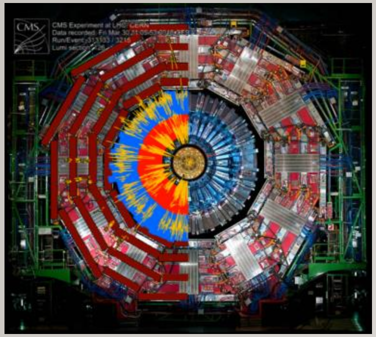
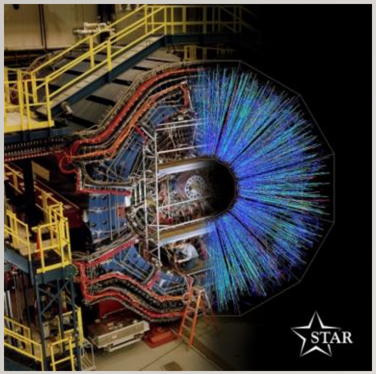
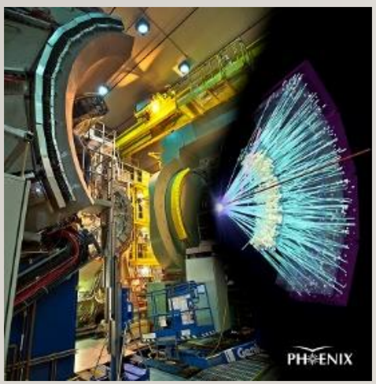
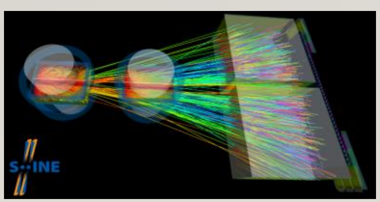




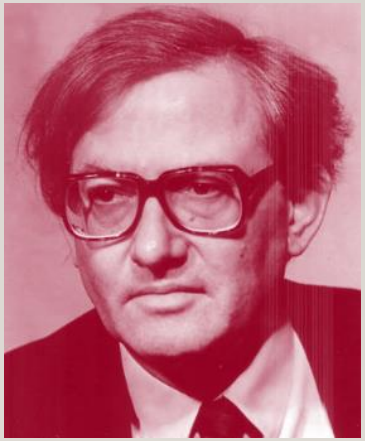




FEMTOSCOPY WITH LÉVY DISTRIBUTIONS FROM SPS TO LHC

CSANÁD MÁTÉ (EÖTVÖS UNIVERSITY)

ZIMÁNYI SCHOOL WINTER WORKSHOP 2022



CONTENTS OF THIS TALK

- Basics of femtoscopy and Lévy sources
- A sample of experimental results
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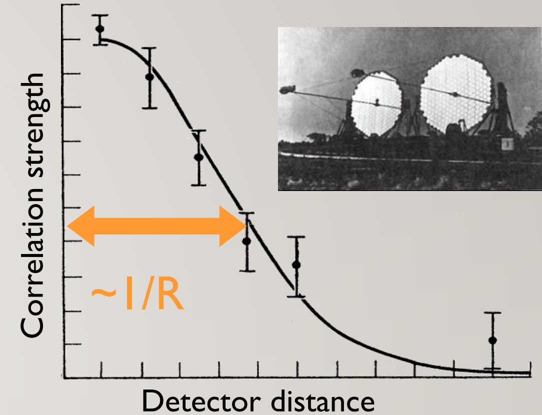
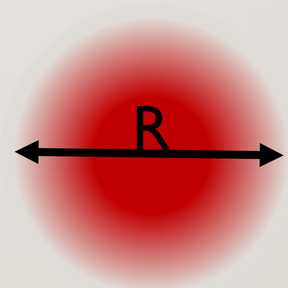
FEMTOSCOPY IN HIGH ENERGY PHYSICS

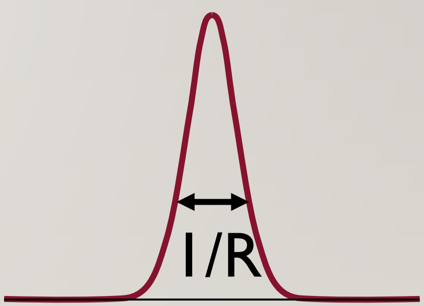
- R. Hanbury Brown, R. Q. Twiss - observing Sirius with radio telescopes
  - Intensity correlations vs detector distance ⇒ source size
  - Measure the sizes of apparently point-like sources!



- Goldhaber et al: applicable in high energy physics
- Understanding: Glauber, Fano, Baym, …
  Phys. Rev. Lett. 10, 84; Rev. Mod. Phys. 78 1267, …
  - Momentum correlation $C(q)$ related to source $S(r)$
  - $C(q) \cong 1 + \left|\int S(r) e^{iqr} dr\right|^2$
    (under some assumptions)
  - Also with distance distribution $D(r)$:
    $C(q) \cong 1 + \int D(r) e^{iqr} dr$
  - Neglected: pair reco., final state int., N-particle correlations, coherence, …



source function S(r)

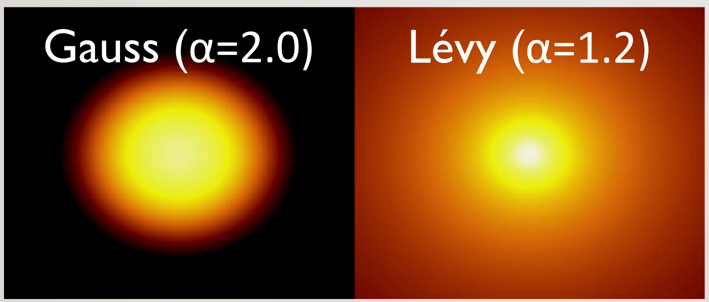


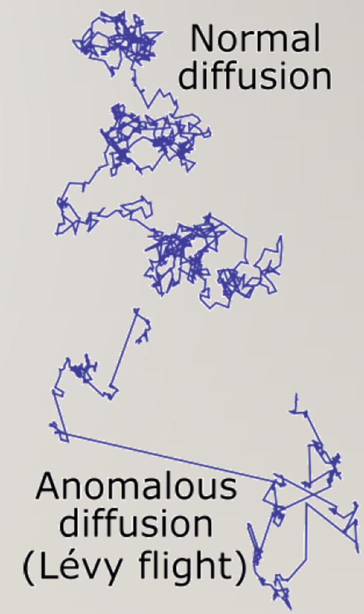
correlation funct. C(q)

- Only way to map out source space-time geometry on femtometer scale!

LÉVY DISTRIBUTIONS IN HEAVY ION PHYSICS

- Central limit theorem (diffusion) and thermodynamics lead to Gaussians
- Measurements suggest phenomena beyond Gaussian distribution
- Lévy-stable distribution: $\mathcal{L}(\alpha, R; r) = (2\pi)^{-3} \int d^3q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$
  - From generalized central limit theorem, power-law tail ~ $r^{-(1+\alpha)}$
  - Special cases: α = 2 Gaussian, α = 1 Cauchy





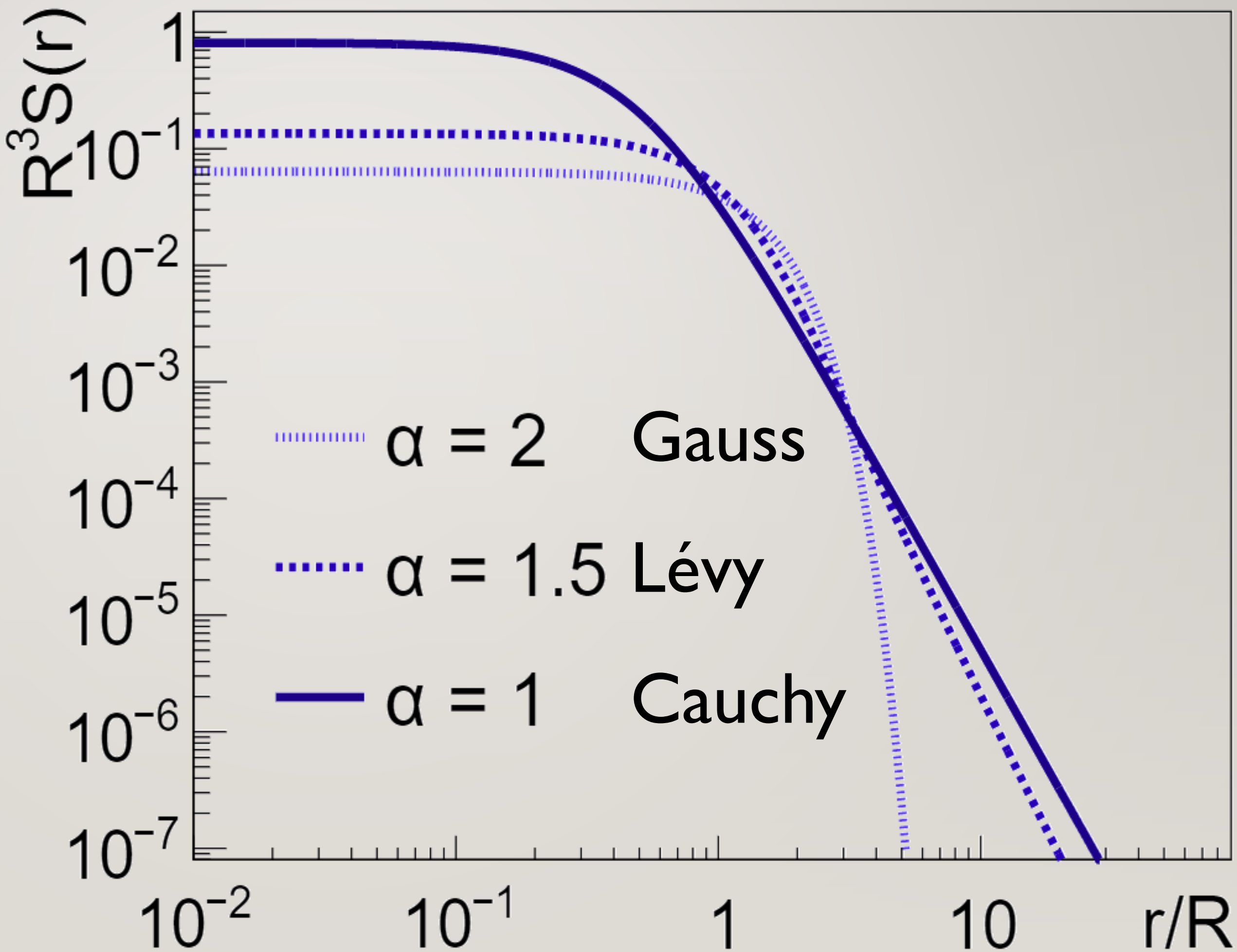
- Shape of the correlation functions with Levy source:
  - $C_2(q) = 1 + \lambda \cdot e^{-|qR|^\alpha}$; $\alpha = 2$: Gaussian; $\alpha = 1$: exponential
    Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67-78
- A possible reason for Levy source: anomalous diffusion, many others

6/28 WHY DOES LÉVY APPEAR, WHY IS IT IMPORTANT?

- A more comprehensive list of possible reasons:
  - Jet fragmentation (Csörgő, Hegyi, Novák, Zajc, Acta Phys.Polon. B36 (2005) 329-337)
  - Critical phenomena (Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828 (2006) no.1, 525-532)
  - Direction averaging and non-sphericality (Cimerman et al., Phys.Part.Nucl. 51 (2020) 282)
  - Event averaging (Cimerman et al., Phys.Part.Nucl. 51 (2020) 282)
  - Resonance decays (Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002; Kincses, Stefaniak, Csanád, Entropy 24 (2022) 308)
  - Hadronic rescattering, Lévy flight (Braz.J.Phys. 37 (2007) 1002; Entropy 24 (2022) 308)
- Importance of utilizing Lévy sources:
  - Measuring α and R
    - Order of quark-hadron transition, critical point search
    - General understanding of source dynamics
  - Measuring λ also requires correct shape assumption
    - In-medium mass modification, coherent pion production

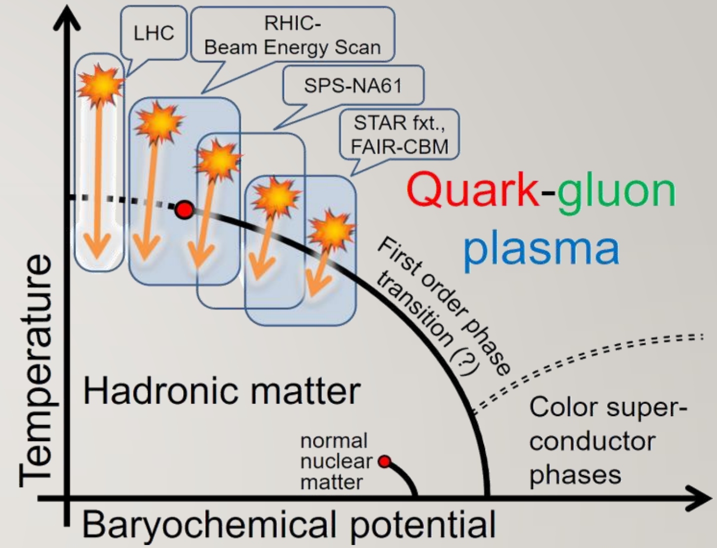
7 /28 LÉVY VERSUS GAUSS VERSUS EXPONENTIAL

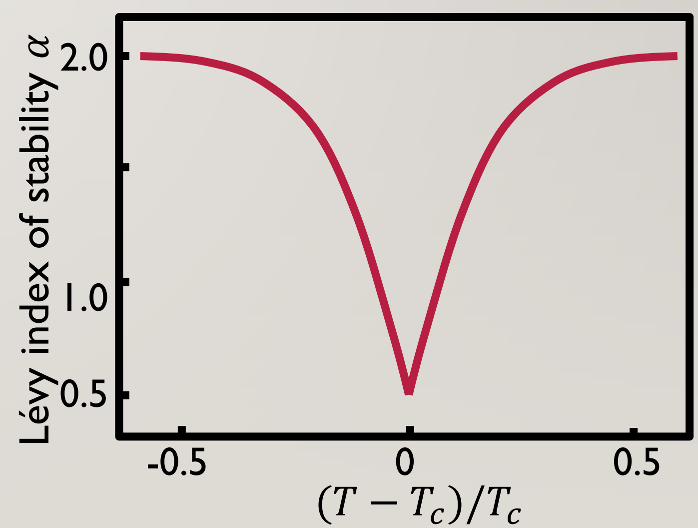
- No tail if $\alpha = 2$, power law if $\alpha < 2$; tail strength depends on α
- If $S(r)$ Lévy, $D(r)$ also Lévy with same α and $R \to 2^{1/\alpha}R$
- In principle, RMS = ∞ if $\alpha < 2$
- In practice, RMS depends on cutoff
- What do Gaussian HBT radii mean?



LÉVY INDEX AS A CRITICAL EXPONENT?

- Critical spatial correlation: $\sim r^{-(d-2+\eta)}$; Lévy source: $\sim r^{-(1+\alpha)}$; $\alpha \Leftrightarrow \eta$?
  Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67,
- QCD universality class ↔ 3D Ising
  Halasz et al., Phys.Rev.D58 (1998) 096007
  Stephanov et al., Phys.Rev.Lett.81 (1998) 4816
- At the critical point:
  - Random field 3D Ising: η = 0.50±0.05
    Rieger, Phys.Rev.B52 (1995) 6659
  - 3D Ising: η = 0.03631(3)
    El-Showk et al., J.Stat.Phys.157 (4-5): 869
- Motivation for precise Lévy HBT!
- Change in α_{Levy} proximity of CEP?
- Finite size/time & non-equilibrium effects → what does power law mean?

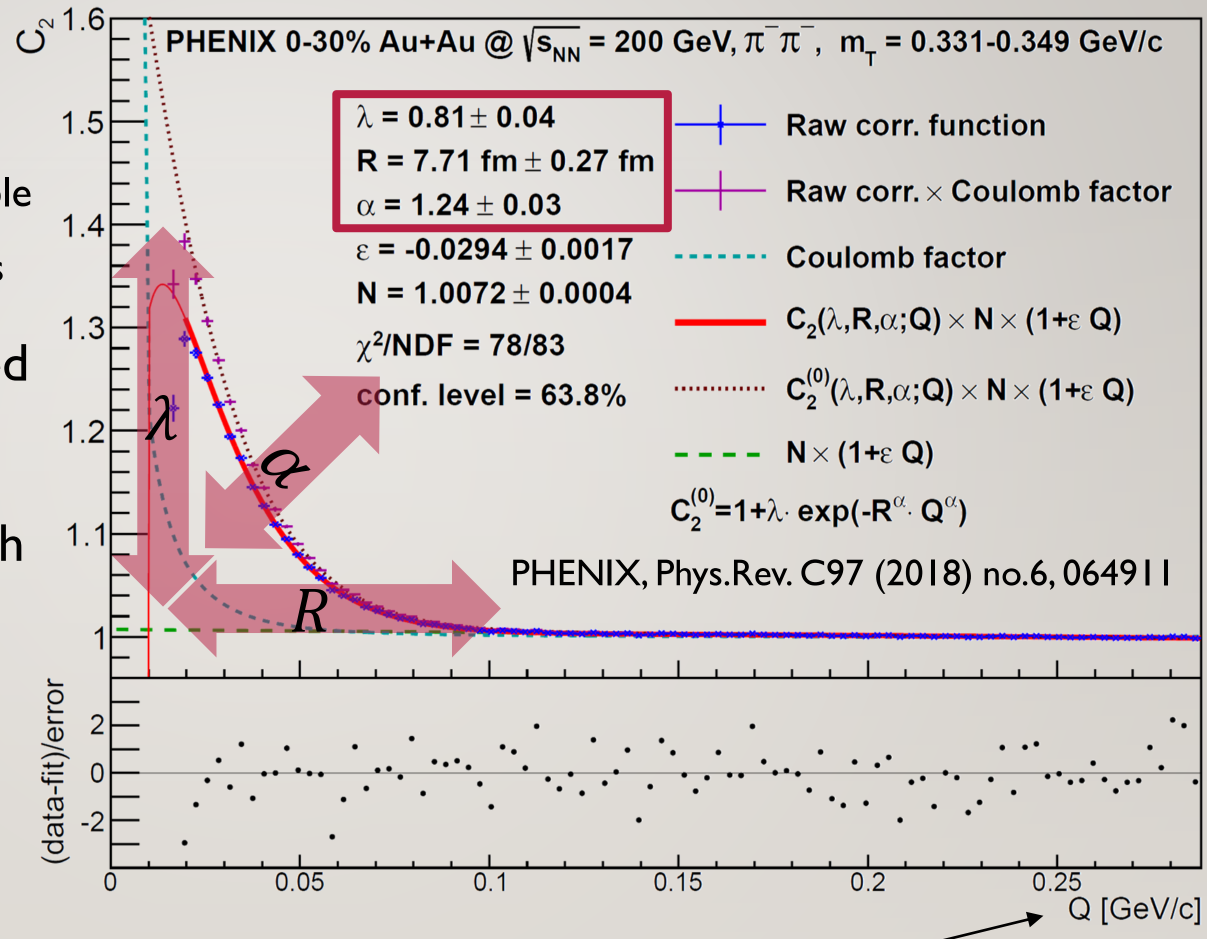




CONTENTS OF THIS TALK

10/28 EXAMPLE $C_2(Q_{LCMS})$ CORRELATION FUNCTION

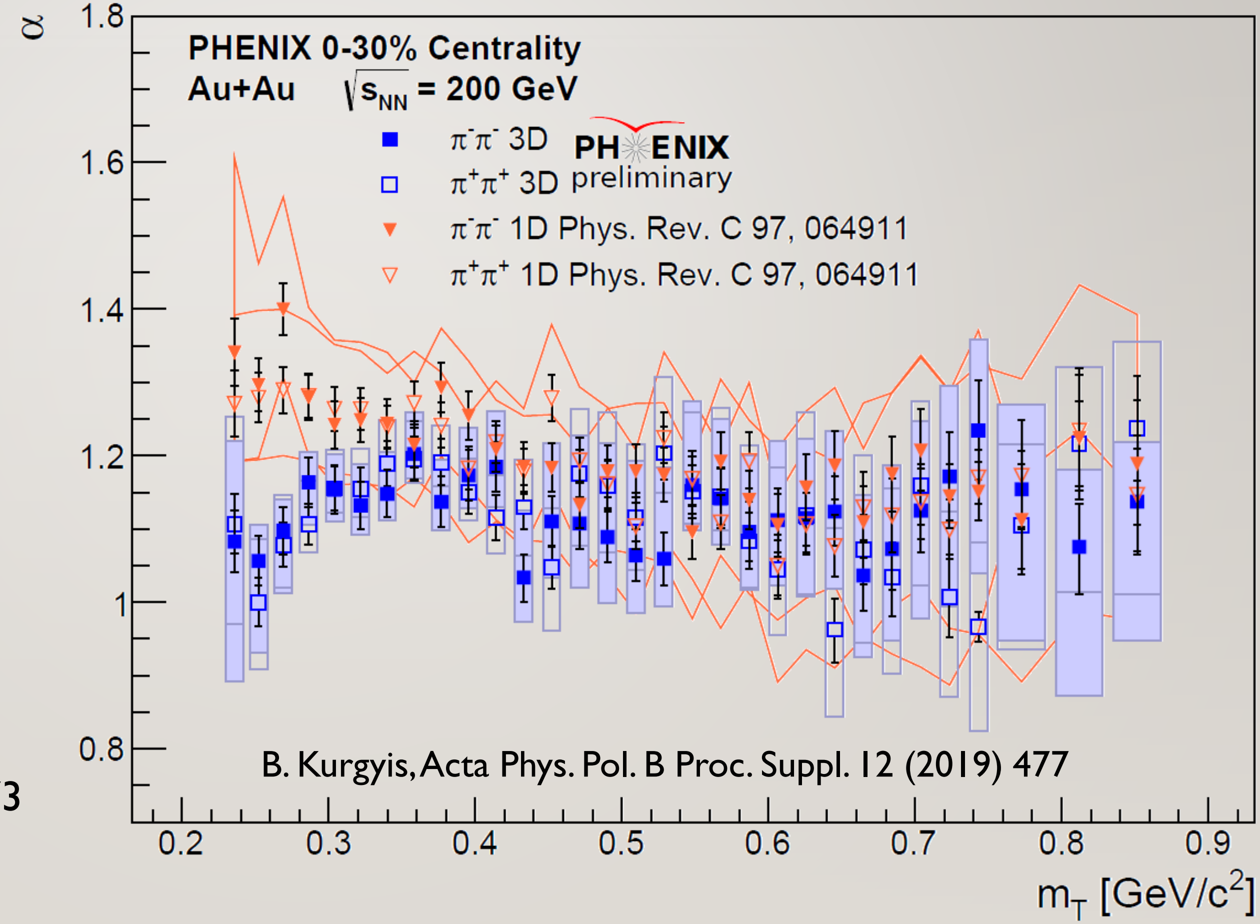
- Correlation function: spherical in LCMS
  - 1D measurement possible
  - Done in several m_T bins
- Fit with calculation based on Lévy distribution
- Only converging fits with good confidence level accepted
- Physical parameters: R, λ, α measured versus pair m_T



