

Antisymmetric galaxy cross-correlations

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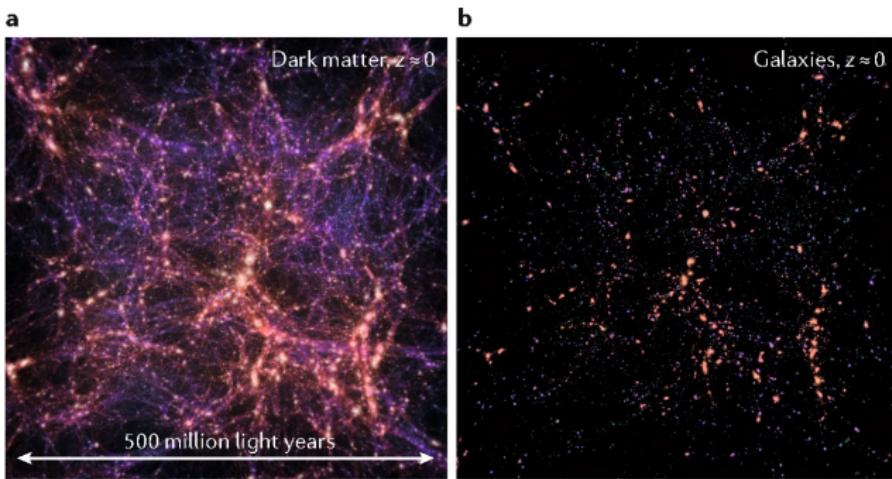
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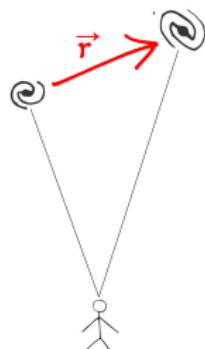
Idea



Robertson *et al.*, Galaxy formation and evolution science in the era of the Large Synoptic Survey Telescope

cross-correlation between **different** tracers
may not be symmetric under exchange

$$r \mapsto -r$$



Antisymmetric galaxy cross-correlations as a cosmological probe

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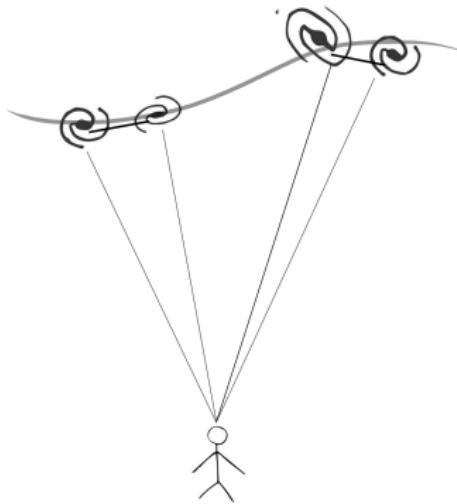
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The auto-correlation between two members of a galaxy population is symmetric under the interchange of the two galaxies being correlated. The cross-correlation between two different types of galaxies, separated by a vector \mathbf{r} , is not necessarily the same as that for a pair separated by $-\mathbf{r}$. Local anisotropies in the two-point cross-correlation function may thus indicate a specific direction which when mapped as a function of position trace out a vector field. This vector field can then be decomposed into longitudinal and transverse components, and those transverse components written as positive- and negative-helicity components. A locally asymmetric cross-correlation of the longitudinal type arises naturally in halo clustering, even with Gaussian initial conditions, and could be enhanced with local-type non-Gaussianity. Early-Universe scenarios that introduce a vector field may also give rise to such effects. These antisymmetric cross-correlations also provide a new possibility to seek a preferred cosmic direction correlated with the hemispherical power asymmetry in the cosmic microwave background and to seek a preferred location associated with the CMB cold spot. New ways to seek cosmic parity breaking are also possible.

