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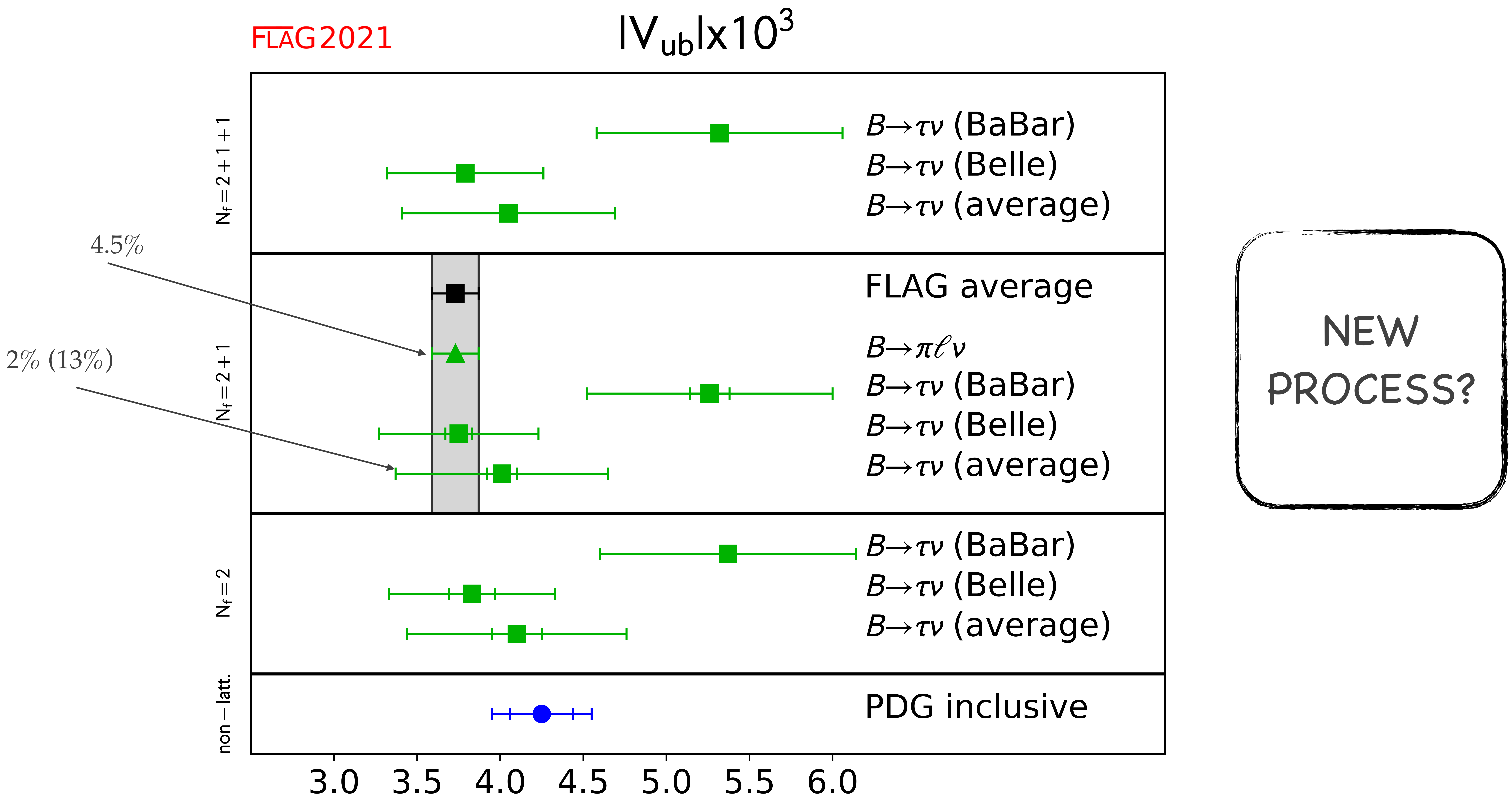
A lattice QCD study of $B \to \pi\pi\ell\bar{\nu}$

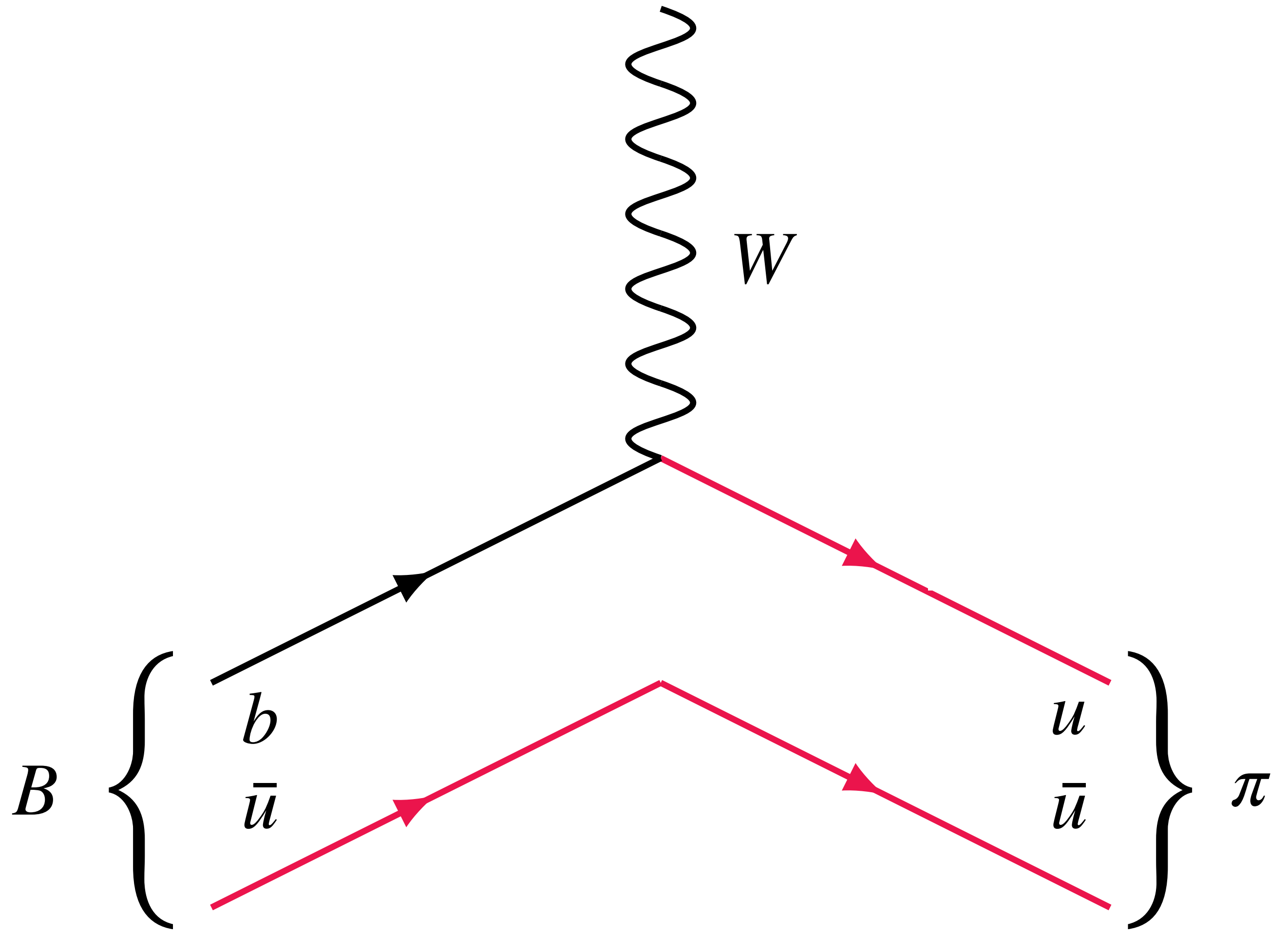
virtually at CERN
14 February, 2023

in collaboration with:
Stefan Meinel, Marcus Petschlies,
Srijit Paul, Gumaro Rendon, John W. Negele,
Andrew Pochinsky

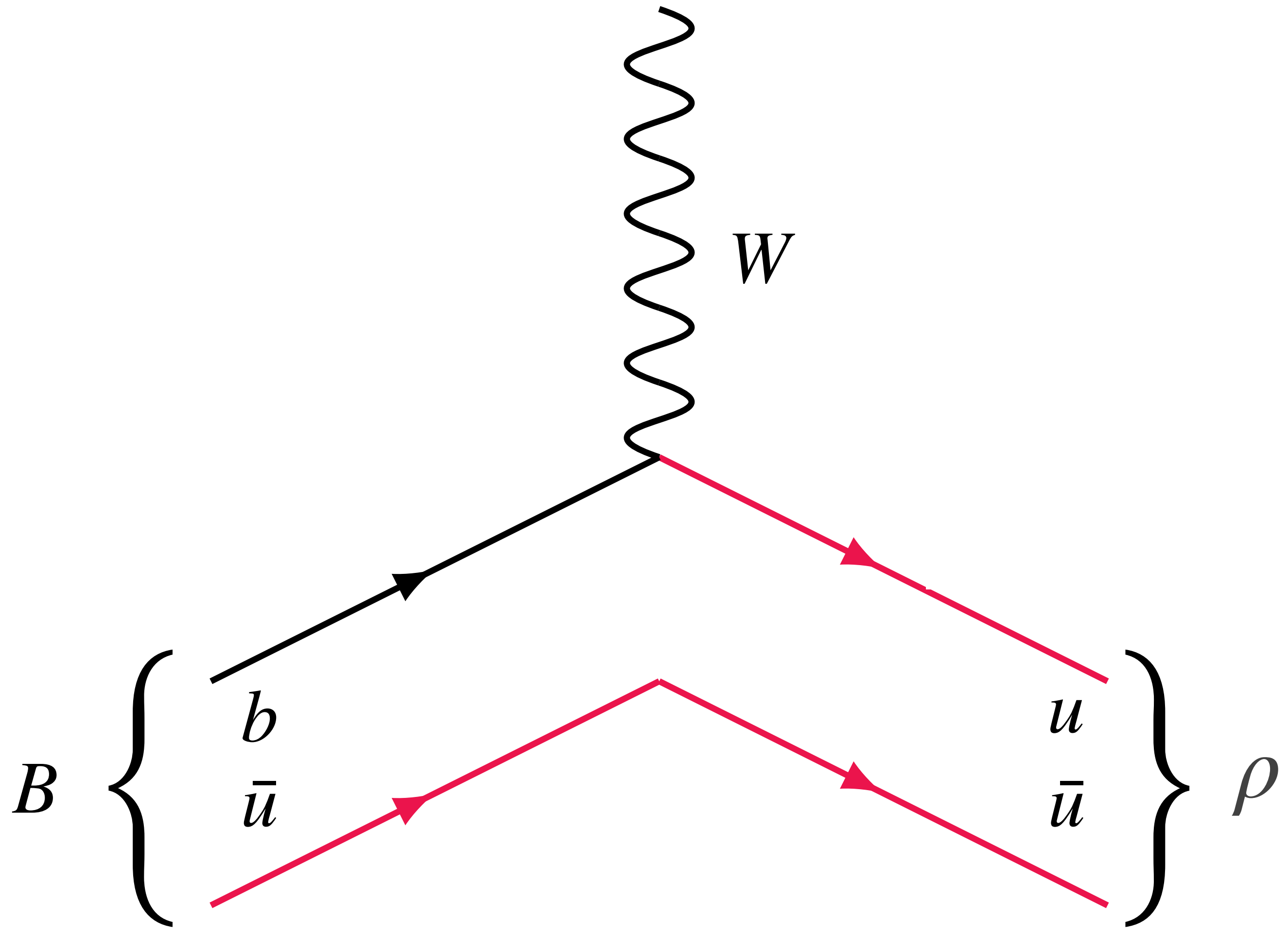


FLAG 2021
|Vub|x10^3
Nf=2+1+1
B→τν (BaBar)
B→τν (Belle)
B→τν (average)
4.5%
2% (13%)
Nf=2+1
FLAG average
B→πℓν
B→τν (BaBar)
B→τν (Belle)
B→τν (average)
Nf=2
B→τν (BaBar)
B→τν (Belle)
B→τν (average)
non−latt.
PDG inclusive
3.0
3.5
4.0
4.5
5.0
5.5
6.0
NEW
PROCESS?

why $B \to \rho \ell \bar{\nu}$?