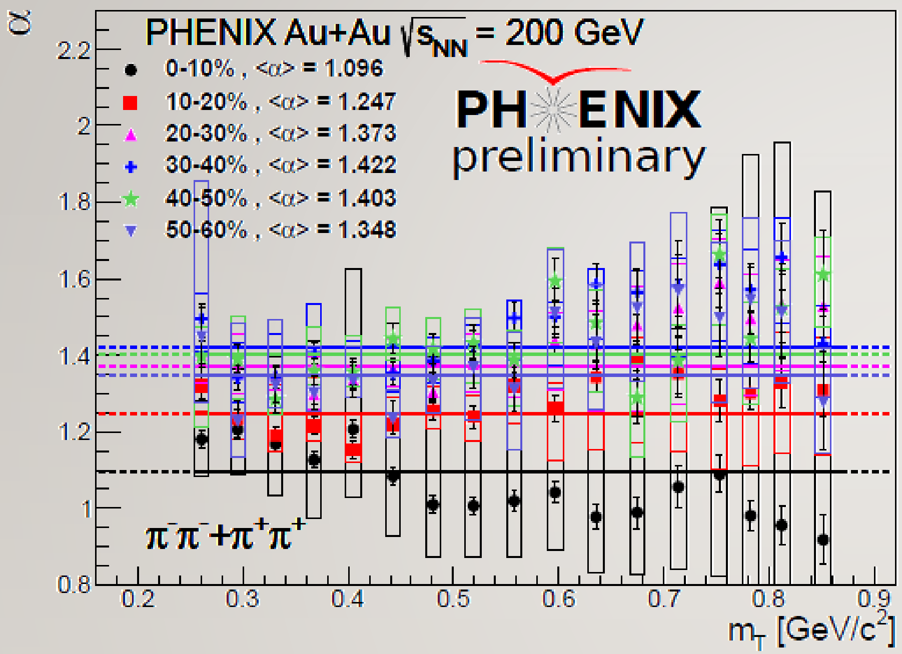
Q: 1D momentum difference in Longitudinally CoMoving System (LCMS)

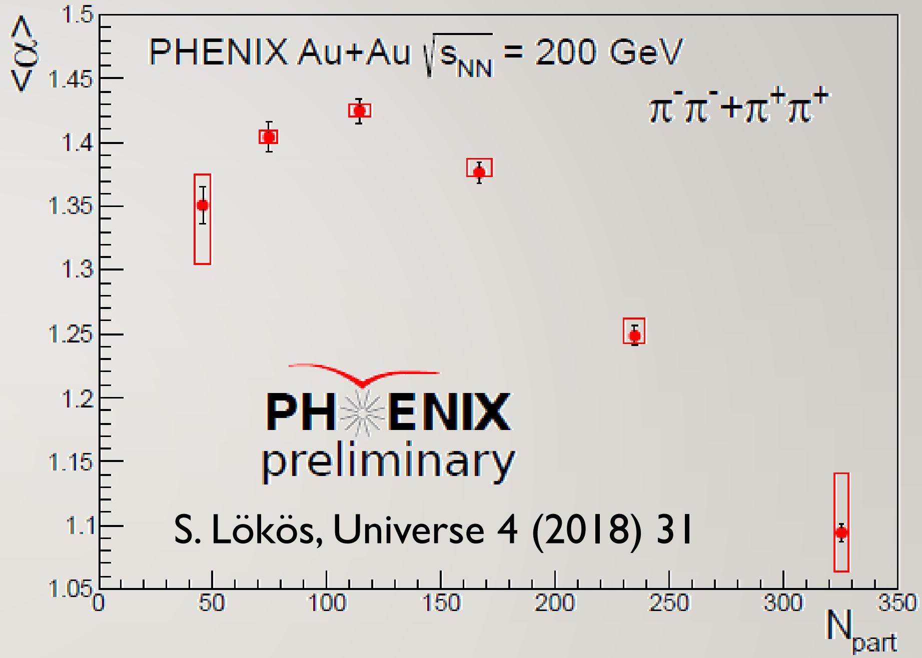
11 /28 LÉVY EXPONENT IN 1D VS 3D

- Lévy exponent α in 3D analysis similar to 1D result
- On average still far from 2
- Observable differences at low m_T
- Maybe due to lack of spherical symmetry?
- Coulomb effect for non-spherical sources?
  - Approximation possible
    B. Kurgyis, arXiv:2007.10173



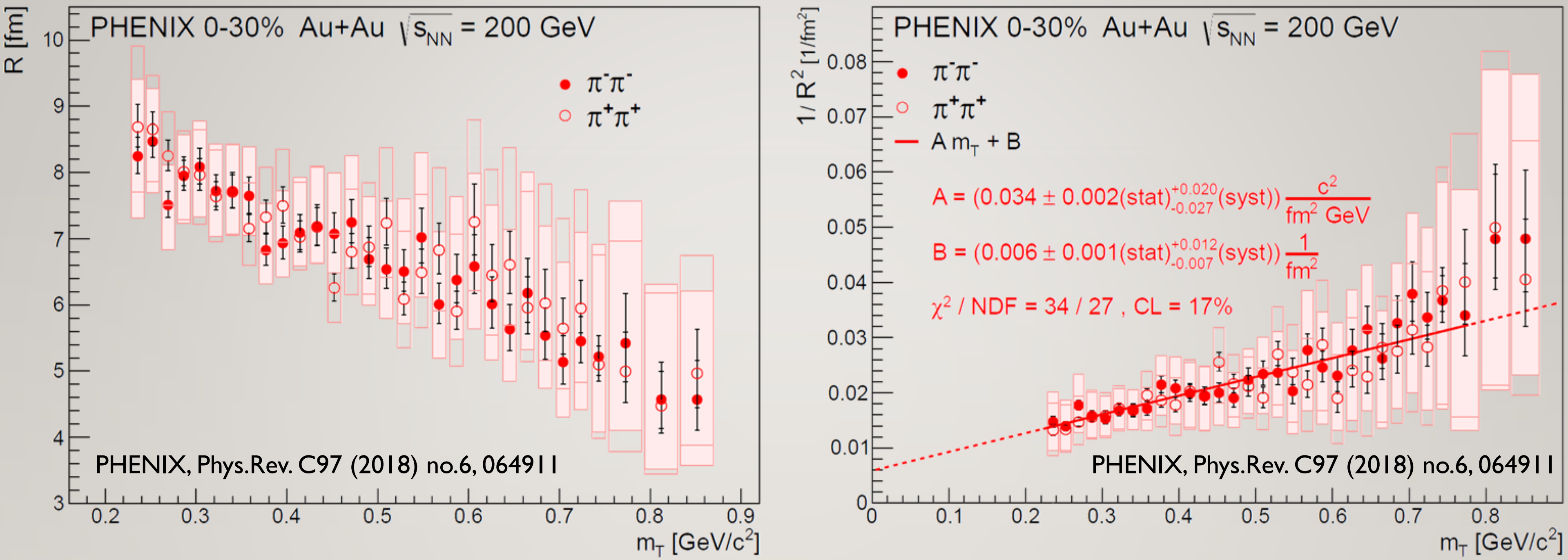
12/28 ANALYZING THE CENTRALITY DEPENDENCE





- Slightly non-monotonic behavior as a function of m_T, averaging still possible
- $\langle\alpha\rangle$ vs N_{part}: slightly non-monotonic behavior versus, decreasing for large N_{part}
- No clear interpretation or understanding of this trend, need theory comparision
- Final data and publication in the works at PHENIX

LÉVY SCALE PARAMETER R AT RHIC

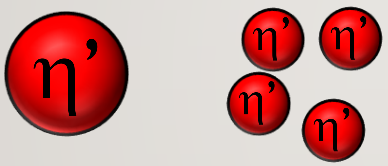


- Similar decreasing trend as Gaussian HBT radii, but it is not an RMS!
  - RMS of a Lévy source: in principle infinity, obtained value depends on cutoff
- What do model calculations, simulations say about this?
- Hydro behavior ($1/R^2 \sim m_T$, predicted for Gaussian case) not invalid

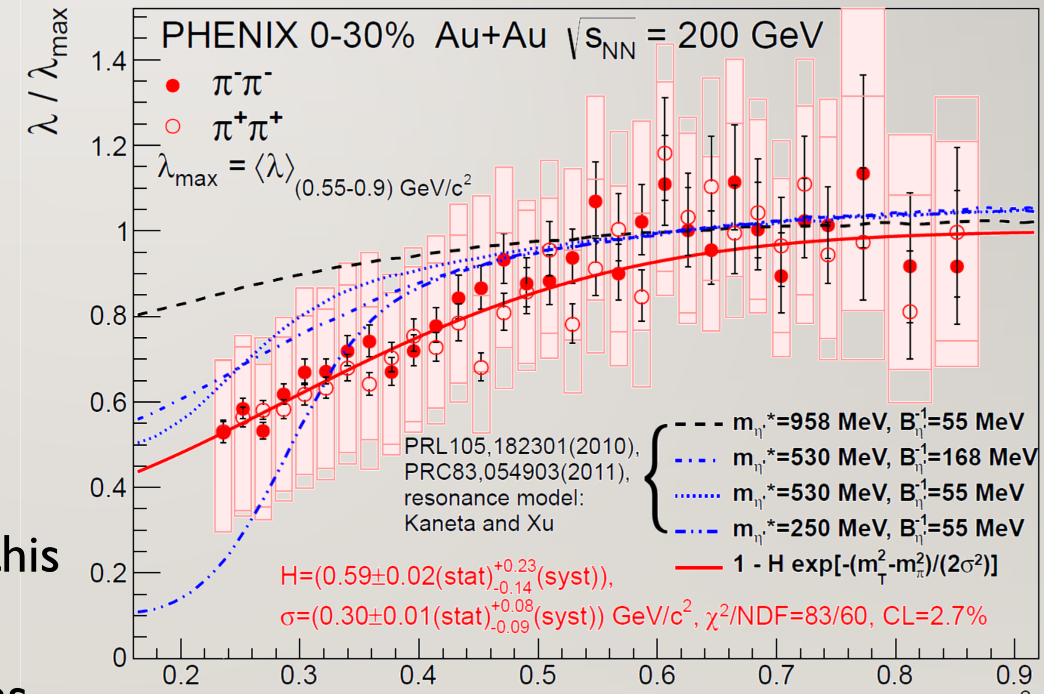
14/28 CORRELATION STRENGTH λ: IN-MEDIUM MASS?

- Connection to chiral restoration
  - Decreased η' mass → more η' produced → more decay pions → λ decreases
  - Kinematics: η' → ππππ with low m_T → decreased $\lambda(m_T)$ specifically at low m_T
  - Dependence on in-medium η' mass?

Kapusta, Kharzeev, McLerran, PRD53 (1996) 5028
Vance, Csörgő, Kharzeev, PRL 81 (1998) 2205
Csörgő, Vértesi, Sziklai, PRL105 (2010) 182301



$$T < T^*$$
$$m_{\eta'} > m^*_{\eta'}$$
$$N_{\eta'} < N^*_{\eta'}$$
$$\lambda > \lambda^*$$



PHENIX, Phys.Rev. C97 (2018) no.6, 064911

- Results not incompatible with this
- 3D results similar to 1D
- Need direct check with photons

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EVENT BY EVENT SHAPE ANALYSIS WITH EPOS

- EPOS model: parton-based Gribov-Regge theory (PBGRT)
  - K. Werner et al., PRC82 (2010) 044904, PRC89 (2014) 064903, …
  - Core-Corona division, viscous hydro evolution (vHLLE), hadronic cascades (UrQMD)
- $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions generated by EPOS359
- Pair distribution calculated: $\boldsymbol{D(r_{LCMS}) = \int d\Omega dt D(t, r_x, r_y, r_z)}$
  angle-averaged radial source distribution of like-sign pion pairs

$$r_{LCMS} = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z_{LCMS})^2}; \Delta z_{LCMS} = \Delta z - \frac{\beta(\Delta t)}{\sqrt{1-\beta^2}}; \beta = \frac{p_{z,1} + p_{z,2}}{E_1 + E_2}$$

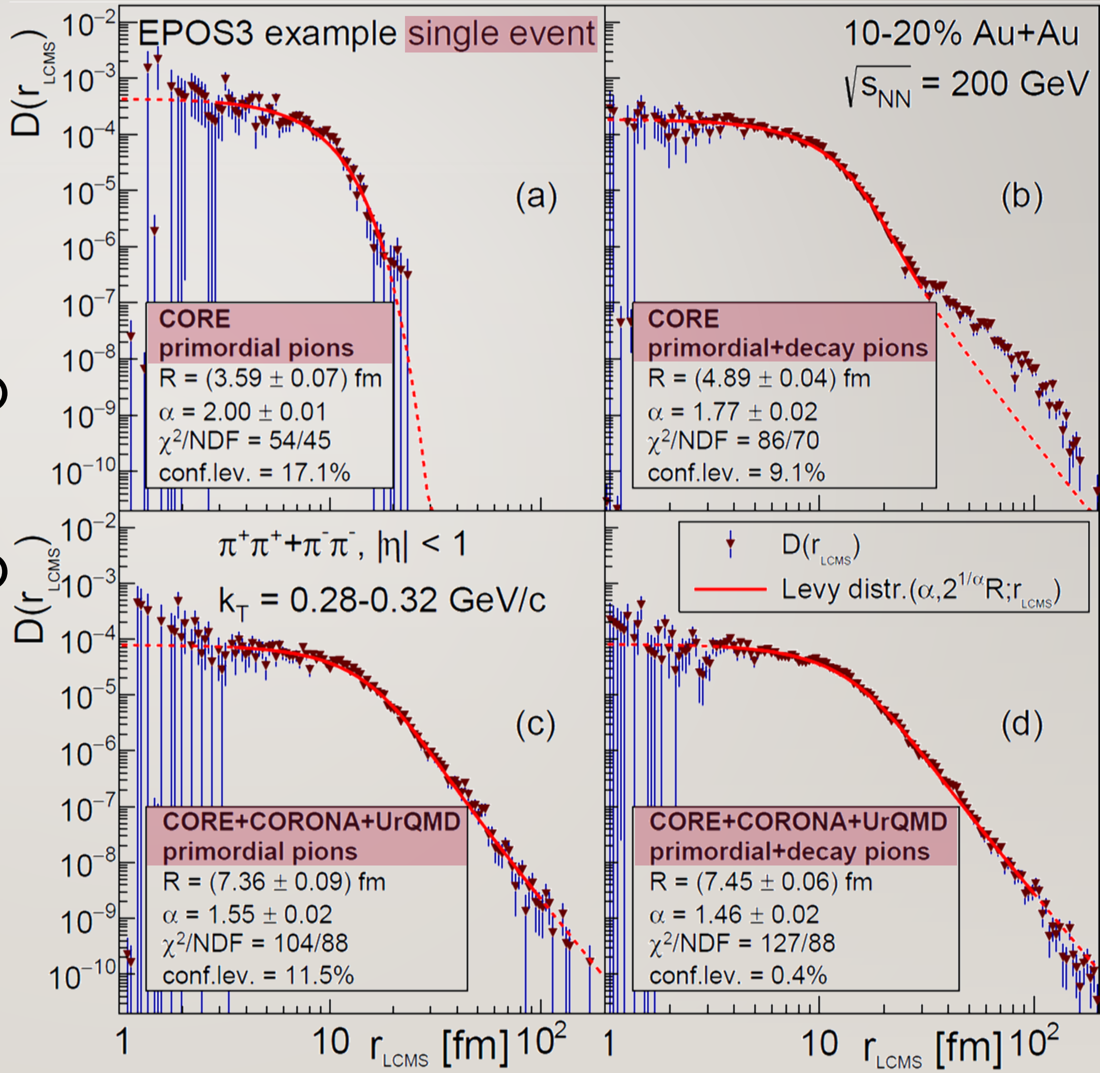
- Investigated cases:
  - CORE, primordial pions only
  - CORE, decay products included
  - CORE+CORONA+UrQMD, primordial pions only
  - CORE+CORONA+UrQMD, decay products included

Kincses, Stefaniak, Csanád, Entropy 24 (2022) 308 [arXiv:2201.07962]

VARIOUS PARTICLE SETS COMPARED

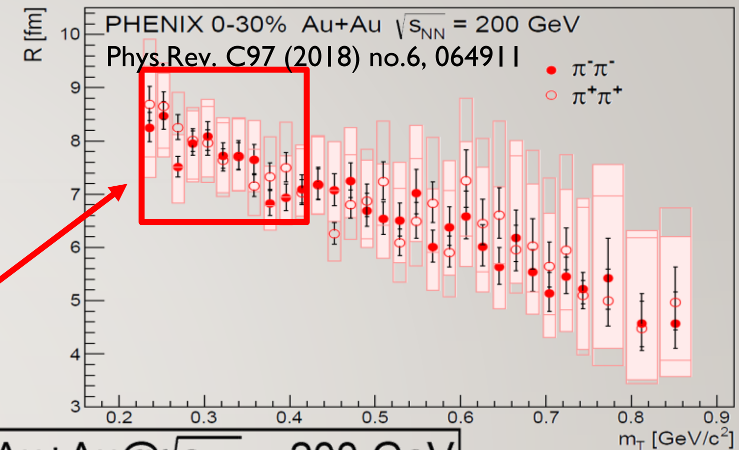
- CORE, primordial pions
  - Gaussian source
- CORE + decays
  - power-law structures
- CORE+CORONA+UrQMD
  - Lévy-shape
- CORE+CORONA+UrQMD +decays
  - Lévy-shape
- Important: Lévy appears in all single events!

