Screening effects at non-zero chemical potential and magnetic field

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Understanding QCD phase diagram $\rightarrow$ one of important goals of high-energy physics

- Most relevant for relativistic heavy-on collisions $\rightarrow$ μ, T plane

Theoretical tools

- Requires non-preturbative treatment
- LQCD $\rightarrow$ first-principle calculations
- Effective models $\rightarrow$ QCD-like theories
  - Extension to large μ
  - Building intuitions $\rightarrow$ Complementary to more advanced methods

Other directions also possible, e.g. quark masses or magnetic field

This talk:

- Chiral phase transition at finite B [1] and μ [2]

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[1] PM Lo, MS, K. Redlich, C. Sasaki, Eur. Phys. J. A (2022) 58:172
[2] PM Lo, MS, K. Redlich, C. Sasaki, work in progress

Why study QCD in strong magnetic field?

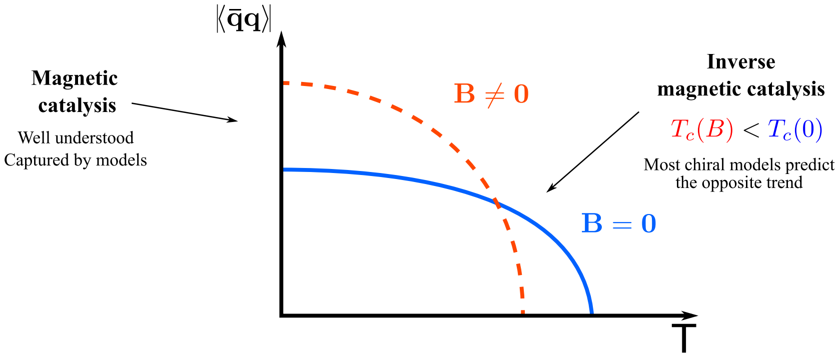
May be important for phenomenology:

- Non-central heavy-ion collisions
- Magnetars

Additional parameter to study QCD under extreme conditions

- Can be probed directly in LQCD simulations
- Possibility to test effective models

Chiral condensate from LQCD



Opposite trends of $T_c(B)$ in LQCD and models $\rightarrow$ Possible missing interactions!

This talk $\rightarrow$ Role of in-medium screening of four-quark interaction

Starting point $\rightarrow$ Chiral model inspired by Coulomb gauge QCD[3]

$$\mathcal{L} = \bar{\psi}(x)(i\partial\!\!\!/ - m_0)\psi(x) + \int d^4y \rho^a(x) V^{ab}(x-y)\rho^b(y)$$

with

- $\rho^a(x) = \bar{\psi}(x)\gamma^0 T^a \psi(x) \rightarrow$ color quark current
- $V^{ab}(x-y) \rightarrow$ Interaction potential

Contact interaction

$$M = m_0 + C_F V_0 \int \frac{d^3q}{(2\pi)^3} \frac{M}{2E} \left(1 - N_{th}(E,\mu) - \bar{N}_{th}(E,\mu)\right)$$

$$E = \sqrt{\vec{q}^2 + M^2}\,, \quad N_{th}(E,\mu) = \frac{1}{e^{\beta(E-\mu)}+1}\,, \quad \bar{N}_{th}(E,\mu) = \frac{1}{e^{\beta(E+\mu)}+1}$$

$$\sum_{a=1}^{N_c^2-1} T^a T^a = C_F \mathcal{I}_{N_c \times N_c}\,, \qquad C_F = \frac{N_c^2-1}{2N_c}$$

[3]See e.g. P. M. Lo, E. S. Swanson Phys. Rev. D **81** 034030 (2010)

Contact model gap equation

$$M = m_0 + C_F V_0 \int \frac{d^3q}{(2\pi)^3} \frac{M}{2E} \left(1 - N_{th}(E,\mu) - \bar{N}_{th}(E,\mu)\right)$$

