# Theories of Neutrino Masses 

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## Oscillating Neutrinos

๕้ Neutrinos oscillate among flavors:

$$
P\left(\bar{\nu}_{e} \rightarrow \bar{\nu}_{e}\right)=1-\sin ^{2} 2 \theta \sin ^{2} \frac{\Delta m^{2} L}{4 E}
$$

Oscillatory behavior observed by Daya Bay $\bar{\nu}_{e}, \operatorname{KamLand} \bar{\nu}_{e}$ and SuperKamiokande atmospheric $\nu_{\mu}$ data


## Current knowledge of 3-neutrino oscillations

|  |  |  |  |  | NuFIT 5.1 (2021) |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Normal Ordering (best fit) |  | Inverted Ordering ( $\left.\Delta \chi^{2}=2.6\right)$ |  |
|  |  | bfp $\pm 1 \sigma$ | $3 \sigma$ range | bfp $\pm 1 \sigma$ | $3 \sigma$ range |
|  | $\sin ^{2} \theta_{12}$ | $0.304_{-0.012}^{+0.013}$ | $0.269 \rightarrow 0.343$ | $0.304_{-0.012}^{+0.012}$ | $0.269 \rightarrow 0.343$ |
|  | $\theta_{12} /^{\circ}$ | $33.44_{-0.74}^{+0.77}$ | $31.27 \rightarrow 35.86$ | $33.455_{-0.74}^{+0.77}$ | $31.27 \rightarrow 35.87$ |
|  | $\sin ^{2} \theta_{23}$ | $0.573_{-0.023}^{+0.018}$ | $0.405 \rightarrow 0.620$ | $0.578_{-0.021}^{+0.017}$ | $0.410 \rightarrow 0.623$ |
|  | $\theta_{23} /^{\circ}$ | $49.22_{-1.3}^{+1.0}$ | $39.5 \rightarrow 52.0$ | $49.5{ }_{-1.2}^{+1.0}$ | $39.8 \rightarrow 52.1$ |
|  | $\sin ^{2} \theta_{13}$ | $0.02220_{-0.00062}^{+0.00068}$ | $0.02034 \rightarrow 0.02430$ | $0.02238_{-0.00062}^{+0.00064}$ | $0.02053 \rightarrow 0.02434$ |
|  | $\theta_{13} /{ }^{\circ}$ | $8.57_{-0.12}^{+0.13}$ | $8.20 \rightarrow 8.97$ | $8.60_{-0.12}^{+0.12}$ | $8.24 \rightarrow 8.98$ |
|  | $\delta_{\mathrm{CP}} /{ }^{\circ}$ | $194{ }_{-25}^{+52}$ | $105 \rightarrow 405$ | $287{ }_{-32}^{+27}$ | $192 \rightarrow 361$ |
|  | $\begin{gathered} \frac{\Delta m_{21}^{2}}{10^{-5} \mathrm{eV}^{2}} \\ \frac{\Delta m_{3 \ell}^{2}}{10^{-3} \mathrm{eV}^{2}} \end{gathered}$ | $7.42_{-0.20}^{+0.21}$ $+2.515_{-0.028}^{+0.028}$ | $6.82 \rightarrow 8.04$ $+2.431 \rightarrow+2.599$ | $7.42_{-0.20}^{+0.21}$ $-2.498_{-0.029}^{+0.028}$ | $6.82 \rightarrow 8.04$ $-2.584 \rightarrow-2.413$ |

Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou (2020)

## Roadmap for Neutrino Models



## Effective Field Theory for neutrino masses

\& Neutrino masses are zero in the Standard Model. Observed oscillations require new physics beyond Standard Model
$\mathscr{\%}$ Neutrino masses and oscillations can be explained in terms of the celebrated $d=5$ Weinberg operator

$$
\mathcal{O}_{1}=\frac{\kappa_{a b}}{2}\left(L_{a}^{i} L_{b}^{j}\right) H^{k} H^{\prime} \epsilon_{i k} \epsilon
$$

\%์ $\kappa^{-1} \sim\left(10^{14}-10^{15}\right) \mathrm{GeV}$ can explain oscillation data
$\mathscr{\%}$ EFT description cannot be the end goal, especially when the coefficients of operators are measured. Without UV completion important phenomena would be missed (e.g. Leptogenesis)
$\mathfrak{H}$ What if neutrinos are Dirac particles? $\mathcal{O}_{1}$ is then the wrong description
\% What if neutrino masses arose from $d=7$ operators or $d=9$ operators in a fundamental theory, and not through $\mathcal{O}_{1}$ ?

