$B_c^+/B^+ \to \tau^+\nu_\tau$ review for finalization

FCC-ee physics performance meeting
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Purpose for this talk

- **The (experimental) analysis of $B^+/B_c^+ \to \tau^+\nu_\tau$ is complete**
  - More work to be done on theory interpretations

- **Seek for review and approval**
  - Crystallize the experimental results
  - Once the theory results are ready, will call for a second round (the final) review
  - Analysis document (paper draft) circulated with flavor and P&P conveners
  - Full set of code at FCCeePhysicsPerformance

- **Table of content**
  - Overview of analysis procedure (just for completeness, will skip during the talk)
  - Major changes since the last version
  - Plan for theory interpretation, and publication

Previous results:

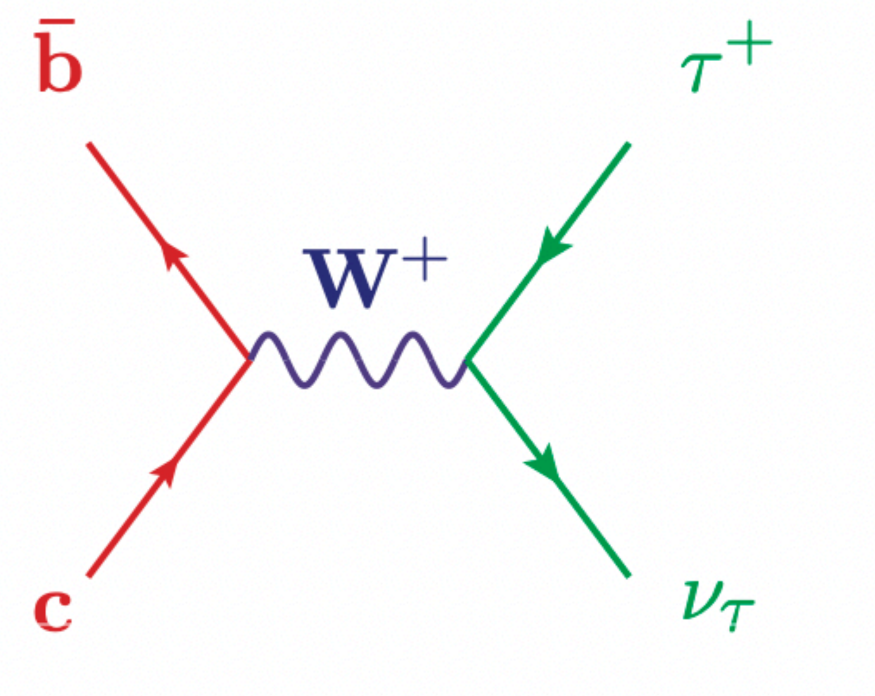
- $B_c^+ \to \tau^+\nu_\tau$: paper
- $B^+/B_c^+ \to \tau^+\nu_\tau$: last iteration

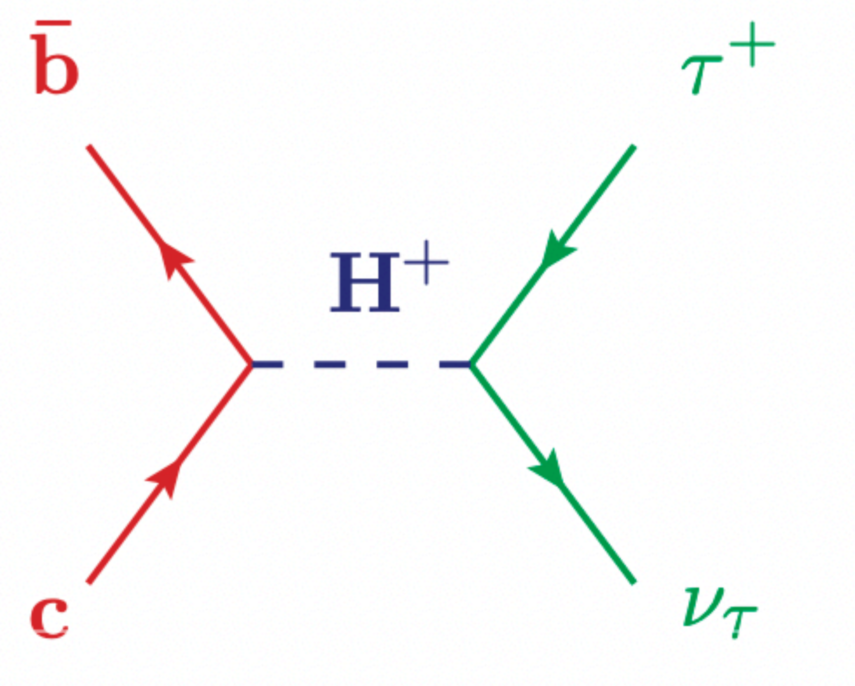
Analysis overview

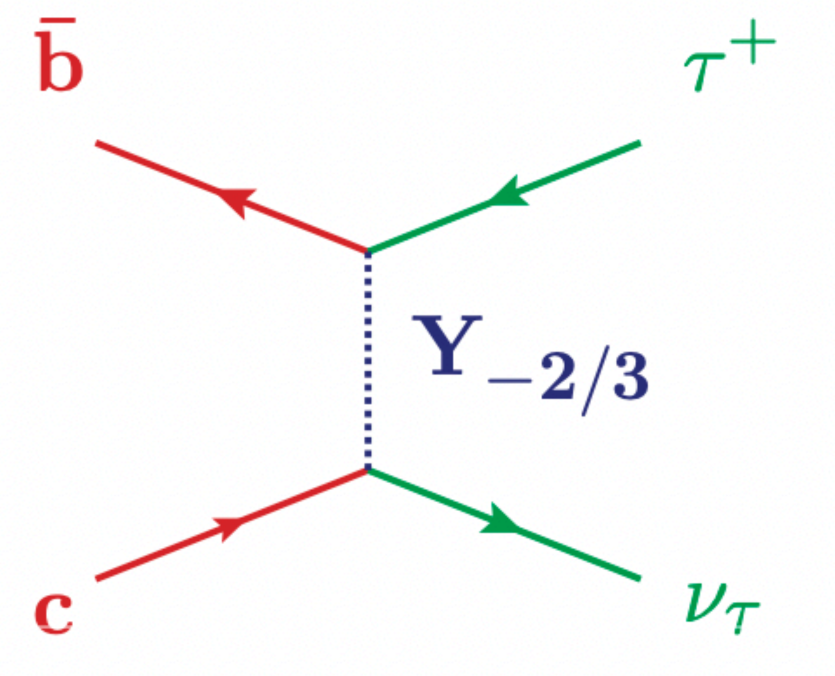
$B_c^+ / B^+ \to \tau^+\nu_\tau$ decays

- Directly related to anomalies in $b \to c\tau\nu_\tau$
- Clean probes to measure $|V_{cb}|$ and $|V_{ub}|$
- Sensitive to BSM physics, like charged Higgs and leptoquarks

- $B_c^+ \to \tau^+\nu_\tau$ and $B^+ \to \tau^+\nu_\tau$ are helicity and CKM-suppressed.
  - $f(B_c^+) \approx 0.04\,\%\,,\quad \mathscr{B}(B_c^+ \to \tau^+\nu_\tau) \approx 1.94\,\%$
  - $f(B^+) \approx 43\,\%\,,\quad \mathscr{B}(B^+ \to \tau^+\nu_\tau) \approx 1.09 \times 10^{-4}$
- In the 5×10^{12} Z events scenario of FCC-ee
  - $B_c^+ \to \tau^+\nu_\tau\ (\pi^+\pi^+\pi^-\bar{\nu}_\tau) \approx$ 1M
  - $B^+ \to \tau^+\nu_\tau\ (\pi^+\pi^+\pi^-\bar{\nu}_\tau) \approx$ 6M





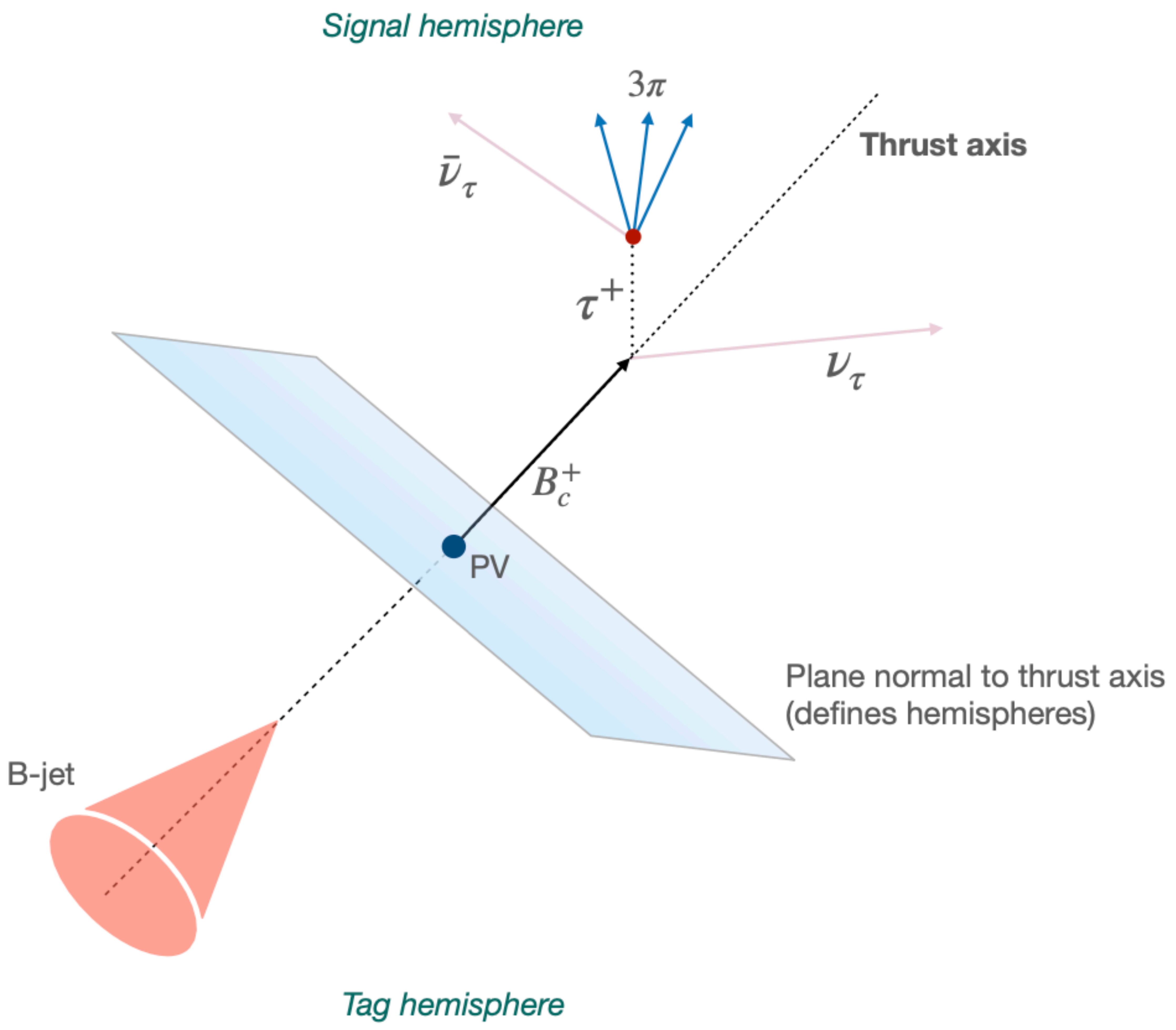


b-decay hemispheres

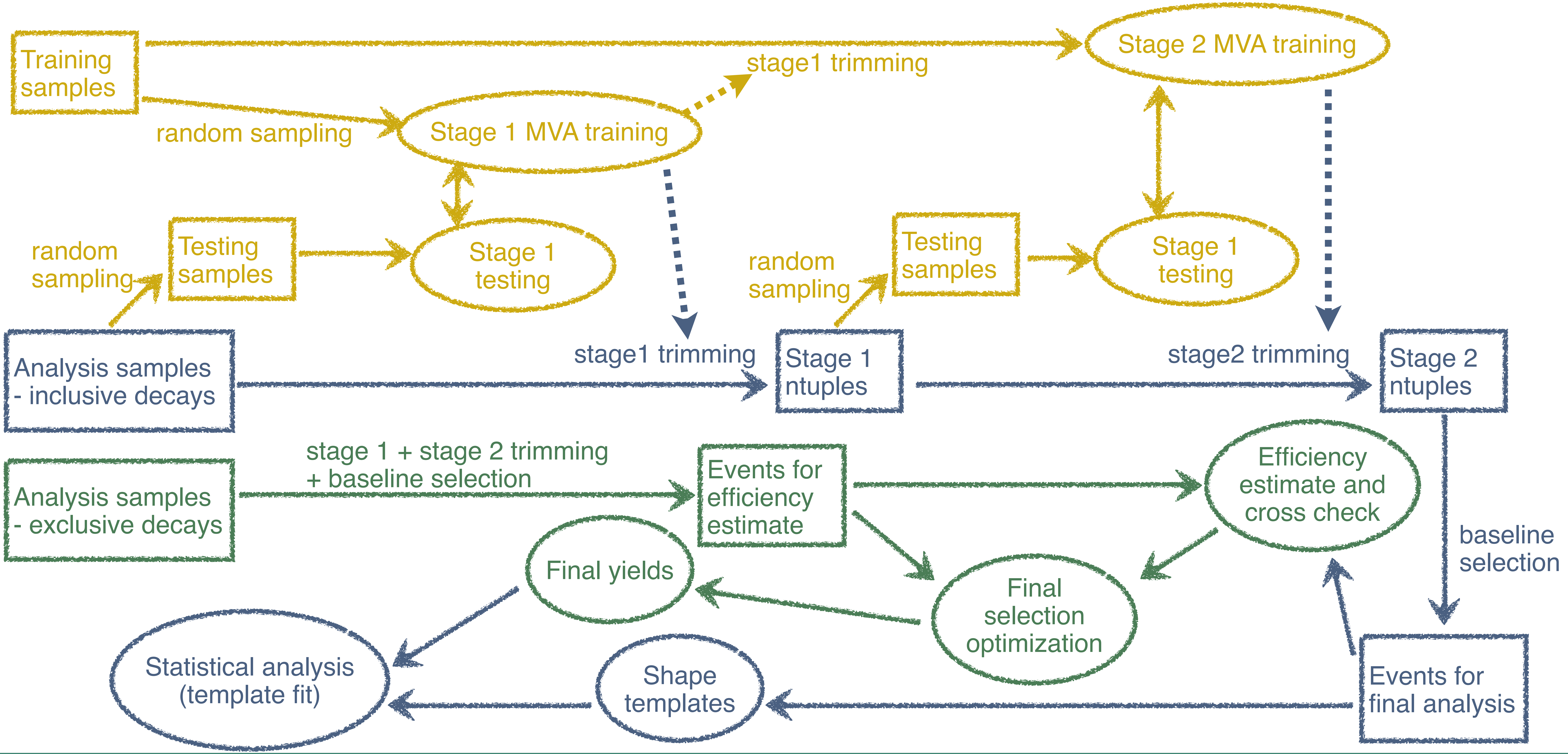
- Focus on three-prong $\tau^+ \to \pi^+\pi^+\pi^-\bar{\nu}_\tau$ decay
- Thrust axis defined as the axis that aligns the most with particle momenta.

$$T_i = \frac{\sum_i |\vec{p_i} \cdot \hat{n}|}{\sum_i |\vec{p_i}|}$$

- Measures the decay axis of $Z \to b\bar{b}$
- Due to high missing energy in the signal decays
  - The thrust axis would be skewed in signal events
  - The two hemispheres would have very different energy distributions.

