The complex potential at *T>0*

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Introduction: heavy quark potential and quarkonia

Lattice calculation of the complex potential at *T>0* in *2+1* flavor QCD
Bala et al (HotQCD), PRD 105 (2022) 054513

Recent progress on fine lattices on complex potential at *T>0*
HotQCD Collaboration, work in progress

22nd Zimányi School, Budapest December 5-9, 2022

Quarkonia and potential models

$m_b, m_c \gg \Lambda_{QCD}$ $\Rightarrow$ non-relativistic bound states, analogs QED positronium

1-gluon exchange, $\alpha_s \sim 0.4$

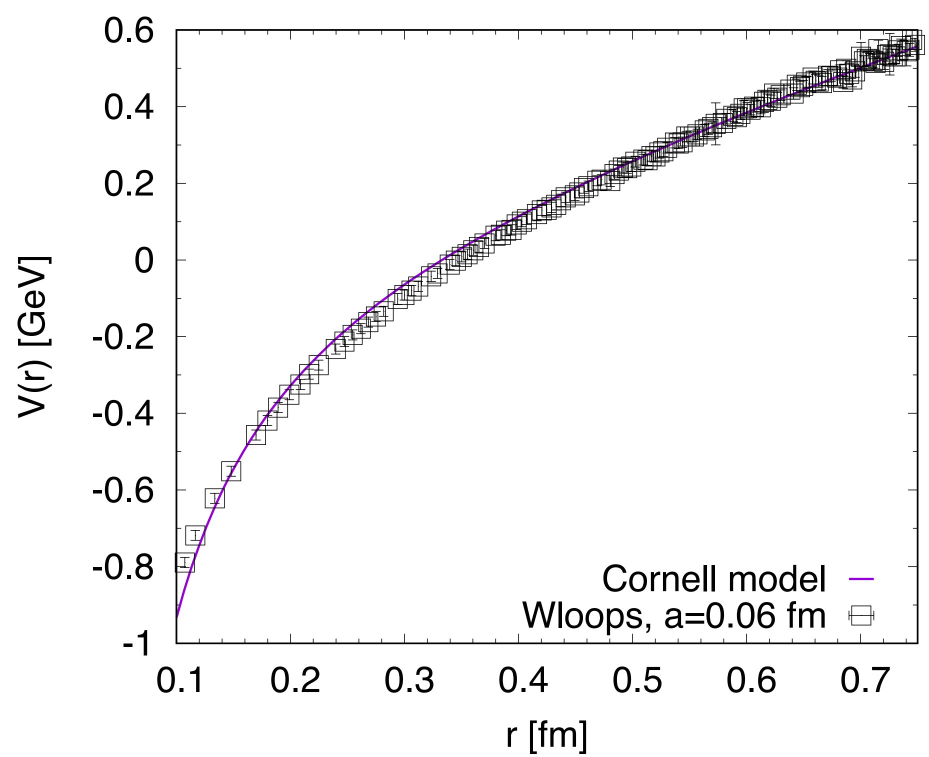
$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r + spin\ dep.$$

Confinement

Eichten et al, PRL 34 (75) 369, PRD 21 (80) 203

Very successful in describing charmonium and bottomonium spectrum below the the open charm and beauty threshold

Nevertheless nearly perfect agreement between the phenomenological and lattice potentials



Problems:

Running of α_s ?

Linear potential valid only for $r \gg 1$ fm,

$$V(r) = \sigma r - \frac{\pi}{12r} + \dots$$

Lüscher term

Soft gluon fields ??

Quarkonia in effective theory approach

$M \gg 1/r \sim Mv \gg Mv^2,\ M = m_{c,b}$ ➡ Effective theory (EFT) approach

Non-relativistic QCD (NRQCD) : EFT at scale $1/r$ (scale M is integrated out):

$$L_{NRQCD} = \psi^\dagger \left(iD_0 - \frac{D_i^2}{2M} \right) \psi + \chi^\dagger \left(iD_0 + \frac{D_i^2}{2M} \right) \chi + \ldots + \frac{1}{4} F_{\mu\nu}^2 + \bar{q}\gamma_\mu D_\mu q$$

Heavy quark fields are Pauli spinors, heavy pair creation is only present implicitly through higher dimension 4-fermion operators

Caswell, Lepage, PLB 167 (86) 437

potential NRQCD (pNRQCD): EFT at scale $E_{bin} \sim Mv^2$ (scale $1/r \sim Mv$ is integrated out):

$$L_{pNRQCD} = \int d^3\mathbf{r} \mathrm{Tr}\Big[S^\dagger \big[i\partial_0 - (\frac{-\nabla_r^2}{M} + V_s(r) + \ldots) \big] S + O^\dagger \big[iD_0 + \frac{-\nabla_r^2}{M} + V_o(r) + \ldots) \big] O \Big]$$
$$+ V_A(r) \mathrm{Tr}\Big[O^\dagger \mathbf{r} g \mathbf{E} S + S^\dagger \mathbf{r} g \mathbf{E} O \Big] + V_B(r) \mathrm{Tr}\Big[O^\dagger \mathbf{r} g \mathbf{E} O + O^\dagger O \mathbf{r} g \mathbf{E} \Big] +$$
$$\mathcal{O}(r^2, \frac{1}{M}) + \frac{1}{4} F_{\mu\nu}^2 + \bar{q}\gamma_\mu D_\mu q$$

Brambilla, Pineda, Soto, Vairo, NPB 566 (00) 275

$S = S(\mathbf{r}, \mathbf{R}, t),\ O = O(\mathbf{r}, \mathbf{R}, t),\ E = E(\mathbf{R}, t)$

Potentials are parameters of the EFT Largangian

Tree level $\leftrightarrow$ potential model $(i\partial_0 + \frac{\nabla_r^2}{M} - V_s(r)) S(\mathbf{r}, \mathbf{R}, t) = 0$

Quark anti-quark potential at *T>0*

Conjecture, Matsui and Satz, PLB 178 (86) 416 $-\frac{4}{3}\frac{\alpha_s}{r} + \sigma r \to -\frac{4}{3}\frac{\alpha_s}{r} e^{-m_D r}, T > T_c$

Extending pNRQCD to *T>0*: the potential is complex, the real part can have thermal correction but is not necessarily screened, except when $r \sim 1/m_D$

Based on weak coupling

Laine, Philipsen, Romatschke, Tassler, JHEP 03 (06) 054
Brambilla, Ghiglieri, PP, Vairo, PRD 78 (08) 014017

Calculate the potential non-perturbatively on the lattice by considering Wislon loops of size $r \times \tau$ at *T>0*

$$W(r,\tau,T) = \int_{-\infty}^{\infty} \rho_r(\omega,T) e^{-\omega\tau}$$

If potential at $T > 0$ exists the $\rho_r(\omega,T)$ should have a well defined peak at $\omega \simeq \mathrm{Re}V(r,T)$, and the width of the peak is $\mathrm{Im}V(r,T)$

Rothkopf, Hatsuda, Sasaki, PRL 108 (2012) 162001

Challenge: reconstruct $\rho_r(\omega,T)$

$$\rho_r(\omega, T=0) = \delta(\omega - V(r)) + \sum_n \delta(\omega - E_n(r))$$

Hybrid potentials, pairs of static-light mesons ...

