# The Weak Gravity Conjecture:
## Quantum Gravity for mortals
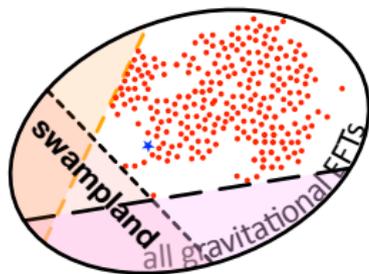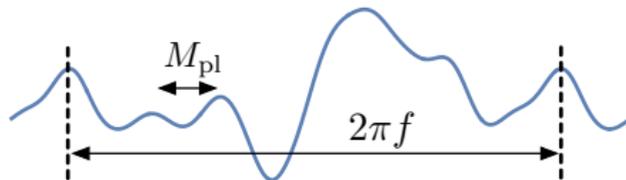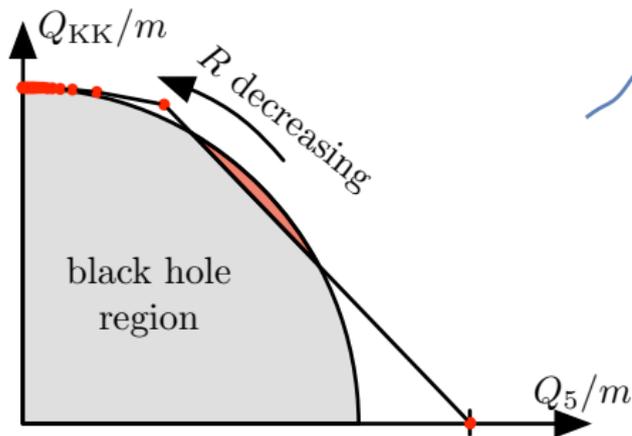


Ben Heidenreich    UMassAmherst | Physics

Pheno 2023, May 10, 2023

$$\text{Quantum Mechanics} \; + \; \text{Gravity} \; = \; ?$$

$$\Delta x \Delta p = \frac{\hbar}{2} \qquad G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Yet to be understood from 1st principles

…but

String theory provides many **indirect** insights and examples
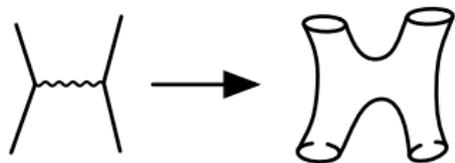
# Insight #1: There are **many** QG theories



The "landscape" of
string compactifications

Standard
Model(?)
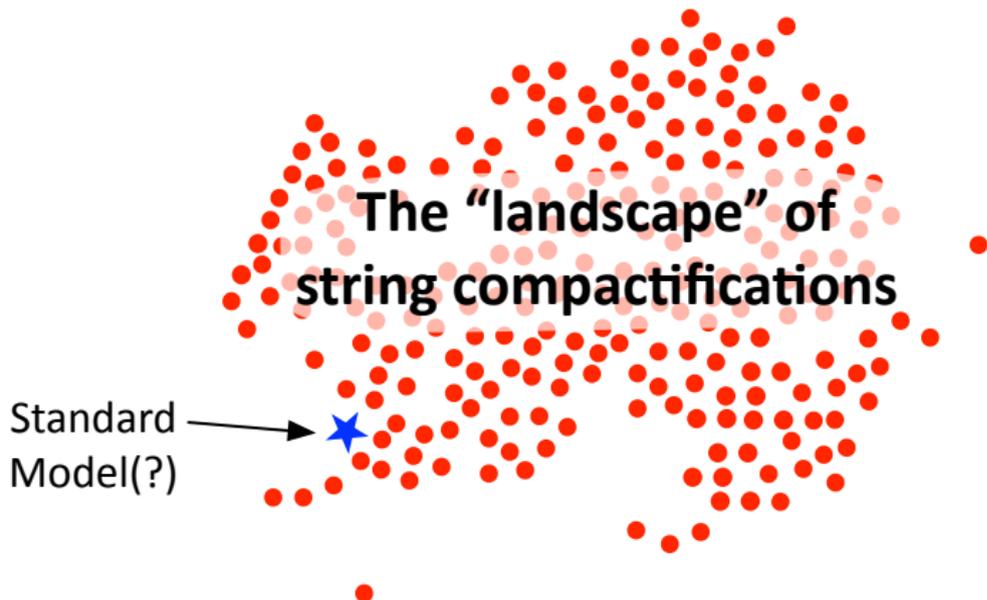
"Theory space"

# Insight #2: …but not everything goes



The "landscape" of string compactifications

"The swampland"

all effective field theories

"Theory space"

02
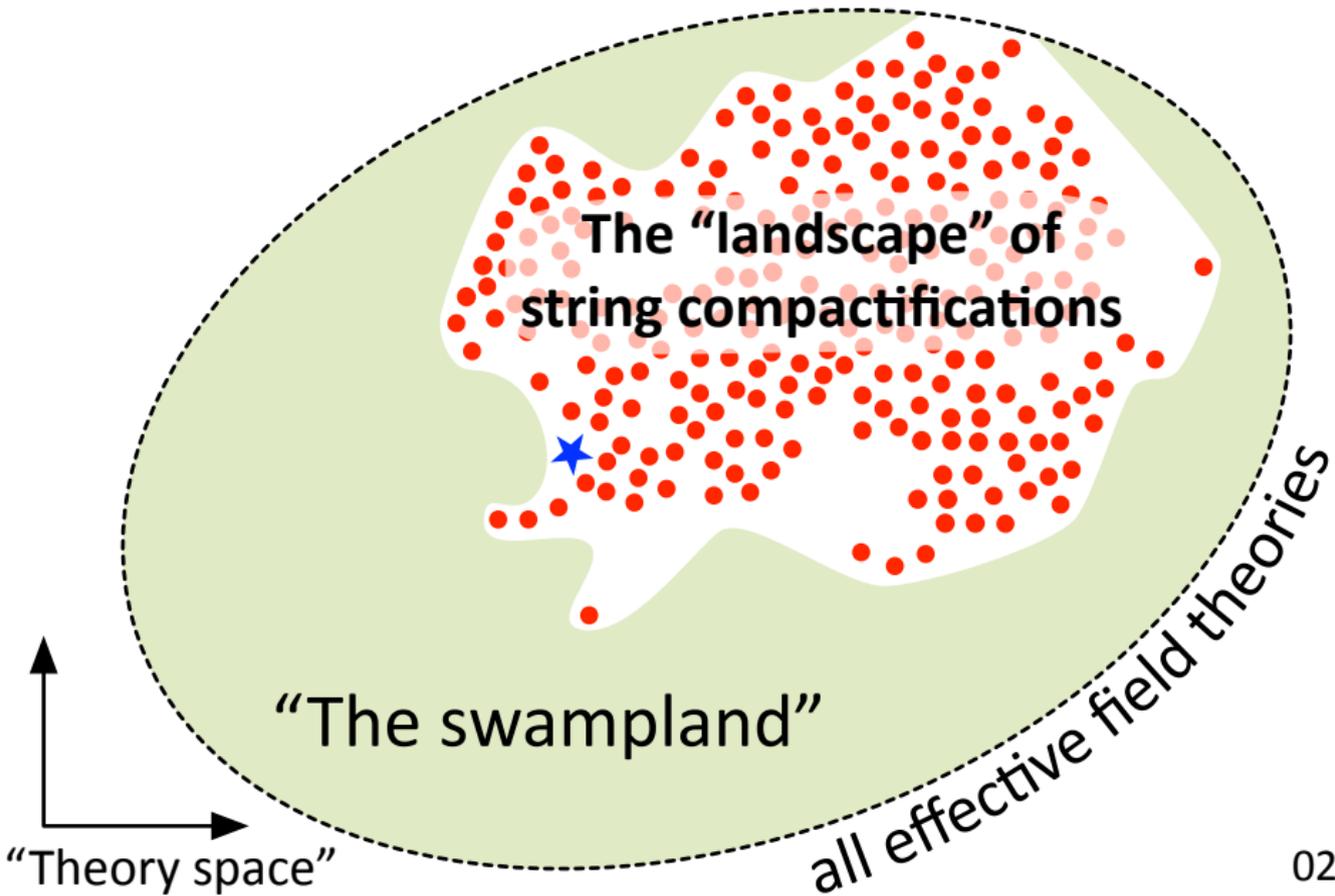
# Strategy: Fencing the swampland
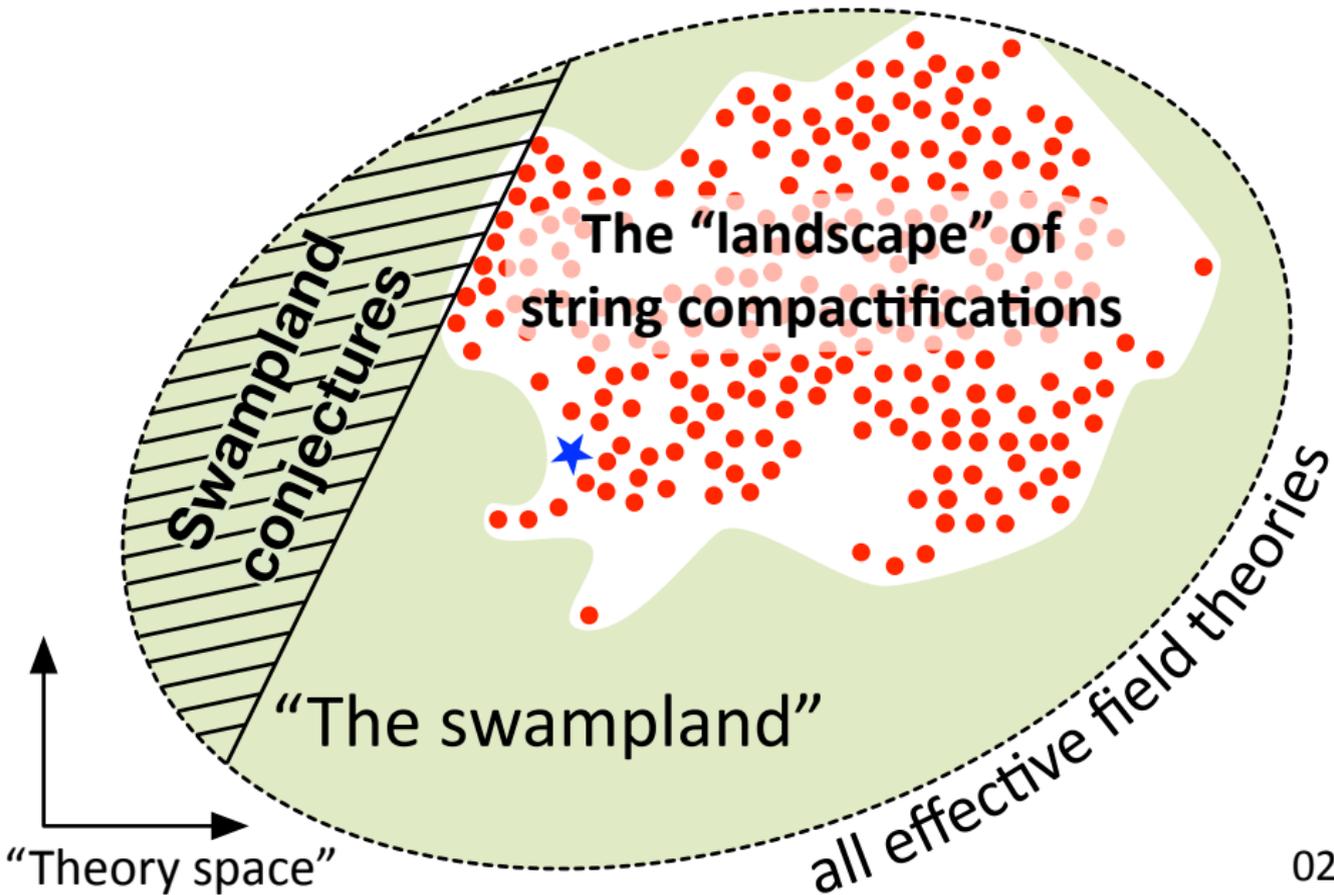
# Ex: "Theorem"

Quantum gravities cannot have
global symmetries (e.g., baryon number)

# "Proof"

Create a black hole by colliding baryons
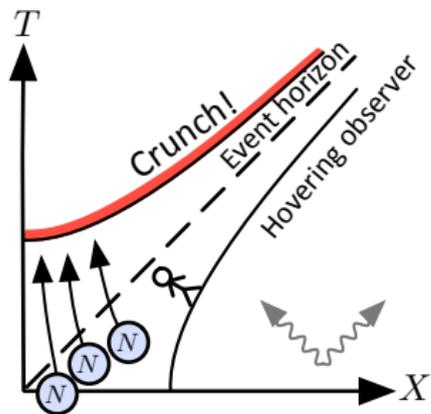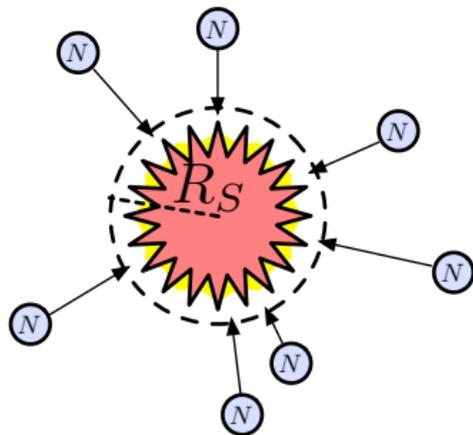
# Quantum black holes **glow**
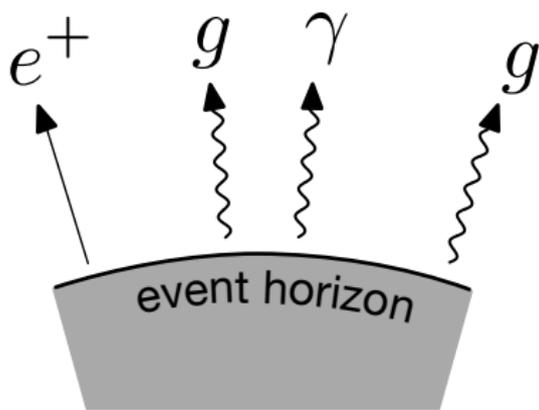


$$k_B T = \frac{\hbar c^3}{8\pi G M}$$

Hawking temperature

$$S = \frac{k_B c^3}{4\hbar G} A_{\text{Horizon}}$$

Bekenstein-Hawking entropy

Over time, they **evaporate**:
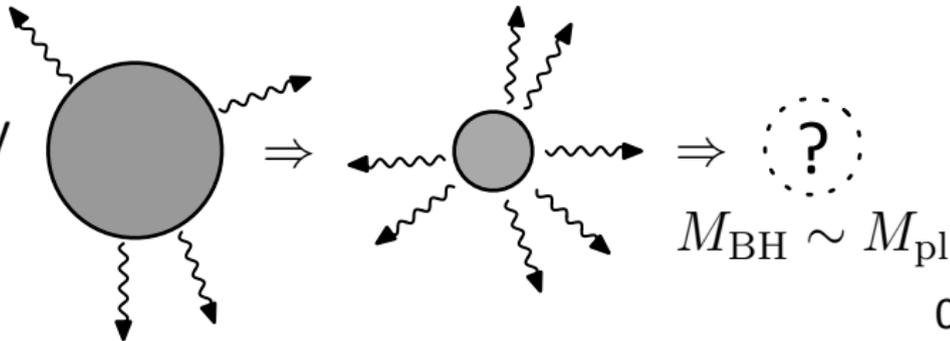


$$M_{\text{BH}} \sim M_{\text{pl}}$$

Remnants

Black Hole Mass

$M_{\rm pl}$

?   ?   ?

