Glueball Resonance Gas Model

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YM thermodynamics & glueball spectrum in LQCD

- Strong evidences for the glueball spectrum in Lattice simulations
- Various simulations with different methods obtain the same states
- Physical masses as well as the thermodynamical quantities are strongly dependent on the lattice parameters

Yang-Mills Thermodynamics
[Borsnayi et al.JHEP07(2012)056]
$T_c = 1.26 \cdot 0.614 \cdot r_0^{-1}$, $T_c = 0.629 \cdot \sqrt{\sigma}$

12 Glueballs Mass Spectrum
[Chen et.al Phys.Rev.D73, 014516
$r_0^{-1} = 410$ MeV , $T_c = 317$ MeV

Glueball Resonance Gas Model
arXiv:2212.xxxxx

22 Glueball Mass Spectrum
[Meyer arXiv:hep-lat/0508002]
$\sqrt{\sigma} = 440$ MeV , $T_c = 276$ MeV

20 Glueballs Mass Spectrum
[A & T JHEP11(2020)172]
$r_0 = 0.472$ fm , $T_c = 322$ MeV

Motivation

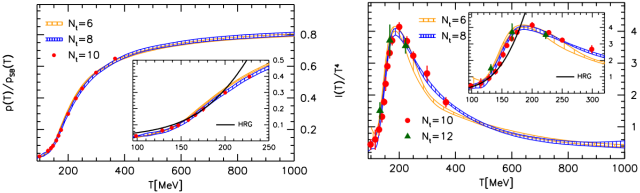
- Considering the Hadron Resonance Gas (HRG) model

$$\frac{p^{HRG}}{T^4} = \frac{1}{VT^3}\left(\sum_{i\in \text{mes}} \log \mathcal{Z}^M(T,V,\mu_{X^a},m_i) + \sum_{i\in \text{bar}} \log \mathcal{Z}^B(T,V,\mu_{X^a},m_i)\right)$$

- Partition function for mesons and baryons $z_i := \exp\left\{\left\{\frac{1}{T}\sum_a X_i^a \mu_{X^a}\right\}\right\}$

$$\log \mathcal{Z}^{M,B}(T,V,\mu_{X^a},m_i) := \mp V d_i \int_0^\infty \frac{dk k^2}{2\pi^2} \log\left(1 \mp z_i e^{-\frac{\sqrt{k^2+m_i^2}}{T}}\right)$$

- All known baryons (≈100) and mesons (≈100) up to 2.5 GeV from PDG within HRG model [Borsnayi et al.JHEP11(2010)077]



- **Pure Yang-Mills sector of QCD within Gluebal Resonance Gas?**

Glueball Spectrum in LQCD

- [Chen et.al Phys.Rev.D73, 014516]
- [Athenodorou and Teper (A & T) JHEP11(2020)172]
- [Meyer arXiv:hep-lat/0508002]

