Zero Modes from Massive Fermions and Axion Strings

Katherine Fraser

Department of Physics
Harvard University
kfraser@g.harvard.edu

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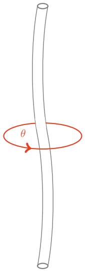
Based on:

[2305.XXXXX with J. Stout, S. Homiller, H. Bagherian]

Outline

1. Intro to Axion Strings
2. Axion String Superconductivity
3. Adding a fermion mass



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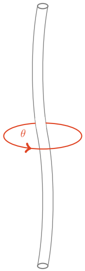
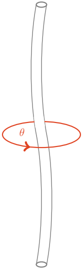


What are Axion Strings?

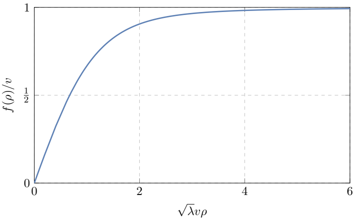
- Axion strings are dynamical objects around which the axion field winds from 0 to 2π.
- We study solitonic axion strings: field configurations that are topologically stable.
- A simple example is the theory



$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |\partial_\mu \Phi|^2 - \lambda(|\Phi|^2 - v^2)^2$$

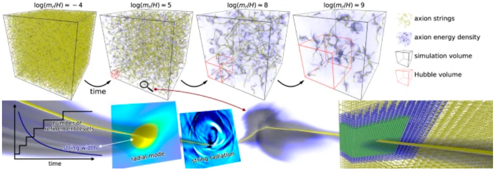
After SSB, there are axion strings $\Phi_n(x) = f(r)e^{in\varphi}$.

The phase of the scalar (the axion) winds around its field space as we move around the string.



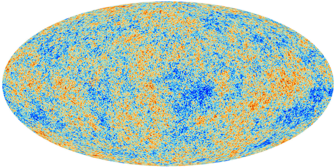
Potential Signals

Axion String Dynamics and Dark Matter Formation



[Buschmann et al: 2108.05368]

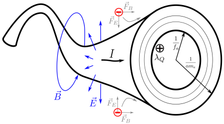
CMB Signals



[Agrawal et al, 1912.02823]

[Image from Planck Satellite]

Zero Mode Collisions



[Agrawal et al, 2010.15848]

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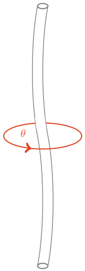
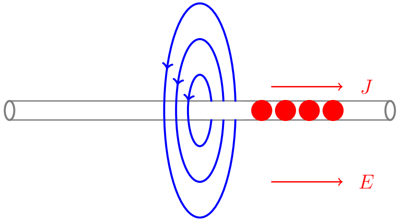


Axion Strings are Superconducting

What happens to bosons and fermions in the axion string background?

- In both cases, there can be massless trapped modes on the string
- These modes lead to superconductivity (a current that grows with imposed electric current)

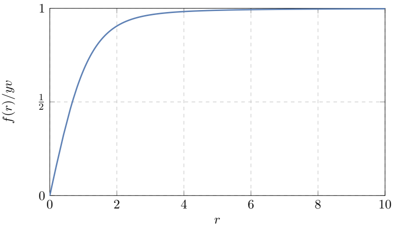


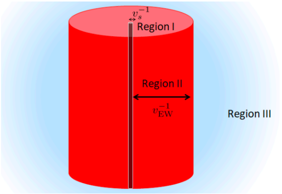
[Witten, Nuclear Physics B 1985]
[Fukuda et al, 2010.02763]

Why do we expect massless modes?

We can formally show superconductivity explicitly using the effective action, or anomaly inflow.

A common explanation is that bulk particles become massless at the core of the string. Is this intitution correct?





For the DFSZ axion, there are electroweak axion strings that superconduct but the EW symmetry is not restored in the core.