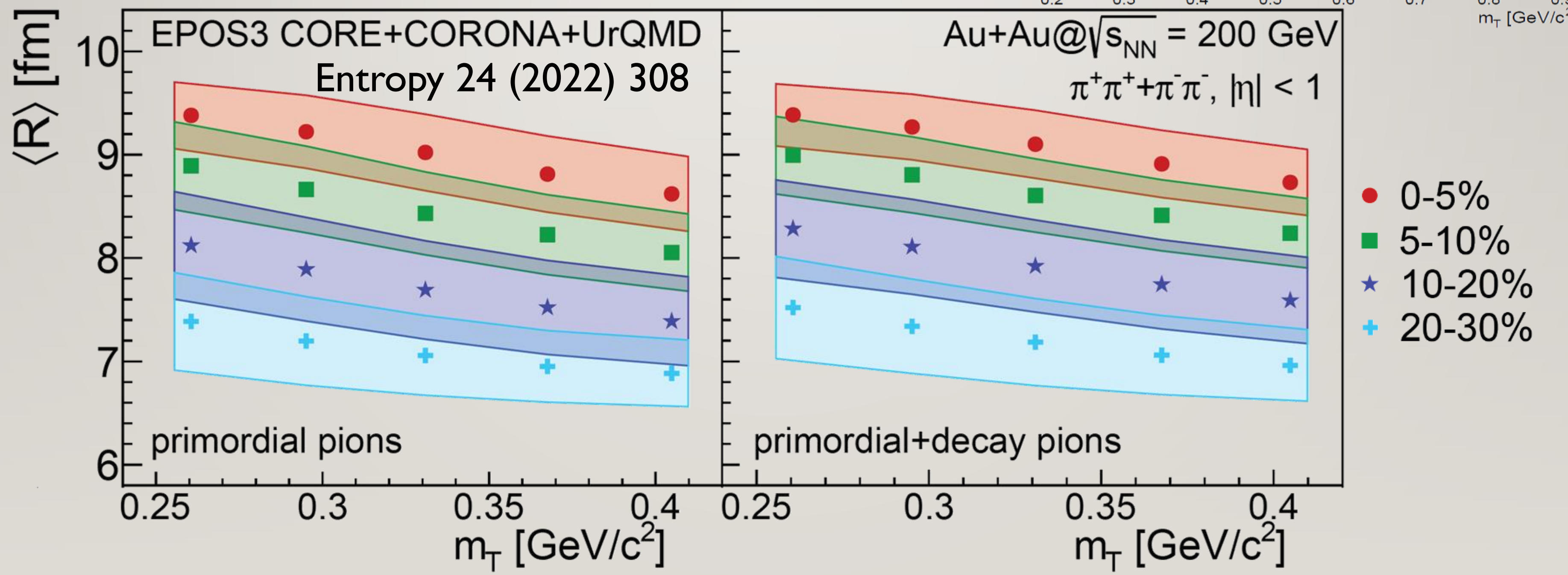
Kincses et al., Entropy 24 (2022) 308



AVERAGE LÉVY SCALE R VS TRANSVERSE MASS

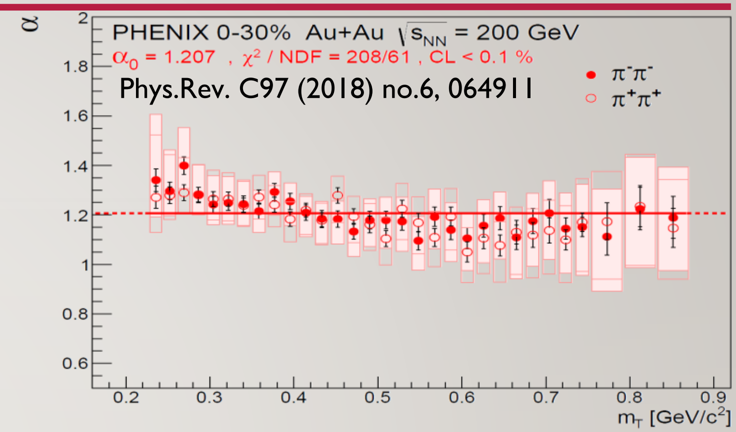
- $\langle R \rangle$ as a function of m_T and centrality
  - Clear dependence on both
  - Distribution width displayed as uncertainty band
- Trends, magnitudes like data
- With decays: slightly higher $\langle R \rangle$ values

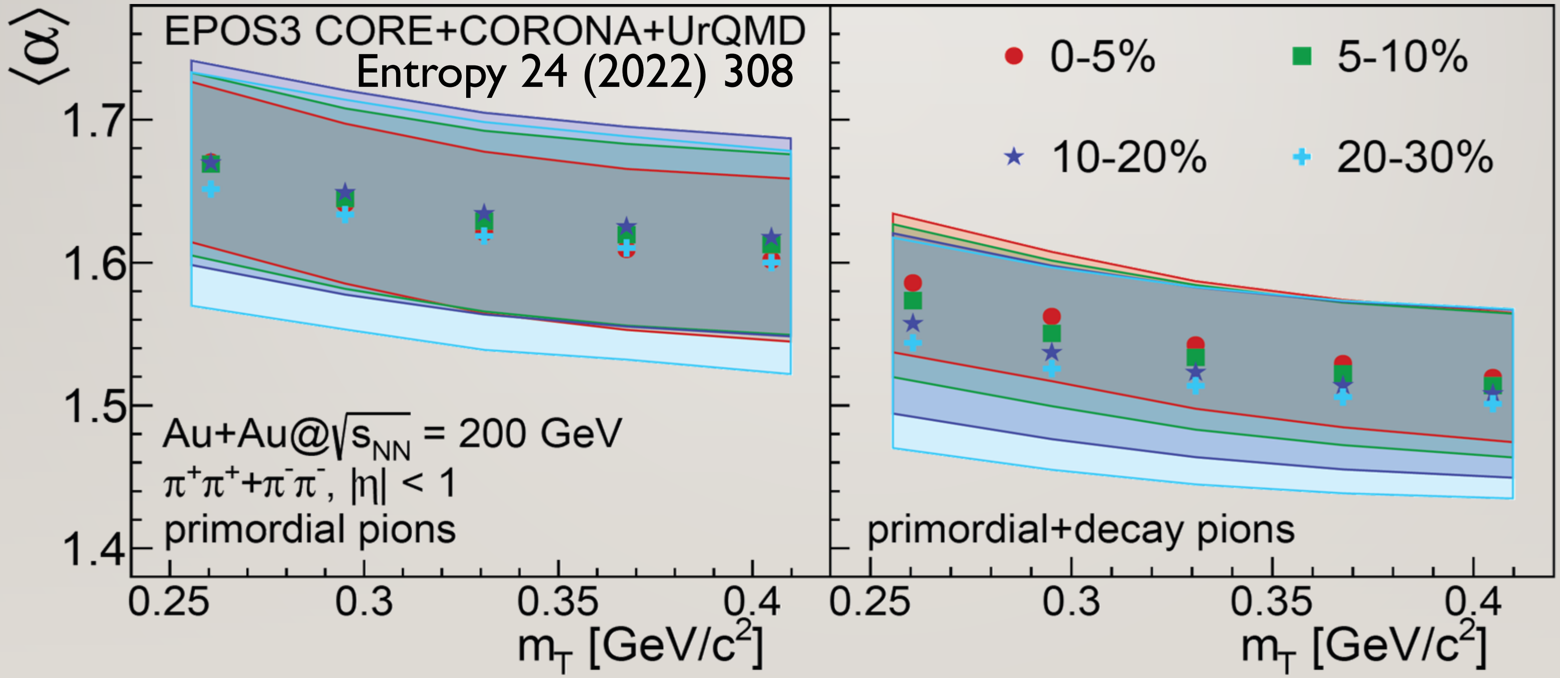




19/28 AVERAGE LÉVY EXPONENT VS TRANSVERSE MASS

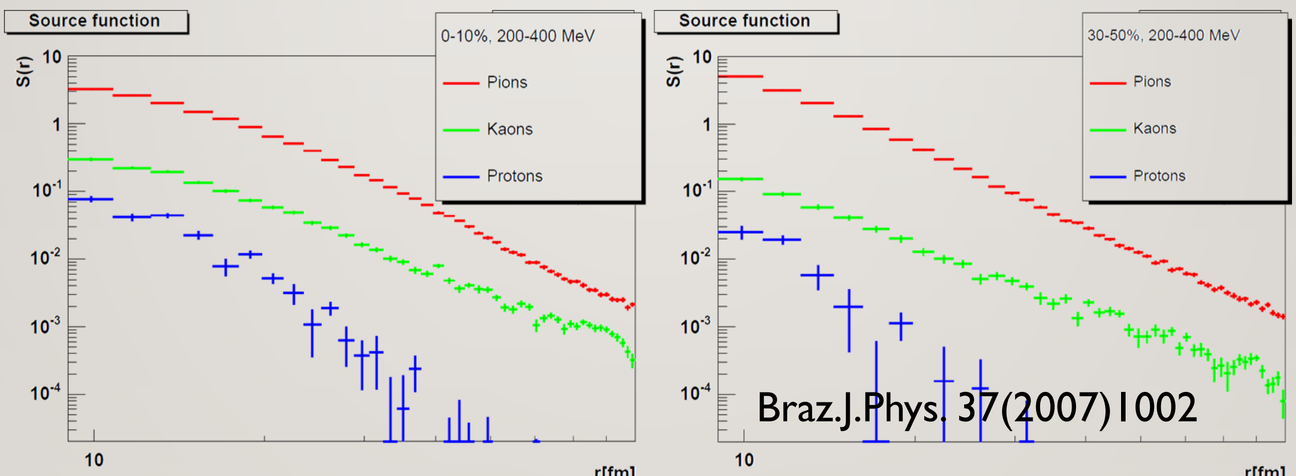
- $\langle\alpha\rangle$ as a function of m_T and centrality
  - Small m_T and centrality dependence
- Both with and without decays: larger than data
- See poster by Dániel Kincses
  + new manuscript at LHC energy





THE IMPORTANCE OF A KAON ANALYSIS

- Kaons: smaller cross-section, larger mean free path
- Heavier power-law tail?
- Prediction for π, K, p based on Humanic's Resonance Model (HRM): anomalous diffusion due to rescattering
  Humanic, Int.J.Mod.Phys. E15 (2006) 197 [nucl-th/0510049]
  Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002 [hep-ph/0702032]



- Kaon HBT radii: m_T scaling or its violation for Lévy scale R?
- Prediction: $\alpha(p) > \alpha(\pi) > \alpha(K)$

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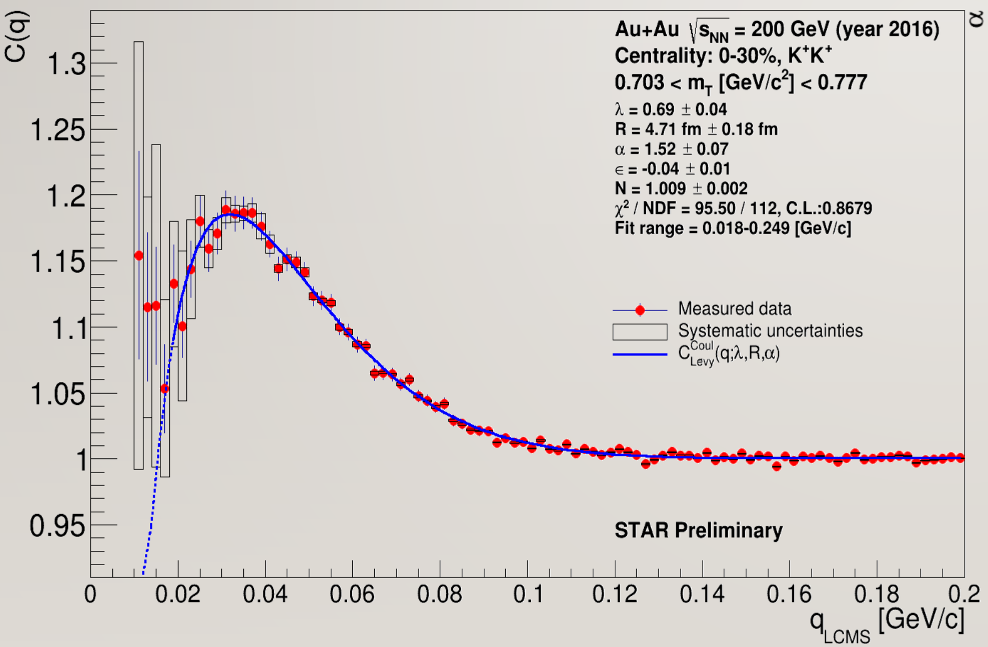
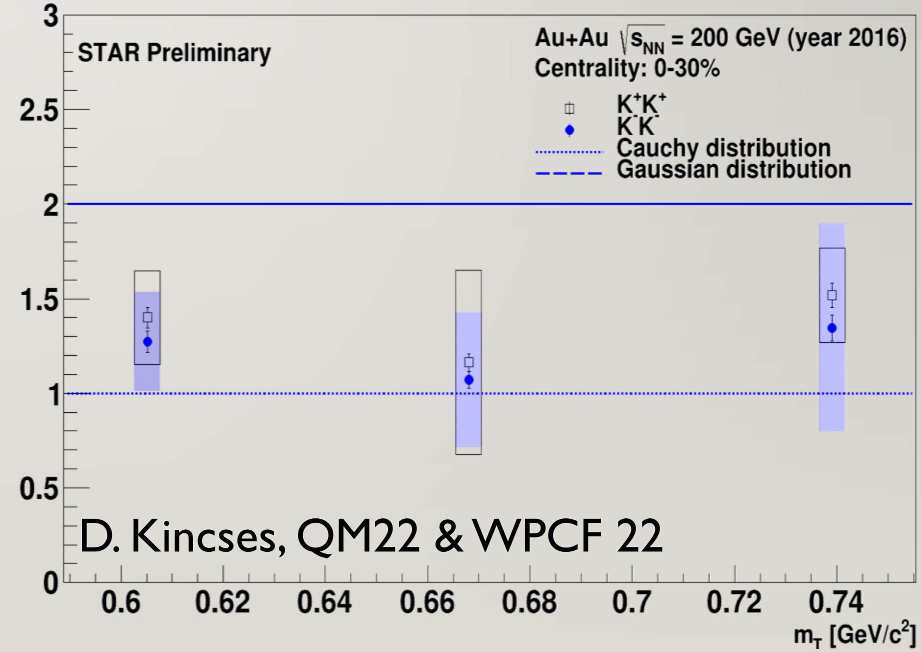
KAON ANALYSIS AT STAR

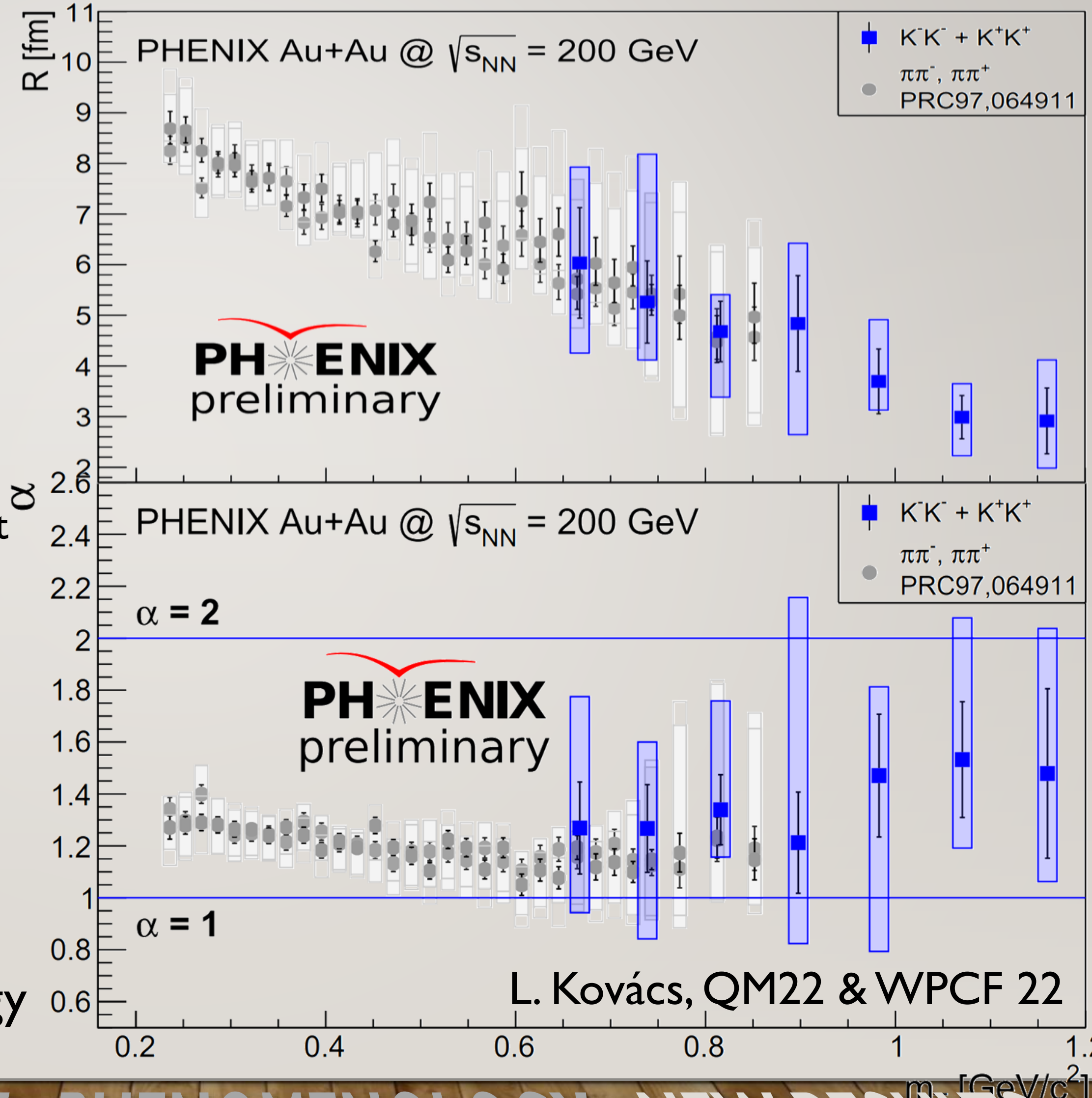
- Data successfully described by Lévy fits
- Lévy-stability parameter α between 1 and 2
- Kaon and pion source of same shape at the same m_T?
- Unlike anomalous diffusion expectation of $\alpha(K) < \alpha(\pi)$





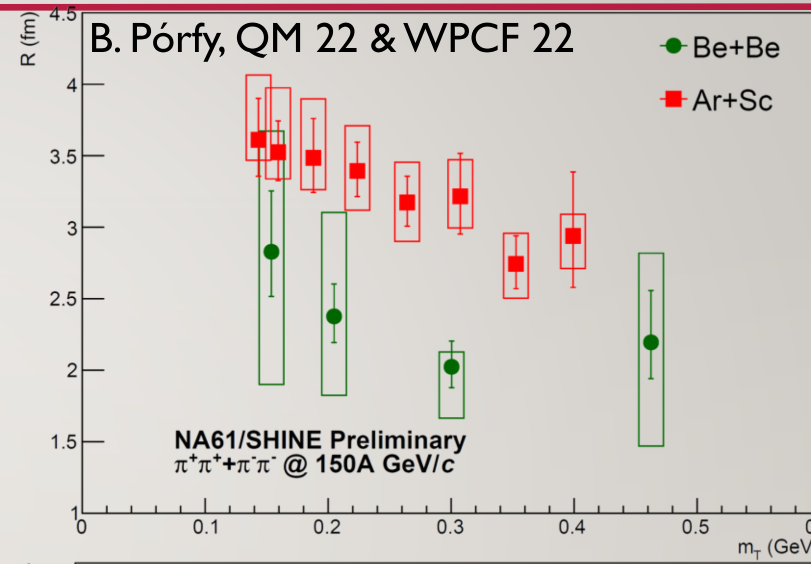
KAON ANALYSIS AT PHENIX

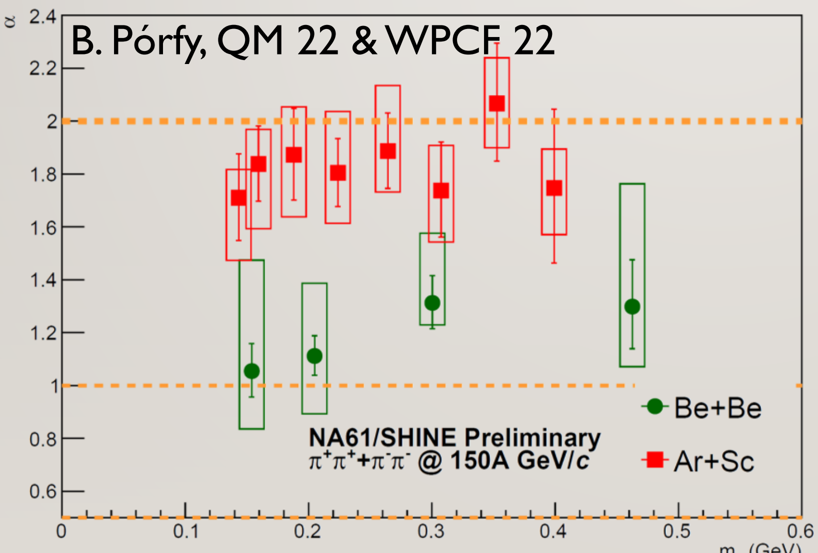
- More detailed analysis performed at PHENIX
- Kaon and pion data seem comtabile at the same m_T
- Lévy scale R shows hydro type of scaling with m_T
  - R depending on m_T but not on particle type separately
- $\alpha(K) \geq \alpha(\pi)$, but anomalous diffusion suggests opposite
- Dominant mechanism?
- See poster by Márton Nagy

