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Medium induced gluon spectrum in dense inhomogeneus matter

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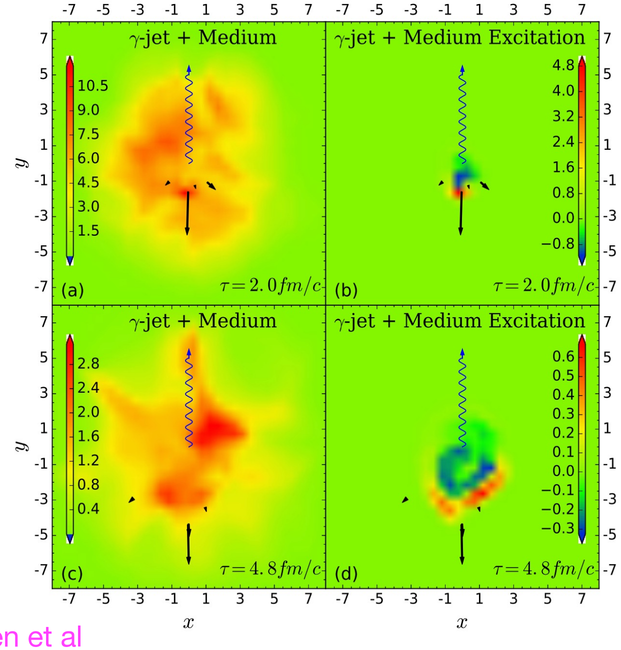
Mainly based on work done with João Barata, Andrey Sadofyev and Carlos Salgado

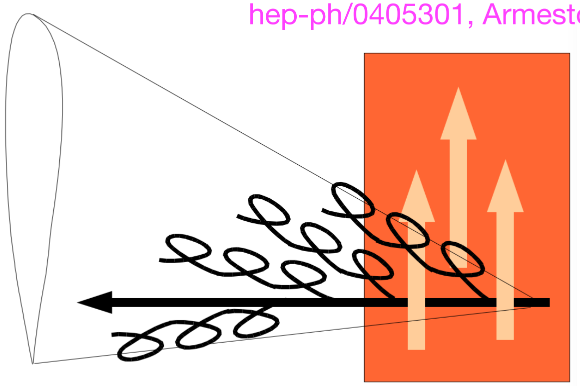
Jet tomography

Jet tomography: jets as differential probes of the spatio-temporal structure of the termal matter created in HIC



1704.03648, Chen et al

hep-ph/0405301, Armesto et al



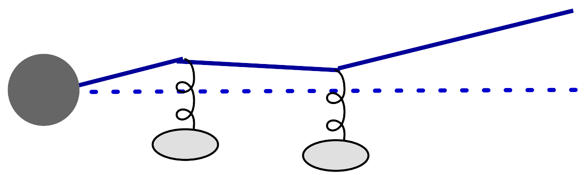
Medium response to **jets**

Jet response to **medium**

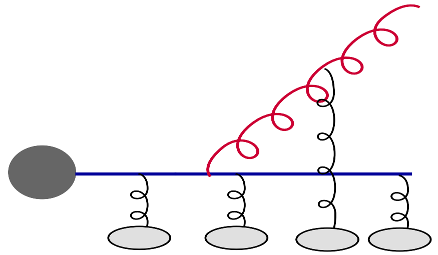
Jets in hot plasma

Focus on leading perturbative processes: Two processes that modify jets.