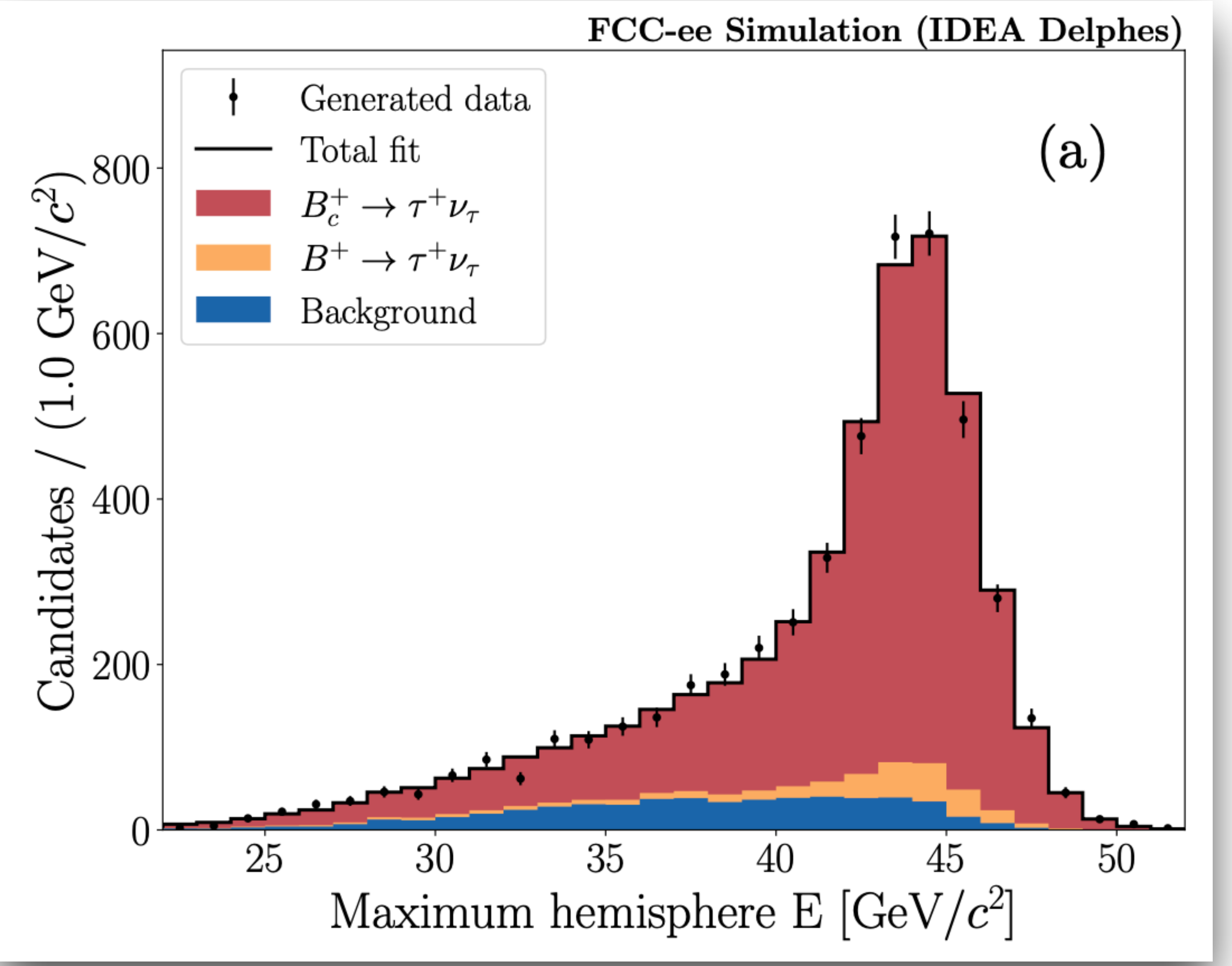


Analysis steps



Previous results

$B_c^+ \to \tau^+\nu_\tau$: paper



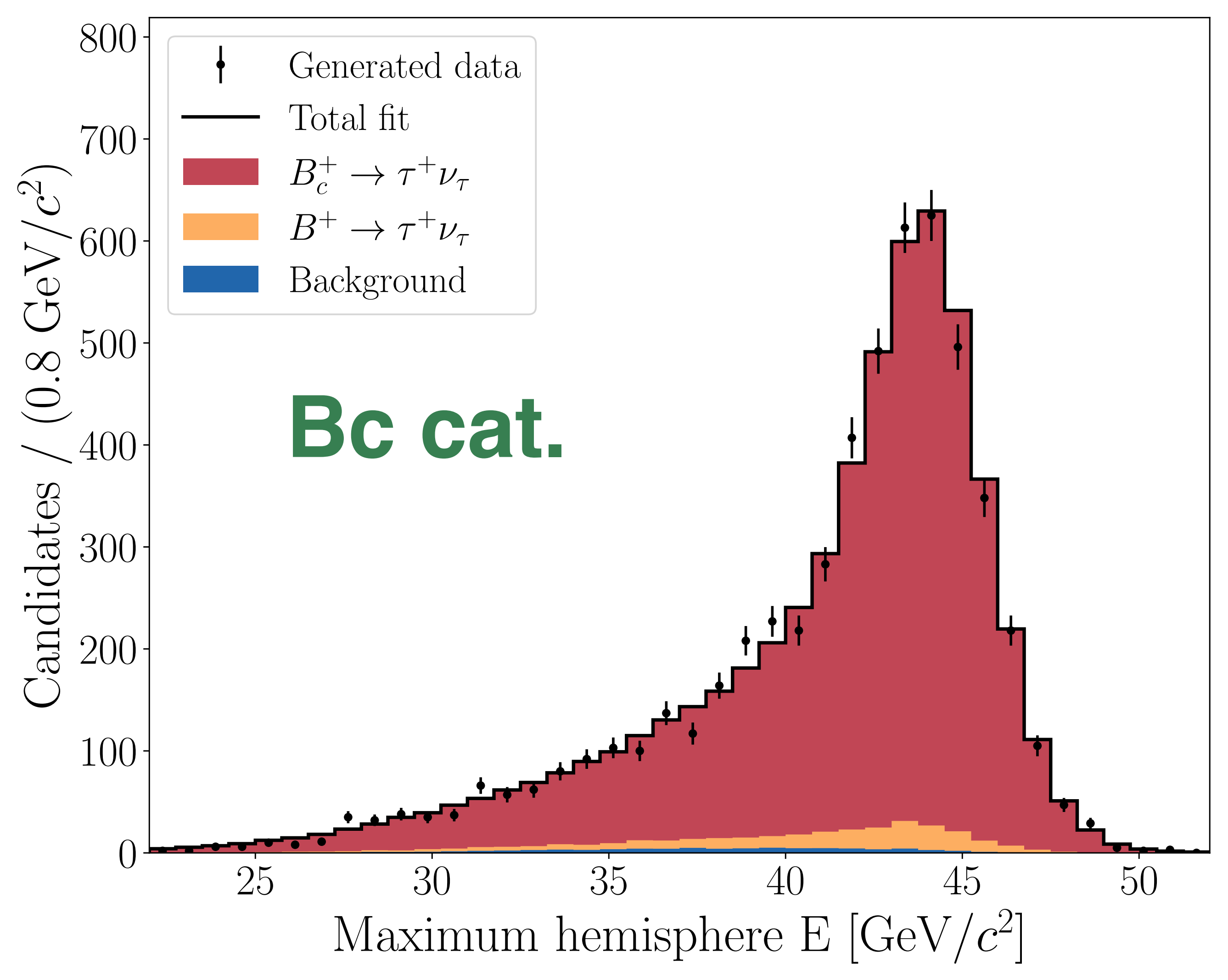
	Bc selection
Exp. Bc events	**4295**
Exp. Bu events	**285**
Exp. bkg events	448

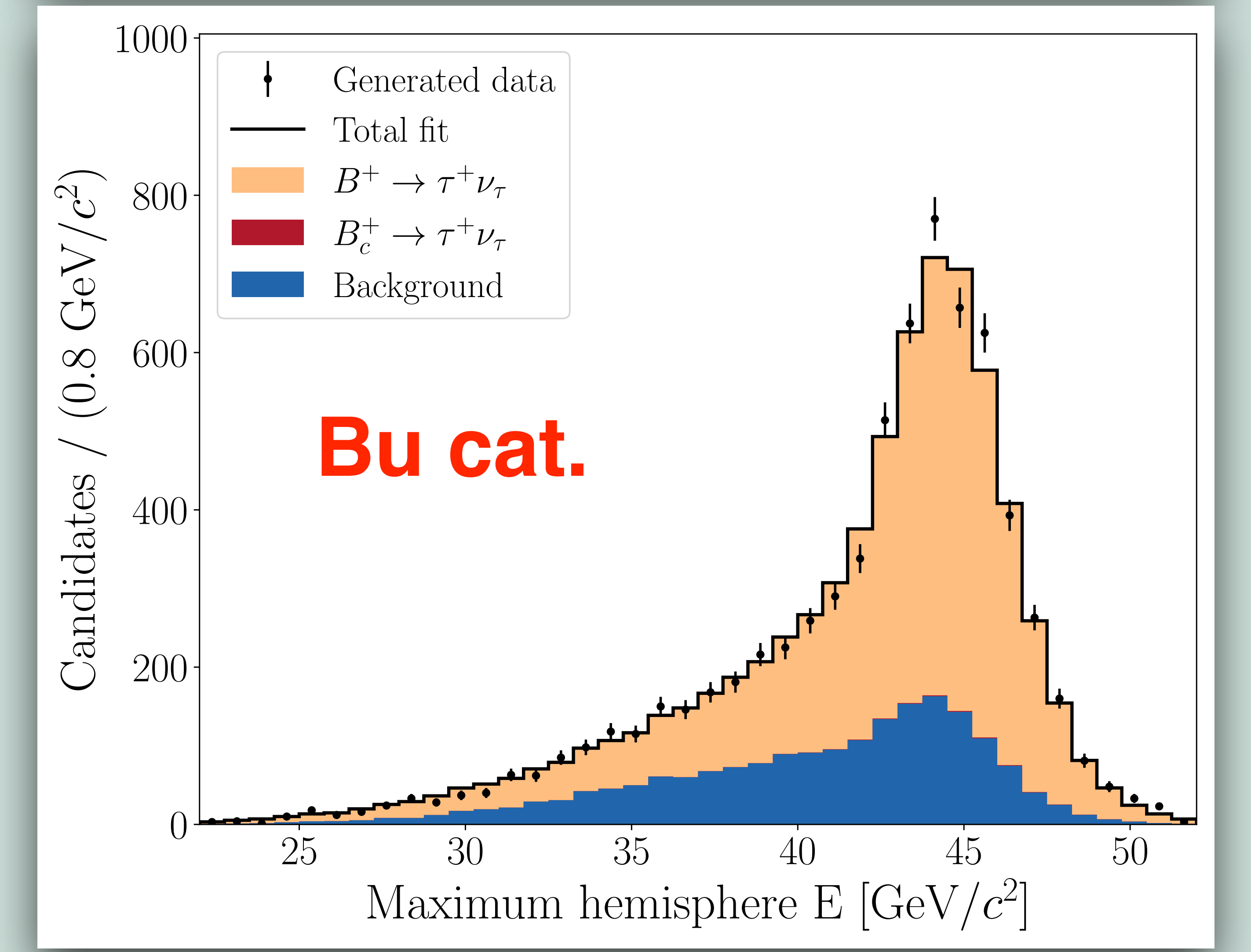
- $\sigma(\mu_{bc})$ = 2.4%

$B^+/B_c^+ \to \tau^+\nu_\tau$: last iteration

	Bc category	Bu category
Exp. Bc events	**5002.2**	**11.14**
Exp. Bu events	**264.6**	**5115.9**
Exp. bkg events	190.4	1806.0