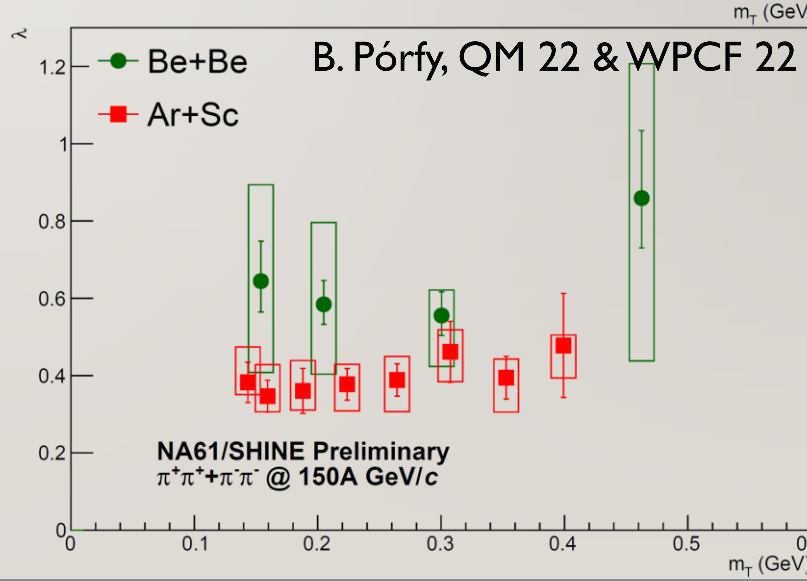


PION ANALYSIS AT SPS NA61/SHINE

- Lévy scale R of Ar+Sc and Be+Be:
  - Compatible with initial geometry factor 1.6
  - Decrease with m_T due to transverse flow?
- No m_T dependence in λ, in contrast to RHIC result – can be turned off?
- Lévy index α: significant difference
- See next talk by Barnabás Pórfy

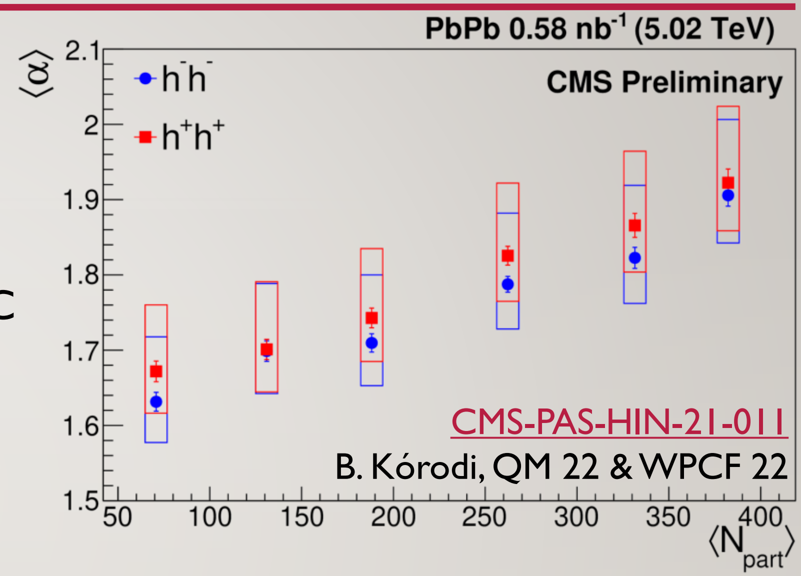


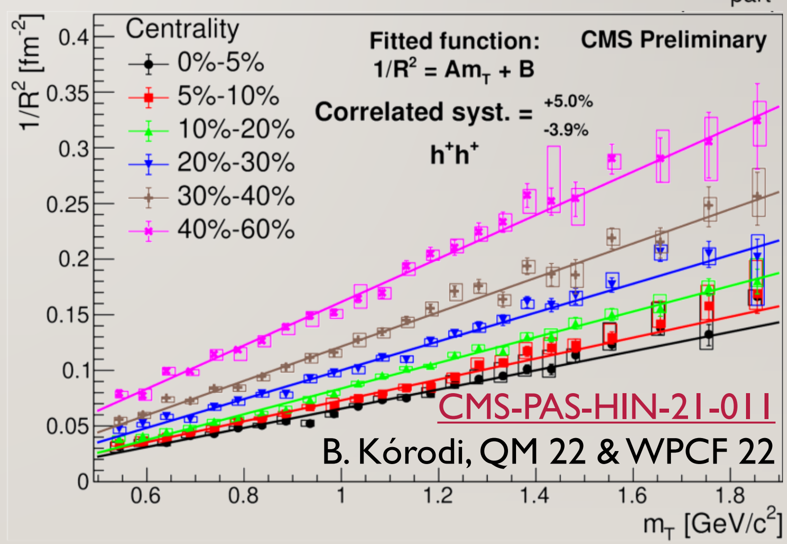




CHARGED HADRON ANALYSIS IN 5 TEV PB+PB

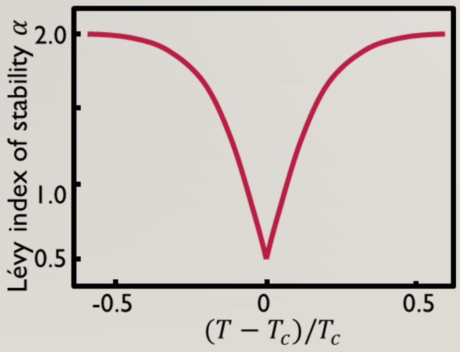
- Lévy index α
  - Far from Cauchy
  - Not exactly Gaussian
  - Closer to Gaussian for large N_{part}, unlike RHIC
- Lévy scale R: hydro scaling confirmed
  - In every centrality class
  - Despite non-Gaussianity
  - Hubble coefficient can be extracted: 0.12-0.18 c/fm
- Correlation strength also analyzed
- Low-Q deviation cross-checked with Monte-Carlo: two-track acceptance
- See poster by Balázs Kórodi

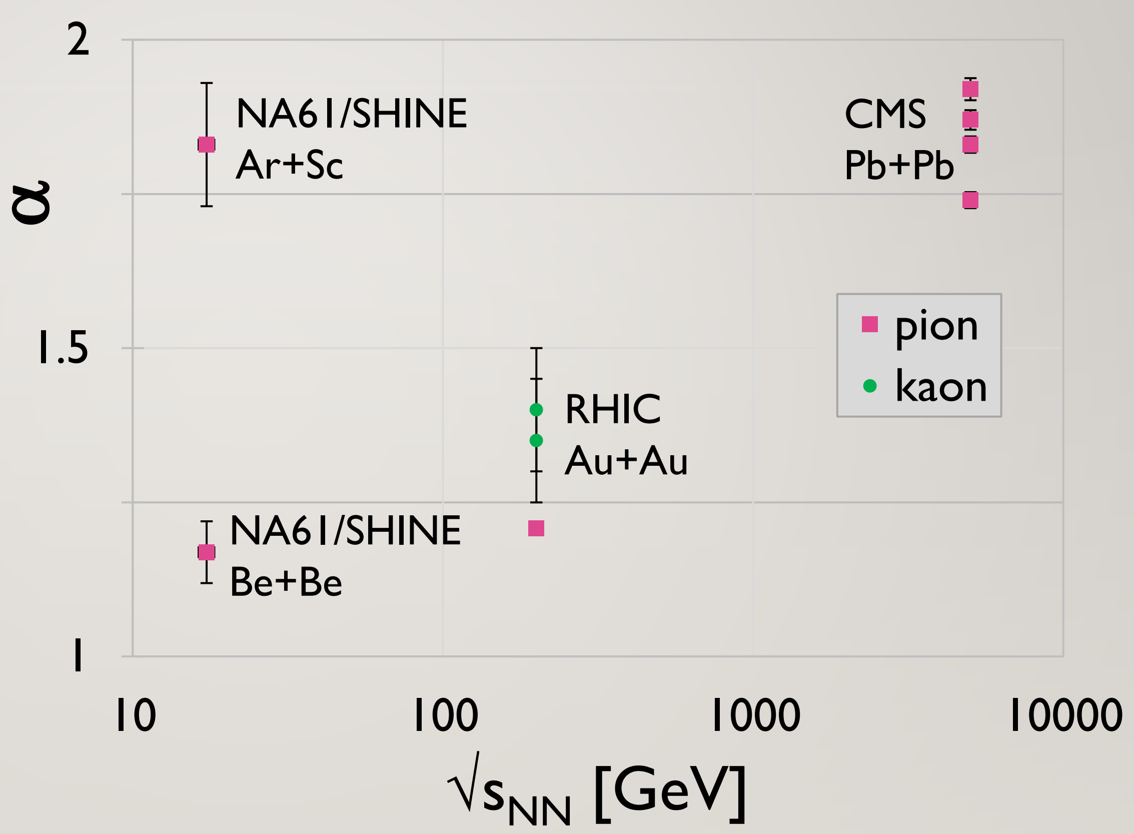




STABILITY PARAMETER α FROM SPS TO LHC

- Different values for small and medium systems at SPS
- Medium and large systems: increasing trend from SPS to RHIC to LHC
- Compare to:





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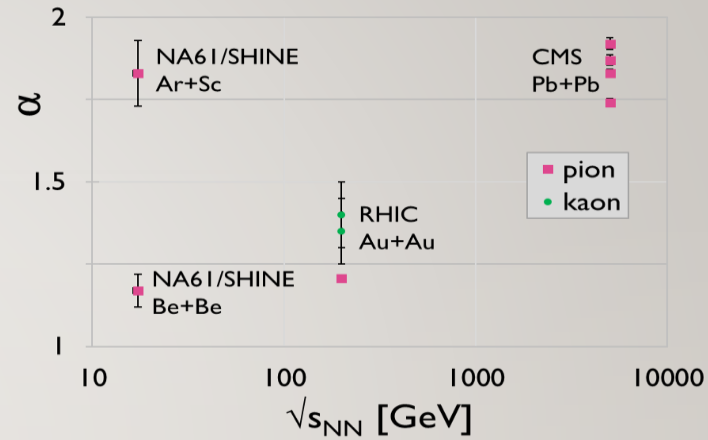
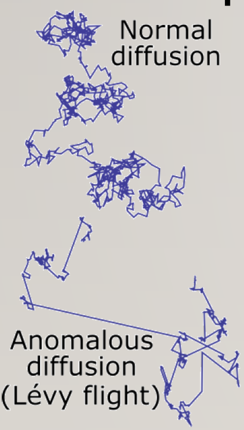
CONCLUSIONS AND OUTLOOK

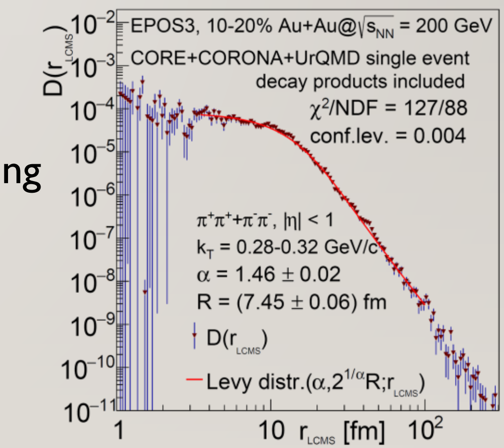
- Lévy sources from SPS to RHIC and LHC
  - **Lévy α**: between 1 and 2, increases with $\sqrt{s_{NN}}$?
    - Contrary to expectations, $\alpha(K) \geq \alpha(\pi)$
  - **Lévy R**: hydro scaling, despite not Gaussian
  - **Lévy λ**: signs of η' in-medium mass modification

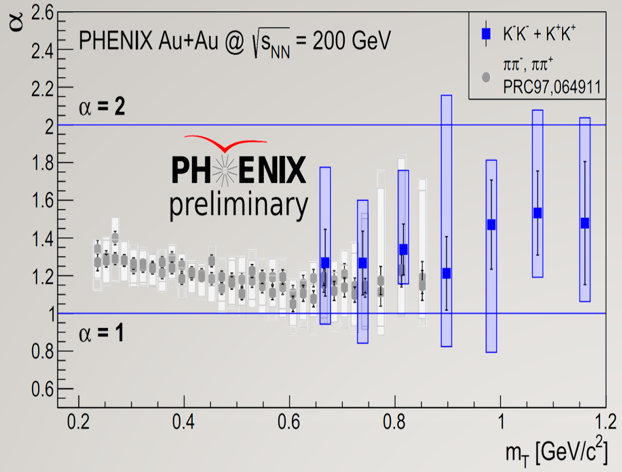


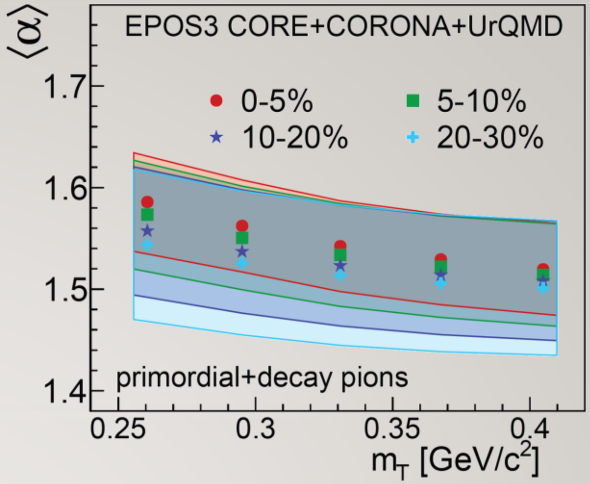
- Possible reasons:
  - Jet fragmentation → not dominant in AA collisions
  - **Critical phenomena** → maybe at lowest RHIC energies and SPS
  - Directional averaging → source is (approx.) spherical in LCMS, 3D cross-check done
  - Event averaging → event-by-event simulations show Lévy
  - **Resonance decays** → part of the reason, not enough alone
  - **Hadronic rescattering, Lévy flight** → $\alpha(K) \geq \alpha(\pi)$ puzzling



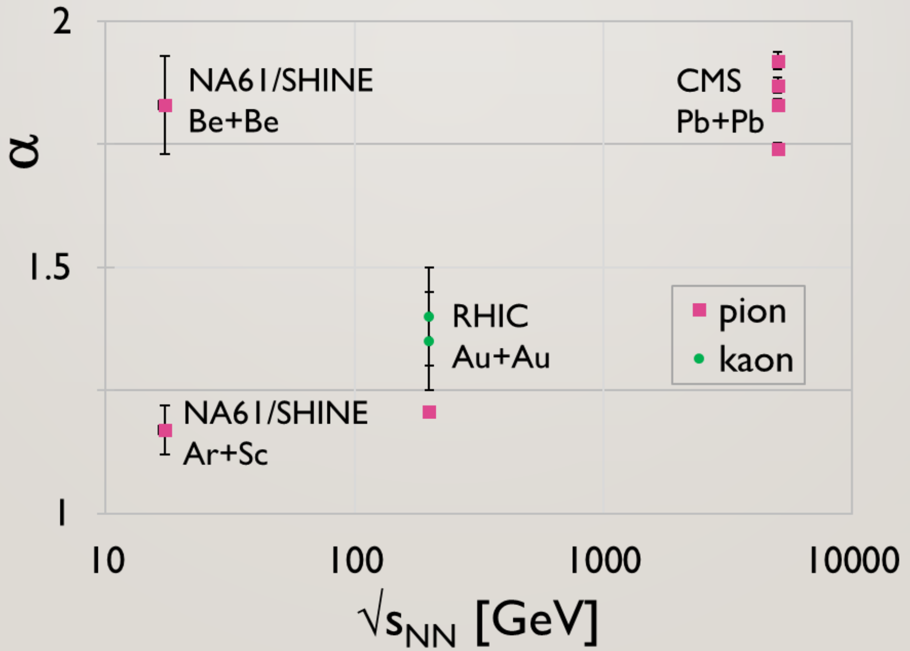
- Questions to be answered:
  - When measuring α, what effects need to be considered?
  - Can there be anomalous diffusion in the quark stage?
  - What is the role of finite size and finite time?







29 THANK YOU FOR YOUR ATTENTION



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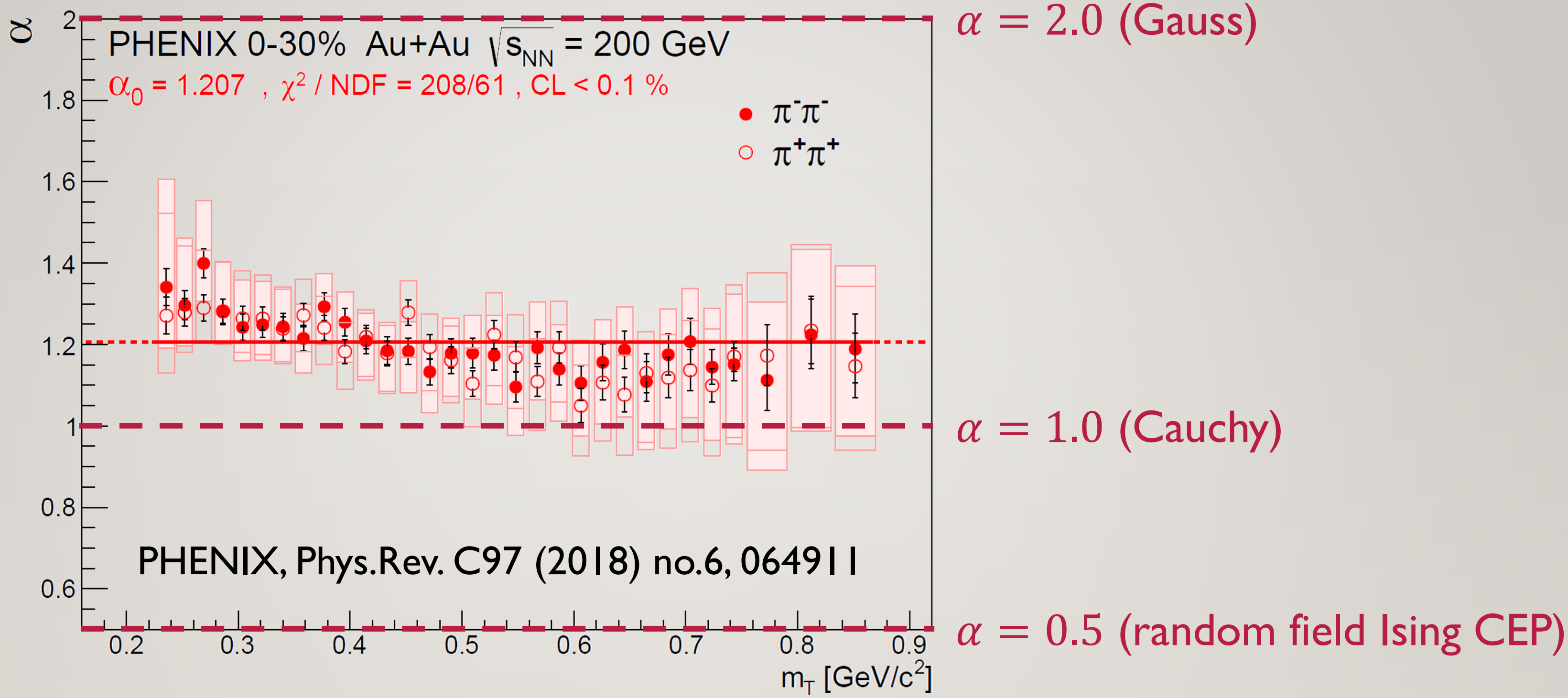
BACKUP

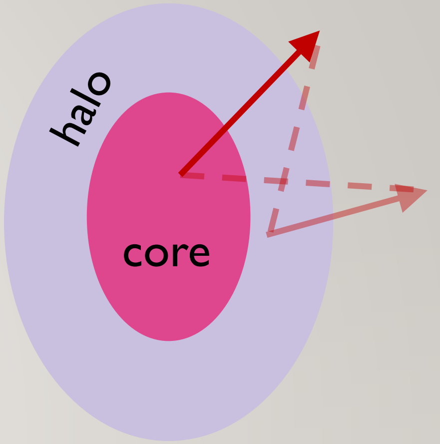
LÉVY EXPONENT α IN 200 GEV AU+AU AT RHIC



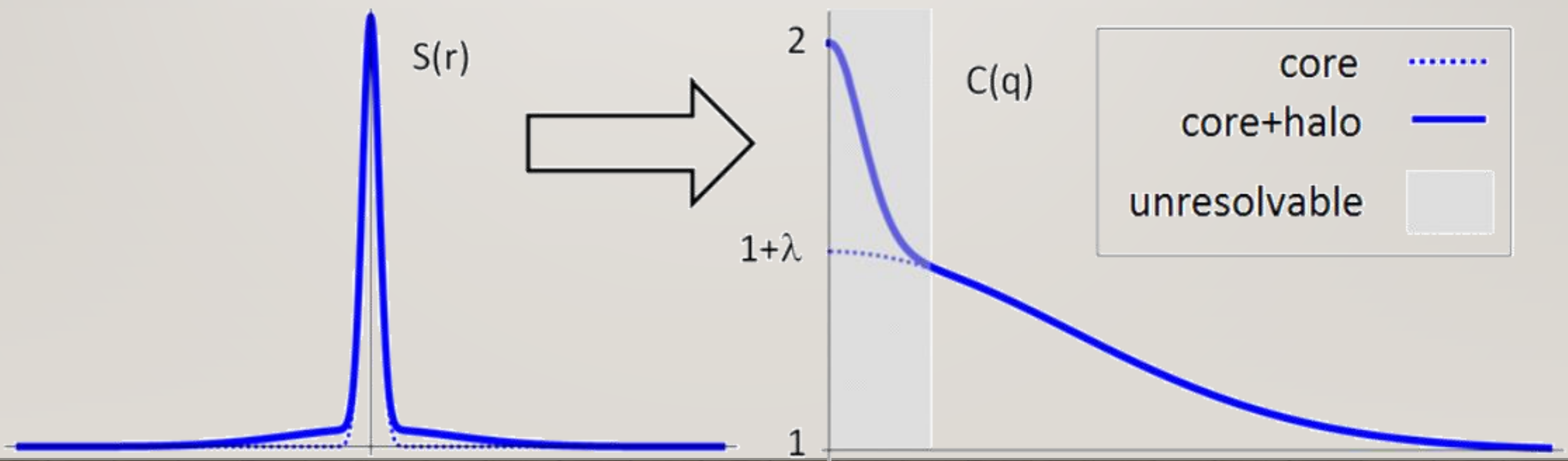
- Measured value far from Gaussian (α = 2), inconsistent with expo. (α = 1)
- Far from random field 3D Ising value at CEP (α = 0.5)
- Approximately constant (at least within systematic uncertainties)
- What do models and calculations say?

