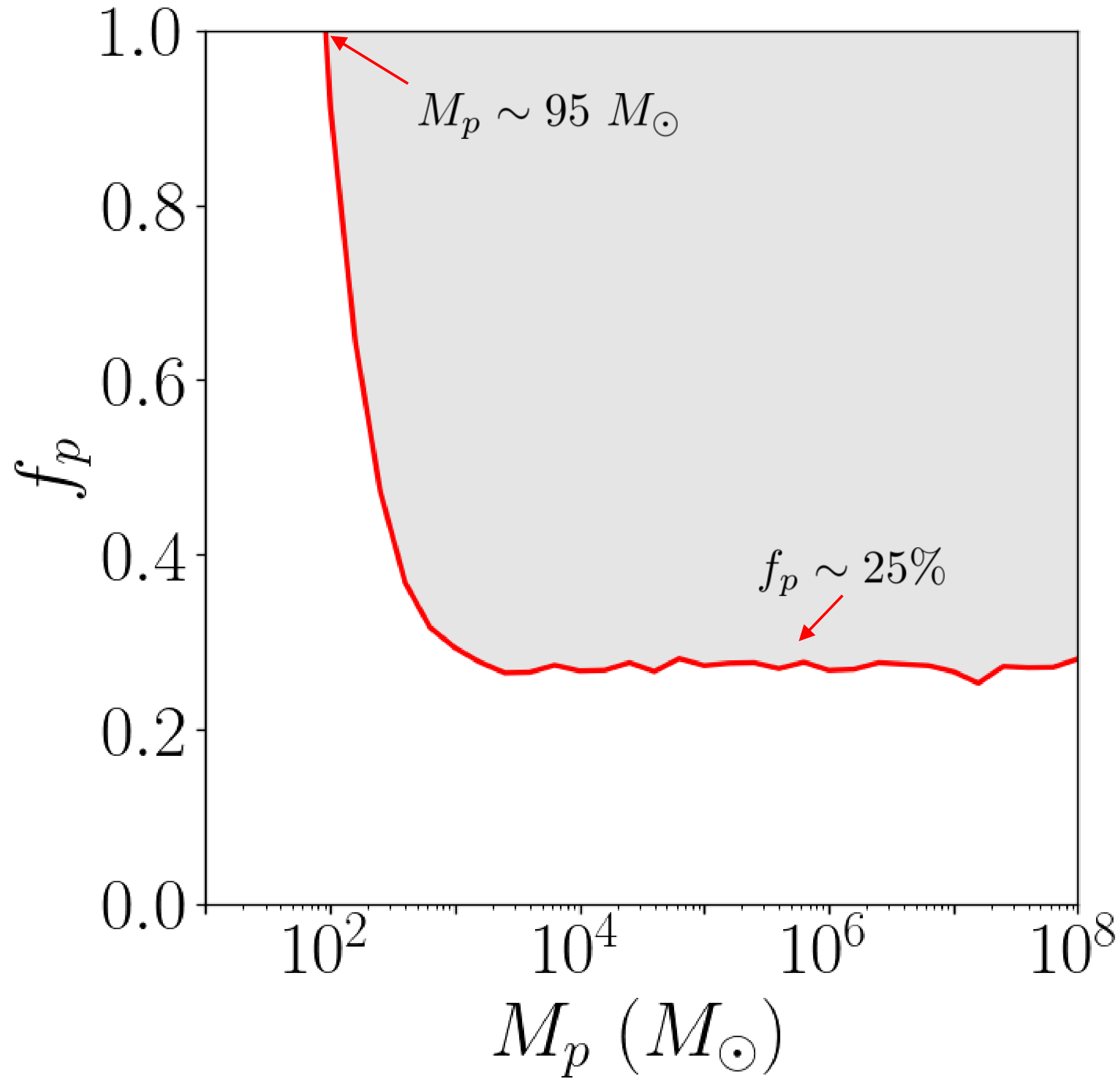
- Before:

$$\begin{cases} M_p = 10^3\ M_\odot \\ R_p = 0.1\ \mathrm{pc} \\ \rho(r) = \mathrm{constant} \end{cases}$$

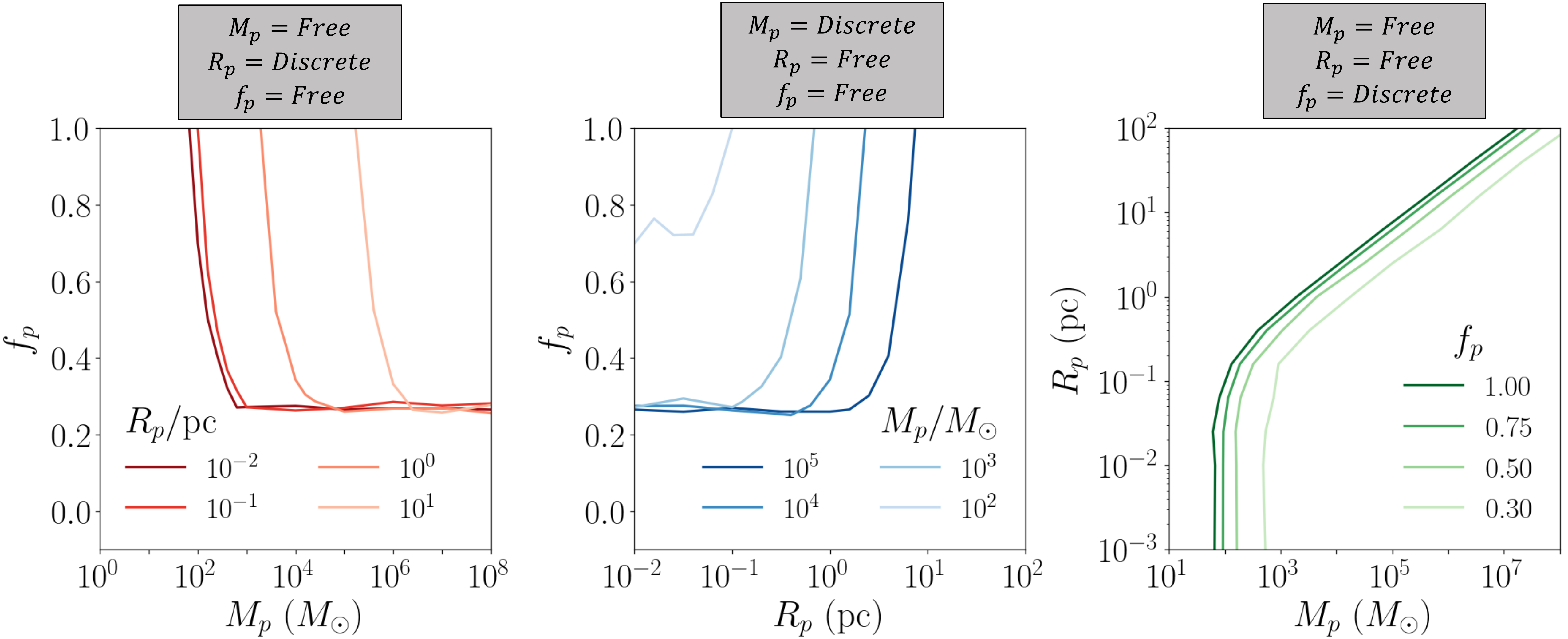
- Now:

  Allow M_p to vary

  - $M_p > 95\ M_\odot$ cannot make up all the dark matter (at 95% level)
  - Can make up at most 25% of dark matter



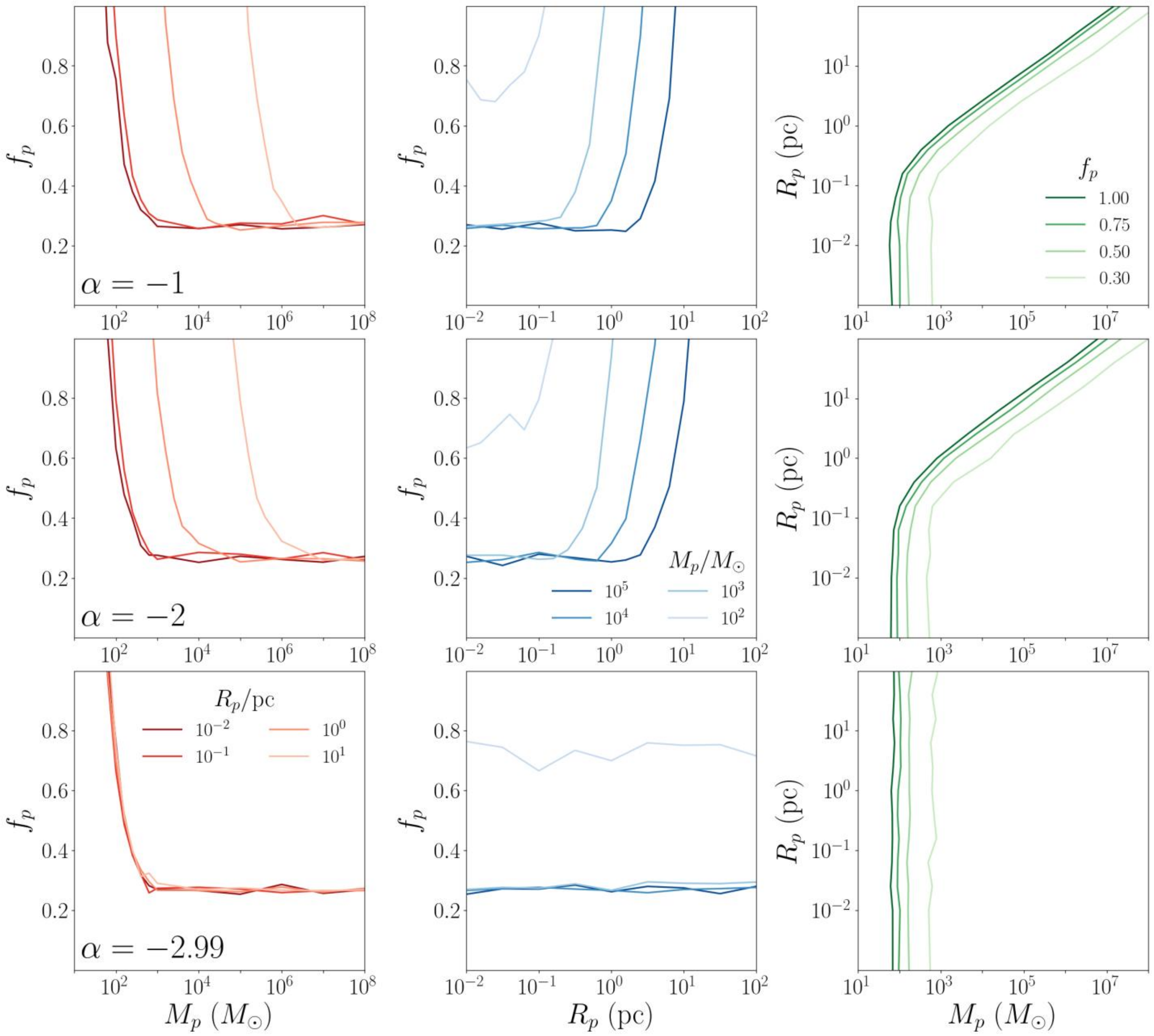
Limits on Uniform-Density Subhalos



Perturbers with Power-Law Density Profiles

- How do limits change with density profile?
  - Consider power-law density profiles:

$$\rho(r;\alpha) = \begin{cases} \rho_0 \left(\frac{r}{R_p}\right)^{\alpha} & , r \leq R_p \\ 0 & , r > R_p \end{cases}$$