[Callan & Harvey, Nuclear Physics B, 1985]
[Harvey & Ruchayskiy, hep-th/0007037]
[Abe, Hamada, Yoshioka, 2010.02834]

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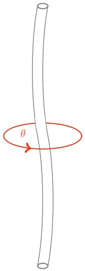


Adding a Mass

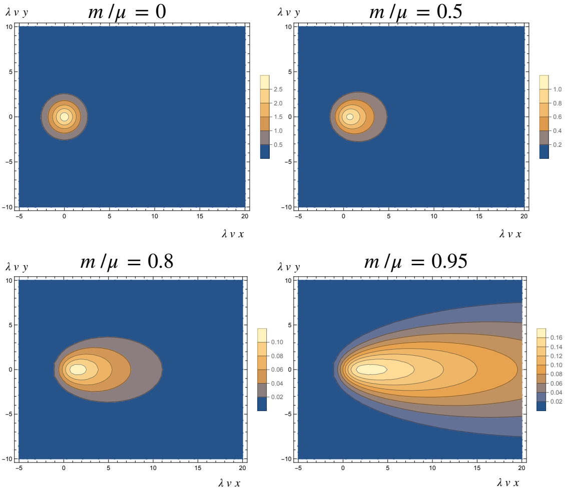
Symmetry restoration is not necessary! There can be zero modes even when bulk particles remain massive everywhere if the explicit PQ breaking $\underbrace{\text{isn't too large}}_{m/\mu \,\leq\, 1 \text{ for } \mu \,=\, y v}$.

<u>Example</u>: massive Dirac fermion in an axion string background

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\not{D} - m)\psi + |\partial_\mu \Phi|^2 + y\overline{\psi}(\Phi_1 + i\gamma^5\Phi_2)\psi - \lambda(|\Phi|^2 - v^2)^2$$

We study trapped fermions by solving the Dirac equation.

Results with a mass



- As we turn on a Dirac mass for the fermions, the zero mode stretches out away from the string, until the critical point $m = \mu$.
- In the critical case ($m = \mu$), the zero modes live on a wedge in the x-y plane about $\varphi = 0$.
- Above the critical point, the zero modes vanish.

Anomaly Inflow: The Big Picture

In the presence of the axion string, the U(1) gauge theory is anomalous unless additional degrees of freedom localized to the string can cancel the anomaly. This is anomaly inflow.

To determine when we have anomaly inflow, we will

1. Integrate out the heavy fermions to get an EFT for the gauge field and axion
2. Do a gauge transformation in the EFT to find the anomaly.

We need zero modes when the path integral in the EFT is not gauge invariant.

[Callan & Harvey, Nuclear Physics B, 1985]

When is there an anomaly?

There is an anomaly when the path integral is not gauge invariant. Since

$$Z_\psi(\varphi) \sim \exp\left[\frac{i}{8\pi^2}\int \delta(\rho,\varphi) F \wedge F\right]$$

gauge invariance is determined by whether $\delta(\varphi) = \arg(m - \mu e^{i\varphi})$ is single or multivalued

If $\delta(\varphi)$ is single valued:

- $\frac{1}{8\pi^2}\int \delta(\varphi) F \wedge F$ is well-defined and gauge invariant

If $\delta(\varphi)$ is multi-valued:

- Need to integrate by parts for a well-defined answer
- Under the U(1) transformation $\delta_\Lambda \psi = ie\Lambda(x)\psi$, $\delta_\Lambda A_\mu = \partial_\mu \Lambda(x)$, the integral is not gauge invariant

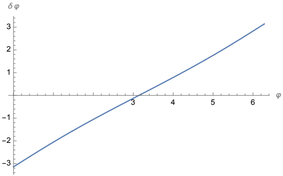
$$-\frac{1}{8\pi^2}\int d\delta \wedge A \wedge F \to \frac{1}{8\pi^2}\int d^2\delta(\varphi) \wedge \Lambda F \neq 0$$

since $d^2\delta(\varphi) \neq 0$ in the presence of an axion string.

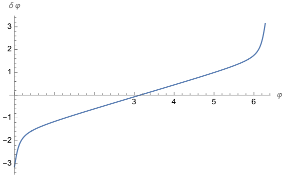
When is there an anomaly?

We see $\delta(\varphi) = \arg(m - \mu e^{i\varphi})$ is multi-valued for $m/\mu \leq 1$ and single valued for $m/\mu > 1$. Therefore, we get zero modes only for $m/\mu \leq 1$, but not for $m/\mu > 1$.