CORRELATION STRENGTH λ: CORE/HALO

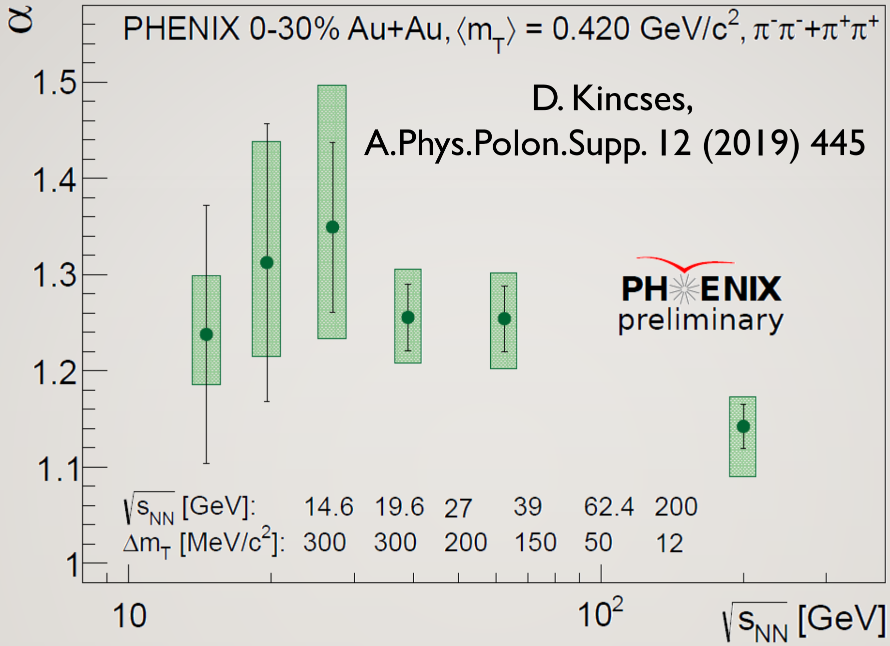
- Two-component core+halo source
  - Core: hydrodynamically expanding, thermal medium
  - Halo: long lived resonances (≳ 10 fm/c, ω,η,η',K_0^S,...)
  - Unresolvable experimentally
  - Define $f_C = N_{\text{core}}/N_{\text{total}}$
- True $q \to 0$ limit: $C(0) = 2$
- Apparently $C(q \to 0) \to 1 + \lambda$
- $\lambda(m_T) = f_C^2(m_T)$



Bolz et al, Phys.Rev. D47 (1993) 3860-3870; Csörgő, Lörstad, Zimányi, Z.Phys. C71 (1996) 491-497



COLLISION ENERGY DEPENDENCE

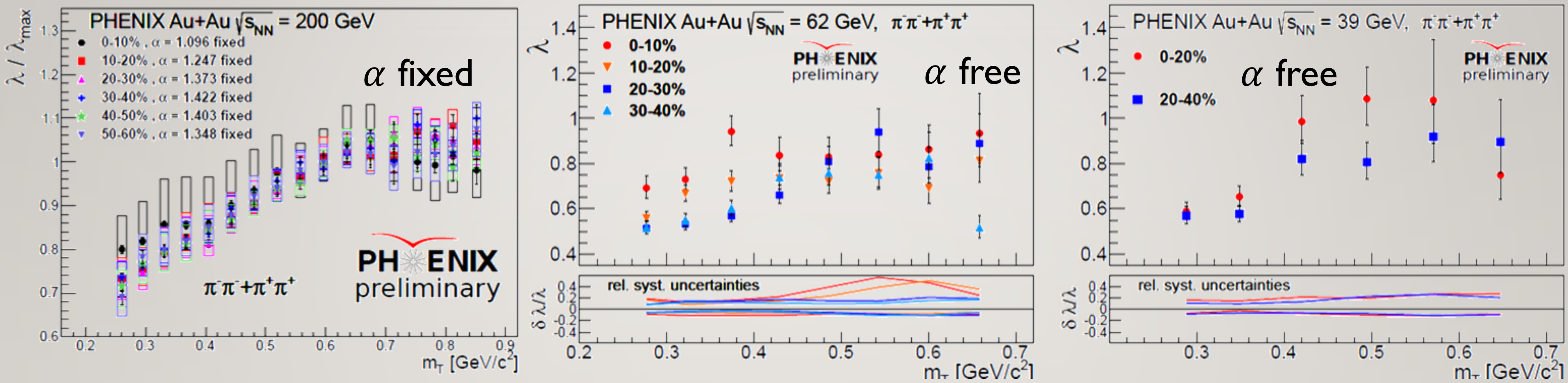


- $\langle\alpha\rangle$ approximately monotonic versus $\sqrt{s_{NN}}$
  - No clear interpretation or understanding of this trend
  - Important w.r.t. shape averaging interpretation of $\alpha \neq 2$
- Lévy exponent α still far from conjectured CEP limit of 0.5
  - Very much dependent on m_T bin width, working on final results…

HOLE IN $\lambda(m_T)$: ALL MEASUREMENTS AT RHIC

- Hole apparent for $\sqrt{s_{NN}} \geq 39$ GeV, ~independently of centrality

S. Lökös, Universe 4 (2018) 31 D. Kincses, Universe 4 (2018) 11 D. Kincses, Universe 4 (2018) 11



- Due to reduced η' mass?
- Sign for chiral restoration?
- To be cross-checked with photons, dileptons, etc.
- Working on finalized PHENIX results

35/28 COHERENCE WITH THREE-PION LÉVY HBT

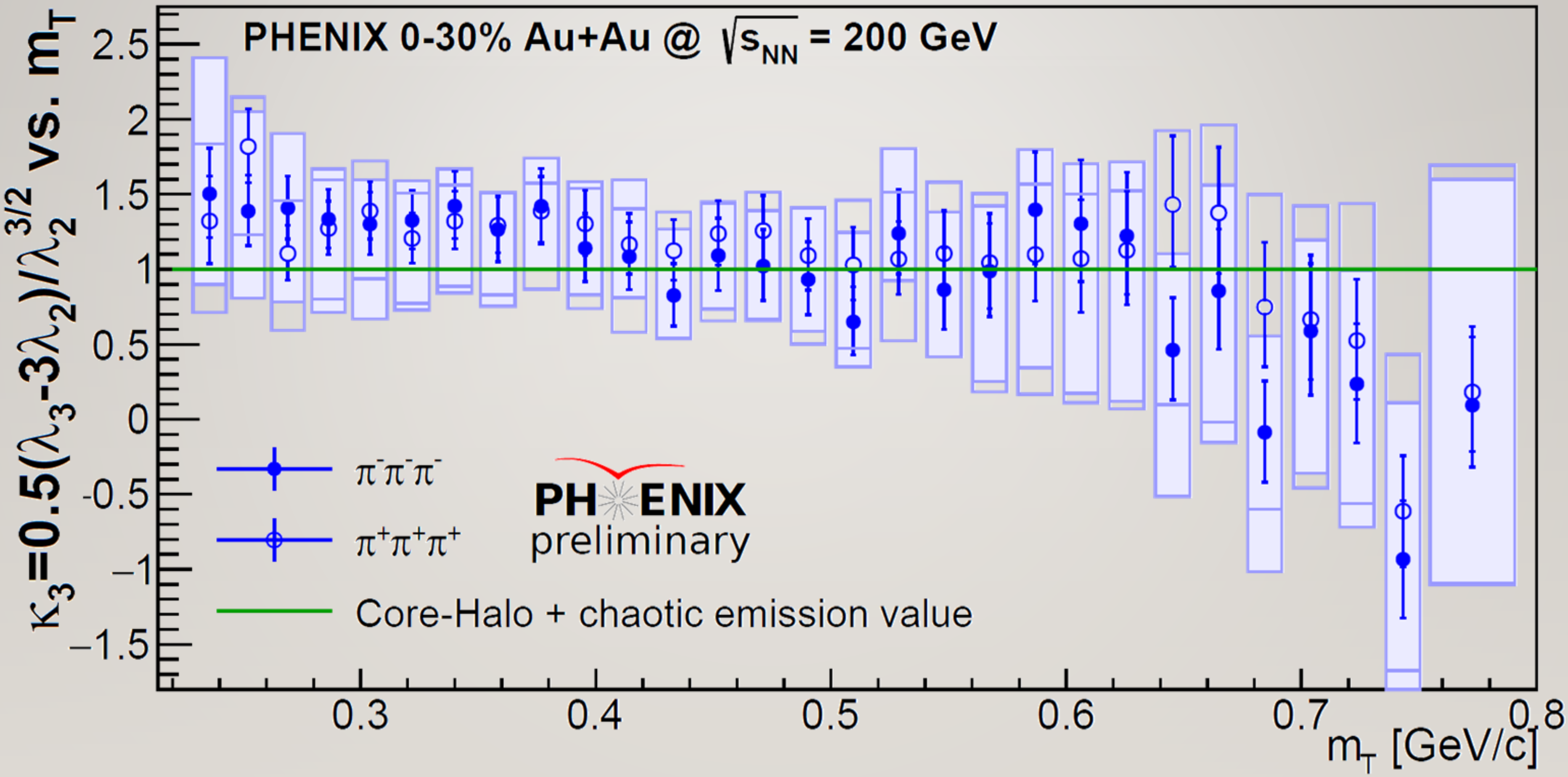
- Recall: two particle correlation strength $\lambda = f_C^2$ where $f_C = N_{\text{core}}/N_{\text{total}}$
- Generalization for higher order correlations: $\lambda_2 = f_C^2, \lambda_3 = 2f_C^3 + 3f_C^2$
- If there is partial coherence (p_C):

$$\lambda_2 = f_C^2[(1-p_C)^2 + 2p_C(1-p_C)]$$
$$\lambda_3 = 2f_C^3[(1-p_C)^3 + 3p_C(1-p_C)^2] + 3f_C^2[(1-p_C)^2 + 2p_C(1-p_C)]$$

- Introduce core-halo independent parameter $\kappa_3 = \frac{\lambda_3 - 3\lambda_2}{2\sqrt{\lambda_2}^3}$
  - does not depend on f_C
  - $\kappa_3 = 1$ if no coherence
- Finite meson sizes?
  - Gavrilik, SIGMA 2 (2006) 074 [hep-ph/0512357]
- Phase shift (a la Aharonov-Bohm) in hadron gas?
  - Random fields create random phase shift, on average distorts Bose-Einstein correlations Csanád et al., Gribov-90 (2021) 261-273 [arXiv:2007.07167]

TEST OF CORE-HALO MODEL / COHERENCE

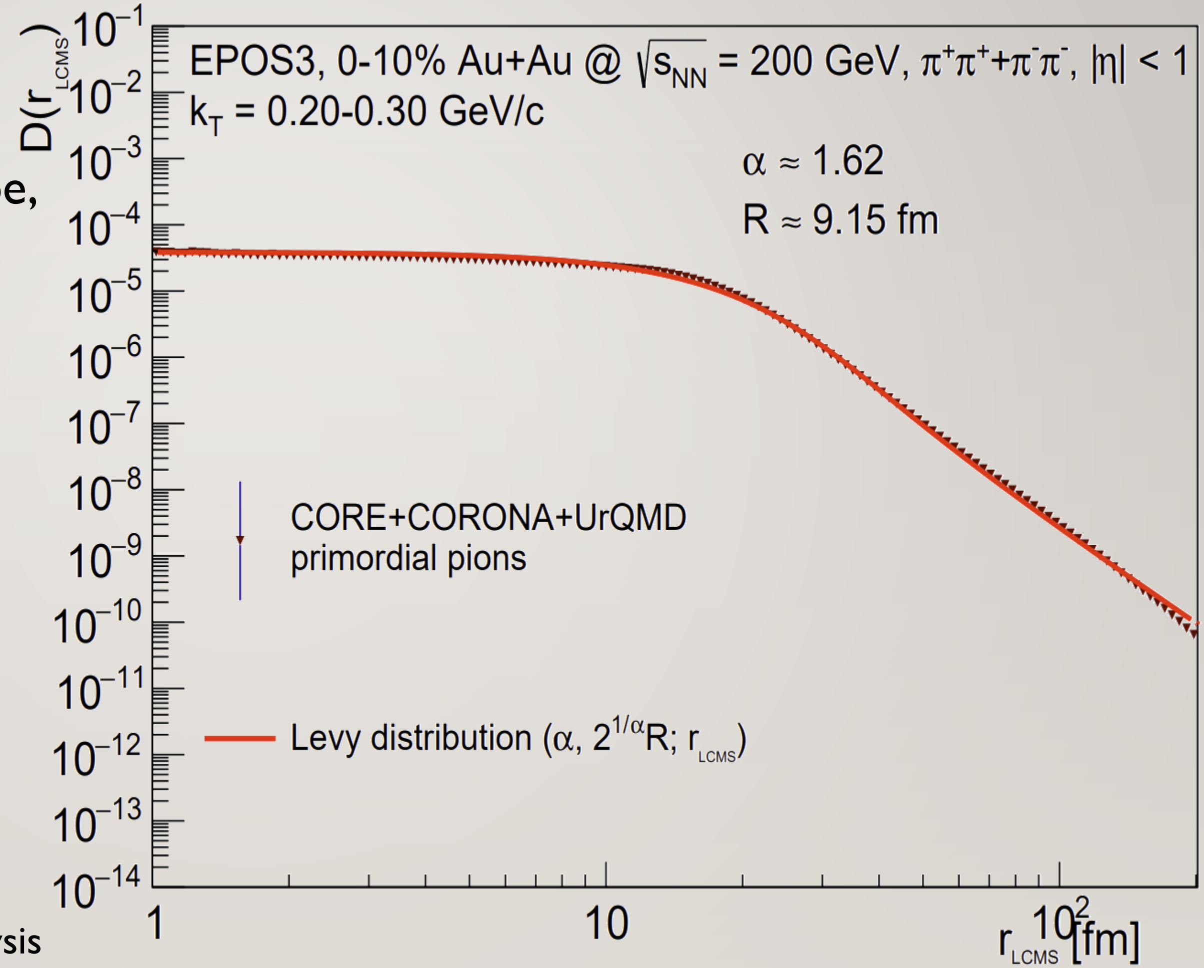
- Recall: $\kappa_3 = 1$ in pure core-halo model, $\kappa_3 \neq 1$ if coherence



WPCF 19, B. Kurgyis, Phys. Part. Nuclei 51 (2020) 263-266

ROLE OF EVENT AVERAGING?

- Event-averaged source also analyzed
- Not perfectly Lévy shape, very large χ^2
- Nevertheless: similar parameters achieved
  - Event averaged: $\alpha \approx 1.62, R \approx 9.15$ fm
  - Event-by-event: $\alpha \approx 1.66, R \approx 8.96$ fm
- More reasonable approach for kaons
  - No event-by-event analysis possible for kaons



SOURCE OR PAIR DISTRIBUTION?

- Under some circumstances (thermal emission, no interactions, …):

$$C_2(q,K) = \int S\left(r_1, K+\frac{q}{2}\right) S\left(r_2, K-\frac{q}{2}\right) |\Psi_2(r_1,r_2)|^2 dr_1 dr_2$$
$$\cong 1 + \left|\int S(r,K)e^{iqr}dr\right|^2$$

- Let us introduce the spatial pair distribution:

$$D(r,K) = \int S\left(\rho+\frac{r}{2}, K\right) S\left(\rho-\frac{r}{2}, K\right) d\rho$$

- Then the Bose-Einstein correlation function becomes:

$$C_2(q,K) \cong \int D(r,K)|\Psi_2(r)|^2 dr = 1 + \int D(r,K)e^{iqr}dr$$

- **Bose-Einstein correlations measure spatial pair distributions!**
- Coulomb and strong Final State Interactions? Under control for Lévy sources

Csanad, Lökös, Nagy, Phys. Part. Nuclei 51 (2020) 238 [arXiv:1910.02231]

Kincses, Nagy, Csanad Phys. Rev. C102, 064912 (2020) [arXiv:1912.01381]

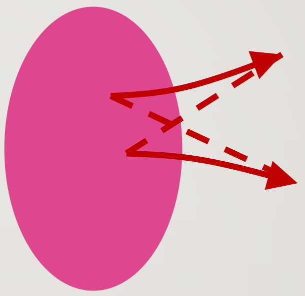
INTERACTIONS: THE COULOMB-EFFECT

- Plane-wave result, based on $\left|\Psi_2^{(0)}(r)\right|^2 = 1 + e^{iqr}$:

$$C_2(q,K) \cong \int D(r,K)\left|\Psi_2^{(0)}(r)\right|^2 dr = 1 + \int D(r,K)e^{iqr}dr$$

- If there is interaction:

$$\Psi_2^{(0)}(r) \rightarrow \Psi_2^{(\mathrm{int})}(r_1, r_2)$$



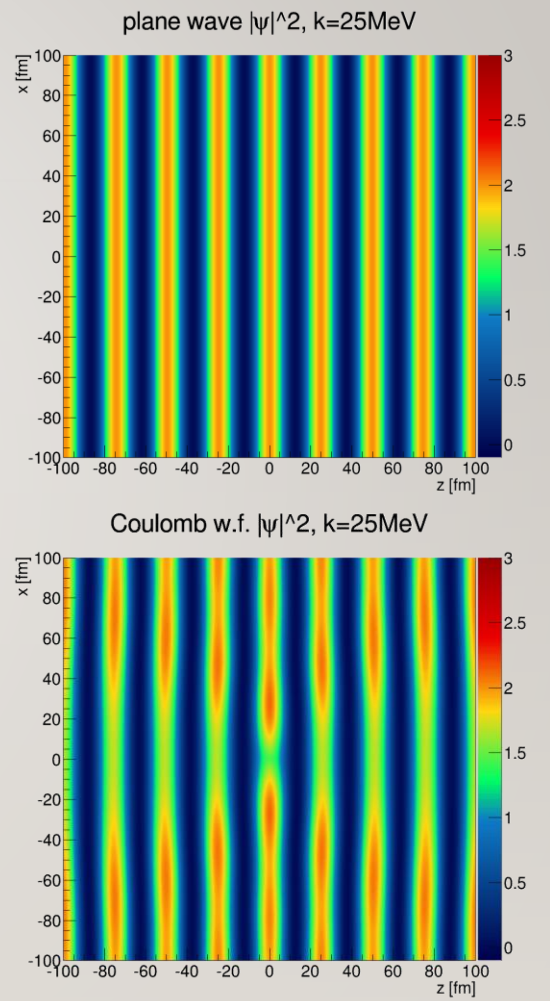
- For Coulomb:

$$\left|\Psi_2^{(C)}(r)\right|^2 = \frac{\pi\eta}{e^{2\pi\eta}-1} \cdot \left(\text{complicated hypergeometric expression}\right)$$

