Deblurring 3D Characteristics of Heavy-Ion Collisions

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Paradigm: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane*
from Theory and Experiment

no control over plane

What is it?!

Paradigm: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane*
from Theory and Experiment

no control over plane

some control, v_n

Still not clear what the system is...

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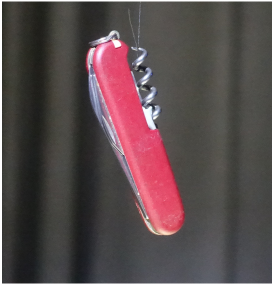
Paradigm: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane*
from Theory and Experiment

no control over plane

some control



full control, $\frac{d^3N}{dp^3}$

Claim: You can go from center to right panel
through deblurring

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Deblurring by Example

Budd, *Crime Fighting Math*, plus.maths.org magazine

Blurred Photo of Moving Car

Deblurred

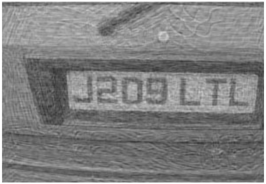
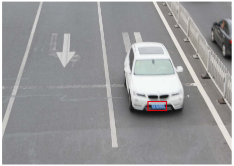


Photo of Parked Car

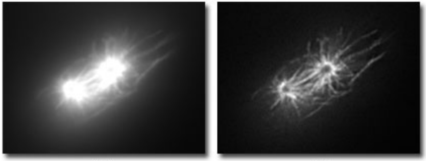
Fast Moving



Lu *et al.*, IEEE Trans Image Processing 25, 2311 (2016)

Deblurring in Optical Microscopy

Before and After Nearest Neighbor Deconvolution Analysis



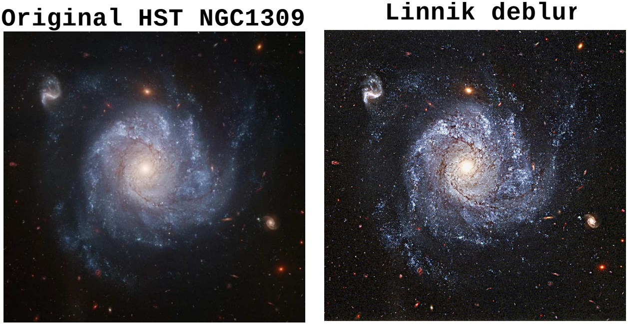
(a) **Figure 1** **(b)**

https://micro.magnet.fsu.edu/primer/digitalimaging/deconvolution

Deblurring in Astronomy

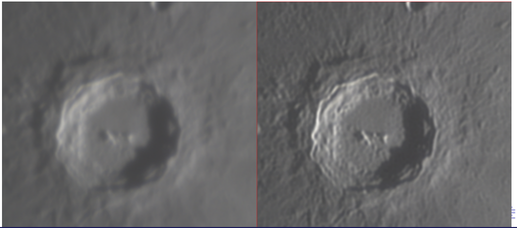
Much advancement in astronomy, in particular spurred by the Hubble Space Telescope (HST) flaw

Carasso, NISTIR 7632 (2009)



Before and after deblurring of image of lunar crater Copernicus

https://en.wikipedia.org/wiki/Deconvolution



Correcting f/Distortions Due to Apparatus or Method

Detector efficiency ϵ, n measured ptcle number, N actual number

$$N \simeq \frac{1}{\epsilon}\, n$$

Typical energy loss in thick target $\overline{\Delta E}$ for detected particle

$$E_{\text{prod}} \simeq E_{\text{det}} + \overline{\Delta E}$$

General problem stated probabilistically, with $P(\zeta|\xi)$ - probability to measure ptcle characteristic to be ζ when it is actually ξ

$$n(\zeta) = \int \mathrm{d}\xi\, P(\zeta|\xi)\, N(\xi)$$

For small distortions, P finite only when ζ little different from ξ.
Optical terminology: P - blurring or transfer function.

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Bayesian Deblurring

Distorted $n(\zeta)$ measured, while pristine $N(\xi)$ sought:

$$n(\zeta) = \int d\xi \, P(\zeta|\xi) \, N(\xi)$$

$P(\zeta|\xi)$ - probability that ptcle with ζ detected while it really has characteristic ξ, understood given the method/apparatus, can be simulated (Geant4) & can depend on N

$Q(\xi|\zeta)$ - complementary probability that ptcle has characteristic ξ while measured at ζ - unknown.