Lattice calculations

Light d.o.f (gluons, u,d,s quarks) are represented by gauge configurations from HotQCD, $m_s = m_s^{phys}$, $m_{u,d} = m_s/20 \leftrightarrow m_\pi = 161$ MeV
$T > 0$: $48^3 \times 12$ lattices, $T_c = 159$ MeV, the temperature is varied by varying $a \leftrightarrow \beta = 10/g^2$ $T = 151 - 1938$ MeV

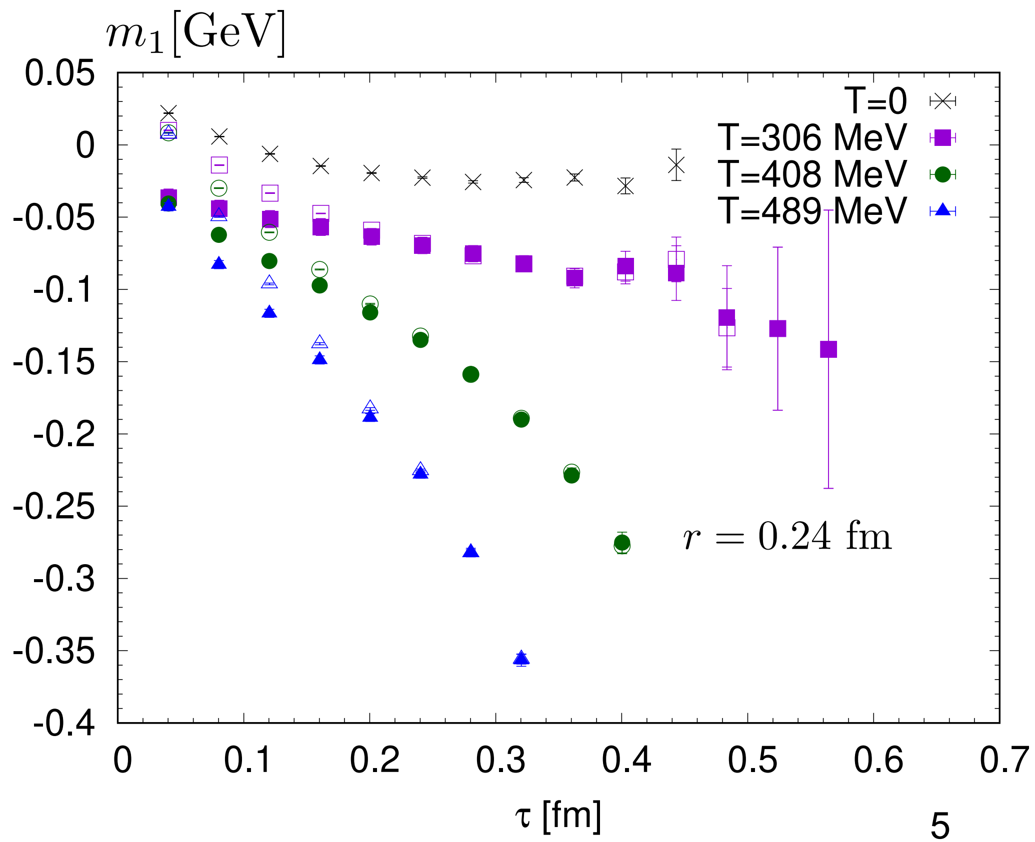
Wilson line correlators in Coulomb gauge for better signal-to-noise ratio (and also Wilson loops with HYP smeared spatial links)

Cumulants:

$$m_1(r,\tau,T) = -\partial_\tau \ln W(r,\tau,T),$$
$$m_n = \partial_\tau m_{n-1}(r,\tau,T), n > 1$$

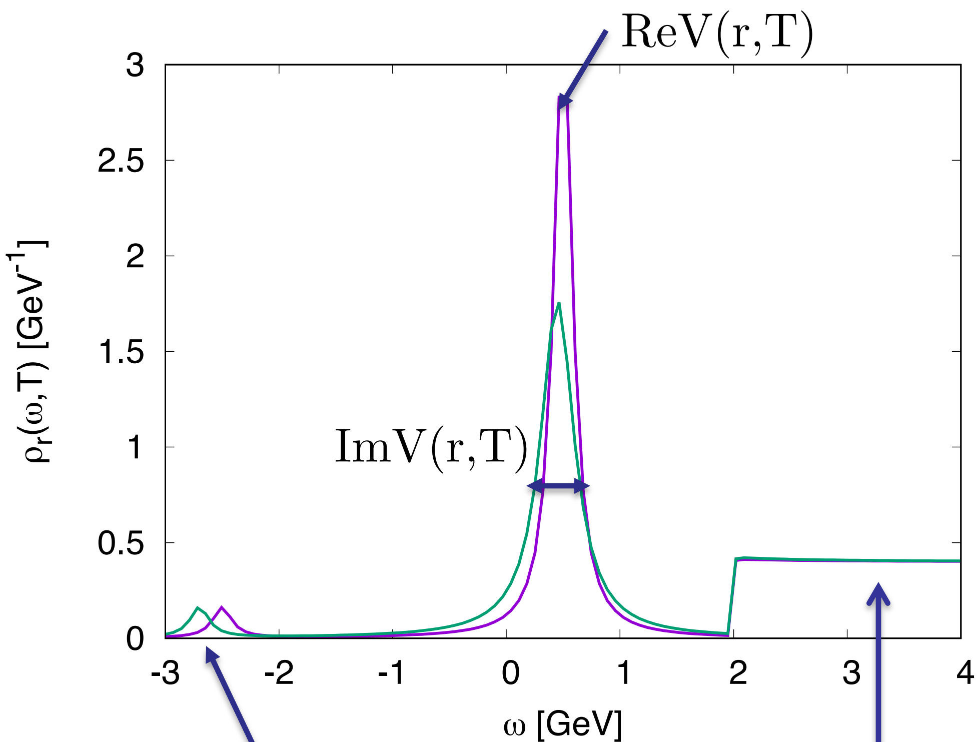
$$m_1(r,\tau,T) = \frac{1}{a} \ln \frac{W(r,\tau,T)}{W(r,\tau+a,T)}$$

$T = 0$: m_1 (effective mass=M_{eff}) reaches a plateau at large τ



- No plateau at $T > 0$ in m_1 at $T > 0$
- Only tiny T-dependence for small τ

Spectral function and the cumulants



$$\rho_r(\omega,T) = \rho_r^{tail}(\omega,T) + \rho_r^{peak}(\omega,T) + \rho_r^{high}(\omega)$$

$$W^{high}(r,\tau) = \int_{-\infty}^{\infty} d\omega \rho_r^{high}(\omega) e^{-\omega\tau}$$

