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Perturbative approaches in relativistic kinetic theory and the emergence of first-order hydrodynamics

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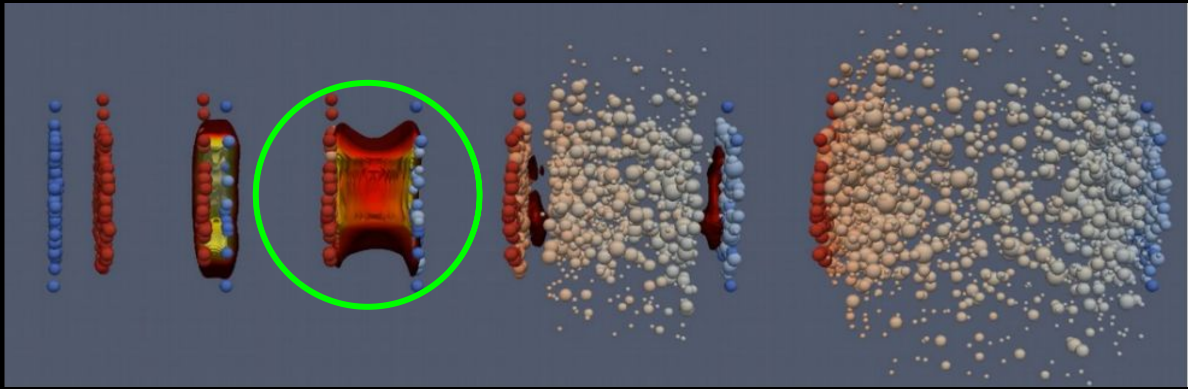
Conselho Nacional de Desenvolvimento Científico e Tecnológico

CAPES

Introduction

- Fluid dynamics: effective theory for collective behavior in systems out of equilibrium in the regime where there is a wide separation of scales $L_{micro} << L_{macro}$

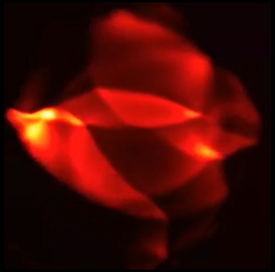
(Credit: Jorge Porto https://www.waves.com.br/expedicao/arraial-do-cabo-de-gala/)



Multistage models for heavy-ion collisions (image: MADAI collaboration)

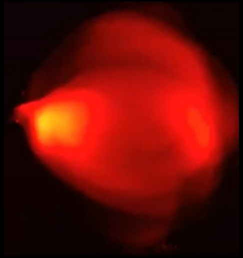
Hydrodynamic variables and EoMs

- Basic hydrodynamics EoMs: local conservation of net charge, energy and momentum $\partial_\mu N^\mu = 0 \quad \partial_\mu T^{\mu\nu} = 0$



- Simplest model: ideal fluid (all cells in local equilibrium)

$$N_E^\mu = n_0 u^\mu,$$
$$T_E^{\mu\nu} = \varepsilon_0 u^\mu u^\nu - P_0 \Delta^{\mu\nu}$$



- Dissipative fluids: Fictitious local equilibrium state

Credit: Chun Shen
https://youtu.be/G-Fbon0YQak

$$N^\mu = N_E^\mu + \tilde{N}^\mu = (n_0 + \delta n) u^\mu + \nu^\mu$$
$$T^{\mu\nu} = T_E^{\mu\nu} + \tilde{T}^{\mu\nu} = (\varepsilon_0 + \delta\varepsilon) u^\mu u^\nu - (P_0 + \Pi)\Delta^{\mu\nu} + h^\mu u^\nu + h^\nu u^\mu + \pi^{\mu\nu}$$

Matching conditions

- The separation is not unique – (α, β, u^μ) are not uniquely defined out of equilibrium

  $\frac{\mu}{T}$ $\frac{1}{T}$

  Kovtun JPA: Math. and Th., 45(47):473001, 2012

- Prescriptions are used to make the definitions – the matching conditions. They usually restrict the conserved tensors

- The most used in HIC is the Landau one: “the comoving observer should see no heat flux”