Bayesian relation: number of times ptcle has characteristic in $d\xi$ while measured in $d\zeta$ is

$$P(\zeta|\xi) \, N(\xi) \, d\xi \, d\zeta = Q(\xi|\zeta) \, n(\zeta) \, d\xi \, d\zeta$$

Hence $\quad N(\xi) = \dfrac{\int d\zeta \, Q(\xi|\zeta) \, n(\zeta)}{\int d\zeta' \, P(\zeta'|\xi)} \, , \quad Q(\xi|\zeta) = \dfrac{P(\zeta|\xi) \, N(\xi)}{\int d\xi' \, P(\zeta|\xi') \, N(\xi')}$

Richardson-Lucy method solves eqs iteratively till stabilization

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Richardson-Lucy (RL) Method from Astronomy

Iterative method, r - iteration index

$$
\begin{aligned}
n^{(r)}(\zeta) &= \int \mathrm{d}\xi \, P^{(r)}(\zeta|\xi) \, N^{(r)}(\xi) \\
A^{(r)}(\xi) &= \frac{\int \mathrm{d}\zeta \, \frac{n(\zeta)}{n^{(r)}(\zeta)} \, P^{(r)}(\zeta|\xi)}{\int \mathrm{d}\zeta' \, P^{(r)}(\zeta'|\xi)} \\
N^{(r+1)}(\xi) &= A^{(r)}(\xi) \, N^{(r)}(\xi)
\end{aligned}
$$

ξ & ζ are binned (pixelated), n & N are arrays and P transformation (transfer) matrix from the method/apparatus. Deblurring amounts to iterative multiplication of arrays by matrices + matrix reconstruction. Typical start: $N^{(1)}(\xi) = n(\xi)$

Richardson JOSA 62(1972)55 ; Lucy AJ 79(1974)745
https://en.wikipedia.org/wiki/Richardson-Lucy_deconvolution

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Other methods include Fourier transformation

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3D Nature of Collisions of Heavy Nuclei

Transport simulation of 2 GeV/nucl Au + Au at $b = 6$ fm

PD *et al.* Science298(2002)1592 z - beam, x - reaction plane

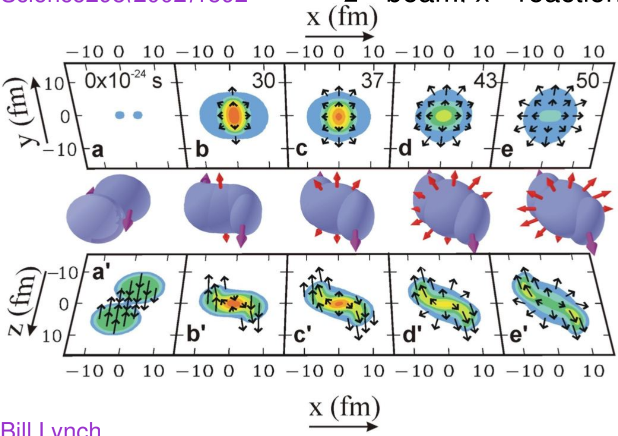

figure by Bill Lynch

Rich 3D structure, but no control over the reaction-plane direction in experiment

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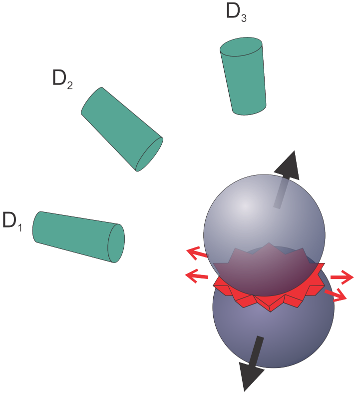
Estimating Reaction-Plane Direction



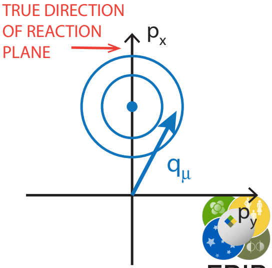
Plane direction estimated with

$$\mathbf{q}_\mu = \frac{1}{N} \sum_{\nu \neq \mu} \omega_\nu \, \mathbf{p}_\nu^\perp \qquad \omega_\nu = \begin{cases} +1, & \text{if } p_\nu^z > 0 \\ -1, & \text{if } p_\nu^z < 0 \end{cases}$$

N - measured particle multiplicity; other ptcles in the event used as reference for μ

PD&Odyniec
PLB157(85)146

Problem: Reference vector $\mathbf{q}_\mu$ Gaussian fluctuates around true plane direction



Any direct record of 3D characteristics will be blurred!