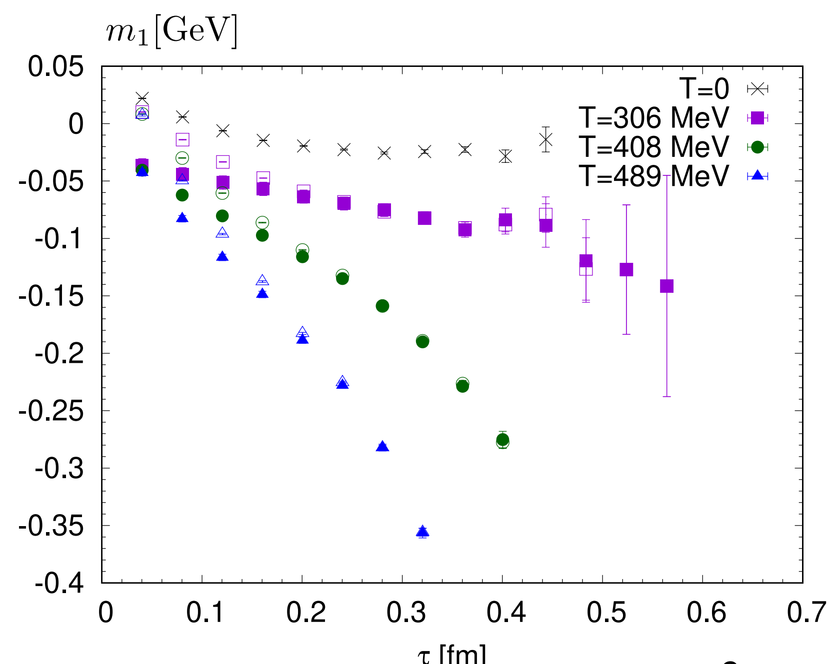
On the lattice:

$$W^{high}(r,\tau) = W(r,\tau,T=0) - A_0 \exp(-V(r)\tau)$$

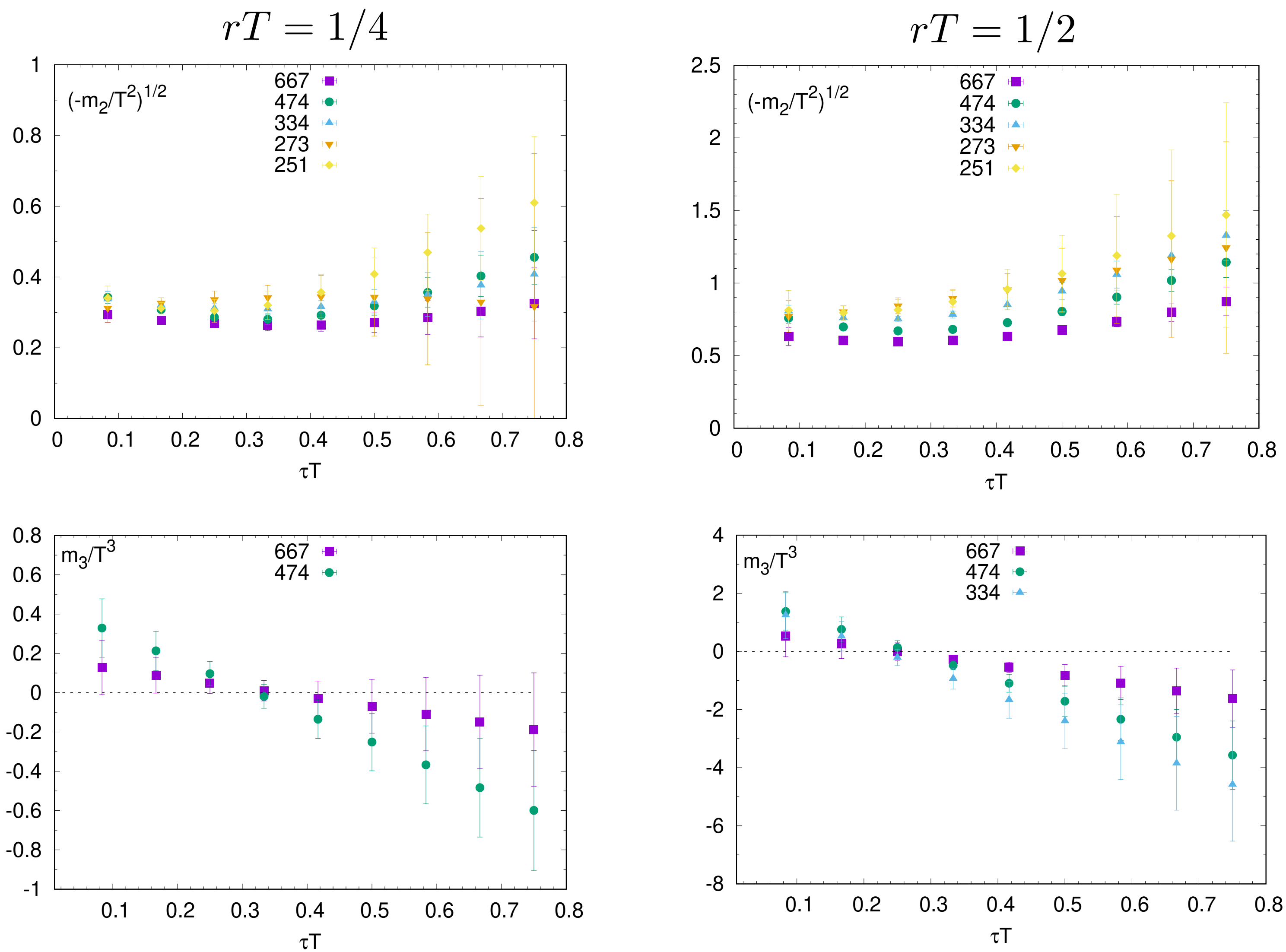
$$W^{sub}(r,\tau,T) = W(r,\tau,T) - W^{high}(r,\tau)$$

Cumulants of $W^{sub}(r,\tau,T)$ carry information about T-dependent part of the spectral function

m_1 for the subtracted correlator has milder τ-dependence, which is approximately linear