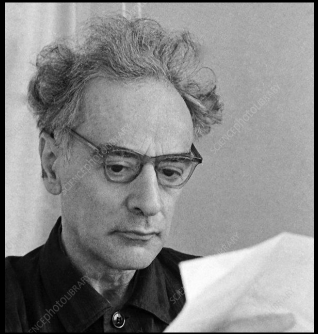
Define a fictitious equilibrium state so that we can define T and μ with equilibrium EoS

$$\delta n = \delta \varepsilon = 0$$

$$\varepsilon = \varepsilon_0(\mu, T),$$

$$n = n_0(\mu, T)$$

$$h^\mu = 0$$



Landau and Lifshitz: Fluid Mechanics - Volume 6 (Course of Theoretical Physics), 1987

Matching conditions

- The separation is not unique – (α, β, u^{μ}) are not uniquely defined out of equilibrium

Kovtun JPA: Math. and Th., 45(47):473001, 2012

$$\frac{\mu}{T} \quad \frac{1}{T}$$

- Prescriptions are used to make the definitions – the matching conditions. They usually restrict the conserved tensors

- The astrophysics community uses mostly Eckart matching: " u^{μ} is the velocity of [one of the] matter currents"

$$\delta n = \delta \varepsilon = 0$$

$$\varepsilon = \varepsilon_0(\mu, T),$$

$$n = n_0(\mu, T)$$

$$\nu^{\mu} = 0$$

Define a fictitious equilibrium state so that we can define T and μ as before

C. Eckart, Physical Review 58, 919 (1940)

Hydrodynamic variables and EoMs

- $\partial_\mu N^\mu = 0 \quad \partial_\mu T^{\mu\nu} = 0$ 5 Eqs for 14 variables (Landau)

- Constitutive relations/further dynamical equations must be derived
  - Navier-Stokes (Landau):

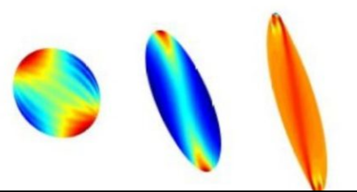
$$\Pi = -\zeta\theta$$

$$\theta \equiv \partial_\mu u^\mu$$

$$\nu^\mu = \kappa \nabla^\mu \alpha$$

$$\nabla^\mu = \Delta^{\mu\nu}\partial_\nu;$$

$$\pi^{\mu\nu} \equiv 2\eta\sigma^{\mu\nu}$$



$$\sigma^{\mu\nu} = \Delta^{\mu\nu\alpha\beta}\partial_\alpha u_\beta$$

Only space-like derivatives!

  - Linearly acausal and unstable EoMs

Pichon, Ann. de l'I.H.P. Phys. théo. 2, 21 (1965)
Hiscock, Lindblom PRD 31, 725 (1985)

  - Possible solutions: IS-like theory; *BDNK theory*

BDN - PRD, 98(10):104064, (2018); PRD 100(10):104020, (2019); PRX 12 2 021044 (2022)
K - JHEP 1910 (2019) 034

Bemfica-Disconzi-Noronha-Kovtun hydro

Idea: Modified constitutive relations

now with *time-like* derivatives $D = u \cdot \partial$

BDN - PRD, 98(10):104064, (2018); PRD 100(10):104020, (2019); PRX 12 2 021044 (2022)
K - JHEP 1910 (2019) 034

General matching:

$$\Pi = \zeta^{(\alpha)} D\alpha - \zeta^{(\beta)} \frac{D\beta}{\beta} - \zeta^{(\theta)}\theta,$$

$$\delta n = \xi^{(\alpha)} D\alpha - \xi^{(\beta)} \frac{D\beta}{\beta} - \xi^{(\theta)}\theta,$$

$$\delta\varepsilon = \chi^{(\alpha)} D\alpha - \chi^{(\beta)} \frac{D\beta}{\beta} - \chi^{(\theta)}\theta,$$

$$\nu^\mu = \kappa^{(\alpha)} \nabla^\mu \alpha - \kappa^{(\beta)} \left(\frac{1}{\beta} \nabla^\mu \beta + Du^\mu \right),$$

$$h^\mu = \lambda^{(\alpha)} \nabla^\mu \alpha - \lambda^{(\beta)} \left(\frac{1}{\beta} \nabla^\mu \beta + Du^\mu \right),$$

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}, \qquad \nabla^\mu = \Delta^{\mu\nu}\partial_\nu; \qquad \sigma^{\mu\nu} = \Delta^{\mu\nu\alpha\beta}\partial_\alpha u_\beta$$

- EoMs are causal, hyperbolic and have non-negative entropy production even w/ GR. [BDN **PRX 12 2 021044**]

- Non-linear causality requires

$$\delta\varepsilon \neq 0 \quad h^\mu \neq 0$$

BDN *PRD* 100(10):104020, (2019)

Unusual definitions of the equilibrium state!