Single particle broadening



Medium induced soft gluon radiation

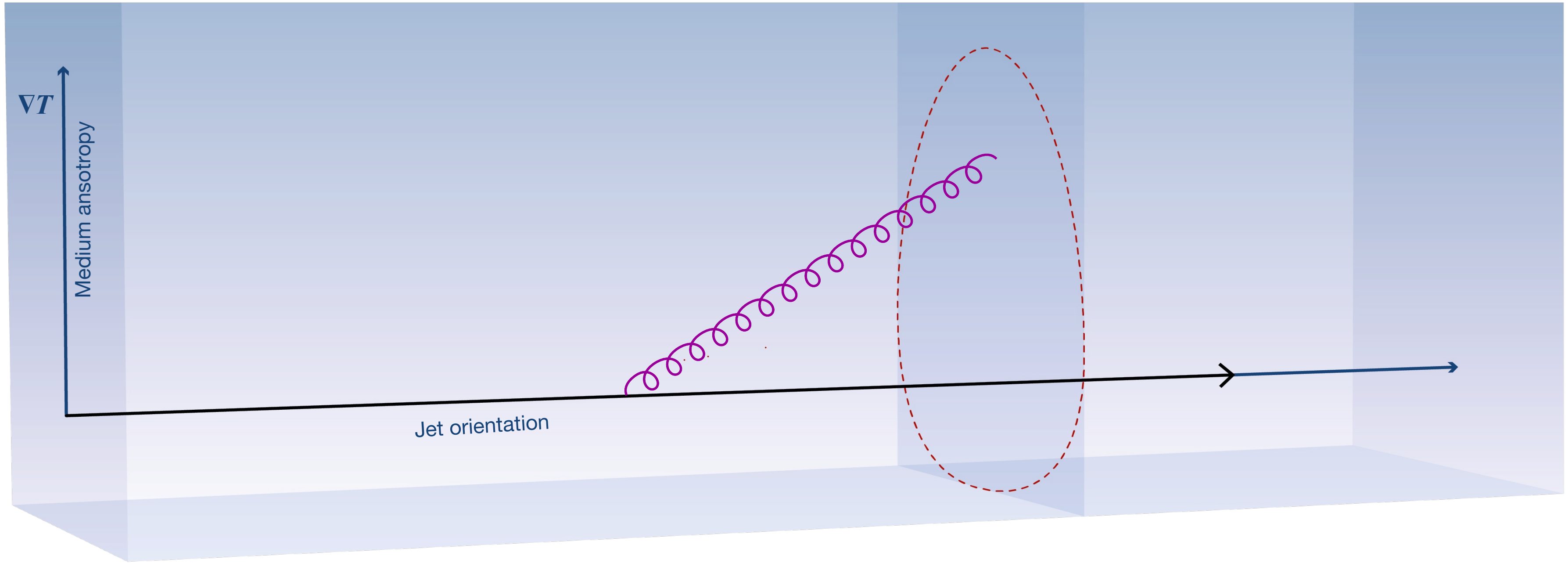


Theoretical formulation of jet quenching require several assumptions to make it tractable. Some of them are:

- Eikonal expansion; only sub-eikonal length enhanced terms are kept
- Medium is modeled by a background field
- In the simplest scenario the medium is static, homogeneous

→ Relaxed

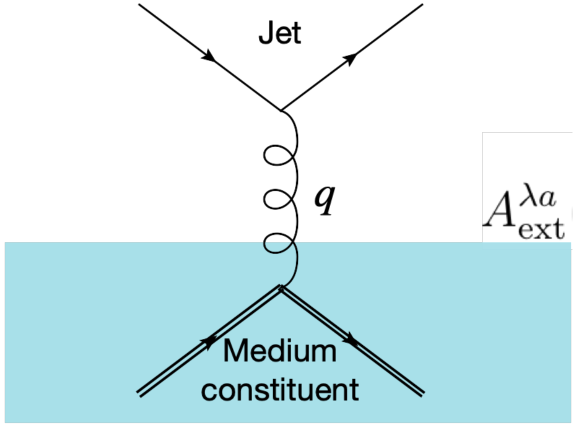
Medium induced gluon spectrum in a dense inhomogeneous static medium



Background colour field

For more information:
2104.09513, Sadofyev et al
2202.08847, Barata et al

The medium is modeled by a classical color field



$$gA^{\lambda}_{ext}(q) = -(2\pi)g^{\lambda 0}\sum_i e^{-i(\mathbf{q}\cdot\mathbf{x}_j + q_z z_j)}\, t^a_j\, v_j(q)\, \delta(q^0)$$

No energy transfer on each scattering with the medium

Model dependent scattering potential for source j

For example the GW model $$v_j(q) = \frac{-g^2}{-q_0^2 + \mathbf{q}^2 + q_z^2 + \mu_j^2 - i\epsilon}$$