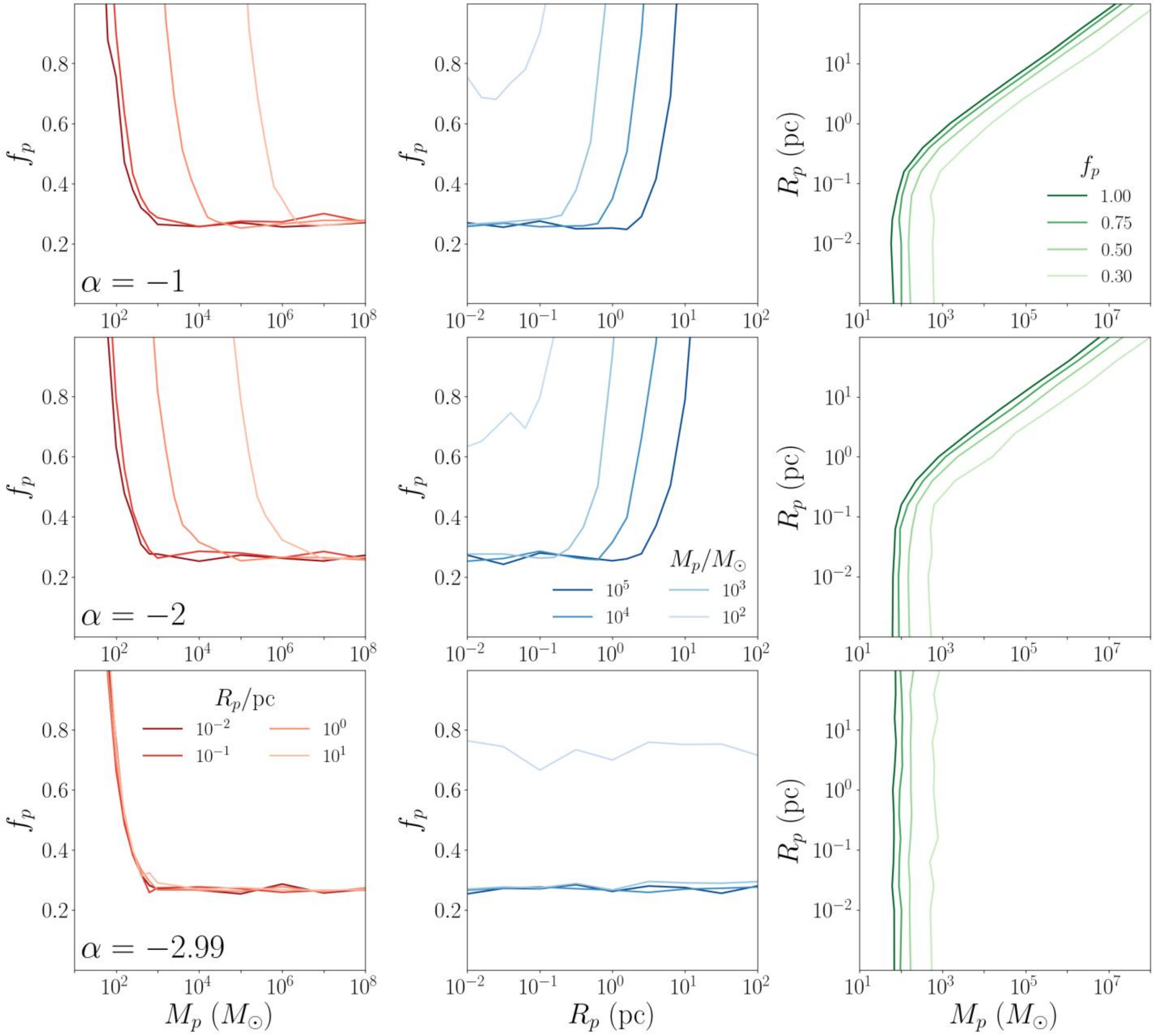


Perturbers with Power-Law Density Profiles

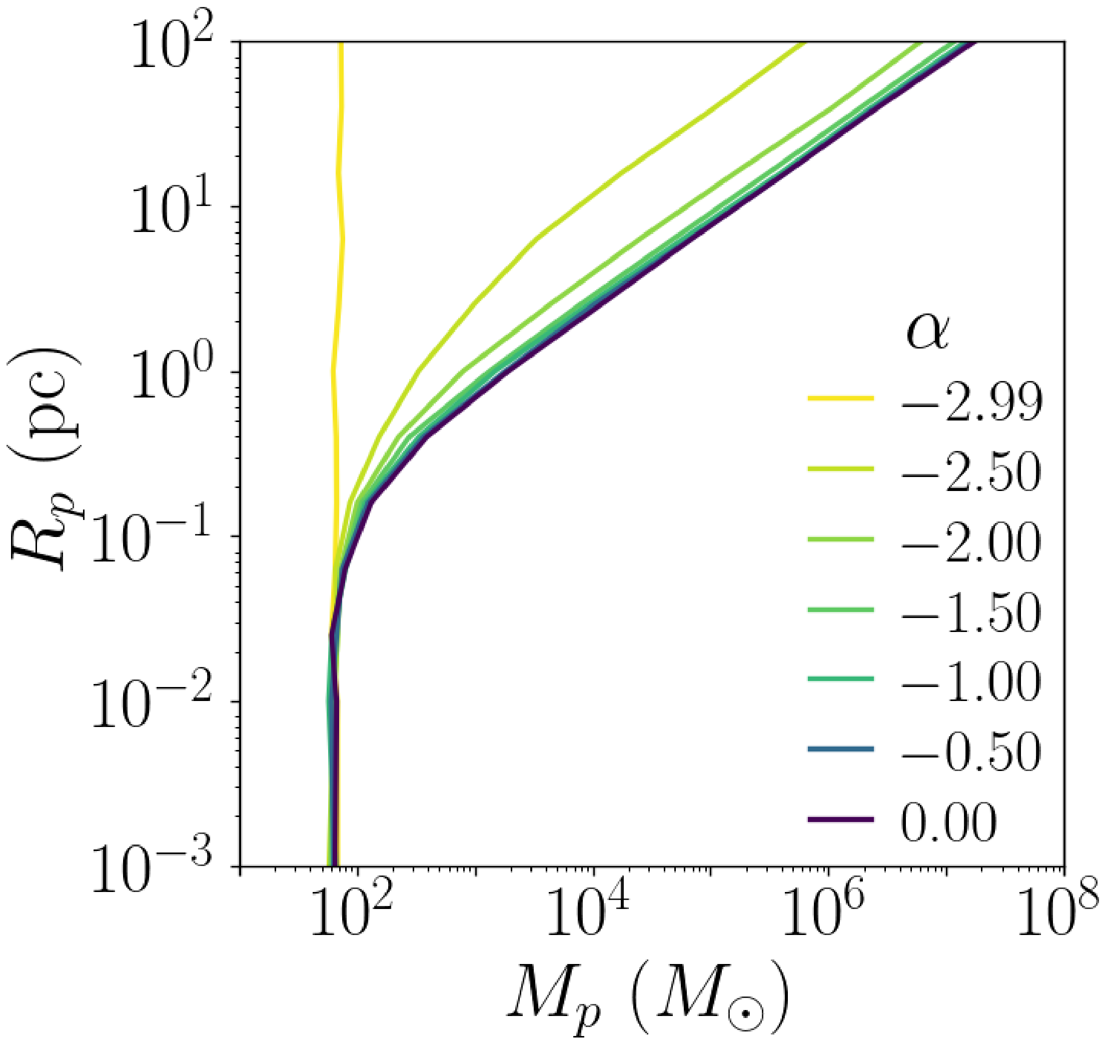
- How do limits change with density profile?
  - Consider power-law density profiles:

Sets mass

$$\rho(r;\alpha) = \begin{cases} \overline{\rho_0} \left(\frac{r}{R_p}\right)^{\alpha} & , r \leq R_p \\ 0 & , r > R_p \end{cases}$$



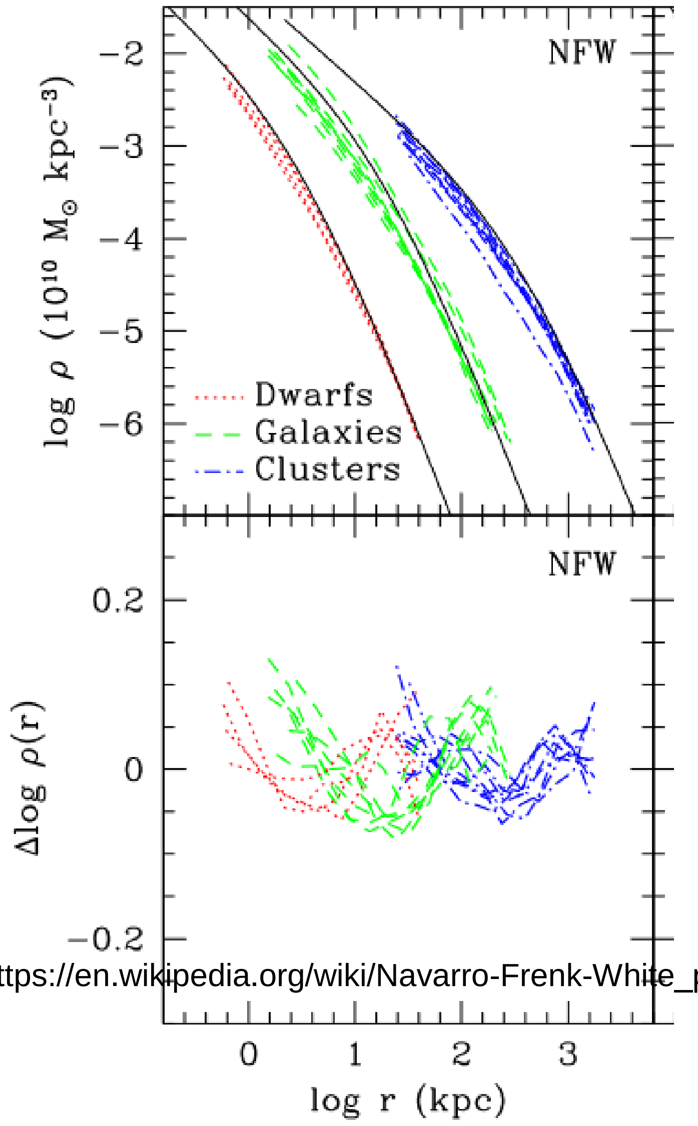
Perturbers with Power-Law Density Profiles