## Origin of neutrino mass: Seesaw mechanism

\% Adding right-handed neutrino $N^{c}$ which transforms as singlet under $S U(2)_{L}$,

$$
\mathcal{L}=f_{\nu}(L \cdot H) N^{c}+\frac{1}{2} M_{R} N^{c} N^{c}
$$

\& Integrating out the $N^{c}, \Delta L=2$ operator is induced:


$$
\mathcal{L}_{\text {eff }}=-\frac{f_{\nu}^{2}}{2} \frac{(L \cdot H)(L \cdot H)}{M_{R}}
$$

$\mathscr{F}$ Once $H$ acquires VEV, neutrino mass is induced:

$$
m_{\nu} \simeq f_{\nu}^{2} \frac{v^{2}}{M_{R}}
$$

Minkowski (1977)
Yanagida (1979)
Gell-Mann, Ramond, Slansky (1980)
Mohapatra \& Senjanovic (1980)
\& For $f_{\nu} v \simeq 100 \mathrm{GeV}, M_{R} \simeq\left(10^{14}-10^{15}\right) \mathrm{GeV}$.

## Baryogenesis via leptogenesis and type-I seesaw

\& In the early history of the universe, a lepton asymmetry may be dynamically generated in the decay of $N$ Fukugita, Yanagida (1986)
\&ะ $N$ being a Majorana fermion can decay to $L+H$ as well as $\bar{L}+H^{*}$

\& Three Sakharov conditions can be satisfied: $B$ violation via electroweak sphaleron, $C$ and $C P$ violation in Yukawa couplings of $N$, and out of equilibrium condition via expanding universe
$\mathscr{\&}$ Lepton asymmetry in decay of $N_{1}$ (with $M_{1} \ll M_{2,3}$ ):

$$
\varepsilon_{1} \simeq \frac{3}{16 \pi} \frac{1}{\left(f_{\nu} f_{\nu}^{\dagger}\right)_{11}} \sum_{i=2,3} \operatorname{Im}\left[\left(f_{\nu} f_{\nu}^{\dagger}\right)_{i 1}^{2}\right] \frac{M_{1}}{M_{i}}
$$

$\% \varepsilon \sim 10^{-6}$ can explain observed baryon asymmetry of the universe
\% Indirect tests in Majorana nature of $\nu$ and in CP violation in oscillations

## Seesaw mechanism (cont.)

Type II seesaw: $\Phi_{3} \sim(1,3,1)$
Mohapatra \& Senjanovic (1980)
Schechter \& Valle (1980)


Lazarides, Shafi, \& Wetterich (1981)

Type III seesaw: $N_{3} \sim(1,3,0)$
Foot, Lew, He, \& Joshi (1989)

Ma (1998)

\& $\Phi_{3}$ abd $N_{3}$ contain charged particles which can be looked for at LHC
\& Eg: $\Phi^{++} \rightarrow \ell^{+} \ell^{+}, \Phi^{++} \rightarrow W^{+} W^{+}$decays would establish lepton number violation

## Neutrinoless Double Beta Decay


\&ะ Majorana vs Dirac neutrinos: Observation of $\beta \beta 0 \nu$ will establish neutrinos are Majorana particles
\% Kamland-Zen collaboration has a limit from ${ }^{136} \mathrm{Xe}$ :

$$
T_{0 \nu}^{1 / 2}>1.07 \times 10^{26} \mathrm{yr} .
$$

\% Constrains effective double beta decay mass of neutrino to be

$$
\begin{gathered}
m_{\beta \beta}<(61-165) \mathrm{meV} \\
m_{\beta \beta}=\left|\sum_{i} U_{e i}^{2} m_{i}\right|=\left|c_{12}^{2} c_{13}^{2} e^{2 i \alpha_{1}} m_{1}+c_{13}^{2} s_{12}^{2} e^{2 i \alpha_{2}} m_{2}+s_{13}^{2} m_{3}\right|
\end{gathered}
$$