Baryon number

Remnants

Black Hole Mass

$M_{\mathrm{pl}}$

stable

$\bar{N}$

$N$

stable

?

?

?

Baryon number

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Remnants

Black Hole Mass

Infinitely many microstates!

(Contradicts Bekenstein-Hawking entropy formula)

stable

$\bar{N}$

$N$

stable

$M_{\text{pl}}$

?

?

?
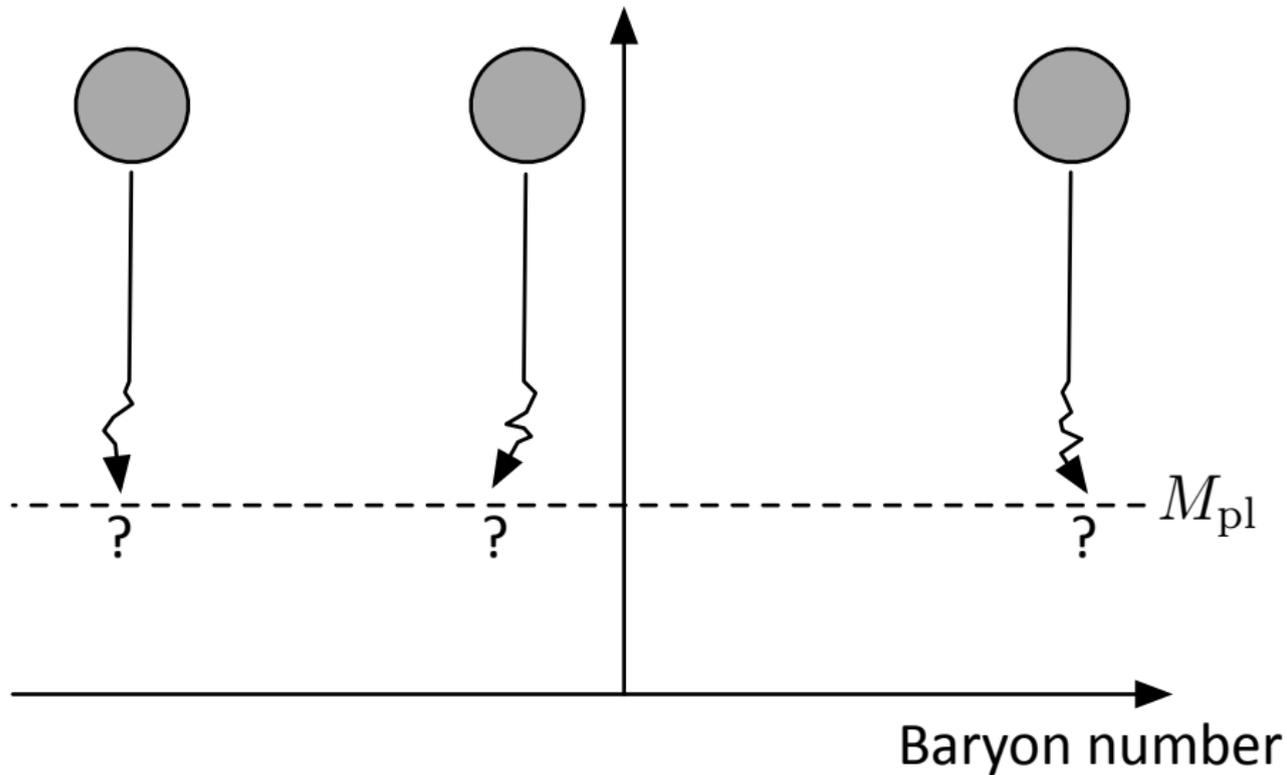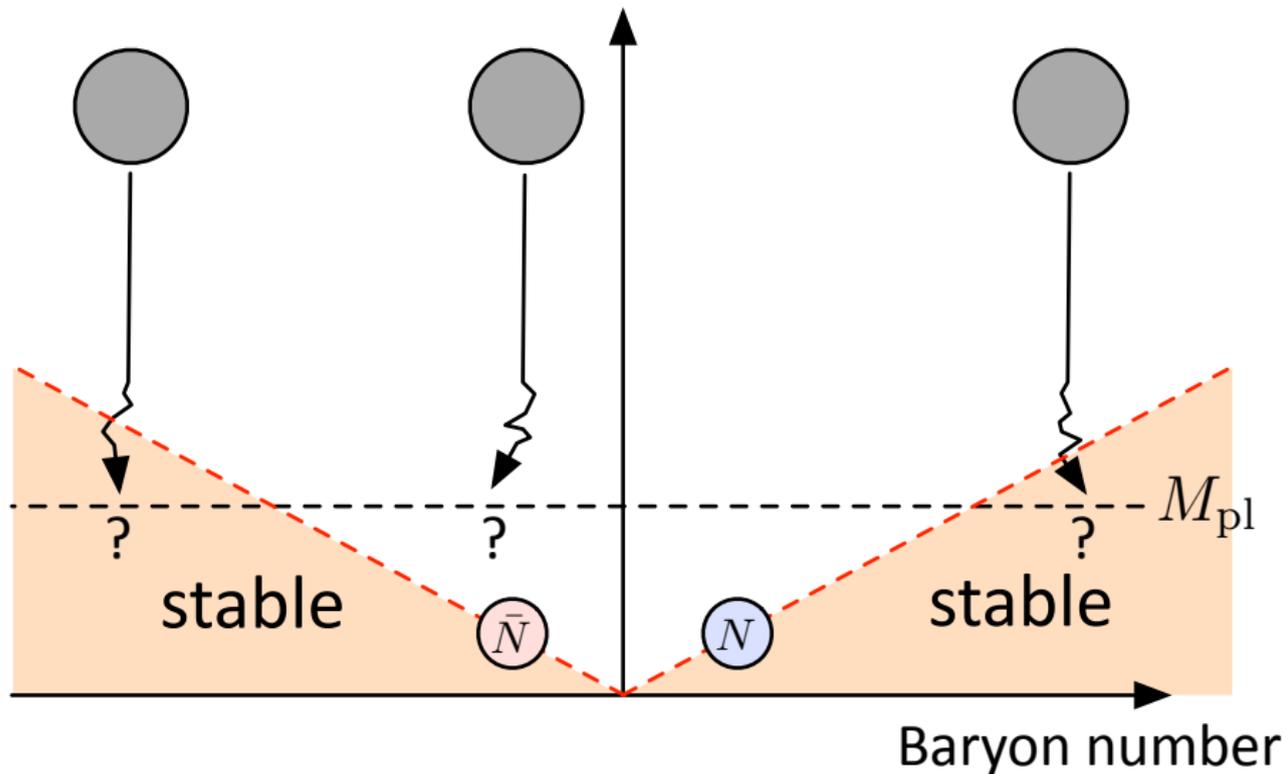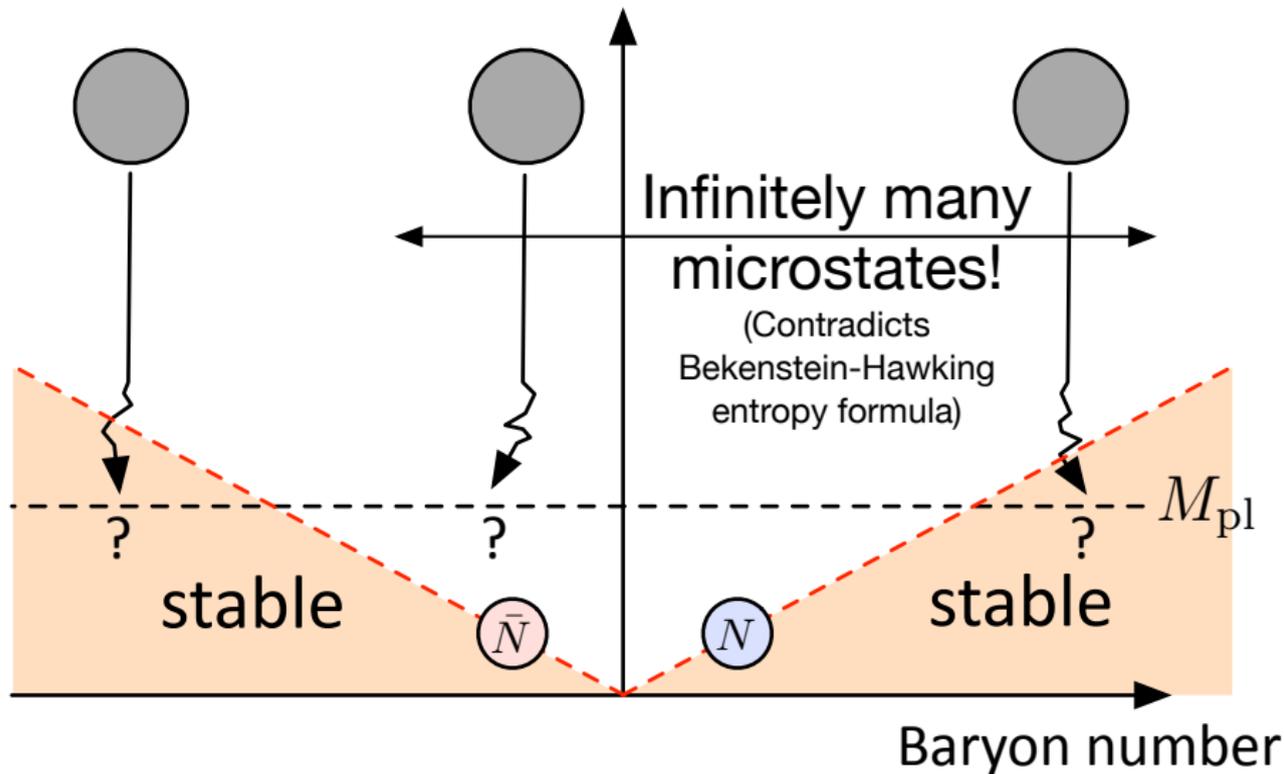
Baryon number

Baryon number is violated!

e.g., $\mathcal{L}_{\mathrm{eff}} \sim \dfrac{1}{M_{\mathrm{pl}}^2} uude$
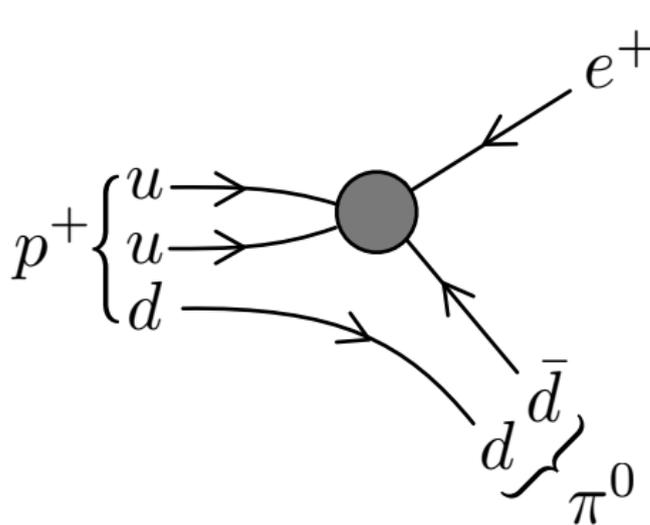


$\tau_{\mathrm{pred}} \sim 10^{45}$ yrs

versus

$\tau_{\mathrm{obs}} \gtrsim 10^{34}$ yrs

…that's ok!

…proton decays!

# Baryon number is violated!

vs. $\mathcal{L}_{\text{eff}} \sim \dfrac{1}{M_{\text{pl}}} u \tilde{u} \tilde{d} e$ (MSSM)
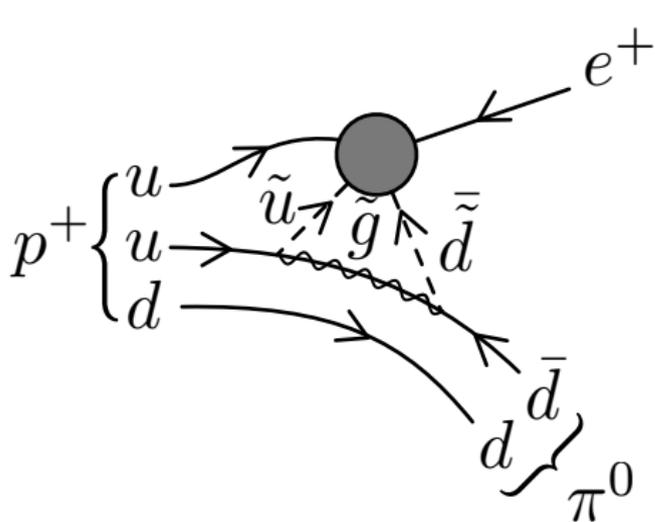


$\tau_{\text{pred}} \sim 10^{20}$ yrs

versus

$\tau_{\text{obs}} \gtrsim 10^{34}$ yrs!

Not ok!

(R-parity does **not** help, but other possible solns exist)

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# Charged remnants?

What if the symmetry is gauged (coupled to a long range force) rather than global?



$R_S$

$Q \neq 0$

$M_{\mathrm{BH}} \sim M_{\mathrm{pl}}$

?