How do the coeffs. depend on μ,T?

Microscopic derivation of BDNK from the Boltzmann eqn.

de Groot, van Leeuwen, van Weert, *Relativistic Kinetic Theory: Principles and Applications* (North-Holland,1980)

- Chapman-Enskog (Navier-Stokes): compatibility conditions imply $D \mapsto \nabla_\mu$

$$\epsilon p^\mu \partial_\mu f_p = C[f_p]$$

$$f_{\mathbf{p}} = \sum_{i=0}^{\infty} \epsilon^i f_{\mathbf{p}}^{(i)}. \qquad Df_p = \sum_{i=0}^{\infty} \epsilon^i (Df_p)^{(i)}$$

"Inhomogeneity must be orthogonal to zero-mode space"

At O(1) $\int dP p^\nu \left(p^\mu \partial_\mu f_{0p} = f_{0p} \hat{L} \phi_p \right) \Rightarrow$ **Euler eqns**

- To derive BDNK, we propose a novel perturbation method GSR, Denicol, Noronha PRD 106, 036010 (2022)

$$\epsilon \int dP P_n^{(\ell)}(\beta E_{\mathbf{p}}) p_{\langle \mu_1} \cdots p_{\mu_\ell \rangle} p^\mu \partial_\mu f_{\mathbf{p}} = \int dP P_n^{(\ell)}(\beta E_{\mathbf{p}}) p_{\langle \mu_1} \cdots p_{\mu_\ell \rangle} C[f_{\mathbf{p}}],$$

Basis, not necessarily orthogonal, conservation laws excluded from the perturbative procedure

Microscopic derivation of BDNK from the Boltzmann eqn.

- Using $P_n^{(\ell)}(\beta E_{\mathbf{p}}) = (\beta E_{\mathbf{p}})^n, \; n = 0, 1, \cdots$

conservation laws excluded from the perturbative procedure

$$\int dP p^{\mu} \partial_{\mu} f_{\mathbf{p}} = 0, \; \int dP E_{\mathbf{p}} p^{\mu} \partial_{\mu} f_{\mathbf{p}} = 0, \; \int dP p_{\langle \mu \rangle} p^{\alpha} \partial_{\alpha} f_{\mathbf{p}} = 0,$$

$$\epsilon \int dP (\beta E_{\mathbf{p}})^n p^{\mu} \partial_{\mu} f_{\mathbf{p}} = \int dP (\beta E_{\mathbf{p}})^n C[f_{\mathbf{p}}], \quad n = 2, 3, 4, \cdots,$$

$$\epsilon \int dP (\beta E_{\mathbf{p}})^n p_{\langle \mu \rangle} p^{\alpha} \partial_{\alpha} f_{\mathbf{p}} = \int dP (\beta E_{\mathbf{p}})^n p_{\langle \mu \rangle} C[f_{\mathbf{p}}], \quad n = 1, 2, 3, \cdots,$$

$$\epsilon \int dP (\beta E_{\mathbf{p}})^n p_{\langle \mu_1} \cdots p_{\mu_\ell \rangle} p^{\mu} \partial_{\mu} f_{\mathbf{p}} = \int dP (\beta E_{\mathbf{p}})^n p_{\langle \mu_1} \cdots p_{\mu_\ell \rangle} C[f_{\mathbf{p}}],$$

$n = 0, 1, 2, \cdots$, for $\ell \geq 2$,

- Another choice $P_m^{(\ell)}(x) = \dfrac{x^{m - m_\ell}}{(1 + x)^{N - n_\ell}}, \; m = 1, \cdots N$

N: Truncation order

Inspired in Arnold, Moore, and Yaffe, JHEP 11, 001 (2000); Arnold, Moore, and Yaffe, JHEP 01, 030 (2003)

Transport coefficients

- First order: two classes of matching conditions

No obvious physical interpretation for both "Exotic Eckart"

(i) $\delta n \equiv 0 \quad \rho_s = \int dP E_{\mathbf{p}}^s \delta f_{\mathbf{p}} \equiv 0 \quad \nu^\mu \equiv 0$

$$\delta \varepsilon = \chi^{(\alpha)} D\alpha - \chi^{(\beta)} \left(\frac{D\beta}{\beta} - \frac{1}{3}\theta \right),$$