Higher cumulants



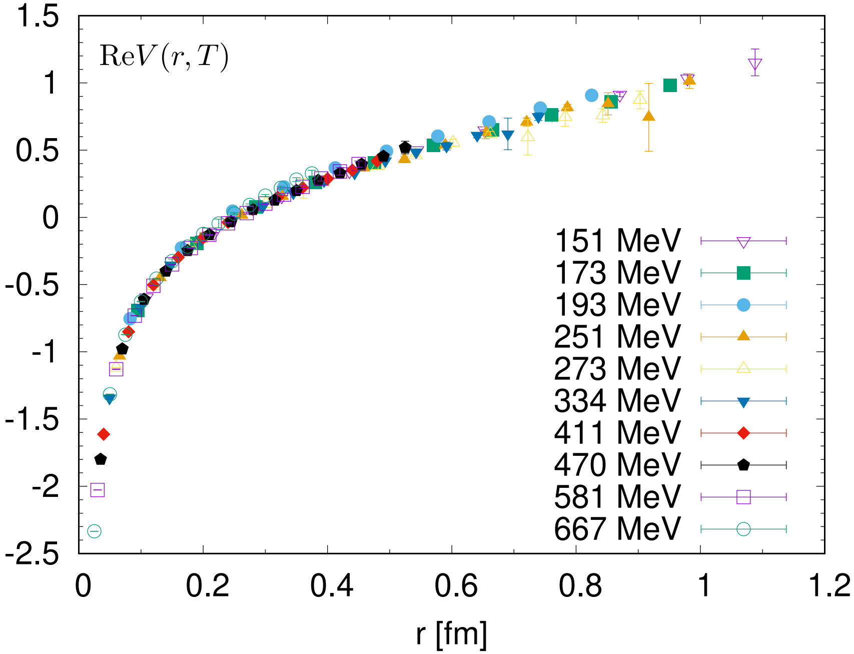
aprox. constant at small τ

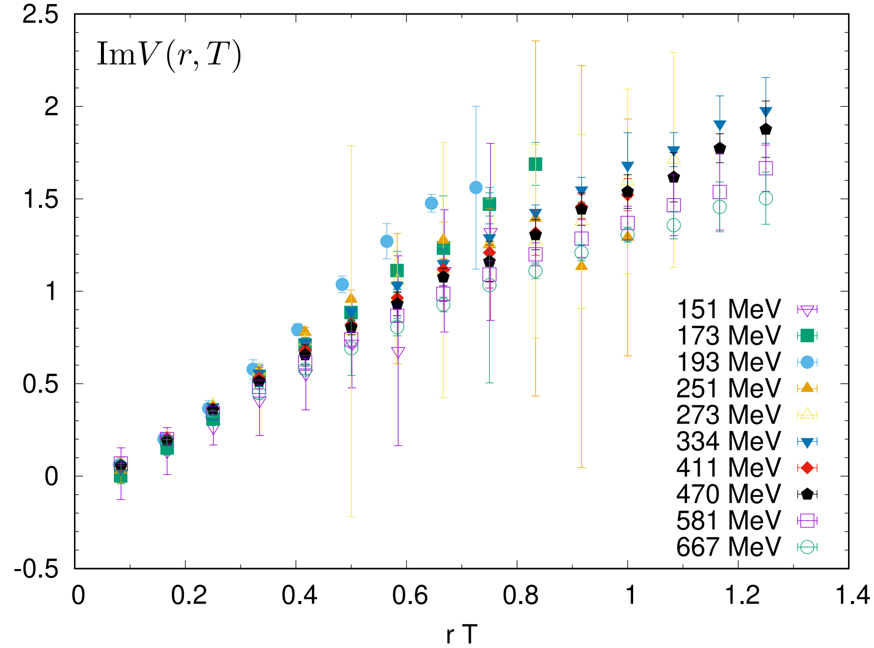
aprox. zero at small τ

Gaussian model

$$\rho_r^{peak}(\omega,T) \sim \exp(-(\omega - \mathrm{Re}V(r,T))^2/(\mathrm{Im}V(r,T))^2)$$

$$\rho_r^{tail}(\omega,T) = A^{tail}\delta(\omega - E^{tail})$$





$\mathrm{Re}V(r,T)$ shows tiny temperature dependence and no hint of screening

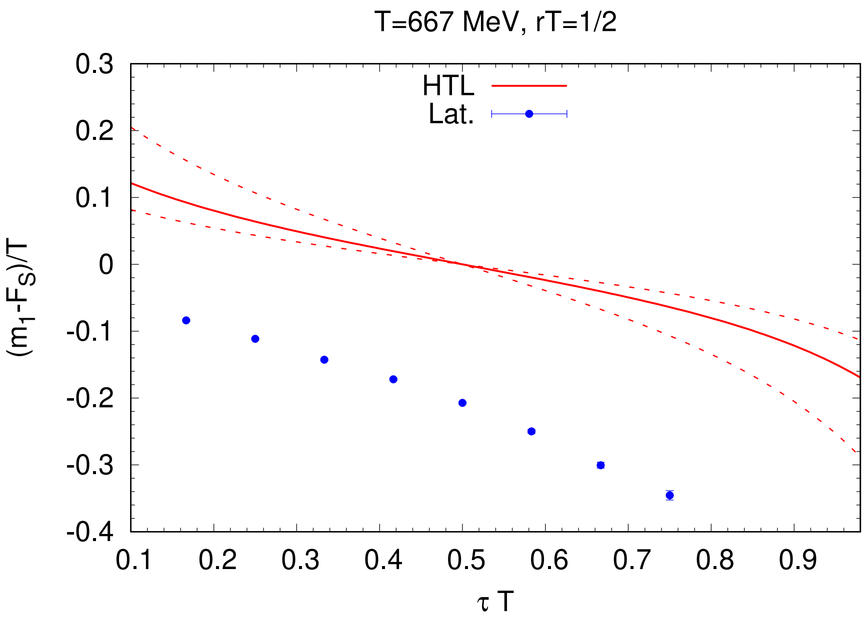
$\mathrm{Im}V(r,T)$ increases with rT and is proportional to T

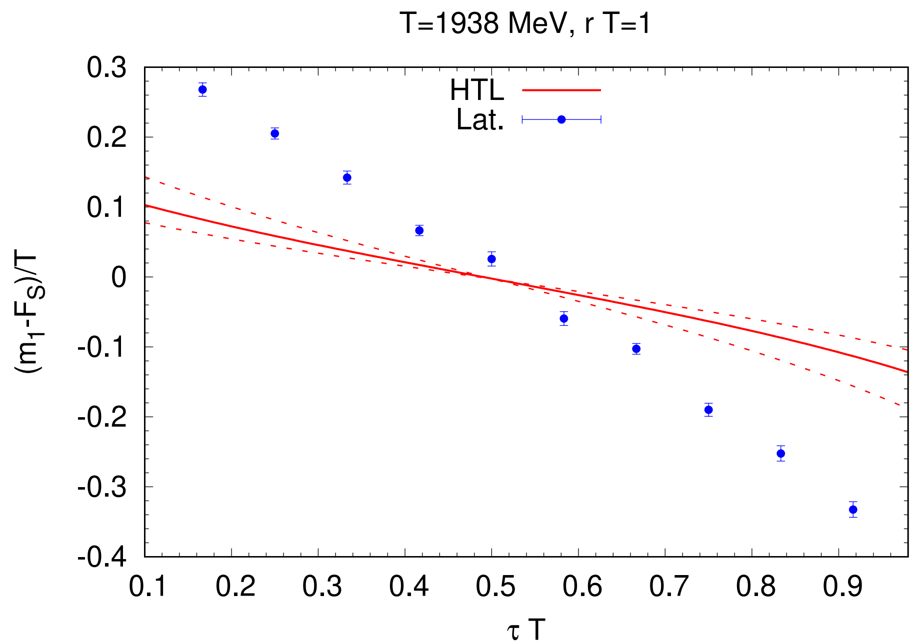
Comparison with HTL perturbation theory

In HTL perturbation theory ReV is screened, but HTL approximation is valid only for $r \sim 1/m_D$ and assumes $m_D \ll T$ (problematic in realistic setup)