Now there is an electric field:

If E-field too strong horizon is lost,
$\Rightarrow$ Q/M **bounded**

# Charged remnants?



$M$

Extremal BHs

No BHs

$T_{\text{Hawking}} = 0$

Extremal BHs

No BHs

$Q$

# Charged remnants?



08

# Charged remnants?



Only possible when $\left|\frac{q}{m}\right| \geq \left|\frac{Q}{M}\right|_{\text{ext BH}}$

08
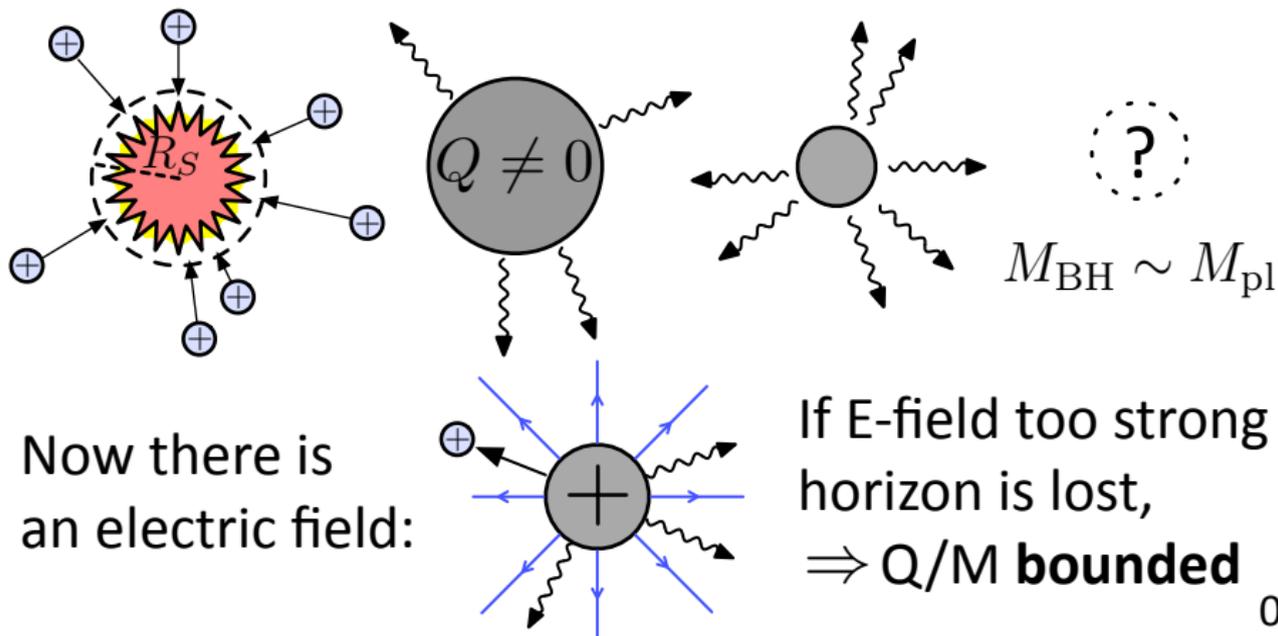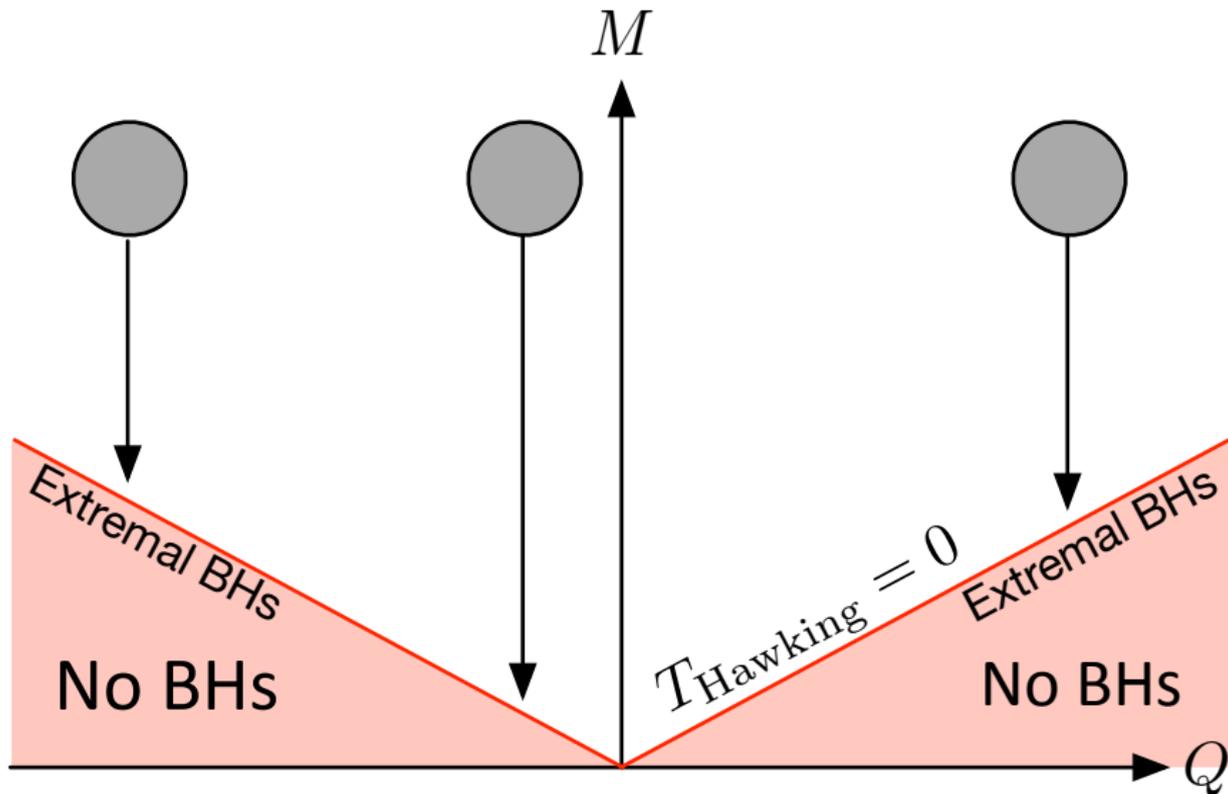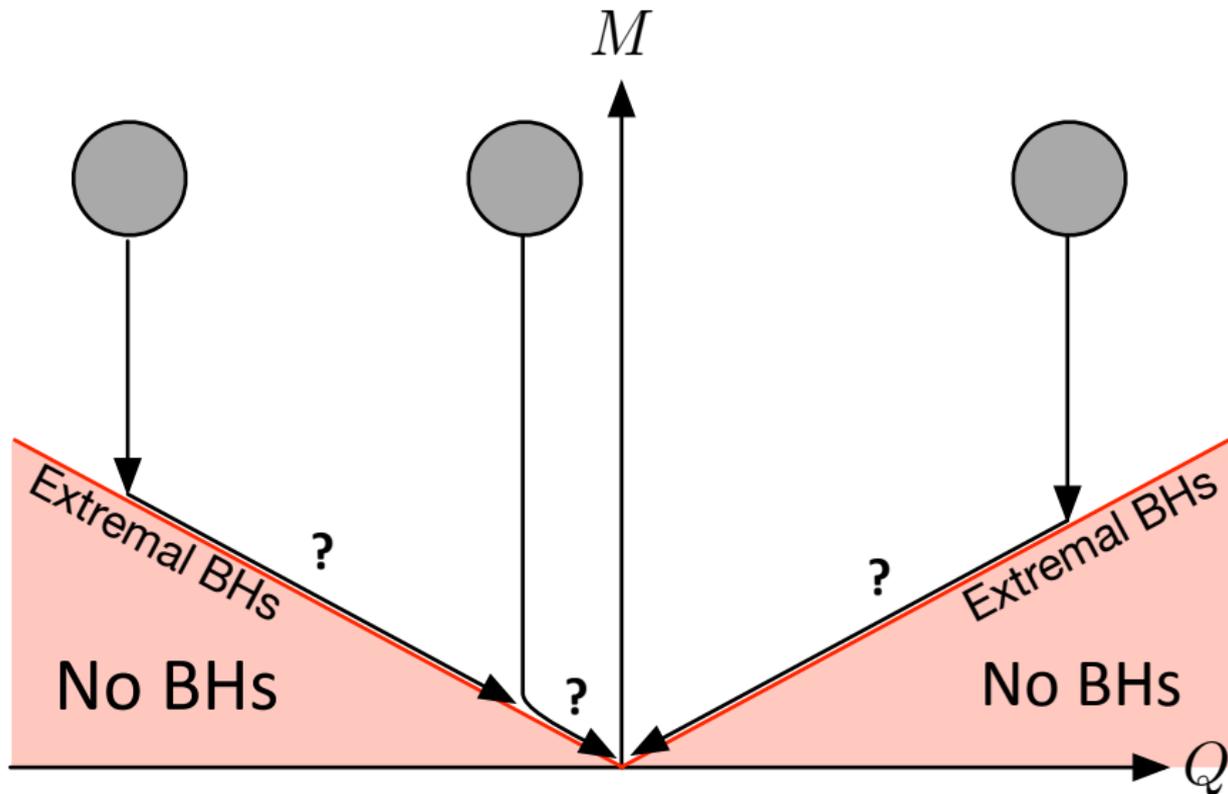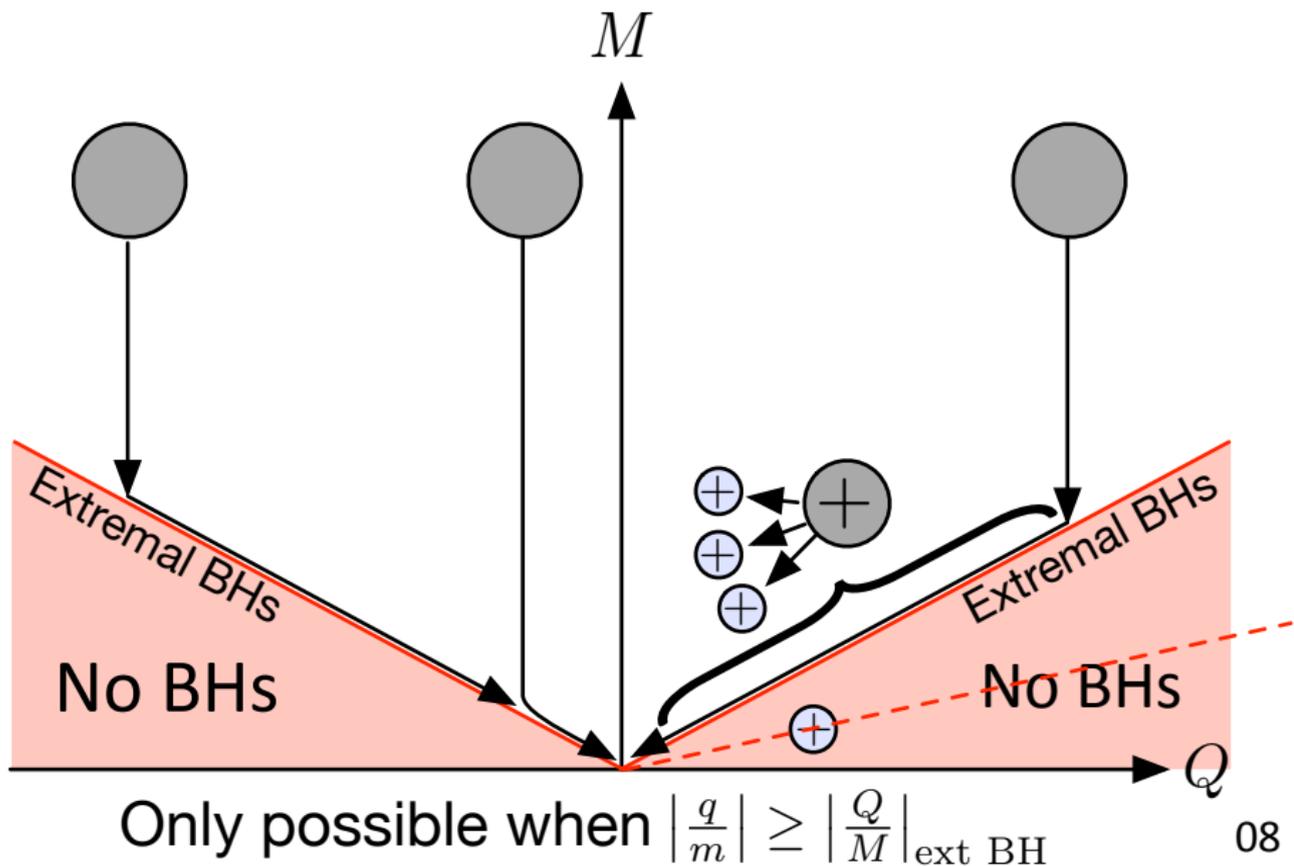
# The Weak Gravity Conjecture (WGC)

(Arkani-Hamed, Motl, Nicolis, Vafa '06)

There is a charged particle with

$$\left| \frac{q}{m} \right| \geq \left| \frac{Q}{M} \right|_{\text{ext BH}}$$

(Otherwise would have charged remnants)

"Gravity is the weakest force!"

$|F_{\text{Coulomb}}| \geq |F_{\text{Newton}}|$ for identical pair

# The Weak Gravity Conjecture (WGC)

e.g., for electromagnetic forces

$$\frac{e^2}{4\pi} \sim \frac{\hbar c}{137} \gg G m_e^2 \sim 10^{-45}\, \hbar c \quad \text{easily satisfied!}$$

but for a "fifth force" such as *B* (really *B-L*)

$$\frac{Q_N^2}{4\pi} \lesssim 10^{-49}\, \hbar c$$

WGC not satisfied
(by nucleons)!
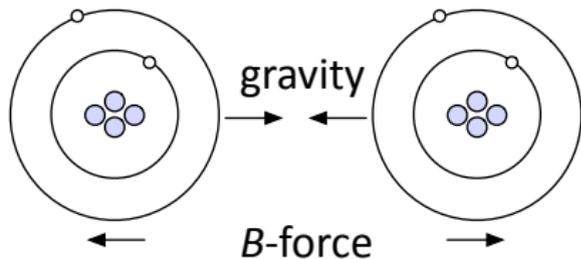
# The Magnetic WGC

No (generalized)
global symmetries

$\Downarrow$

Magnetic
monopoles exist!

