Thermalization of gluons in spatially homogeneous systems

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in collaboration with

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Introduction

- After a heavy-ion collision, an out-of-equilibrium system of gluons is produced.

  Phys. Rev. D 55 (1997). Jalilian-Marian et al.
  Nucl. Phys. B 529 (1998). Kovchegov and Mueller

- In the weak coupling limit, the thermalization follows a bottom-up fashion.

  Phys. Lett. B 502 (2001). Baier et al.

- The only tool used for a quantitative study of these systems before is the Effective Kinetic Theory (EKT).

  JHEP 01 (2003). Arnold, Moore, and Yaffe

- Our study uses the Boltzmann Equation in Diffusion Approximation (BEDA) as an alternative approach.

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BEDA I

- The QCD Boltzmann equation at leading order:

$$(\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{x}}) f = C^{2\leftrightarrow 2}[f] + C^{1\leftrightarrow 2}[f]$$

- The thermalization process is mainly determined by the Debye mass and the jet quenching parameter.

$$m_D^2 = 8\pi\alpha_s N_c \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{f}{|\mathbf{p}|} \quad , \quad \hat{q} = 8\pi\alpha_s^2 N_c^2 \ln\frac{\langle p_t^2\rangle}{m_D^2} \int \frac{d^3\mathbf{p}}{(2\pi)^2} f(1+f)$$

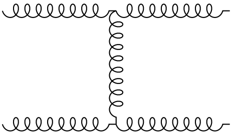
BEDA II. Elastic collision kernel

- In diffusion approximation, the $2 \leftrightarrow 2$ collision kernel can be rewritten as a Fokker-Planck equation.

Phys. Lett. B 475 (2000). Mueller

$$C^{2\leftrightarrow 2} = \frac{1}{4}\hat{q}(t)\nabla_{\mathbf{p}} \cdot \left(\nabla_{\mathbf{p}} f + \frac{\mathbf{v}}{T_*(t)} f(1+f) \right)$$

$$T_*(t) \equiv \frac{\hat{q}}{2\alpha_s N_c m_D^2 \ln \frac{\langle p_t^2 \rangle}{m_D^2}}$$



- The function f is known to diverge at small p for over-occupied systems, which is interpreted as the onset of Bose-Einstein Condensation (BEC).

Nucl. Phys. A 920 (2013). Blaizot, Liao, and McLerran

- The presence of BEC can be study numerically by choosing the appropriate boundary conditions.

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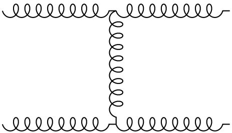

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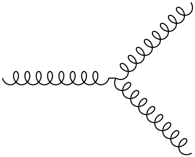
BEDA III. Inelastic collision kernel

- The $1 \leftrightarrow 2$ kernel is computed considering the LPM splitting rate.

Nucl. Phys. B 483 (1997). Baier et al.

$$C^{1\leftrightarrow 2}[f] = \int \frac{d^3\mathbf{p}'}{(2\pi)^3} \int_0^1 dx \frac{d^2 I(\mathbf{p}')}{dx dt} \times \left\{ \begin{matrix} \text{Statistics} \\ \text{contribution} \end{matrix} \right\}$$



$$\frac{d^2 I(p)}{dx dt} = \frac{\alpha_s N_c}{\pi} \frac{\left(1 - x + x^2\right)^{\frac{5}{2}}}{\left(x - x^2\right)^{\frac{3}{2}}} \sqrt{\frac{\hat{q}}{p}}$$

- When we include the inelastic collisions, no BEC appears.

Rapid thermalization of the soft sector

- At very early times, the inelastic kernel dominates for small p.

$$f(p) \approx \frac{T_*}{p} \text{ for } p \lesssim p_*$$

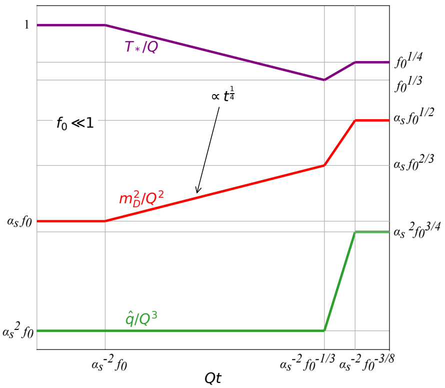
where the momentum scale p_* has been introduced.

$$p_* \equiv \left(\hat{q} m_D^4 t^2\right)^{\frac{1}{5}}$$

This means that the soft sector is always in a thermal distribution.

Under-populated scenario I

Three different stages for thermalization.



Parametric estimation for $f_0 \ll 1$

1. Soft gluon radiation and overheating.
    - T_* is almost constant.
2. Cooling and overcooling of soft gluons.
    - Soft gluons start to contribute to m_D^2.
    - T_* decreases.
3. Reheating of soft gluons and mini-jet quenching.
    - $\hat{q}$ receives dominant contribution from soft gluons.
    - T_* increases until it reaches T_{eq}.