Case of biased halo clustering



$$\delta_i = b_i \delta + c_i \delta^2 + \dots$$

$$P^A = (b_2 c_1 - b_1 c_2) \frac{\partial P(k_1)}{\partial k_1} P(k_3) \frac{k_1 \cdot k_3}{k_1} \quad k_3 \ll k_1$$

k_3 long-wavelength mode, k_1 short-wavelength mode

Adding RSD and f_{NL}

- bias at second order:

$$\delta_g = b_1 \delta + \frac{b_2}{2} \delta^2 + b_{K^2} K^2 \quad \text{with } K_{ij}(k) = \frac{2}{3\Omega_m \mathcal{H}^2} \partial_i \partial_j \Phi - \frac{1}{3} \delta_{ij} \delta = \left[\frac{k_i k_j}{k^2} - \frac{1}{3} \delta_{ij} \right] \delta(k)$$

- RSD at second order:

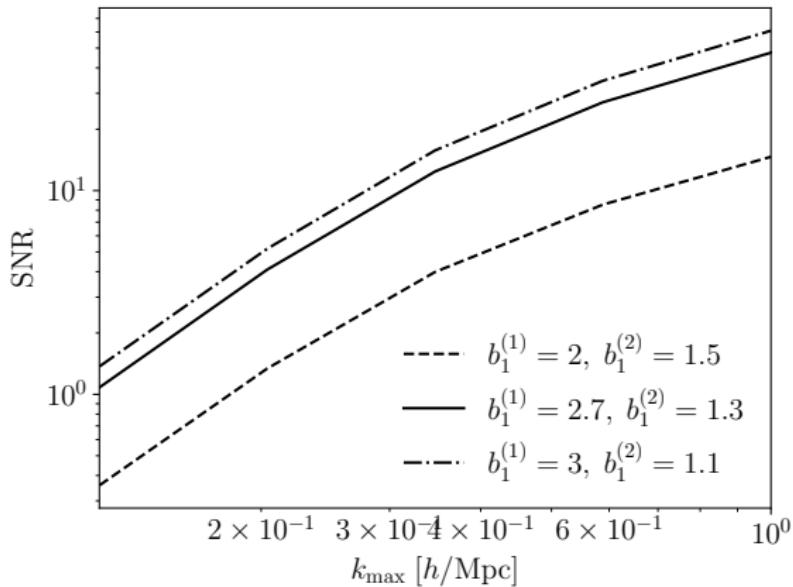
$$\begin{aligned} \delta_s &\simeq \delta - \frac{1}{\mathcal{H}} \partial_r v - \frac{1}{\mathcal{H}r} \left(\frac{r}{\bar{n}} \frac{\partial \bar{n}}{\partial r} + 2 \right) v \\ &+ \frac{1}{2\mathcal{H}^2} \partial_r^2 v^2 - \frac{1}{\mathcal{H}} \partial_r (\delta v) \\ &- \frac{1}{\mathcal{H}r} \left(\frac{r}{\bar{n}} \frac{\partial \bar{n}}{\partial r} + 2 \right) \delta v + \frac{1}{\mathcal{H}^2 r^2} \left(\left(\frac{r}{\bar{n}} \frac{\partial \bar{n}}{\partial r} \right)^2 - \frac{r^2}{2\bar{n}} \nabla^2 \bar{n} + 2 \frac{r}{\bar{n}} \frac{\partial \bar{n}}{\partial r} + 3 \right) v^2 \\ &+ \frac{1}{\mathcal{H}^2} \partial_r \left[\left(\frac{r}{\bar{n}} \frac{\partial \bar{n}}{\partial r} + 2 \right) \frac{v^2}{r} \right] \end{aligned}$$

neglecting Doppler term and selection effects:

$$\begin{aligned} Z_1(\mathbf{k}) &= \mathbf{b}_1 + f \mu^2 \\ Z_2(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) &= \frac{\mathbf{b}_2}{2} + b_1 F_2(\mathbf{k}_1, \mathbf{k}_2) + b_{K^2} \left(\mu_{12}^2 - \frac{1}{3} \right) + f \mu^2 G_2(\mathbf{k}_1, \mathbf{k}_2) \\ &+ \frac{k \mu f}{2} \left(\frac{\mu_1}{k_1} \left(b_1 + f \mu_2^2 \right) + \frac{\mu_2}{k_2} \left(b_1 + f \mu_1^2 \right) \right) \end{aligned}$$

Signal-to-noise?