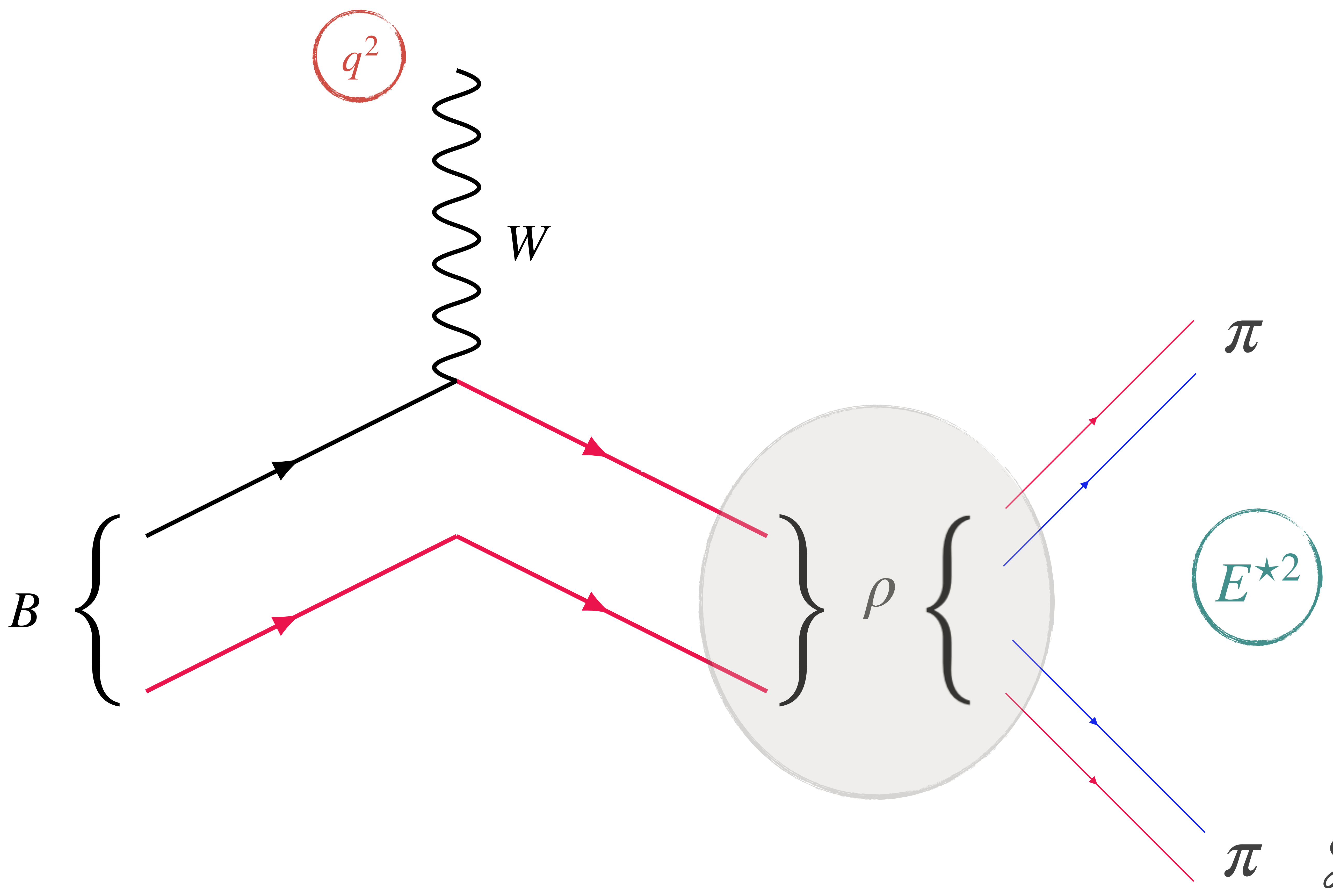
$J^P = 0^-$

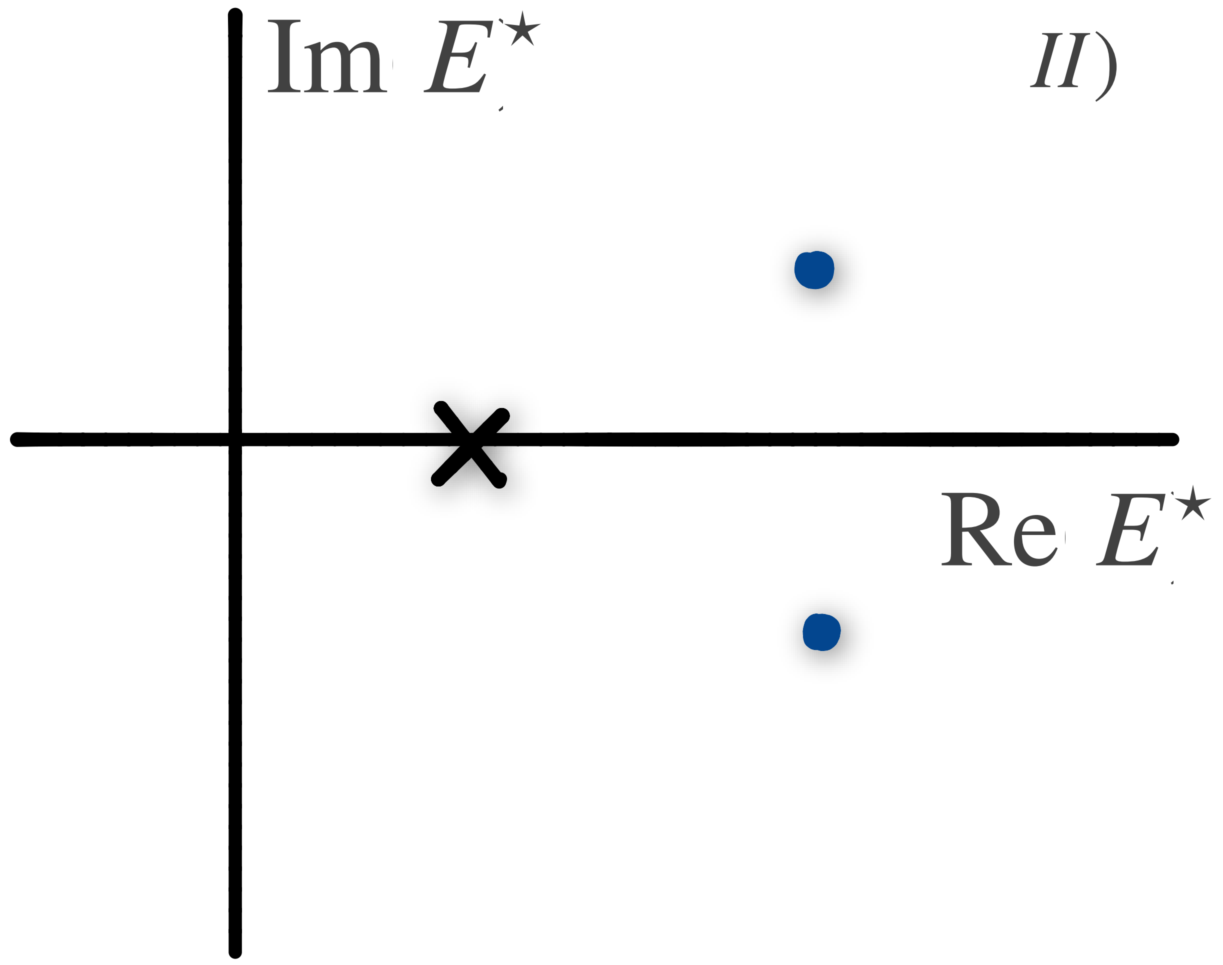


$J^P = 1^-$

- $B \to \pi \ell \nu$
  - f_+, f_0
  - established
- $B \to \rho \ell \nu$
  - V, A_0, A_1, A_2
  - new

about $B \to \rho\ell\nu$





poles

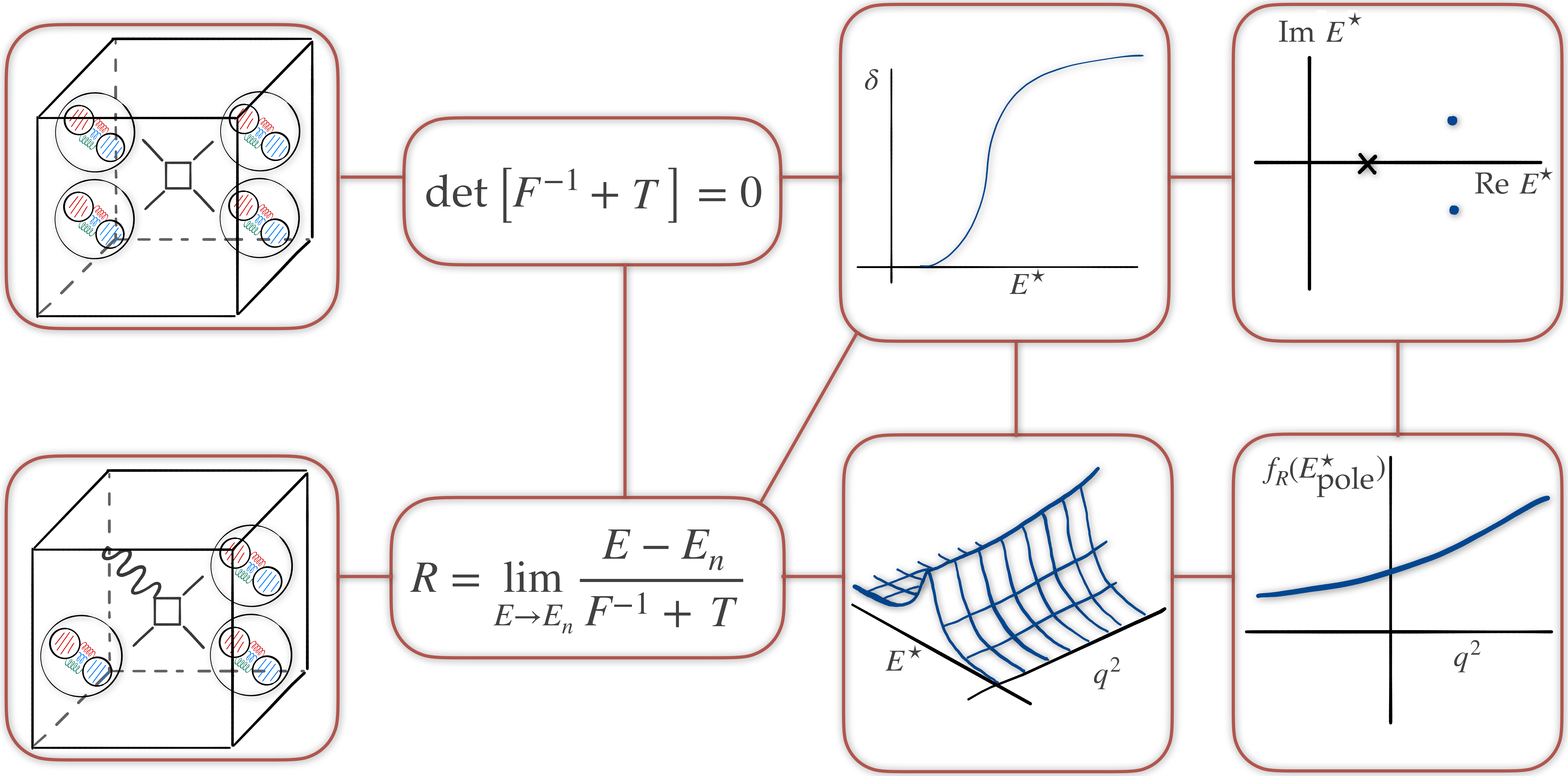
smooth

poles

$$\mathscr{H}^{\mu}_{1,m_\ell}(q^2, E^{\star 2}) = \mathscr{A}^{\mu}_{1,m_\ell}(q^2, E^{\star 2})\frac{T(E^{\star 2})}{k}$$

"form factor"

$B \to \rho \ell \nu$ on the lattice

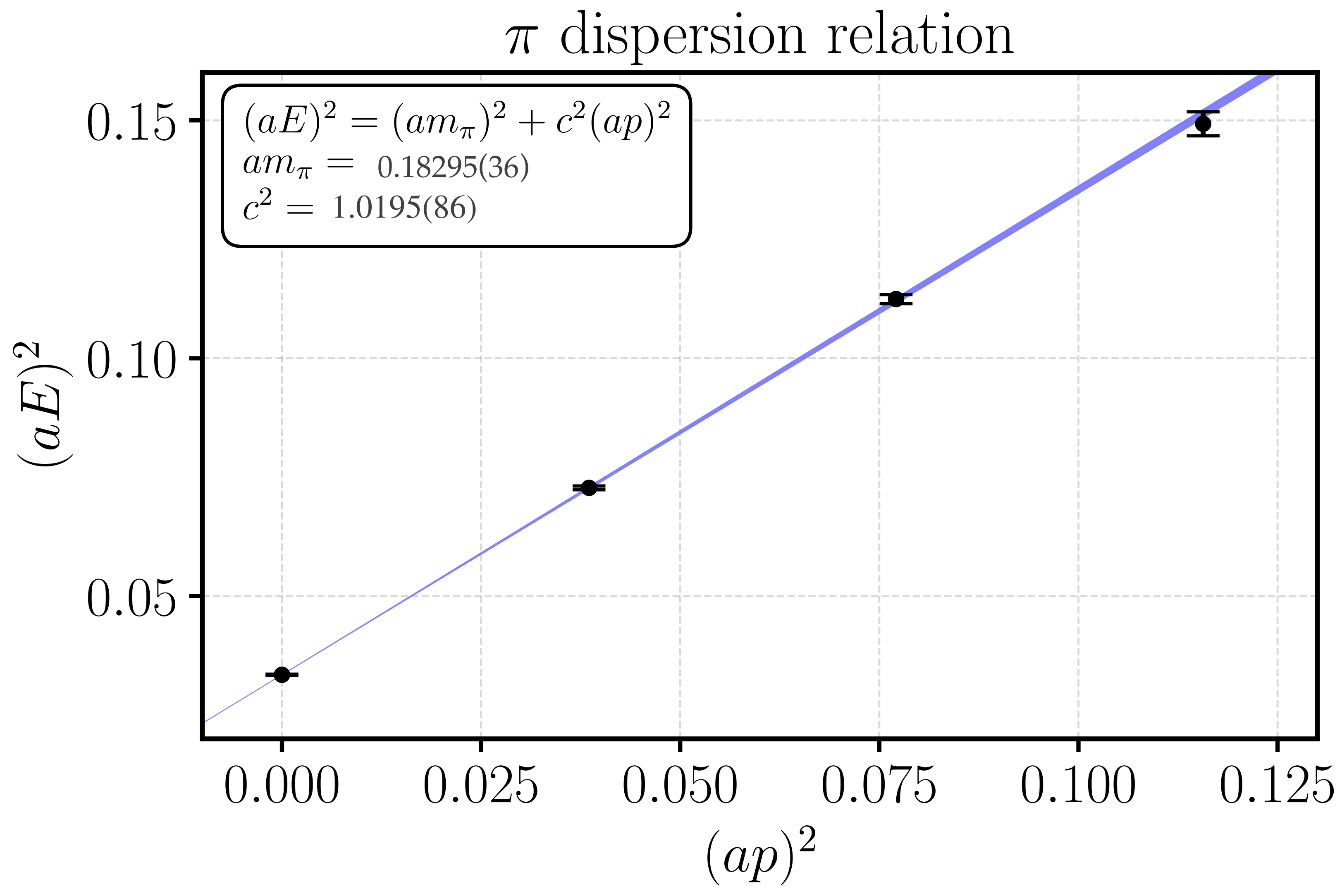


"Briceno-Hansen-Walker-Loud way of doing it"

