# Generalized Symmetry in Particle Physics

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# References and Collaborators

"Exploring 2-group Global Symmetry" [CC-Dumitrescu-Intriligator]

"Axions Higher Groups and Emergent Symmetry" [Brennan-CC]

"Non-Invertible Global Symmetries in the Standard Model" [Choi-Lam-Shao]

"Non-invertible Chiral Symmetry and Exponential Hierarchies" [CC-Ohmori]

"Neutrino Masses from Generalized Symmetry Breaking"
[CC-Hong-**Koren**-Ohmori]

"Higher Flavor Symmetries in the Standard Model" [CC-**Koren**]

# Higher Form Symmetry

[Gaiotto-Kapustin-Seiberg-Willet].

# Higher Symmetry

Main idea: develop concepts of symmetry that involve extended operators.

Language: p-form global symmetry → charged operators p-dimensional

Ordinary (0-form) symmetry has charged local operators (create particles)

1-form global symmetry has charged line operators (create strings)

2-form global symmetry has charged surface operators (create domain wall)

**1-form common in gauge theory. 2-form common in models of axions**

Currents antisymmetric with p+1 indices. 1-form symmetry: $\partial^{\mu} J_{\mu\nu} = 0$

# Charged Lines

Basic example of a theory with a 1-form symmetry is a $U(1)$ gauge theory

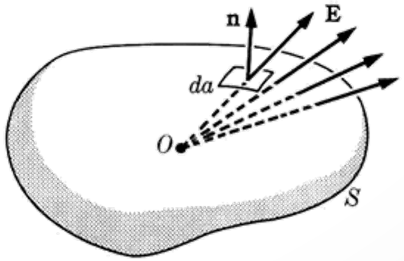
Two varieties of one-form symmetry:

electric symmetry measures charges of Wilson lines. Current $J_{\mu\nu} = F_{\mu\nu}$

magnetic symmetry measures charges of 't Hooft lines. Current $J_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}$

Can alternatively discuss the charge defined by integrating over a surface.

Independent of shape (topological)




Gauss' law: $\int_S \vec{E} \cdot da \sim q$

# 1-Form Symmetry Breaking

For ordinary symmetries simply add symmetry violating operators to action

How are higher-form symmetries broken in QFT?

**New effect: all local operators are uncharged under higher-form symmetry**

Impossible to break them by adding operator deformations. In the leading Euler-Heisenberg deformation of Maxwell theory:

$$S = \frac{1}{e^2} \int d^4x \;\, F_{\mu\nu} F^{\mu\nu} + \frac{1}{\Lambda^4} (F_{\mu\nu} F^{\mu\nu})^2$$

Bianchi (magnetic): $\partial^\mu \epsilon_{\mu\nu\rho\sigma} F_{\rho\sigma} = 0$ , EOM (electric): $\partial^\mu (F_{\mu\nu} + \frac{1}{\Lambda^4} F_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}) = 0$

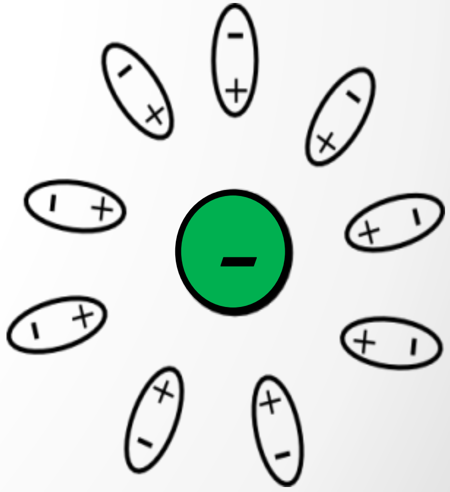
# 1-Form Breaking & Screening

Break one-form symmetry with **charged matter fields via screening**

$$\partial^\mu F_{\mu\nu} \sim J_\nu^{elec}$$

With mobile charged particles (mass $m$, charge $n \in \mathbb{Z}$) Gauss' law breaks

vacuum has virtual charges that screen Wilson line charge $q$



Equivalently the charge $q(R)$ depends on the distance $R$ from the source. Quantify via effective potential:

$$q(R)\big/q(\infty) = 1 - e^2 n^2 \log(mR) + e^2(const. + O(mr))$$

# Higher Group Symmetry

[CC-Dumitrescu-Intriligator, CC-Koren, CC-Brennan]