(ii) $\delta \varepsilon \equiv 0 \quad \rho_s = \int dP E_{\mathbf{p}}^s \delta f_{\mathbf{p}} \equiv 0 \quad \nu^\mu \equiv 0$

$$\delta n = \xi^{(\alpha)} D\alpha - \xi^{(\beta)} \left(\frac{D\beta}{\beta} - \frac{1}{3}\theta \right),$$

$$h^\mu = \lambda^{(\alpha)} \nabla^\mu \alpha - \lambda^{(\beta)} \left(\frac{1}{\beta} \nabla^\mu \beta + D u^\mu \right), \qquad \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu},$$

For both classes

Massless limit, relaxation time approximation, constant relaxation time

GSR, Denicol, Noronha PRL 127, 042301 (2021)

(i)

Trsp. cff. / N	10
$\chi^{(\alpha)}/(P_0\tau_R)$ $(s=3)$	1.50
$\chi^{(\beta)}/(P_0\tau_R)$ $(s=3)$	7.50
$\chi^{(\alpha)}/(P_0\tau_R)$ $(s=4)$	1.00
$\chi^{(\beta)}/(P_0\tau_R)$ $(s=4)$	6.00

(ii)

Trsp. cff. / N	10
$\xi^{(\alpha)}/(P_0\tau_R)$ $(s=3)$	-1.00
$\xi^{(\beta)}/(P_0\tau_R)$ $(s=3)$	-5.00
$\xi^{(\alpha)}/(P_0\tau_R)$ $(s=4)$	-0.50
$\xi^{(\beta)}/(P_0\tau_R)$ $(s=4)$	-3.00

(i) and (ii)

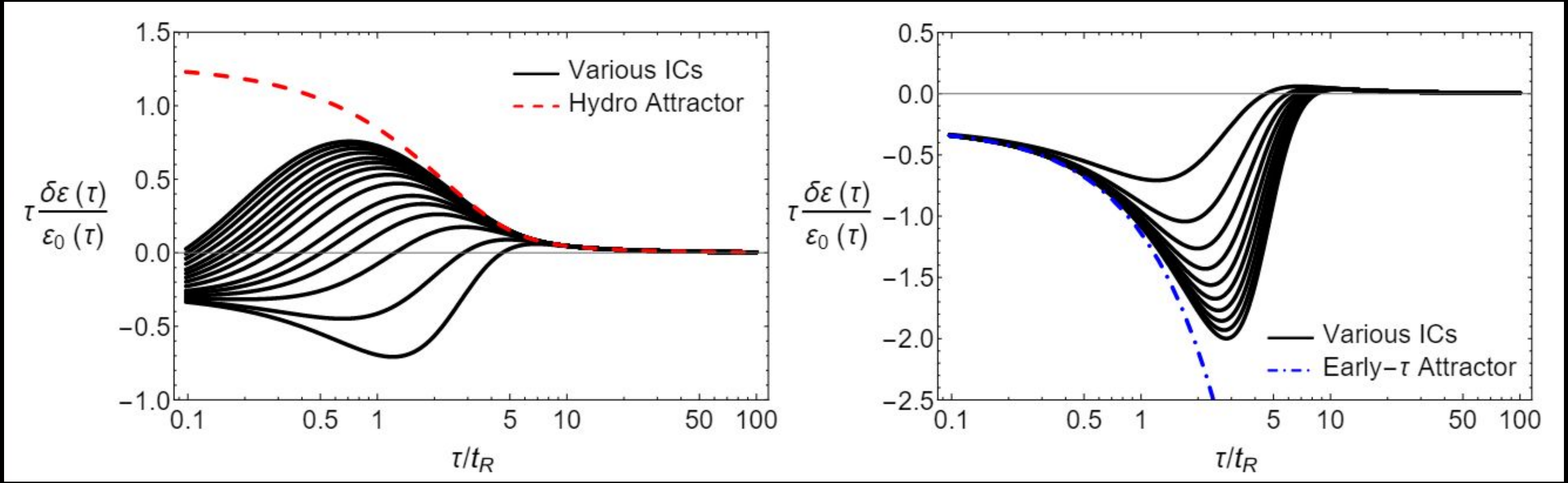
Trsp. cff. / N	10
$\eta/(P_0\tau_R)$	0.80
$\lambda^{(\alpha)}/(P_0\tau_R)$	1.33
$\lambda^{(\beta)}/(P_0\tau_R)$	4.00