$f_p = 1$ contours

Milky Way-like Subhalos

[1606.07790]

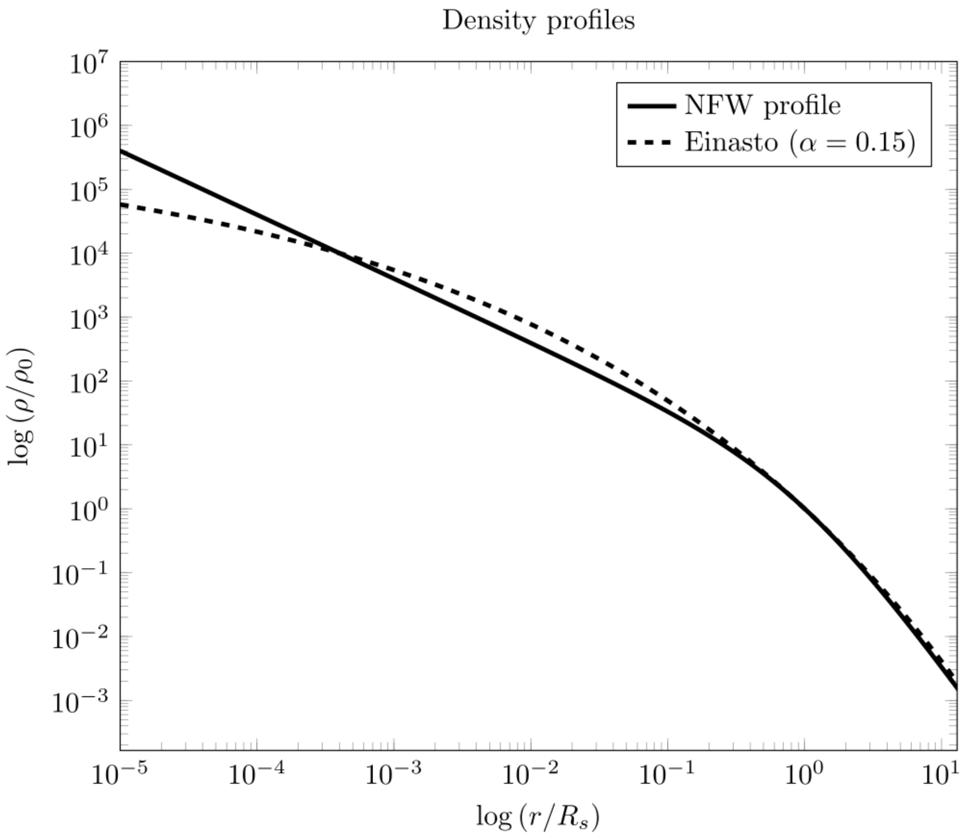


https://en.wikipedia.org/wiki/Navarro-Frenk-White_profile

- NFW Profile

$$\rho_{\mathrm{NFW}}(r) = \rho_0 \left(\frac{r}{R_s}\right)^{-1} \left(1 + \frac{r}{R_s}\right)^{-2}$$

Milky Way-like Subhalos



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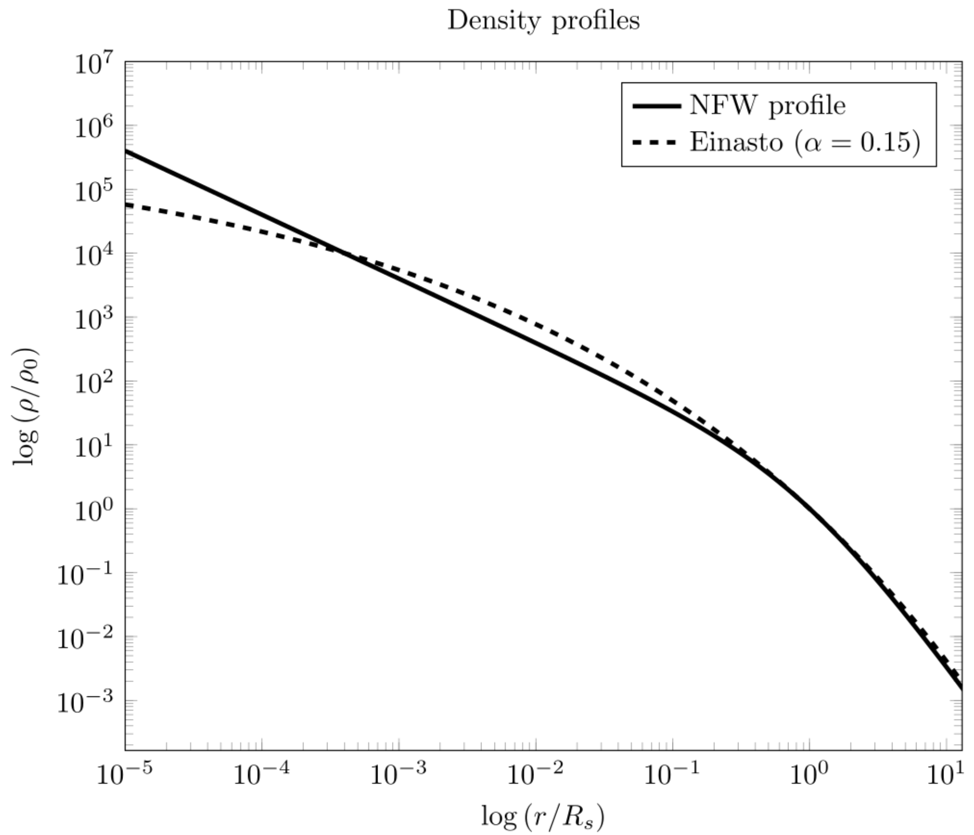
Milky Way-like Subhalos