## Neutrinoless Double Beta Decay (cont.)

\&ะ Largest uncertainty from unknown Majorana phases $\alpha_{1}, \alpha_{2}$ and $\theta_{23}$
\% For normal hierarchy cancellation possible

$$
m_{\beta \beta} \simeq\left|c_{13}^{2} s_{12}^{2} \sqrt{\Delta m_{s}^{2}} e^{2 i \alpha_{2}}+s_{13}^{2} \sqrt{\Delta m_{a}^{2}}\right|<4 \times 10^{-3} \mathrm{eV}
$$

$\%$ For inverted hierarchy no such cancellation possible

$$
m_{\beta \beta} \simeq \sqrt{\Delta m_{a}^{2}} \sqrt{1-\sin ^{2} 2 \theta_{12} \sin ^{2}\left(\alpha_{2}-\alpha_{1}\right)}, 2 \times 10^{-2} \leq m_{\beta \beta} \leq 5 \times 10^{-2} \mathrm{eV}
$$



Giunti, Bilenki (2014)

## Lepton Number Violation at the LHC

\% Classic way to establish Majorana nature of neutrino is to observe neutrinoless double beta decay (Schechter, Valle, 1981)
\%i $p p \rightarrow \ell^{ \pm} \ell^{ \pm}+$jets process can also establish $L$ violation by two units, and hence Majorana nature of neutrino (Keung, Senjanovic, 1983)
\% This is realized in type-II seesaw model (Babu, Barman, Gonçalves, Ismail, 2022; Cai, Han, Li, Ruiz, 2018)


## L-violation in type-II Seesaw at LHC



Figure: $p p \rightarrow \ell^{ \pm} \ell^{\prime \pm}+$ jets

(Babu, Barman, Gonçalves, Ismail, 2022)

## Dirac Neutrino Models

\&์ Neutrinos may be Dirac particles without lepton number violation
\%ٌ Oscillation experiments cannot distinguish Dirac neutrinos from Majorana neutrinos
$\%$ Spin-flip transition rates (in stars, early universe) are suppressed by small neutrino mass:

$$
\Gamma_{\text {spin-flip }} \approx\left(\frac{m_{\nu}}{E}\right)^{2} \Gamma_{\text {weak }}
$$

ซ: If neutrinos are Dirac, it would be nice to understand the smallness of their mass
\& Models exist which explain the smallness of Dirac $m_{\nu}$
\% "Dirac leptogenesis" can explain baryon asymmetry
Dick, Lindner, Ratz, Wright (2000)

## Dirac Seesaw Models

$\mathscr{E}$ Dirac seesaw can be achieved in Mirror Models Lee, Yang (1956); Foot, Volkas (1995); Berezhiani, Mohapatra (1995), Silagadze(1997) and Left-Right Symmetric Models Mohapatra (1988); Babu, He (1989); Babu, He, Su, Thapa (2022)
\&: Mirror sector is a replica of Standard Model, with new particles transforming under mirror gauge symmetry:

$$
L=\binom{\nu}{e}_{L} ; \quad H=\binom{H^{+}}{H^{0}} ; \quad L^{\prime}=\binom{\nu^{\prime}}{e^{\prime}}_{L} ; \quad H^{\prime}=\binom{H^{\prime+}}{H^{\prime 0}}
$$

Effective dimension-5 operator induces small Dirac mass:

\& $B-L$ may be gauged to suppress Planck-induced Weinberg operator $(L L H H) / M_{P l}$ that would make neutrino pseudo-Dirac particle

## Unification of Forces \& Matter in SO(10)

16 members of a family fit into a spinor of $S O(10)$

| $u_{r}:\{-+++-\}$ | $d_{r}:\{-++-+\}$ | $u_{r}^{c}:\{+--++\}$ | $d_{r}^{c}:\{+----\}$ |
| :---: | :---: | :---: | :---: |
| $u_{b}:\{+-++-\}$ | $d_{b}:\{+-+-+\}$ | $u_{b}^{c}:\{-+-++\}$ | $d_{b}^{c}:\{-+---\}$ |
| $u_{g}:\{++-+-\}$ | $d_{g}:\{++--+\}$ | $u_{g}^{c}:\{--+++\}$ | $d_{g}^{c}:\{--+--\}$ |
| $v:\{---+-\}$ | $e:\{----+\}$ | $v^{c}:\{+++++\}$ | $e^{c}:\{+++--\}$ |