# Symmetry of 4d Massless QED

When QFT has higher symmetry of different degrees, mixture is possible

Consider e.g. $U(1)$ gauge theory with $N_f$ massless fermions of charge $Q$:

- $SU(N_f)_L \times SU(N_f)_R$ ordinary symmetry acting on the fermions

- $U(1)$ 1-form magnetic symmetry (via Bianchi). Charged 't Hooft lines

Mixing between 0-form and 1-form symmetry encoded in $< J_\mu J_\nu J_{\rho\sigma} > \sim \kappa \in \mathbb{Z}$

Non-zero $\kappa$ defines "two-group" global symmetry

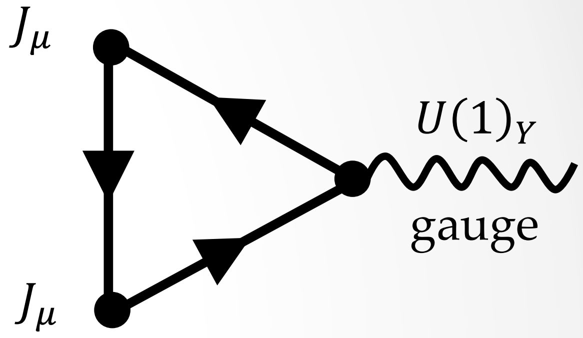
Analog: structure constants for ordinary symmetry encoded in $< J_\mu J_\nu J_\sigma >$

# Two-Group Global Symmetry

In weakly coupled gauge theory characterize using triangles

Implications depend on vertices:

$\kappa \sim \sum q_f^2 \, q_g \in \mathbb{Z}$



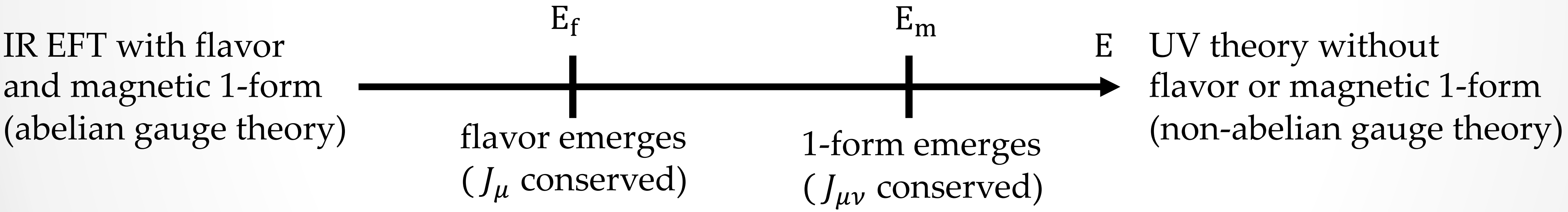

- g-g-g: theory mathematically inconsistent
- g-g-f: flavor symmetry broken quantum mechanically (**MAYBE?! see later!**)
- g-f-f: flavor symmetry forms current algebra with magnetic one-form sym
- f-f-f: flavor symmetry has ‘t Hooft anomaly (obstruction to gauging)

# Higher Groups and Emergence

Basic observation from Ward. OPE: $J_\mu(x) J_\nu(0) \supseteq \kappa\, J_{\mu\nu}(0)$

**For $\kappa \neq 0$ the flavor symmetry generates the one-form symmetry**

Interesting when both symmetries are accidental/emergent in IR




**Universal inequality:** $E_f \lesssim E_m$ (squiggles since approximately defined)

For IR abelian gauge theory emerging via Higgsing at $E_m$, flavor broken by $E_m$

# Toy Example of Unification

Return to massless QED now viewed as IR model

|  | $U(1)_g$ | $SU(N_f)_L$ | $SU(N_f)_R$ |
|---|---|---|---|
| $\chi^+$ | $+1$ | $\square$ | – |
| $\chi^-$ | $-1$ | – | $\square$ |

L and R flavor symmetries have $\kappa = \pm 1$ (diag $\kappa = 0$ )

Any UV where $U(1)_g$ emerges via higgsing breaks flavor symmetry.