# The Magnetic WGC

Monopole self-energy

$$m \gtrsim \frac{q_{\mathrm{mag}}^2}{e^2} \Lambda$$

WGC

$$m < \sqrt{2}\, \frac{q_{\mathrm{mag}}}{e} M_{\mathrm{pl}}$$

$$\Longrightarrow \boxed{\Lambda \lesssim e M_{\mathrm{pl}}}$$



$\Lambda^{-1}$
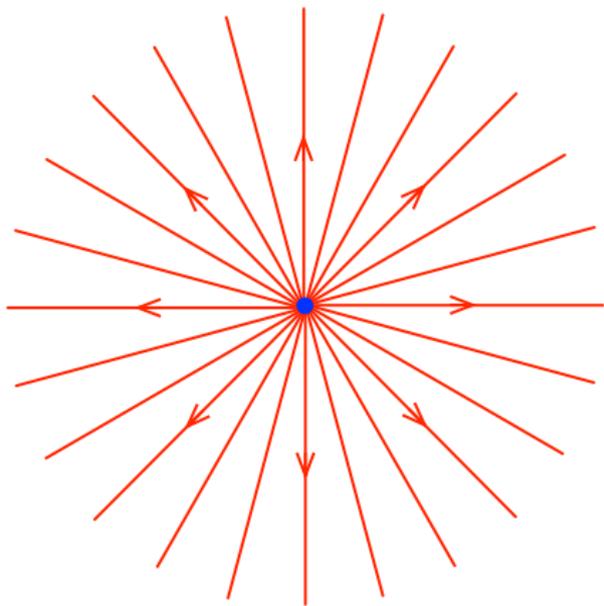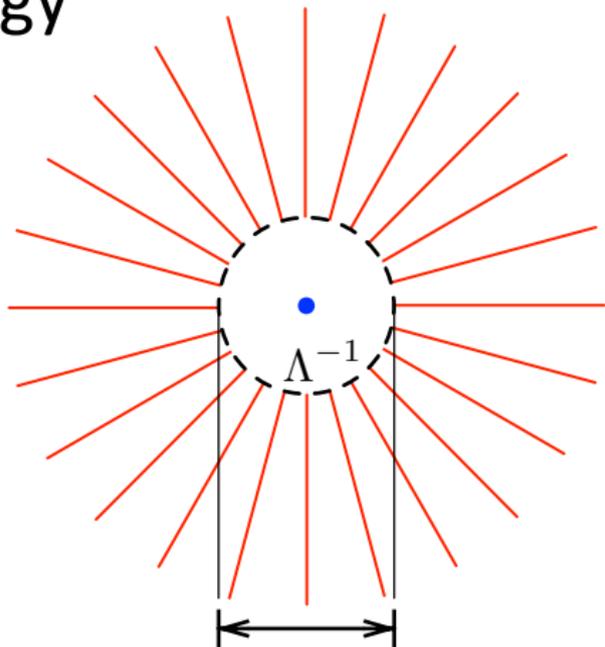
Scale of new physics

# The Magnetic WGC

Monopole self-energy

$$m \gtrsim \frac{q_{\mathrm{mag}}^2}{e^2} \Lambda$$

WGC

$$m < \sqrt{2}\, \frac{q_{\mathrm{mag}}}{e} M_{\mathrm{pl}}$$

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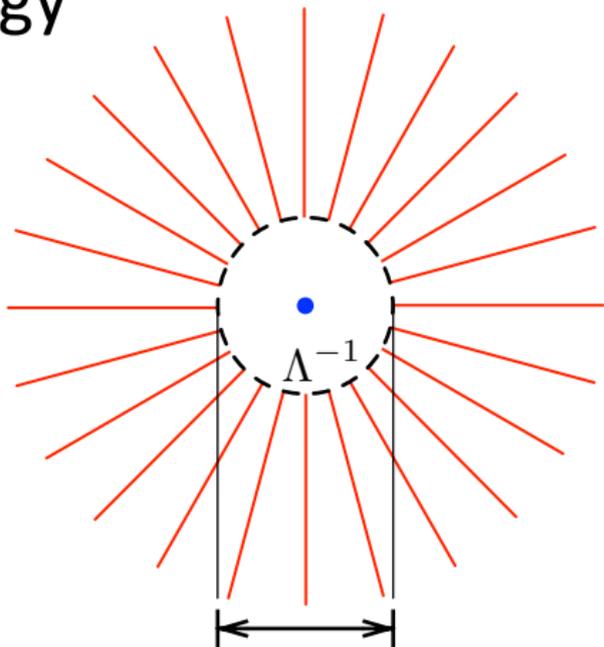
**What happens at new physics scale $\Lambda$?**

# WGC w/ multiple photons
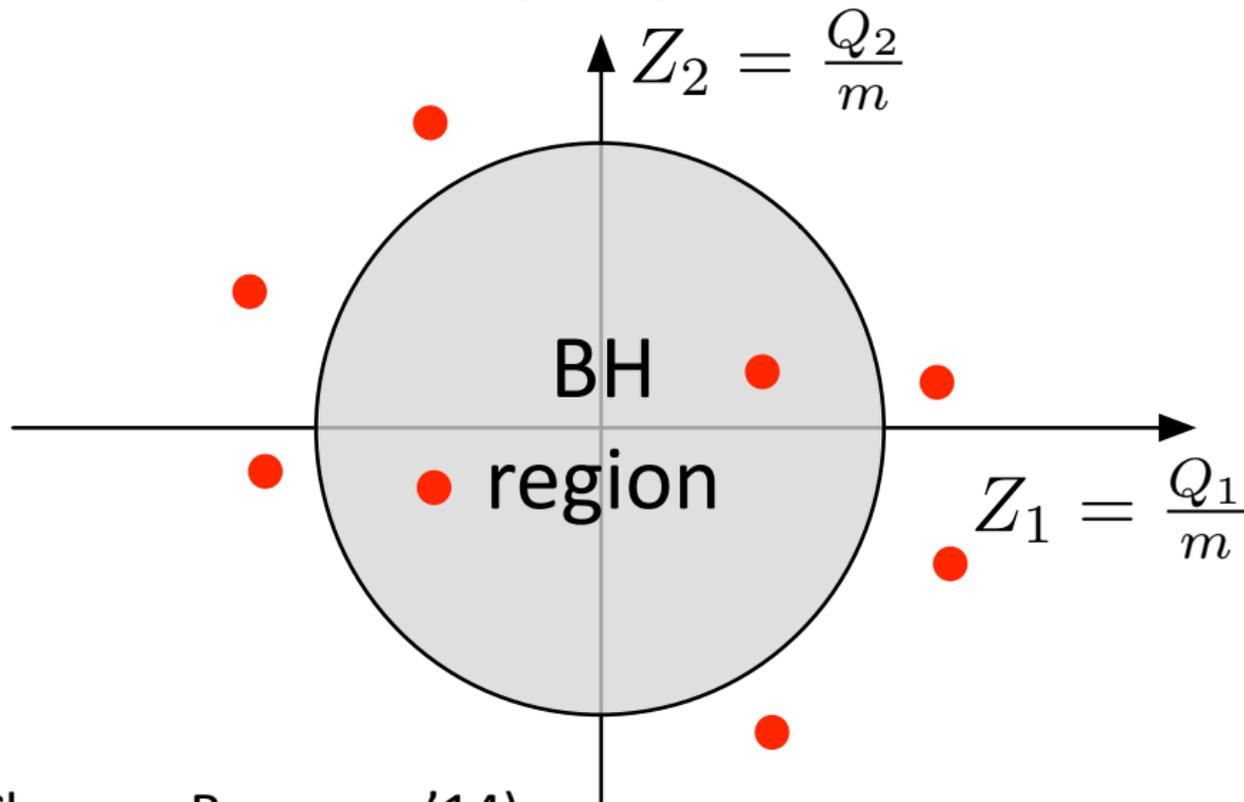


(Cheung, Remmen '14)

# WGC w/ multiple photons



(Cheung, Remmen '14)

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# WGC w/ multiple photons



$Z_2 = \frac{Q_2}{m}$

$Z_1 = \frac{Q_1}{m}$

stable

BH region

stable
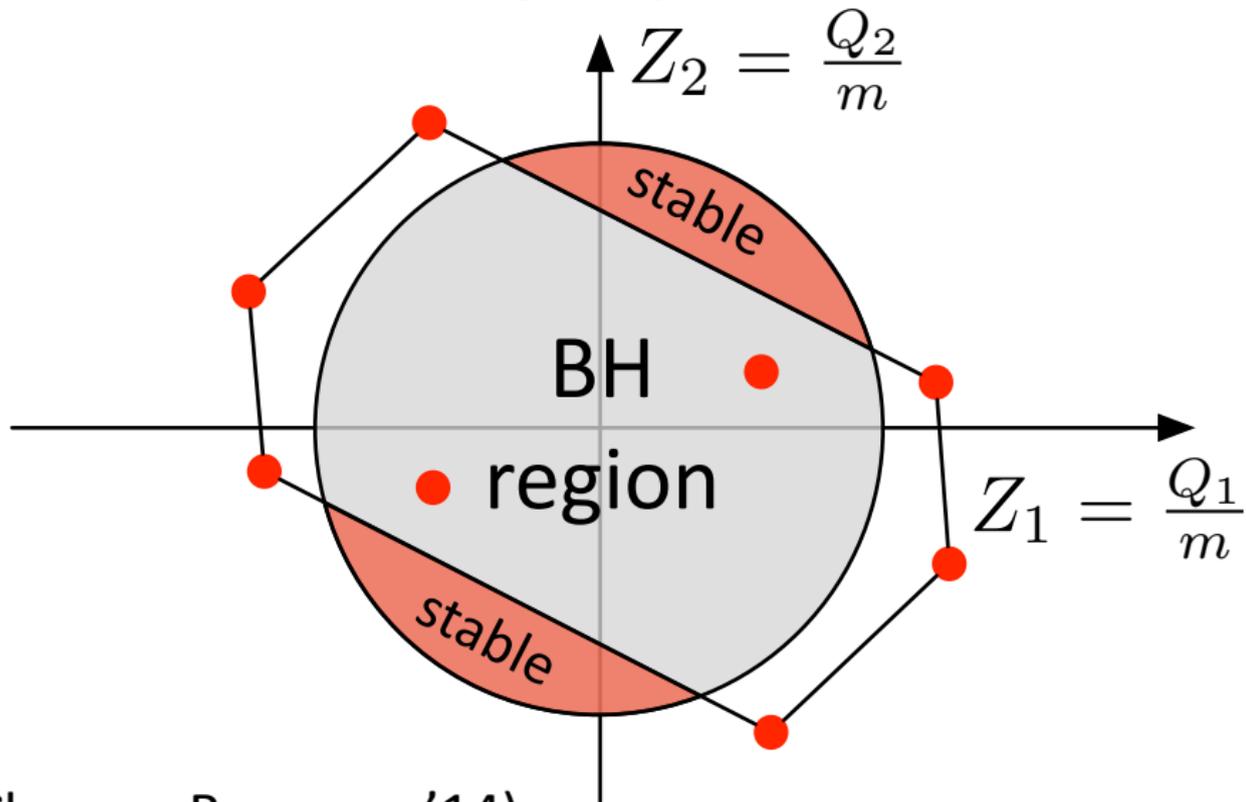
(Cheung, Remmen '14)

# The WGC with Moluli

String compactifications typically contain
many light, neutral scalars ("moduli") with
vevs that control various coupling constants

(In simple/unrealistic exs, $V_{\mathrm{moduli}} = 0$,
protected by unbroken SUSY.)

# The WGC with Moluli

BH region expands

$Z_2 = \frac{Q_2}{m}$

BH region

$Z_1 = \frac{Q_1}{m}$
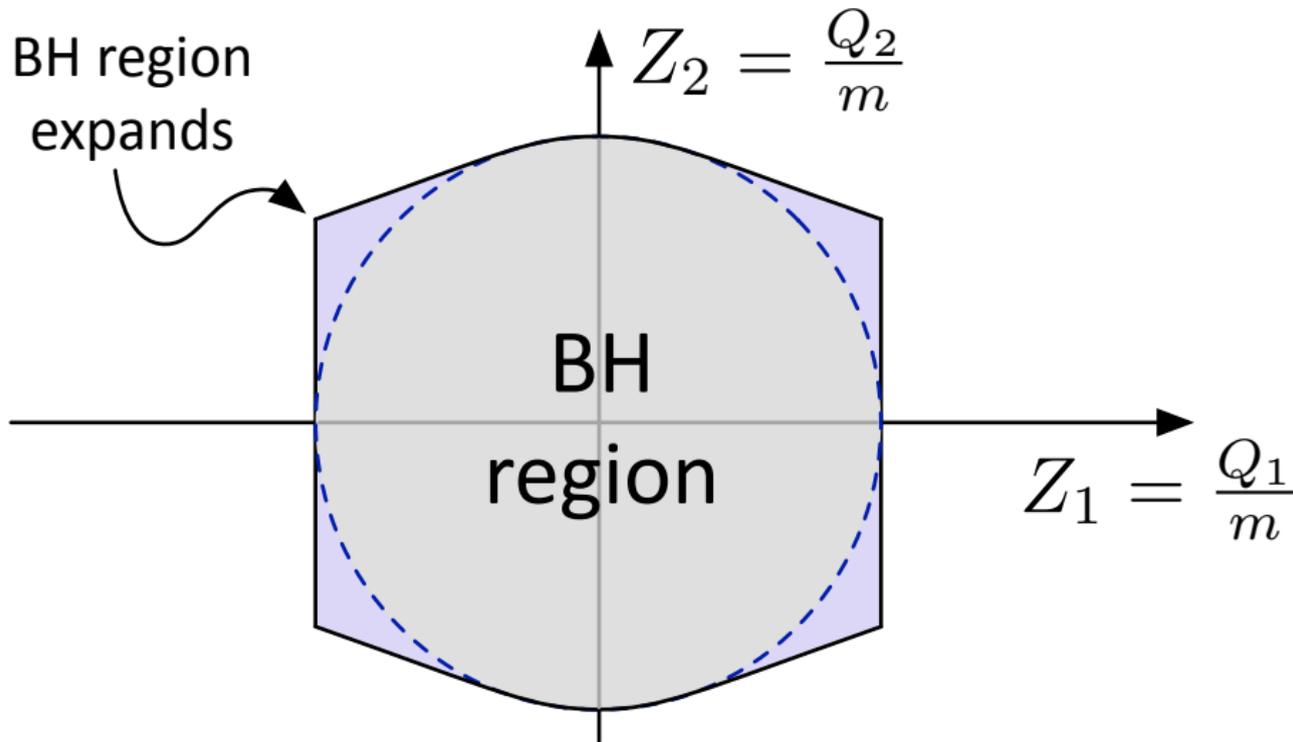
(Shape depends on moduli couplings)

# Circle compactification



Graviton/photon KK modes

$Z_{KK}$

BH region

4d RN region

$Z_{5d}$   $Z$

Charged particle KK 0-mode

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# Circle compactification



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$Q_{\mathrm{KK}}/m$

BH region

5d spectrum:

$Q, m$

$Q_5/m$

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$Q_{\mathrm{KK}}/m$

$R \to 0$

5d spectrum:

●— $Q, m$

BH region

$Q_5/m$

16

$Q_{KK}/m$

BH region

$Q_5/m$

5d spectrum:

● $2Q, 2m$

● $Q, m$
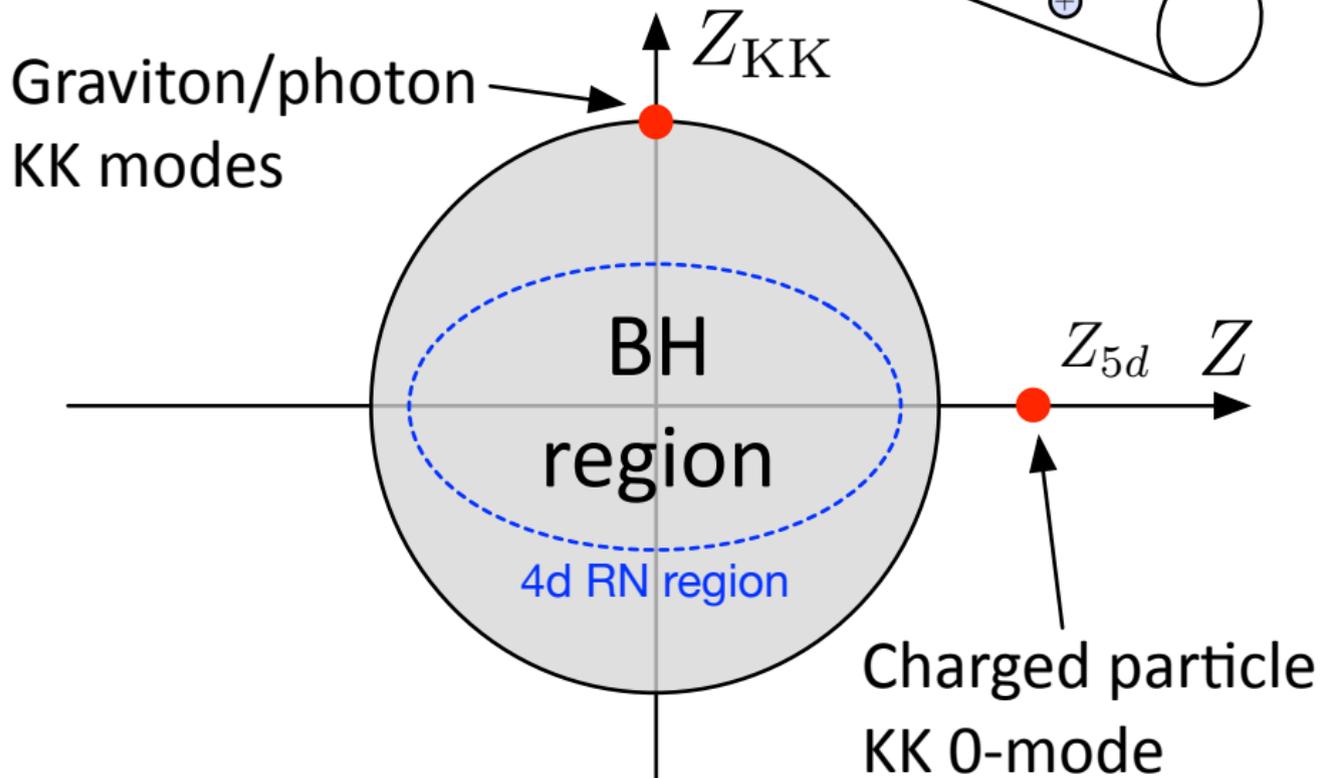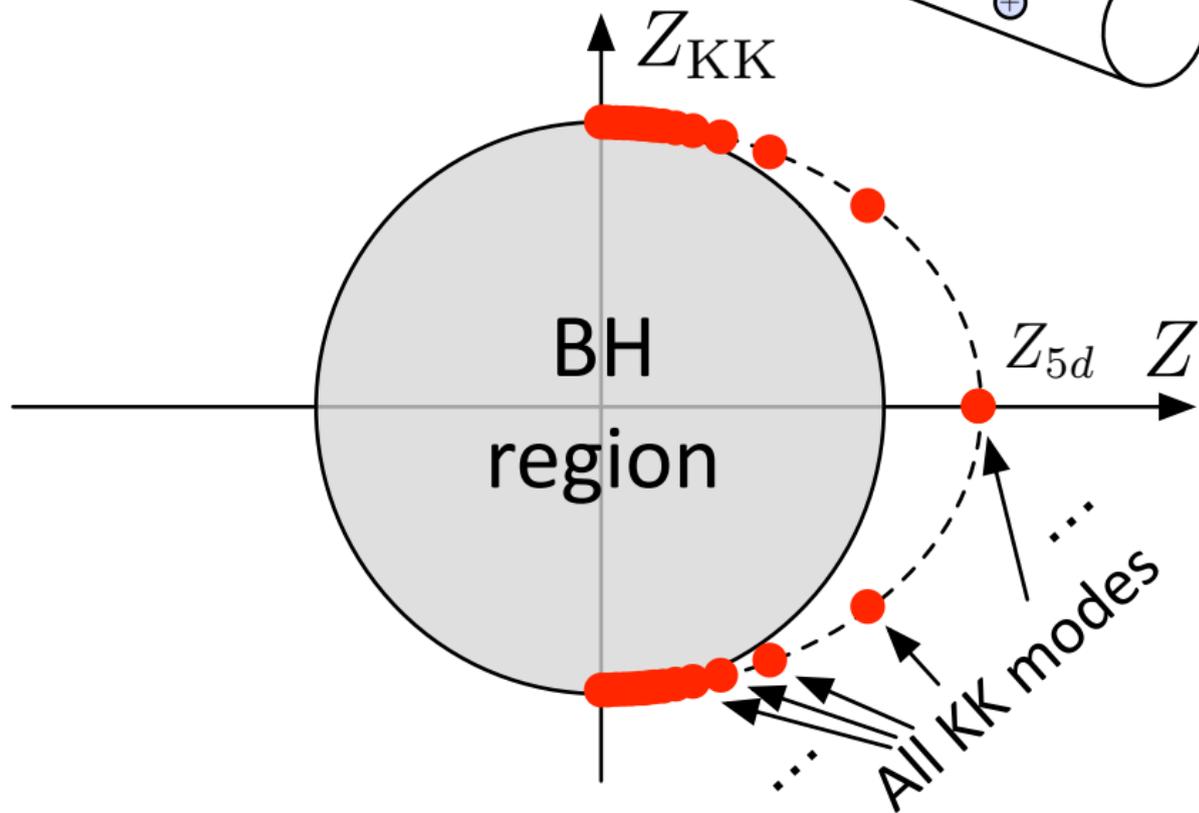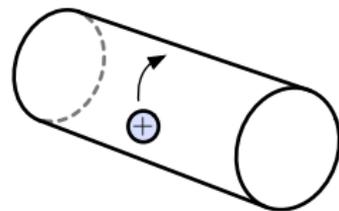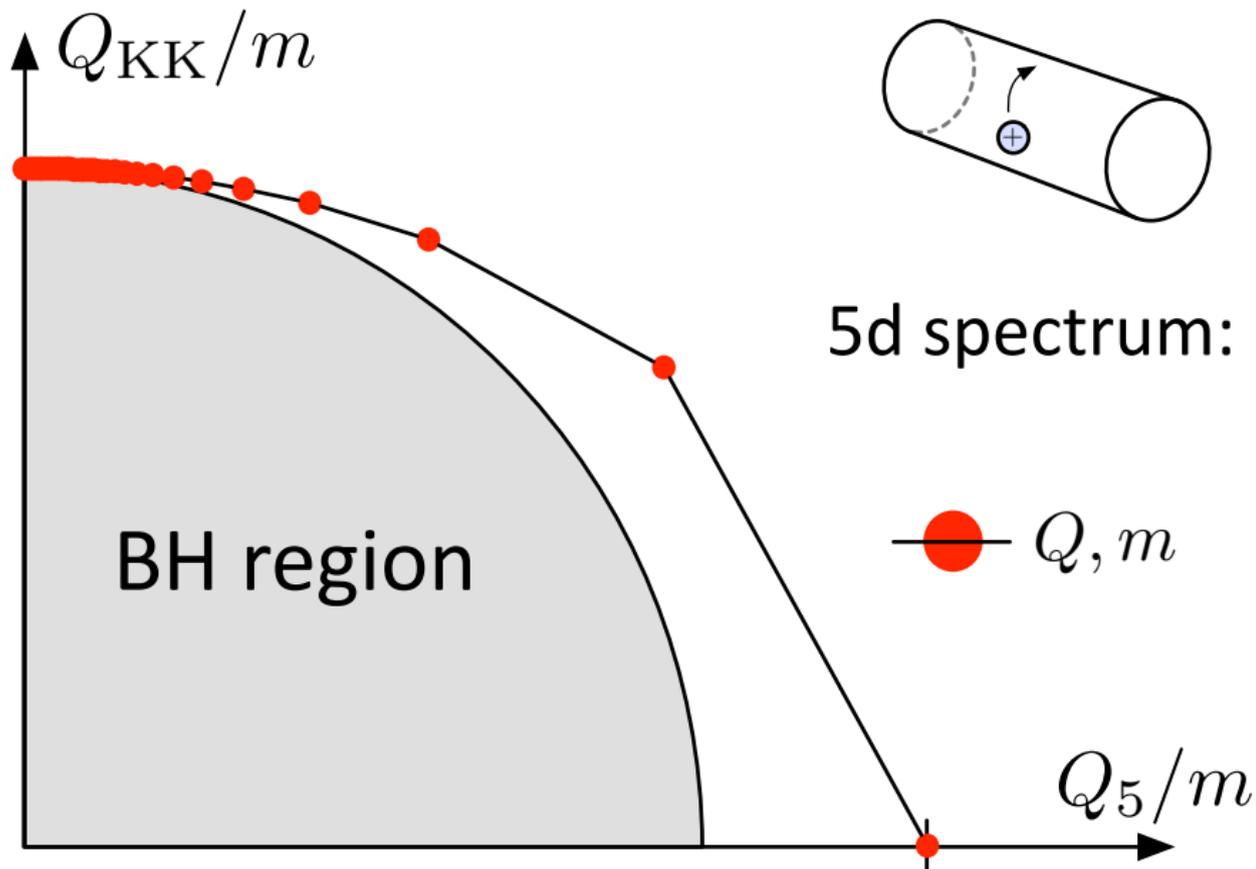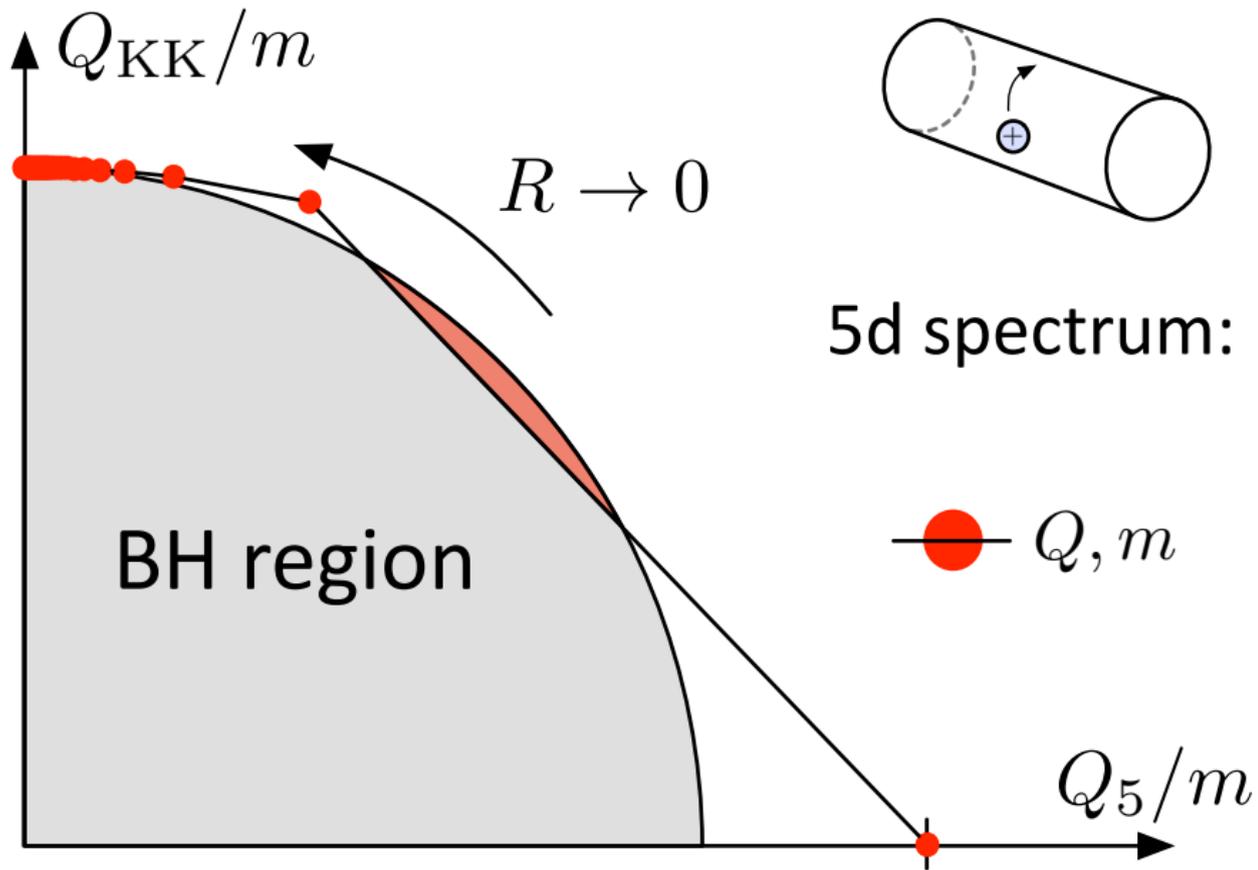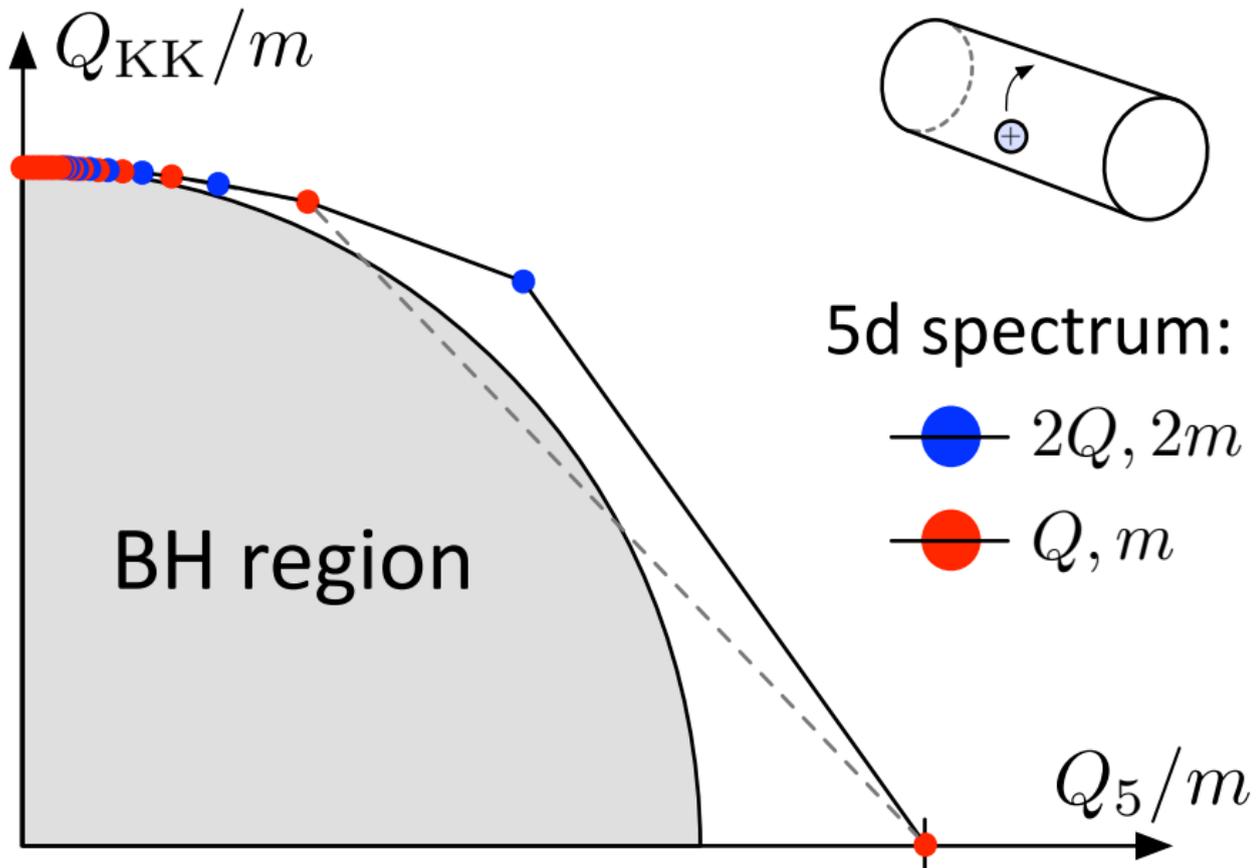
# WGC$_{d-1}$ requires superextremal resonances of arbitrarily large charge!
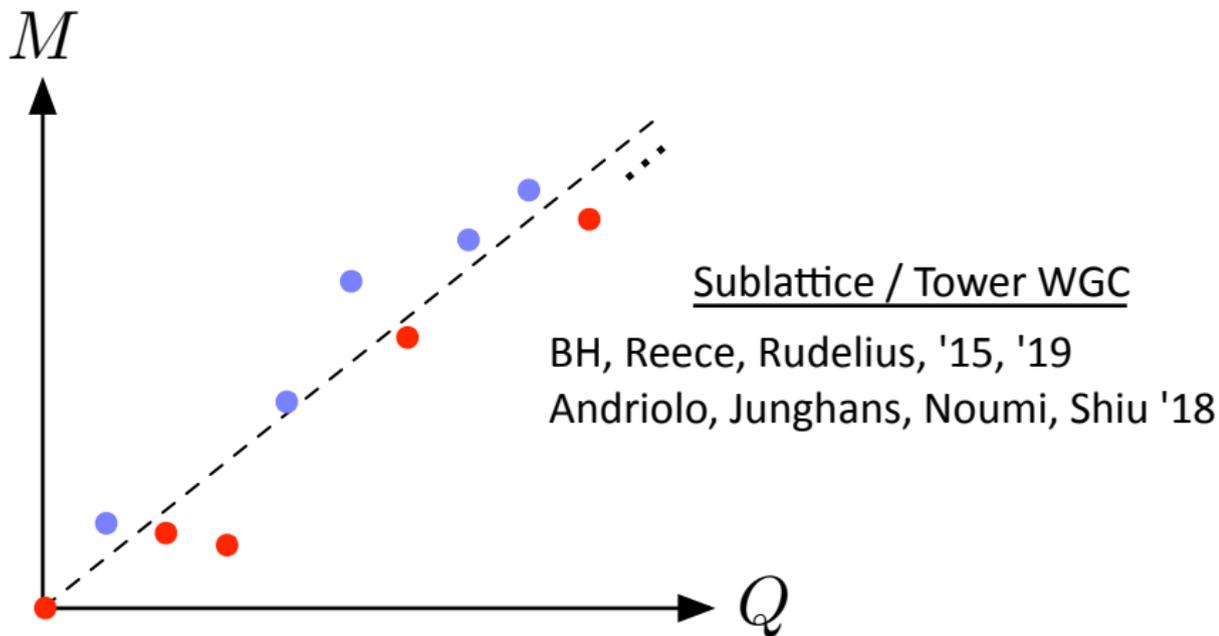