First 3 spins refer to color, last two are weak spins

$$
Y=\frac{1}{3} \Sigma(C)-\frac{1}{2} \Sigma(W)
$$



## Disparity in Quark \& Lepton Mixings



## Yukawa Sector of Minimal $S O(10)$

$$
16 \times 16=10_{s}+120_{a}+126_{s}
$$

$\%$ At least two Higgs fields needed for family mixing
$\%$ Symmetric $10_{H}$ and $\overline{126}$ is the minimal model

$$
\begin{aligned}
W_{S O(10)} & =16^{T}\left(Y_{10} 10_{H}+Y_{126} \overline{126}_{H}\right) 16 . \\
M_{U} & =v_{u}^{10} Y_{10}+v_{u}^{126} Y_{126} \\
M_{D} & =v_{d}^{10} Y_{10}+v_{d}^{126} Y_{126} \\
M_{E} & =v_{d}^{10} Y_{10}-3 v_{d}^{126} Y_{126} \\
M_{\nu_{D}} & =v_{u}^{10} Y_{10}-3 v_{u}^{126} Y_{126} \\
M_{R} & =Y_{126} V_{R}
\end{aligned}
$$

## Minimal Yukawa sector of SO(10)

\& 12 parameters plus 7 phases to fit 18 observed quantities
$\%$ This setup fits all obsevables quite well
\% Large neutrino mixings coexist with small quark mixings
\% $\theta_{13}$ prediction turned out to be correct


Babu, Mohapatra (1993); Bajc, Senjanovic, Vissani (2001); (2003); Fukuyama, Okada (2002); Goh, Mohapatra, Ng (2003); Bajc, Melfo, Senjanovic, Vissani (2004); Bertolini, Malinsky, Schwetz (2006); Babu, Macesanu (2005); Dutta, Mimura, Mohapatra (2007); Aulakh et al (2004); Bajc, Dorsner, Nemevsek (2009); Joshipura, Patel (2011); Dueck, Rodejohann (2013); Ohlsson, Penrow (2019); Babu, Bajc, Saad (2018); Babu, Saad (2021)