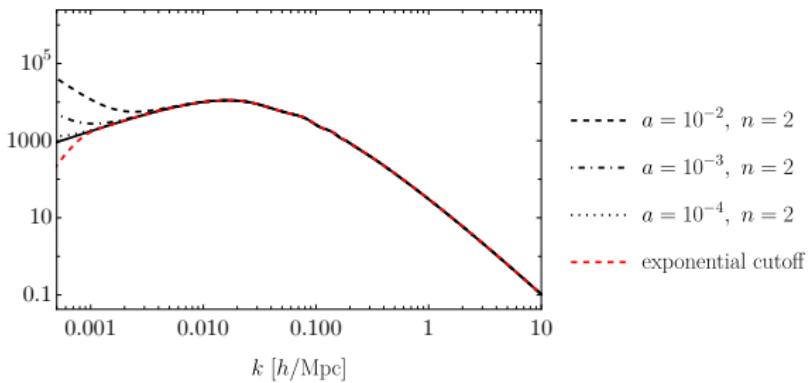
$$\left(\frac{S}{N}\right)^2 = \frac{V_s}{(2\pi)^3} \int_{k_{\min}}^{k_{\max}} k^2 dk \int_0^{2\pi} d\phi \int_0^1 d\mu \left[\int_{k_{\text{horizon}}}^{k/10} K^2 dK 2\pi \int_{-1}^{+1} d\mu_{13} \left(\frac{S}{N}\right)_{k,K}^2 \right]$$



What can use this for?

- **exotic models**, e.g., a two-component dark matter model with a relative preferred direction
- **features** of the power-spectrum at large scales, e.g.,
 $P(k) + a/k^n$
 $P(k) (1 - \exp(-(k/k_c)^\alpha))$

...



Thank you for your attention.

Literature

- clustering fossils: Jeong & Kamionkowski 2012 [1203.0302](#)
- antisymmetric: Dai *et al.* 2015 [1507.05618](#)

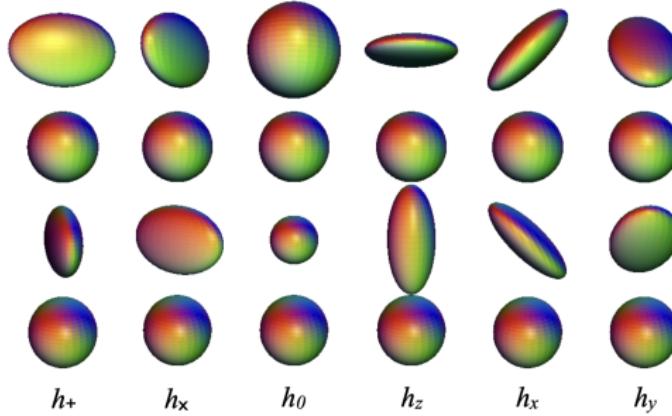
Symmetric case

Assume we have a new field with Fourier modes $h_p(k_3)$ and polarization p . Global statistical isotropy requires that the new field induces a correlation:

$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2) \rangle|_{h_p(k_3)} = f_p(\mathbf{k}_1, \mathbf{k}_2) h_p^*(\mathbf{k}_3) \epsilon_{ij}^p k_1^i k_2^j \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$

$\epsilon_{ij}^p(k)$? Most general 3×3 symmetric tensor can be decomposed into 6 orthogonal polarization states:

$$p = \{+, \times, 0, z, x, y\} \quad \epsilon_{ij}^p \epsilon^{p',ij} = 2\delta_{pp'}$$