Sublattice / Tower WGC
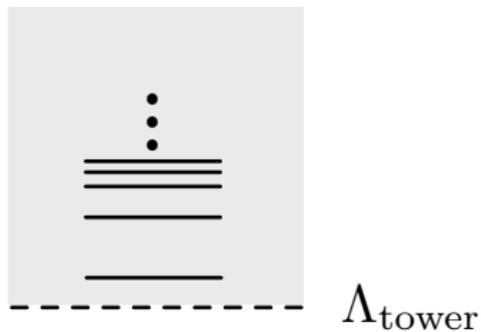
BH, Reece, Rudelius, '15, '19
Andriolo, Junghans, Noumi, Shiu '18

# sL/TWGC versus magnetic WGC



$\Lambda_{\text{tower}}$

Light states

# sL/TWGC versus magnetic WGC



Assuming tower starts at
$$q \simeq O(1)$$

$eM_{\mathrm{pl}}$

——— Light states

# sL/TWGC versus magnetic WGC



$eM_{\mathrm{pl}}$
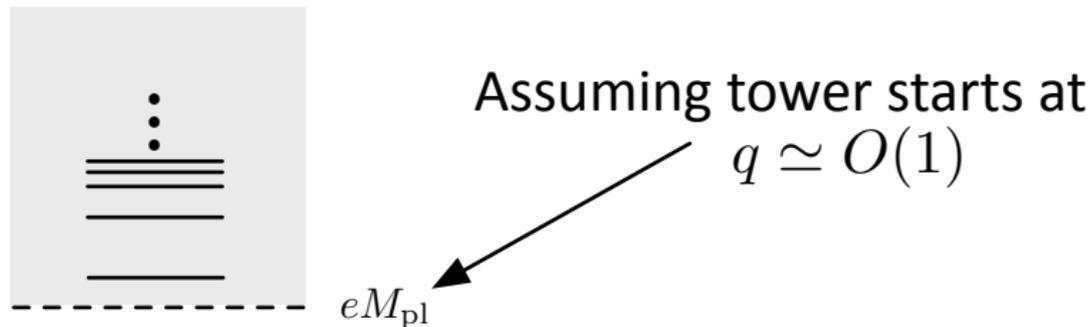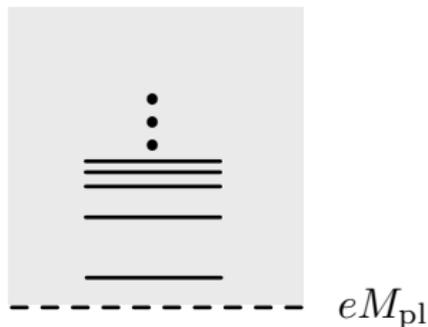
Light states
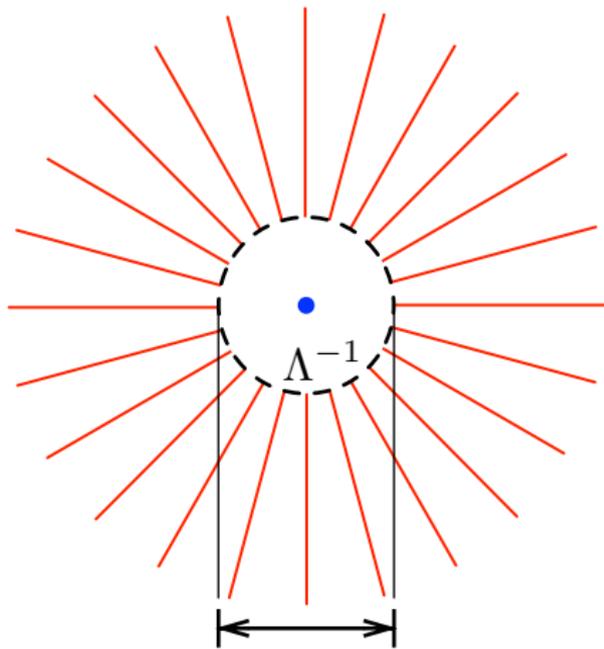
Tower provides required new physics!

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# Emergence



⋮

—————— $5eM_{\rm pl}$

—————— $4eM_{\rm pl}$

—————— $3eM_{\rm pl}$

—————— $2eM_{\rm pl}$

—————— $eM_{\rm pl}$
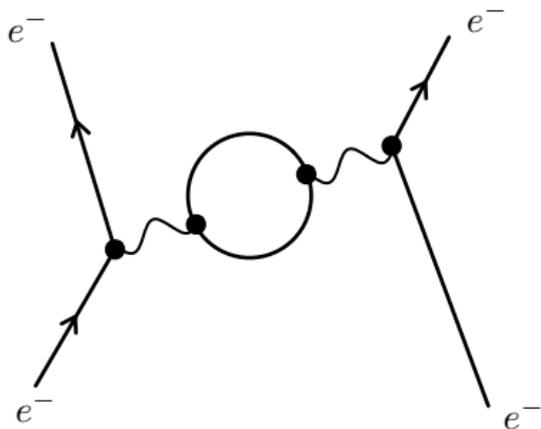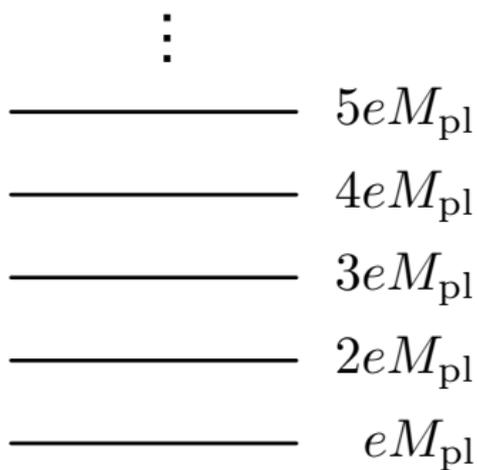
Loops of tower resonances
renormalize elementary charge

$$e \to \infty \text{ as } \Lambda \to e_{\rm IR}^{1/3} M_{\rm pl}$$

# Emergence



$\vdots$

$5eM_{\rm pl}$

$4eM_{\rm pl}$

$3eM_{\rm pl}$

$2eM_{\rm pl}$

$eM_{\rm pl}$
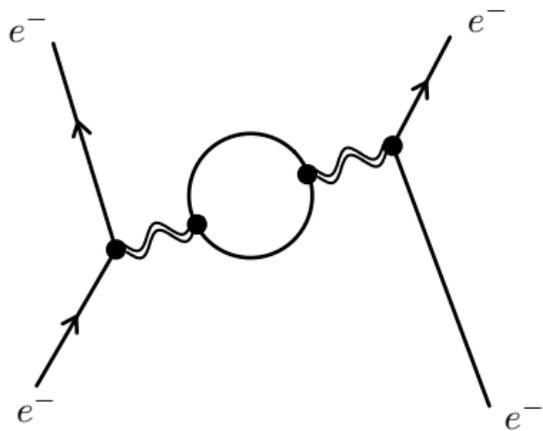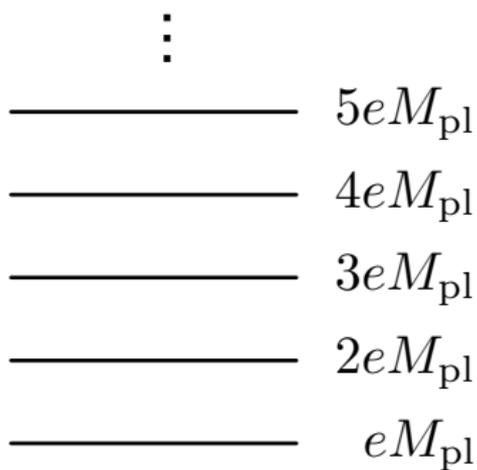
Gravitational interactions
also get stronger

$$M_{\rm pl} \to M_{\rm pl}/\sqrt{N} \qquad \text{for } N \text{ light species}$$

# Emergence
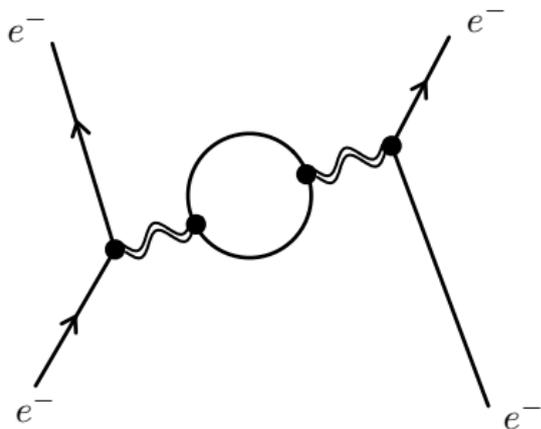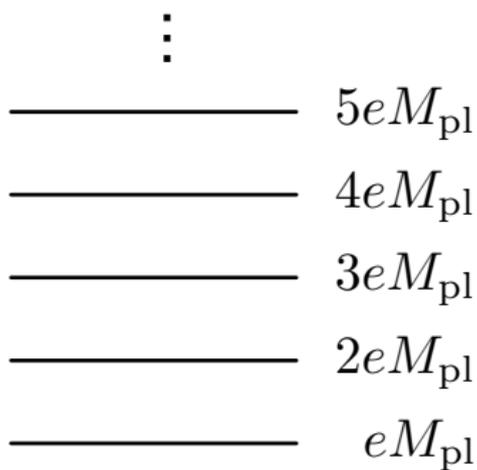


These scales match!

$$M_{\rm pl}/\sqrt{N} \simeq e_{\rm IR}^{1/3} M_{\rm pl}$$

# Emergence

Gravity and electromagnetism both become strongly coupled at

$$\Lambda_{\mathrm{QG}} \simeq e_{\mathrm{IR}}^{1/3} M_{\mathrm{pl}}$$

# Emergence

Gravity and electromagnetism both become highly non-linear at

$$\Lambda_{\mathrm{QG}} \simeq e_{\mathrm{IR}}^{1/3} M_{\mathrm{pl}}$$

Expectation: notion of spacetime breaks down at length scale $\ell_{\mathrm{QG}} = \hbar c / \Lambda_{\mathrm{QG}}$

Spacetime concepts, like local (gauge) symmetries must disappear above this scale. They "emerge" for $\ell \gg \ell_{\mathrm{QG}}$, due to screening effects of the tower of resonances.

# Emergence

Conversely, if gravity and electromagnetism both become highly non-linear at a common scale "$\Lambda_{QG}$" then the WGC (approximately) follows!

Emergence implies the WGC!

Harlow '15
BH, Reece, Rudelius '17

# Emergence

Conversely, if gravity and electromagnetism both become highly non-linear at a common scale "$\Lambda_{\mathrm{QG}}$" then the WGC (approximately) follows!