For example unifying $\chi^\pm$ into a doublet of $SU(2)$ breaks L and R (but preserves diagonal)

|  | $SU(2)_g$ | $SU(N_f)$ |
|---|---|---|
| $\chi_i = (\chi_i^+, \chi_i^-)$ | $\square$ | $\square$ |

Key point: within weakly coupled gauge theory, allowed UV flavor symmetry tells us about which IR fermions can be unified into UV gauge multiplets

# Higher Flavor Symmetry in Nature

SM flavor symmetry for each fermion species violated by yukawa couplings:

$$\mathcal{L} \supseteq y_u \tilde{H} Q \bar{u} + y_d H Q \bar{d} + y_e H L \bar{e}$$

When $y_i \rightarrow 0$: flavor symmetries $SU(3)_Q, SU(3)_u, SU(3)_d, SU(3)_L, SU(3)_e$

Each flavor symmetry has a non-zero f-f-g $\kappa$. So the SM has (approximate) two-group symmetry

Useful to understand structure of possible GUT models by constraining their flavor symmetry. This controls how SM fermions are fused into GUT multiplets

| Flavor$^2$ | $U(1)_Y$ |
|---|---|
| $SU(3)_Q^2$ | $+1 \cdot 2 \cdot N_c$ |
| $SU(3)_u^2$ | $-4 \cdot N_c$ |
| $SU(3)_d^2$ | $+2 \cdot N_c$ |
| $SU(3)_L^2$ | $-3 \cdot 2$ |
| $SU(3)_e^2$ | $+6$ |

f-f-g coeffs

# GUTs from Higher Symmetry

In a GUT model $U(1)_Y$ emerges from nonabelian gauge group so magnetic symmetry broken

UV flavor symmetries are diagonal combinations with vanishing total f-f-g coefficient $\kappa$

| | | | |
|---|---|---|---|
| One | None | | |
| Two | $\{L, Q\}$ | $\{L, \bar{d}\}$ | $\{L, \bar{e}\}$ |
| Three | $\{\bar{u}, \bar{d}, \bar{e}\}$ | $\{\bar{u}, \bar{e}, Q\}$ | $\{\bar{u}, \bar{d}, Q\}$ |
| Four | None | | |
| Five | $\{Q, \bar{u}, \bar{d}, L, \bar{e}\}$ | | |

Combinations with vanishing f-f-g

Familiar examples of GUT manifest this organizational principle:

$SO(10)$: all fermions (+ $N$) unified into single multiplet $\{Q, \bar{u}, \bar{d}, L, \bar{e}, N\}$

$SU(5)$: two multiplets $\bar{5} = \{L, \bar{d}\}$ and $10 = \{Q, \bar{u}, \bar{e}\}$

$SU(4)\times SU(2)\times SU(2)$: two multiplets (4,2,1)= $\{Q, L\}$ and $(\bar{4}, 1, 2) = \{\bar{u}, \bar{d}, \bar{e}, N\}$

# Non-Invertible Chiral Symmetry

[CC-Ohmori, Choi-Lam-Shao]

# Chiral Symmetry in Massless QED

Return to massless $U(1)$ gauge theory. Now with single light fermion $\chi^{\pm}$