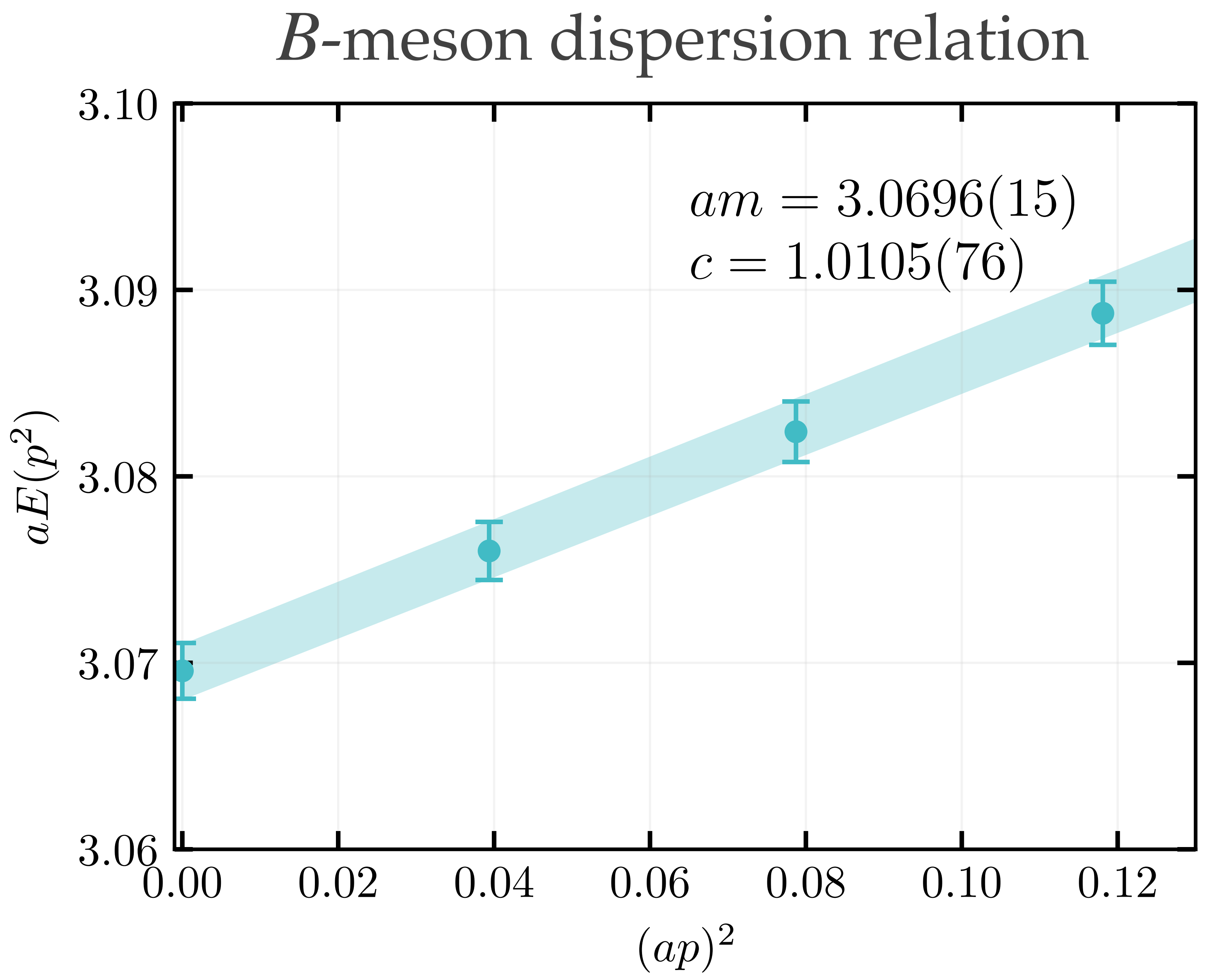
Briceno, Hansen, Walker-Loud 1406.5965

ensemble

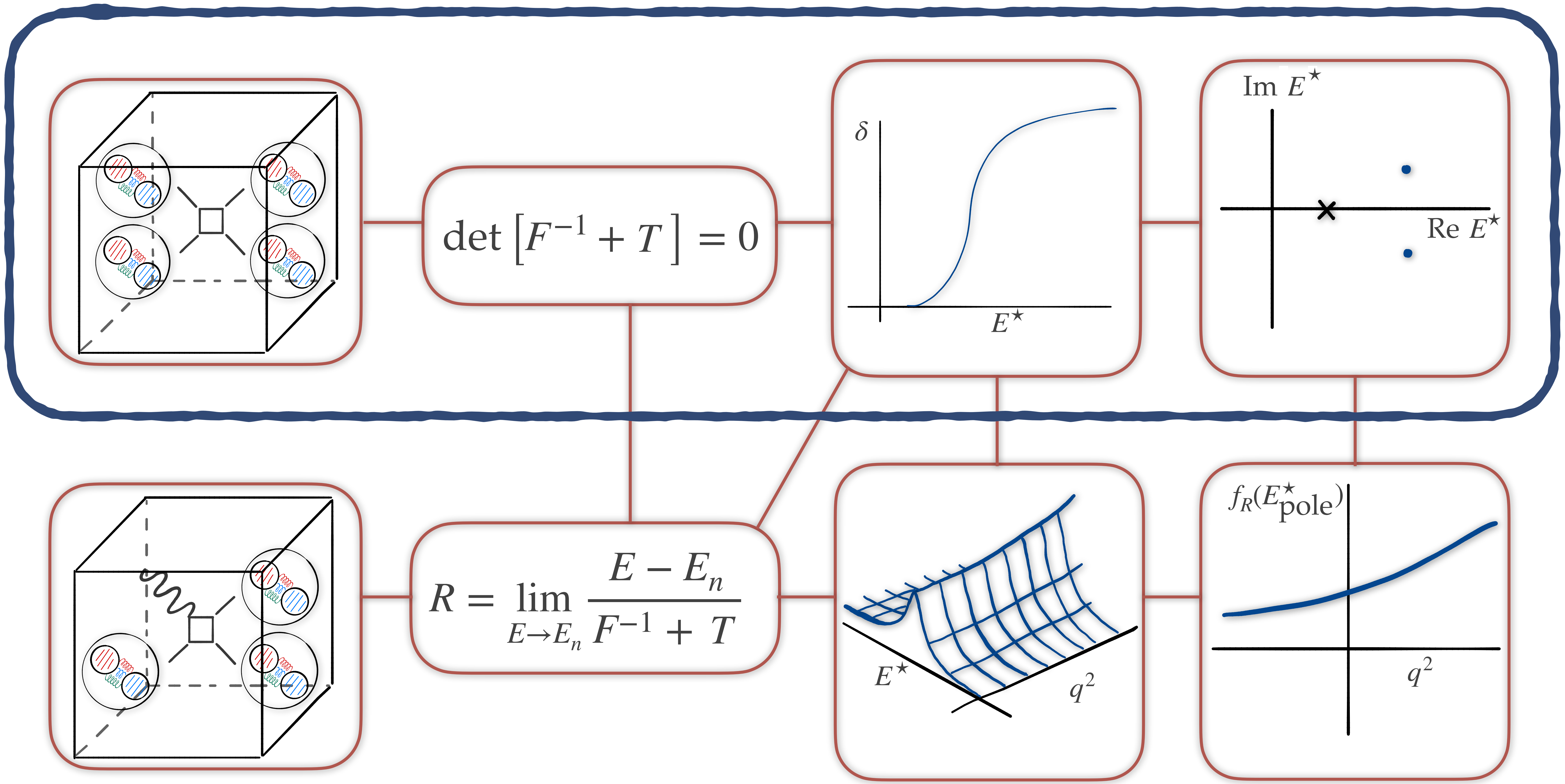
- $N_f = 2 + 1$
- $32^3 \times 96$
- $L \approx 3.6$ fm
- clover-Wilson light q
- RHQ heavy q
- $m_\pi \approx 320$ MeV
- $m_B = 5319.8(2.6)$ MeV
- $O(a)$ current
- $a \approx 0.11$ fm