Lattice results on $W(r,\tau,T)$ can be compared with the HTL calculations

Burnier, Rothkopf, PRD 87 (2013) 114019

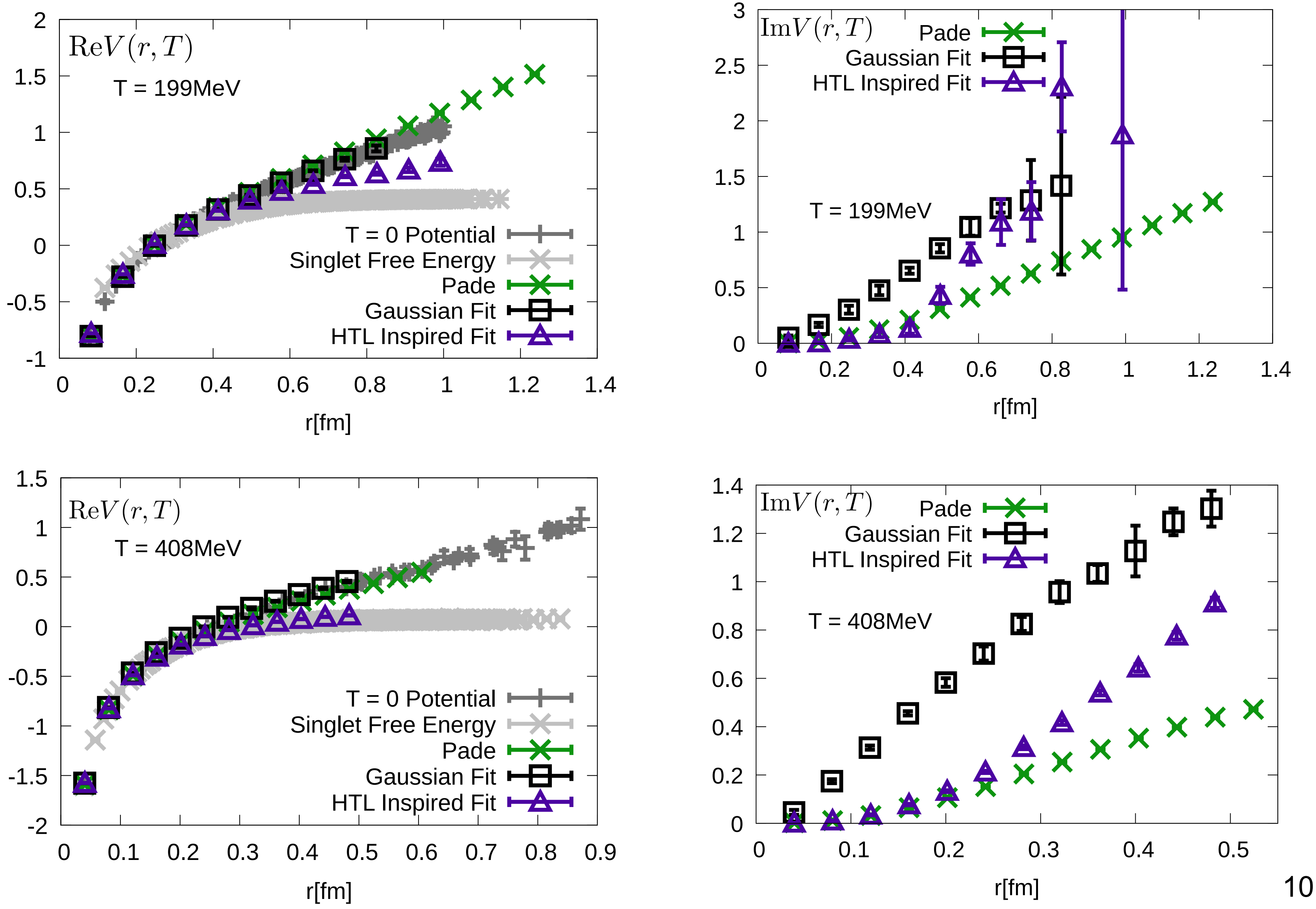




In HTL approximation Re$V(r,T) = F_S(r,T)$

No agreement between the lattice and the HTL results even at the highest T

Quark anti-quark potential at *T>0* using different methods



Calculations on fine lattices (work in progress)

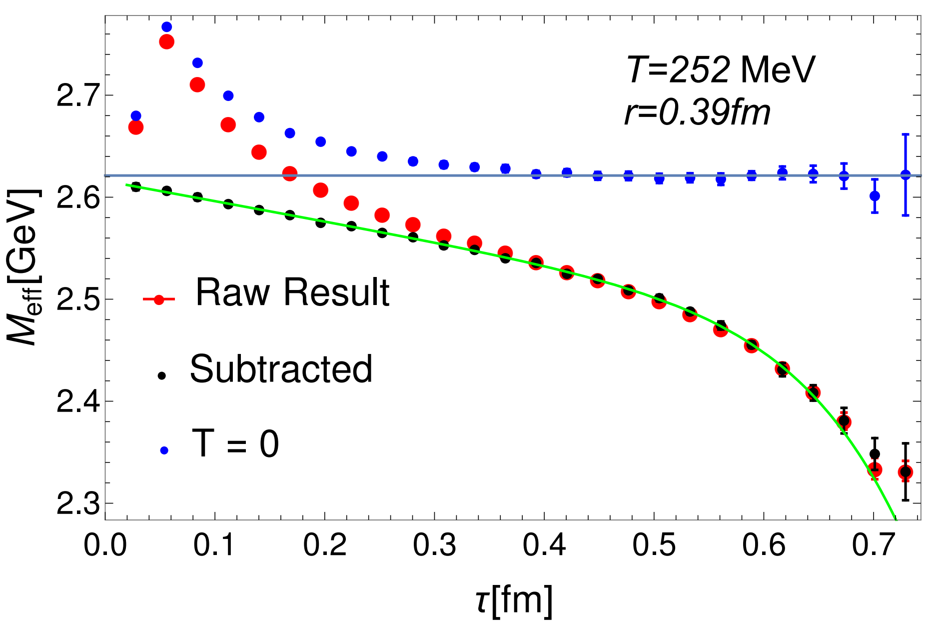
$2+1$ f QCD, $m_\pi = 300$ MeV $\quad T = 126,\ 196,\ 220,\ 252,\ 294,\ 354$ MeV

