Deuteron production in heavy-ion collisions

Boris Tomášik

Fakulta jaderná a fyzikálně inženýrská, České vysoké učení technické, Praha, Czech Republic
and Univerzita Mateja Bela, Banská Bystrica, Slovakia

boris.tomasik@fjfi.cvut.cz

Zimányi School
Budapest, Hungary

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Get introduced: the deuteron

Wave function(s) (r is the distance between proton and neutron):

- Hulthen form ($\alpha = 0.23$ fm^{-1}, $\beta = 1.61$ fm^{-1})

$$\varphi_d(r) = \sqrt{\frac{\alpha\beta(\alpha+\beta)}{2\pi(\alpha-\beta)^2}}\,\frac{e^{-\alpha r} - e^{-\beta r}}{r}$$

- spherical harmonic oscillator ($d = 3.2$ fm)

$$\varphi_d(r) = (\pi d^2)^{-3/4} \exp\left(-\frac{r^2}{2d^2}\right)$$

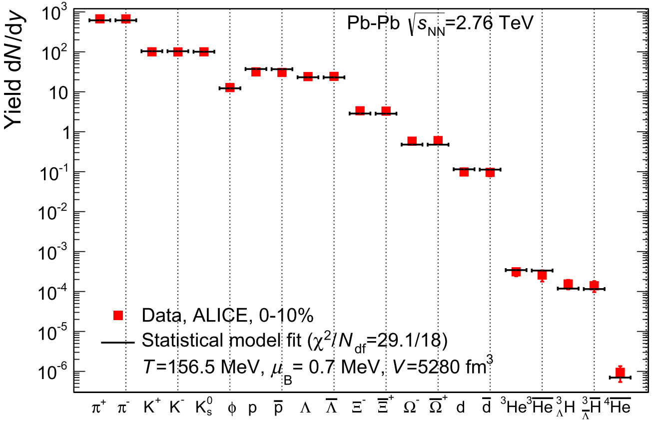
rms radius: 1.96 fm
Binding energy 2.2 MeV
spin 1

Clusters and statistical model: a neat coincidence

Cluster abundancies fit into a universal description with the statistical model

Is this robust feature, or is this a result of fine-tuning?

What does it actually tell us?



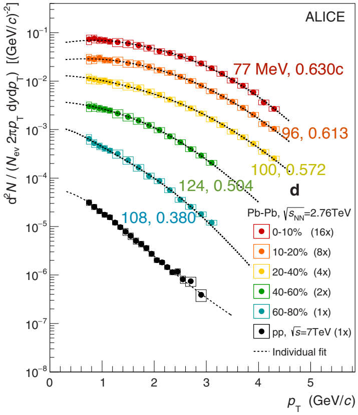
[A. Andronic *et al.*, J. Phys: Conf. Ser 779 (2017) 012012]

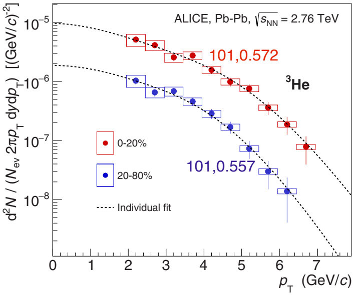
This is (a part of the) motivation to look at clusters,
although clusters actually carry *femtoscopic* information about the freeze-out.

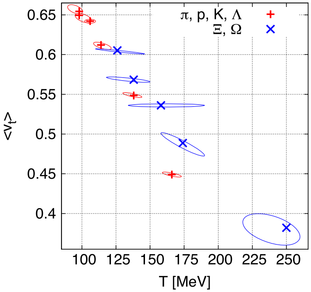
Kinetic freeze-out of clusters: ALICE

[J. Adam *et al.* [ALICE collab], Phys. Rev. C 93 (2016) 024917]

p_t spectra of d and ^{3}He fitted individually with the blast-wave formula







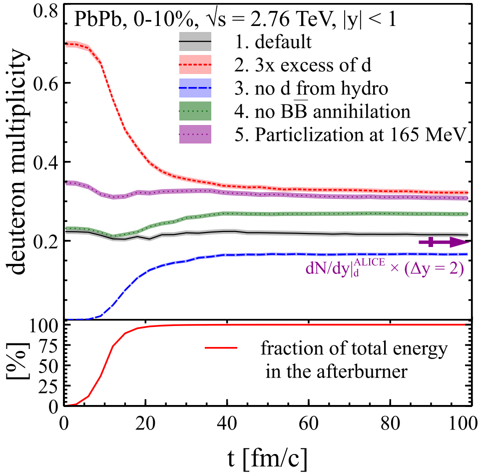
[I. Melo, B. Tomášik, J. Phys. G. 43 (2016) 015102]

Deuteron regeneration

[D. Oliinychenko, *et al.*, Phys. Rev. C 99 (2019) 044907]

If deuterons were produced at hadronisation, they are continuously destroyed and regenerated.