The same form as the NJL model if $C_F V_0 \to 4N_c N_f (2G_{NJL})$

However

- NJL $\to$ Scalar-scalar interaction

$$\mathcal{L}_{NJL} = \mathcal{L}_0 + G_{NJL}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right]$$

- Current model $\to$ Vector-vector interaction
  - Systematic improvements possible
  - This talk $\to$ dressing by polarization

This talk

- Dressing by polarization, ring diagram approximation

$$\tilde{V}_0^{-1} = V_0^{-1} - \frac{1}{2} N_f \Pi_{00}(p_0, \vec{p}) \quad \Rightarrow \quad \tilde{V}_0 = \frac{1}{V_0^{-1} - \frac{1}{2} N_f \Pi_{00}(p_0, \vec{p})}$$

Static limit

$$m_{el}^2 = -\frac{1}{2} N_f \times \Pi_{00}(p_0 = 0, \vec{p} \to 0)$$

Screening $\to$ Medium-dependent coupling

Coupling to the Polyakov loop → Statistical confinement

- Pure gluon system → Deconfinement order parameter
- Effective models → Accounts for non-preturbative gluon dynamics

$$N_{th}(E,\mu) \to N_{th}(E,\ell,\bar{\ell},\mu) = \frac{\ell e^{-\beta(E-\mu)} + 2\bar{\ell} e^{-2\beta(E-\mu)} + e^{-3\beta(E-\mu)}}{1 + 3\ell e^{-\beta(E-\mu)} + 3\bar{\ell} e^{-2\beta(E-\mu)} + e^{-3\beta(E-\mu)}}$$

$$= \begin{cases} \dfrac{1}{1+e^{3\beta(E-\mu)}}\,, & \ell = \bar{\ell} = 0\,, \quad \text{baryon-like} \\ \dfrac{1}{1+e^{\beta(E-\mu)}}\,, & \ell = \bar{\ell} = 1\,, \quad \text{quark-like} \end{cases}$$

Two additional gap equations

$$\frac{\partial}{\partial \ell}(\mathcal{U}_G + \mathcal{U}_Q) = 0 \qquad \frac{\partial}{\partial \bar{\ell}}(\mathcal{U}_G + \mathcal{U}_Q) = 0$$

- $\mathcal{U}_G$ – pure gauge potential[4]
- $\mathcal{U}_Q$ – quark-gluon interaction

[4] P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, Phys. Rev. D **88**, 074502 (2013)

Final set of gap equations:

$$M = m_0 + C_F \tilde{V}_0(M,\ell) \left[I_{vac} - \int \frac{d^3q}{(2\pi)^3} \frac{M}{2E} \left(N_{th}(E,\ell,\bar{\ell},\mu) + \bar{N}_{th}(E,\ell,\bar{\ell},\mu) \right) \right]$$

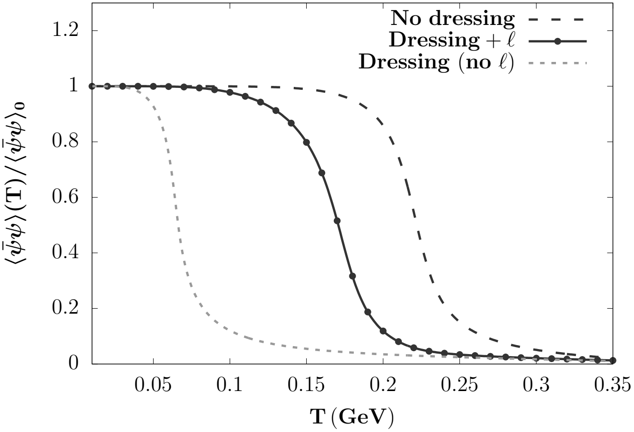
$$\tilde{V}_0(M,\ell,\bar{\ell}) = \frac{1}{V_0^{-1} + m_{el}^2(T,M,\ell,\bar{\ell})}$$