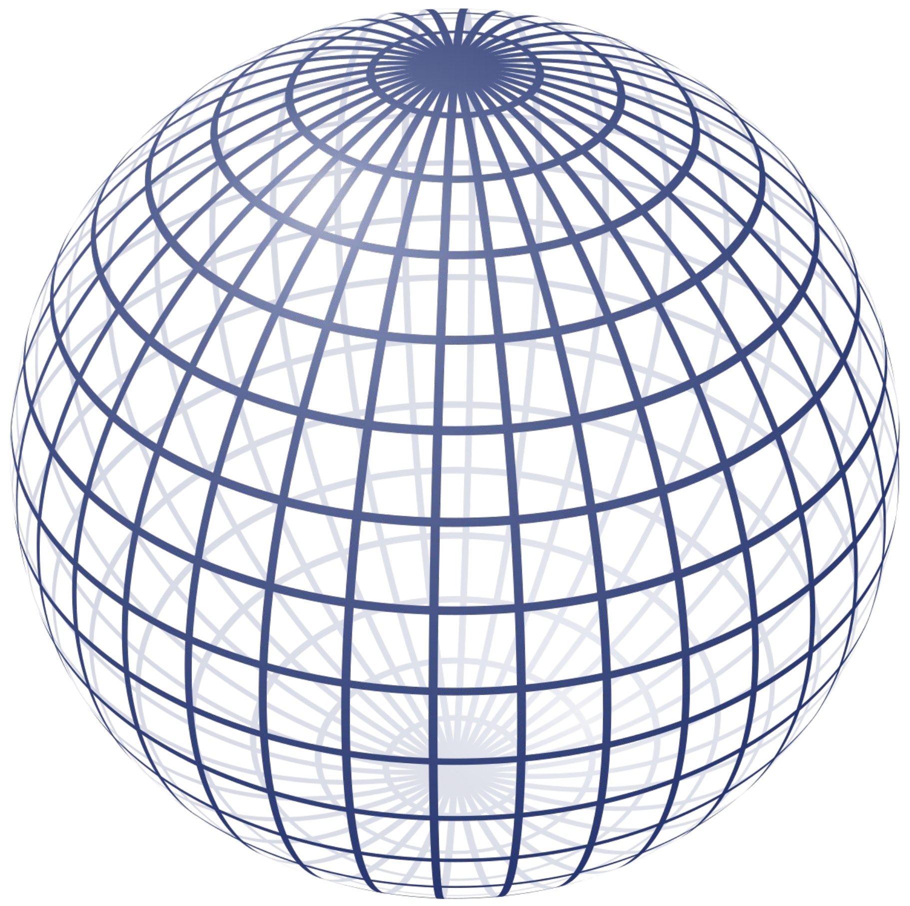
$$(aE)^2 = (am)^2 + c^2(ap)^2$$



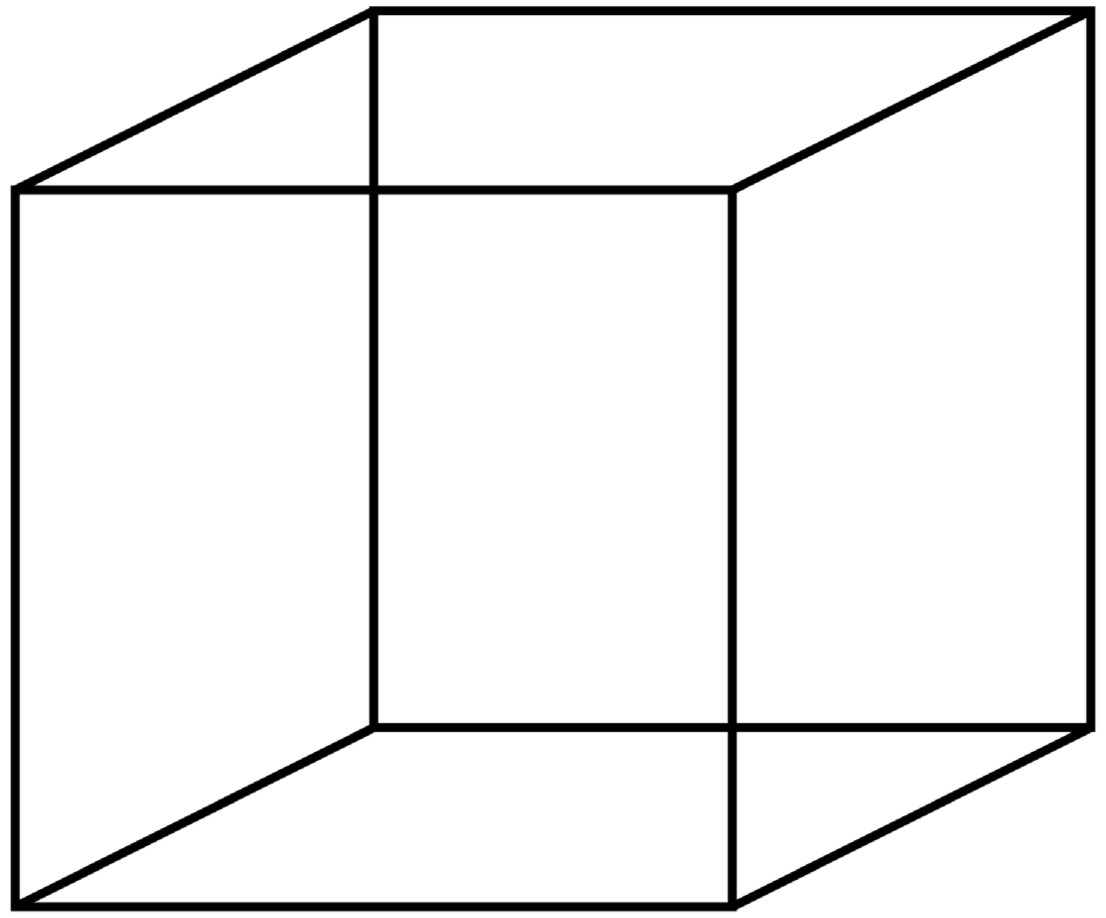
$B \to \rho \ell \nu$ on the lattice



ρ with lattice QCD



many-to-one

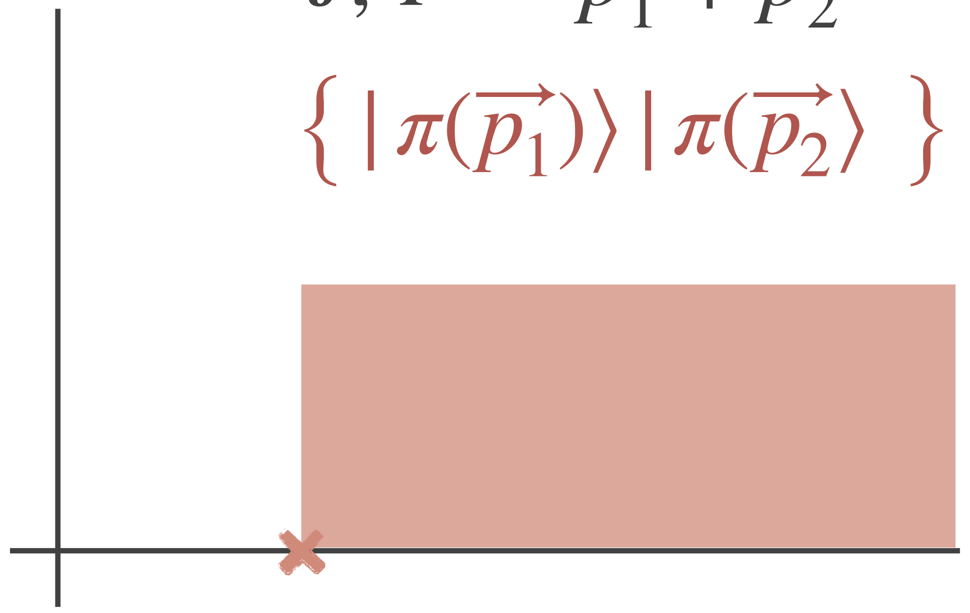


infinite volume:

- $O(3)$ symmetry
- infinite irreps (J^P)

$J, \vec{P} = \vec{p}_1 + \vec{p}_2$

$\{\, |\pi(\vec{p}_1)\rangle\, |\pi(\vec{p}_2)\rangle \,\}$



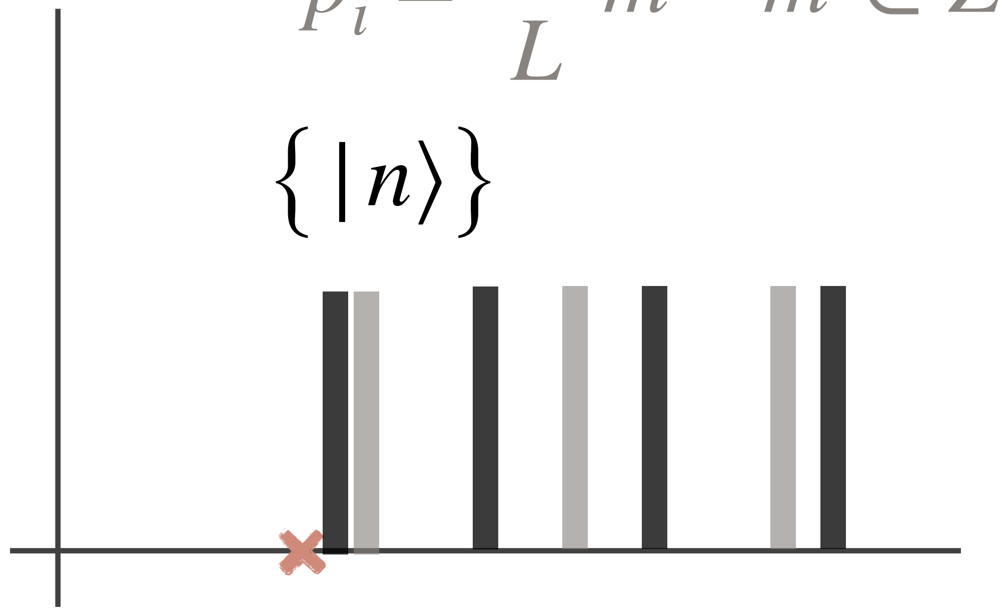
finite volume:

- discrete symmetries, Λ

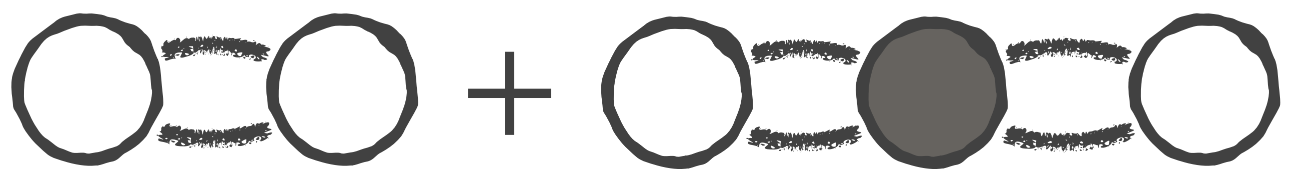
$L, \Lambda, \vec{P} = \vec{p}_1 + \vec{p}_2$

$\vec{p}_i = \frac{2\pi}{L}\vec{m} \quad \vec{m} \in Z^3$

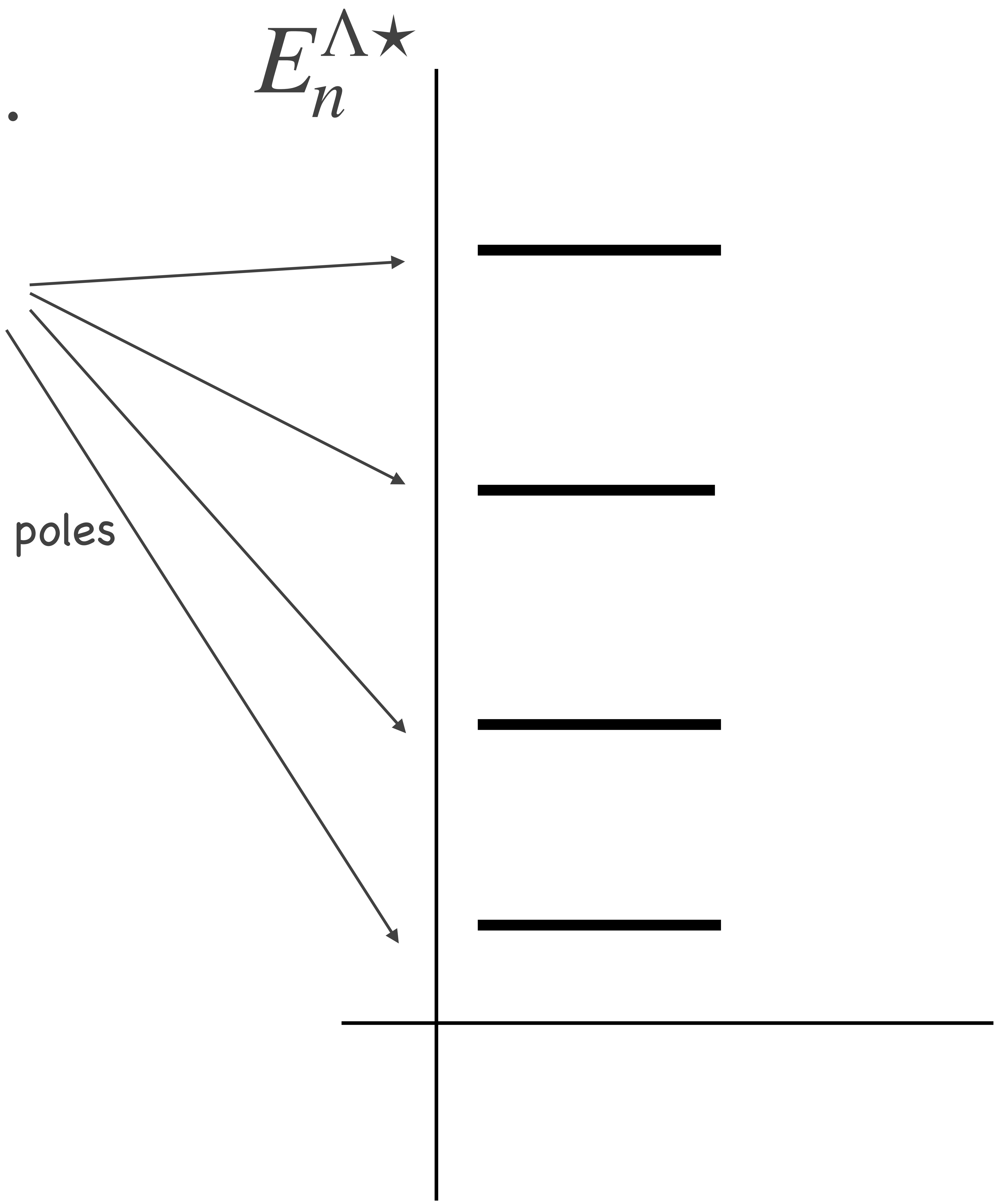
$\{\, |n\rangle \,\}$



ρ with lattice QCD

$C_L^{(2)} =$  $+ \ldots$

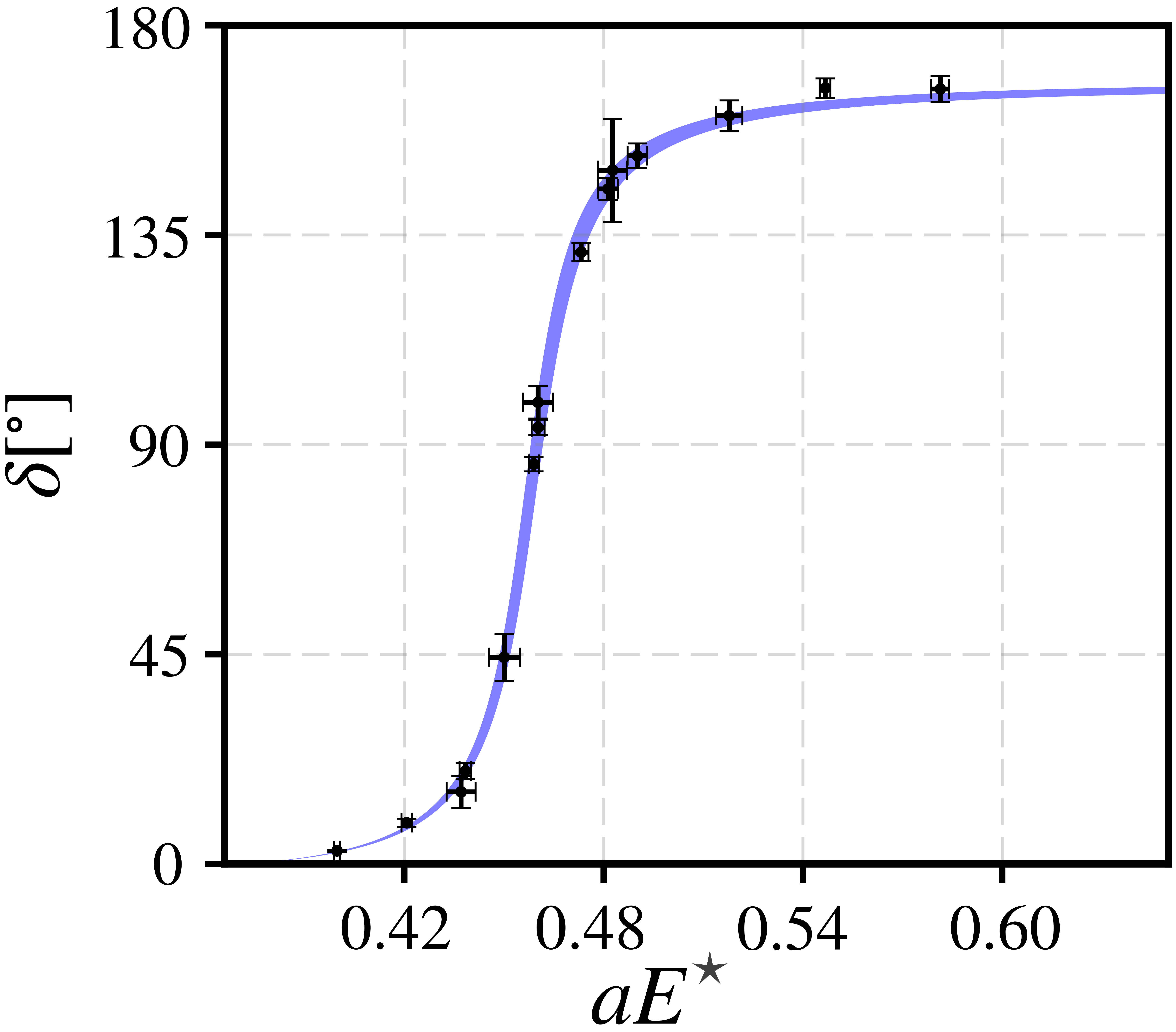
$$C_L^{(2)} = C_\infty^{(2)} - A' \frac{1}{F^{-1}(E^\star) + T(E^\star)} A$$