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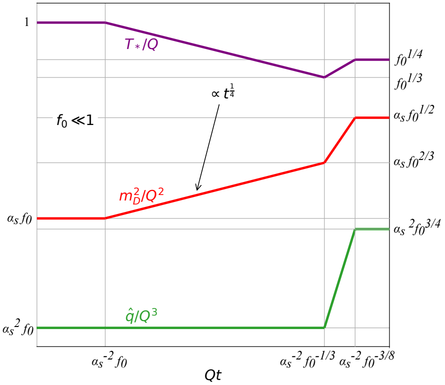

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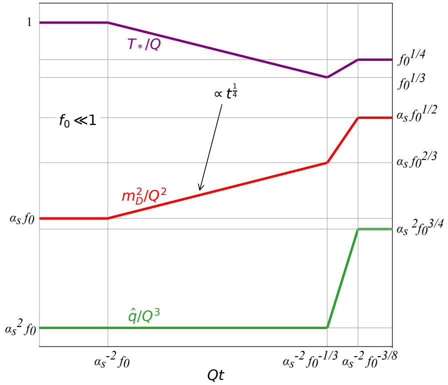

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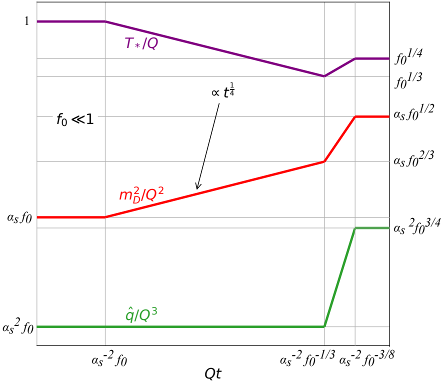
Under-populated scenario II



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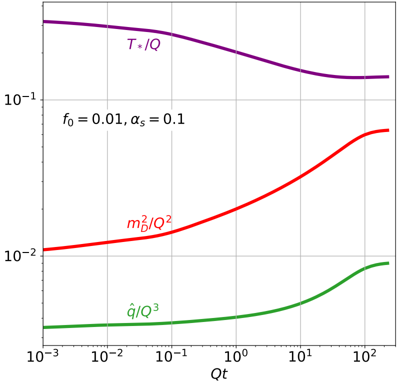
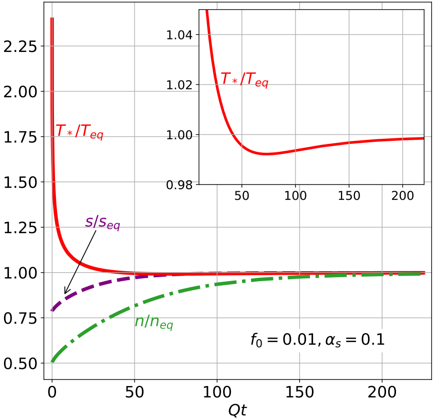


Numerical results for $f_0 = 0.01$

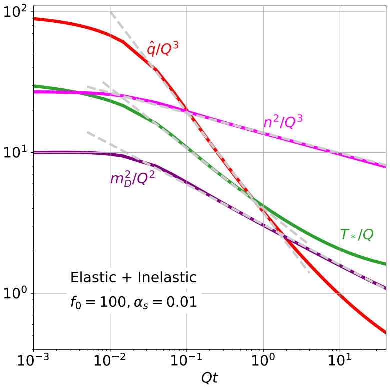
Under-populated scenario III



Time evolution of T_*, entropy density s and gluon number density n.

Over-populated scenario

Two-stage thermalization



Parametric estimation for $f_0 \ll 1$

1. Soft gluon radiation and overheating.
    - T_* is almost constant
2. Momentum broadening and cooling (no overcooling)
    - T_* starts to decrease until it reaches thermal equilibrium.
    - All the quantities evolve according the universal scaling solution (dashed lines).

See also:

Phys. Rev. D 86 (2012). Kurkela and Moore

Phys. Rev. D 89.7 (2014). Abraao York et al.

Over-populated scenario

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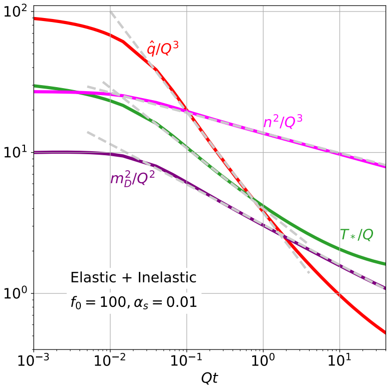

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Summary and conclusions

- The Boltzmann Equation in Diffusion Approximation (BEDA) provides a framework to study the thermalization of a system of gluons.
- The qualitative features of thermalization described by the BEDA agree with previous studies using EKT.
- The soft sector quickly achieves a thermal distribution due to inelastic processes.
- We identify the reheating of the gluons, which agrees with the increasing of the temperature identified in the bottom-up scenario for initially under-populated systems.

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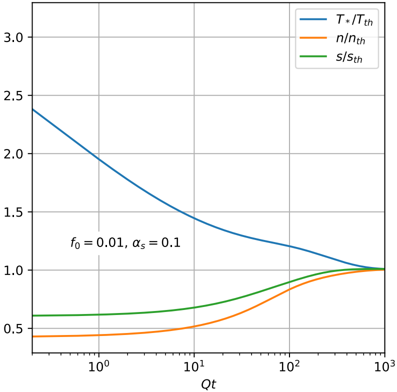
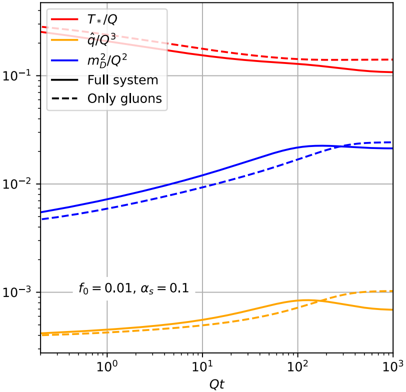
Outlook. Including quarks

- Currently, we are including quarks and antiquarks in our calculations for BEDA.





Thank you for your attention