$$\frac{\partial}{\partial \ell} (\mathcal{U}_G + \mathcal{U}_Q) = 0$$

$$\frac{\partial}{\partial \bar{\ell}} (\mathcal{U}_G + \mathcal{U}_Q) = 0$$

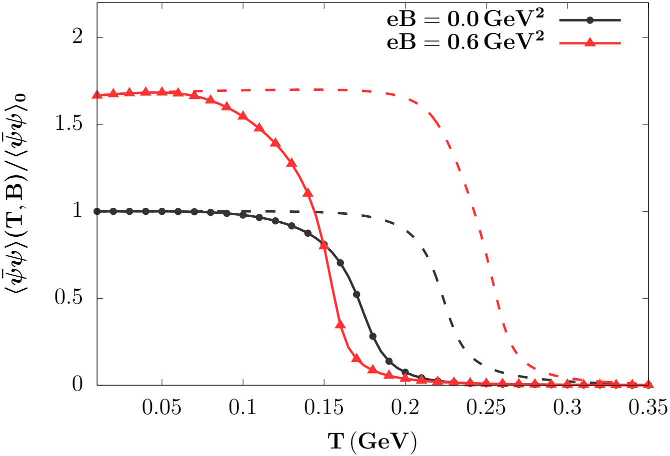
Regularization

$$I_{vac} = \int \frac{d^3q}{(2\pi^3)} \frac{M}{2E} \to \int_{1/\Lambda^2}^{\infty} \frac{ds}{16\pi^2} \frac{1}{s^2} e^{-M^2 s}$$

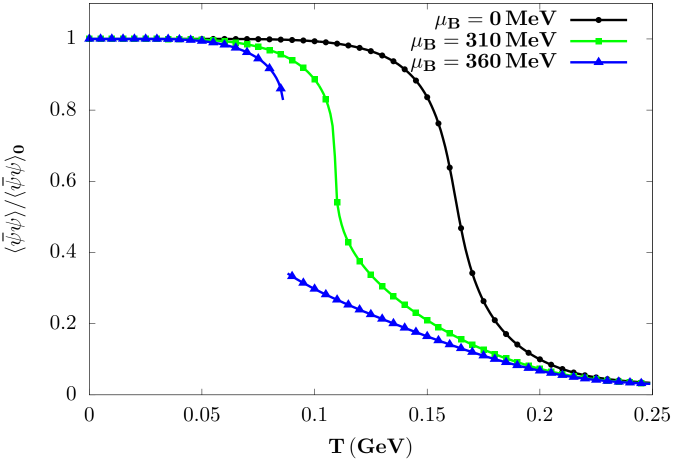


Screening → pseudocritical temperature reduced

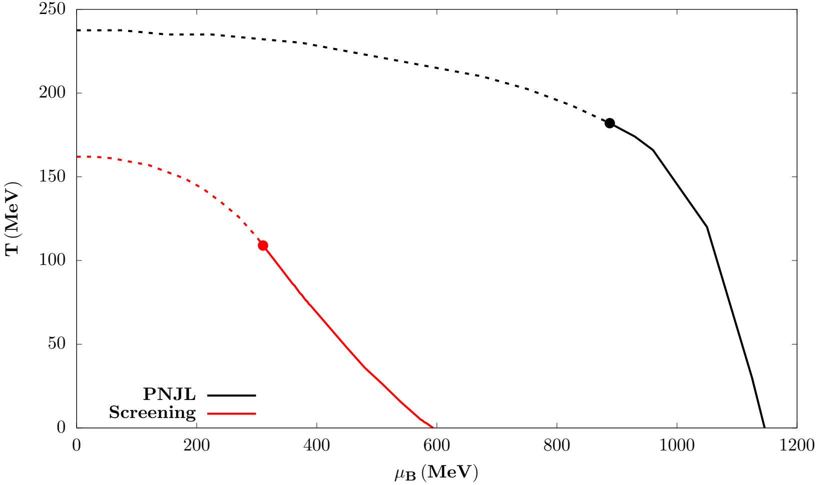
- $T_c \sim 230\,\text{MeV} \rightarrow \sim 160\,\text{MeV}$
- No additional modification of model parameters necessary
- Role of Polyakov loop → suppression of the screening strength