$n\,J^{PC}$	M[MeV]			$n\,J^{PC}$	M[MeV]		
	Chen et.al.	A & T	Meyer		Chen et.al.	A & T	Meyer
$1\,0^{++}$	1710(50)(80)	1651(23)	1475(30)(65)	$1\,1^{--}$	3830(40)(190)		3240(330)(150)
$2\,0^{++}$		2840(40)	2755(30)(120)	$1\,2^{--}$	4010(45)(200)	3920(90)	3660(130)(170)
$3\,0^{++}$	3650(60)		3370(100)(150)	$2\,2^{--}$			3740(200)(170)
$4\,0^{++}$			3990(210)(180)	$1\,3^{--}$	4200(45)(200)	4030(70)	4330(260)(200)
$1\,2^{++}$	2390(30)(120)	2376(32)	2150(30)(100)	$1\,0^{+-}$	4780(60)(230)		
$2\,2^{++}$		3300(50)	2880(100)(130)	$1\,1^{+-}$	2980(30)(140)	2944(42)	2670(65)(120)
$1\,3^{++}$	3670(50)(180)	3740(70)	3385(90)(150)	$2\,1^{+-}$		3800(60)	
$1\,4^{++}$		3690(80)	3640(90)(160)	$1\,2^{+-}$	4230(50)(200)	4240(80)	
$1\,6^{++}$			4360(260)(200)	$1\,3^{+-}$	3600(40)(170)	3530(80)	3270(90)(150)
$1\,0^{-+}$	2560(35)(120)	2561(40)	2250(60)(100)	$2\,3^{+-}$			3630(140)(160)
$2\,0^{-+}$		3540(80)	3370(150)(150)	$1\,4^{+-}$		4380(80)	
$1\,2^{-+}$	3040(40)(150)	3070(60)	2780(50)(130)	$1\,5^{+-}$			4110(170)(190)
$2\,2^{-+}$		3970(70)	3480(140)(160)				
$1\,5^{-+}$			3942(160)(180)				
$1\,1^{-+}$		4120(80)					
$2\,1^{-+}$		4160(80)					
$3\,1^{-+}$		4200(90)					

- The same states with different physical masses because of the lattice parameters
- Different choice of parameters leads different T_c's within GRG
- More number of glueball operators leads new states missing in other works

Interesting fact about Glueball masses in [JHEP11(2020)172]

- Mass relation for the ground states ≤ 3 GeV

$$M^2(n, J^{PC}) = a(n+J) + b_{J^{PC}}$$

- Use χ^2-fit, to obtain the parameters for radially excited states

$$\chi^2(a, b_{0^{++}}, b_{2^{++}}, b_{0^{-+}}, b_{2^{-+}}, b_{1^{+-}}) = \sum_{J^{PC}} \left(\frac{M(n, J^{PC}) - M^{\text{lat}}(n, J^{PC})}{\delta M^{\text{lat}}(n, J^{PC})} \right)^2$$

Glueball spectrum compared to the fit				Parameters [GeV^2]
$n\ J^{PC}$	m [GeV]	Fit [GeV]	χ_i^2	
1 0^{++}	1.653(26)	1.647(25)	0.04	$b_{0^{++}} = -2.78 \pm 0.21$
2 0^{++}	2.842(40)	2.865(30)	0.3	
1 2^{++}	2.376(32)	2.367(30)	0.08	$b_{2^{++}} = -10.87 \pm 0.57$
2 2^{++}	3.30(5)	3.33(3)	0.38	
1 0^{-+}	2.561(40)	2.572(38)	0.08	$b_{0^{-+}} = 1.12 \pm 0.27$
2 0^{-+}	3.54(8)	3.48(4)	0.57	
1 2^{-+}	3.07(6)	3.11(5)	0.52	$b_{2^{-+}} = -6.79 \pm 0.66$
2 2^{-+}	3.97(7)	3.90(4)	1.10	
1 1^{+-}	2.944(42)	2.955(37)	0.07	$b_{1^{+-}} = -2.25 \pm 0.45$
2 1^{+-}	3.80(6)	3.77(3)	0.23	
			$\chi_{tot}^2 = 3.38$	$a = 5.49 \pm 0.17$

- Use the same parameter "a" for other ground states obtained in LQCD

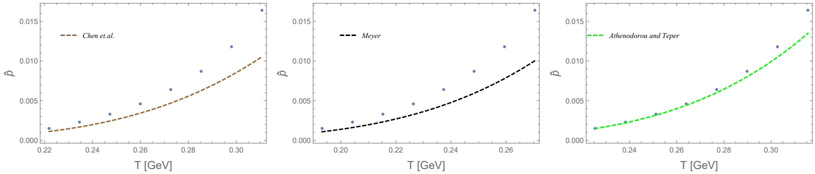
Results compared to LQCD in [Borsnayi et al.JHEP07(2012)056]