- $\sigma(\mu_{bc})$ = 2.2%
- $\sigma(\mu_{bu})$ = 3.9%





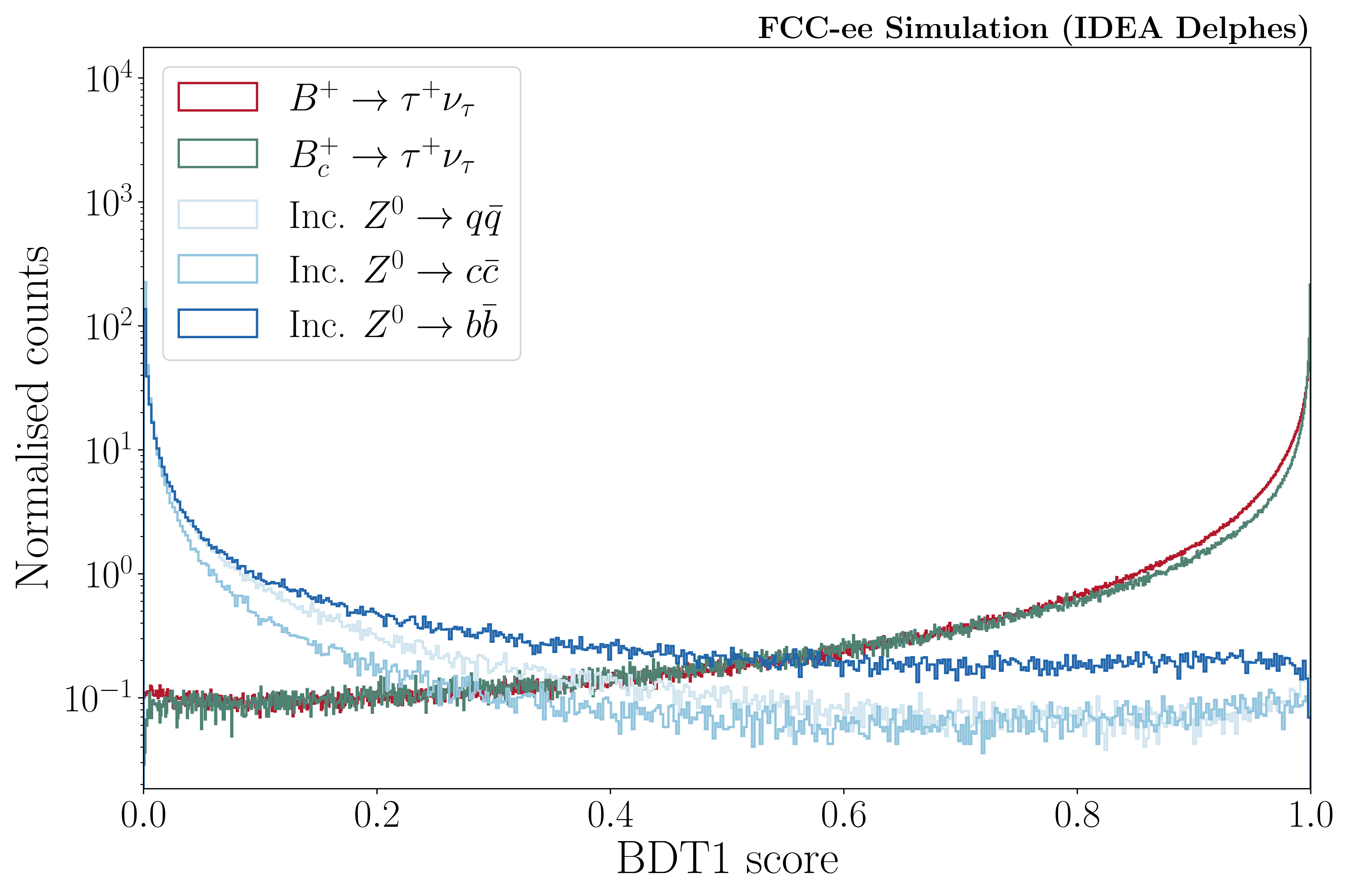
Major changes in analysis

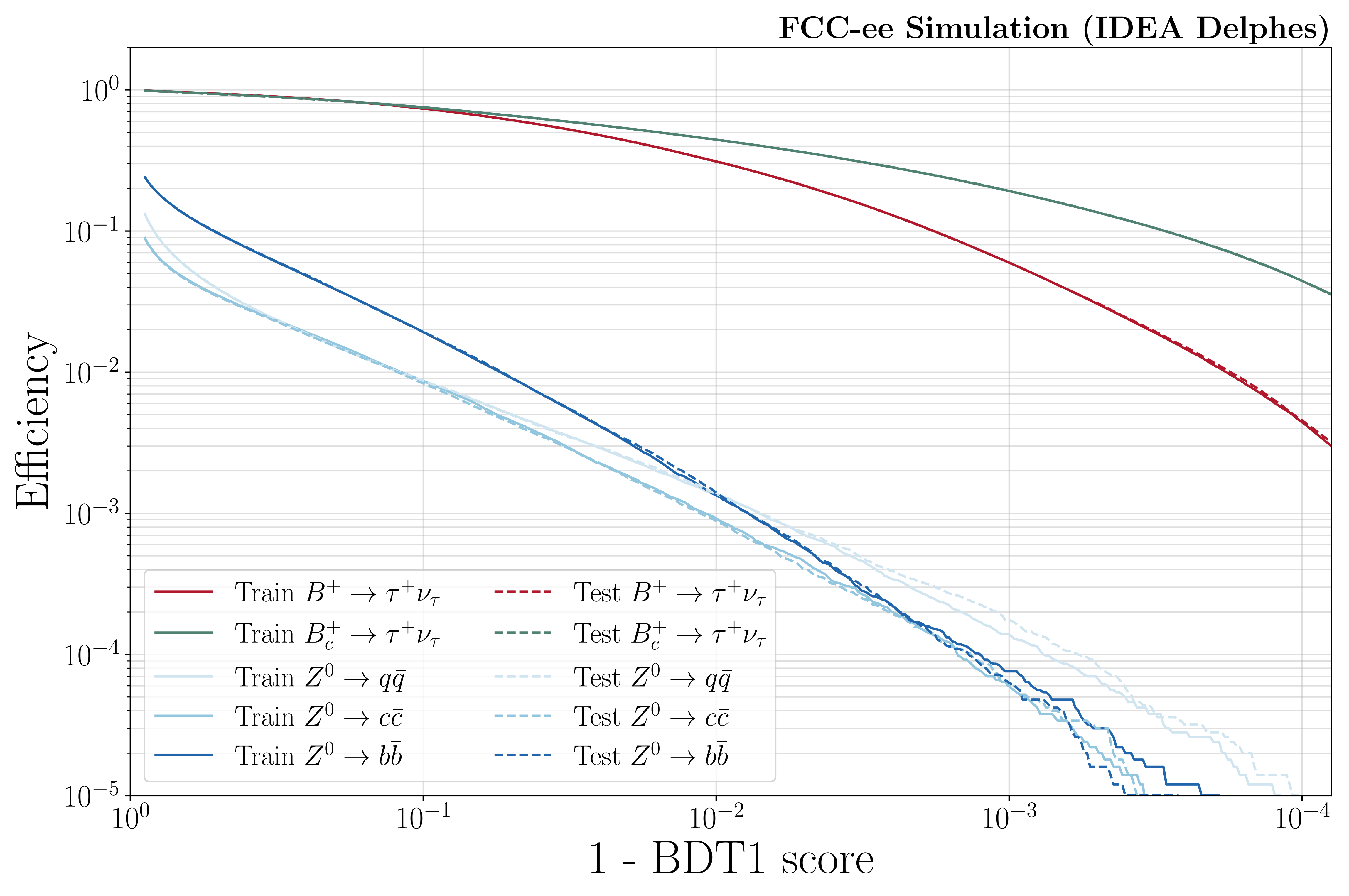
with respect to previous iteration

- Validation for MVA overtraining
- Additional exclusive samples
- Procedure for background yield and shape estimate
- Estimate of systematic uncertainty impacts

Stage 1 performance

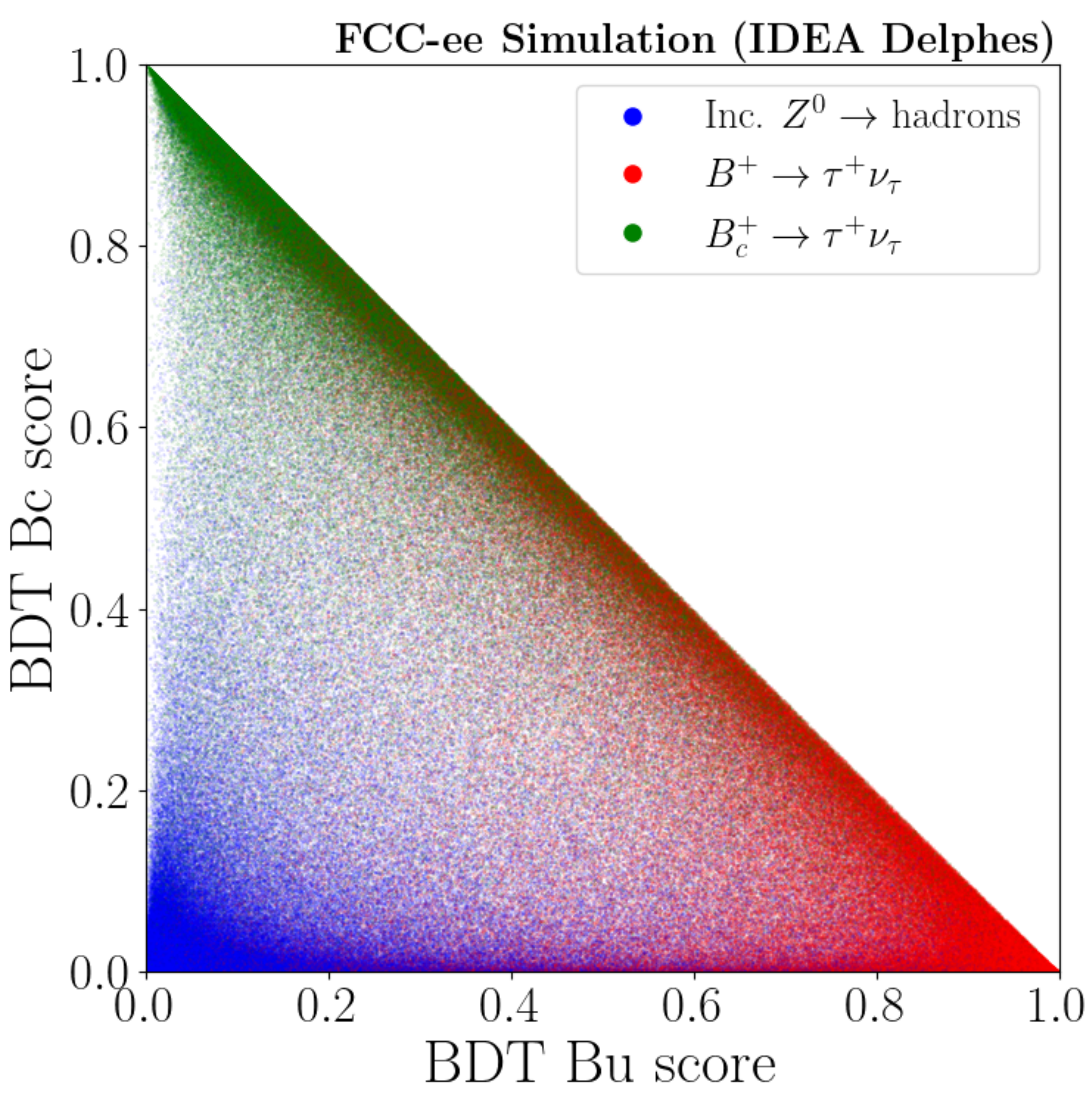
- No significant change in the training configuration.
- Only addition is the performance on testing samples