Attractor structure of BDNK theory in Bjorken flow

Inspired in BDN *PRD* 100(10):104020, (2019)

- $\delta n \equiv 0, \delta\varepsilon \neq 0$ – both hydro and early time attractors

(i) Dynamics independent of ‘s’ $\rho_s = \left\langle E_{\mathbf{p}}^s \phi_{\mathbf{p}} \right\rangle_0 \equiv 0$



Late time attractor

Pullback attractor

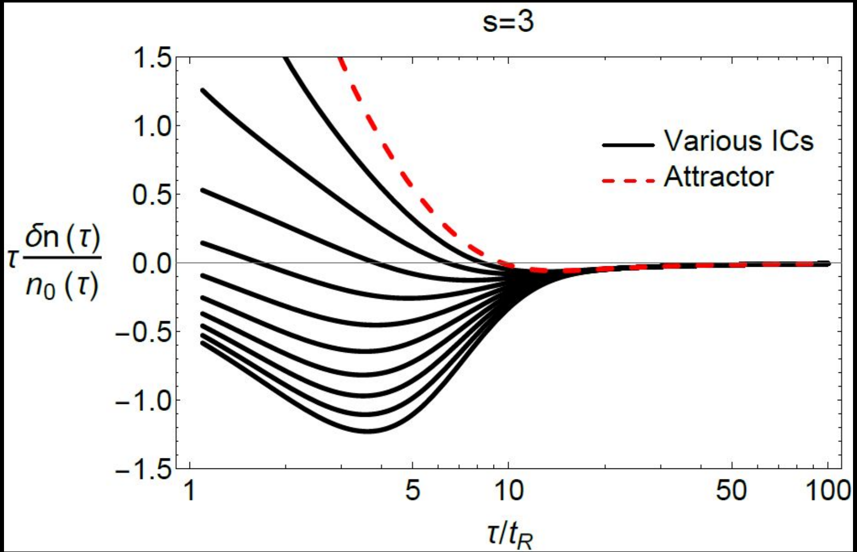
A. Behtash, S. Kamata, M. Martinez, and H. Shi, Phys. Rev. D 99, 116012 (2019)

Attractor structure of BDNK theory in Bjorken flow

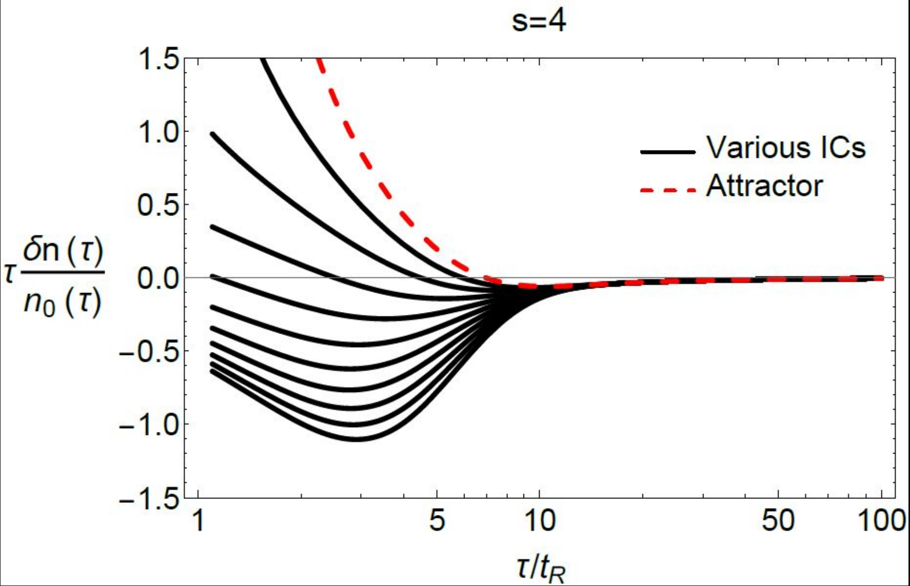
- $\delta n \neq 0, \delta\varepsilon \equiv 0$ No pullback attractor $\rho_s = \langle E_{\mathbf{p}}^s \phi_{\mathbf{p}} \rangle_0 \equiv 0$