- Pressure $\hat{p} = \sum_{i=1}^{N} p_i / T^4$ and energy density $\hat{\epsilon} = \sum_{i=1}^{N} \epsilon_i / T^4$

$$p_i = -(2J_i + 1) T \int_0^\infty \frac{k^2}{2\pi^2} \ln\left(1 - e^{-\frac{\sqrt{k^2+m_i^2}}{T}}\right) dk$$

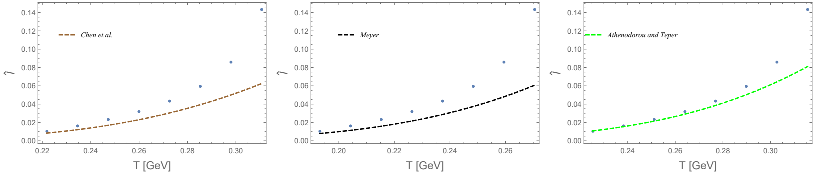
$$\epsilon_i = (2J_i + 1) \int_0^\infty \frac{k^2}{2\pi^2} \frac{\sqrt{k^2 + m_i^2}}{\exp\left[\frac{\sqrt{k^2+m_i^2}}{T}\right] - 1} dk$$

- GRG pressure for three different sets of the glueball masses compared

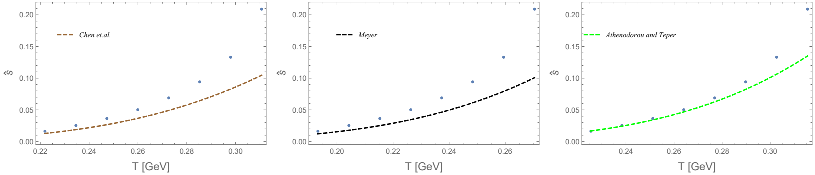


- Recent glueball masses describe YM thermodynamics well

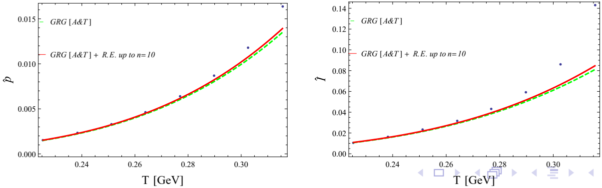
- Free Anomaly $\hat{I} = \hat{\epsilon} - 3\hat{p}$



- Free Entropy $\hat{s} = \hat{p} + \hat{\epsilon}$



- Assuming the Regge trajectories for the glueballs analogous to mesons



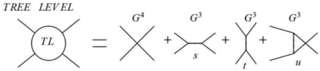
Scalar Glueball Interaction

- Scalar field G that describes both the scalar glueball and the trace anomaly

$$\mathcal{L}_{\text{dil}} = \frac{1}{2}\left(\partial_\mu G\right)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda_G^2}\left(G^4 \ln\left|\frac{G}{\Lambda_G}\right| - \frac{G^4}{4}\right)$$

- Amplitude can be decomposed in terms of the Legendre polynomials

$$A(s,\cos\theta) = \sum_{\ell=0}^{\infty}(2\ell+1)A_\ell(s)P_\ell(\cos\theta)$$



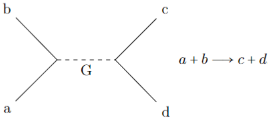
- Partial waves in terms of phase shift

$$\frac{e^{2i\delta_\ell(s)}-1}{2i} = \frac{k}{16\pi\sqrt{s}}A_\ell(s) \longrightarrow \delta_\ell(s) = \frac{1}{2}\arg\left[1+2i\frac{k}{16\pi\sqrt{s}}A_\ell(s)\right]$$

- Unitarized Amplitude $A_\ell^U(s) = \left(A_\ell^{-1}(s) - \Sigma(s)\right)^{-1}$ [Giacosa et.al. Eur.Phys.J.C 82 (2022) 5, 487]

Tensor Glueball Interaction (details in poster by E.Trotti)