## Best fit values for fermion masses and mixings

| Observables <br> $($ masses in GeV$)$ | SUSY |  |  | non-SUSY |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Input | Best Fit | Pull | Input | Best Fit | Pull |
| $m_{u} / 10^{-3}$ | $0.502 \pm 0.155$ | 0.515 | 0.08 | $0.442 \pm 0.149$ | 0.462 | 0.13 |
| $m_{c}$ | $0.245 \pm 0.007$ | 0.246 | 0.14 | $0.238 \pm 0.007$ | 0.239 | 0.18 |
| $m_{t}$ | $90.28 \pm 0.89$ | 90.26 | -0.02 | $74.51 \pm 0.65$ | 74.47 | -0.05 |
| $m_{b} / 10^{-3}$ | $0.839 \pm 0.17$ | 0.400 | -2.61 | $1.14 \pm 0.22$ | 0.542 | -2.62 |
| $m_{s} / 10^{-3}$ | $16.62 \pm 0.90$ | 16.53 | -0.09 | $21.58 \pm 1.14$ | 22.57 | 0.86 |
| $m_{b}$ | $0.938 \pm 0.009$ | 0.933 | -0.55 | $0.994 \pm 0.009$ | 0.995 | 0.19 |
| $m_{e} / 10^{-3}$ | $0.3440 \pm 0.0034$ | 0.344 | 0.08 | $0.4707 \pm 0.0047$ | 0.470 | -0.03 |
| $m_{\mu} / 10^{-3}$ | $72.625 \pm 0.726$ | 72.58 | -0.05 | $99.365 \pm 0.993$ | 99.12 | -0.24 |
| $m_{\tau}$ | $1.2403 \pm 0.0124$ | 1.247 | 0.57 | $1.6892 \pm 0.0168$ | 1.688 | -0.05 |
| $\left\|V_{u s}\right\| / 10^{-2}$ | $22.54 \pm 0.07$ | 22.54 | 0.02 | $22.54 \pm 0.06$ | 22.54 | 0.06 |
| $\left\|V_{c b}\right\| / 10^{-2}$ | $3.93 \pm 0.06$ | 3.908 | -0.42 | $4.856 \pm 0.06$ | 4.863 | 0.13 |
| $\left\|V_{u b}\right\| / 10^{-2}$ | $0.341 \pm 0.012$ | 0.341 | 0.003 | $0.420 \pm 0.013$ | 0.421 | 0.10 |
| $\delta_{C K M}^{\circ}$ | $69.21 \pm 3.09$ | 69.32 | 0.03 | $69.15 \pm 3.09$ | 70.24 | 0.35 |
| $\Delta m_{21}^{2} / 10^{-5}\left(e V^{2}\right)$ | $8.982 \pm 0.25$ | 8.972 | -0.04 | $12.65 \pm 0.35$ | 12.65 | -0.01 |
| $\Delta m_{31}^{2} / 10^{-3}\left(\mathrm{eV} \mathrm{V}^{2}\right)$ | $3.05 \pm 0.04$ | 3.056 | 0.02 | $4.307 \pm 0.059$ | 4.307 | 0.006 |
| $\sin ^{2} \theta_{12}$ | $0.318 \pm 0.016$ | 0.314 | -0.19 | $0.318 \pm 0.016$ | 0.316 | -0.07 |
| $\sin ^{2} \theta_{23}$ | $0.563 \pm 0.019$ | 0.563 | 0.031 | $0.563 \pm 0.019$ | 0.563 | 0.01 |
| $\sin ^{2} \theta_{13}$ | $0.0221 \pm 0.0006$ | 0.0221 | -0.003 | $0.0221 \pm 0.0006$ | 0.0220 | -0.16 |
| $\delta_{C P}^{\circ}$ | $224.1 \pm 33.3$ | 240.1 | 0.48 | $224.1 \pm 33.3$ | 225.1 | 0.03 |
| $\chi^{2}$ | - | - | 7.98 | - | - | 7.96 |

Babu, Saad (2021)

## Dirac CP phase

Multiple $\chi^{2}$ minima make $\delta_{C P}$ prediction difficult


Babu, Bajc, Saad (2018)

## Proton decay predictions

$\mathscr{\&}$ Proton decay branching ratios determined by neutrino oscillation fits
\& Mediated by superheavy gauge bosons
$\mathscr{\&}$ Lifetime has large uncertainties, $\tau_{p} \approx\left(10^{32}-10^{36}\right)$ yrs.

## Prediction of branching ratios

$$
\begin{aligned}
\Gamma\left(p \rightarrow \pi^{0} e^{+}\right) & \rightarrow 47 \% \\
\Gamma\left(p \rightarrow \pi^{0} \mu^{+}\right) & \rightarrow 1 \% \\
\Gamma\left(p \rightarrow \eta^{0} e^{+}\right) & \rightarrow 0.20 \% \\
\Gamma\left(p \rightarrow \eta^{0} \mu^{+}\right) & \rightarrow 0.00 \% \\
\Gamma\left(p \rightarrow K^{0} e^{+}\right) & \rightarrow 0.16 \% \\
\Gamma\left(p \rightarrow K^{0} \mu^{+}\right) & \rightarrow 3.62 \% \\
\Gamma\left(p \rightarrow \pi^{+} \bar{\nu}\right) & \rightarrow 48 \% \\
\Gamma\left(p \rightarrow K^{+} \bar{\nu}\right) & \rightarrow 0.22 \%
\end{aligned}
$$

Nemesvek, Bajc, Dorsner (2009)
Babu, Khan (2015)