- Direct fit with this, or the usual iterative Coulomb-correction:

$$C_{\mathrm{Bose-Einstein}}(q)K(q), \text{ where } K(q) = \frac{\int D(r,K)\left|\Psi_2^{(C)}(r)\right|^2 dr}{\int D(r,K)\left|\Psi_2^{(0)}(r)\right|^2 dr}$$

- Complication: need for integrating power-law tails
- In this analyis: assuming spherical source
- Parametrization possible Csanád, Lökös, Nagy, Phys.Part.Nucl. 51 (2020) 238

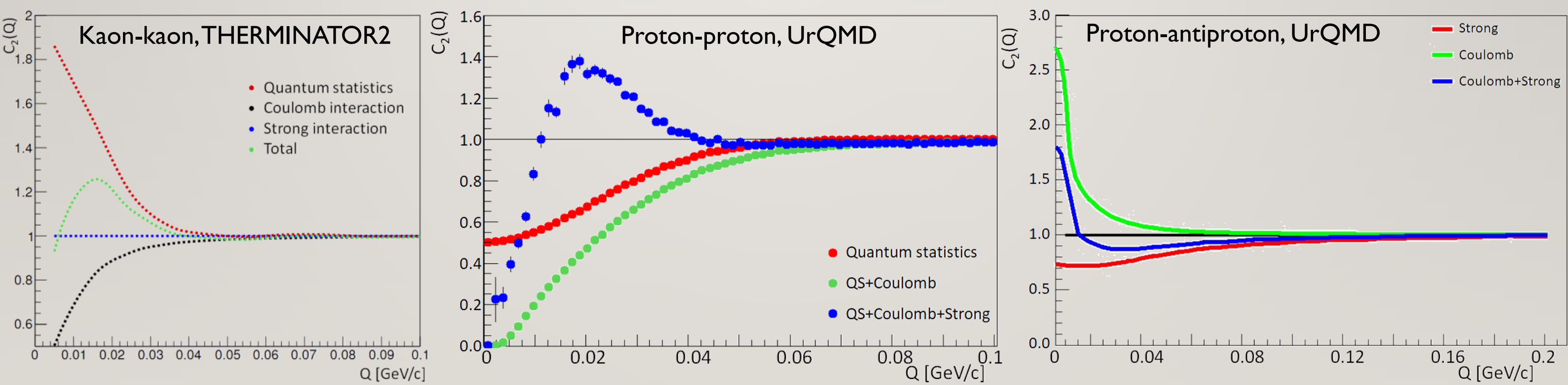


40/28 ROLE OF THE STRONG INTERACTION

- In case of other interactions or not identical bosons, the formula still works:

$$C_2(q,K) \cong \int D(r,K)|\Psi_2(r)|^2 dr$$

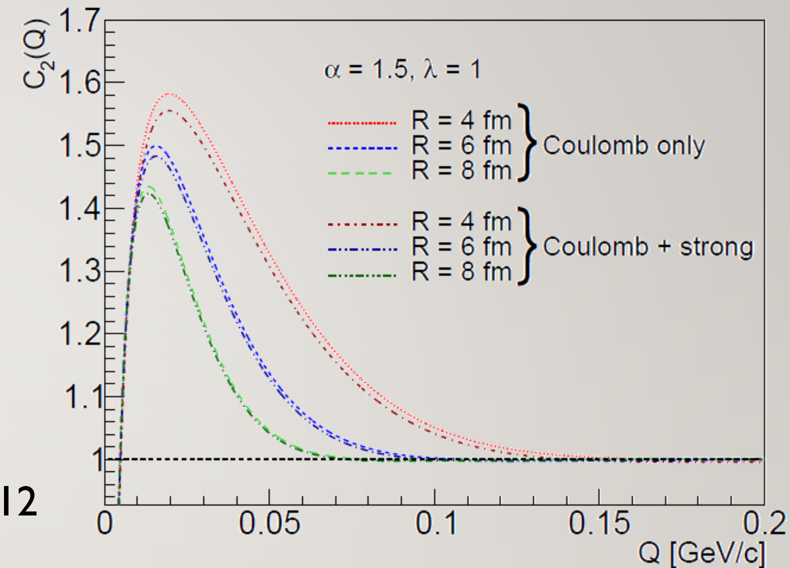
- Pair wave function determines $D \leftrightarrow C_2$ connection
- Mesons, baryons: strong interaction; fermions: anticorrelation
- Non-identical pairs: interaction modifies wave function

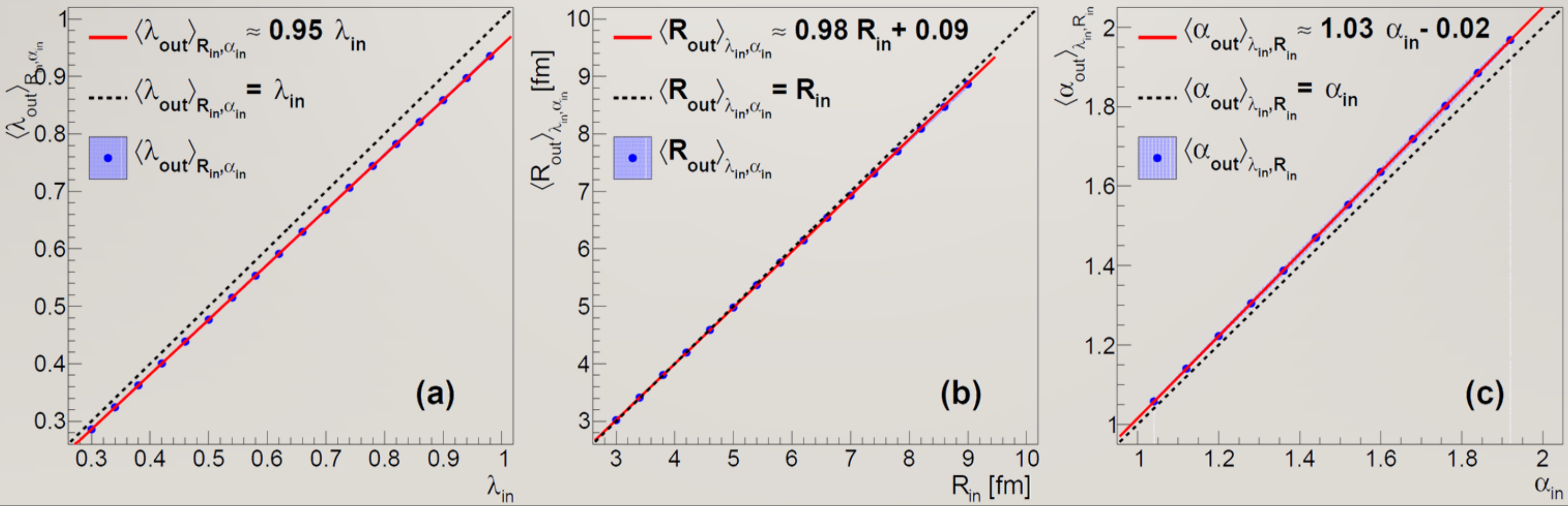


From e.g. H. Zbroszczyk's talk at Zimányi School 2019

STRONG INTERACTION FOR PION PAIRS

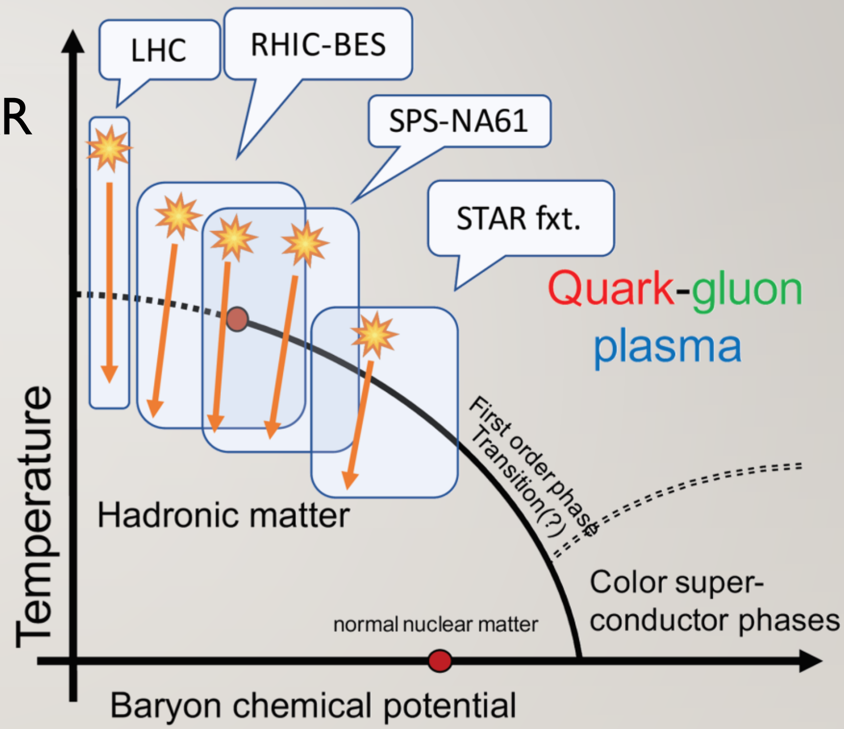
- Additional potential appearing
- Possible handling: strong phase shift, Modify s-wave component in wave func.
  R. Lednicky, Phys. Part. Nucl.40, 307 (2009)
- Small difference in case of pions
- Few percent modification in λ, α
  Kincses, Nagy, Csanád, Phys.Rev.C 102 (2020) 064912





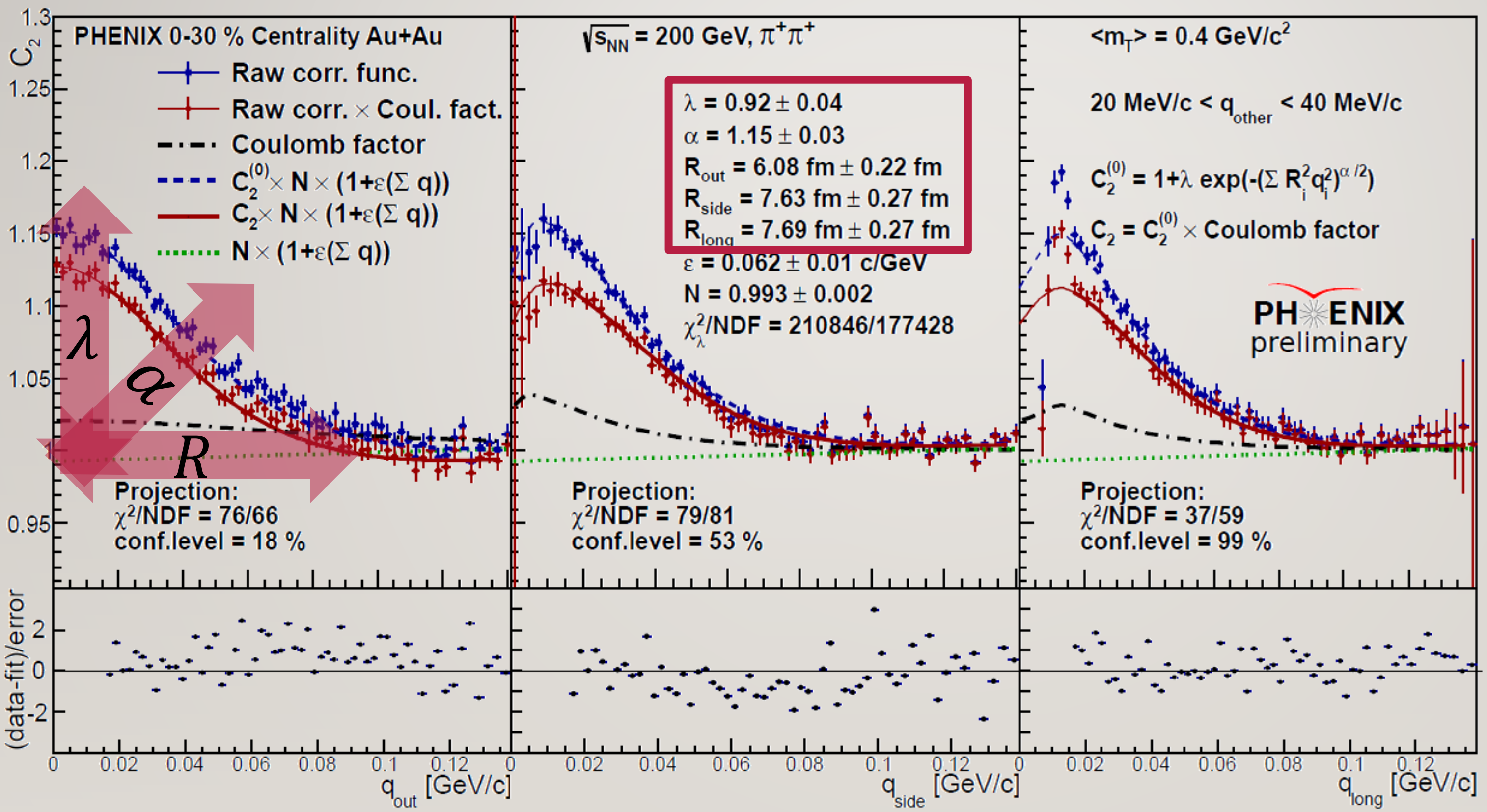
HBT MEASUREMENTS AND THE PHASE DIAGRAM

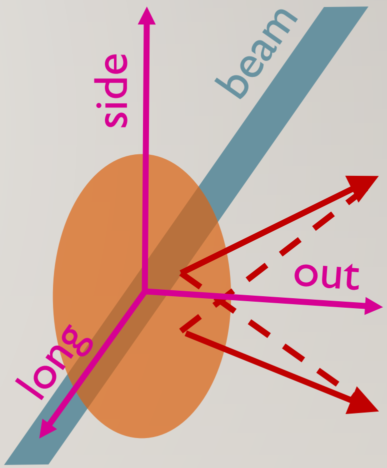
- LHC: measurement at CMS
  - 2-5 ATeV energy, p+p & Pb+Pb
- RHIC: measurement at PHENIX+STAR
  - 10-200 AGeV energy, Au+Au
- SPS: measurement at NA61
  - 17 AGeV energy, Be+Be
- Phase diagram can be investigated



A CROSS-CHECK: 3D LÉVY FEMTOSCOPY

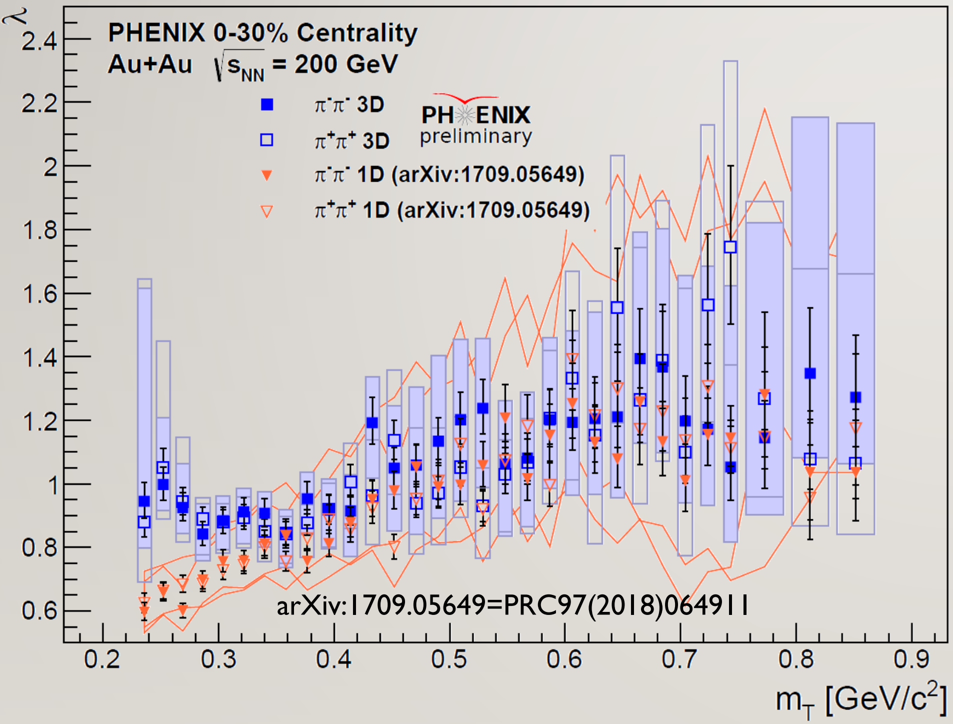
- Femtoscopy done in 3D: Bertsch-Pratt pair frame (out/side/long coordinates)
- Physical parameters: $R_{out/side/long}$ λ, α measured versus pair m_T
- Fit in this case: modified log-likelihood (small statistics in peak range)

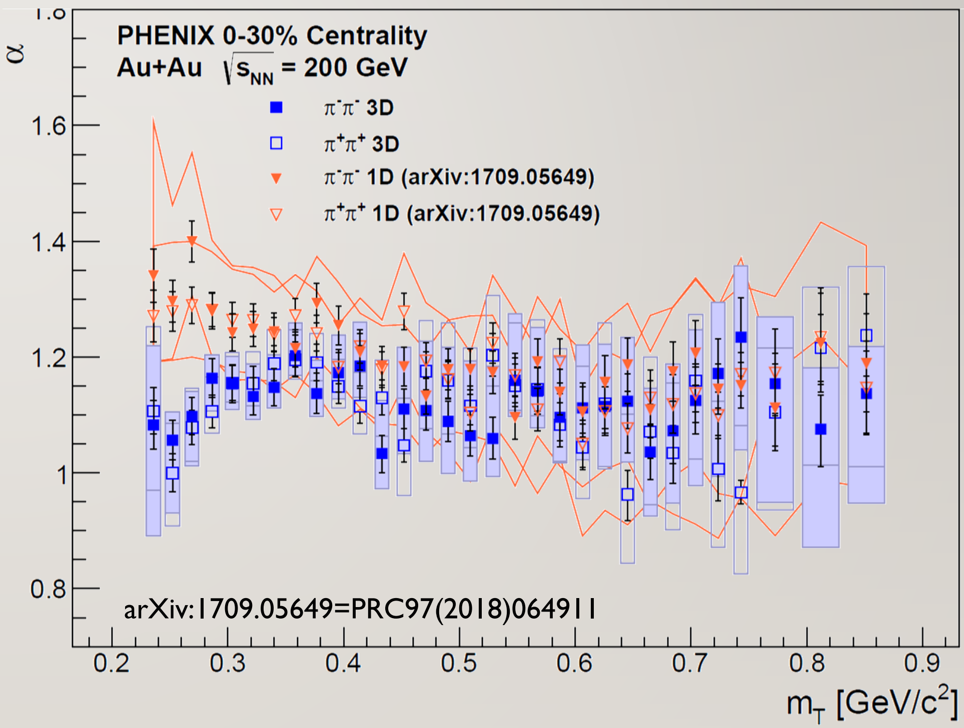




3D VERSUS 1D: STRENGTH λ AND SHAPE α

- Compatible with 1D (Q_{LCMS}) measurement of PRC97(2018)064911
- Small discrepancy at small mT: due to large Rlong at small mT?