discrete spectrum where:

$$\det\left[F^{-1}(E^\star) + T(E^\star)\right] = 0$$

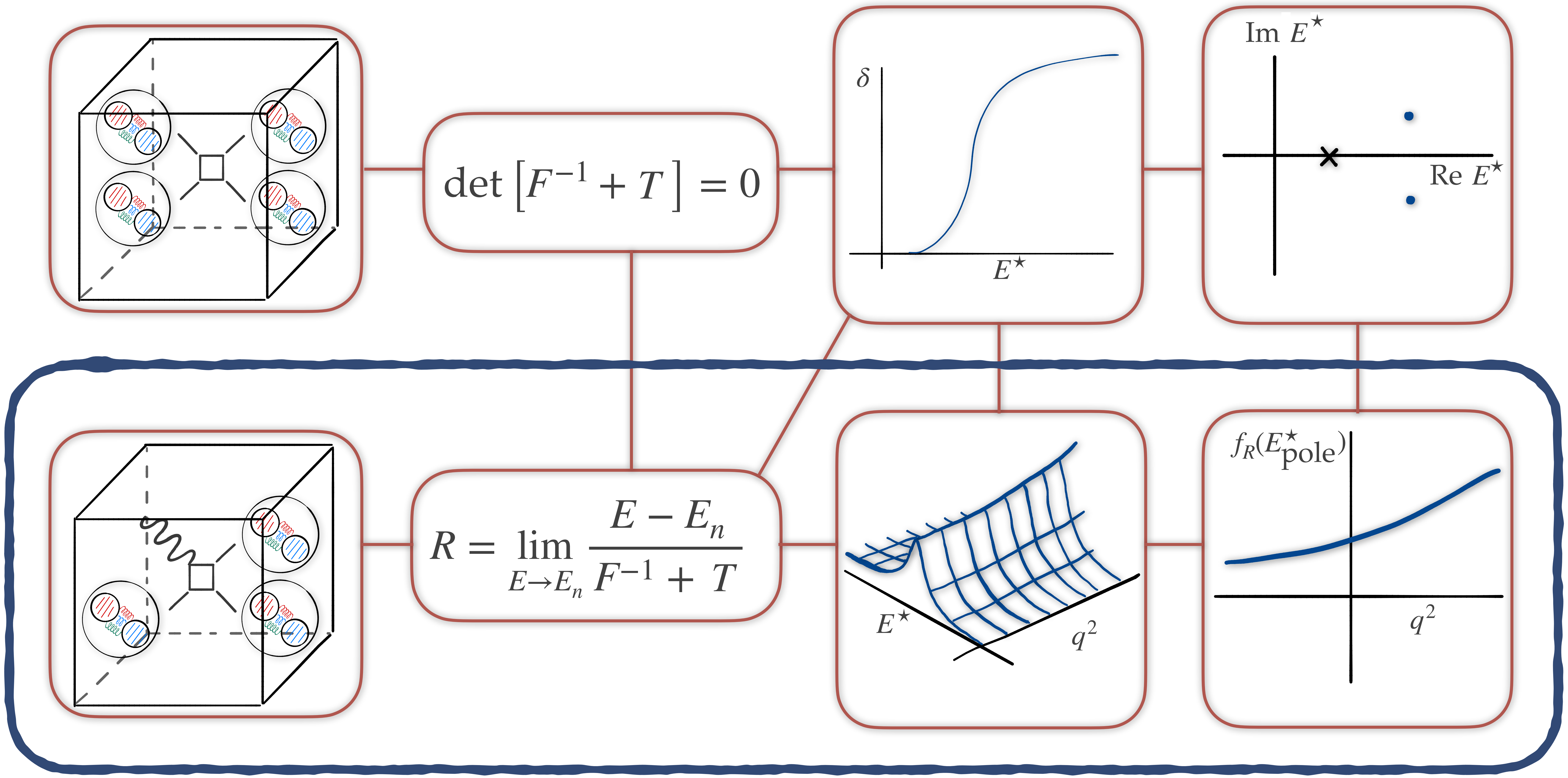
Luscher NPB354
Rummukainen, Gottlieb hep-lat/9503028
Kim, Sharpe, Sachrajda hep-lat/0507006
Briceno 1401.3312
Woss, Wilson, Dudek 2001.08474
Briceno, Dudek, Young 1706.06223
[and many more]



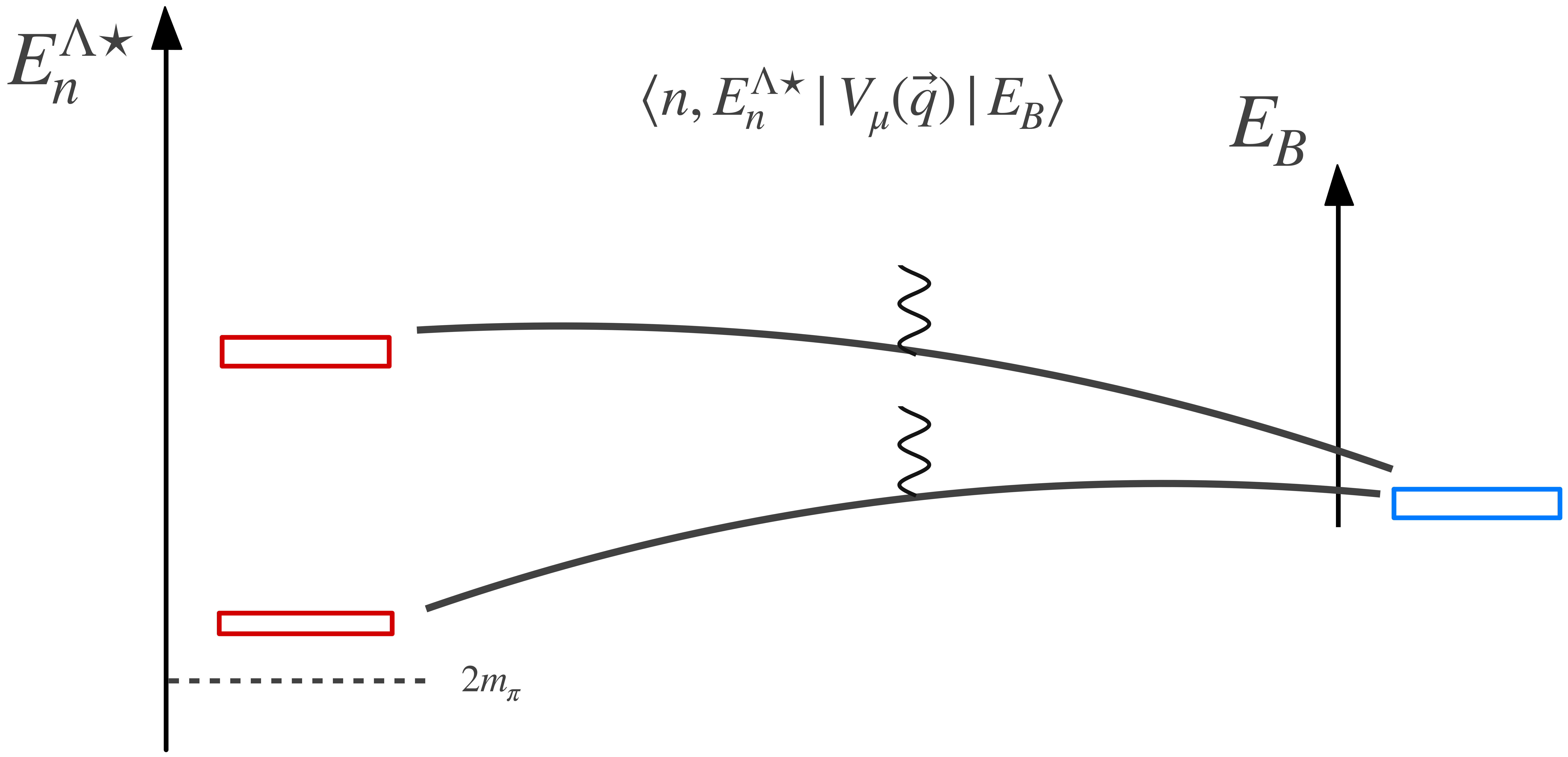
$$T(E^\star) = \frac{1}{\rho}\frac{1}{\cot\delta - i}$$

1704.05439

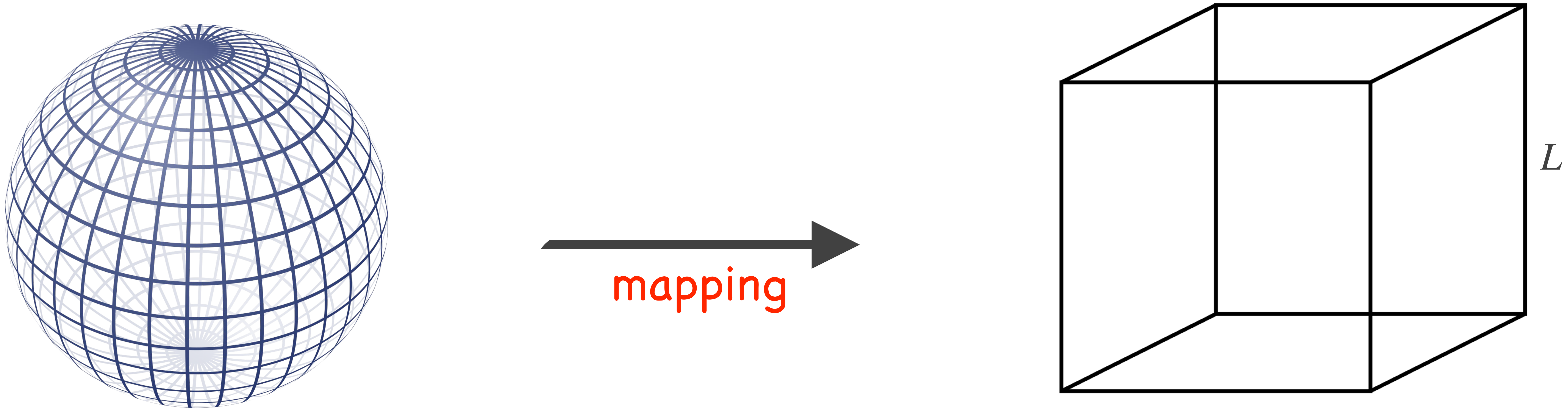
$B \to \rho\ell\nu$ on the lattice



transitions on the lattice



ρ with lattice QCD



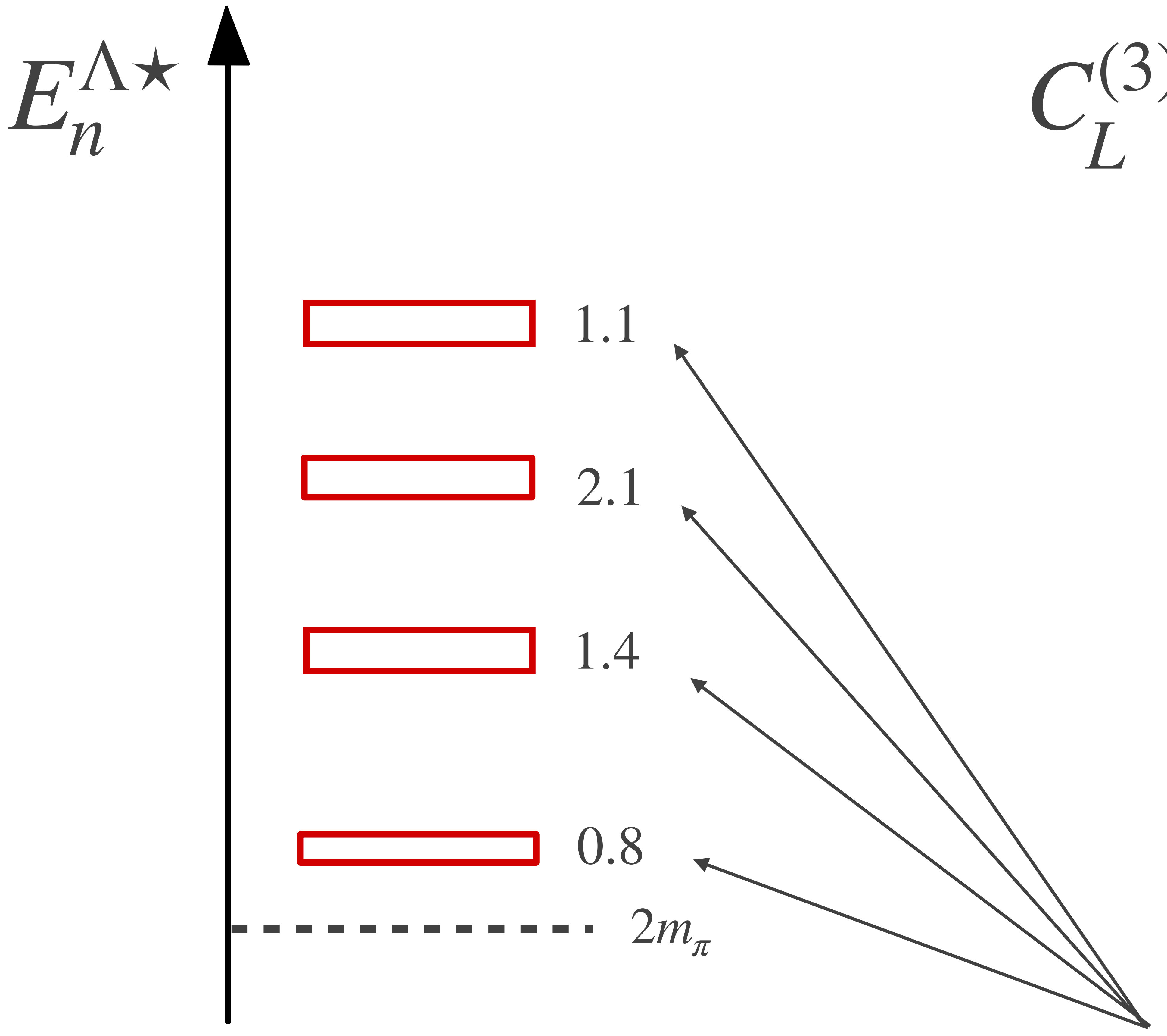
$$\{\, |\pi(\vec{p}_1)\rangle\, |\pi(\vec{p}_2)\rangle \,\}$$