- SMASH: hybrid model
- deuterons generated at particlisation ($T = 155$ MeV)
- dominant reactions at high energies: $d\pi \leftrightarrow pn\pi$
- Hadronic reactions can keep the deuteron multiplicity constant after chemical freeze-out.



Production mechanism: coalescence

[R. Scheibl, U. Heinz, Phys. Rev. C 59 (1999) 1585]

Projection of the deuteron density matrix onto two-nucleon density matrix
Deuteron spectrum:

$$E_d \frac{dN_d}{d^3P_d} = \frac{3}{8(2\pi)^3} \int_{\Sigma_f} P_d \cdot d\Sigma_f(R_d)\, f_p\left(R_d, \frac{P_d}{2}\right) f_n\left(R_d, \frac{P_d}{2}\right) \mathcal{C}_d(R_d, P_d)$$

QM correction factor

$$\mathcal{C}_d(R_d, P_d) \approx \int d^3r \frac{f(R_+, P_d/2) f(R_-, P_d/2)}{f^2(R_d, P_d/2)} |\varphi_d(\vec{r})|^2$$

r relative position, R_+, R_-: positions of nucleons
approximation: narrow width of deuteron Wigner function in momentum

Correction factor: limiting cases

$$\mathcal{C}_d(R_d, P_d) \approx \int d^3r \frac{f(R_+, P_d/2) f(R_-, P_d/2)}{f^2(R_d, P_d/2)} \left|\varphi_d(\vec{r})\right|^2$$

- Large homogeneity region for nucleon momentum $P_d/2$: $L \gg d$ (L is the scale on which $f(R, P_d/2)$ changes)

$$\mathcal{C}_d(R_d, P_d) \approx \int d^3r \left|\varphi_d(\vec{r})\right|^2 = 1$$

  No correction! Just product of nucleon source functions.

- Small homogeneity region for nucleon momentum $P_d/2$: $L \ll d$
  $f(R, P_d/2)$ effectively limits the integration to Ω

$$\mathcal{C}_d(R_d, P_d) \approx \int_\Omega d^3r \left|\varphi_d(\vec{r})\right|^2 = \mathcal{C} < 1$$

Interesting regime: $L \approx d$

Analytical approximation of the (average) correction factor

[R. Scheibl, U. Heinz, Phys. Rev. C 59 (1999) 1585]

$$\langle \mathcal{C}_d \rangle (P_d) = \frac{\int_{\Sigma_f} P_d \cdot d\Sigma_f(R_d) f^2 \left(R_d, \frac{P_d}{2} \right) \mathcal{C}_d(R_d, P_d)}{\int_{\Sigma_f} P_d \cdot d\Sigma_f(R_d) f^2 \left(R_d, \frac{P_d}{2} \right)}$$

Approximations:

- Gaussian profile in rapidity and in the transverse direction,
- weak transverse expansion ($\Rightarrow$ no p_t dependence)
- saddle point integration

$$\langle \mathcal{C}_d \rangle \approx \left\{ \left(1 + \left(\frac{d}{2\mathcal{R}_\perp(m)} \right)^2 \right) \sqrt{1 + \left(\frac{d}{2\mathcal{R}_\parallel(m)} \right)^2} \right\}^{-1}$$

Homogeneity lengths:

$$\mathcal{R}_\perp = \frac{\Delta\rho}{\sqrt{1 + (m_t/T)\eta_f^2}} \qquad \mathcal{R}_\parallel = \frac{\tau_0\, \Delta\eta}{\sqrt{1 + (m_t/T)(\Delta\eta)^2}}$$

The invariant coalescence factor B_2

[R. Scheibl, U. Heinz, Phys. Rev. C 59 (1999) 1585]

$$E_d \frac{dN_d}{d^3P_d} = B_2 E_p \frac{dN_p}{d^3P_p} E_n \frac{dN_n}{d^3P_n}\bigg|_{P_p=P_n=P_d/2}$$