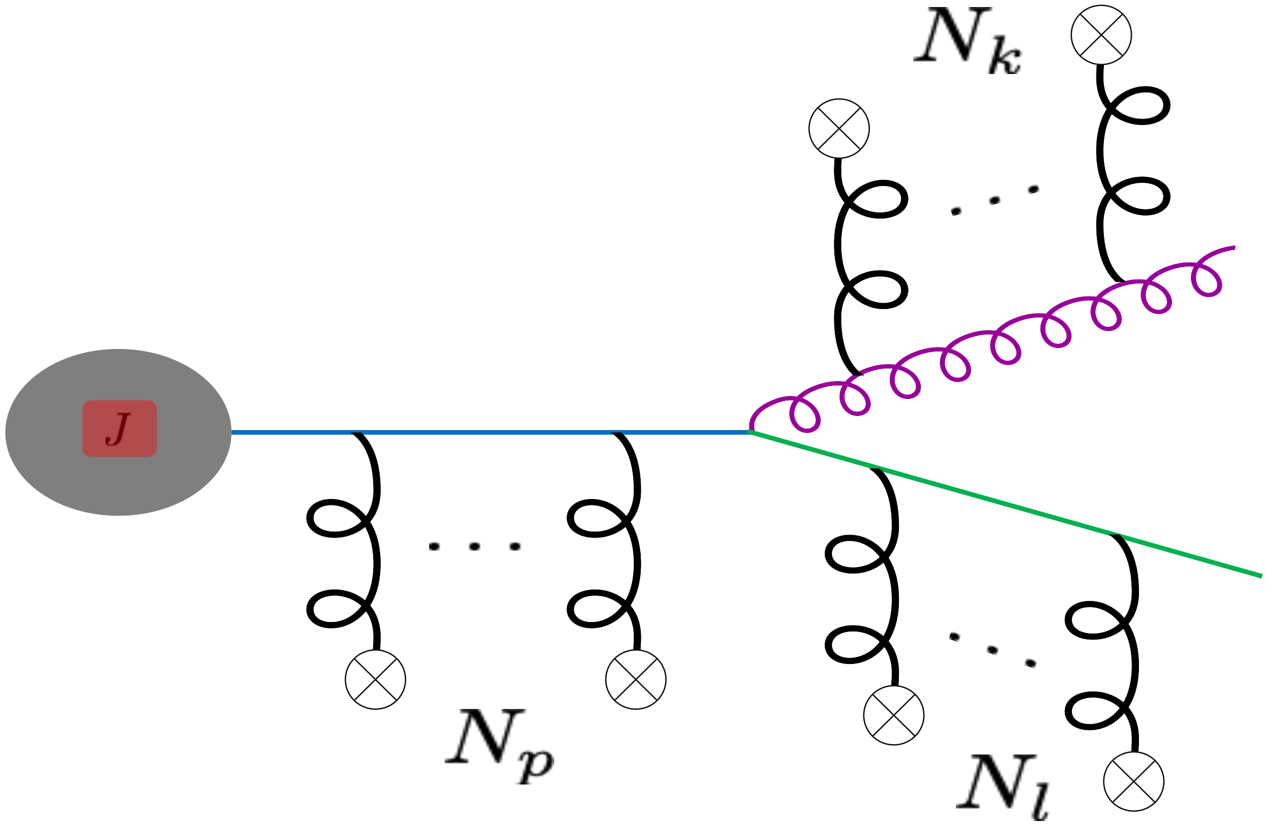
The scattering potential enters the effective propagators

$$G(\mathbf{x}_L, L; \mathbf{x}_0, 0) = \int_{\mathbf{x}_0}^{\mathbf{x}_L} \mathcal{D}\mathbf{r} \exp\left(i\frac{E}{2}\int_0^L d\tau\, \dot{\mathbf{r}}^2\right) \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{proj} v^a(\mathbf{r}(\tau),\tau)\right) \qquad \mathcal{W}(\mathbf{x}_1; L, 0) = \mathcal{P}\exp\left(i\int_0^L d\tau\, t^a_{proj} v^a(\mathbf{x}_1,\tau)\right)$$

MI radiation in inhomogeneus media

J. Barata, XML, A. Sadofyev, C. Salgado

The relevant diagram is



The amplitude can be ressumed to the simple form

$$iR \simeq -\frac{g}{\omega}\int_0^{\infty} dz_s \int d^2x_{in}\, e^{-i\mathbf{x}_{in}\cdot\mathbf{l}_f}\, J(\mathbf{x}_{in})\, \mathcal{W}(\mathbf{x}_{in};L,z_s)\, t^a_{proj}\, \mathcal{W}(\mathbf{x}_{in};z_s,0)\, e^{i\frac{\mathbf{k}^2_{f\perp}}{2\omega}L}\, \left[\epsilon\cdot\nabla_{\mathbf{x}_{in}}\mathcal{G}^{ba}\left(\mathbf{k}_f,L;\mathbf{x}_{in},z_s\right)\right]$$