- Consider $\frac{\alpha}{2} G^2 G_{\mu\nu} G^{\mu\nu}$ and doing the shift $\alpha \Lambda G G_{\mu\nu} G^{\mu\nu}$ where $\alpha = \frac{m_{g_2}^2}{\Lambda^2}$



- For $A = \frac{-4(\alpha\Lambda)^2}{s - m_G^2}$, $B = \frac{-4(\alpha\Lambda)^2}{t - m_G^2}$, $C = \frac{-4(\alpha\Lambda)^2}{u - m_G^2}$

$$\langle cd | T | ab \rangle = A\delta^{ab}\delta^{cd} + B\delta^{ac}\delta^{bd} + C\delta^{ad}\delta^{bc}$$

- Non-zero terms $\left\langle J^{cd}, m_J^{cd} \right| T \left| J^{ab}, m_J^{ab} \right\rangle$, for which $J^{cd} = J^{ab}$ and $m_J^{cd} = m_J^{ab}$
- We obtain $T^0 = 5A + B + C$, $T^1 = T^3 = B - C$, $T^2 = T^4 = B + C$

$$T_l^J = \frac{1}{2} \int_{-1}^{+1} d(\cos\theta) P_l(\cos\theta) T^J(A, B, C)$$

- Amplitude

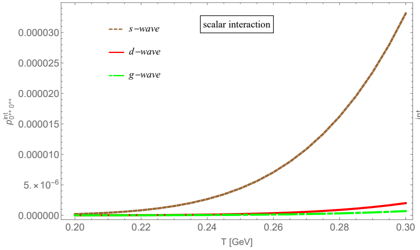
$$\mathcal{A}(s,t,u) = \sum_{l=0}^{\infty} \sum_{J=0}^{4} (2l+1)(2J+1) T_l^J P_l \cos\theta$$

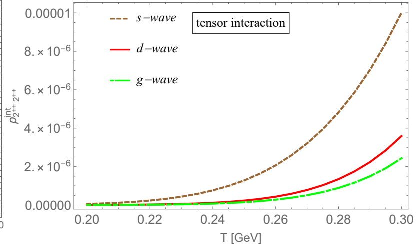
Change in Pressure from Glueball Interactions

- Phase shift approach to the interaction pressure [Dashen et.al Phys.Rev.187.345 (1969)],[Dashen and Rajaraman Phys.Rev.D10.694 (1974)], [Samanta and Giacosa arXiv:2110.14752]

$$p^{\rm int}_{0^{++}0^{++}} = -\frac{1}{T^3}\sum_{l=0}^{\infty}\int_{m_{0^{++}}}^{\infty} dx\,\frac{2l+1}{\pi}\,\frac{d\delta_l^{0^{++}0^{++}}(x)}{dx}\int\frac{d^3k}{(2\pi)^3}\ln\left(1-e^{-\beta\frac{\sqrt{k^2+x^2}}{T}}\right)$$

$$p^{\rm int}_{2^{++}2^{++}} = -\frac{1}{T^3}\sum_{J=0}^{4}\sum_{l=0}^{\infty}\int_{m_2^{++}}^{\infty} dx(2J+1)\,\frac{2l+1}{\pi}\,\frac{d\delta_l^{2^{++}2^{++},J}(x)}{dx}\int\frac{d^3k}{(2\pi)^3}\ln\left(1-e^{-\beta\frac{\sqrt{k^2+x^2}}{T}}\right)$$





Conclusion

- Glueball Resonance Gas model is revisited
- Glueballs are used as hadronic degrees of freedom
- Location of the T_c strongly depends on the lattice parameters
- Recent Glueball spectrum works well below T_c
- It implies $T_c \approx 320 \pm 20$ MeV for pure YM thermodynamics
- Assuming further glueballs via Regge trajectories leads only negligible difference
- Contribution from the interaction of scalar and tensor glueballs are also small

Thank you for the attention!