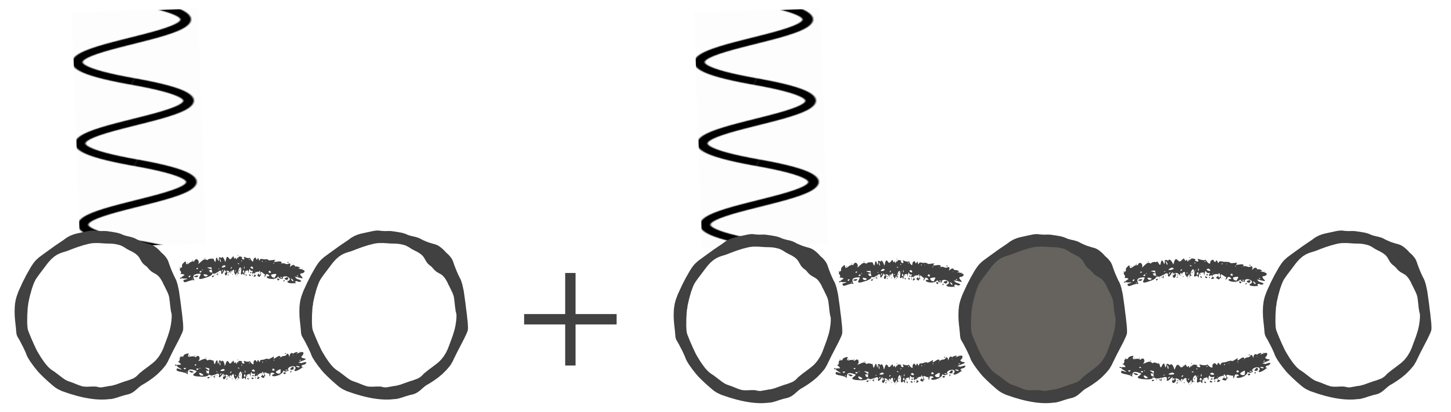
$$\{\, |n\rangle \,\}$$

- one particle normalization
  $\langle \pi, p \,|\, \pi, p' \rangle = 2E_\pi (2\pi)^3 \delta^3(\vec{p} - \vec{p}\,')$

- one particle normalization
- normalization due to strong interaction

the finite volume



$C_L^{(3)} =$  $+ \ldots$

$$C_L^{(3)} = C_\infty^{(3)} - A'RA$$

$$R_n = \lim_{E \to E_n} \frac{E - E_n}{F^{-1} + T}$$

Lellouch, Luscher hep-lat/0003023
Lin, Sachrajda, Testa hep-lat/0104006
…
Briceno, Hansen, Walker-Loud 1406.5965
Briceno, Hansen 1502.04314
Briceno, Dudek, LL 2105.02017

normalization of finite-volume states

$$|E_n^{\Lambda\star}\rangle_L \sim \sqrt{R_n}\,|\pi\pi(E^\star = E_n^{\Lambda\star})\rangle_\infty$$

on the R "matrix"

$$|E_n^{\star\Lambda}\rangle_L \sim \sqrt{R_n}\,|\pi\pi(E^\star = E_n^{\star\Lambda})\rangle_\infty$$

$$R_n \approx \frac{1}{F^{-1} + T}$$

on the R "matrix"

$$|\, E_n^{\star\Lambda} \rangle_L \sim \sqrt{R_n}\, |\, \pi\pi(E^\star = E_n^{\star\Lambda}) \rangle_\infty \qquad R_n \approx \frac{1}{F^{-1} + T}$$

$$F + T^{-1} = F \left(F^{-1} + T \right) T^{-1}$$ at the energies $E_n^{\star\Lambda}$

Briceno, Dudek, LL 2105.02017

on the R "matrix"

$$| E_n^{\star\Lambda} \rangle_L \sim \sqrt{R_n}\, | \pi\pi(E^\star = E_n^{\star\Lambda}) \rangle_\infty$$

$$R_n \approx \frac{1}{F^{-1} + T}$$

$$F + T^{-1} = F \left(F^{-1} + T \right) T^{-1}$$

at the energies $E_n^{\star\Lambda}$

$$F + T^{-1} = \frac{1}{\mu_0^{\star\,\prime}} \mathbf{w}_0 \mathbf{w}_0^T \Big|_{E_n^{\star\Lambda}}$$

Briceno, Dudek, LL 2105.02017

on the R "matrix"

$$|E_n^{\star\Lambda}\rangle_L \sim \sqrt{R_n}\, |\pi\pi(E^\star = E_n^{\star\Lambda})\rangle_\infty \qquad R_n \approx \frac{1}{F^{-1}+T}$$

$$F + T^{-1} = F\left(F^{-1}+T\right)T^{-1}$$ at the energies $E_n^{\star\Lambda}$

$$F + T^{-1} = \frac{1}{\mu_0^{\star\,\prime}}\mathbf{w}_0\mathbf{w}_0^T\Big|_{E_n^{\star\Lambda}}$$

F, T, L

"form factors"

$$|\langle ME\rangle_L| = \frac{1}{\sqrt{2E_B}\sqrt{2E_n^\Lambda}}\sqrt{\frac{2E_n^\star}{-\mu_0^{\star\,\prime}}}\;\mathbf{w_0}^{\mathbf{T}}\cdot\mathscr{A}$$

Briceno, Dudek, LL 2105.02017

on the R "matrix"

$$|E_n^{\star\Lambda}\rangle_L \sim \sqrt{R_n}\,|\pi\pi(E^\star = E_n^{\star\Lambda})\rangle_\infty \qquad R_n \approx \frac{1}{F^{-1}+T}$$

$$F + T^{-1} = F\left(F^{-1}+T\right)T^{-1}$$ at the energies $E_n^{\star\Lambda}$

$$F + T^{-1} = \frac{1}{\mu_0^{\star\,\prime}}\mathbf{w}_0\mathbf{w}_0^T\Big|_{E_n^{\star\Lambda}}$$

channel/partial wave space vectors

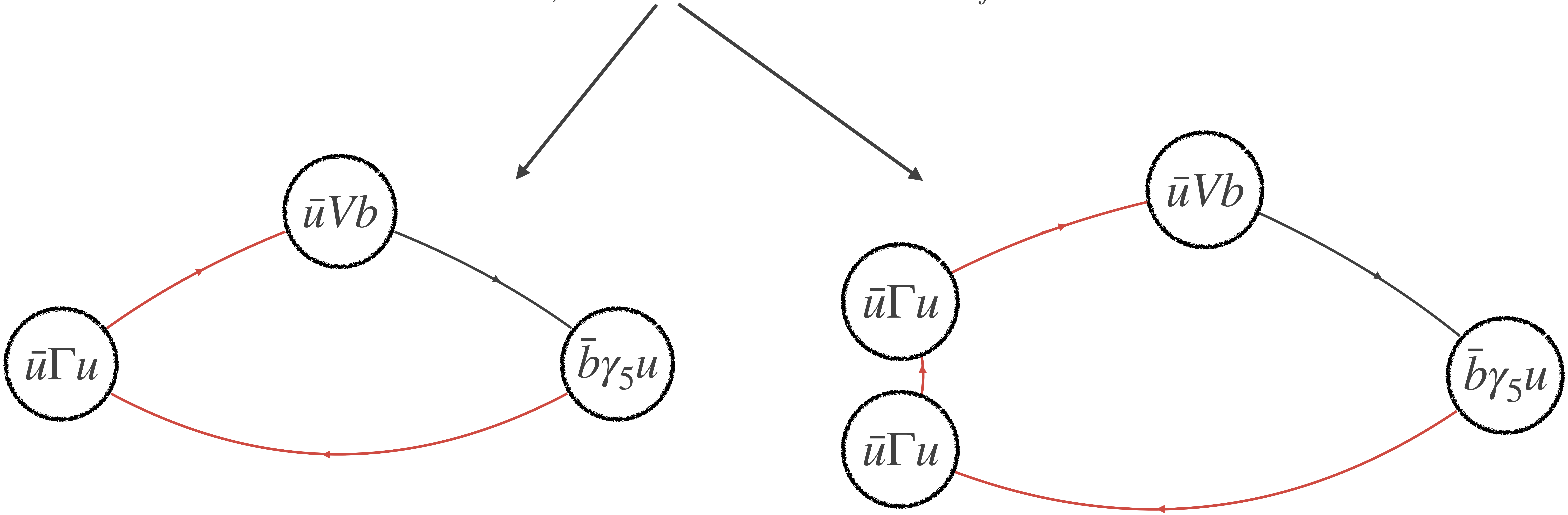
$$|\langle ME\rangle_L| = \frac{1}{\sqrt{2E_B}\sqrt{2E_n^\Lambda}}\sqrt{\frac{2E_n^\star}{-\mu_0^{\star\,\prime}}}\;\mathbf{w_0}^{\mathbf{T}}\cdot\mathscr{A}$$

note: some factors skipped for simplicity!

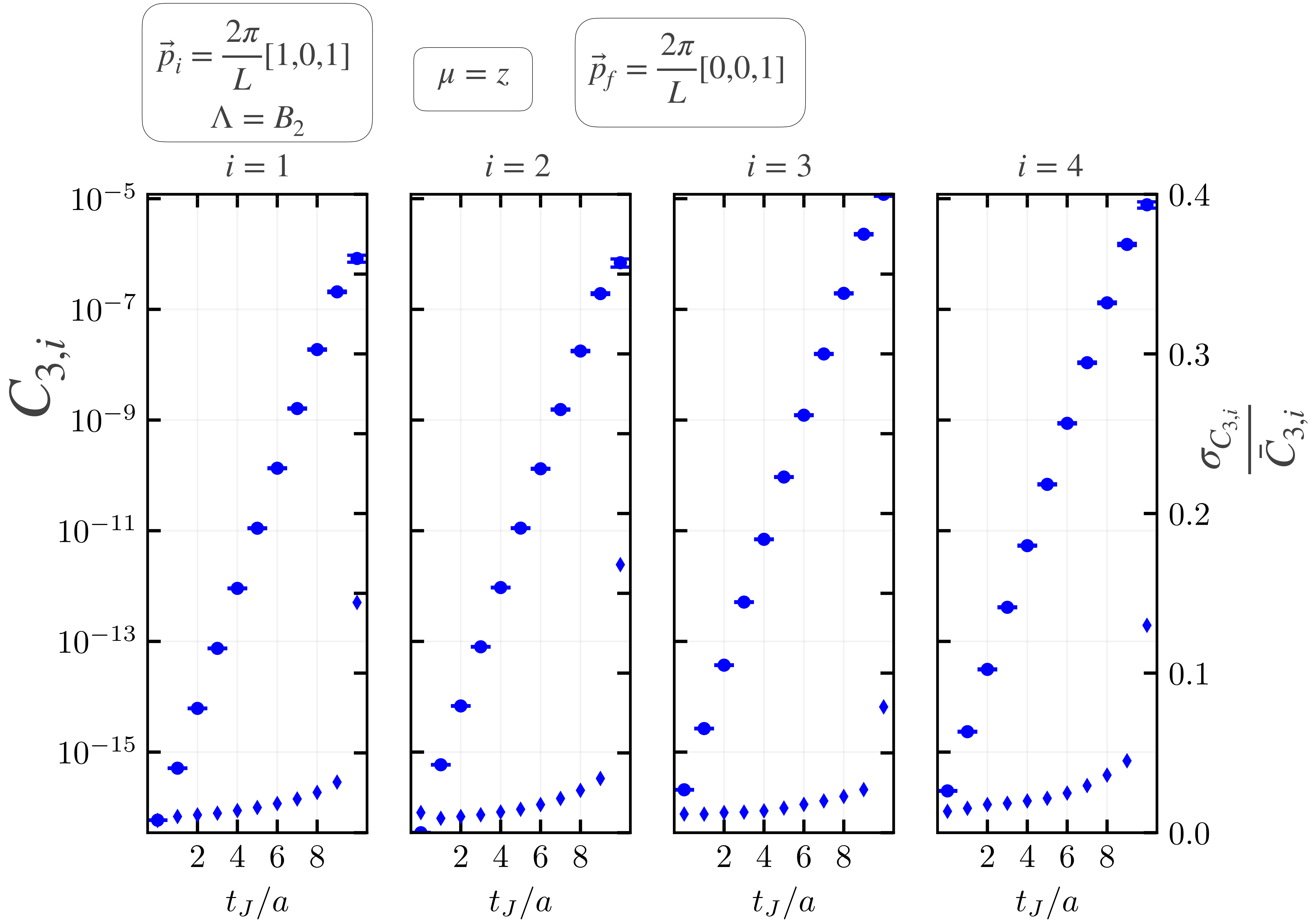
Briceno, Dudek, LL 2105.02017

the calculation

$$C_{3,i} = \langle\ O_i(\vec{p}_i, \Lambda)\ \ V^\mu\ \ O_B(\vec{p}_f)\ \rangle$$



the 3-point functions



$$C_{3,i} = \langle O_i(\vec{p}_i, \Lambda)\; V^\mu\; O_B(\vec{p}_f)\rangle$$

$$C_{3,i} = \sum_{m\in B}\sum_{n\in[\pi\pi]} \langle 0|O_i|n\rangle\langle n|V|m\rangle\langle m|O_B|0\rangle \frac{e^{-E_n(t_f-t)}e^{-E^B_m(t-t_i)}}{2E_n 2E^B_m},$$

$$\boxed{C_3^n = v_i^n C_{3,i}}$$

weights from $\pi\pi$ GEVP

$$C_3^n = \langle n|V|B\rangle\langle B|O_B|0\rangle \frac{e^{-E_n(t_f-t)}e^{-E_B(t-t_i)}}{2E_n 2E_B}$$