Results at $\mu = 0$, $B > 0$:
Dashed line – no dressing, $T_c(B) > T_c(0)$
Solid line with symbols – dressing, $T_c(B) < T_c(0)$



Results at $\mu > 0$, $B = 0$ (screening only)



250
200
150
100
50
0
T (MeV)
PNJL
Screening
0
200
400
600
800
1000
1200
μ_B (MeV)

Conclusions and outlook

- Effect of the screening of 4-quark interaction
  - $B = 0$: $T_C^{no\ screening} \approx 230\,\mathrm{MeV} \rightarrow T_C^{screening} \approx 160\,\mathrm{MeV}$
  - $B \neq 0$: IMC due to screening
  - $\mu_B > 0$: CP located at lower μ_B than in PNJL models

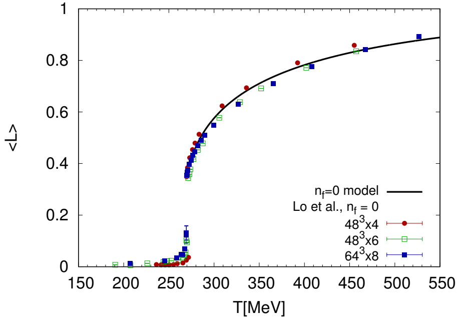
  No need for artificial rescaling of the parameters or fitting the coupling
- Future prospects
  - Investigation of fluctuations
  - Going beyond contact interaction

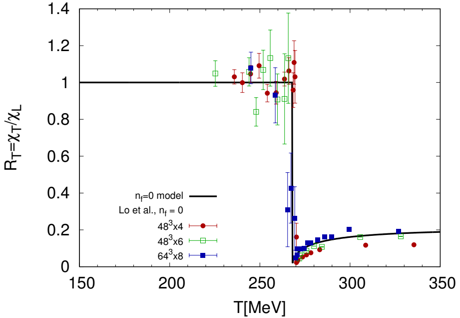
Appendix

Pure gauge part $\rightarrow$ Polyakov loop potential[1]

$$\frac{\mathcal{U}_G}{T^4} = -\frac{1}{2}a(T)\ell\bar{\ell} + b(T)\ln M_H(\ell,\bar{\ell}) + \frac{1}{2}c(T)(\ell^3+\bar{\ell}^3) + d(T)(\ell\bar{\ell})^2$$