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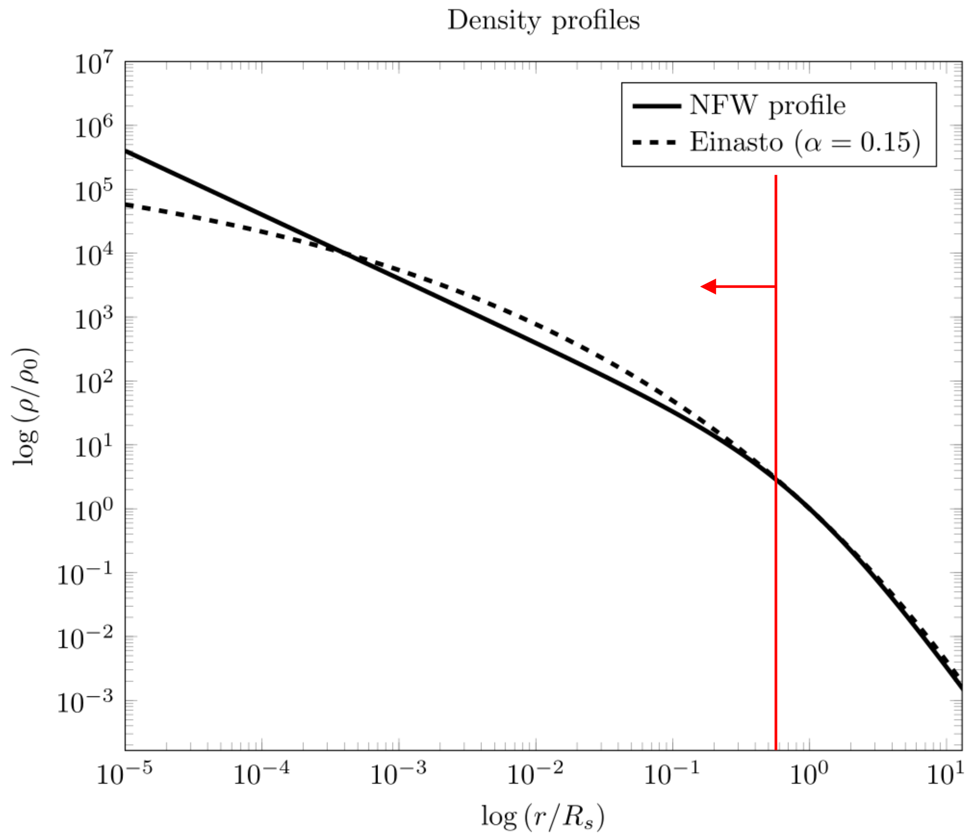
- NFW Profile

$$\rho_{\mathrm{NFW}}(r) = \underbrace{\rho_0}_{\text{Sets mass}} \left(\frac{r}{R_s}\right)^{-1} \left(1 + \frac{r}{R_s}\right)^{-2}$$

Milky Way-like Subhalos



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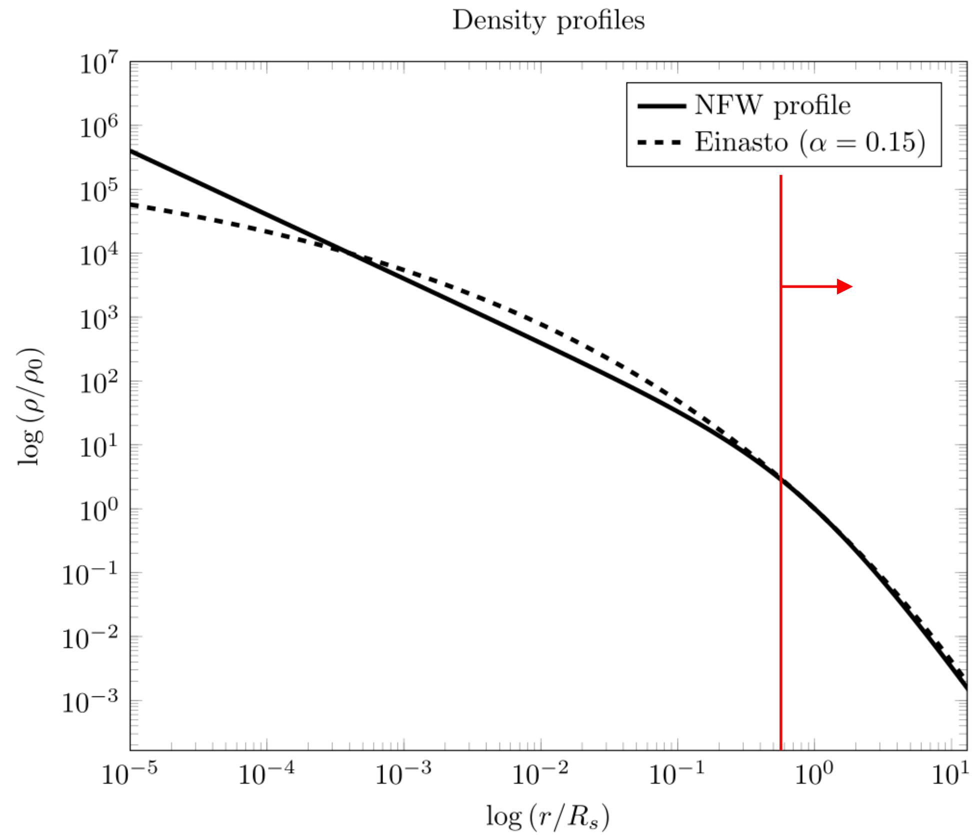
- NFW Profile

$$\rho_{\mathrm{NFW}}(r) = \rho_0 \left(\frac{r}{R_s}\right)^{-1} \left(1 + \frac{r}{R_s}\right)^{-2}$$

$r \ll R_s$:

$\rho(r) \sim r^{-1}$

Milky Way-like Subhalos

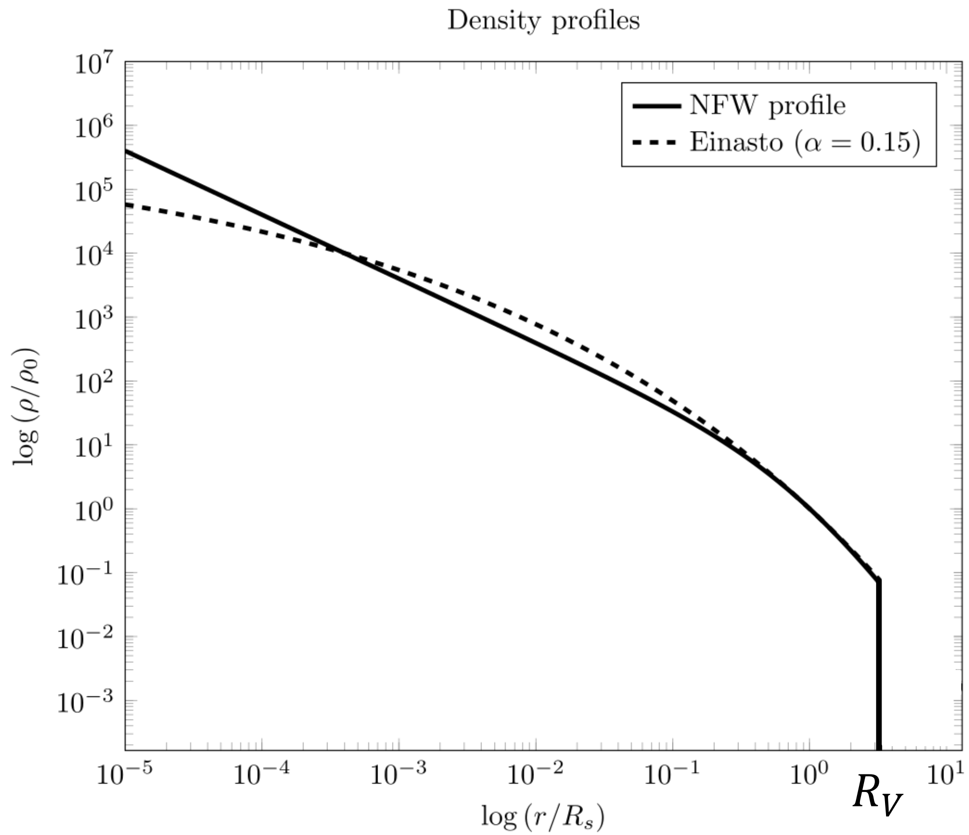


https://en.wikipedia.org/wiki/Navarro-Frenk-White_profile

- NFW Profile

$$\rho_{\mathrm{NFW}}(r) = \rho_0 \left(\frac{r}{R_s}\right)^{-1} \underbrace{\left(1 + \frac{r}{R_s}\right)^{-2}}_{\substack{r \gg R_s: \\ \rho(r) \sim r^{-3}}}$$

Milky Way-like Subhalos



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- NFW Profile

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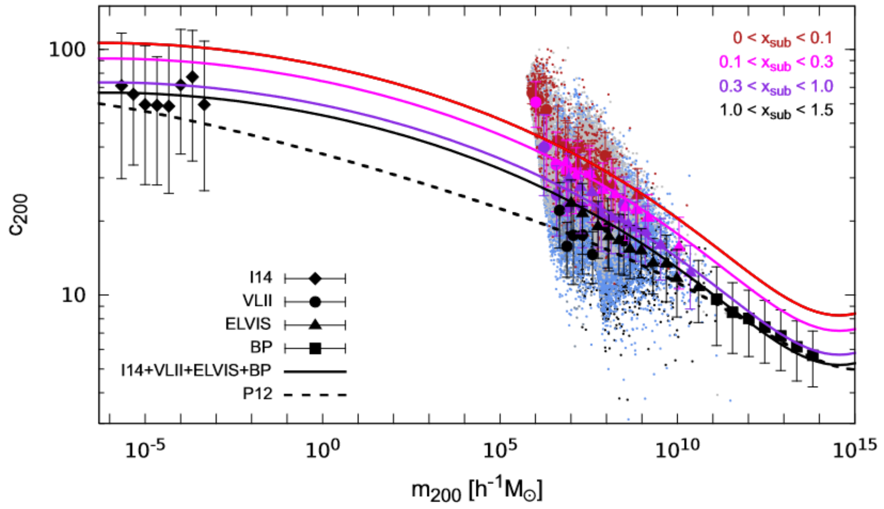
Issue:
For mass to be finite, we truncate profile at the radius R_V

$$M_V = \int_0^{R_V} 4\pi r^2 \rho_{\mathrm{NFW}}(r) dr$$

Milky Way Subhalos from Simulation

- VL-2 and ELVIS subhalo simulations
  - R_s, M_V are correlated
  - Density specified by R_V, M_V
  - Caveats
- Set limits on subhalos with mass and density profiles consistent with simulations



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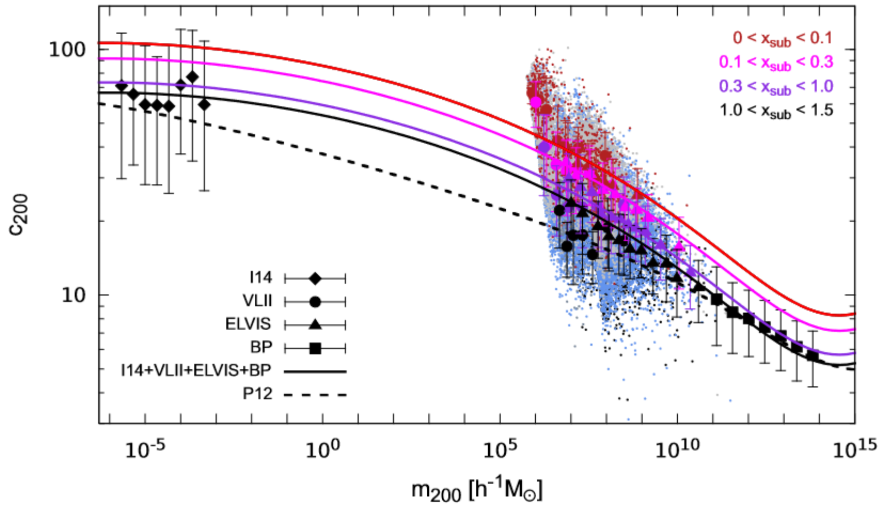

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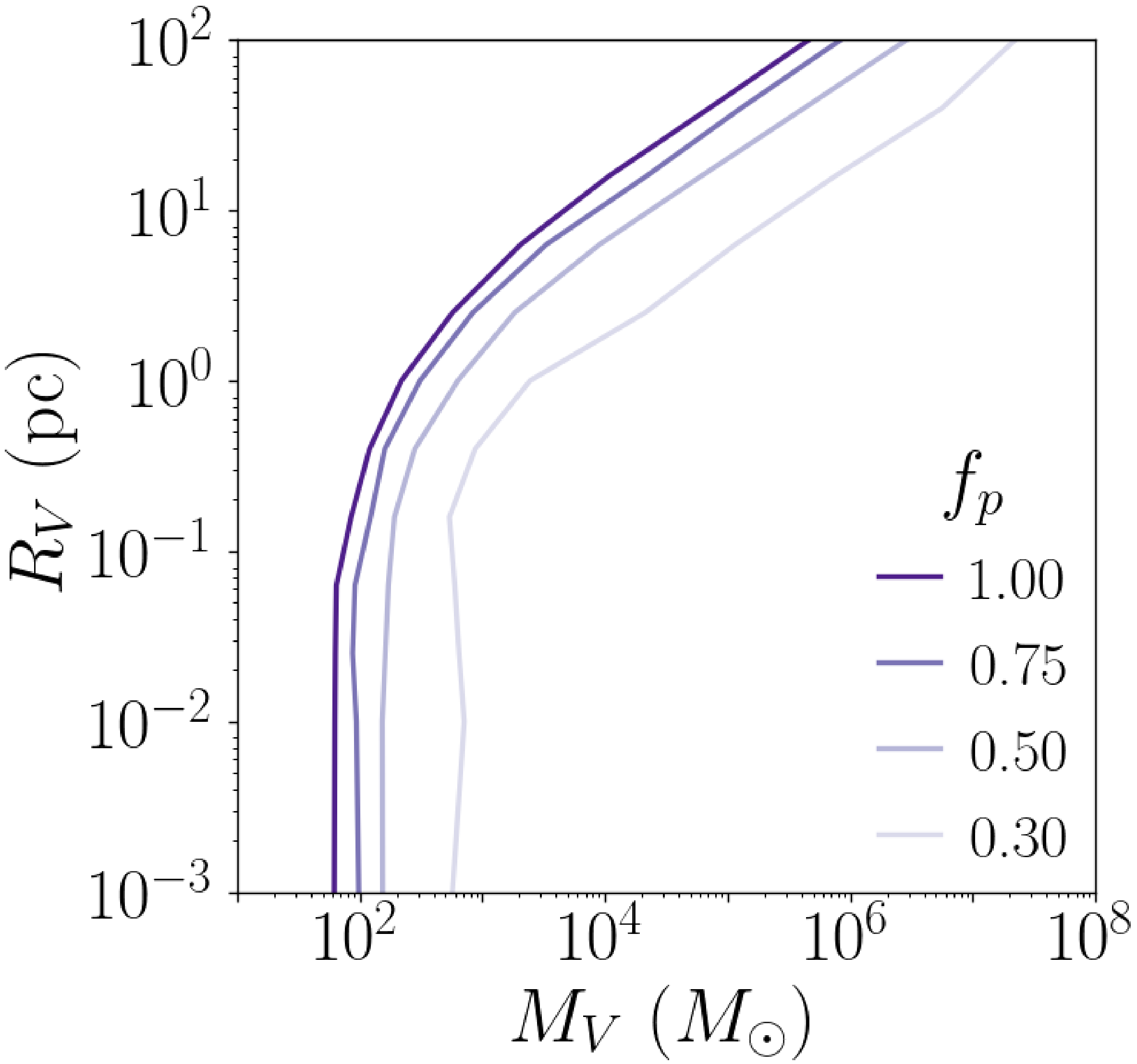
Limits on Milky Way-like Subhalos