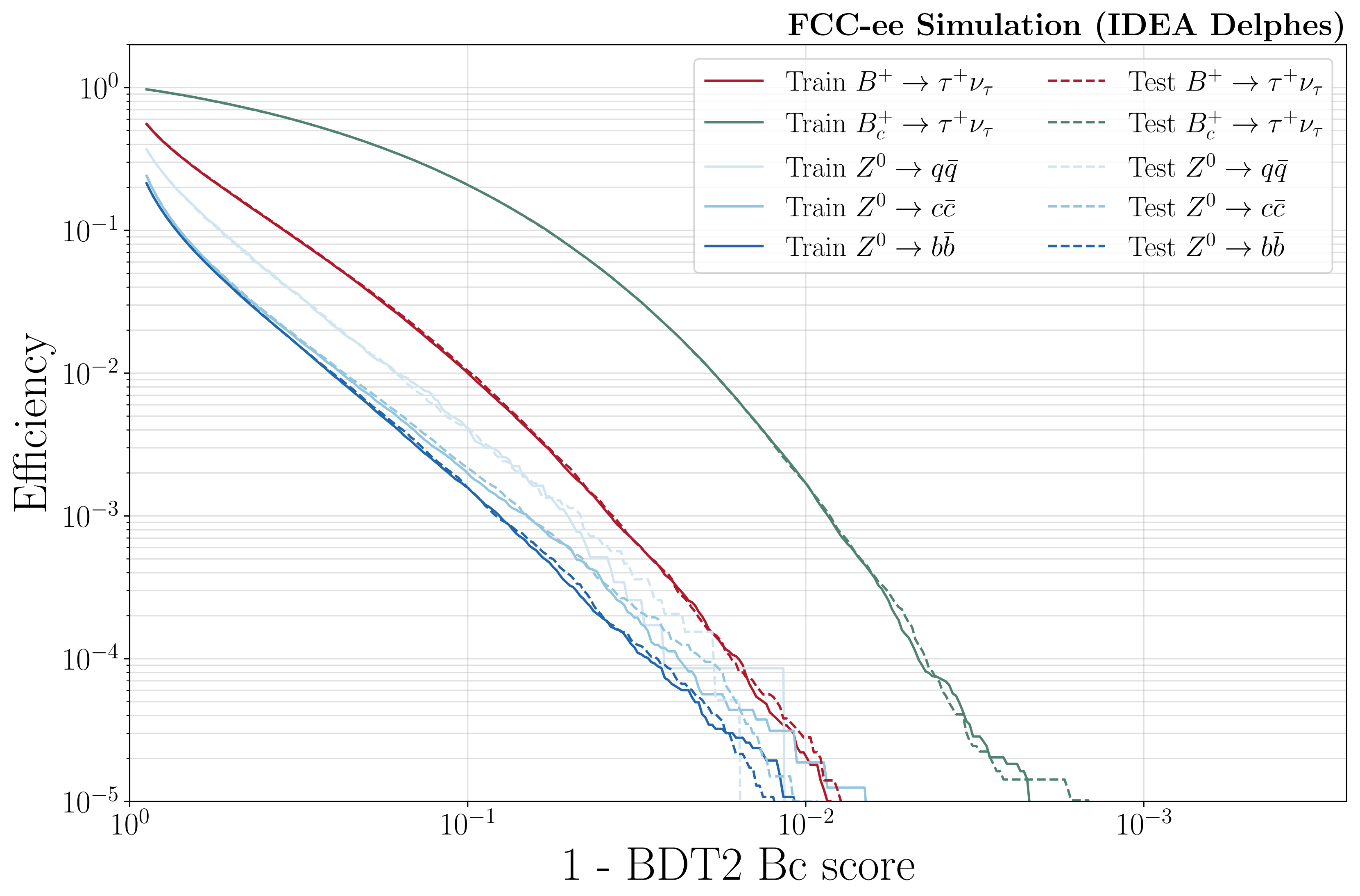


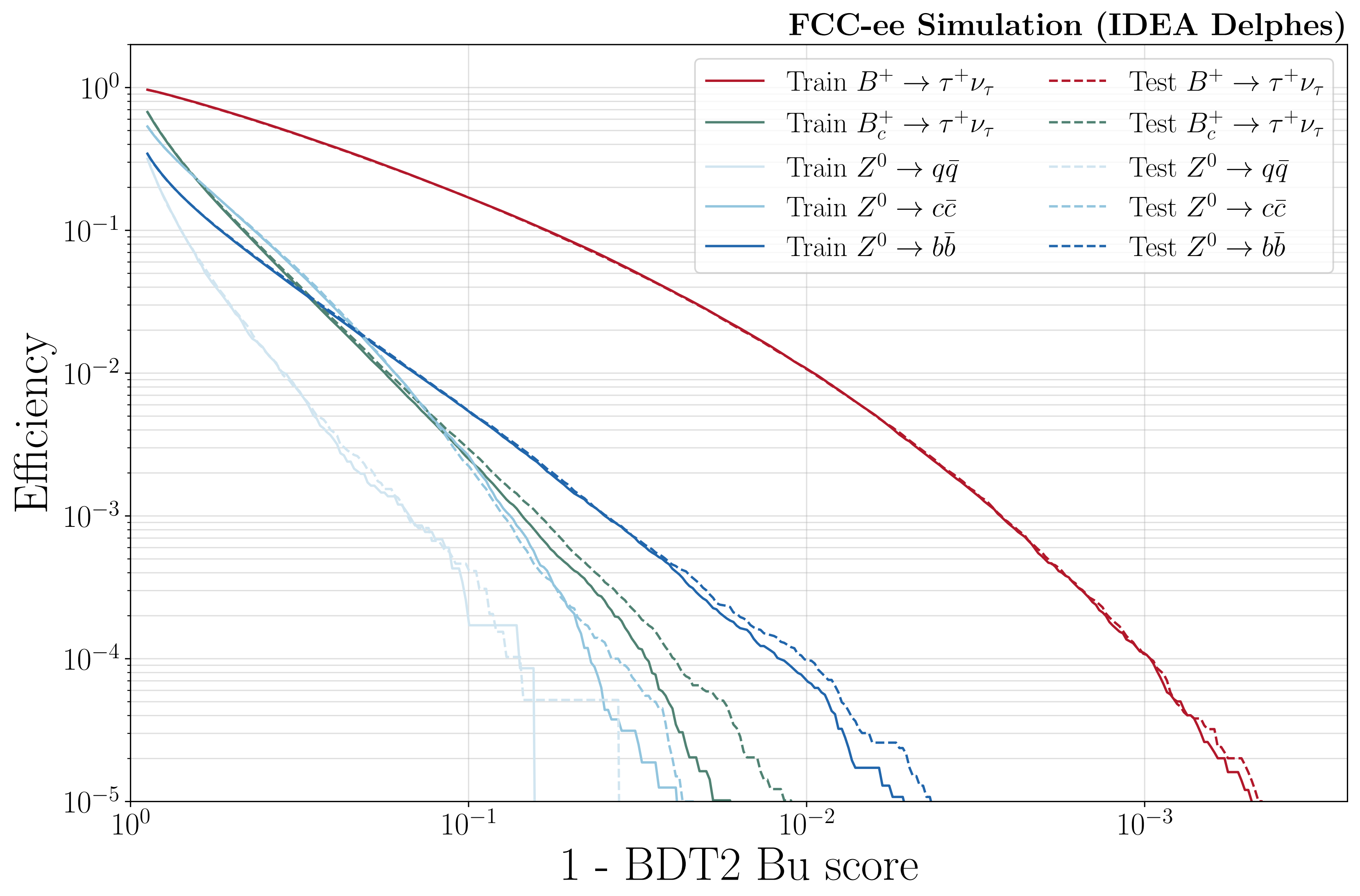


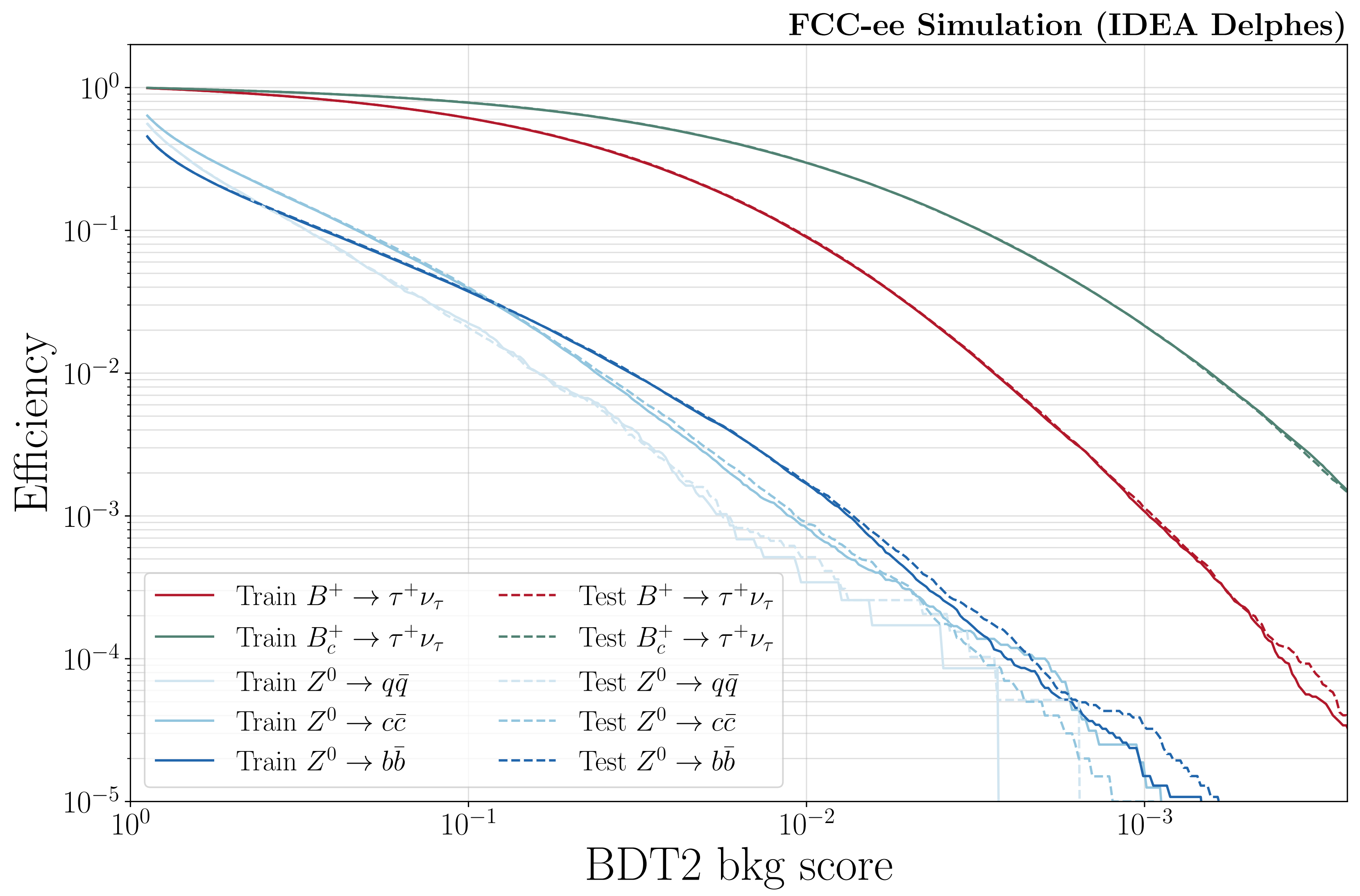
Stage 2 performance

- No significant change in the training configuration.
- Only addition is the performance on testing samples









Exclusive samples

$Z \to b\bar{b}$

Only additional ones shown here

(Original set attached in backup)

Decay	Number of events
$B^0 \to D^- e^+ \nu_e$	1×10^8
$B^0 \to D^{*-} e^+ \nu_e$	1×10^8
$B^0 \to D^- \mu^+ \nu_\mu$	1×10^8
$B^0 \to D^{*-} \mu^+ \nu_\mu$	1×10^8
$B^+ \to \bar{D}^0 e^+ \nu_e$	1×10^8
$B^+ \to \bar{D}^{*0} e^+ \nu_e$	1×10^8
$B^+ \to \bar{D}^0 \mu^+ \nu_\mu$	1×10^8
$B^+ \to \bar{D}^{*0} \mu^+ \nu_\mu$	1×10^8
$B_s^0 \to D_s^- e^+ \nu_e$	1×10^8
$B_s^0 \to D_s^{*-} e^+ \nu_e$	1×10^8
$B_s^0 \to D_s^- \mu^+ \nu_\mu$	1×10^8
$B_s^0 \to D_s^{*-} \mu^+ \nu_\mu$	1×10^8
$\Lambda_b^0 \to \Lambda_c^- e^+ \nu_e$	1×10^8
$\Lambda_b^0 \to \Lambda_c^{*-} e^+ \nu_e$	1×10^8
$\Lambda_b^0 \to \Lambda_c^- \mu^+ \nu_\mu$	1×10^8
$\Lambda_b^0 \to \Lambda_c^{*-} \mu^+ \nu_\mu$	1×10^8

$Z \to c\bar{c}$

$Z \to c\bar{c}$ bkg considered negligible in the Bc paper and no exclusive sample was generated

Decay	Number of events
$D^+ \to \tau^+ \nu_\tau$	1×10^8
$D^+ \to K^0 \pi^+ \pi^+ \pi^-$	1×10^8
$D_s^+ \to \tau^+ \nu_\tau$	1×10^8
$D_s^+ \to \rho^+ \eta'$	1×10^8
$\Lambda_c^+ \to e^+ \nu_e$	1×10^8
$\Lambda_c^+ \to \mu^+ \nu_\mu$	1×10^8
$\Lambda_c^+ \to \Lambda^0 \rho^0 \pi^+$	1×10^8
$\Lambda_c^+ \to \Sigma^+ \pi^+ \pi^-$	1×10^8

Decays of c-hadrons are complicated

Modes of $D^+ \to K^0 3\pi$, $D_s^+ \to \rho^+ \eta'$ etc,

are chosen to model generic hadronic decays

Background estimate

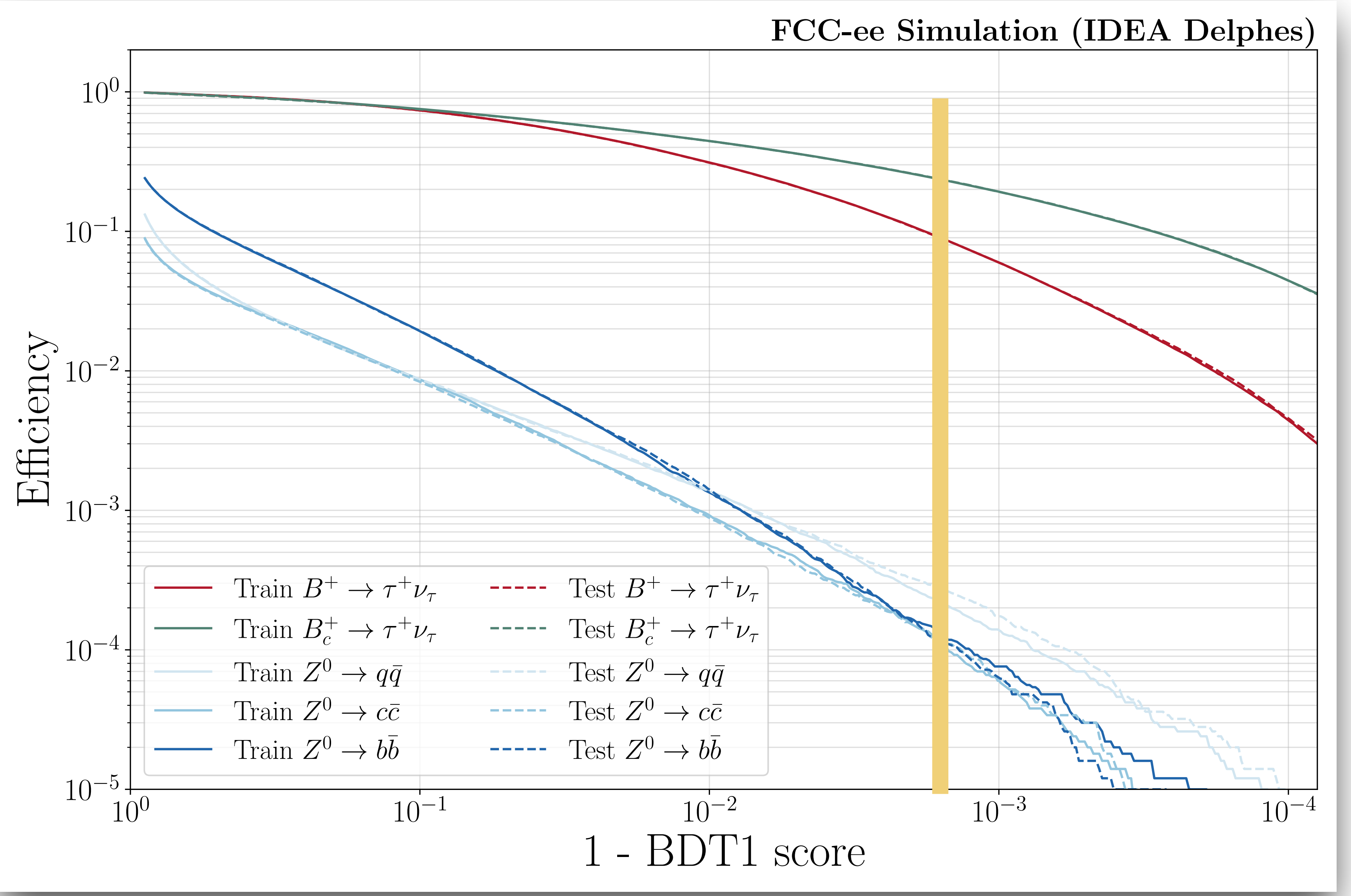
Context:

- The final selection is made on 2 BDTs (1-D cut on BDT1, 2-D cut on BDT2)
- In the final selection, backgrounds are rejected at the level of 10^{-10}. The inclusive samples $(10^9$ events each) are not enough to estimate the final background yields.
- Estimate approach similar to the Bc paper, with many details reformed

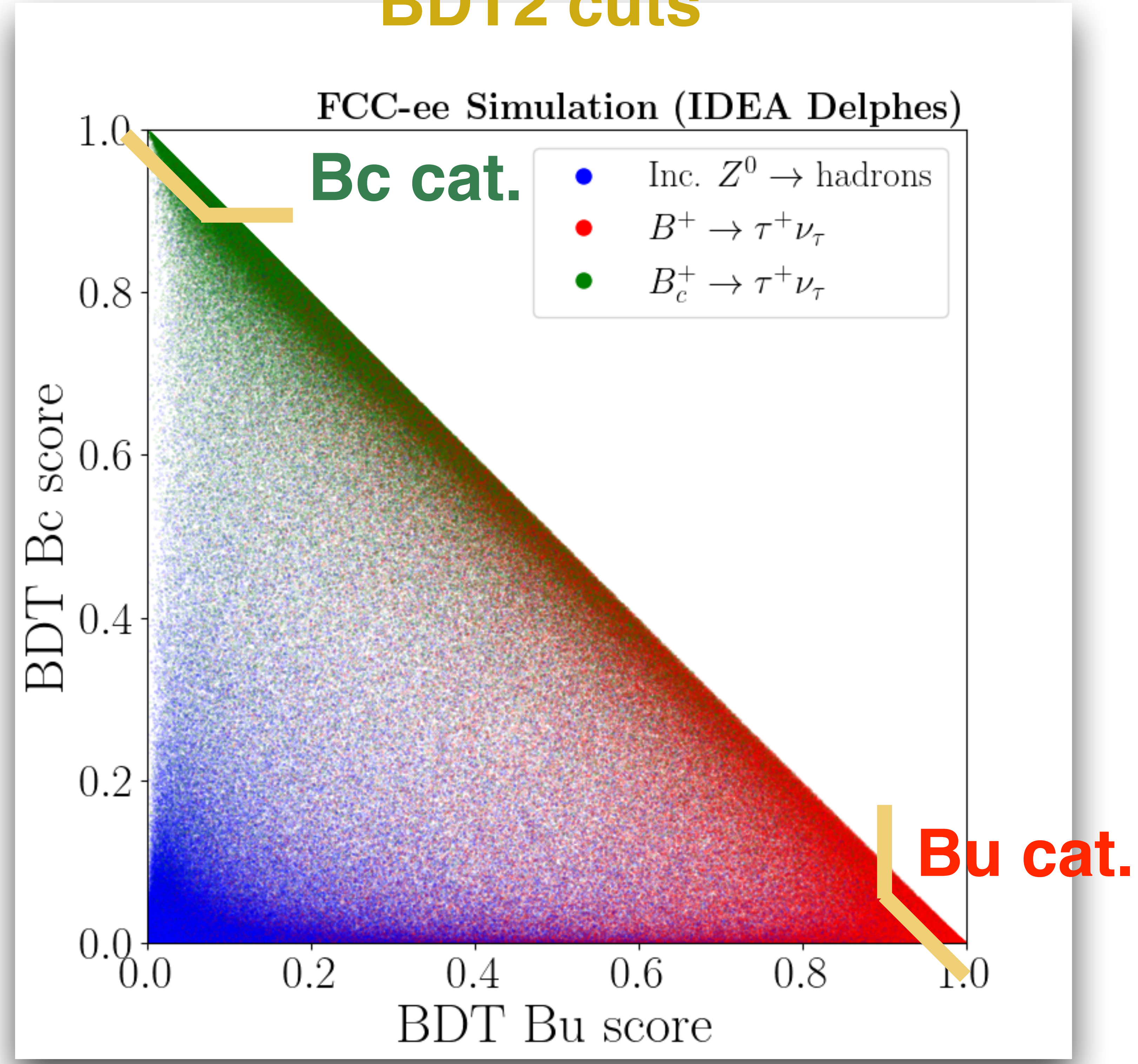
Approach:

- Estimate yields at relatively loose selection with inclusive samples, and efficiencies at further selections with exclusive samples.
- Details in the next slide

BDT1 cut



BDT2 cuts



Efficiency estimate

Underlying assumptions:

- Efficiencies in exclusive samples represent the efficiency of the inclusive process
- BDT1 and BDT2 are not correlated toward the very tight BDT region

Approach:

- Estimate yields at relatively loose selection (**baseline**) with inclusive samples.
- Efficiencies at tighter selections relative to the baseline selection are compared between the inclusive and exclusive samples.
- The **tight** selection is chosen at a level where there are enough events in exclusive samples for direct efficiency estimates.
- Efficiencies further than tight selection are evaluated with exclusive samples individually for BDT1 and BDT2 with smoothed splines. The **final** selection is optimized with scans on splines.

$$\epsilon_1(\alpha) = \epsilon(\mathrm{BDT1} > \alpha \mid \text{tight selection}),$$
$$\epsilon_2(\beta,\gamma) = \epsilon(\mathrm{BDT2}_{\mathrm{sig}} > \beta, \mathrm{BDT2}_{\mathrm{bkg}} < \gamma \mid \text{tight selection}),$$
$$\epsilon_{tot}(\alpha,\beta,\gamma) = \epsilon_1(\alpha) \times \epsilon_2(\beta,\gamma)$$