## Energy-dependent oscillation parameters

$\mathscr{R}^{\circ}$ In presence of light new physics coupled to neutrinos, oscillation angles at production and detection may not coincide
$\mathscr{L}$ Quantum corrections can lead to observable signals in neutrino oscillations. Babu, Brdar, de Gouvea, Machado (2021); (2022)
\& For neutrino produced in pion decay, $Q_{p}^{2}=m_{\pi}^{2}$, but if detected via $\nu+n \rightarrow e^{-}+p, Q_{d}^{2} \approx m_{n} E_{\nu}$. For two flavors,

$$
\begin{gathered}
P_{e \mu}=P_{\mu e}=\sin ^{2}\left(\theta_{p}-\theta_{d}\right)+\sin 2 \theta_{p} \sin 2 \theta_{d} \sin ^{2}\left(\frac{\Delta m^{2} L}{4 E}+\frac{\beta}{2}\right) \\
\theta\left(Q_{p}^{2}\right) \equiv \theta_{p}, \quad \theta\left(Q_{d}^{2}\right) \equiv \theta_{d}, \quad \text { and } \quad \tilde{\beta}\left(Q_{d}^{2}\right)-\tilde{\beta}\left(Q_{p}^{2}\right) \equiv \beta
\end{gathered}
$$

\&: $\theta_{p} \neq \theta_{d}$ if there are light states in the mass range $Q_{p}$ and $Q_{d}$


## Addressing MiniBoone Anomaly

\%์ Active neutrinos assumed to mix with two sterile neutrinos:

$$
\begin{aligned}
& M_{\nu}=\left(\begin{array}{ccccc}
x & x & x & \mu_{e} & 0 \\
x & x & x & \mu_{\mu} & 0 \\
x & x & x & \mu_{\tau} & 0 \\
\mu_{e} & \mu_{\mu} & \mu_{\tau} & 0 & M \\
0 & 0 & 0 & M & 0
\end{array}\right) \\
& \tan \theta_{14} \simeq \frac{\mu_{e}}{M}, \\
& \tan \theta_{24} \simeq \frac{\mu_{\mu}}{M}
\end{aligned}
$$

\&์ If $N_{i}$ couple to a light gauge boson, $M$ will decrease with energy, and thus $\theta_{14}$ and $\theta_{24}$ will decrease

$$
M(\mu)=M\left(\mu_{0}\right)\left(1-\frac{5 g^{\prime}\left(\mu_{0}\right)^{2}}{24 \pi^{2}} \ln \left(\frac{\mu}{\mu_{0}}\right)\right)^{9 / 4}
$$

\% Tension between appearance and disappearance experiments can be relaxed by this running effect

## Addressing MiniBoone Anomaly: Results




## MiniBoone Results



Babu, Brdar, de Gouvea, Machado (2022)

## Radiative neutrino mass generation

$\%$ An alternative to seesaw is radiative neutrino mass generation, where neutrino mass is absent at tree level, but arises via quantum loop corrections
$\mathfrak{2}$ The smallness of neutrino mass is explained by loop and chiral suppressions
$\mathfrak{\&}$ Loop diagrams may arise at 1-loop, 2-loop or 3-loop levels
\% New physics scale typically near TeV and thus accessible to LHC
๕์ Further tests in observable LFV processes and as nonstandard neutrino interaction (NSI) in oscillations

## Effective $\Delta L=2$ Operators

$$
\begin{aligned}
& \mathcal{O}_{1}=L^{i} L^{j} H^{k} H^{\prime} \epsilon_{i k} \epsilon_{j l} \\
& \mathcal{O}_{2}=L^{i} L^{j} L^{k} e^{c} H^{\prime} \epsilon_{i j} \epsilon_{k l} \\
& \mathcal{O}_{3}=\left\{L^{i} L^{j} Q^{k} d^{c} H^{\prime} \epsilon_{i j} \epsilon_{k l}, L^{i} L^{j} Q^{k} d^{c} H^{\prime} \epsilon_{i k} \epsilon_{j j}\right\} \\
& \mathcal{O}_{4}=\left\{L^{i} L^{j} \bar{Q}_{i} \bar{u}^{c} H^{k} \epsilon_{j j}, \quad \quad^{i} L^{j} \bar{Q}_{k} \bar{u}^{c} H^{k} \epsilon_{i j}\right\} \\
& \mathcal{O}_{5}=L^{i} L^{j} Q^{k} d^{c} H^{\prime} H^{m} \bar{H}_{i} \epsilon_{j j} \epsilon_{k m} \\
& \mathcal{O}_{6}=L^{i} L^{j} \bar{Q}_{k} \bar{u}^{c} H^{\prime} H^{k} \bar{H}_{i j l} \epsilon_{j l} \\
& \mathcal{O}_{7}=L^{i} Q^{j} \overline{j^{c}} \bar{Q}_{k} H^{k} H^{\prime} H^{m} \epsilon_{i l} \epsilon_{j m} \\
& \mathcal{O}_{8}=L^{i} \bar{e}^{c} \bar{u}^{c} d^{c} H^{j} \epsilon_{i j} \\
& \mathcal{O}_{9}=L^{i} L^{j} L^{k} e^{c} L^{\prime}{ }^{c} \epsilon_{i j} \epsilon_{k l} \\
& \mathcal{O}_{1}^{\prime}=L^{i} L^{j} H^{k} H^{\prime} \epsilon_{i k} \epsilon_{j l} H^{* m} H_{m}
\end{aligned}
$$