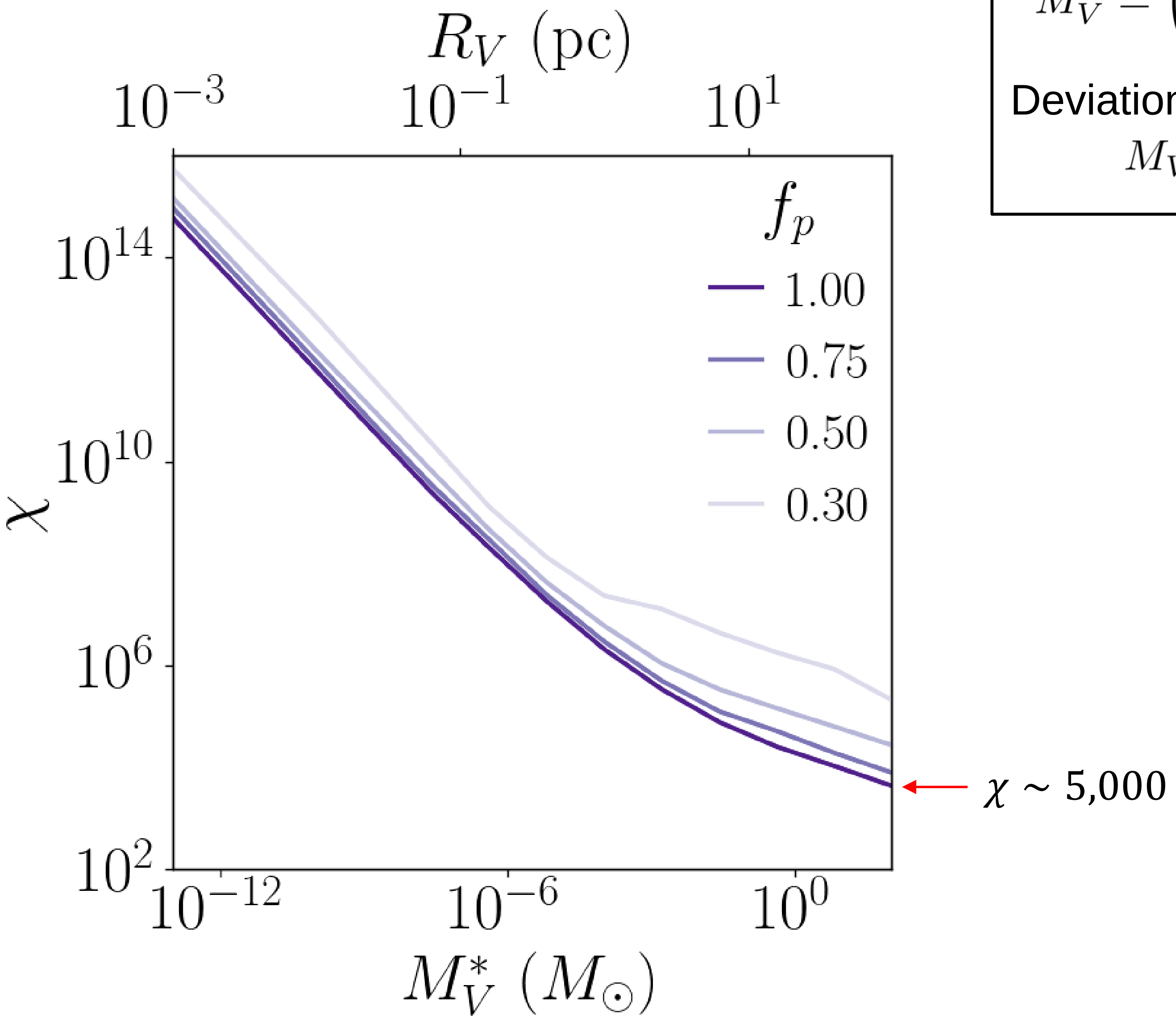
Limits on Milky Way-like Subhalos

Canonical NFW Mass:

$$M_V^* = \left(\frac{4\pi R_V^3}{3}\right)\rho_c\Delta$$

Deviation from Canonical:

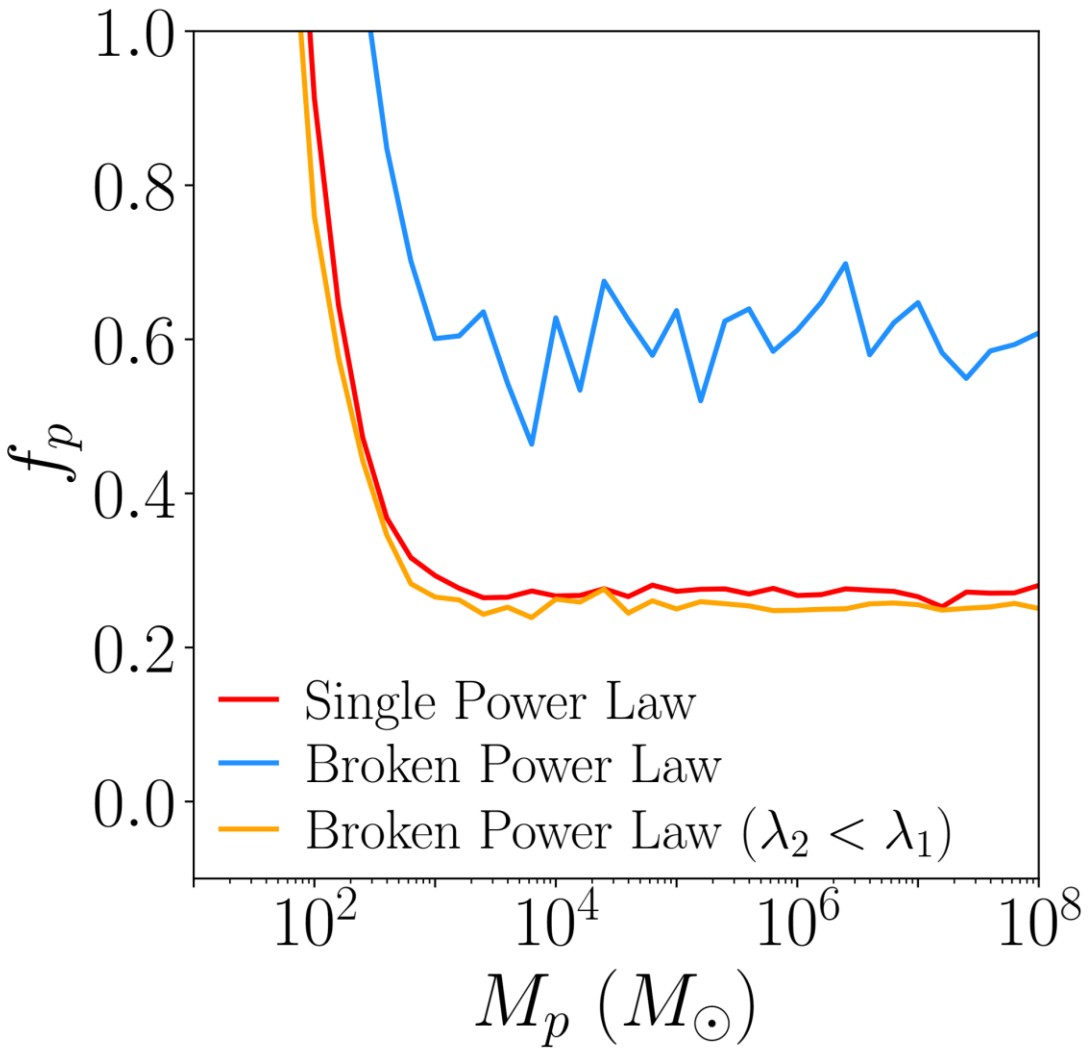
$$M_V \equiv \chi M_V^*$$



Extras

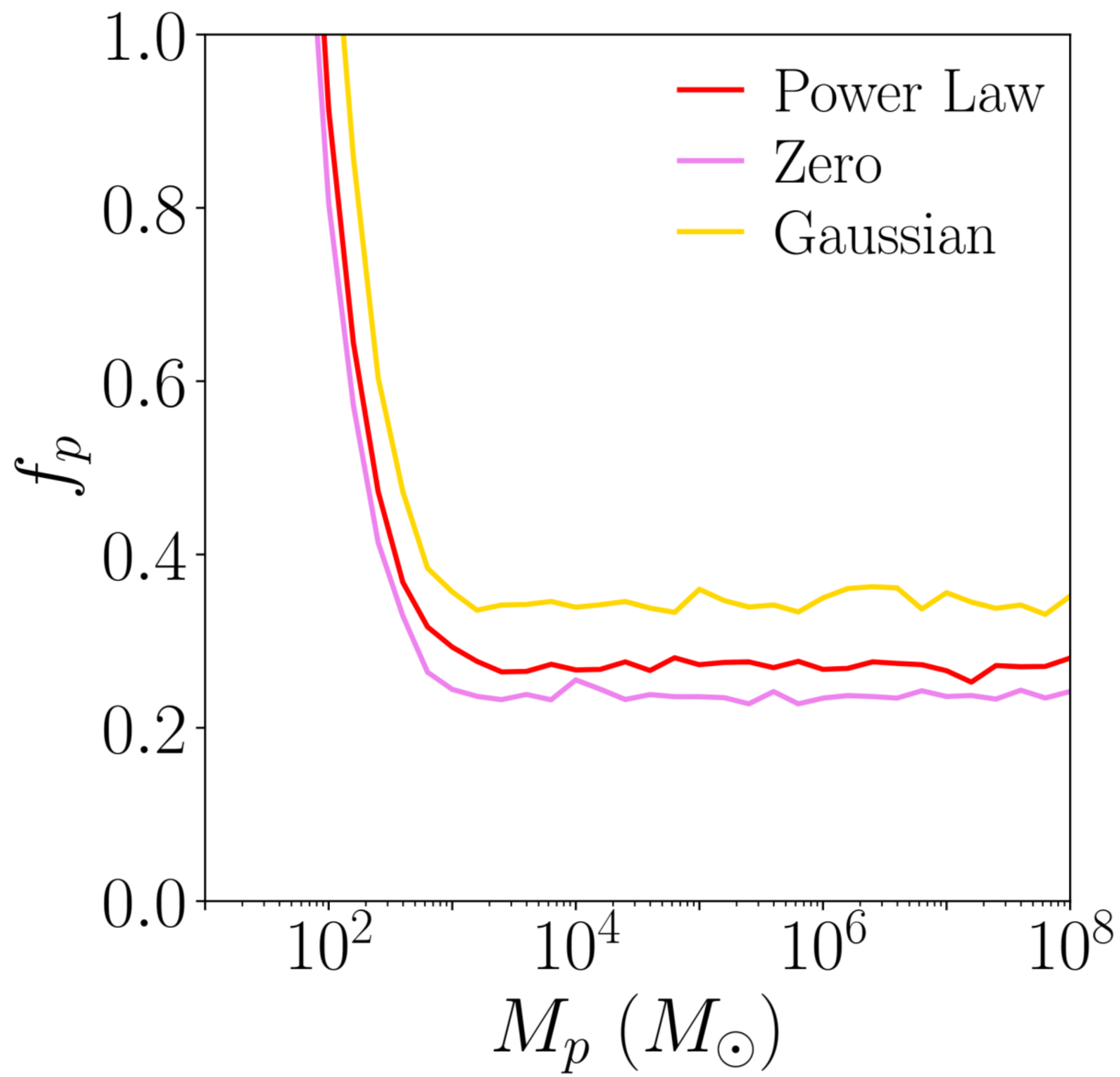
Alternative Models for the Initial Semimajor Axis Distribution

$$\phi_0(a_0) \propto \left(\frac{a_0}{a_b}\right)^{\lambda_1} \left\{\frac{1}{2}\left[1+\left(\frac{a_0}{a_b}\right)^{1/\Delta}\right]\right\}^{(\lambda_2-\lambda_1)\Delta}$$



Alternative Chance-alignment Modelling

$$\phi_c(s) = \frac{1}{\sqrt{2\pi\sigma_c^2}} \exp\left[-\frac{1}{2}\left(\frac{s-\mu_c}{\sigma_c}\right)^2\right]$$



Extension to Arbitrary Mass Functions

- Can rewrite Monte Carlo simulations to generate subhalos with non-monochromatic mass functions

$$\psi(M_p) \propto M_p \; dn/dM_p \quad : \quad f_\psi \equiv \int dM_p \; \psi(M_p)$$

- Alternative: Derive non-monochromatic constraints from the monochromatic functions

$$f_p(M_p) \leq f_{\max}(M_p),$$

$$\int dM_p \frac{\psi(M_p)}{f_{\max}(M_p)} \leq 1$$