Current Solution: Angular Moments of Distributions

Solution: average angular moments (azimuthal Fourier coefficients)

$$v_n = \langle \cos n\phi \rangle$$

ϕ - angle relative to true reaction plane

Voloshin&Zhang ZfPhC70(1996)665

v_n derived from average scalar products/contractions, e.g.,

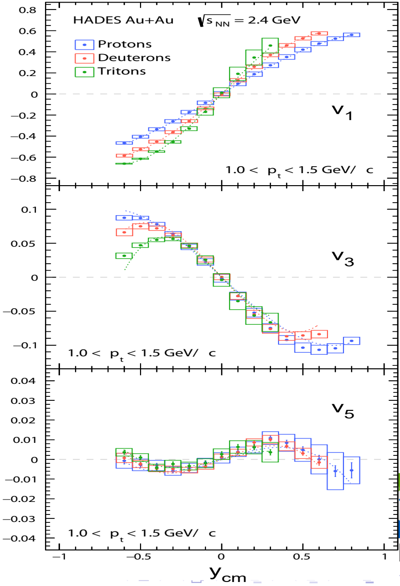
$$\langle \mathbf{p}^{\perp}_{\mu} \cdot \mathbf{q}_{\mu} \rangle \simeq p^{\perp} \langle q^{x} \rangle \langle \cos \phi \rangle$$

for different $p^{\perp}$, y and ptcle ID

Problem: unclear physics in v_n especially for higher n

1.23 GeV/nucl Au + Au $b \simeq 6$ fm

HADES PRL125(2020)262301



Schematic 1D Model

Proposition: Carry out as good determination of 3D info as you can & refine with deblurring. ~~v_n~~?

First 1D deblurring test.

Projectile at unknown velocity V deexcites emitting $N = 10$ ptcles distributed with box-like dN/dv in projectile cm. Task: Measuring ptcles in lab, determine dN/dv. Cm velocity V' estimated from remaining ptcles, so V' & dN/dv' smeared:

$$\frac{dN}{dv'} = \int dV' \, \frac{dP}{dV'} \, \frac{dN}{dv}$$

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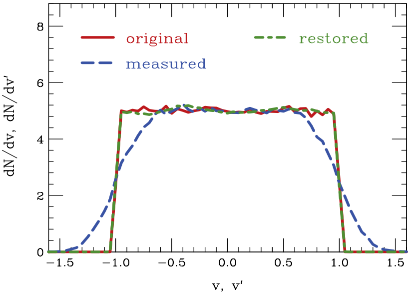

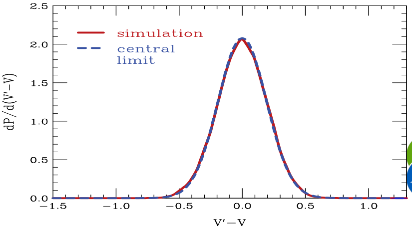

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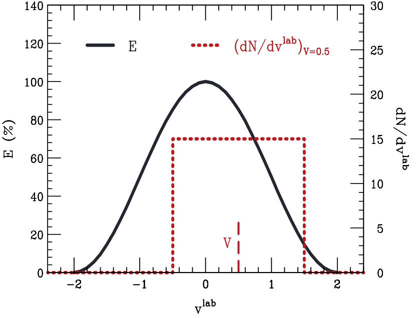
→Central-limit smear + RL deblur

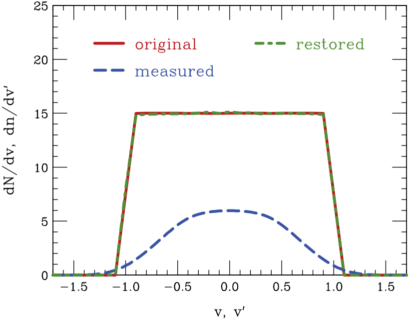
1D Model with Detector Inefficiencies

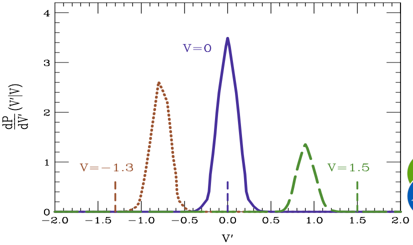
Again projectile at unknown velocity V deexcites emitting N particles, but now measured w/detector of strongly changing efficiency E. Find dN/dv.

$N = 30$ unknown.