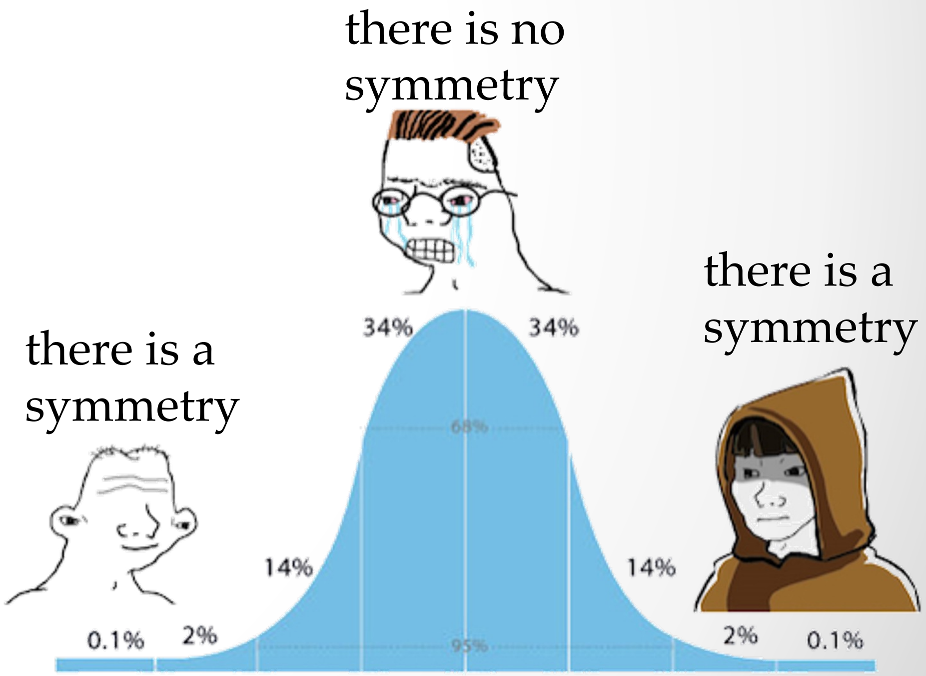
$J_\mu$ classical chiral current has ABJ anomaly f-g-g: $\partial^\mu J_\mu \sim \epsilon_{\nu\rho\sigma\tau} F^{\nu\rho} F^{\sigma\tau}$

Does $J_\mu$ generate a conserved charge?

NO: at quantum level $J_\mu$ not conserved

YES: Conservation only violated by abelian Instantons and on S⁴ (regulated spacetime) there are no such configurations ['t Hooft].

**symmetry holds in scattering-matrix**



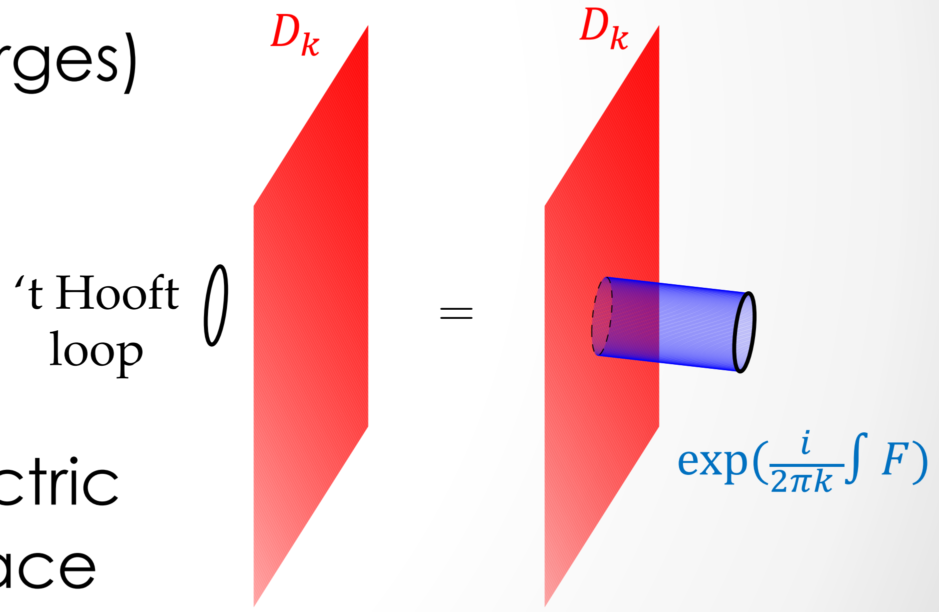

Clay's path to enlightenment

# Define Chiral Symmetry

Resolution: dress chiral charge by fractional hall state (abelian anyons)

Acts on local operators as chiral rotation e.g. fermion mass $m\, \chi^{+}\chi^{-}$ forbidden

Acts unusually on ‘t Hooft lines (magnetic charges)




**magnetic charges create the topology needed to activate abelian instantons**

Finite chiral transformation adds fractional electric charge to the ‘t Hooft line. Attaches to a surface