Efficiency validations

Efficiencies in agreement between inclusive and exclusive samples

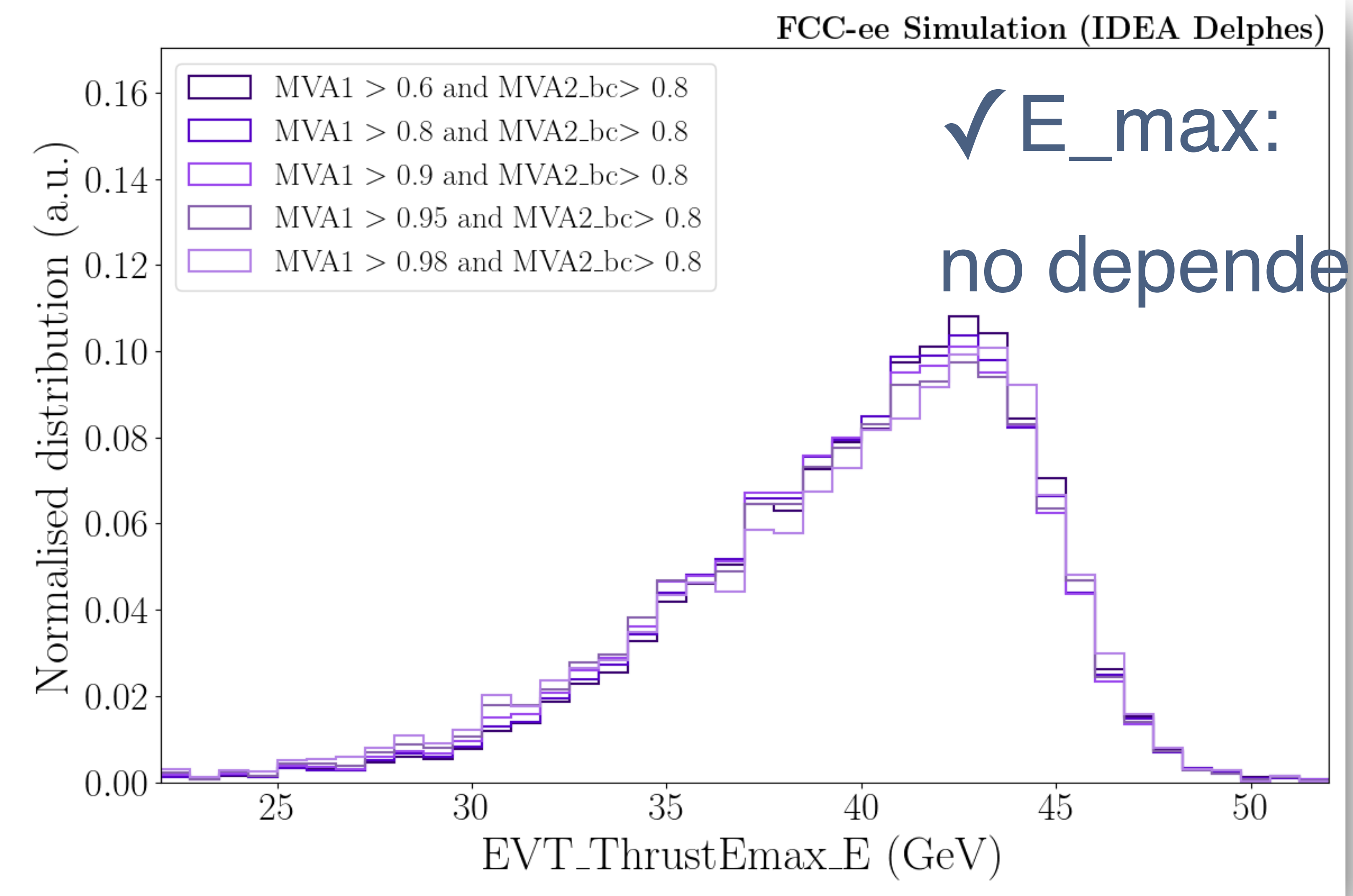
- Can trust exclusive samples for further estimates

Example of efficiencies for BDT2 cuts

(relative to baseline selection)

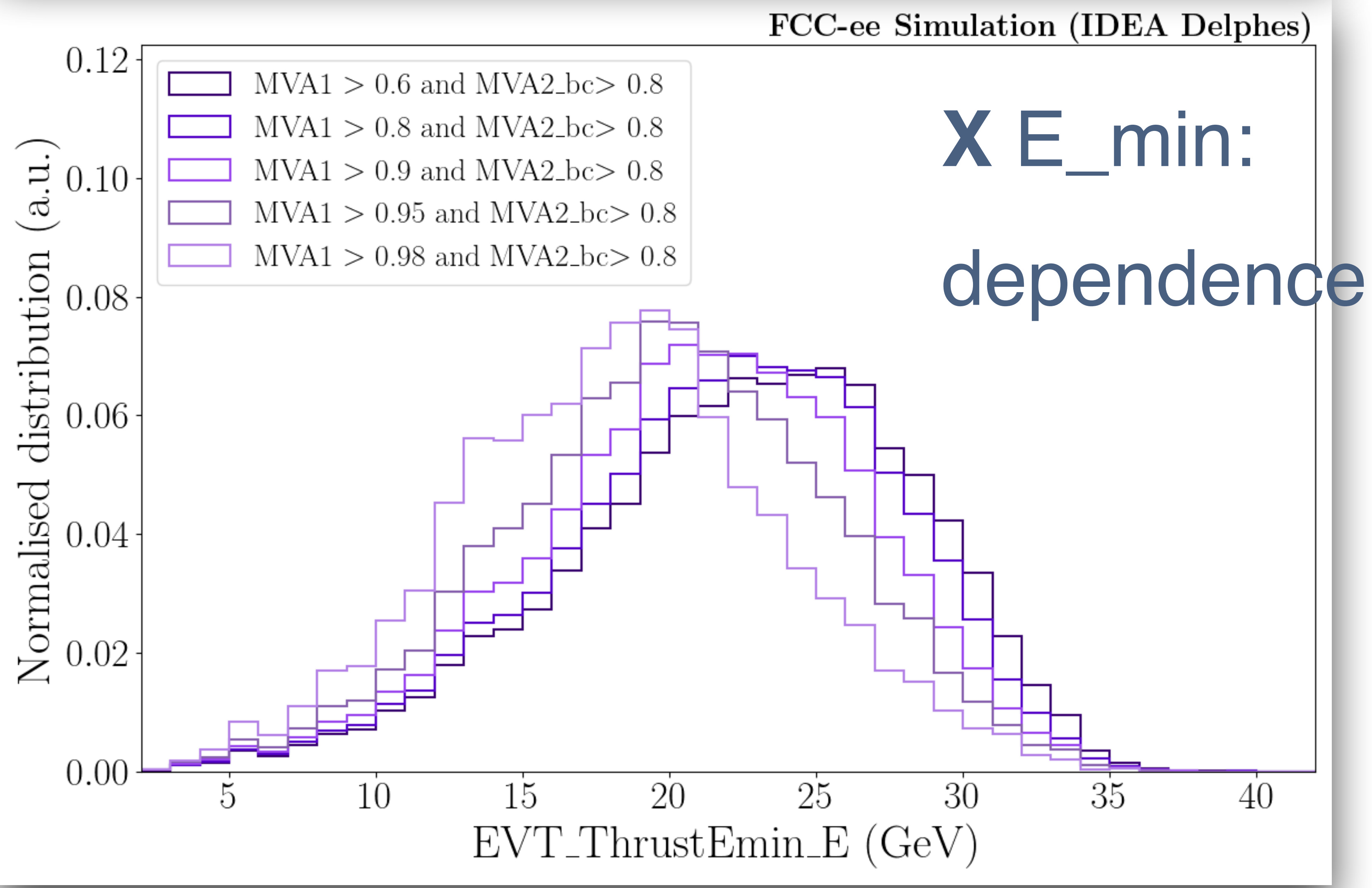
Sample	MVA2 Bc > 0.9	MVA2 Bc > 0.95	MVA2 Bc > 0.98
Zbb incl.	25.9%	6.63%	0.847%
Zbb excl.	24.4%	5.78%	0.774%
Zcc incl.	27.6%	7.14%	0.893%
Zcc excl.	27.1%	7.59%	0.895%

Fit variable



✓ E_max:

no dependence on BDT



X E_min:

dependence on BDT

Final selections

Final selections are decided by scanning splines to find the combination that achieves the best signal purity.

Final choice of selections

	Bc cat	Bu cat
Baseline	BDT1 > 0.9 BDT2_Bc > 0.8	BDT1 > 0.9, BDT2_Bu > 0.8
tight	BDT1 > 0.98, BDT2_Bc > 0.9	BDT1 > 0.98, BDT2_Bu > 0.9
Final BDT1	BDT1 > 0.99988	BDT1 > 0.99961
Final BDT2	BDT2_bkg < 0.0028	BDT2_bkg < 0.0132

Selection efficiency at each step

Selection stage	Bc category		Bu category	
	$Z \to b\bar{b}$	$Z \to c\bar{c}$	$Z \to b\bar{b}$	$Z \to c\bar{c}$
Baseline selection	9.28×10^{-6}	2.58×10^{-6}	3.01×10^{-5}	5.64×10^{-6}
Tight selection	1.15×10^{-1}	1.57×10^{-1}	1.09×10^{-1}	6.77×10^{-2}
Final selection BDT1	5.21×10^{-3}	2.41×10^{-2}	6.74×10^{-4}	8.42×10^{-2}
Final selection BDT2	6.93×10^{-2}	3.16×10^{-2}	7.62×10^{-2}	1.44×10^{-4}
Total	3.88×10^{-10}	3.07×10^{-10}	1.68×10^{-10}	4.65×10^{-12}

Note: the scan for BDT2 is a 2D scan, but the final decision touches the boundary of BDT2 sig > 0.9

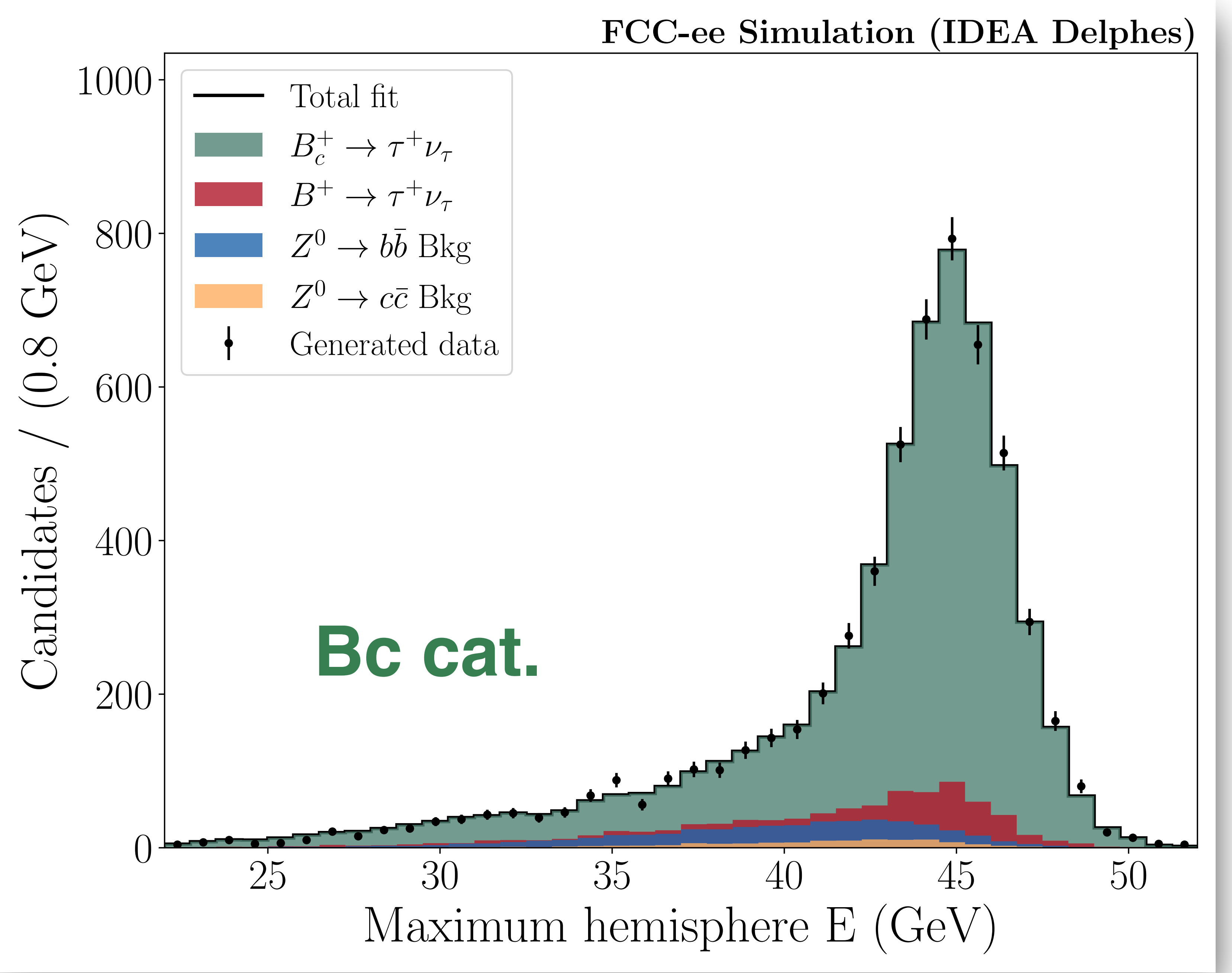
Final yields

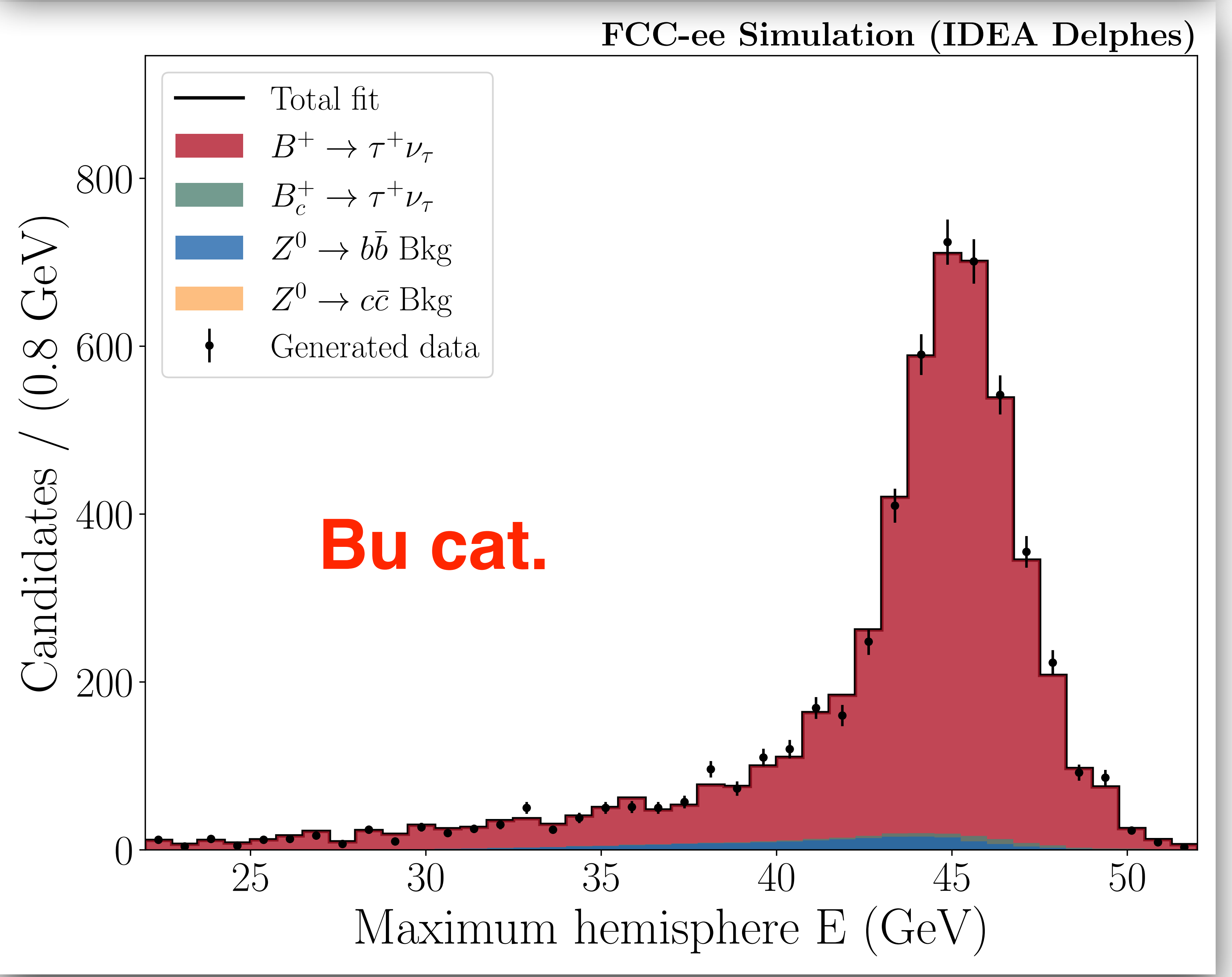
During the 1D⊗2D scan,

- Bkg eff can become 0 with very tight cuts. Required O(10) remaining bkg MC events to avoid over-aggressive estimates.
- There are usually a few local minima with similar signal purities.