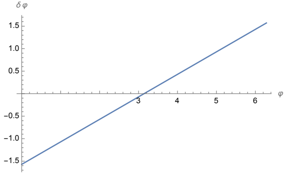
$m/\mu = 0.1$



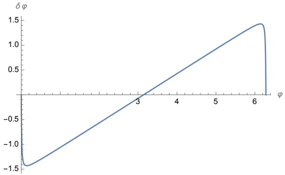
$m/\mu = 0.91$



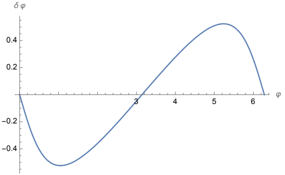
$m/\mu = 1$



$m/\mu = 1.01$



$m/\mu = 2$



Summary and Future Directions

Axion strings can have massless modes even when bulk fermions are massive. In the simplest case, these zero modes become less localized to the string as the mass is increased, up until a critical value where they disappear.

We expect similar arguments can help us understand axion strings in DFSZ models. This can tell us about the existance and profiles of zero modes in phenomologically interesting models, which could have physical consequences.

Back Up Slides

Solving PDEs

- Chebyshev interpolation is a useful tool for solving partial differential equations.
- It allows us to turn PDEs into algebraic matrix equations by expanding in a convenient set of polynomials and discretizing

$$0 = F_1(\psi_0, \psi_3, \partial_i \psi_3)$$
$$0 = F_2(\psi_0, \psi_3, \partial_i \psi_0)$$

$$\psi_i(\zeta) = \sum_k \overbrace{\psi_{i,k}}^{\text{Constants}} p_k(\zeta) \longrightarrow$$

Matrix Equation; solve for constants $\psi_{i,k}$ numerically

Sum over Chebyshev Nodes

$$\zeta_k = \cos\left(\frac{\pi(2k+1)}{2(N+1)}\right), \text{k} = 0, \ldots, \text{N}$$

Use Chebyshev polynomials:

$$p_k(\zeta) = \frac{\Pi_{k\neq n}(\zeta - \zeta_k)}{\Pi_{k\neq n}(\zeta_n - \zeta_k)}$$

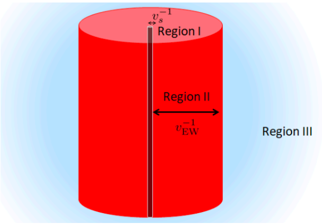
- There are other choices of nodes and functions, but the Chebyshev nodes have nice numerical properties.
- Since $\zeta \in [-1,1]$, we also need change variables before we can use them

[Trefethen - "Approximation Theory and Practice"]
[1908.10370 - Stout et al]

Overview: The DFSZ Case

- The DFSZ model is like a 2HDM with an extra complex scalar. The axion is a massless pseudoscalar that is a combination of H_1, H_2, S.
- In this model, there are three types of strings with the same global winding but different gauge winding. Only the lightest is topologically stable.
- Type C strings are superconducting:

[2010.02834 - Abe, Hamada, Yoshioka]



$$S = v_s e^{i\theta}\phi(r)$$

$$H_1 = \frac{1}{2} v_1 e^{i\theta} \begin{pmatrix} f(r)e^{i\theta} - h(r)e^{-i\theta} \\ f(r)e^{i\theta} + h(r)e^{-i\theta} \end{pmatrix}$$

$$H_2 = \frac{1}{2} v_2 e^{-i\theta} \begin{pmatrix} h(r)e^{i\theta} - f(r)e^{-i\theta} \\ h(r)e^{i\theta} + f(r)e^{-i\theta} \end{pmatrix}$$

$$Z_i, W_i^1 \neq 0, A_i = 0$$

$$\phi(0) = f(0) = 0 \quad \partial_r h|_{r=0} = 0$$

Particle mass doesn't vanish in the core.

DFSZ Solutions

Superconducting!

Type A

$$S = v_s e^{i\theta}\phi(r)$$

$$H_1 = v_1 e^{i\theta}\begin{pmatrix}0\\ f(r)\end{pmatrix}$$

$$H_2 = v_2 e^{-i\theta}\begin{pmatrix}0\\ h(r)\end{pmatrix}$$

$$Z_i \neq 0, W_i^{\pm} = A_i = 0$$

Type B

$$S = v_s e^{i\theta}\phi(r)$$

$$H_1 = v_1 e^{2i\theta}\begin{pmatrix}0\\ f(r)\end{pmatrix}$$

$$H_2 = v_2 \begin{pmatrix}0\\ h(r)\end{pmatrix}$$

$$Z_i \neq 0, W_i^{\pm} = A_i = 0$$

Type C

$$S = v_s e^{i\theta}\phi(r)$$

$$H_1 = \frac{1}{2} v_1 e^{i\theta}\begin{pmatrix}f(r)e^{i\theta} - h(r)e^{-i\theta}\\ f(r)e^{i\theta} + h(r)e^{-i\theta}\end{pmatrix}$$