Babu \& Leung (2001)
de Gouvea \& Jenkins (2008)
Angel \& Volkas (2012)
Cai, Herrero-Garcia, Schmidt, Vicente, Volkas (2017)
Lehman (2014) - all d=7 operators
Li, Ren, Xiao, Yu, Zheng (2020); Liao, Ma (2020) - all d=9 operators

## Operator $\mathcal{O}_{2}$ and the Zee model

\% Introduce a singly charged scalar and a second Higgs doublet to standard model:

$$
\begin{gathered}
\mathcal{L}=f_{i j} L_{i}^{a} L_{j}^{b} h^{+} \epsilon_{a b}+\mu H^{a} \Phi^{b} h^{-} \epsilon_{a b}+\text { h.c. } \\
\Downarrow \\
\mathcal{O}_{2}=L^{i} L^{j} L^{k} e^{c} H^{\prime} \epsilon_{i j} \epsilon_{k l}
\end{gathered}
$$

Zee (1980)
\% Neutrino mass arises at one-loop.

\% A minimal version of this model in which only one Higgs doublet couples to a given fermion sector with a $Z_{2}$ symmetry yields:

Wolfenstein (1980)

$$
m_{\nu}=\left(\begin{array}{ccc}
0 & m_{e \mu} & m_{e \tau} \\
m_{e \mu} & 0 & m_{\mu \tau} \\
m_{e \tau} & m_{\mu \tau} & 0
\end{array}\right), \quad m_{i j} \simeq \frac{f_{i j}}{16 \pi^{2}} \frac{\left(m_{i}^{2}-m_{j}^{2}\right)}{\Lambda}
$$

It requires $\theta_{12} \simeq \pi / 4 \rightarrow$ ruled out by solar + KamLAND data.
Koide (2001); Frampton et al. (2002); He (2004)

## Neutrino oscillations in the Zee model

\&ะ Neutrino oscillation data can be fit to the Zee model consistently without the $Z_{2}$ symmetry
\% Some benchmark points for Yukawa couplings of second doublet:

$$
\begin{aligned}
\text { BP I }: Y & =\left(\begin{array}{ccc}
Y_{e e} & 0 & Y_{e \tau} \\
0 & Y_{\mu \mu} & Y_{\mu \tau} \\
0 & Y_{\tau \mu} & Y_{\tau \tau}
\end{array}\right) \\
\text { BP II : } Y & =\left(\begin{array}{ccc}
0 & Y_{e \mu} & Y_{e \tau} \\
Y_{\mu e} & 0 & Y_{\mu \tau} \\
0 & Y_{\tau \mu} & Y_{\tau \tau}
\end{array}\right) \\
\text { BP III : } Y & =\left(\begin{array}{ccc}
Y_{e e} & 0 & Y_{e \tau} \\
0 & Y_{\mu \mu} & Y_{\mu \tau} \\
Y_{\tau e} & 0 & Y_{\tau \tau}
\end{array}\right)
\end{aligned}
$$

Babu, Dev, Jana, Thapa (2019)

## Neutrino fit in the Zee model






Babu, Dev, Jana, Thapa (2019)

## Measuring L-violation in the Zee Model at LHC

\% $L$ violation in Zee model occurs via mixing of two charged scalars which carry different lepton number
\&ะ $p p \rightarrow e^{+} \mu^{+}+$jets occurs:
Babu. Barman. Goncalves, Ismail, 2022