Jeong et al., 1203.0302

Extension to antisymmetric case

More general case: $\epsilon_{ij}^p(k_3)$ may be antisymmetric \Rightarrow nine degrees of freedom! Three new polarizations $p = \{L, x, y\}$.

$$\begin{aligned}\langle \delta_1(\mathbf{k}_1) \delta_2(\mathbf{k}_2) \rangle &= P(k_1) \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2) \\ &+ \sum_{\mathbf{k}_3} \sum_p f_p(k_1, k_2, \mu) h_p^*(\mathbf{k}_3) \epsilon_{ij}^p(\mathbf{k}_3) k_1^i k_2^j \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \\ &+ \sum_{\mathbf{k}_3, p} f_p(k_1, k_2, \mu) h_p^*(\mathbf{k}_3) \hat{\epsilon}_p \cdot (\mathbf{k}_1 - \mathbf{k}_2) \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\end{aligned}$$

Parametrization allows for a global preferred direction (exotic new physics).
In our case: in any small volume, the cross-correlation could “point” in some given direction and this direction could be spatially dependent, in such a way that global statistical isotropy is still preserved on sufficiently large scales.

Adding RSD and f_{NL} (I)

- Bias at second order:

$$\delta_g = b_1 \delta + \frac{b_2}{2} \delta^2 + b_{K^2} K^2 \quad \text{with } K_{ij}(k) = \frac{2}{3\Omega_m \mathcal{H}^2} \partial_i \partial_j \Phi - \frac{1}{3} \delta_{ij} \delta = \left[\frac{k_i k_j}{k^2} - \frac{1}{3} \delta_{ij} \right] \delta(k)$$

- RSD at second order:

$$\begin{aligned} \delta_s \simeq & \delta - \frac{1}{\mathcal{H}} \partial_r v - \frac{1}{\mathcal{H}r} \left(\frac{r}{\bar{n}} \frac{\partial \bar{n}}{\partial r} + 2 \right) v \\ & + \frac{1}{2\mathcal{H}^2} \partial_r^2 v^2 - \frac{1}{\mathcal{H}} \partial_r (\delta v) \\ & - \frac{1}{\mathcal{H}r} \left(\frac{r}{\bar{n}} \frac{\partial \bar{n}}{\partial r} + 2 \right) \delta v + \frac{1}{\mathcal{H}^2 r^2} \left(\left(\frac{r}{\bar{n}} \frac{\partial \bar{n}}{\partial r} \right)^2 - \frac{r^2}{2\bar{n}} \nabla^2 \bar{n} + 2 \frac{r}{\bar{n}} \frac{\partial \bar{n}}{\partial r} + 3 \right) v^2 \\ & + \frac{1}{\mathcal{H}^2} \partial_r \left[\left(\frac{r}{\bar{n}} \frac{\partial \bar{n}}{\partial r} + 2 \right) \frac{v^2}{r} \right] \end{aligned}$$

Neglecting Doppler term and selection effects, the new kernels are:

$$\begin{aligned} Z_1(\mathbf{k}) = & \mathbf{b}_1 + f \mu^2 \\ Z_2(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) = & \frac{\mathbf{b}_2}{2} + b_1 F_2(\mathbf{k}_1, \mathbf{k}_2) + b_{K^2} \left(\mu_{12}^2 - \frac{1}{3} \right) + f \mu^2 G_2(\mathbf{k}_1, \mathbf{k}_2) \\ & + \frac{k \mu f}{2} \left(\frac{\mu_1}{k_1} \left(b_1 + f \mu_2^2 \right) + \frac{\mu_2}{k_2} \left(b_1 + f \mu_1^2 \right) \right) \end{aligned}$$