$$H_2 = \frac{1}{2} v_2 e^{-i\theta}\begin{pmatrix}h(r)e^{i\theta} - f(r)e^{-i\theta}\\ h(r)e^{i\theta} + f(r)e^{-i\theta}\end{pmatrix}$$

$$Z_i, W_i^1 \neq 0, A_i = 0$$

Type A: $\phi(0) = f(0) = h(0) = 0$

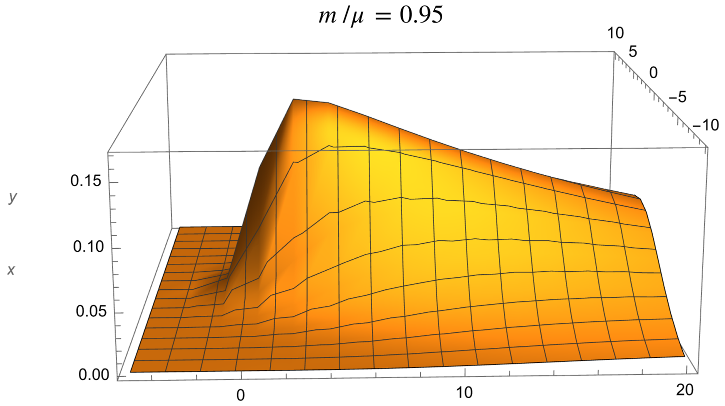
Types B, C: $\phi(0) = f(0) = 0, \partial_r h|_{r=0} = 0$

All: $\phi(\infty) = f(\infty) = h(\infty) = 1$

We see massless modes even though the particle mass doesn't vanish at the core

[2010.02834 - Abe, Hamada, Yoshioka]

Visualizing the wedge



Explicit Superconductivity Example

To see superconductivity more explicitly, it is easiest to study $U(1) \times \tilde{U}(1)$ gauge strings, where only $\tilde{U}(1)$ is broken.

Study fluctuations of scalar σ, which is charged under $U(1)$:

$$\sigma(x, y, z, t) = e^{i\theta(z,t)}\sigma_0(x, y)$$

Then $\theta(z, t)$ has effective action

$$S_\theta = K \int dz dt (\partial_i \theta + eA_i)^2$$

This has current

$$J_i(z, t) = -\delta S_\theta / \delta A_i = 2Ke(\partial_i \theta + eA_i)$$

which we can show is persistent by relating it to the topological quantity N, which is the integral of the derivative of θ around a loop

[Witten, Nuclear Physics B 1985]

The critical case

In the critical case ($m = \mu$), there are still zero modes, but they are no longer localized to the string. Instead, they live on a wedge in the x-y plane about $\varphi = 0$.

To see this, we search for solutions that propagate in x and z

$$\psi(\vec{x}) = \psi(x, y) e^{-i\omega t + ik_x x + ik_z z}$$

We find zero mode solutions for ψ_0, ψ_3

$$\psi_{0,3}(\vec{x}) \sim e^{-y^2 \mu/(2x) - i\omega t + ik_x x + ik_z z}$$

Integrating out the fermions

- In order to integrate out the fermions, we want to compute

$$Z_\psi(\varphi) = \int \mathscr{D}\overline{\psi}\,\mathscr{D}\psi \,\mathrm{exp}\left[i\int d^4x\,\overline{\psi}(i\not{D} - m + yf(\rho)e^{i\gamma^5\varphi})\psi\right]$$

- Performing the path integral gives

$$Z_\psi(\varphi) = \frac{\mathrm{det}(i\not{D} - M(\rho,\varphi)e^{i\gamma^5\overbrace{\delta(\rho,\varphi)}^{\mathrm{arg}(m - yf(\rho)e^{i\varphi})}})}{\mathrm{det}(i\not{D}) - \tilde{M}}$$

- We can simplify this with a spatially dependent chiral transformation $\psi \to e^{-i\gamma^5\delta(\varphi)/2}\psi$

$$Z_\psi(\varphi) = \frac{\mathrm{det}(i\not{D} - M(\rho,\varphi))}{\mathrm{det}(i\not{D}) - \tilde{M}}\,\exp\left[\frac{i}{8\pi^2}\int \delta(\rho,\varphi) F\wedge F\right]$$

where we have assumed there are no zero modes in the core to affect the transformation