Novel algebra between chiral and 1-form symmetry. **Beyond groups**

# Symmetry Breaking by Monopoles

Chiral symmetry with abelian ABJ anomaly in algebra with magnetic charge

Dynamical monopoles break one-form and chiral symmetry by screening

Estimate size of effects of monopoles using U(1) EFT [Fan-Fraiser-Reece-Stout]

monopole mass $\mathrm{m} \sim v/g$, cutoff $\sim \delta t^{-1} \sim vg$ ($v$ higgs scale, $g$ coupling)

monopole loop action: . $\exp(-S) \sim \exp(-m\,\delta t) \sim \exp(-\#/g^2)$

**Loops of monopoles will violate chiral symmetry non-perturbatively**

monopole loops give IR description of UV non-abelian instantons (exist on $S^4$)

# Exponential Hierarchy

Protect a physical effect by a symmetry with an abelian ABJ anomaly.
Embed in UV with monopoles to generate natural exponential corrections

Simple toy example: SU(2) + 2 Weyl doublets $\chi_1, \chi_2$ + adjoint Higgs $\Phi$

Condense $\Phi$: IR is U(1) + 2 electron flavors $\chi_1^{\pm}, \chi_2^{\pm}$

Emergent non-invertible chiral symmetry violated by UV 't Hooft vertex:

$$\exp(-\frac{8\pi^2}{g^2})\,\chi_1\chi_2 \rightarrow \exp\left(-\frac{8\pi^2}{g^2}\right)(\chi_1^+\chi_2^- - \chi_1^-\chi_2^+)$$

Exponentially suppressed

# Neutrino Masses from Monopoles

[CC-Hong-Koren-Ohmori]

# Neutrino Mass Redux

Majorana mass. Effective dimension five Weinberg operator

$$\mathcal{L} \supseteq \frac{1}{\Lambda}\, y_{ij}^{N}\, (\widetilde{H}L_i)(\widetilde{H}L_j) \rightarrow \frac{v^2}{\Lambda}\, y_{ij}^{N} L_i L_j$$

Dirac masses. New right-handed neutrinos $N_i$

$$\mathcal{L} \supseteq\; y_{ij}^{N}\, \widetilde{H} L_i N_j$$

Challenge for Dirac masses: tiny dimensionless coupling $\;{}^{y_\nu}/_{y_\tau} \sim 10^{-11}$

Goal: Dirac model where ${}^{y_\nu}/_{y_\tau}$ exponentially suppressed via monopole loops

# $Z'$ Lepton Family Difference Model

Consider $Z'$ (additional $U(1)$ gauge group). Non-invertible symmetry generic

SM lepton family differences are symmetries. Use $L_\mu - L_\tau$

Classical Lagrangian & Symmetry: $\mathcal{L} = y_\tau H L \bar{e}$

$\tilde{L}$ SM Lepton #, $N$ RH Neutrino #, $L = \tilde{L} - N$

Quantum symmetry (including anomalies):

$\mathbb{Z}_3^L$ forbids Majorna masses $\sim (\tilde{H}L)^2$, $NN$

$\mathbb{Z}_3^{\tilde{L}}$ forbids Dirac masses $\sim \tilde{H}LN$

| | $SU(3)_H$ | $U(1)_{\mu-\tau}$ | $U(1)_{\tilde{L}}$ | $U(1)_N$ |
|---|---|---|---|---|
| $\mathbf{L}$ | $3$ | $\begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} = \begin{pmatrix} 0 \\ +1 \\ -1 \end{pmatrix}$ | $+1$ | $0$ |
| $\bar{\mathbf{e}}$ | $\bar{3}$ | $\begin{pmatrix} \bar{e} \\ \bar{\mu} \\ \bar{\tau} \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ +1 \end{pmatrix}$ | $-1$ | $0$ |
| $\mathbf{N}$ | $\bar{3}$ | $\begin{pmatrix} N_e \\ N_\mu \\ N_\tau \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ +1 \end{pmatrix}$ | $0$ | $+1$ |