Adding RSD and f_{NL} (II)

$$\begin{aligned}
 P^A = & P(k_1)P(k_3) (1 + f\mu_3^2) \left\{ \left[2 \left(b_1^{(2)} b_{K^2}^{(1)} - b_1^{(1)} b_{K^2}^{(2)} \right) \mu_{13} (-\mu_{13}^2 + 1) + \right. \right. \\
 & + \left(b_2^{(1)} - b_2^{(2)} \right) f\mu_1 \frac{k_3}{k_1} (-\mu_{13}\mu_1 + \mu_3) - \left(b_{K^2}^{(1)} - b_{K^2}^{(2)} \right) \frac{2f\mu_1}{3} \frac{k_3}{k_1} (6\mu_{13}^3\mu_1 - 4\mu_{13}\mu_1 - 3\mu_{13}^2\mu_3 + \mu_3) \\
 & - \left(b_1^{(1)} - b_1^{(2)} \right) f\mu_1 \frac{k_3}{k_1} [f\mu_1\mu_{13} (-\mu_1^2 + \mu_3^2) + (\mu_{13}\mu_1 - \mu_3)(-f\mu_1^2 + 2F_2(-\mathbf{k}_2, -\mathbf{k}_3) + 2G_2(-\mathbf{k}_1, -\mathbf{k}_3))] \Big] \\
 & - \left(b_1^{(1)} - b_1^{(2)} \right) f\mu_1^2 (F_2(-\mathbf{k}_1, -\mathbf{k}_3) - F_2(-\mathbf{k}_2, -\mathbf{k}_3) - G_2(-\mathbf{k}_1, -\mathbf{k}_3) + G_2(-\mathbf{k}_2, -\mathbf{k}_3)) \Big] \\
 & + \frac{\partial P(k_1)}{\partial k_1} P(k_3)\mu_{13}k_3 (1 + f\mu_3^2) \left\{ \frac{1}{2} \left(b_1^{(2)} b_2^{(1)} - b_1^{(1)} b_2^{(2)} \right) + \left(b_1^{(1)} b_{K^2}^{(2)} - b_1^{(2)} b_{K^2}^{(1)} \right) \left(\frac{1}{3} - \mu_{13}^2 \right) + \frac{1}{2} \left(b_2^{(1)} - b_2^{(2)} \right) f\mu_1^2 \right. \\
 & + \left(b_{K^2}^{(1)} - b_{K^2}^{(2)} \right) f\mu_1^2 \left(\mu_{13}^2 - \frac{1}{3} \right) \\
 & \left. - \frac{1}{2} \left(b_1^{(1)} - b_1^{(2)} \right) f\mu_1^2 (f\mu_1^2 - f\mu_3^2 - 2F_2(-\mathbf{k}_2, -\mathbf{k}_3) + 2G_2(-\mathbf{k}_2, -\mathbf{k}_3)) \right\}
 \end{aligned}$$

...and main contribution from f_{NL} :

$$\begin{aligned}
 & - \frac{1}{k_1 k_3} \frac{3}{4} \frac{f_{\text{NL}} H_0^2 \Omega_{m,0}}{D_{\text{md}}(\tau) (T(k_1))^2} P(k_3) (1 + f\mu_3^2) \left\{ \left[\frac{\partial P(k_1)}{\partial k_1} k_1 \mu_{13}^2 \left(\left(b_\phi^{(1)} b_1^{(2)} - b_\phi^{(2)} b_1^{(1)} \right) + 2 \left(b_\phi^{(1)} b_{K^2}^{(2)} - b_\phi^{(2)} b_{K^2}^{(1)} \right) + \left(b_\phi^{(1)} - b_\phi^{(2)} \right) f\mu_1^2 \right) \right. \right. \\
 & + P(k_1) \left(2 \left(b_1^{(1)} b_\phi^{(2)} - b_1^{(2)} b_\phi^{(1)} \right) \mu_{13} - 8 \left(b_\phi^{(1)} b_{K^2}^{(2)} - b_\phi^{(2)} b_{K^2}^{(1)} \right) \mu_{13} - 2 \left(b_\phi^{(1)} - b_\phi^{(2)} \right) f\mu_1 (2\mu_1\mu_{13} - \mu_3) \right] T(k_1) \\
 & \left. \left. + P(k_1) k_1 \mu_{13} \left(\left(b_\phi^{(1)} b_1^{(2)} - b_\phi^{(2)} b_1^{(1)} \right) - 2 \left(b_\phi^{(1)} b_{K^2}^{(2)} - b_\phi^{(2)} b_{K^2}^{(1)} \right) + \left(b_\phi^{(1)} - b_\phi^{(2)} \right) f\mu_1^2 \right) \frac{\partial T(k_1)}{\partial k_1} \right\}
 \end{aligned}$$