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Harlow '15
BH, Reece, Rudelius '17

e.g., Kaluza-Klein theory:

$$e_{\mathrm{KK}}^{1/3} M_4 = \sqrt{2}\pi^{1/3} M_5$$

$$\Longrightarrow \Lambda_{\mathrm{QG}} \simeq M_5!$$

# The Lattice WGC

Electric/magnetic charges are restricted to a discrete **lattice** by Dirac quantization

Guess: there is a superextremal charged particle at every site in this lattice

(BH, Reece, Rudelius, '15)

# The Lattice WGC

Electric/magnetic charges are restricted to a discrete **lattice** by Dirac quantization

Guess: there is a superextremal charged particle at every site in this lattice (BH, Reece, Rudelius, '15)

True in **many**, but **not all** string compactifications

BH, Reece
Rudelius, '16

(Known counterexs are all **orbifolds**)



$n_2 = 1$

$n_2 = 2$
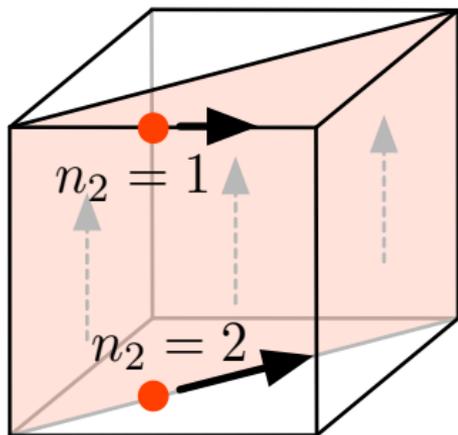
# The **sub**lattice WGC (BH, Reece, Rudelius, '16)

Fix this with a qualified statement:

<u>Sublattice WGC</u>: There is a superextremal charged particle at every site on some (equal dimension) **sub**lattice of the charge lattice

This is a **theorem** in tree-level string theory
(for electric NSNS sector gauge bosons)

# Proof via modular invariance

(BH, Reece, Rudelius '16; Montero, Shiu, Soler '16; BH, Lotito 2307.xxxxx)

Tree-level spectrum:

$$\frac{\alpha'}{4}m^2 = \frac{1}{2}Q_L^2 + T_L = \frac{1}{2}Q_R^2 + T_R$$

Partition function (torus 0-pt ampl):

$$Z(\mu, \tau) = \mathrm{Tr}(q^{T+\frac{1}{2}Q^2} y^Q)$$

$$q = e^{2\pi i \tau}$$

$$Z(\mu + \rho) = Z(\mu), \qquad \rho \in \Gamma^*$$

$$y = e^{2\pi i \mu}$$

Benjamin, Dyer, Fitzpatrick, Kachru '16:

$$Z(\mu/\tau, -1/\tau) = e^{\pi i \frac{\mu^2}{\tau}} Z(\mu, \tau)$$

$$Z = \text{Tr}(q^{T+\frac{1}{2}Q^2} y^Q)$$
$$= \text{Tr}(q^{T+\frac{1}{2}(Q+k\rho)^2} y^{Q+k\rho})$$
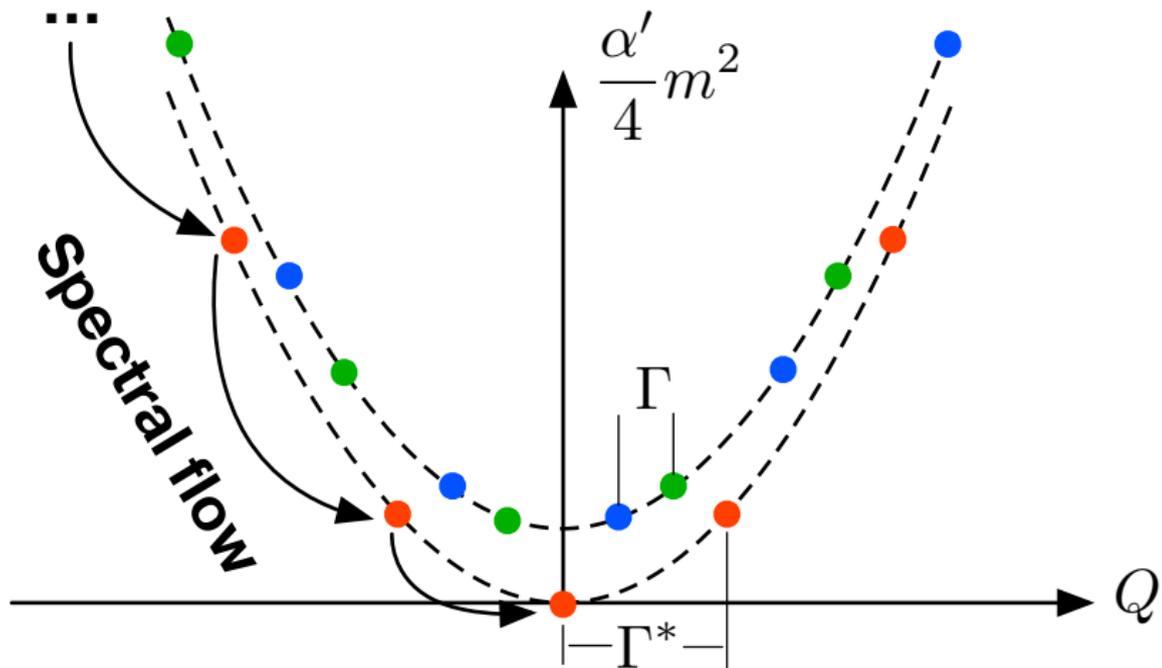
**Spectral flow**

$\frac{\alpha'}{4}m^2$

$\Gamma$

$Q$

$|-\Gamma^*-|$

# The extremality bound & the RFC

BH, Reece, Rudelius '19; BH, Lotito 2307.xxxxx

Found a tower of resonances with:

$$\frac{\alpha'}{4} m^2 = \frac{1}{2} \max(Q_L^2, Q_R^2)$$

What is the relation to extremal BHs?

# The extremality bound & the RFC

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What is the relation to extremal BHs?

Detour: "sublattice RFC"

> $\exists$ a finite-index sublattice $\Gamma_0 \subseteq \Gamma$
> s.t. $\forall Q \in \Gamma_0$, $\exists$ a **self-repulsive**
> charged particle with charge $Q$

# The extremality bound & the RFC

BH, Reece, Rudelius '19; BH, Lotito 2307.xxxxx

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Detour: "sublattice RFC"

Above tower
is self-repulsive

BH, Lotito
2307.xxxxx

$\exists$ a finite-index sublattice $\Gamma_0 \subseteq \Gamma$
s.t. $\forall Q \in \Gamma_0$ a **self-repulsive**
charged particle with charge $Q$

Proved in
tree-level ST

# The extremality bound & the RFC

BH, Reece, Rudelius '19; BH, Lotito 2207.xxxxx

Found a tower of resonances with:

$$\frac{\alpha'}{4}m^2 = \frac{1}{2}\max(Q_L^2, Q_R^2) \qquad (*)$$

What is the relation to extremal BHs?

Detour: "sublattice RFC"

Above tower is self-repulsive

$\exists$ a finite-index sublattice $\Gamma_0 \subseteq \Gamma$
s.t. $\forall Q \in \Gamma_0$ a self-repulsive
charged particle with charge $Q$

BH, Lotito
2207.xxxxx

Proved in tree-level ST

…actually this proves $(*)$ is extremal

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# The extremality bound & the RFC

Found a tower of resonances with:

$$\frac{\alpha'}{4} m^2 = \frac{1}{2} \max(Q_L^2, Q_R^2) \qquad (*)$$

What is the relation to extremal BHs?

A particle that is self-repulsive throughout moduli space is superextremal

(BH, Reece, Rudelius '19)

(Harlow, BH, Reece, Rudelius 2201.08380)

# Nonpert. evidence via BPS particles

Alim, BH, Rudelius 2108.08309
Gendler, BH, McAllister, Moritz, Rudelius 2212.10573

BPS prediction:



Infinite BPS towers predicted inside BPS=BHE cone

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M2

Genus 0 GV invariants:

| $q_Y \setminus q_X$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | | 640 | 10032 | 288384 |
| 1 | 64 | 6912 | 742784 | 75933184 |
| 2 | 0 | 14400 | 8271360 | 2445747712 |
| 3 | 0 | 6912 | 31344000 | 26556152064 |
| 4 | 0 | 640 | 48098560 | 130867460608 |
| 5 | 0 | 0 | 31344000 | 329212616704 |
| 6 | 0 | 0 | 8271360 | 445404149568 |
| 7 | 0 | 0 | 742784 | 329212616704 |
| 8 | 0 | 0 | 10032 | 130867460608 |
| 9 | 0 | 0 | 0 | 26556152064 |
| 10 | 0 | 0 | 0 | 2445747712 |
| 11 | 0 | 0 | 0 | 75933184 |
| 12 | 0 | 0 | 0 | 288384 |
| 13 | 0 | 0 | 0 | 0 |

$\tilde{X}$

$\tilde{Y}$

BPS=BHE cone

# Nonpert. evidence via BPS particles

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WGC

$\Downarrow$

Highly nontrivial
predictions
about CY geometry

Verified in
>1000 geometries



M2

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$\tilde{X}$

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BPS=BHE cone

# Nonpert. evidence via BPS particles

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**Every example we tested actually satisfies the Lattice WGC?!**
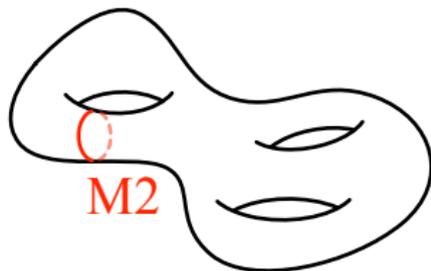
(Hint that LWGC violations are somehow rare / associated to special structures e.g., orbifolds.)



M2

Genus 0 GV invariants:

| $q_Y \setminus q_X$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | | 640 | 10032 | 288384 |
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$\tilde{X}$

$\tilde{Y}$

BPS=BHE cone

# Application: Transplanckian scalars!?

Can a scalar field have a flat potential over a super-Planckian field range?

# Application: Transplanckian scalars!?

Can a scalar field have a flat potential over a super-Planckian field range?



$V(\phi)$

$\gtrsim M_{\text{pl}}$

$\phi$

start          end
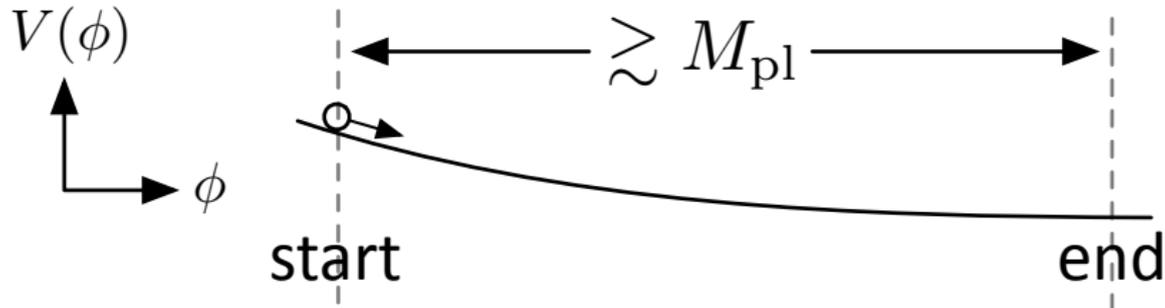
Generically **no**:

$$V_{\text{eff}}(\phi) \sim V_0(\phi) + \phi^5/M_{\text{pl}} + \phi^6/M_{\text{pl}}^2 + \ldots$$

$\sim M_{\text{pl}}$
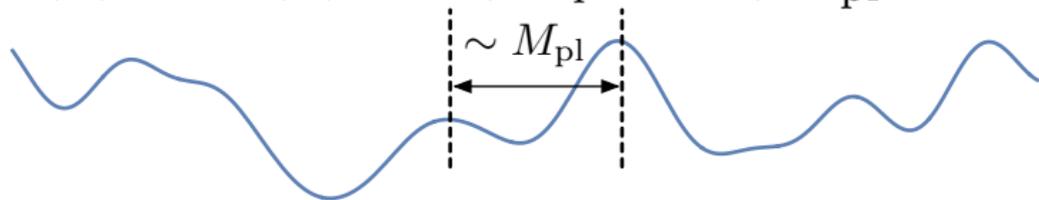
# Axion loophole? $\qquad \phi \cong \phi + 2\pi f$

$$V_{\text{eff}}(\phi) \sim V_0(\phi) + \phi^5/M_{\text{pl}} + \ldots$$

Example:



photon $\longrightarrow$ photon+axion
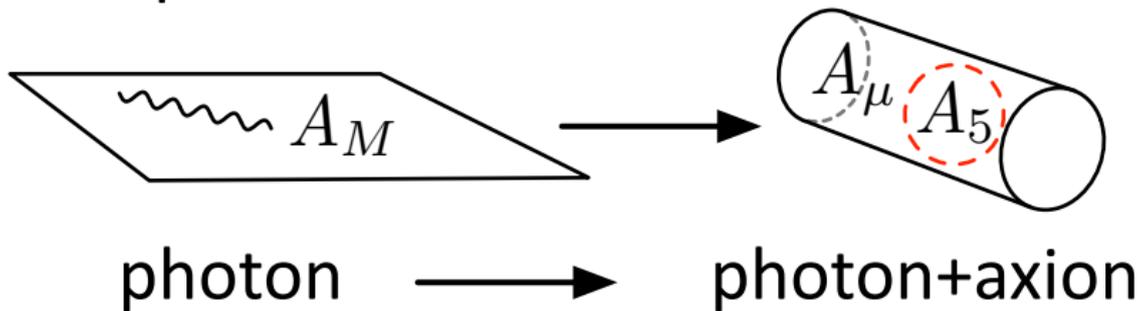
<u>string theory axions</u>
<u>often realized this way</u>

# Axion Potential

Casimir effect

$$\sum_{\text{species}} \sum_{n}$$



$n$

$A_5$
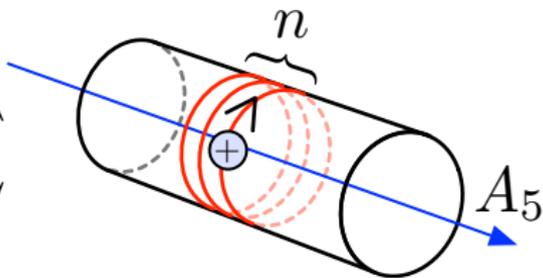
$$V = \sum_i c_i \underbrace{\cos(q_i \phi / f)}_{\text{Aharonov-Bohm phase}} e^{-2\pi R m_i} + \ldots$$