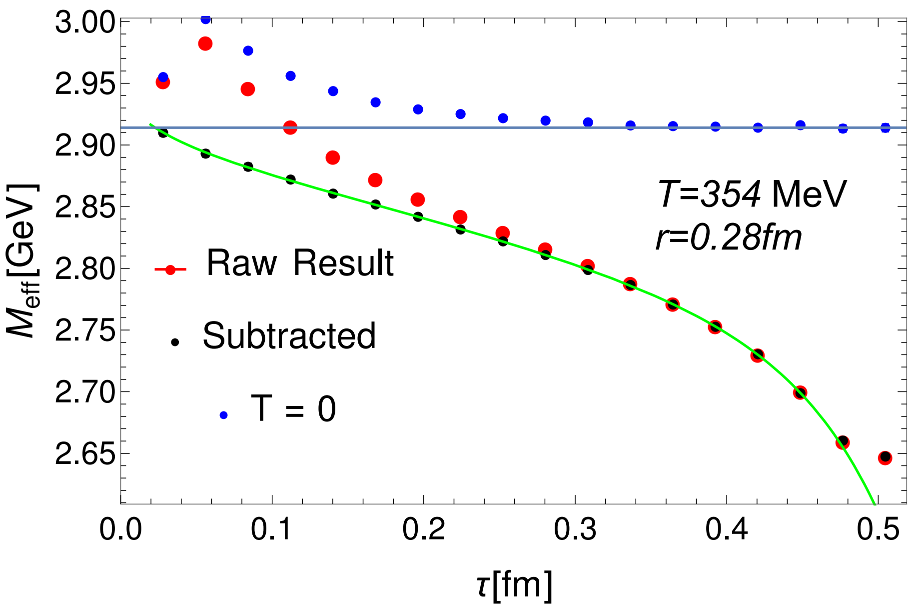
$a = 0.028$ fm, $96^3 \times N_\tau, N_\tau = 56,\ 36,\ 32,\ 28,\ 24,\ 20$

Gradient flow for noise reduction:

$\tau_F = 0.125\ (N_\tau = 20,\ 24),\ \tau_F = 0.2\ (N_\tau = 28),\ \tau_F = 0.4\ (N_\tau = 32,\ 36)$

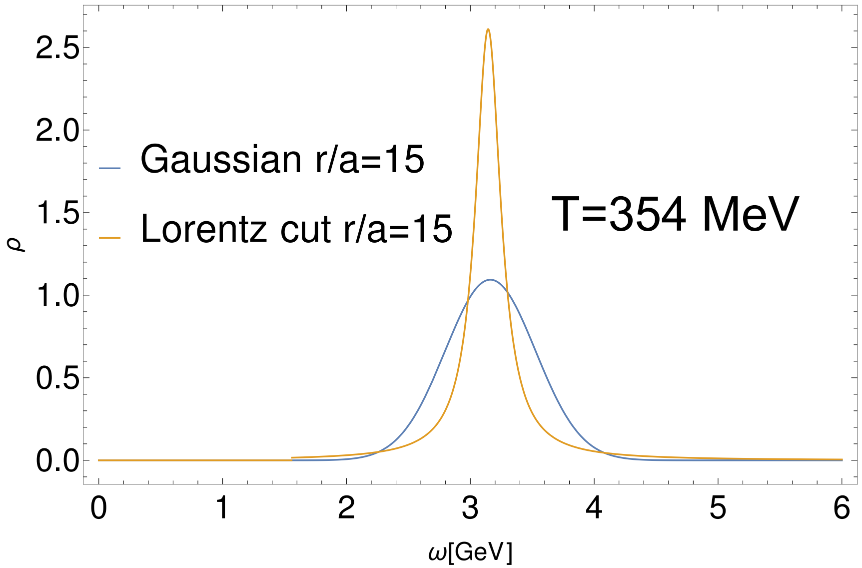
$\sqrt{8\tau_F}T = 0.04 - 0.05$



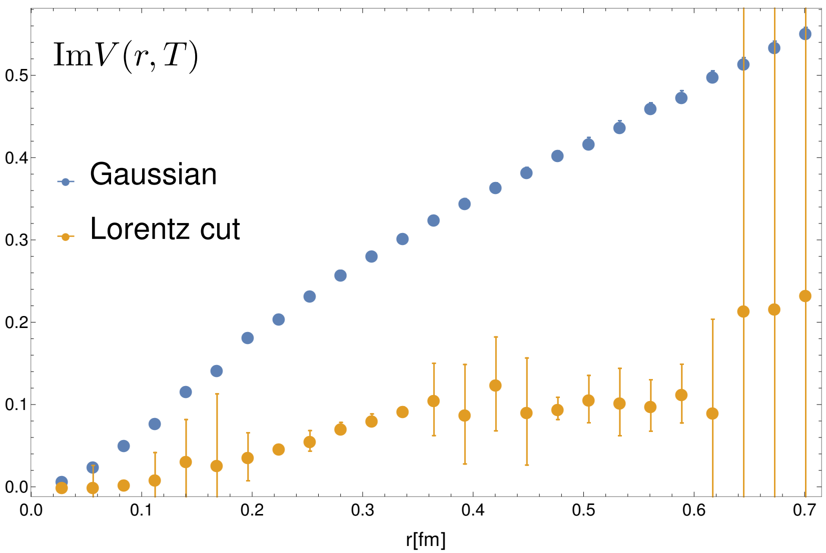


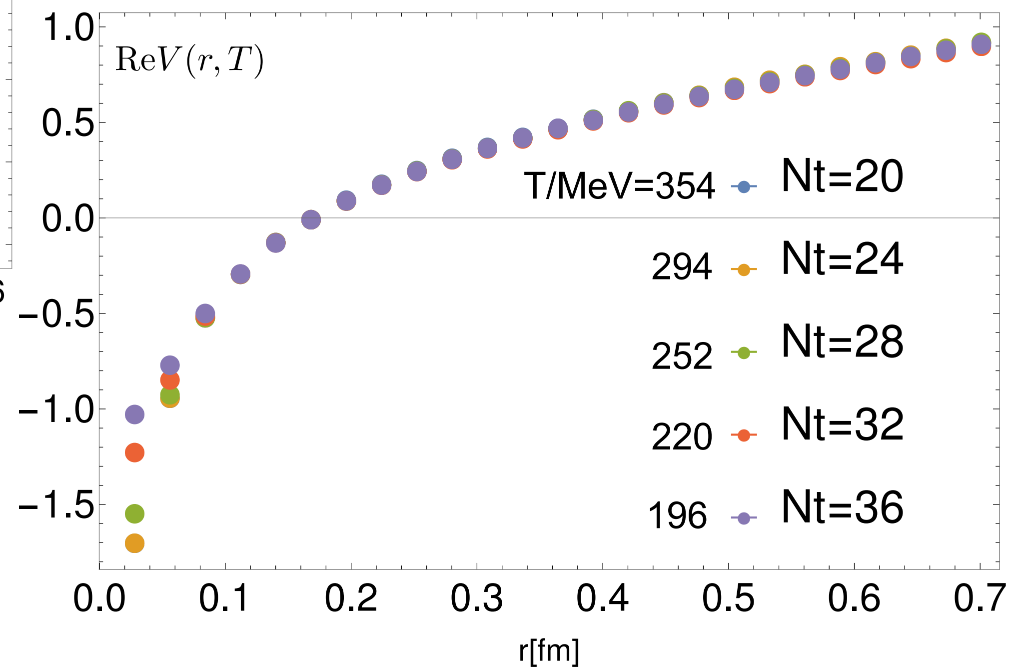
Non-linear behavior at short time and high T => need Lorentzian fits

Calculations on fine lattices (work in progress)



The peak position (real part of the potential)l does not depend on the spectral form