(Approximations with corrections for box profile)

$$\boxed{B_2 = \frac{3\pi^{3/2}\langle \mathcal{C}_d \rangle}{2m_t \mathcal{R}_\perp^2(m_t)\mathcal{R}_\parallel(m_t)} e^{2(m_t - m)(1/T_p^* - 1/T_d^*)}}$$

where the effective temperatures are

$$T_p^* = T + m_p \eta_f^2 \qquad T_d^* = T + \frac{M_d}{2}\eta_f^2$$

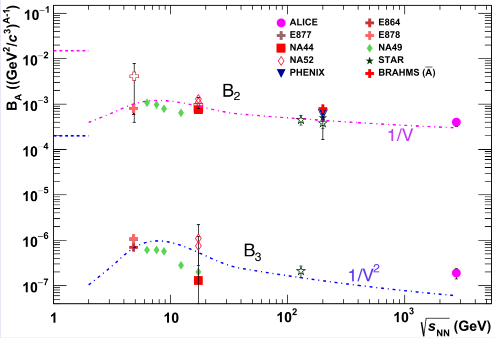
Approximate behaviour: $B_2 \approx 1/volume$

It works!

B_2 as function of $\sqrt{s_{NN}}$

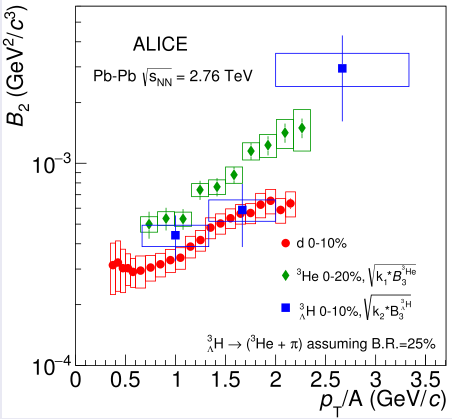
[P. Braun-Munzinger, B. Dönigus, Nucl. Phys. A987 (2019) 144]

Compared with $\propto 1/V$



B_2 as function of p_t

[J. Adam, *et al.* [ALICE coll.], Phys. Lett. B754 (2016) 360]



consistent with decreasing homogeneity volume

Difference to thermal production

[F. Bellini, A. Kahlweit, Phys. Rev. C 99 (2019) 054905]

For coalescence use

$$B_2 = \frac{3\pi^{3/2}\langle \mathcal{C}_d \rangle}{2m_t R^3(m_T)}$$

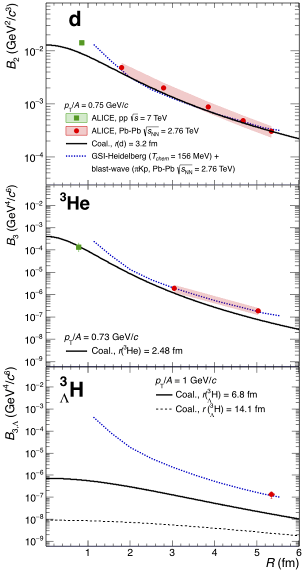
generalized

$$B_A = \frac{2J_A+1}{2^A} \frac{1}{\sqrt{A}} \frac{1}{m_T^{A-1}} \left(\frac{2\pi}{R^2 + (r_A/2)^2} \right)^{\frac{3}{2}(A-1)}$$

with

$$R = (0.473\,\mathrm{fm})\langle dN_{ch}/d\eta \rangle$$

Difference between coalescence and blast-wave for small source sizes.

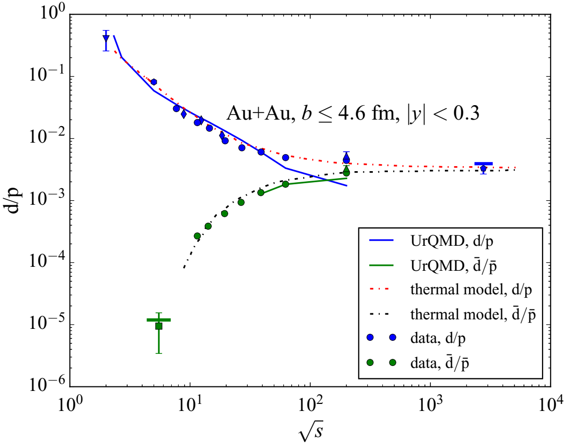


UrQMD with coalescence

[S. Sombun, *et al.*, Phys. Rev C 99 (2019) 014901]

- protons and neutrons followed in UrQMD
- after finishing of the fireball evolution, merge those which are close in phase-space
- $\Delta r_{max} = 3.575$ fm
  $\Delta p_{max} = 0.285$ GeV$/c$
- spin-isospin factor $3/8$

Deuteron production reasonably reproduced.