(ii)

's'-dependent dynamics



s=3



s=4

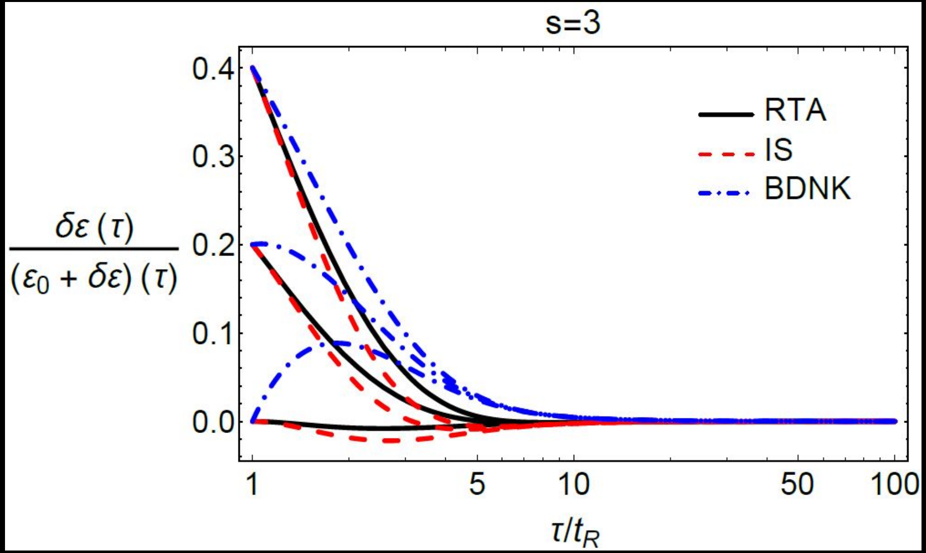
Comparison of attractor structures in Bjorken flow

GSR & Denicol, PRD 104, 9, 096016, (2021).

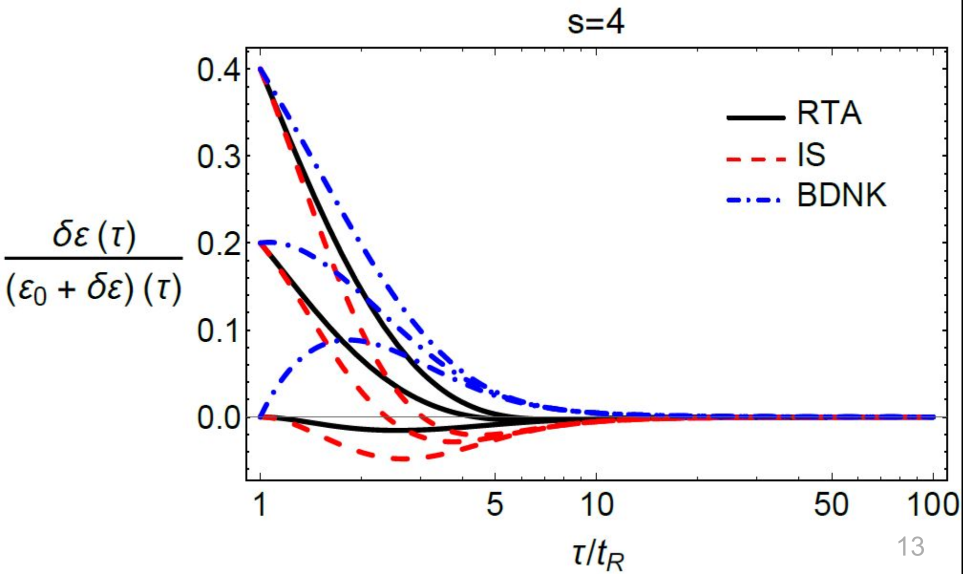
- We compare the evolution under the Boltzmann moment equation, IS, and BDNK in Bjorken flow for the alternative matching conditions