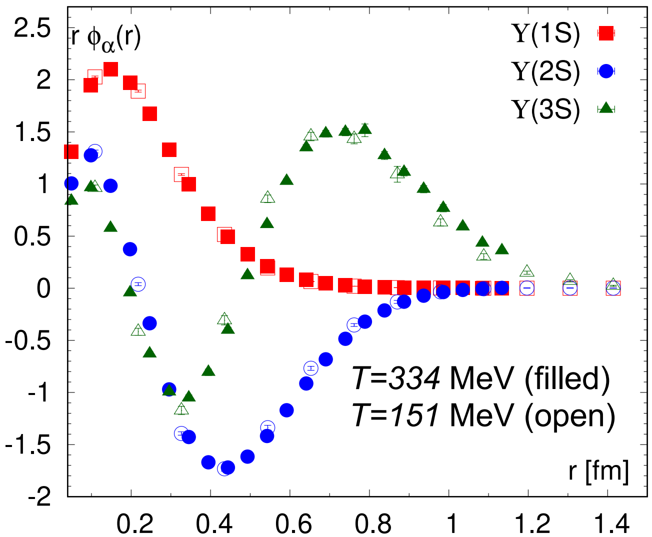


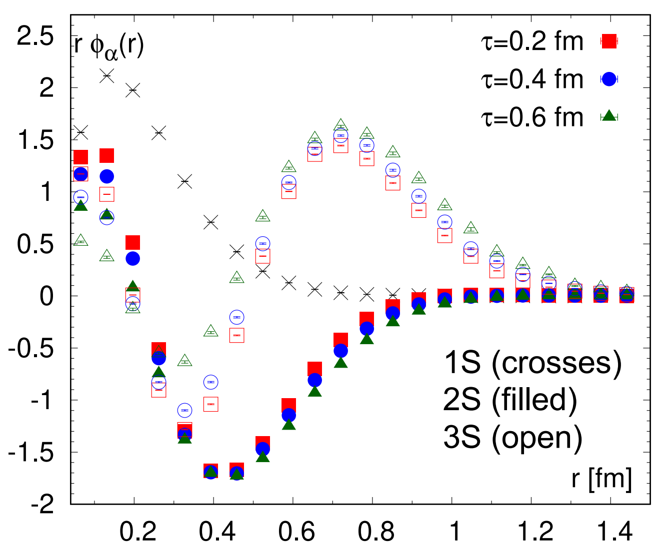


No T-dependence, no screening of the real part of the potential

Bethe-Salpeter amplitude at *T>0* and potential model

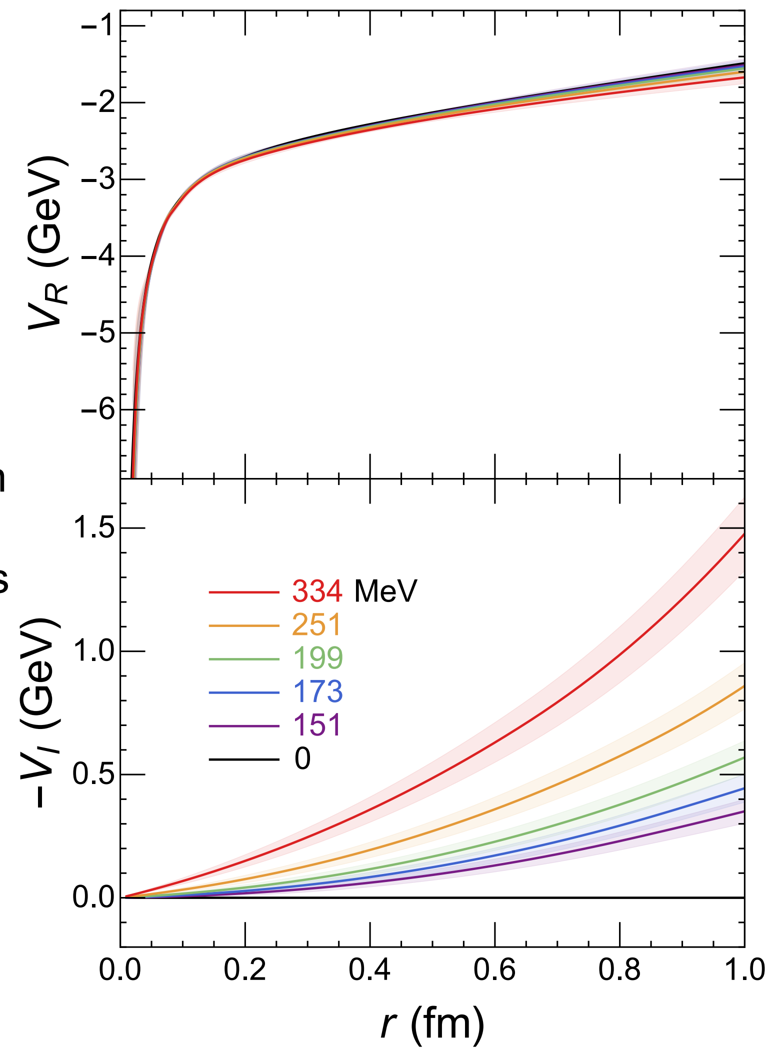
Larsen, Meinel, Mukherjee, PP, PRD 102 (20)114508





potential model
with inverse problem
+
Thermal width /mass
from lattice
+
Machine learning

Shi et al, PRD 105 (2022) 014017



Summary

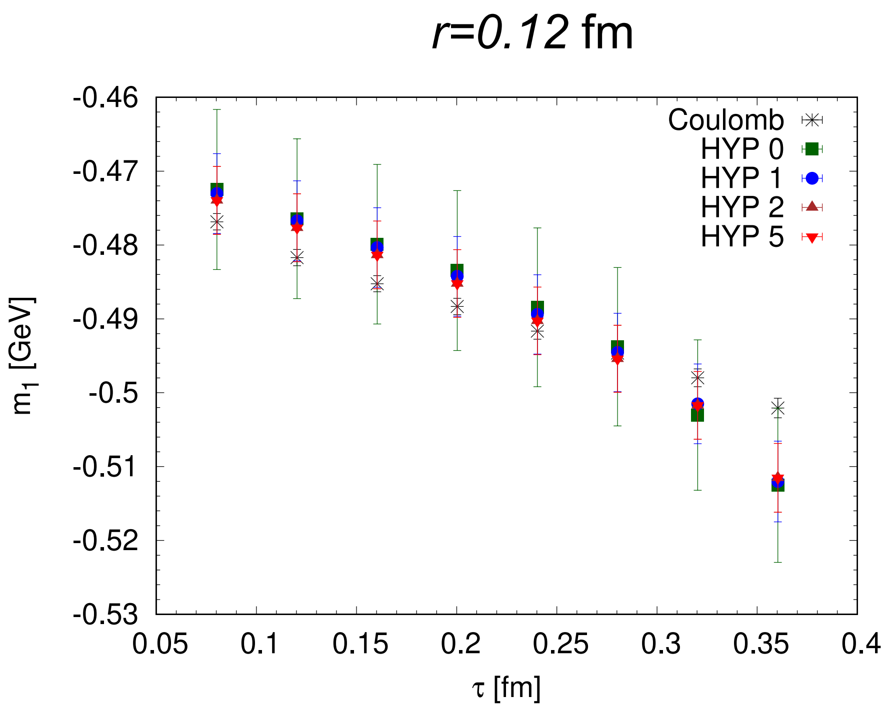
- The complex potential at $T>0$ can be obtained from the lattice calculations of Wilson line correlator (Wilson loops) using a simple parametrization of the corresponding spectral function