How to get the yields consistent with the statistical model?

Assume thermal source function (Boltzmann)

$$f_N(p_N, x) = 2\exp\left(-\frac{p_n \cdot u + \mu_N}{T}\right) H(r,\phi,\eta)$$

coalescence:

$$E_d \frac{dN_d}{d^3P_d} = \frac{3}{4}\int_{\Sigma_f} \frac{P_d \cdot d\Sigma_f(R_d)}{(2\pi)^3} \left(2\exp\left(-\frac{p_n \cdot u + \mu_N}{T}\right)\right)^2 (H(r,\phi,\eta))^2\, \mathcal{C}_d(R_d, P_d)$$

thermal production:

$$E_d \frac{dN_d}{d^3P_d} = 3\int_{\Sigma_f} \frac{P_d \cdot d\Sigma_f(R_d)}{(2\pi)^3} \exp\left(-\frac{P_d \cdot u + \mu_d}{T}\right) H(r,\phi,\eta)$$

they are equal if

- volume is large, i.e. $\mathcal{C}_d(R_d, P_d) = 1$
- $\mu_d = 2\mu_N$, and μ_N guarantees right number of nucleons - PCE
- $H^2(r,\phi,\eta) = H(r,\phi,\eta)$, fulfilled for box profile

see also [X. Xu, R. Rapp, Eur. Phys. J. 55 (2019) 68]

Lesson from coalescence

- deuteron spectrum sensitive to the shape of the density profile, through $(H(r,\phi,\eta))^2$
- proton spectrum sensitive to $H(r,\phi,\eta)$
- effects for homogeneity lengths comparable with the size of the cluster

$\Rightarrow$ femtoscopy probe

- elliptic flow of deuterons - probes finer changes in homogeneity lengths

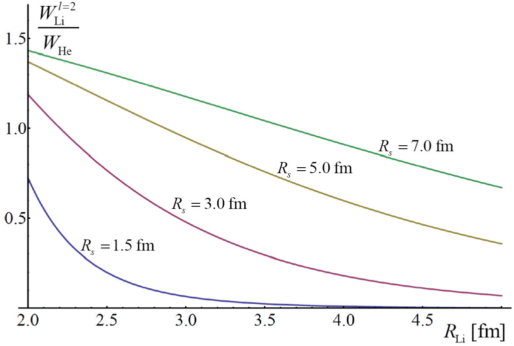
see also [A. Polleri, *et al.*, Phys. Lett. B 473 (2000) 193]

Distinguishing coalescence: large clusters

[S. Bazak, S. Mrówczyński, Mod. Phys. Lett. A 33 (2018) 1850142]

Compare production of clusters similar in masses: ^{4}He and ^{4}Li.

- Thermal model prediction: 5 times more ^{4}Li (spin 2) than ^{4}He (spin 0)
- Coalescence: uncertainty due to unknown size of ^{4}Li



clearly smaller yield than for thermal model

Experimental challenge: measure ^{4}Li from ^{3}He-p correlation function

Distinguishing coalescence: d number fluctuations

[Z. Fecková, *et al.*, Phys. Rev. C 93 (2016) 054906]

Measure the fluctuations of deuteron number.

- Thermal model prediction: Poissonian fluctuations
- Coalescence: protons and neutrons Poissonian, deuterons fluctuate more

Model A: fully correlated proton and neutron numbers: $\lambda_d = Bn_i^2$

$$P_d(n_d|n_i) = \lambda_d^{n_d} \frac{e^{-\lambda_d}}{n_d!} = (Bn_i^2)^{n_d} \frac{e^{-Bn_i^2}}{n_d!}$$

$$P_d(n_d) = \sum_{n_i \geq n_d} P_d(n_d|n_i) P_i(n_i)$$

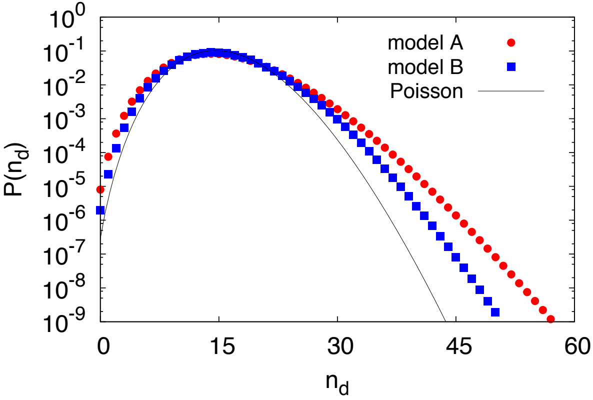
Model B: independent proton number n_i and neutron number n_j: $\lambda_d = Bn_i n_j$