(i) $\delta n \equiv 0, \delta\varepsilon \neq 0 \quad \rho_s = \langle E_{\mathbf{p}}^s \phi_{\mathbf{p}} \rangle_0 \equiv 0$

s=3



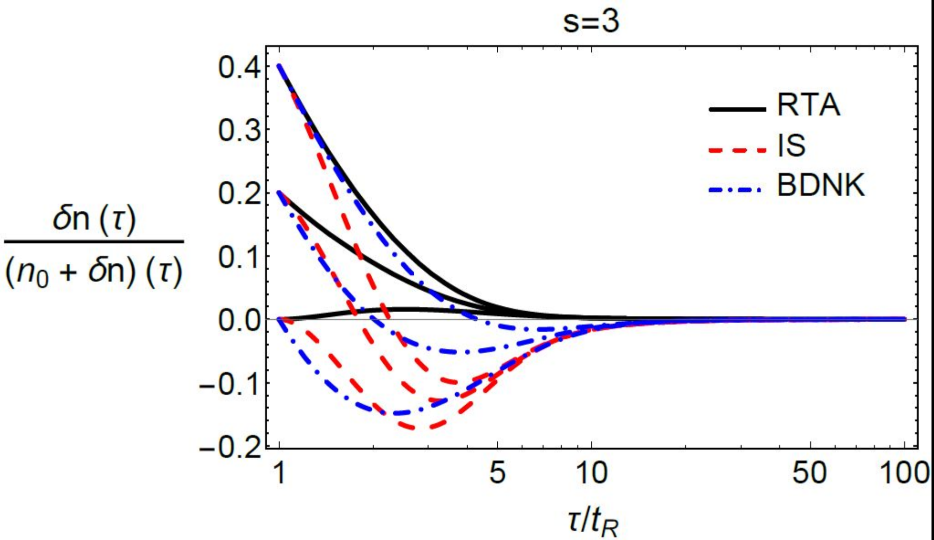
s=4

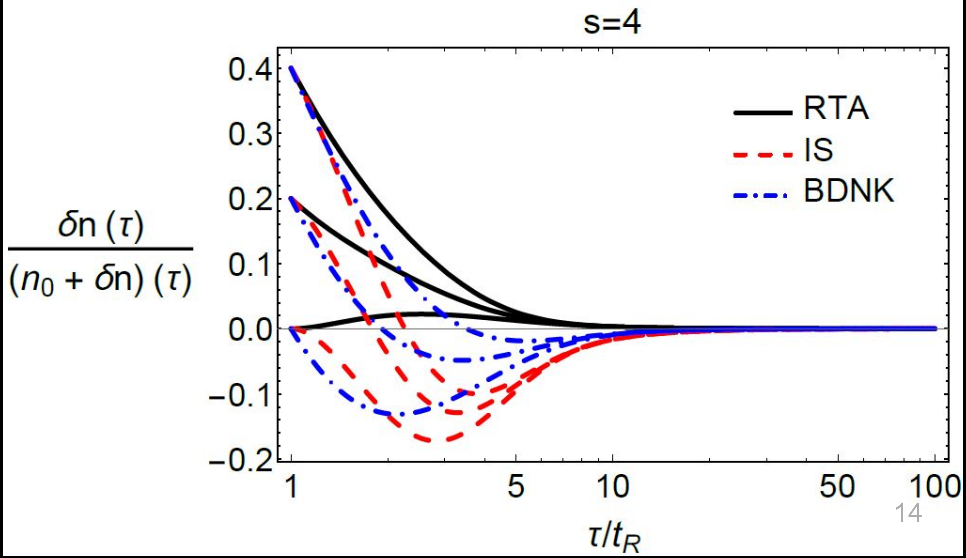


Comparison of attractor structures in Bjorken flow

- We compare the evolution under the Boltzmann moment equation, IS, and BDNK in Bjorken flow for the alternative matching conditions

s=3 **(ii)** $\delta n \neq 0, \delta\varepsilon \equiv 0 \quad \rho_s = \left\langle E_{\mathbf{p}}^s \phi_{\mathbf{p}} \right\rangle_0 \equiv 0$ s=4





Conclusions

- We proposed a novel perturbative procedure to derive BDNK hydrodynamics from Kinetic Theory;
- We analytically compute the attractors of BDNK theory and compared it with Boltzmann moments and IS EoMs in Bjorken flow;

https://dailynewshungary.com/19-photos-that-will-make-you-regret-you-did-not-visit-budapest-in-the-winter/

- We intend to generalize to other backgrounds and for momentum-dependent relaxation times;
- Improve hydro IS-like truncation;
- Application in thermal mass models;

THANK YOU FOR THE ATTENTION!