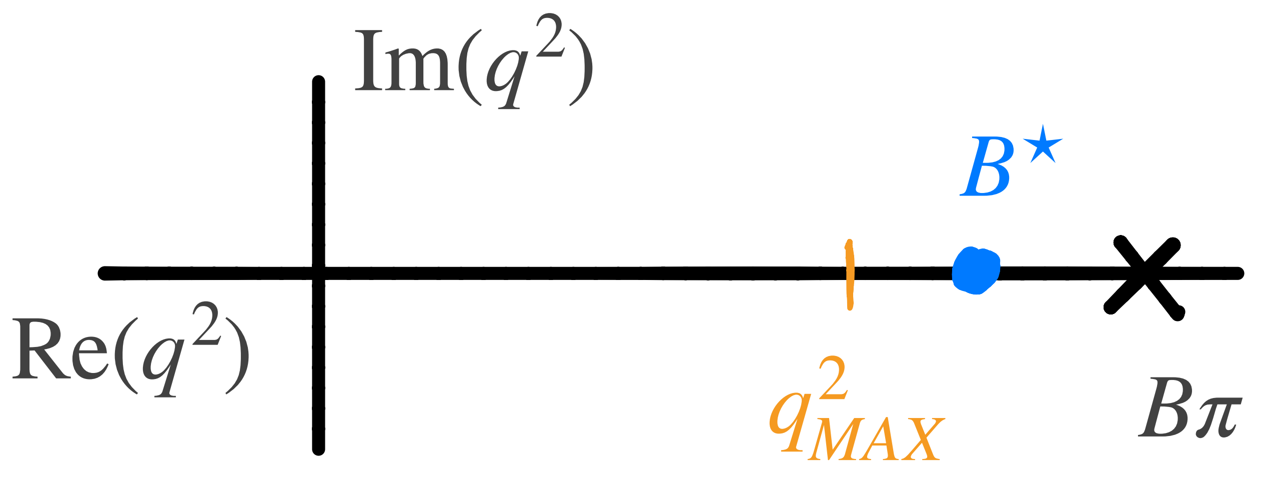
$+$ excited state cont.

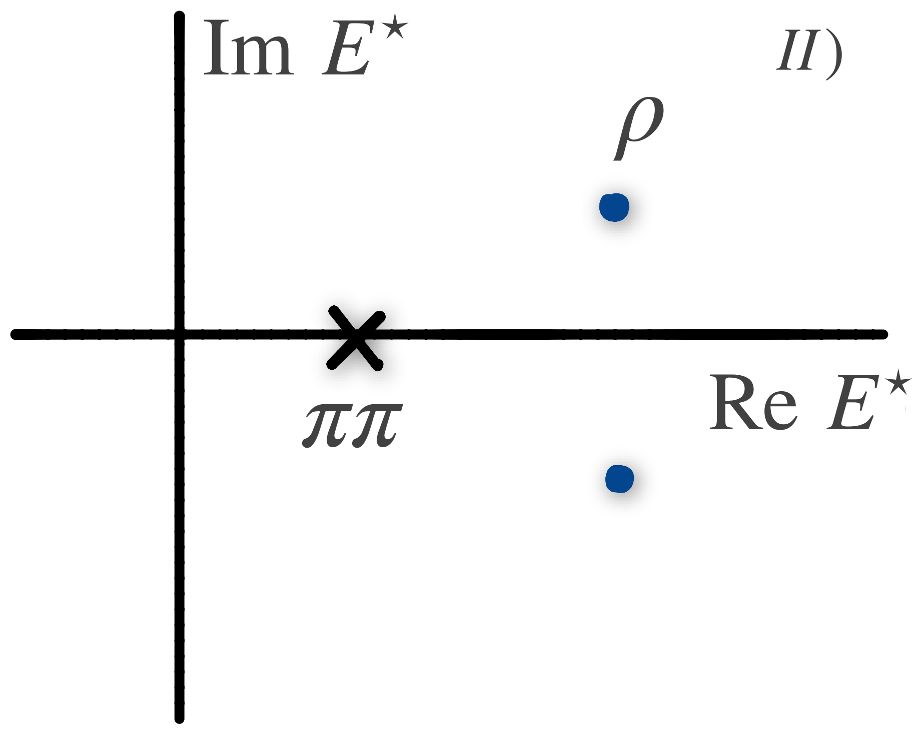
$\langle n\,|\,V\,|\,B\rangle(t)$ with excited state contr.
different models for the fit
0.65
0.60
0.55
0.50
0.45
$\langle \mathcal{V} \rangle$ re
2 4 6
2 4 6 8
2 4 6 8 10
t_J/a
AKAIKE average
ratio $\propto \sqrt{\langle n\,|\,V\,|\,B\rangle(t)\langle B\,|\,V\,|\,n\rangle(dt-t)}$

$B \to \rho(\to \pi\pi)\ell\bar{\nu}$ vector transition amplitude

$$\mathcal{H}^{\mu}_{1,m_\ell} = \langle \pi\pi(\varepsilon(m_\ell), p_f) \,|\, \bar{q}\gamma^\mu b \,|\, B(p_i)\rangle = \frac{2iV(q^2, E^\star)}{m_B + 2m_\pi}\epsilon_{\mu\nu\alpha\beta}\,\varepsilon^{\nu *}(1, m_\ell)\, p_i^\alpha p_f^\beta$$

$$V(q^2, E^\star) = \mathcal{A}(q^2, E^\star)\frac{T(E^\star)}{k}$$





- $F(q^2, E^\star)$:
  - smooth in E
  - q^2 has poles and thresholds
  - use z-expansion

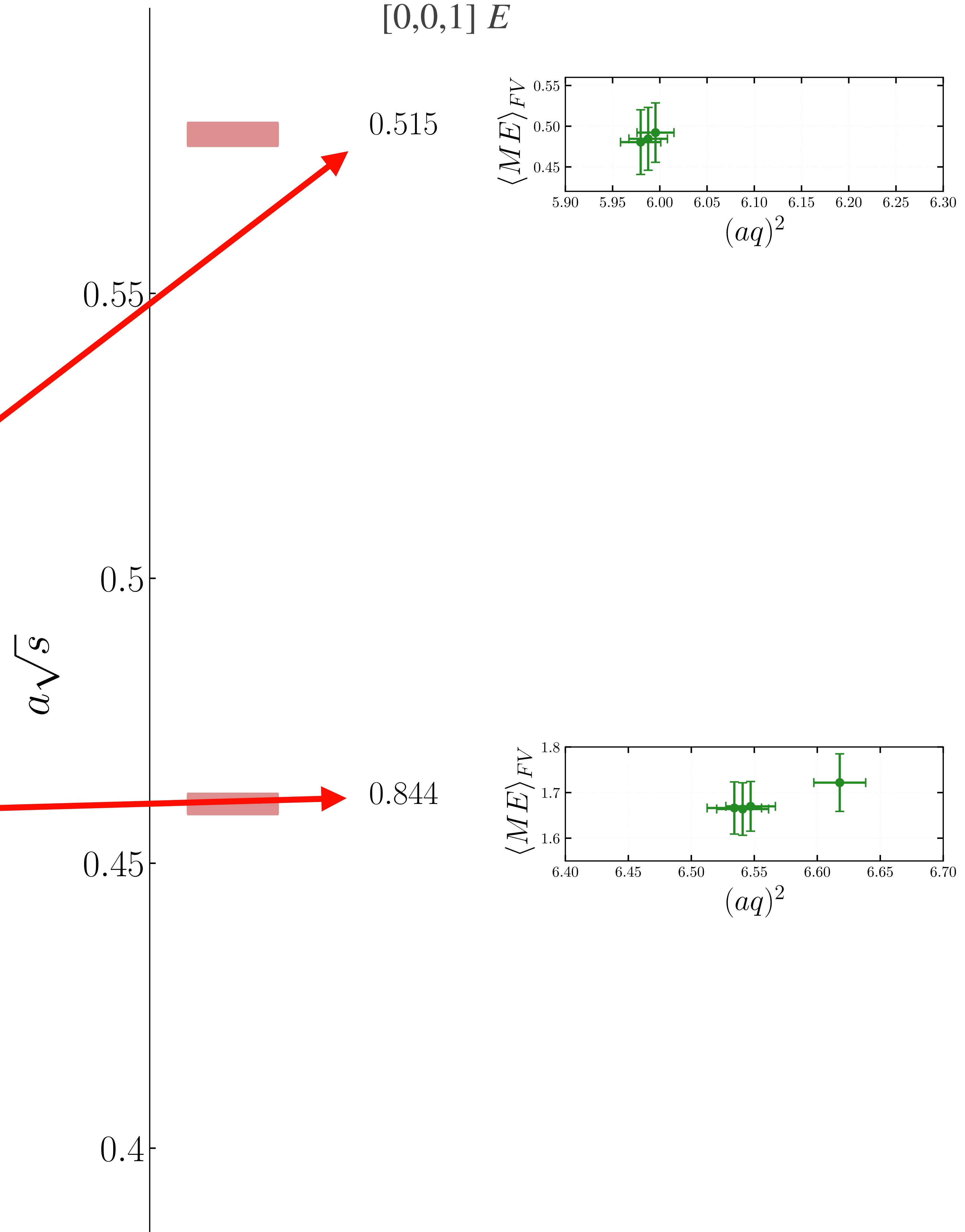
Boyd, Grinstein, Lebed hep-ph/9412324
Bourrely, Caprini, Lellouch 0807.2722
Alexandrou, LL, Meinel et al. 1807.08357

- $T(E^\star)$:
  - $\pi\pi$ threshold
  - ρ pole

matrix element and transition amps

normalization of
finite-volume states

$$|\langle ME \rangle_L| = \frac{1}{\sqrt{2E_B}\sqrt{2E_n^{\Lambda}}} \sqrt{\frac{2E_n^{\star}}{-\mu_0^{\star\,\prime}}}\; w_0^T \cdot F$$