## Neutrino Non-Standard Interactions (NSI)

$\mathfrak{\&}$ Neutrino oscillation picture would change if there are non-standard interactions
\& M Modification of matter effects most important
\& EFT for neutrino NSI:

$$
\begin{aligned}
& \mathcal{L}_{\mathrm{NSI}}^{\mathrm{NC}}=-2 \sqrt{2} G_{F} \sum_{f, x, \alpha, \beta} \varepsilon_{\alpha \beta}^{f x}\left(\bar{\nu}_{\alpha} \gamma^{\mu} P_{L} \nu_{\beta}\right)\left(\bar{f} \gamma_{\mu} P_{X} f\right), \\
& \mathcal{L}_{\mathrm{NSI}}^{\mathrm{CC}}=-2 \sqrt{2} G_{F} \sum_{f, f^{\prime}, x, \alpha, \beta} \varepsilon_{\alpha \beta}^{f f^{\prime} x}\left(\bar{\nu}_{\alpha} \gamma^{\mu} P_{L} \ell_{\beta}\right)\left(\bar{f}^{\prime} \gamma_{\mu} P_{X} f\right)
\end{aligned}
$$

Wolfenstein (1978)
Effective Hamiltonian for neutrino propagation in matter is now:

$$
H=\frac{1}{2 E} U\left(\begin{array}{ccc}
0 & 0 & 0 \\
0 & \Delta m_{21}^{2} & 0 \\
0 & 0 & \Delta m_{31}^{2}
\end{array}\right) U^{\dagger}+\sqrt{2} G_{F} N_{e}(x)\left(\begin{array}{ccc}
1+\varepsilon_{e e} & \varepsilon_{e \mu} & \varepsilon_{e \tau} \\
\varepsilon_{e \mu}^{\star} & \varepsilon_{\mu \mu} & \varepsilon_{\mu \tau} \\
\varepsilon_{e \tau}^{\star} & \varepsilon_{\mu \tau}^{\star} & \varepsilon_{\tau \tau}
\end{array}\right)
$$

\&่ย $\epsilon_{\alpha \beta}$ measure of NSI normalized to weak interaction strength

## Neutrino NSI in the Zee model

\&์ The two charged scalars of the Zee model mediate NSI

(a)

(b)

(c)

(d)
\% The NSI parameters are given by:

$$
\varepsilon_{\alpha \beta}=\frac{1}{4 \sqrt{2} G_{F}} Y_{\alpha e} Y_{\beta e}^{*}\left(\frac{\sin ^{2} \varphi}{m_{h^{+}}^{2}}+\frac{\cos ^{2} \varphi}{m_{H^{+}}^{2}}\right)
$$

\% Constrained by LHC and LEP direct limits; cLFV; precision electroweak tests; neutrino oscillation data; and theory

Babu, Dev, Jana, Thapa (2019)

## LEP and LHC constraints on Charged Scalar




## Diagonal NSI in Zee model



## Summary of NSI in radiative models



Babu, Dev, Jana, Thapa (2019)

## NSI from Light Mediators

\%ٌ With light mediators NSI induced may be more compatible with charged lepton flavor violation
\% Several models have been proposed Gavela, Hernandez, Ota, Winter (2009); Farzan, Heeck (2016); Babu, Friedland, Machado, Mocioiu (2017); Denton, Farzan, Shoemaker (2018)

$(B-L)_{3}$ Model: Babu, Friedland, Machado, Mocioiu (2017)

## Conclusions

$\mathscr{R}$ : EFT description alone in neutrino sector is inadequate; we may miss important phenomena such as leptogenesis
\& Grand Unification provides powerful tools to interconnect neutrino sector with quark sector
$\mathscr{\&}$ Neutrino may very well be Dirac particles; interesting models of Dirac neutrino exist
\& Lepton number violation by two units is accessible at the LHC, which would imply Majorana nature of neutrino
\& Energy-dependent oscillation angles can arise in presence of light fields coupled to neutrinos
$\mathscr{\&}$ Various $d=7$ and $d=9$ lepton number violating EFT operators can lead to interesting neutrino mass models