(The yields listed correspond to the cuts in the previous slide. Similar yields can be achieved with some other cut choices.)

Process	Bc category	Bu category
$N(B_c^+ \to \tau^+\nu_\tau)$	5110.0	51.8
$N(B^+ \to \tau^+\nu_\tau)$	370.8	5051.2
$N(Z \to b\bar{b})$	293.0	127.4
$N(Z \to c\bar{c})$	184.6	2.8





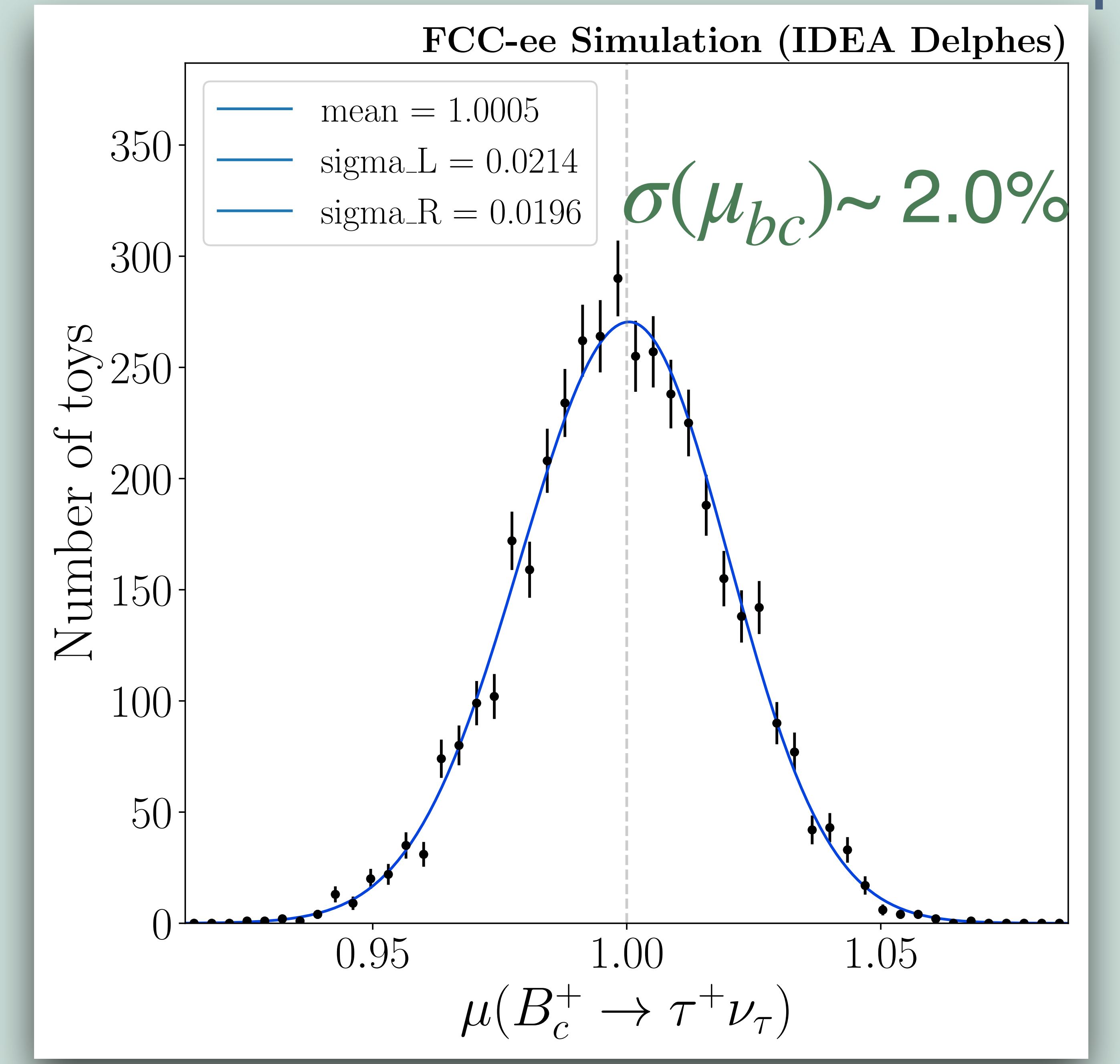
Fit approach

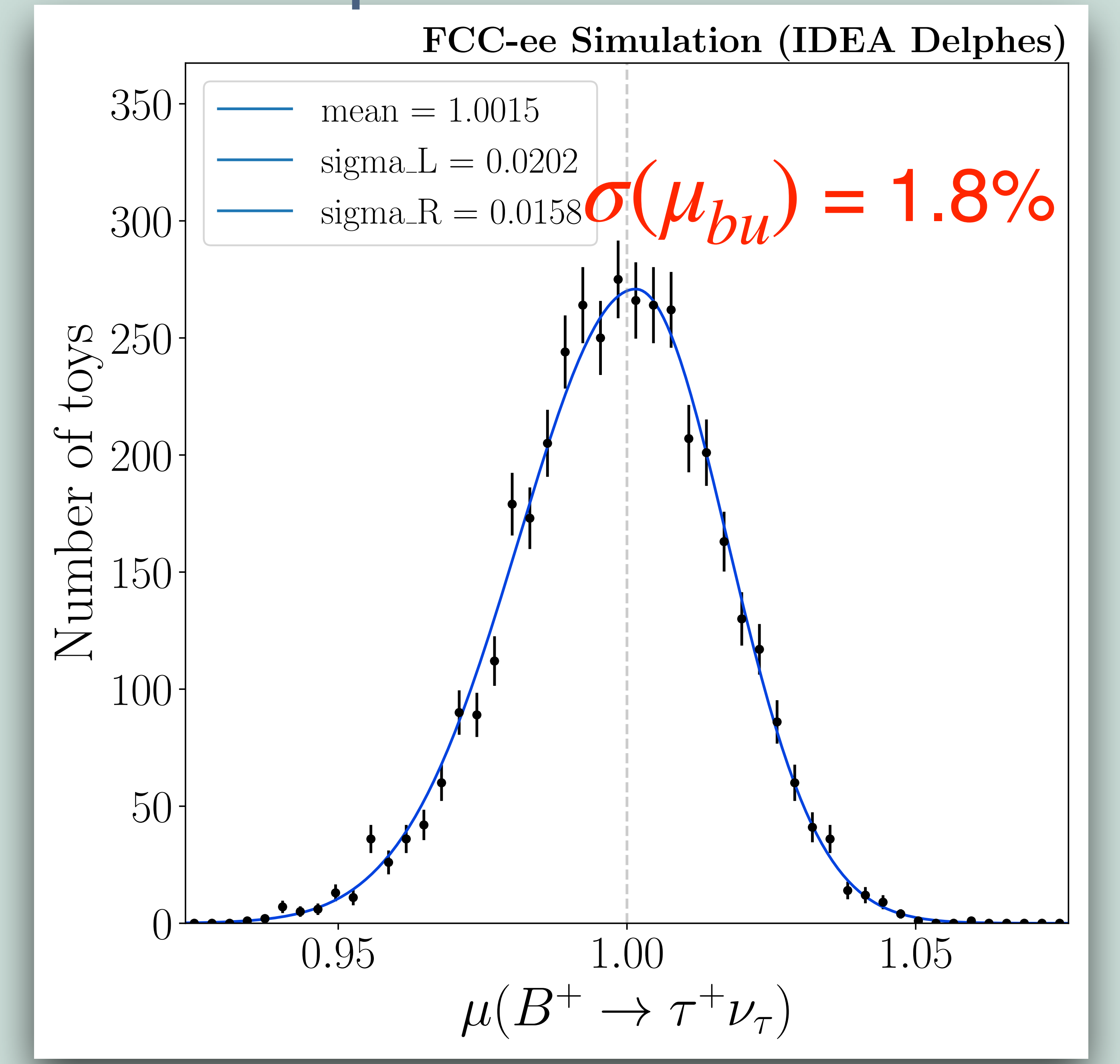
Binned maximum likelihood fit, 6 strength modifiers

- Sig strengths correlated across categories, bkg strengths uncorrelated
- Sig strengths fully floating, bkg strengths floating with a lognormal penalty (corresponding to having an uncertainty equal to the expected yield)

	Bc cat	Bu cat
$\mu(B_c^+)$	✔	✔
$\mu(B^+)$	✔	✔
$\mu_{Bc}(Zbb)$	✔	
$\mu_{Bu}(Zbb)$		✔
$\mu_{Bc}(Zcc)$	✔	
$\mu_{Bu}(Zcc)$		✔

Tested with 4000 pseudo-experiments

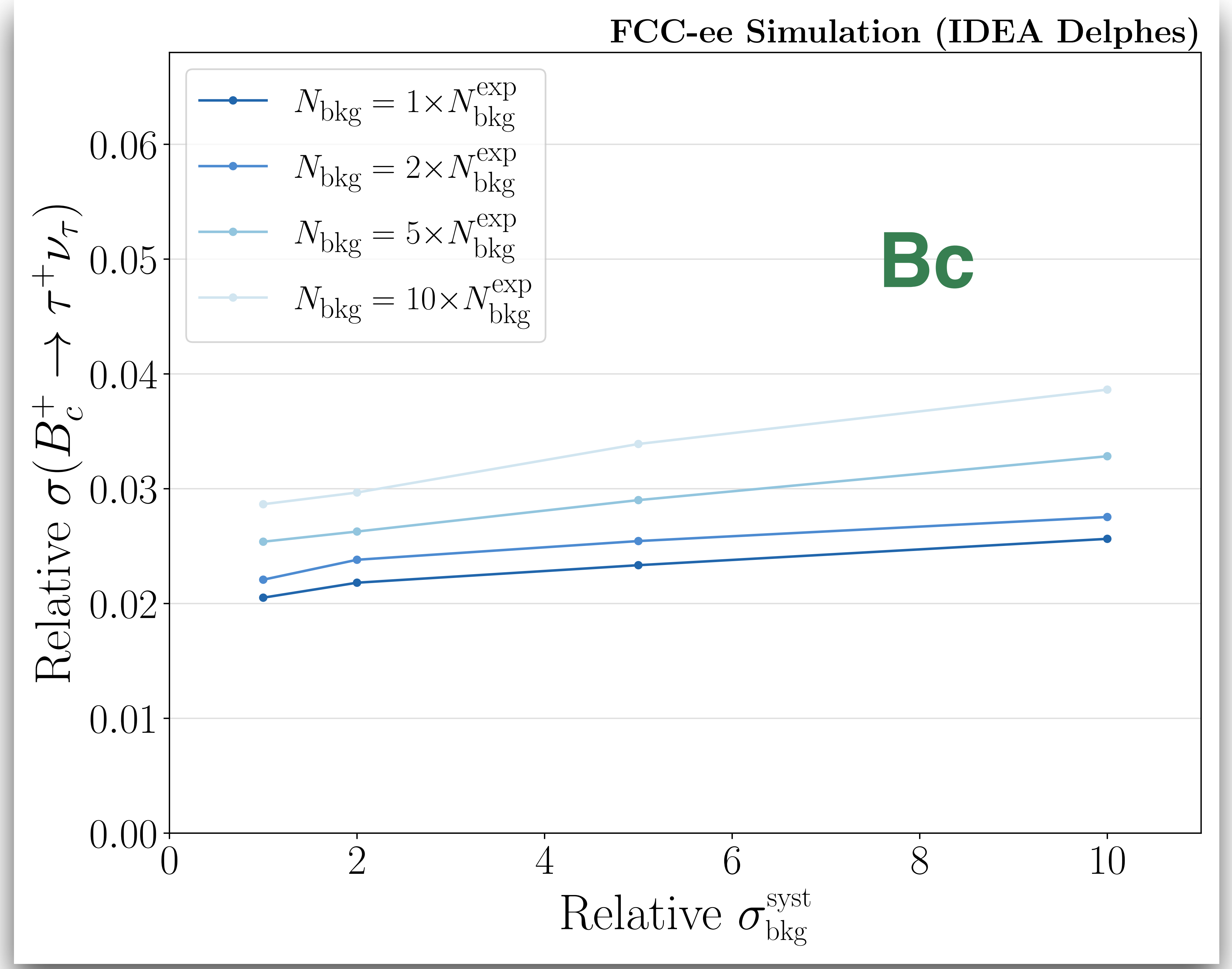


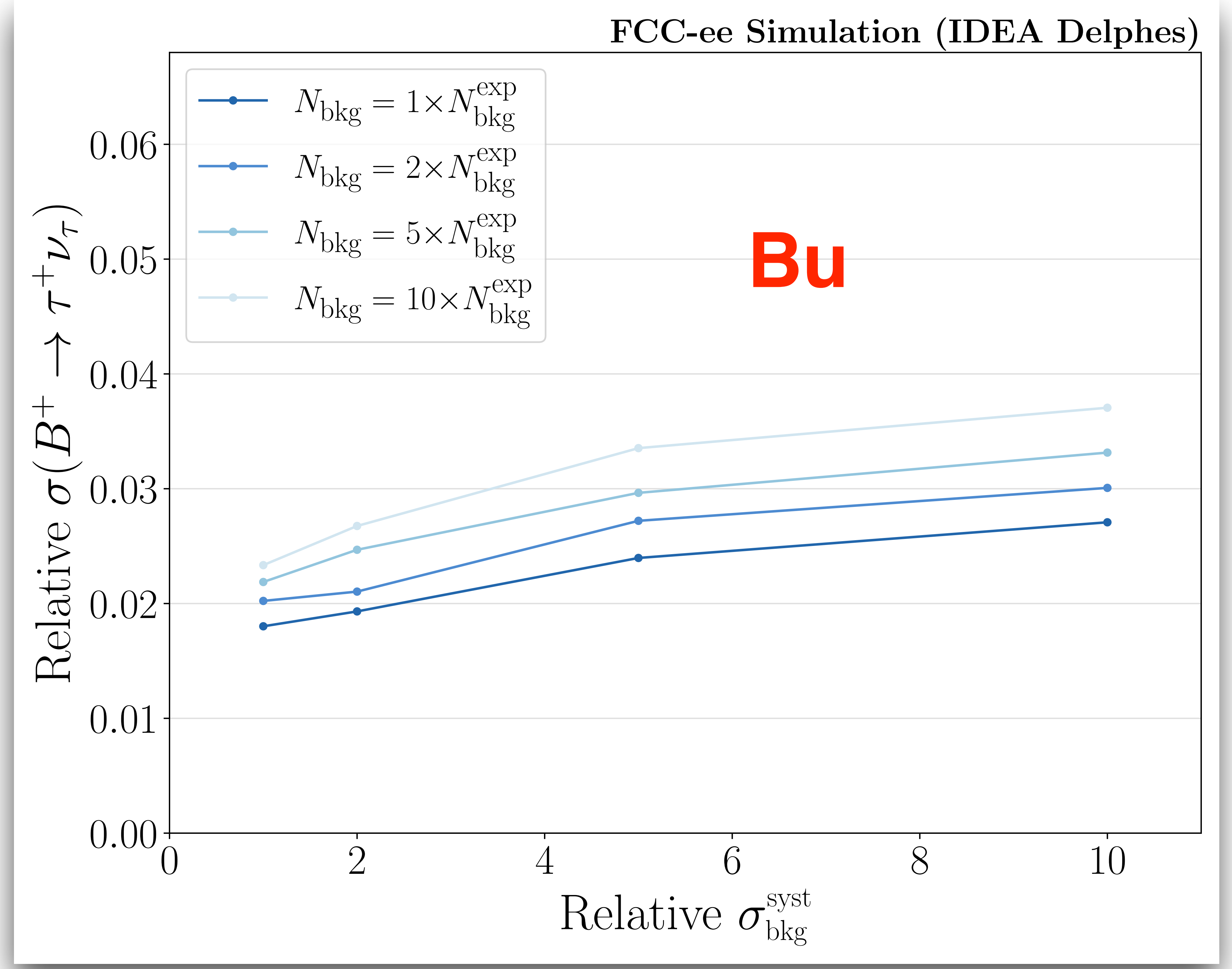


Estimates of uncertainties and inflated backgrounds

Background estimate relies on assumptions. Consider two impacts

- Bkg inflation: an exaggeration of expected bkg yields. Take scenarios of $N_{\text{bkg}} = [1,2,5,10] \times N_{\text{bkg}}^{\text{exp}}$
- Bkg uncertainty: a random fluctuation on bkg yields. Take scenarios of $\sigma_{\text{bkg}}^{\text{syst}} = [1,2,5,10]$, where $\sigma_{\text{bkg}}^{\text{syst}}$ is a lognormal uncertainty on the bkg yield, relative to the expected yield.