Primordial non-Gaussianity f_{NL}

In the presence of **local-type primordial non-Gaussianity**, the Eulerian basis of operators in the bias expansion must be augmented by additional terms: $f_{\text{NL}}\phi(q)$ at first order and $f_{\text{NL}}\delta(x)\phi(q)$ at second order, with ϕ the Bardeen potential.

The redshift space kernels in Fourier space become:

$$Z_{1,f_{\text{NL}}}^{\text{tr}}(k) = f_{\text{NL}} b_\phi \mathcal{M}^{-1}(k)$$

$$\begin{aligned} Z_{2,f_{\text{NL}}}^{\text{tr}}(k, k_1, k_2) &= f_{\text{NL}} b_\phi \frac{k_1 \cdot k_2}{2} \left(\frac{1}{k_1^2} \mathcal{M}^{-1}(k_2) + \frac{1}{k_2^2} \mathcal{M}^{-1}(k_1) \right) \\ &\quad + f_{\text{NL}} b_\phi \delta \frac{1}{2} \left(\mathcal{M}^{-1}(k_1) + \mathcal{M}^{-1}(k_2) \right) \\ &\quad + f_{\text{NL}} b_\phi \frac{k \mu f}{2} \left(\frac{\mu_2}{k_2} \mathcal{M}^{-1}(k_1) + \frac{\mu_1}{k_1} \mathcal{M}^{-1}(k_2) \right) \end{aligned}$$

$$\delta^{(1)}(k, \tau) = \mathcal{M}(k, \tau) \phi(k) \qquad \mathcal{M}(k, \tau) = \frac{2}{3} \frac{k^2 T(k) D_{\text{md}}(\tau)}{\Omega_{\text{m},0} H_0^2}$$

Estimator for the Fourier amplitude $\delta(K)$

Jeong *et al.* 1203.0302 & Dai *et al.* 1507.05618

$$\frac{1}{2} [\delta_1(\mathbf{k}_1)\delta_2(\mathbf{k}_2) - \delta_1(\mathbf{k}_2)\delta_2(\mathbf{k}_1)] = V_s \delta_{\mathbf{k}_1, \mathbf{k}_2, K} \delta^*(K) f_L^A(\mathbf{k}_1, \mathbf{k}_2) \hat{\mathbf{K}} \cdot (\mathbf{k}_1 - \mathbf{k}_2)$$

Each pair $\mathbf{k}_1, \mathbf{k}_2$ provides an estimator:

$$\widehat{\delta(K)} = \frac{1}{2} [\delta_1(\mathbf{k}_1)\delta_2(\mathbf{k}_2) - \delta_1(\mathbf{k}_2)\delta_2(\mathbf{k}_1)] \left[f_L^A(\mathbf{k}_1, \mathbf{k}_2) \hat{\mathbf{K}} \cdot (\mathbf{k}_1 - \mathbf{k}_2) \right]^{-1}$$

with variance

$$\frac{V_s}{2} \left[f_L^A(\mathbf{k}_1, \mathbf{k}_2) \hat{\mathbf{K}} \cdot (\mathbf{k}_1 - \mathbf{k}_2) \right]^{-2} (P_1(k_1)P_2(k_2) + P_1(k_2)P_2(k_1) - 2P_{12}(k_1)P_{12}(k_2))$$