Lepton sector

# Instanton Vertices

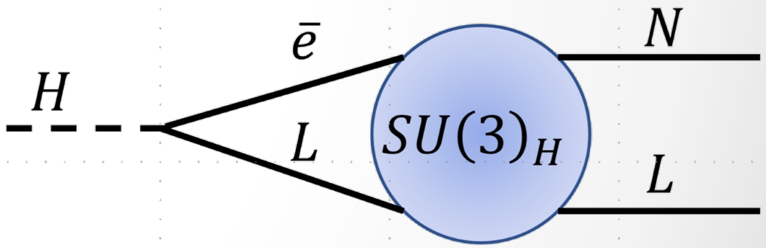
$\mathbb{Z}_3^{\tilde{L}}$ forbidding Dirac mass has f-g-g anomaly with $L_\mu - L_\tau$. So $\mathbb{Z}_3^{\tilde{L}}$ Non-invertible

Anticipate, universally, a UV where Dirac masses generated by monopoles

Simple possibility $U(1)_{\mu-\tau} \subset SU(3)_H$ at scale $v_\Phi$. Generations into triplets

Each $\bar{3}/3$ has one zero mode. 't Hooft instanton vertex:

$$\mathcal{L} \sim v_\Phi^{-2} \exp(-2\pi/\alpha_H)\, L\bar{e}LN$$




Composite interaction

Combine with Lepton yukawa to get Dirac $\nu$ mass:

$$\mathcal{L} \sim y_\tau \exp\left(-\frac{2\pi}{\alpha_H}\right) \tilde{H}LN$$

# Running Commentary

Running neutrino mass interaction to the $Z'$ scale gives relation:

$$v_\Phi^2 \sim M_{Z'}^2 \left(\frac{m_\nu}{m_\tau}\right)^{\frac{3}{2}} \exp\left(\frac{3\pi}{4\alpha_{\mu\tau}\,(Z')}\right)$$

Given $Z'$ mass, coupling, UV higgsing scale $v_\Phi$ where $SU(3)_H \to U(1)_{\mu-\tau}$ fixed

Novel at low scale, strong coupling: $M_{Z'} \sim 1\ \text{TeV},\ \alpha_{\mu\tau}\,(Z') \sim \frac{1}{20} \implies v_\Phi \sim 100\ \text{TeV}$

**Neutrino Texture?:** Use scalar Higgs sector which breaks $SU(3)_H \to U(1)_{\mu-\tau} \to \emptyset$

Other entries of mass matrix proportional to $y_\tau\, v \exp\left(-\frac{2\pi}{\alpha_H}\right)$

# Conclusions

Theme: algebra between higher symmetry and ordinary symmetry

Gave interplay between symmetry breaking effects:

| | | |
|---|---|---|
| local operator deformations (ordinary symmetry) | $\Longleftrightarrow$ | dynamical charged particles (one-form symmetry) |

Symmetry is THE way (old school) to understand questions of naturalness

**Early days of higher symmetry in particle physics, much more to discover!**

**Thanks for Listening!**

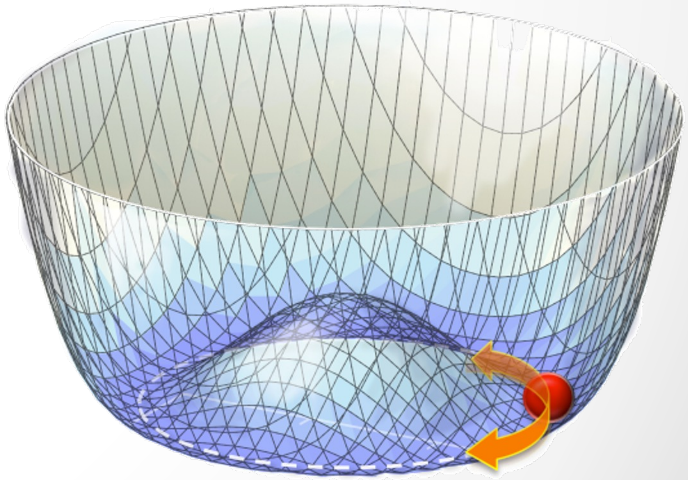
# A Theory of Light

When the higher-current ($F_{\mu\nu}$) acts on the vacuum it creates photons:

$$\langle 0|F_{\mu\nu}(x)|\epsilon,p\rangle \sim (\epsilon_\mu p_\nu - \epsilon_\nu p_\mu)\exp(ipx)$$