Further plans

Ideas of interpretation

- Constrains on BSM models, as done in Bc paper
  - Theory group at KIT is interested and available
  - Bu signal, can be directly used for interpretations
  - Bc is better measured with a normalization mode, to decouple from $f(B_c^+)$
    - $B_c^+ \to J/\psi\mu^+\nu_\mu$ was used before, maybe consider other modes?
- Measurements on CMK elements
  - S. Monteil agreed to help
  - Need LQCD inputs for $f(B_c^+)$

More to be discussed and arranged with FCC conveners and other collaborators

Toward publication

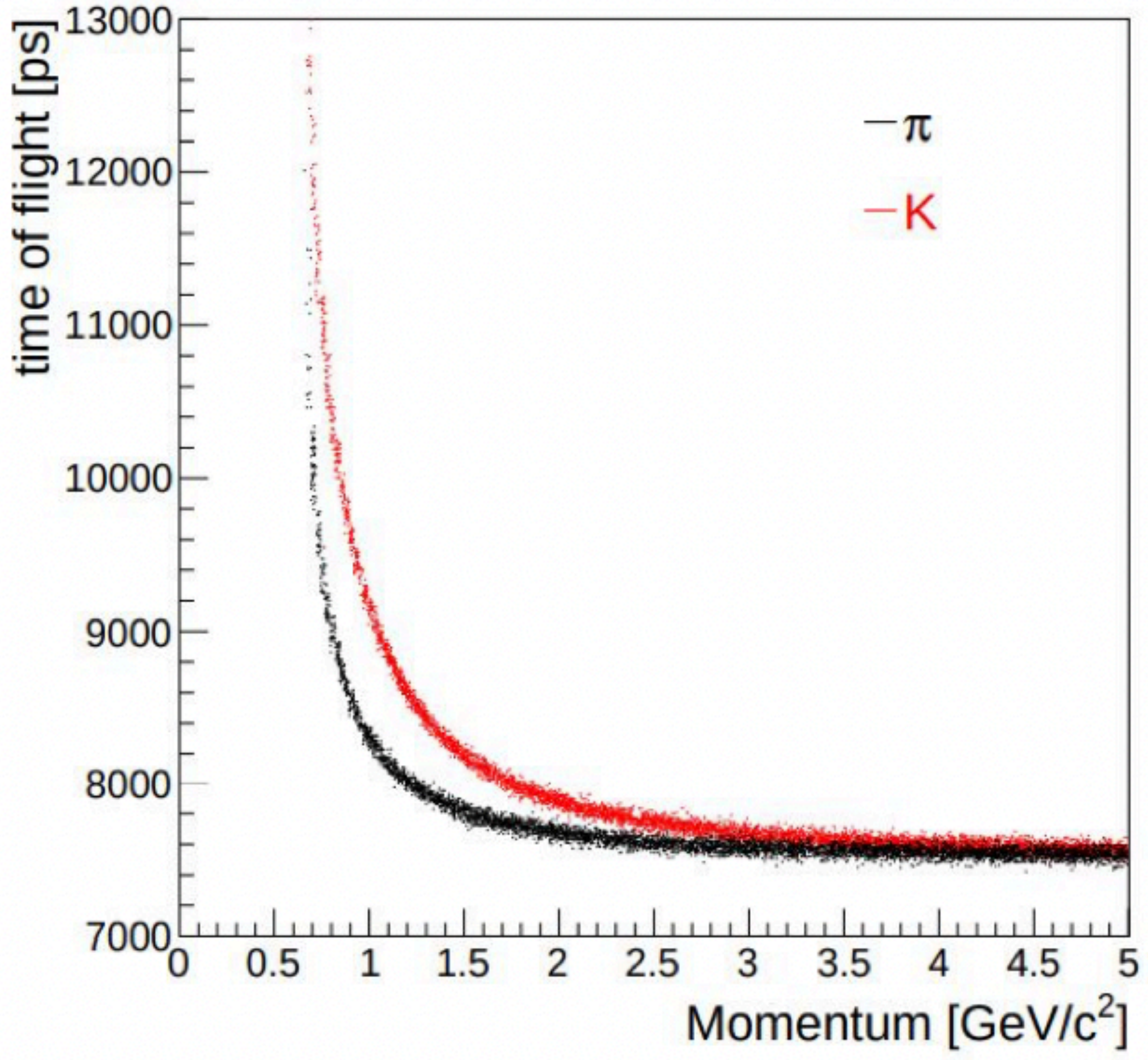
- Analysis results ready
  - One small thing to test (slide 15): the final selection touches the boundary of BDT2 sig cut, relaxing this cut may improve final sensitivity further.
    - Only impacts the sensitivity results. Does not change strategy.
    - Can be updated in short terms
  - All current results fully documented
- Theory results to be added
  - In parallel, prepare full paper draft (introduction, FCC description etc.)

Backup

Kaon vs Pion ID

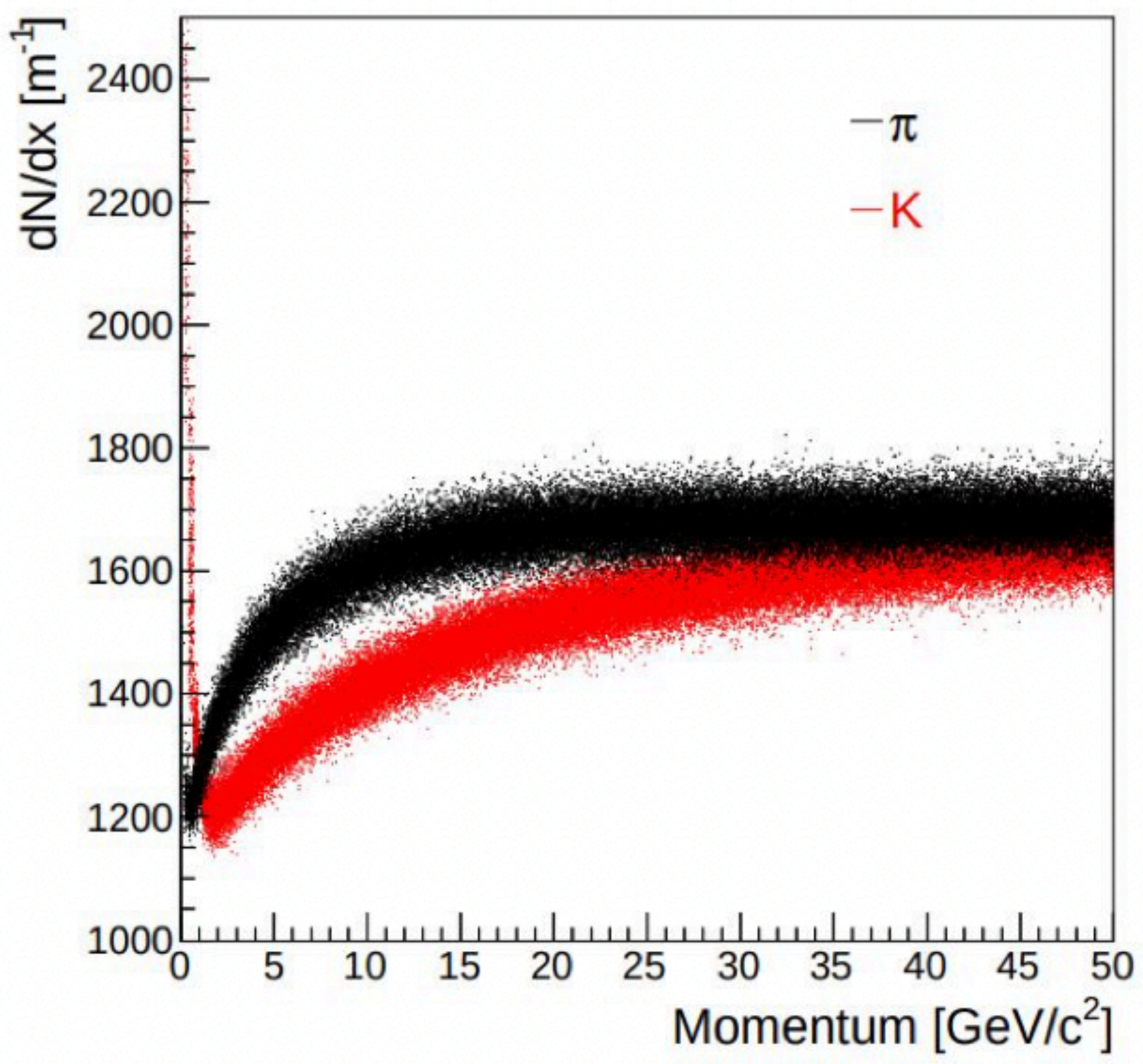
Time of Flight

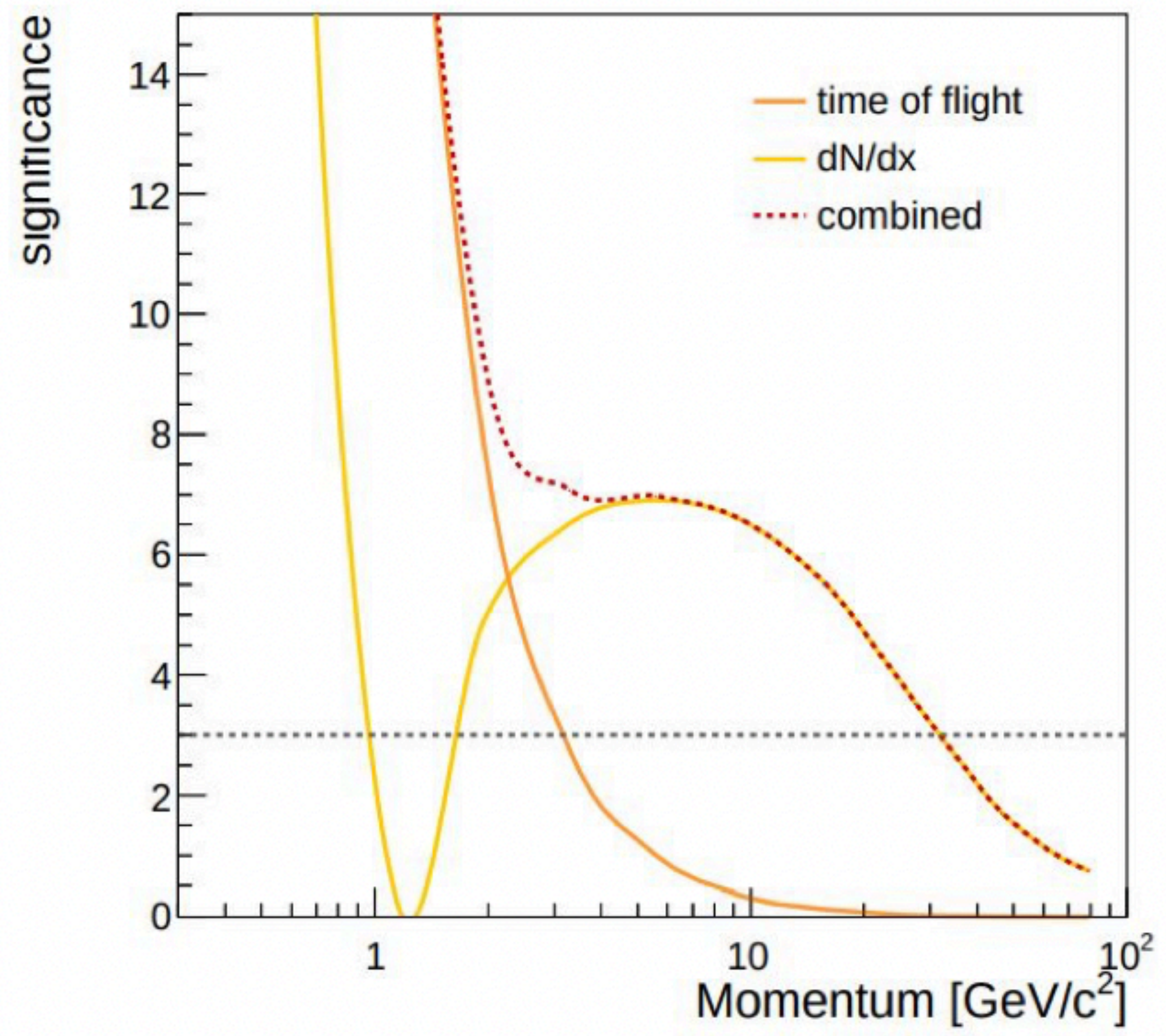
- Good K/π separation at low momenta



Cluster Counting

- Count number of primary ionization clusters along track path





3-sigma separation for tracks with **p < 30GeV**

Assume perfect ID in the kinematic region (p < 30 GeV) of study.