"keyhole view"

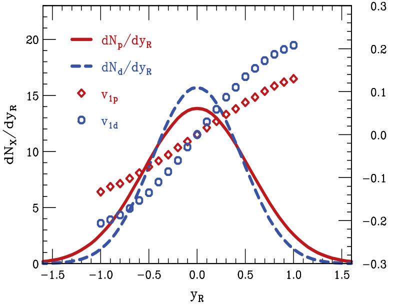




⇒ RL deblur w/s-consistent transfer mtx

3D Model for Collisions

Customary thermal model with flow, N, d, t, ^{3}He, ^{4}He. $\langle Z_{Tot} \rangle = 50$
Rapidity dstr, temperature & flow typical for semicentral collisions at 300 MeV/nucl

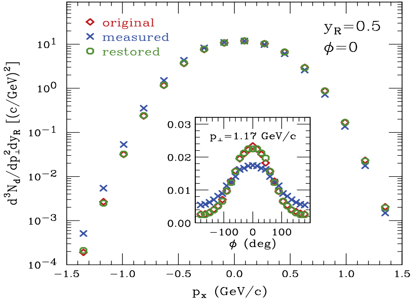


$$\frac{dN}{d\phi'} = \int d\Phi' \, \frac{dP}{d\Phi'} \, \frac{dN}{d\phi}$$

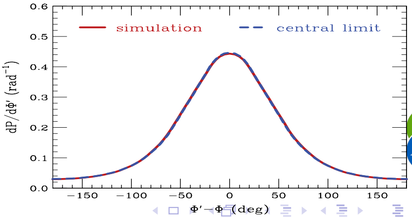
$$\phi' + \Phi' = \phi + \Phi$$

RL deblur + central-limit
Strong anisotropies restored!

Triple differential spectrum in reaction plane:



Uncertainty in reaction plane:

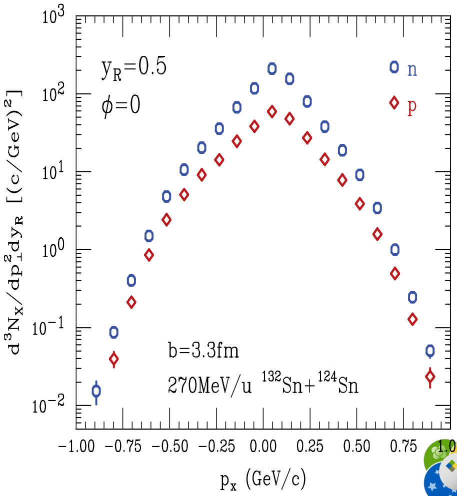


Why 3D Characteristics?

Transport-model simulation of 270 MeV/nucl ^{132}Sn + ^{124}Sn collision at $b = 3.3$ fm

3-differential spectrum for the same conditions as in thermal model, but looks very different. Not parabolic, i.e., Gaussian, cusps, different left-right slopes, knees. Steeper slope on spectator side, softer on participant. Physics??

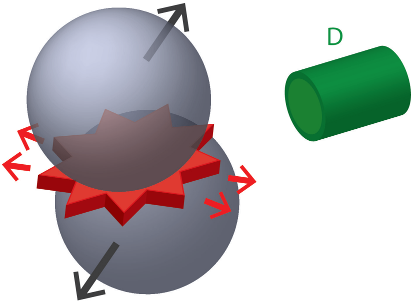
Averaged over ϕ, the spectrum would look thermal and no obvious sign in v_n...

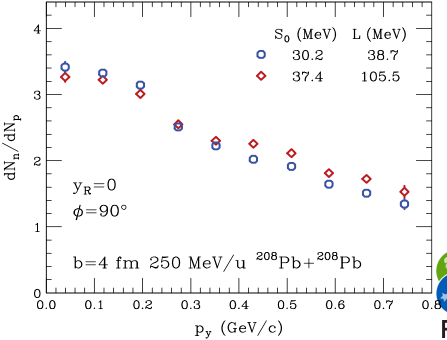


Symmetry energy at $\rho > \rho_0$?

Deblurring allows to effectively look into the heart of matter, unobscured high-density central region in the collisions

Transport-model simulations of 250 MeV/nucl ^{208}Pb + ^{208}Pb collisions w/medium-soft & stiff symmetry energy. n/p yield ratio at $\phi = 90°$, perp to reaction plane, and $y_{cm} = 0$.





Conclusions

- Deblurring: strong record in optics & fields that heavily rely on optics: forensic science, astronomy & microscopy
- Deblurring can expand the reach of measurement ahead of any comparison to theory
- No reason for deblurring to confine to photons and not extend to other particles - its domain are probabilities
- Deblurring should effectively allow to control reaction plane in energetic heavy-ion collisions, hopefully expand horizons
- Nuclear problems where deblurring started producing results: $^{26}\text{O} \rightarrow ^{24}\text{O} + n + n$ decay, source-imaging from 2-particle correlations in heavy-ion collisions, triple-differential distributions in heavy-ion collisions

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