Aharonov-Bohm phase

$$(Q_i \propto q_i \in \mathbb{Z}, \ A_5 \propto \phi)$$

$$V = \sum_i c_i \cos(q_i \phi / f) \, e^{-2\pi R m_i} + \dots$$

Suppose $m_1 \ll m_{i\neq 1}$, $q_1 = 1$
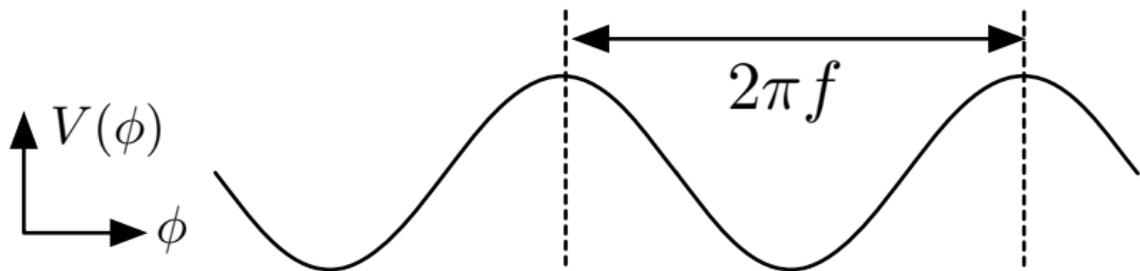


"Natural inflation" (Freese, Frieman, Olinto '90)

Needs $f \gtrsim 5\text{–}10 \ M_{\mathrm{pl}}$ for successful inflation

$$V = \sum_i c_i \cos(q_i \phi / f)\, e^{-2\pi R m_i} + \dots$$

(s)LWGC implies particles with
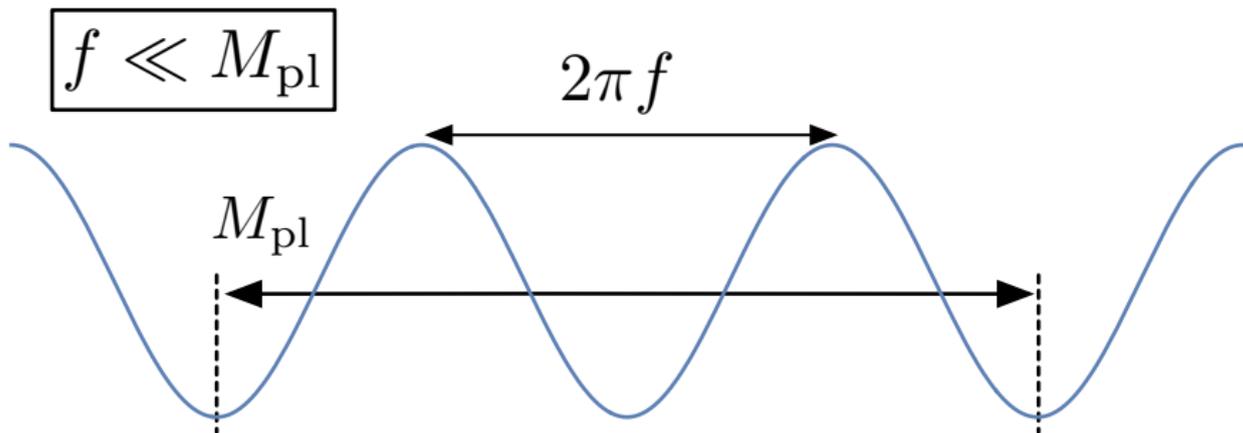
$$2\pi R m_i \lesssim \frac{M_{\mathrm{pl}}}{f} q_i \text{ for every } q_i \in \mathbb{Z}$$



$\boxed{f \ll M_{\mathrm{pl}}}$

$2\pi f$

$M_{\mathrm{pl}}$

$$V = \sum_i c_i \cos(q_i \phi / f) \, e^{-2\pi R m_i} + \dots$$

(s)LWGC implies particles with

$$2\pi R m_i \lesssim \frac{M_{\mathrm{pl}}}{f} q_i \ \text{for every} \ q_i \in \mathbb{Z}$$

$\boxed{f \gtrsim M_{\mathrm{pl}}}$



$M_{\mathrm{pl}}$

$2\pi f$
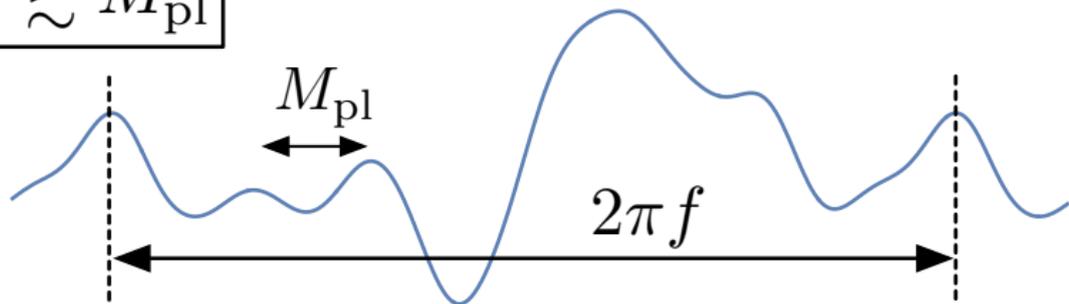
# The generalized WGC

Charged worldline wrapped around circle generates an "instanton"

"Action" is
$$S = 2\pi R m \lesssim M_{\mathrm{pl}}/f$$



$A_5$
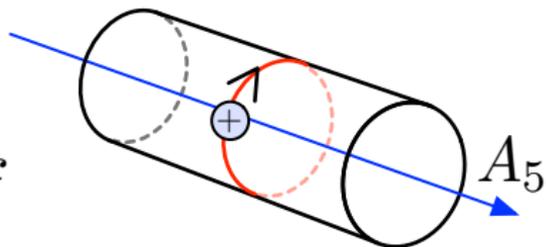
Tower of instanton corrections unsuppressed when $f \gtrsim M_{\mathrm{pl}}$

Generates harmonics in the potential, reducing effective field range!

ST: $f \lesssim M_{\mathrm{pl}}$ (Banks, Dine, Fox, Gorbatov '03)

# 1 axion: $f \lesssim M_{\mathrm{pl}}$ (Banks, Dine, Fox, Gorbatov '03)
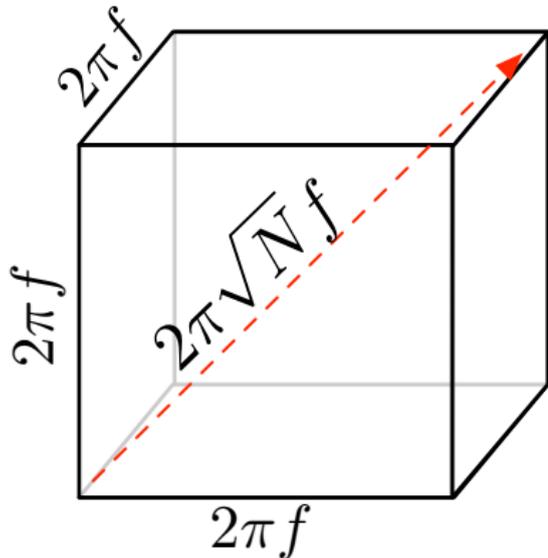
## $N \gg 1$ axions?



$$f_{\mathrm{eff}} = \sqrt{N} f$$

(Dimopoulos, Kachru, McGreevy, Wacker '05)

**1 axion:** $f \lesssim M_{\mathrm{pl}}$   (Banks, Dine, Fox, Gorbatov '03)

$N \gg 1$ **axions?**
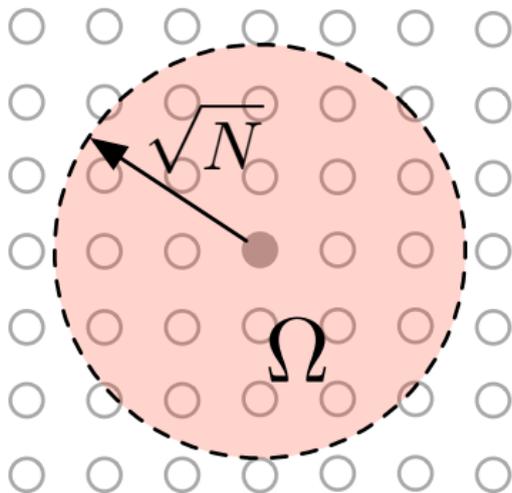


Many instantons can contribute!

# Random axion potentials

BH, Long, McAllister, Rudelius, Stout '19

$$\mathcal{L} = -\frac{1}{2}\delta_{ab}\,\partial_\mu \phi^a\,\partial^\mu \phi^b - \Lambda^4 \sum_{\mathbf{Q}\in\Gamma} Z_{\mathbf{Q}} \exp\left(i\mathbf{Q}\cdot\boldsymbol{\phi}\right)$$

$$\langle |Z_{\mathbf{Q}}|^2 \rangle = e^{-2\mu|\mathbf{Q}|}$$

## Focusing on potential along a ray:

$$\sigma_n^2(\bar{\mathbf{e}}) = \sum_{\substack{\mathbf{Q}\in\Gamma}}^{\mathbf{Q}\cdot\bar{\mathbf{e}}=n} e^{-2\mu|\mathbf{Q}|}$$

## Using a continuum approximation

$$\mathrm{vol}\,\boldsymbol{\phi} \lesssim \mathrm{vol}\,D_N(2\mu)$$ **Ball, not cube!**

# Random axion potentials

BH, Long, McAllister, Rudelius, Stout '19
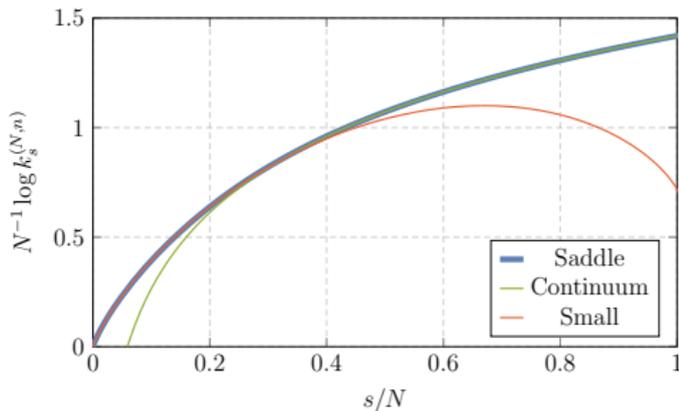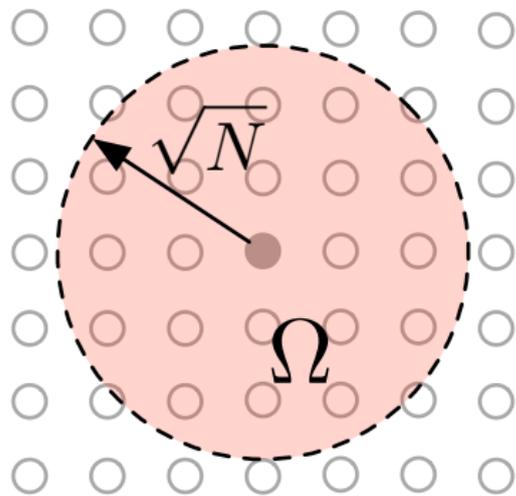


Can analyze potential more systematically via a saddle-point approx

$$\text{Still find: } f_{\text{eff}} \lesssim M_{\text{pl}}$$

# Alignment

(Kim, Nilles, Peloso '04)



5d charge spectrum:

$(K, 1)$

$(1, 0)$
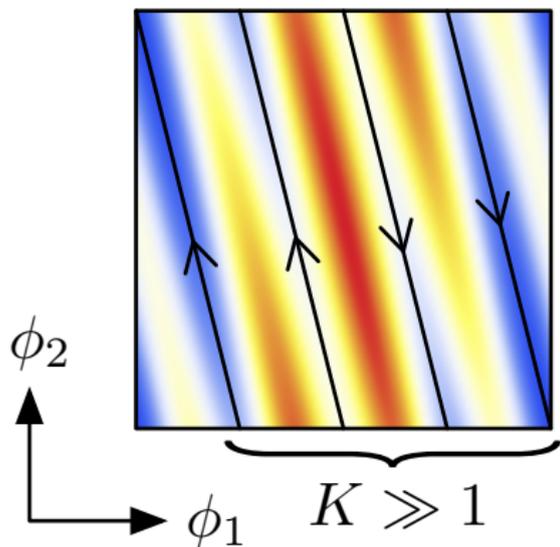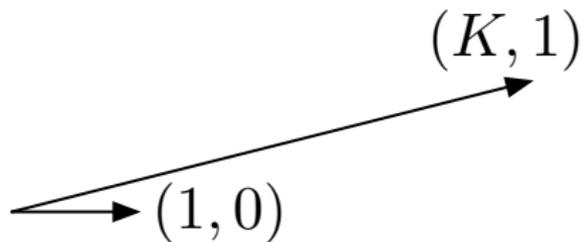
$$f_{\mathrm{eff}} = \sqrt{K^2 + 1}\, f \simeq Kf$$

# Alignment

5d charge spectrum:

$(K, 1)$

$(1, 0)$

$\phi_2$

$\phi_1$

$K \gg 1$
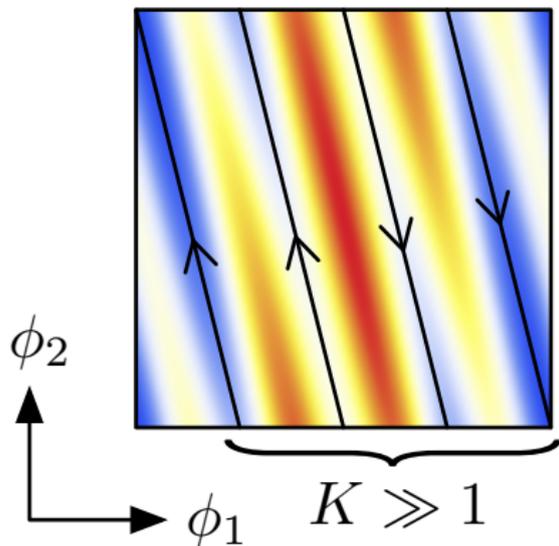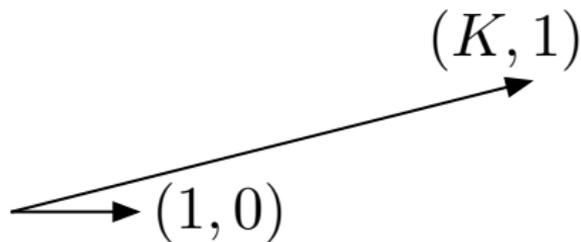
Requires tuning of charged spectrum

Potential alternative: kinetic alignment

# Summary

The WGC is a constraint on QGs supported by highly non-trivial string theory evidence which…

• makes potentially falsifiable predictions,

• while also hinting at the emergent nature of QG,

• as well as providing a template for further exploration of the swampland and the landscape,

• even though there is still much more to be learned about the WGC itself!