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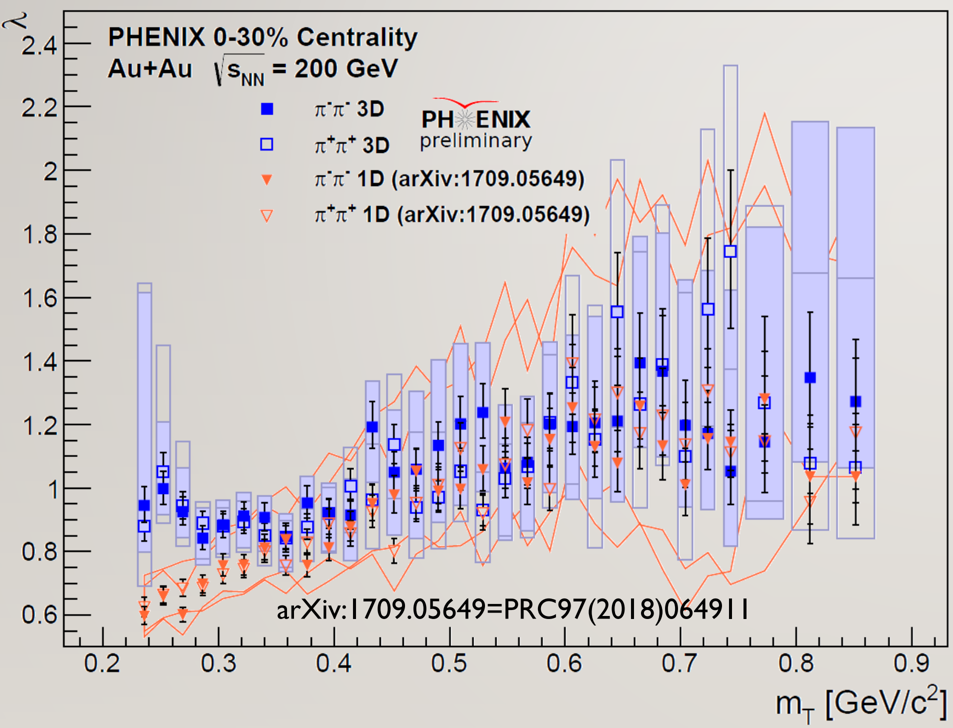
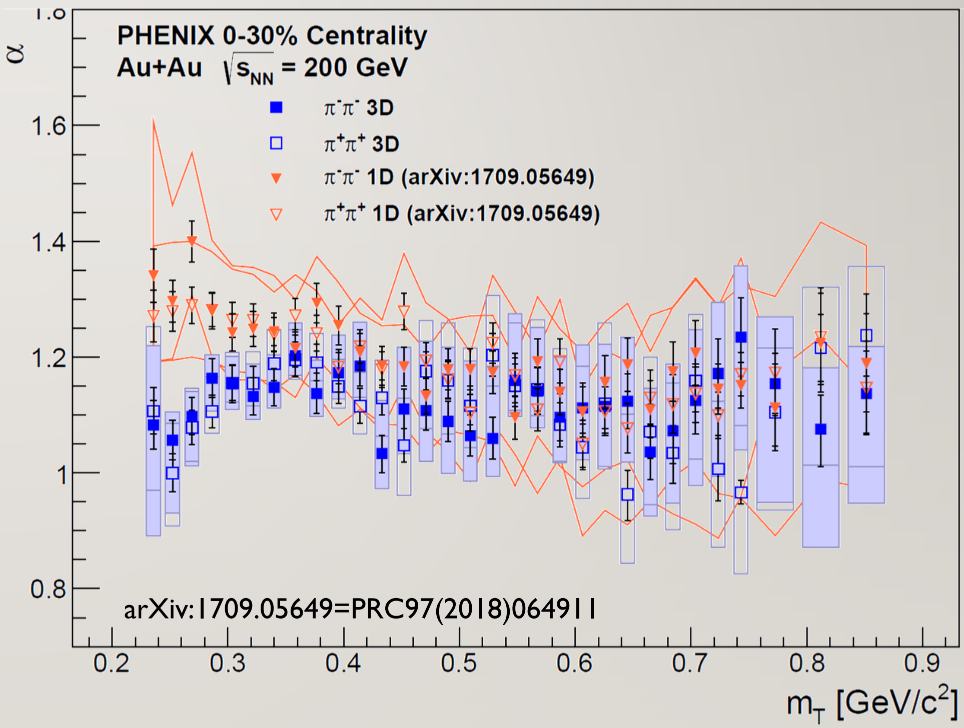
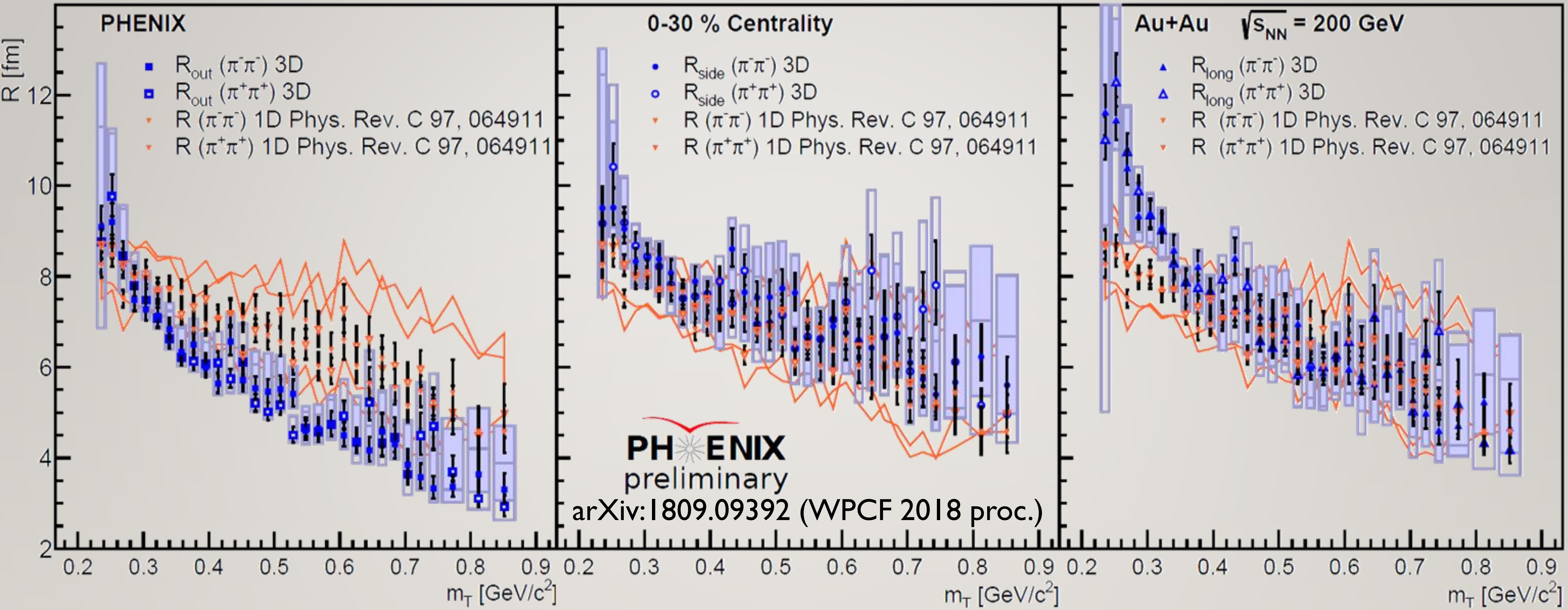




46/28 LÉVY SCALES IN 3D



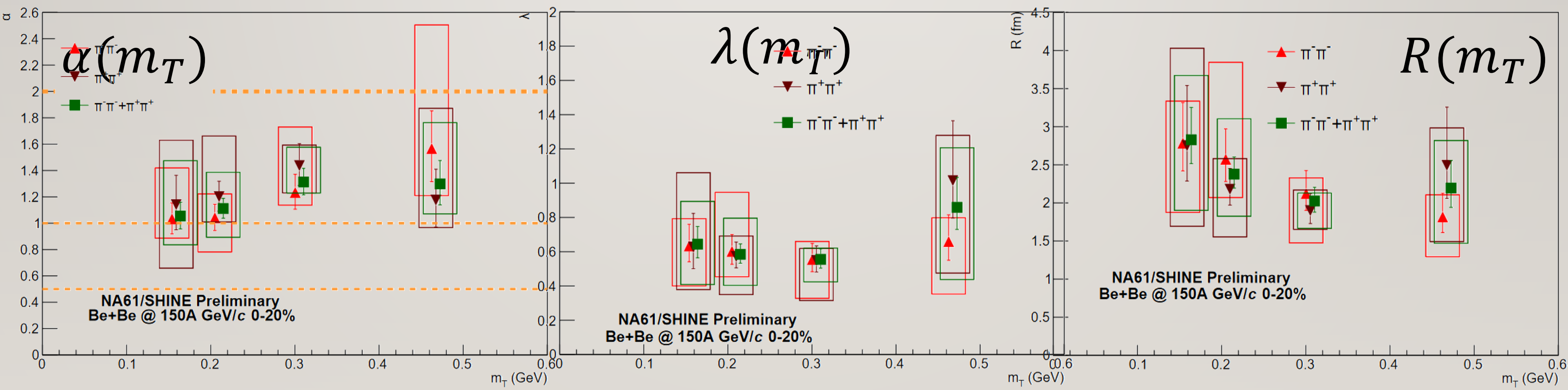
- Compatibility with 1D Lévy analysis
- Similar decreasing trend as Gaussian HBT radii, but it is not an RMS radius!
  - There is no 2nd moment (variance or root mean square) for Lévy distributions with α<2!
- Asymmetric source for small m_T, validity of Coulomb-approximation?

OPEN QUESTIONS

- Collision energy and centrality dependence of Lévy parameters?
  - Non-monotonicity in $\alpha\left(\sqrt{s_{NN}}\right)$ or $\alpha(\text{centrality})$?
  - Hole in $\lambda(m_T)$ at low $\sqrt{s_{NN}}$? Really due to η'?
- Reason for the appearance of Lévy distributions for pions?
  - What is the Lévy exponent for kaons?
  - Kaons have smaller total cross-section thus larger mean free path, heavier tail?
  - Does m_T scaling hold for Lévy scale R?
- Correlation strength versus core-halo picture: are there other effects?
  - Three-particle correlations may show if coherence or other effects play a role
  - Other effects may also play a role (finite meson sizes, random field phase shift, etc)

RESULTS AT NA61/SHINE

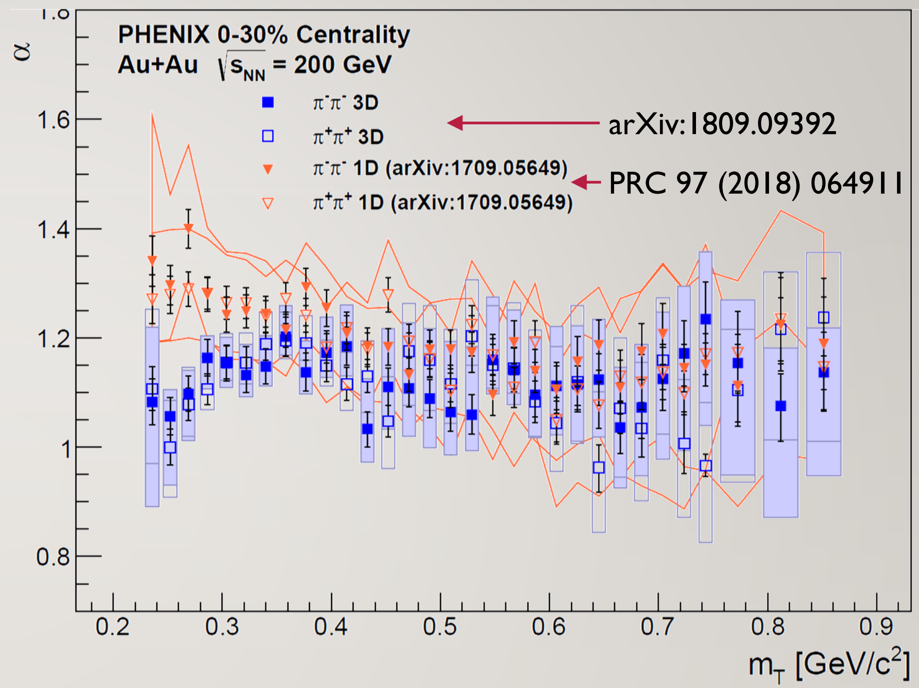
- Be+Be collisions at 150 AGeV beam momentum (17.3 AGeV in c.m.s.)
- Lévy fits describe correlation functions
  - Shape parameter α: far from Gaussian and CEP conjecture
  - Strength parameter λ: nearly constant as previous SPS results, unlike RHIC
  - Spatial scale R: weakly decreasing trend → hydro
- Plans: particle identification, Ar+Sc analysis, different energies

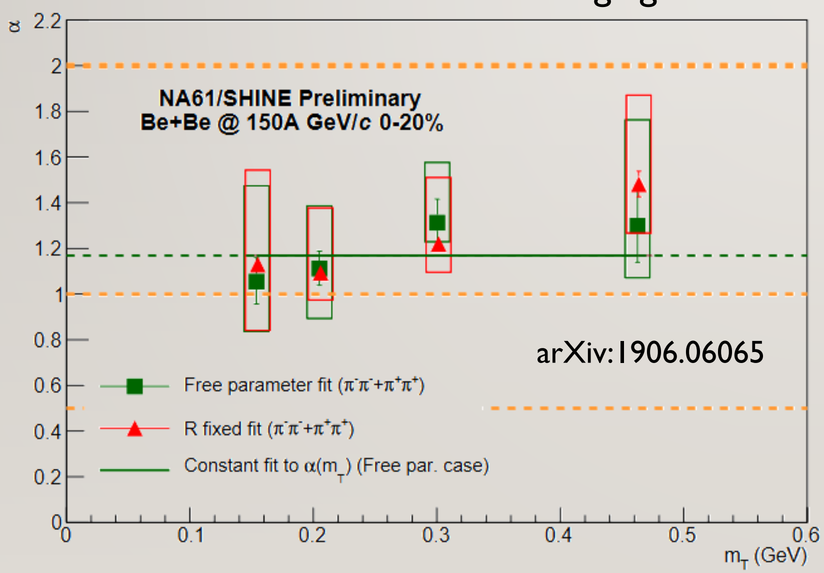


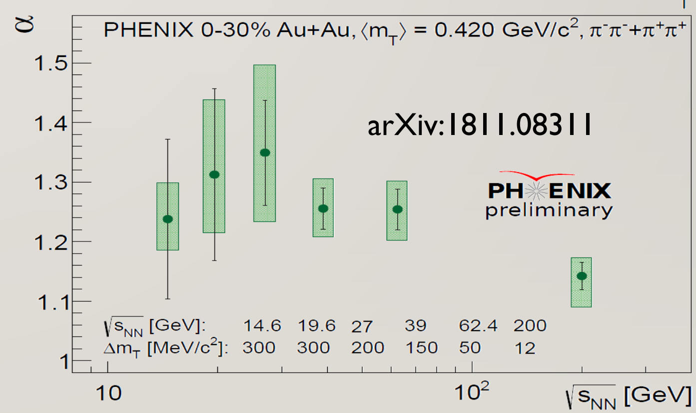
B. Porfy [NA61] WPCF19 & PoS CORFU2018 (2019) 184

49/28 LÉVY HBT MEASUREMENTS

- Many experimental results
  - PHENIX Au+Au: $\alpha \approx 1 - 1.5$
  - STAR Au+Au: ongoing
  - NA61 Be+Be: $\alpha \approx 1 - 1.5$
  - CMS Pb+Pb: $\alpha = 1$ fixed
- Where does this Lévy shape come from? What does it mean?
  - Role of event class averaging?







THE EPOS MODEL

- **E**nergy conserving quantum-mechanical multiple scattering approach, based on **P**artons ladders, **O**ff-shell remnants, and **S**plitting of parton ladders
  - K. Werner et al., PRC82 (2010) 044904, PRC89 (2014) 064903, …
- Based on Monte-Carlo simulation
- Theoretical framework: parton-based Gribov-Regge theory (PBGRT)
- Three main parts of the model:
  - Core-Corona division (based on dE/dx of string segments)
  - Hydrodynamical evolution (vHLLE 3D+1 viscous hydro)
  - Hadronic cascades (UrQMD afterburner)
- Effects/components to be turned on or off (on top of Core):
  - Corona
  - Rescattering
  - Decays

TWO-PARTICLE SPATIAL CORRELATIONS

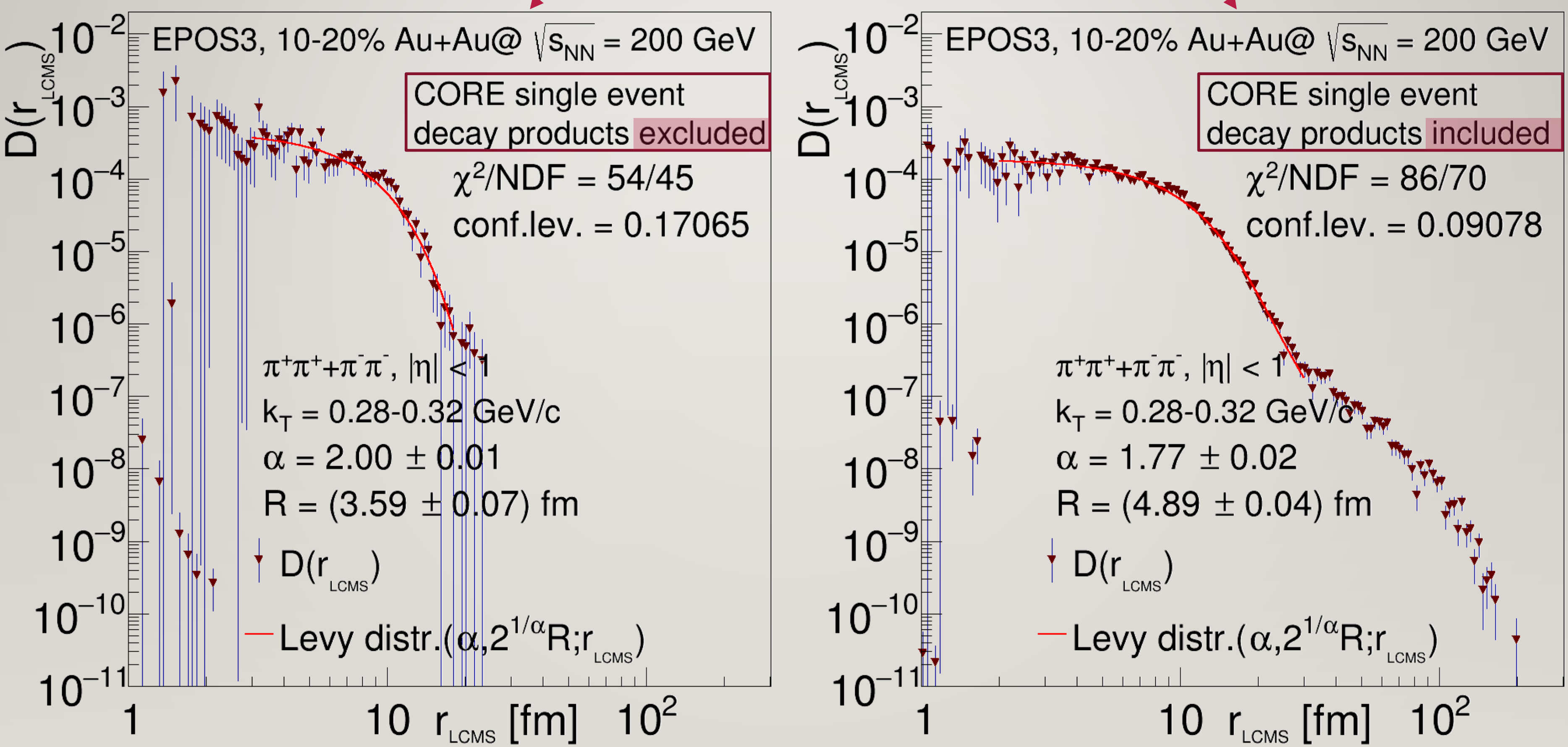
- Object to be investigated: two-particle source

$$D(r,K) = \int d^4\rho S\left(\rho + \frac{r}{2}, K\right) S\left(\rho - \frac{r}{2}, K\right)$$

- Experimental results measure power-law tails, Lévy shapes
  - Measure momentum-space correlations, reconstruct $D(r)$ or fit its parameters
- Why do these Lévy shapes appear?
  - What physics does contribute to it? Rescattering, decays?
  - What role does event averaging have in it?
    Cimerman, Plumberg, Tomasik, Phys.Part.Nucl. 51 (2020) 282, PoS ICHEP2020 538
  - What do specific α values mean?
- Event generator models (like EPOS) – direct access to pair-source!
  - Phenomenological investigations of $D(r)$ possible
  - Effects can be turned off or on, investigated separately