Compare to a matrix element for a spontaneously broken current $J_\mu(x)$:

$$\langle 0|\,J_\mu(x)|p\rangle \sim p_\mu \exp(ipx)$$



Higher-symmetry spontaneously broken
Photon is the goldstone mode

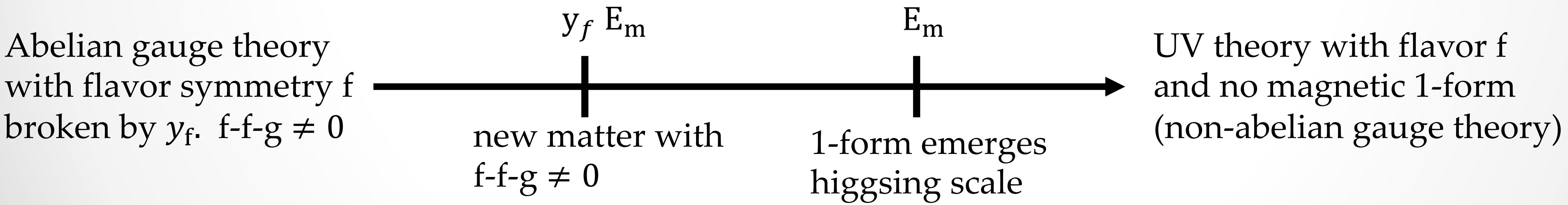
**Symmetry-based explanation for masslessness**

# Approximate Symmetry

Often have approximate flavor symmetry, f, broken by coupling $y_{\mathrm{f}}$ (yukawa)

Additional contributions to g-f-f triangles from matter which is massive but whose masses go to zero parametrically as $y_{\mathrm{f}} \to 0$ (chiral under flavor)

These additional matter fields allow us to preserve f in the UV. But if $y_{\mathrm{f}}$ is small they are parametrically lighter than the unification scale




As $y_{\mathrm{f}} \to 0$ the total g-f-f anomaly $\kappa$ must vanish to preserve f in the UV

# Approximate Symmetry & Trinification

Another GUT is trinification with UV gauge group $SU(3)\times SU(3)\times SU(3)$

Fermions in $\Psi_Q = (3,\bar{3},1) \supseteq \{Q,d'\}$, $\Psi_{\bar{Q}}= (\bar{3},1,3) \supseteq \{\bar{u},\bar{d},\bar{d}'\}$, $\Psi_L= (1,3,\bar{3}) \supseteq \{\bar{e},L,N\}$.
Where $(d',\bar{d}')$ are an extra vector like pair of down quarks

Appears to be an illegal pattern of flavor symmetry! But, without interactions this does not reproduce the SM spectrum.

This scenario is expected for an approximate flavor symmetry. $(d',\bar{d}')$ contribute to a g-f-f triangle. Lifting them ties their mass to the coupling breaking the flavor:

$$L \supseteq y\Phi\, \Psi_Q\Psi_{\bar{Q}} \supseteq y\Lambda\, d'\bar{d}' + y\widetilde{H}Q\bar{u} + yHQ\bar{d}$$

# 2-Group Ward Identities

Ward identities relating $< J_\mu J_\nu J_{\rho\sigma} >$ and $< J_{\rho\sigma} J_{\tau\upsilon} >$. On the locus in momentum space $p^2 = q^2 = (p+q)^2$ with M a mass scale:

$$< J_{\rho\sigma}(p) J_{\tau\upsilon}(-p) > = \frac{1}{p^2} f\left(\frac{p^2}{M^2}\right) tensor_{\rho\sigma\tau\upsilon}$$

$$< J_\mu^k(p) J_\nu^l(q) J_{\rho\sigma}(-p-q) > = \kappa \, \frac{\delta^{kl}}{p^2} f\left(\frac{p^2}{M^2}\right) tensor'_{\mu\nu\rho\sigma}$$

4d QED realizes identities with $\kappa = Q$, $J_\mu^k$ either chiral $SU(N_f)$, $J_{\rho\sigma} = (* F)_{\rho\sigma}$

Alternatively in OPE: $\partial^\mu J_\mu(x) J_\nu(0) \sim \kappa \, \partial^\sigma \delta(x) J_{\sigma\nu}(0)$ (compare $\kappa$ to $f^{ijk}$)