- The real part of the potential is NOT screened, while the imaginary part of the potential is large and increases with distance

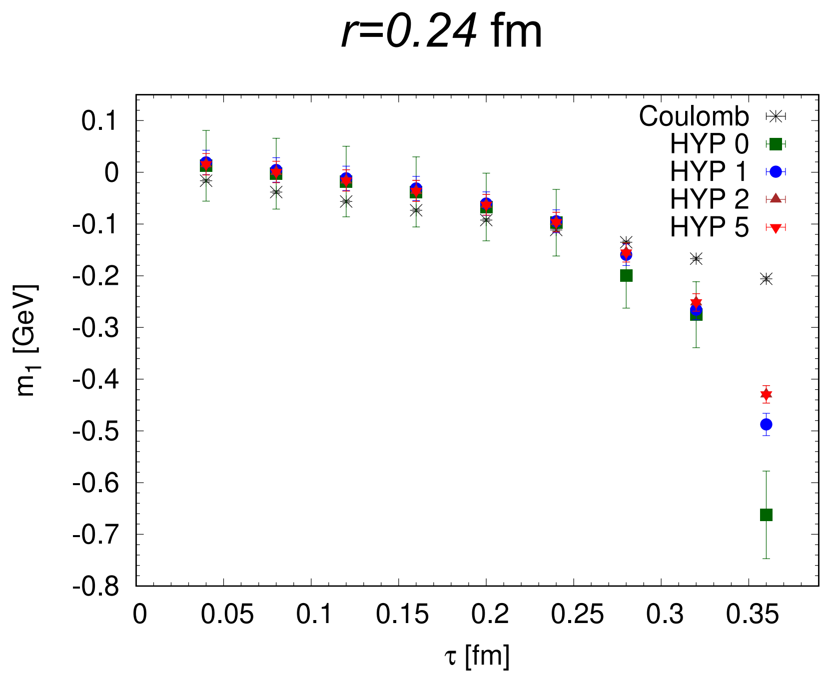
- The HTL perturbative calculation for the Wilson line correlator does not agree with the lattice results even at $T=1938$ MeV

- The results on the complex potential are corroborated by the results on the bottomonium Bethe-Salpeter amplitude obtained on the lattice as well as by the in-medium bottomonium masses

Back-up:

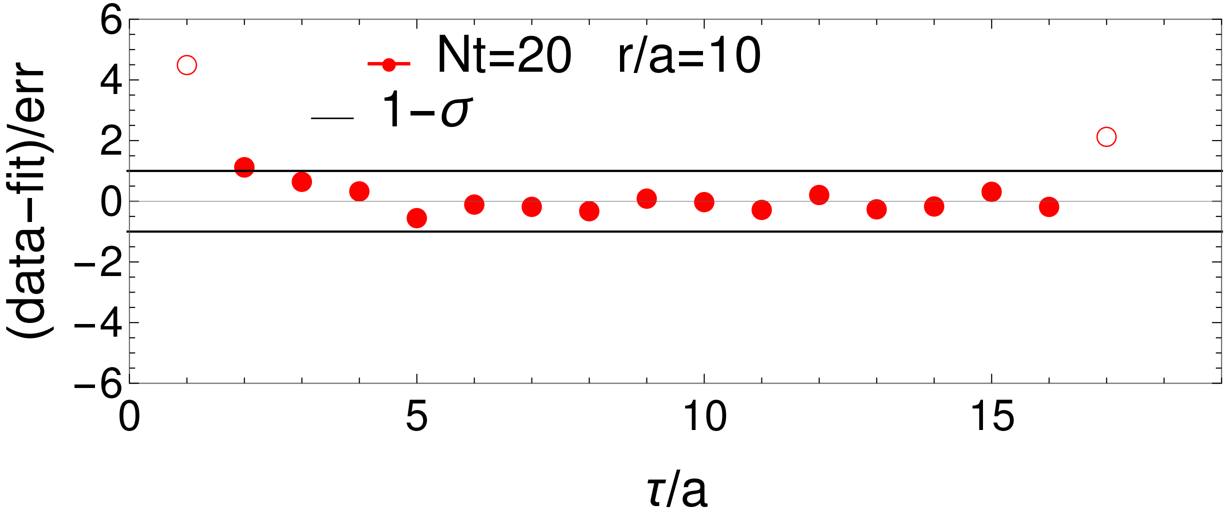


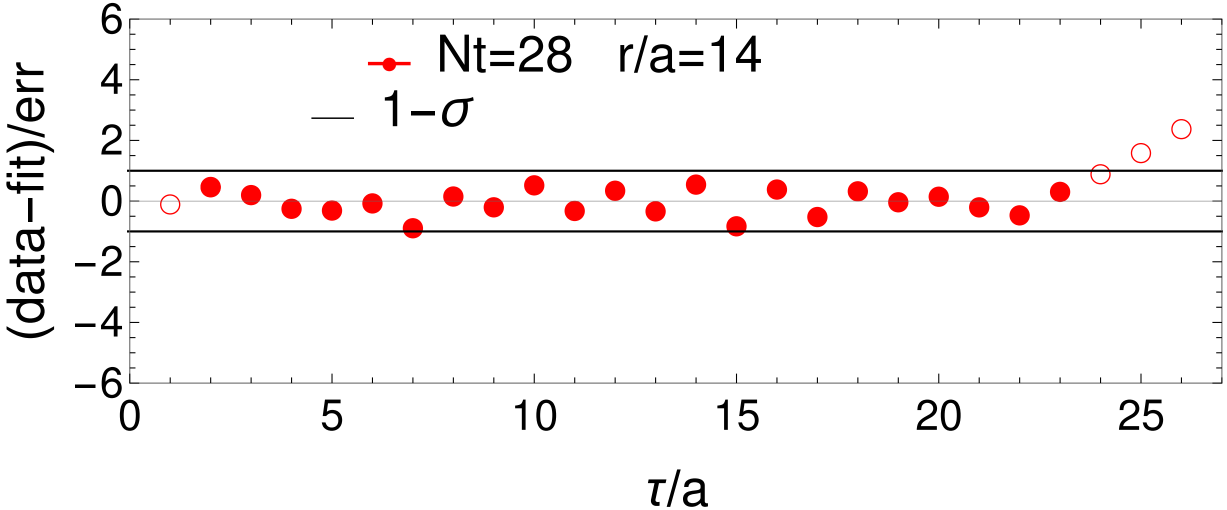
r=0.12 fm
Coulomb
HYP 0
HYP 1
HYP 2
HYP 5
m1 [GeV]
τ [fm]
-0.46
-0.47
-0.48
-0.49
-0.5
-0.51
-0.52
-0.53
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4



r=0.24 fm
Coulomb
HYP 0
HYP 1
HYP 2
HYP 5
m1 [GeV]
τ [fm]
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35

Gaussian fits





Lorentzian fit