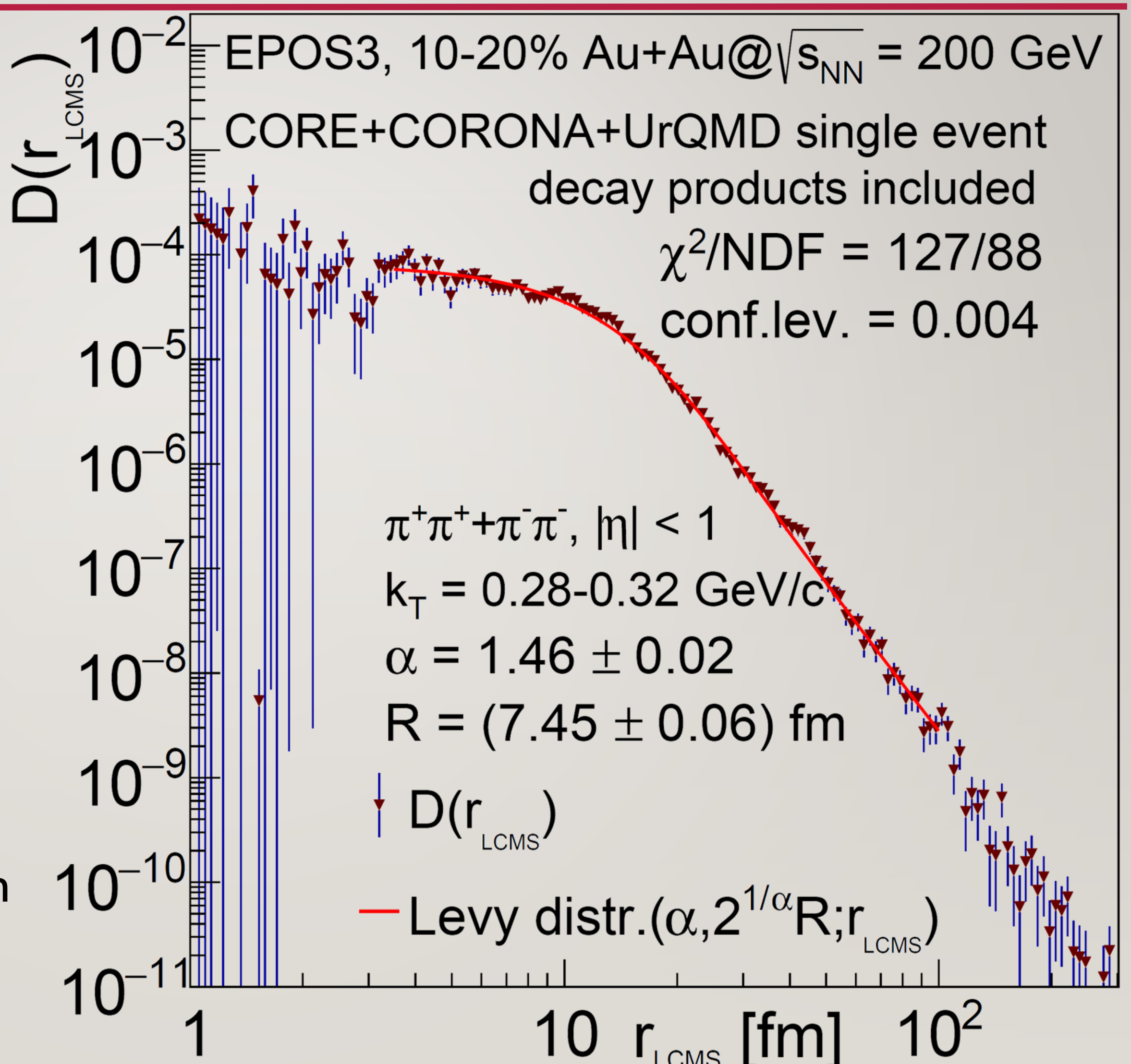
52/28 EXAMPLE SINGLE EVENT, CORE ONLY

- Gaussian shape without decays, additional structure with decays



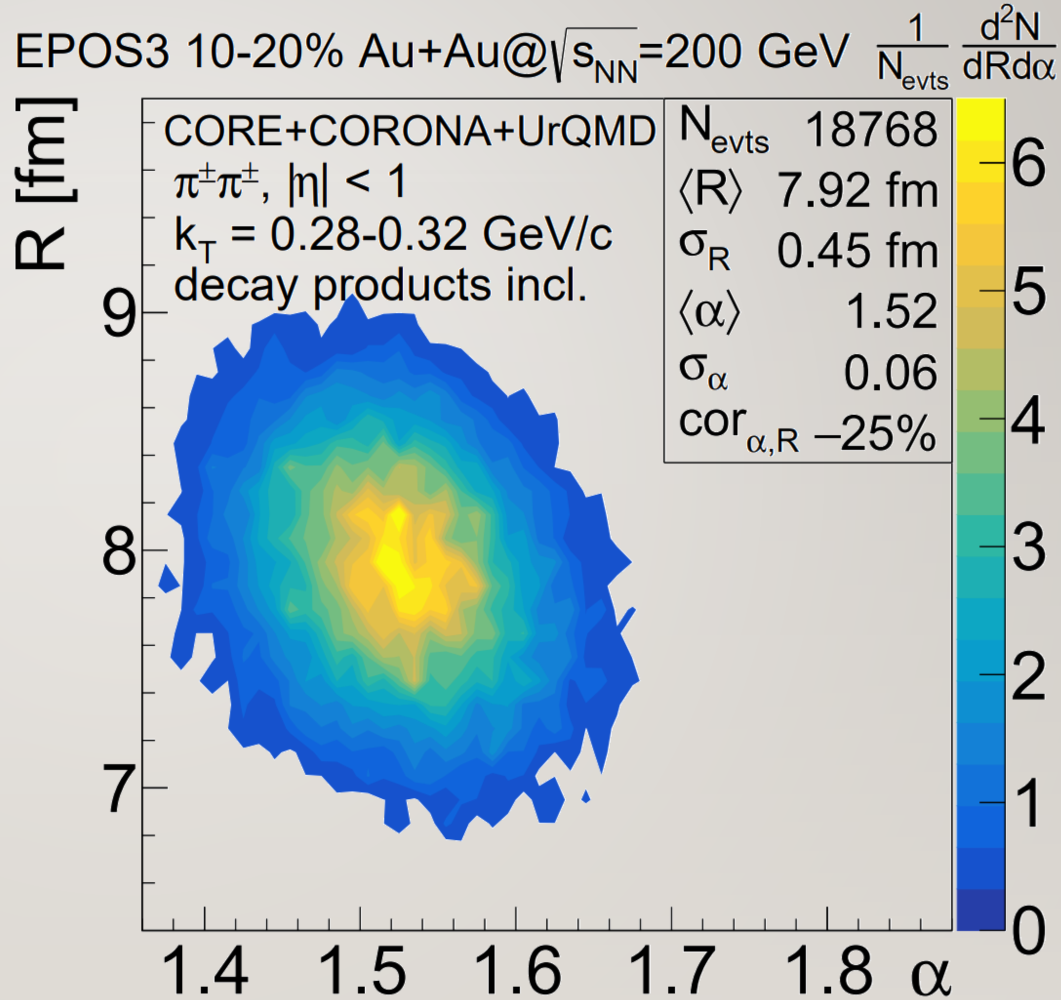
53/28 EXAMPLE EVENT, CORE+CORONA+URQMD

- Investigating $D(r)$ event-by-event
- Lévy-fits provide good description (2-100 fm range)
- Repeat such fits for thousands of events
- Extract α, R distribution



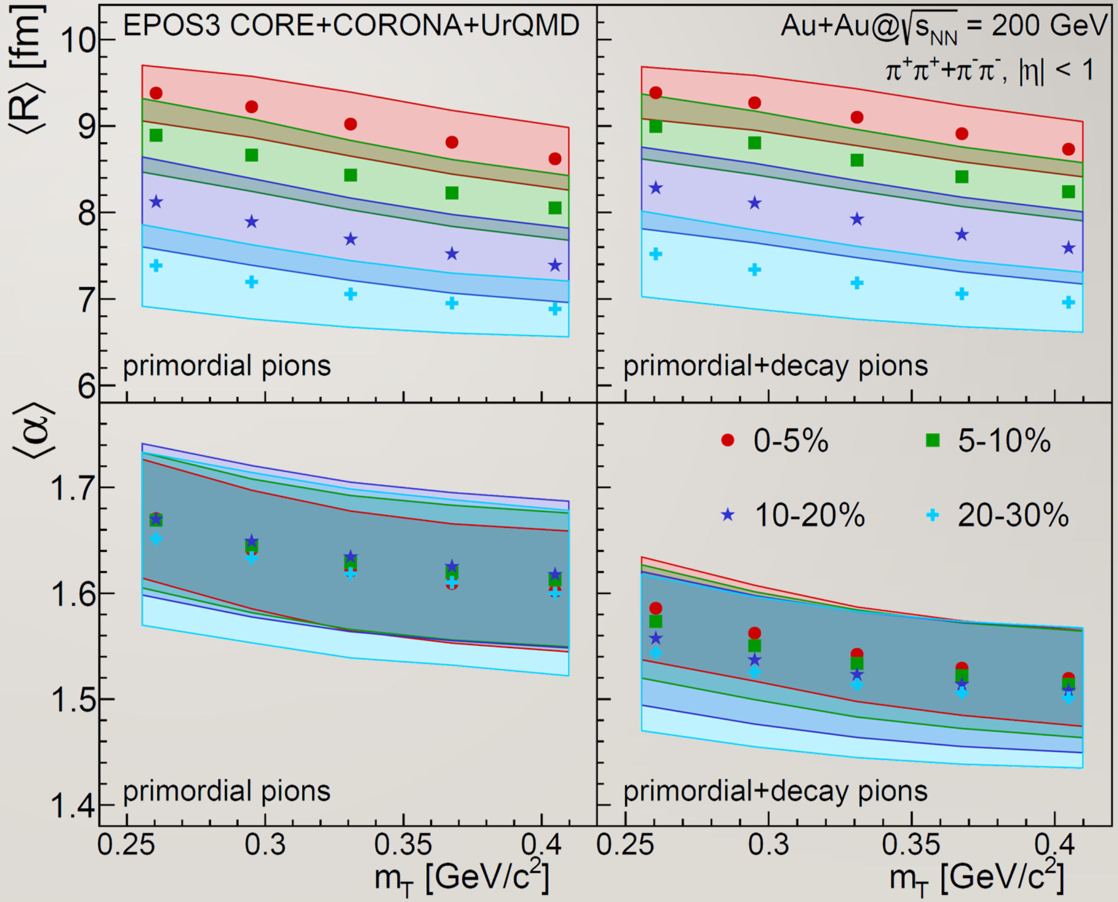
54/28 DISTRIBUTION OF α, R PARAMETERS

- Normal distribution of α, R for given centrality & k_T
- Extract mean and std.dev,
- Investigate centrality & k_T dependence
- kT dependence investigated around the peak of the pair-kT distr. to have adequate stat.



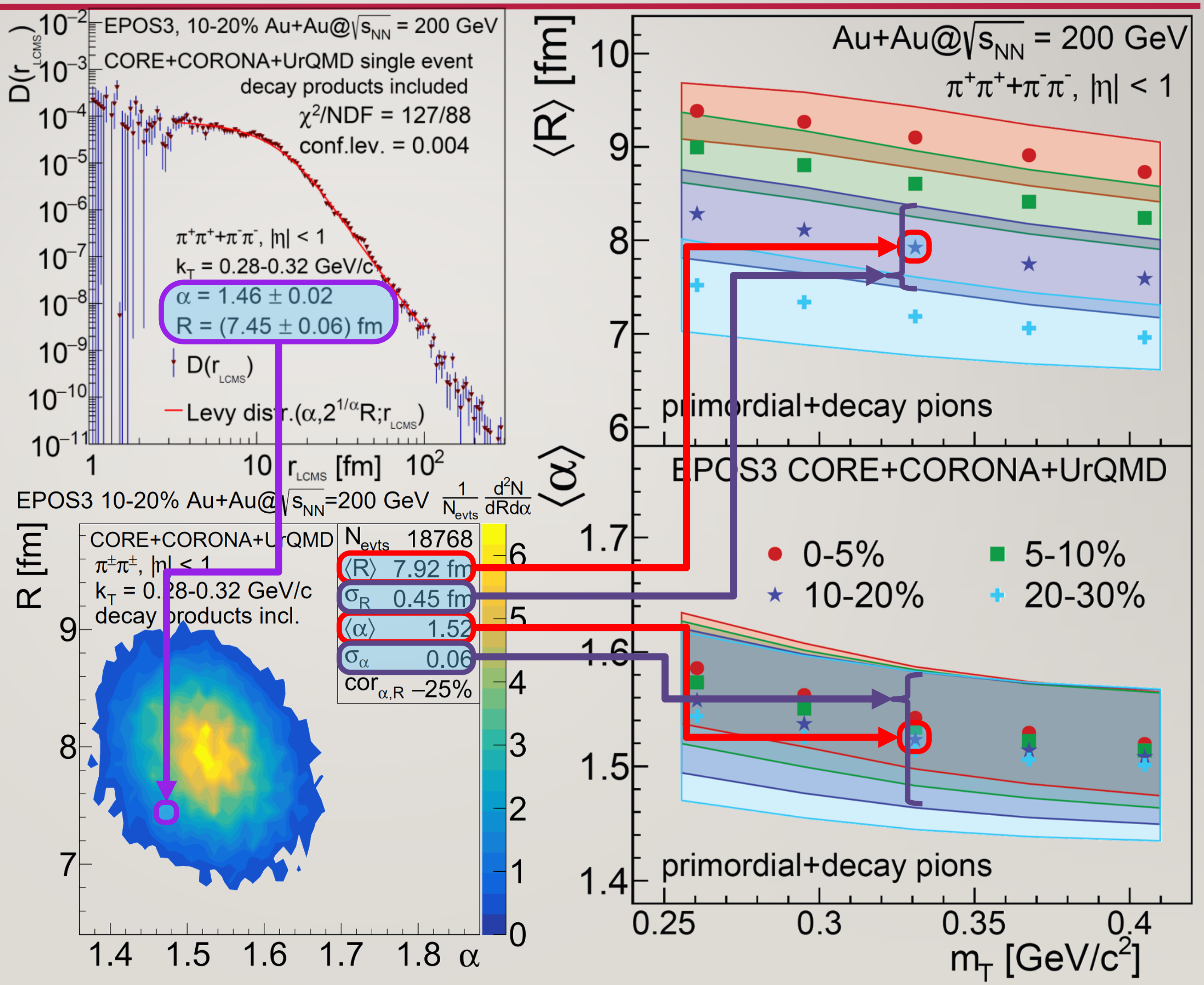
CONCLUSIONS AND OUTLOOK

- Lévy fits done to event-by-event EPOS spatial distributions, good description
- Power-law tail strongly affected by rescattering and decays
- Lévy R in EPOS: similar to data
- Lévy α in EPOS: larger than data
- Details in:
  - Entropy 24 (2022) 308 [arXiv:2201.07962]
- Next steps:
  - Multiple dimensions
  - Different particle species
  - Correlation function



SUMMARY

- D(r) calculated in EPOS evt-by-evt
- Lévy fits done evt-by-evt
- Non-Gaussianity in single events
- Extracting mean, & std.dev. of R, α
- m_T & centrality dependence



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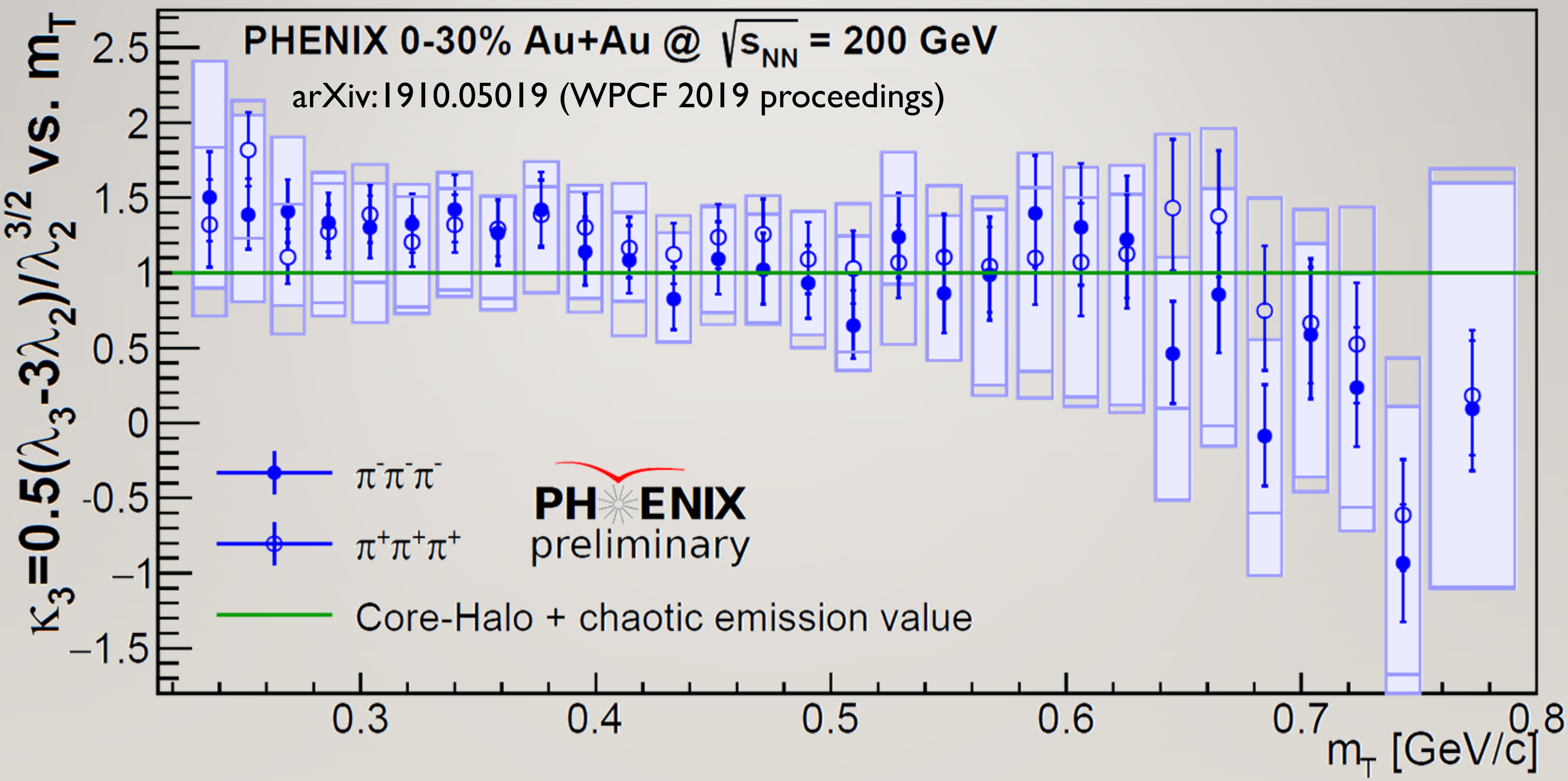
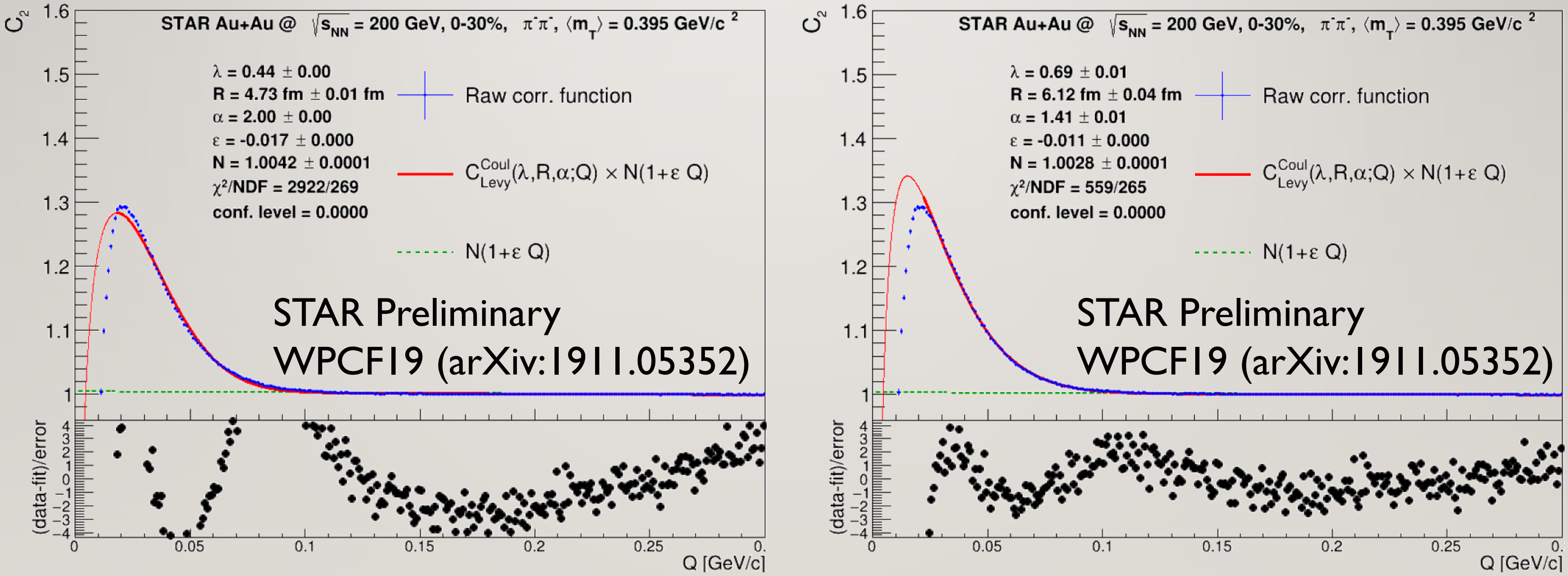
TEST OF CORE-HALO MODEL / COHERENCE

- Recall: $\kappa_3 = 1$ in pure core-halo model, $\kappa_3 \neq 1$ if coherence



SHAPE ANALYSIS AT STAR

- Gaussian fit: unacceptable description
- Levy fit somewhat better, but still additional effects present
- Low Q behavior not captured by any of the two



D. Kincses, Phys. Part. Nuc. 51 (2020) 267–269