Theory predictions

$$\mathcal{B}(B_c^+ \to \tau^+\nu_\tau)^{\text{SM}} = \tau_{B_c} \frac{G_F^2 |V_{cb}|^2 f_{B_c}^2 m_{B_c} m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_{B_c}^2}\right)^2$$

- $|V_{cb}| = 39.09(68) \times 10^{-3}$ from $B \to D^{(*)} l\nu$
- $f_{B_c} = 427(6)$ MeV from LQCD

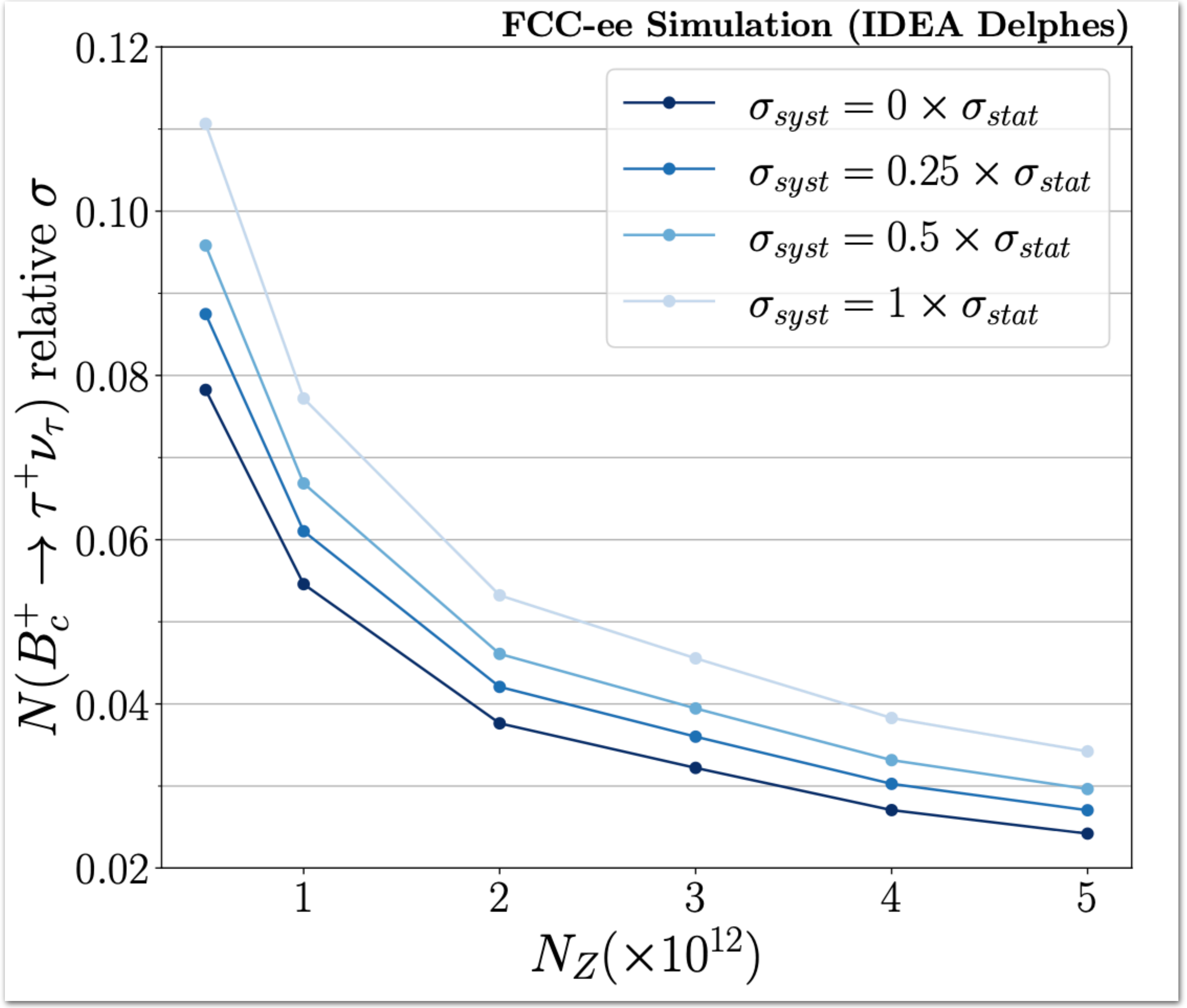
Samples

- $B_c^+ \to \tau^+\nu_\tau$ and $B^+ \to \tau^+\nu_\tau$ signals, 10M each
- Inclusive $Z \to b\bar{b}, c\bar{c}, q\bar{q}$ processes, 1B each
- Exclusive B decays backgrounds, 200M each
- All events generated with Pythia and simulated in DELPHES with IDEA detector

Decay mode	N(expected)	N(generated)	Expected / Generated	Final ϵ
$B^+ \to \bar{D}^0\tau^+\nu_\tau$	5.01×10^9	2×10^8	25.0	1.46×10^{-9}
$B^+ \to \bar{D}^{*0}\tau^+\nu_\tau$	1.22×10^{10}	2×10^8	61.1	1.1×10^{-9}
$B^+ \to \bar{D}^0 3\pi$	3.64×10^9	1.9×10^8	19.2	1.56×10^{-9}
$B^+ \to \bar{D}^{*0} 3\pi$	6.7×10^9	2×10^8	33.5	1.04×10^{-9}
$B^+ \to \bar{D}^0 D_s^+$	5.85×10^9	2×10^8	29.3	2.52×10^{-10}
$B^+ \to \bar{D}^{*0} D_s^+$	4.94×10^9	1.75×10^8	28.2	2.72×10^{-10}
$B^+ \to \bar{D}^{*0} D_s^{*+}$	1.11×10^{10}	2×10^8	55.6	2.42×10^{-10}
$B^0 \to D^-\tau^+\nu_\tau$	7.02×10^9	2×10^8	35.1	2.69×10^{-9}
$B^0 \to D^{*-}\tau^+\nu_\tau$	1.02×10^{10}	2×10^8	51.0	1.25×10^{-9}
$B^0 \to D^- 3\pi$	3.9×10^9	2×10^8	19.5	3.4×10^{-9}
$B^0 \to D^{*-} 3\pi$	4.69×10^9	2×10^8	23.4	9.84×10^{-10}
$B^0 \to D^- D_s^+$	4.68×10^9	2×10^8	23.4	3.23×10^{-10}
$B^0 \to D^{*-} D_s^+$	5.2×10^9	2×10^8	26.0	2.32×10^{-10}
$B^0 \to D^{*-} D_s^{*+}$	1.15×10^{10}	2×10^8	57.5	2.35×10^{-10}
$B_s^0 \to D_s^-\tau^+\nu_\tau$	3.53×10^9	2×10^8	17.6	3.71×10^{-9}
$B_s^0 \to D_s^{*-}\tau^+\nu_\tau$	2.35×10^9	2×10^8	11.8	2.27×10^{-9}
$B_s^0 \to D_s^- 3\pi$	8.85×10^8	2×10^8	4.4	5.53×10^{-9}
$B_s^0 \to D_s^{*-} 3\pi$	1.05×10^9	2×10^8	5.2	3.38×10^{-9}
$B_s^0 \to D_s^- D_s^+$	6.39×10^8	2×10^8	3.2	4.09×10^{-10}
$B_s^0 \to D_s^{*-} D_s^+$	2.02×10^9	2×10^8	10.1	3.17×10^{-10}
$B_s^0 \to D_s^{*-} D_s^{*+}$	2.09×10^9	2×10^8	10.5	2.56×10^{-10}
$\Lambda_b^0 \to \Lambda_c^-\tau^+\nu_\tau$	1.83×10^9	2×10^8	9.1	1.36×10^{-9}
$\Lambda_b^0 \to \Lambda_c^{*-}\tau^+\nu_\tau$	1.83×10^9	2×10^8	9.1	9.44×10^{-10}
$\Lambda_b^0 \to \Lambda_c^- 3\pi$	4.31×10^8	2×10^8	2.2	5.58×10^{-9}
$\Lambda_b^0 \to \Lambda_c^{*-} 3\pi$	4.31×10^8	2×10^8	2.2	9.21×10^{-10}
$\Lambda_b^0 \to \Lambda_c^- D_s^+$	6.15×10^8	2×10^8	3.1	3.46×10^{-10}
$\Lambda_b^0 \to \Lambda_c^{*-} D_s^+$	6.15×10^8	2×10^8	3.1	2.72×10^{-10}
$\Lambda_b^0 \to \Lambda_c^{*-} D_s^{*+}$	6.15×10^8	2×10^8	3.1	2.5×10^{-10}

Systematic uncertainty

- By design, syst. uncert. at FCC-ee is expected to be constrained to the level comparable to stat. uncert. of EW precision measurements, and not a major concern for this result.
- Current analysis relies on strong assumptions in the background estimate method. Hard to estimate the uncertainty from these assumptions.
- Consider a few scenarios $\sigma_{syst} = [0,\ 0.25,\ 0.5,\ 1.0] \times \sigma_{stat}$



$|V_{cb}|$ and $|V_{ub}|$

- By taking leptonic decay constant f_B as input, the $B_c^+ \to \tau^+\nu_\tau$ and $B^+ \to \tau^+\nu_\tau$ results can be used to determine $|V_{cb}|$ and $|V_{ub}|$
  - Clean measurement with high experimental precision
  - Theoretical uncertainty (from lattice QCD) to be studied

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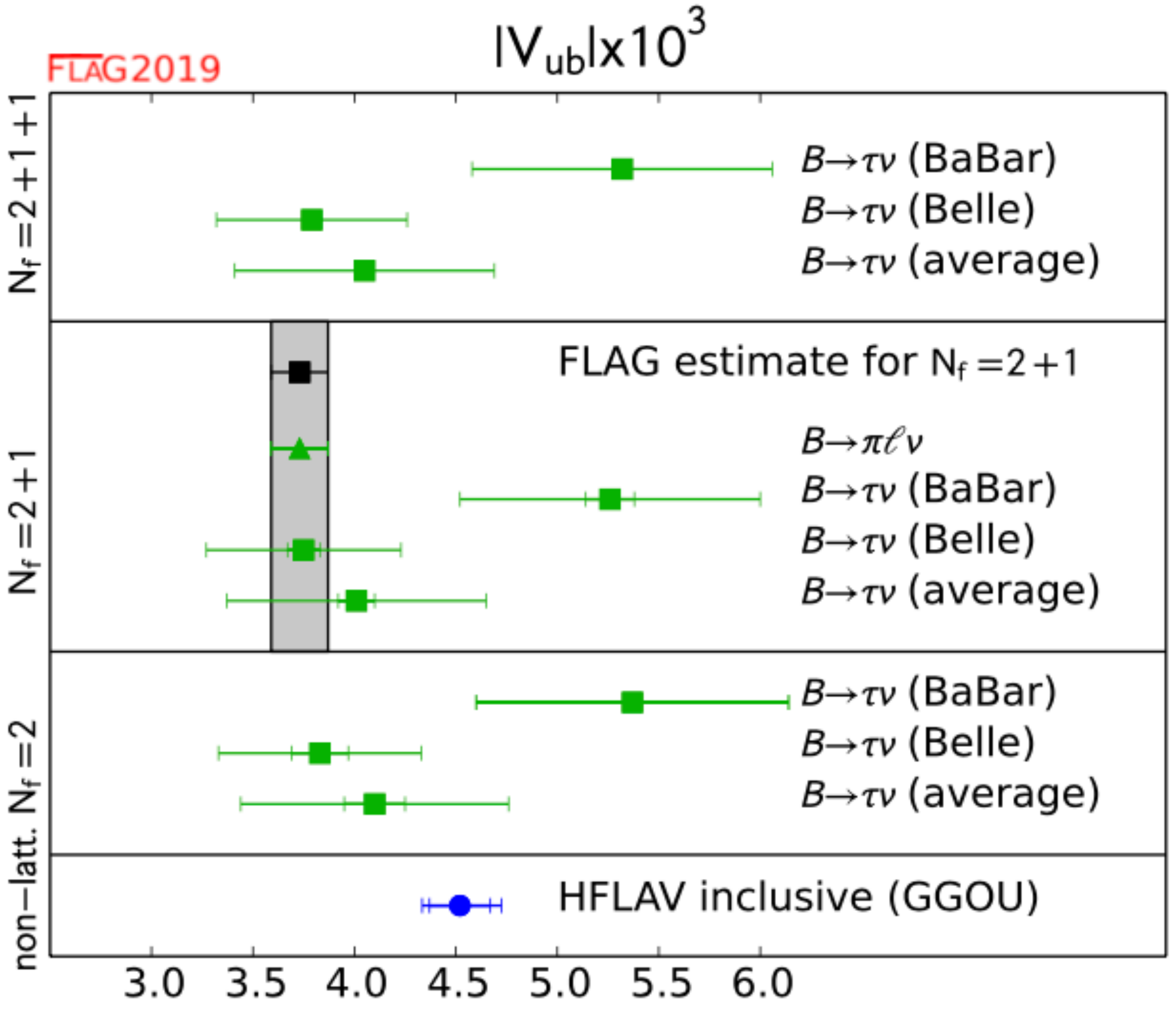
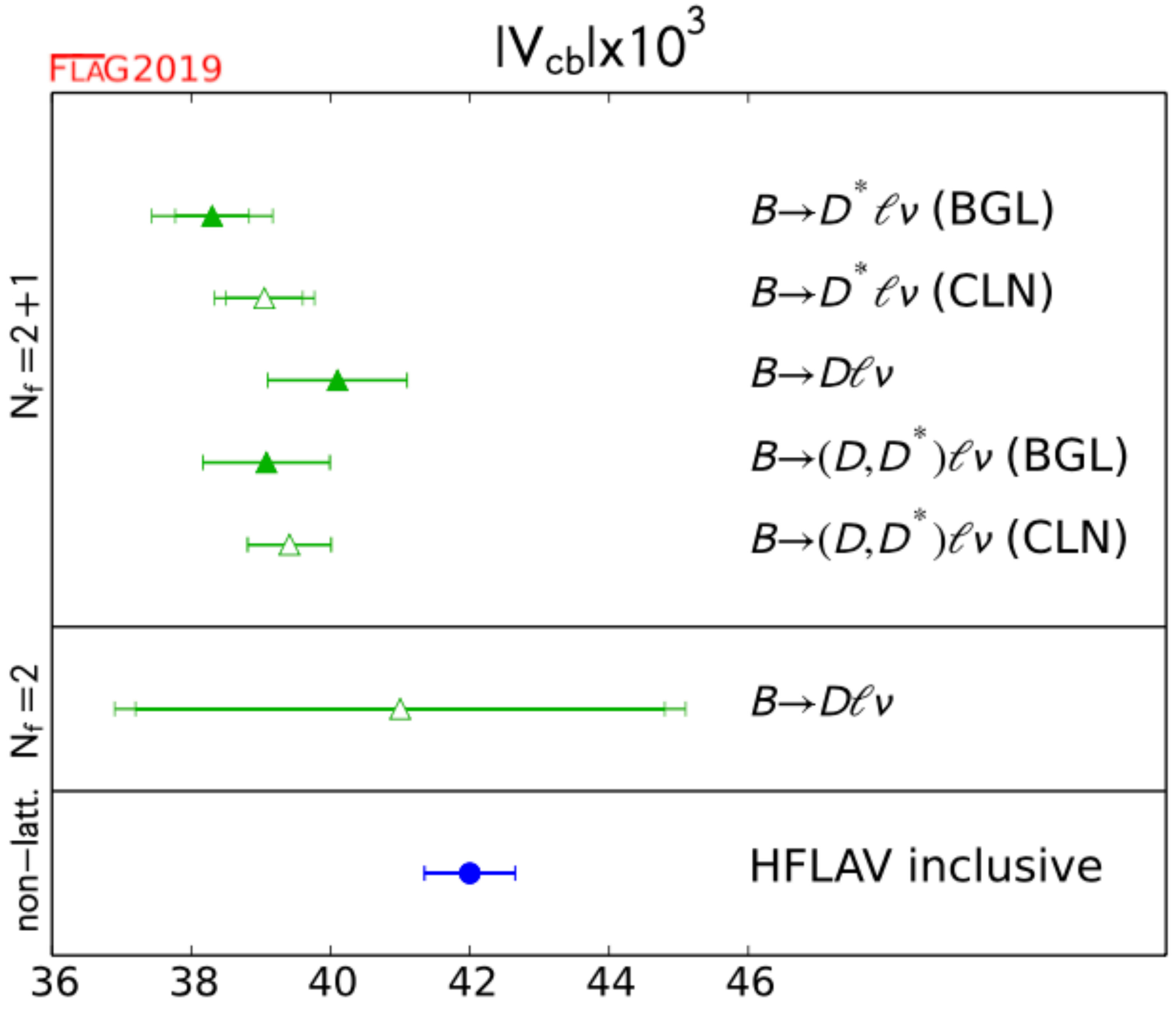


$\sigma \sim 4\%$



$\sigma \sim 2\%$