Minimum variance estimator obtained by summing over all $(\mathbf{k}_1, \mathbf{k}_2)$ with inverse-variance weighting:

$$\widehat{\delta(K)} = P_n(K) \sum_k \frac{\left[f_L^A(\mathbf{k}_1, \mathbf{k}_2) \hat{\mathbf{K}} \cdot (\mathbf{k}_1 - \mathbf{k}_2) \right]}{\frac{V_s}{2} (P_1(k_1)P_2(k_2) + P_1(k_2)P_2(k_1) - 2P_{12}(k_1)P_{12}(k_2))} \frac{1}{2} [\delta_1(\mathbf{k}_1)\delta_2(\mathbf{k}_2) - \delta_1(\mathbf{k}_2)\delta_2(\mathbf{k}_1)]$$
$$P_n(K) = \left[\sum_k \frac{\left[f_L^A(\mathbf{k}_1, \mathbf{k}_2) \hat{\mathbf{K}} \cdot (\mathbf{k}_1 - \mathbf{k}_2) \right]^2}{\frac{V_s}{2} (P_1(k_1)P_2(k_2) + P_1(k_2)P_2(k_1) - 2P_{12}(k_1)P_{12}(k_2))} \right]^{-1}$$

Estimator for the amplitude A

Since $\langle \widehat{|\delta(K)|^2} \rangle = V_s(P(K) + P_n(K))$, if one parametrizes $P(K) = AP_f(K)$, each K provides an estimator for the amplitude:

$$\hat{A}_K = P_f(K)^{-1} \left(V_s^{-1} \widehat{|\delta(K)|^2} - P_n(K) \right)$$

$$\hat{A} = \sigma^2 \sum_K \frac{P_f(K)}{2(P_n(K))^2} \left(V_s^{-1} \widehat{|\delta(K)|^2} - P_n(K) \right)$$

$$\sigma^{-2} = \sum_K \frac{(P_f(K))^2}{2(P_n(K))^2}$$

SNR

$$\widehat{P^A}(k, K) \equiv \frac{1}{2} (\delta_1(k)\delta_2(K-k) - \delta_1(K-k)\delta_2(k))$$

under the null hypothesis:

$$\text{Cov}(k, k')_K = \frac{1}{2} [P_{11}(k)P_{22}(k) - P_{12}(k)P_{12}(k)] [\delta_{k+k'}^D - \delta_{k-k'}^D]$$

but since $P^A(-k) = -P^A(k)$, one can consider only one emisphere in k space and then combine the contribution from both k and $-k$ mode:

$$[\langle P^A(k)^2 \rangle - \langle P^A(k) \rangle^2] - [\langle P^A(k)P^A(-k) \rangle - \langle P^A(k) \rangle \langle P^A(-k) \rangle]$$

$$\boxed{\text{Cov}^{\text{emi}}(k)_K = \frac{1}{2} \left(P_{11}(k)P_{22}(|K-k|) + P_{11}(|K-k|)P_{22}(k) \right) - P_{12}(k)P_{12}(|K-k|)}$$

$$\left(\frac{S}{N}\right)^2 = \sum_{(k_1, k_2, k_3)} \int_{-1}^{+1} d\mu \int_0^{2\pi} d\phi \frac{1}{s_B V_s} \frac{\left(P^A(k_1, k_2, k_3)\right)^2}{\text{Var}(P^A(k_1, k_2, k_3))} \prod_{i=1}^3 \left(\frac{dk_i \Delta k_i}{k_f^2} \right) \times \begin{cases} \frac{\pi}{2\pi} & k_i = k_j + k_k \\ 2\pi & \text{otherwise} \end{cases}$$