- Polyakov loop & fluctuations determined from LQCD

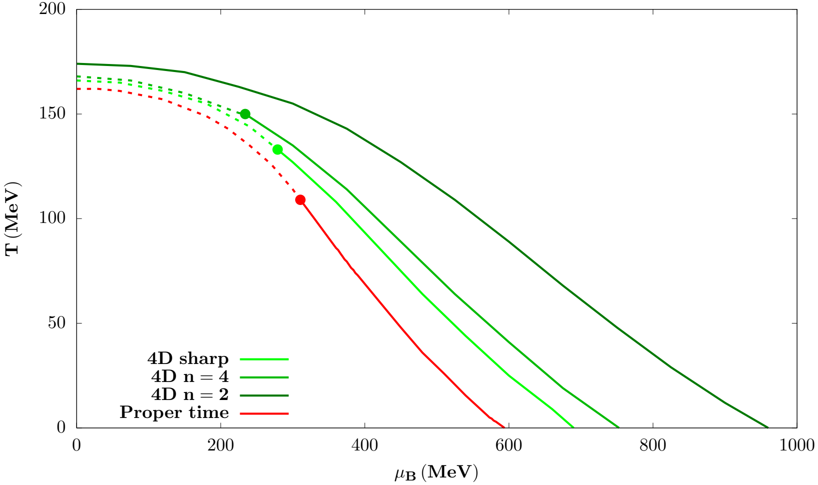




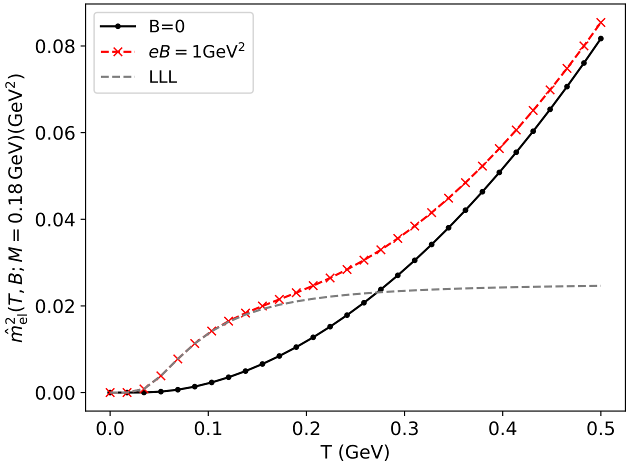
Quark-gluon interaction

$$\mathcal{U}_Q = -2T\int\frac{d^3q}{(2\pi)^3}2\ln\left(1+3\ell e^{-\beta E}+3\ell e^{-2\beta E}+e^{-3\beta E}\right)$$

[1] *P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, Phys. Rev. D* **88***, 074502 (2013)*



T (MeV)
μ_B (MeV)
200
150
100
50
0
0
200
400
600
800
1000
4D sharp
4D n = 4
4D n = 2
Proper time



Interesting limits

- $m_q = 0$: $m_{el}^2 = N_f \times (T^2/6 + \mu^2/(2\pi^2))$
- Large B: $m_{el}^2 \sim qB$

Electric mass

$$m_{el}^2 = -\frac{1}{2} N_f \times \Pi_{00}(p_0 = 0, \vec{p} \to 0) = \frac{1}{2} N_f \times \int \frac{d^3 q}{(2\pi)^3} 4\beta N_{th}(1 - N_{th})$$

External magnetic field $\to$ Landau quantization

$$2 \int \frac{d^3 p}{(2\pi)^3} \to \frac{|qB|}{2\pi} \sum_{k=0}^{\infty} (2 - \delta_{k,0}) \int_{-\infty}^{\infty} \frac{dp_z}{2\pi}$$

$$E_k{}^2 = m^2 + p_z^2 + 2k|q_f B| \,,$$

Electric mass (per flavor)

$$m_{el}^2 = \frac{1}{2} \frac{|q_f B|}{2\pi} \sum_{k=0}^{\infty} (2 - \delta_{k,0}) \int \frac{dq_z}{2\pi} 4\beta N_{th}(E_k)(1 - N_{th}(E_k))$$

$$\approx \frac{1}{2} \frac{|q_f B|}{4\pi} \int \frac{dq_z}{2\pi} \frac{4\beta e^{\beta\sqrt{(q_z)^2 + m^2}}}{(e^{\beta\sqrt{(q_z)^2+m^2}} + 1)^2} \,, \qquad |q_f B| \gg T^2$$

$$\mathcal{L} = \bar{\psi}(x)(i\partial\!\!\!/ - m_0)\psi(x) + \int d^4y \rho^a(x) V^{ab}(x-y)\rho^b(y)$$

with

- $\rho^a(x) = \bar{\psi}(x)\gamma^0 T^a \psi(x) \rightarrow$ color quark current
- $V^{ab}(x-y) \rightarrow$ Interaction potential

Instantaneous & color-diagonal potential:

$$V^{ab}(x-y) = \delta(x_0 - y_0) \times \delta^{ab} \times V(\vec{x} - \vec{y})$$

$$S^{-1}(p) = p\!\!\!/ - m_0 - C_F \int \frac{d^4q}{(2\pi)^4} V(\vec{p} - \vec{q}) i\gamma^0 S(q)\gamma^0$$

$$\sum_{a=1}^{N_c^2-1} T^a T^a = C_F \mathcal{I}_{N_c \times N_c}\,, \qquad C_F = \frac{N_c^2 - 1}{2N_c}$$