$$P_d(n_d|n_i, n_j) = \lambda_d^{n_d} \frac{e^{-\lambda_d}}{n_d!} = (Bn_i n_j)^{n_d} \frac{e^{-Bn_i n_j}}{n_d!}$$

$$P_d(n_d) = \sum_{n_i, n_j \geq n_d} P_d(n_d|n_i, n_j) P_i(n_i) P(n_j)$$

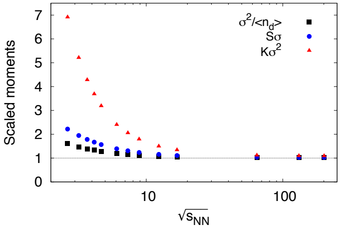
An example of deuteron number distribution

calculated for $\sqrt{s_{NN}} = 2.6\,\text{GeV}$
correlated p and n: $\sigma^2/\langle n_d \rangle = 1.609$, $S\sigma = 2.218$, $\kappa\sigma^2 = 6.915$
independent p and n: $\sigma^2/\langle n_d \rangle = 1.308$, $S\sigma = 1.616$, $\kappa\sigma^2 = 3.422$
Poissonian values are 1.

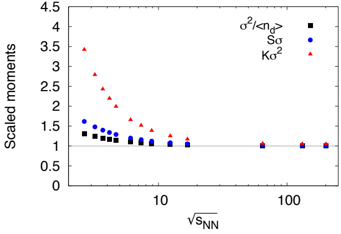


Predictions for the deuteron scaled moments

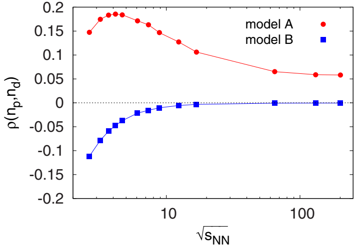
correlated p and n number



independent p and n number



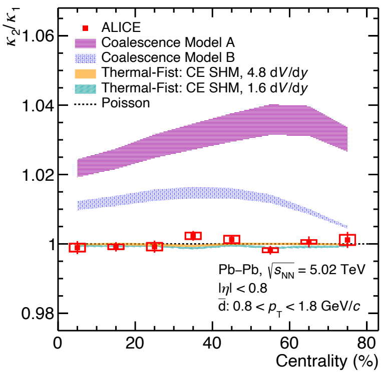
correlation coefficient
of d and p



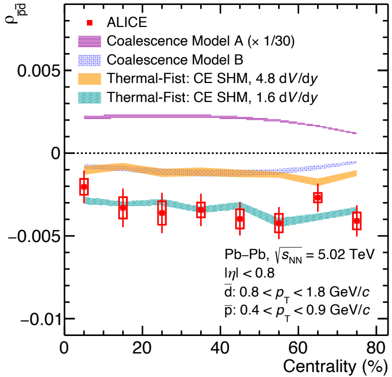
Distinguishing production mechanism from fluctuations: data by ALICE

[ALICE collaboration: arxiv 2204.10166 (PRL in press)]

(anti)d number fluctuations



correlation coefficient



Thermal-FIST: Statistical Hadronisation Model with Canonical Ensemble and correlation volume V_c for baryon number conservation.

[V. Vovchenko, B. Dönigus, H. Stöcker, Phys. Lett. B 785 (2018) 171]

Conclusions

- Is it coalescence or statistical model?
- Existence of clusters in the fireball?
  [V. Kireyeu, et al., Phys. Rev. C 105 (2022) 044909]
- coalescence is sensitive to space-time and dynamics features of the fireball at freeze-out—femtoscopic probe
- interesting femtoscopic application: homogeneity regions of the size of cluster wave functions
- Tasks:
  - review B_A calculations for sources with strong expansion
  - calculate/simulate v_2 of clusters (more detailed probe; no, it is not $Av_2(p_t/A)$!)
  - derive conditions for the freeze-out to match the thermal model abundances
  - explain fluctuations

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