Lellouch, Luscher hep-lat/0003023
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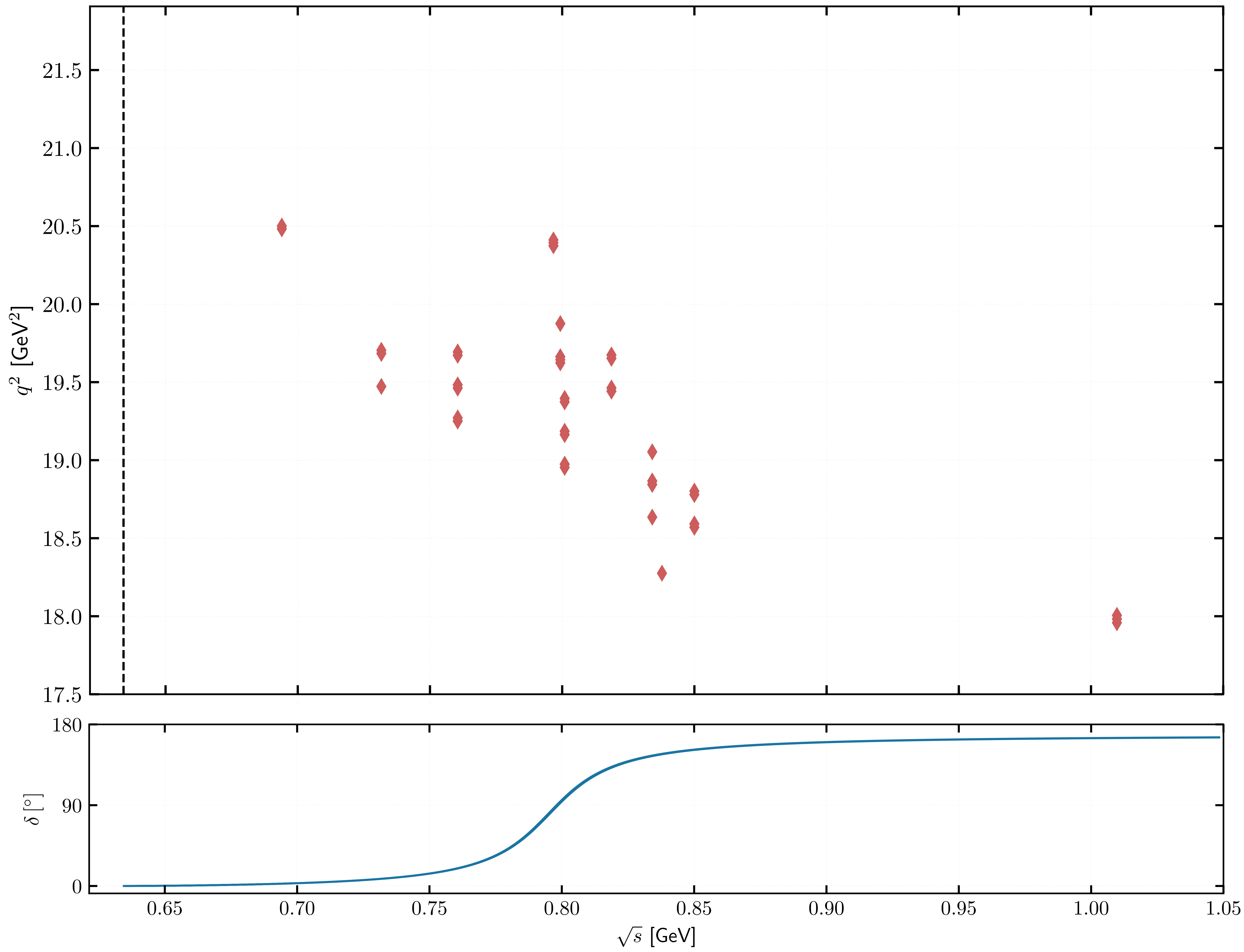
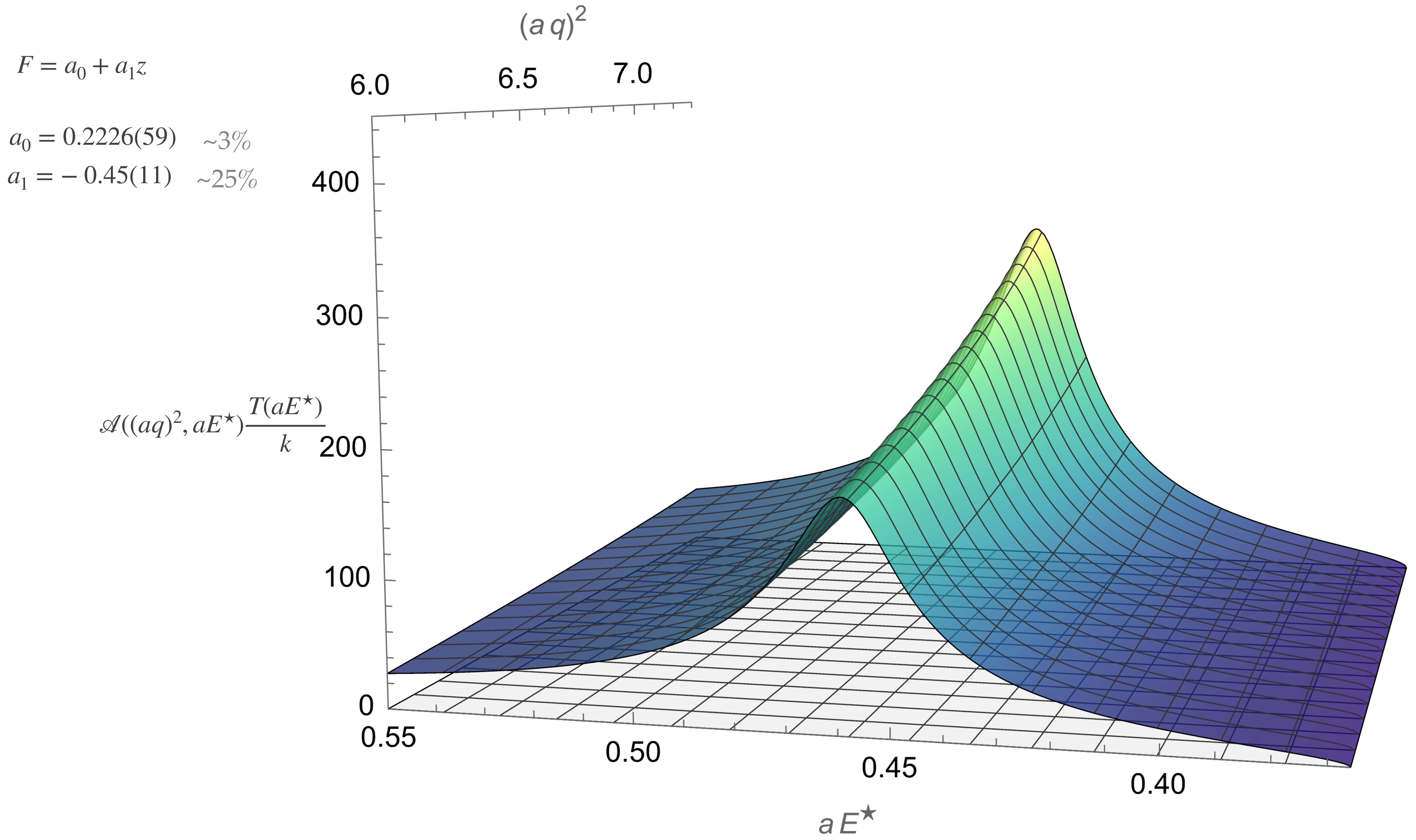
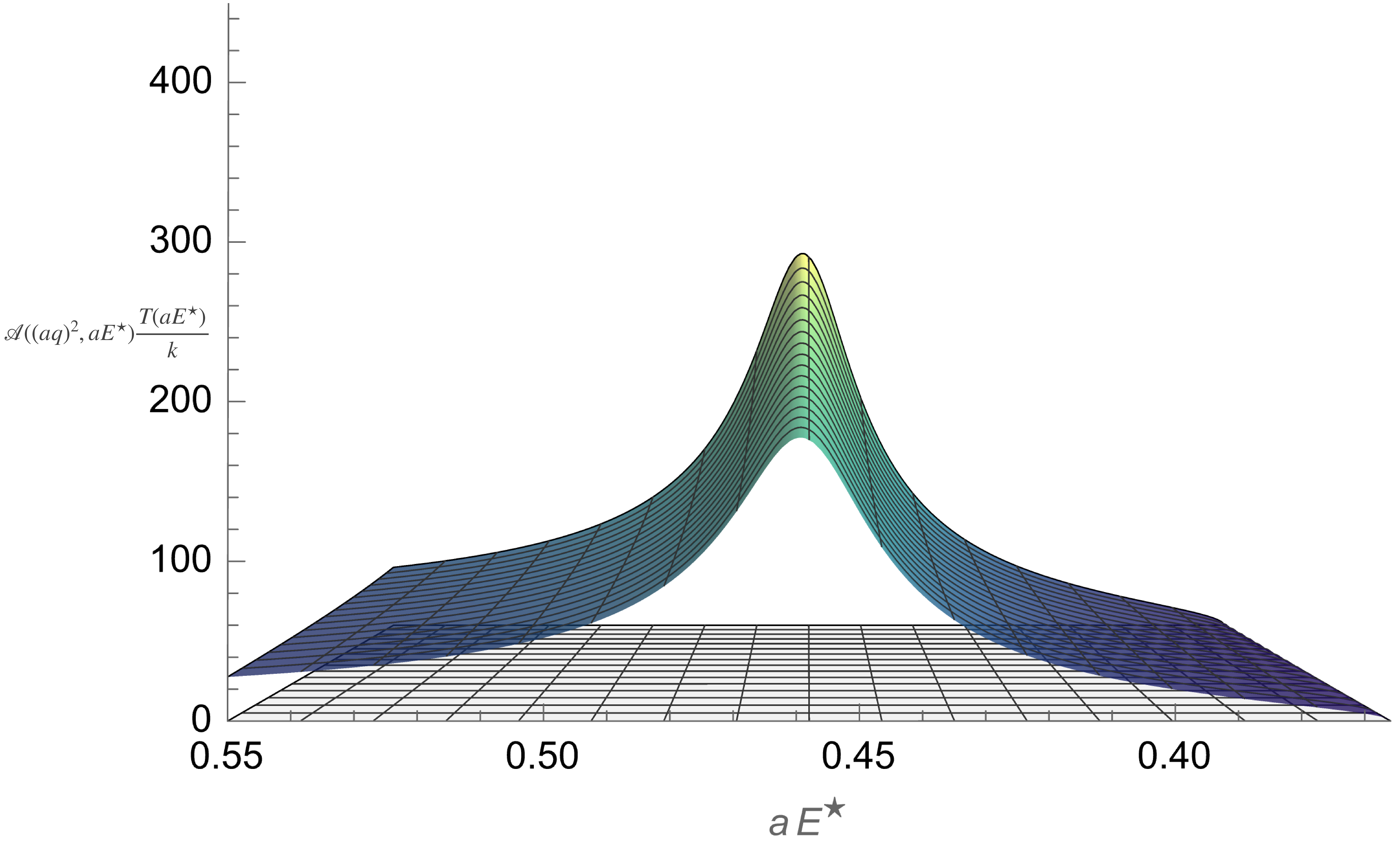


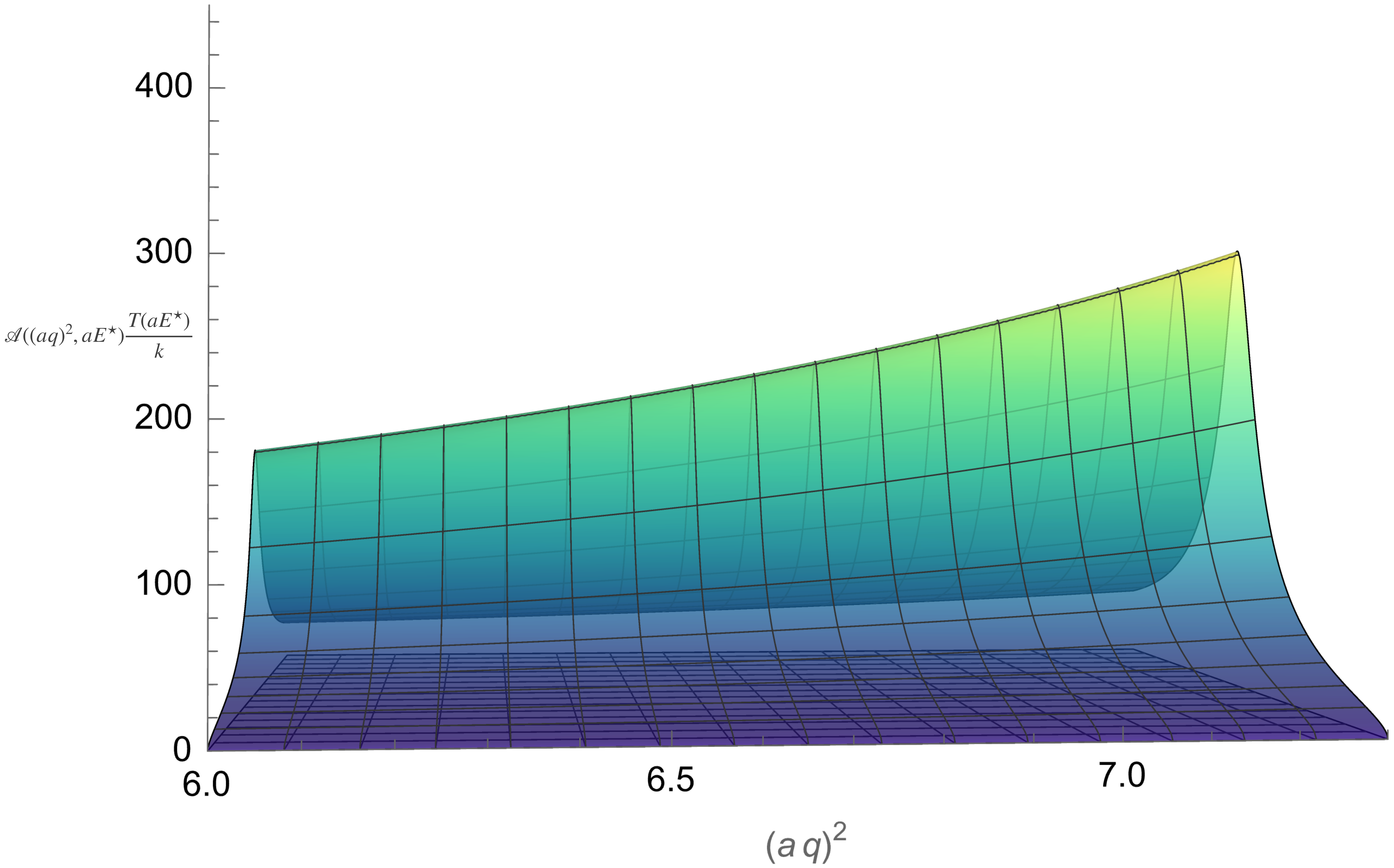
q^2 [GeV^2]
21.5
21.0
20.5
20.0
19.5
19.0
18.5
18.0
17.5
δ [°]
180
90
0
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
1.05
$\sqrt{s}$ [GeV]



F = a_0 + a_1 z
a_0 = 0.2226(59) ~3%
a_1 = − 0.45(11) ~25%
(a q)^2
6.0
6.5
7.0
400
300
200
100
0
𝒜((aq)^2, aE*) T(aE*)/k
0.55
0.50
0.45
0.40
a E*



400
300
200
100
0
$\mathscr{A}((aq)^2, aE^\star)\frac{T(aE^\star)}{k}$
0.55
0.50
0.45
0.40
$a\,E^\star$


400
300
200
100
0
$\mathscr{A}((aq)^2, aE^\star)\frac{T(aE^\star)}{k}$
6.0
6.5
7.0
$(a\,q)^2$

outlook

- first look into B-meson decays to resonances
- new process for $|V_{ub}|$
- combining flavor physics with hadronic physics
- exciting times ahead

Thank you!