MI radiation in inhomogeneus media

J. Barata, XML, A. Sadofyev, C. Salgado

The average of two scattering potentials in two different points is See 2202.08847, Barata et al

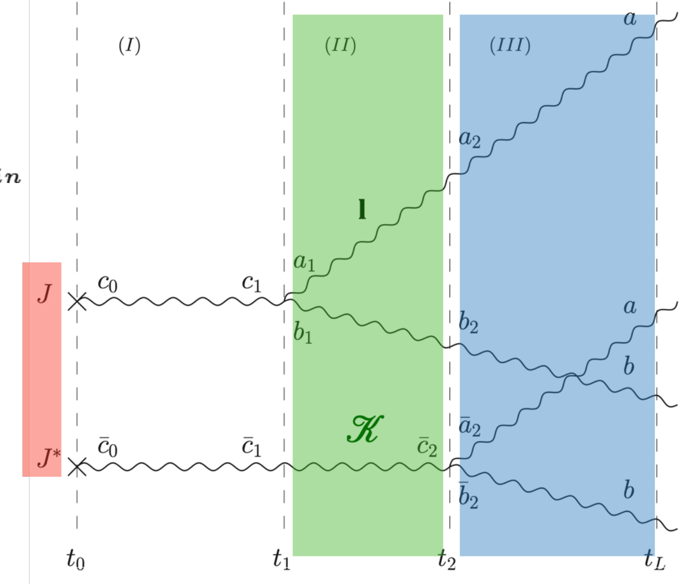
$\hat{\mathbf{g}}$

$$\langle v^a(\mathbf{x}, z) v^{\dagger b}(\bar{\mathbf{x}}, \bar{z})\rangle \simeq \frac{\delta^{ab}}{2C_{\bar{R}}} g^4\,\delta(z-\bar{z})\left(1+\frac{\mathbf{x}+\bar{\mathbf{x}}}{2}\cdot\left(\nabla\rho\frac{\delta}{\delta\rho}+\nabla\mu^2\frac{\delta}{\delta\mu^2}\right)\right)\int\frac{d^2q}{(2\pi)^2}\frac{\rho(z)\,e^{i\mathbf{q}\cdot(\mathbf{x}-\bar{\mathbf{x}})}}{(\mathbf{q}^2+\mu^2(z))^2}$$

Local in z-component which enables to split averages on different regions and write the spectrum

1209.4585, Blaizot et al

$$dN = C_F\frac{g^2}{w^2}2\Re\int_{\bar{z},z,\mathbf{x}_{in},\mathbf{y}}\left|J(\mathbf{x}_{in})\right|^2(\nabla_{\mathbf{x}}\cdot\nabla_{\bar{\mathbf{x}}})\,\mathcal{K}(\mathbf{y},\mathbf{x}_{in},\bar{z};\mathbf{x},\mathbf{x}_{in},z)\,S^{(2)}(\mathbf{k}_f,\mathbf{k}_f,L;\mathbf{y},\bar{\mathbf{x}},\bar{z})\Big|_{\mathbf{x}=\bar{\mathbf{x}}=\mathbf{x}_{in}}$$



Radiative kernel

$$\mathcal{K}(\mathbf{y},\mathbf{x}_{in},\bar{z};\mathbf{x},\mathbf{x}_{in},z) = \frac{1}{N_c^2-1}\left\langle\mathcal{G}^{ba}(\mathbf{y},\bar{z};\mathbf{x},z)\,\mathcal{W}_A^{\dagger\,ab}(\mathbf{x}_{in};\bar{z},z)\right\rangle$$

2-point function

$$S^{(2)}(\mathbf{k}_f,\mathbf{k}_f,L;\mathbf{y},\bar{\mathbf{x}},\bar{z}) = \frac{1}{N_c^2-1}\left\langle\mathcal{G}^{bc}(\mathbf{k}_f,L;\mathbf{y},\bar{z})\,\mathcal{G}^{\dagger\,cb}(\mathbf{k}_f,L;\bar{\mathbf{x}},\bar{z})\right\rangle$$

The two point function

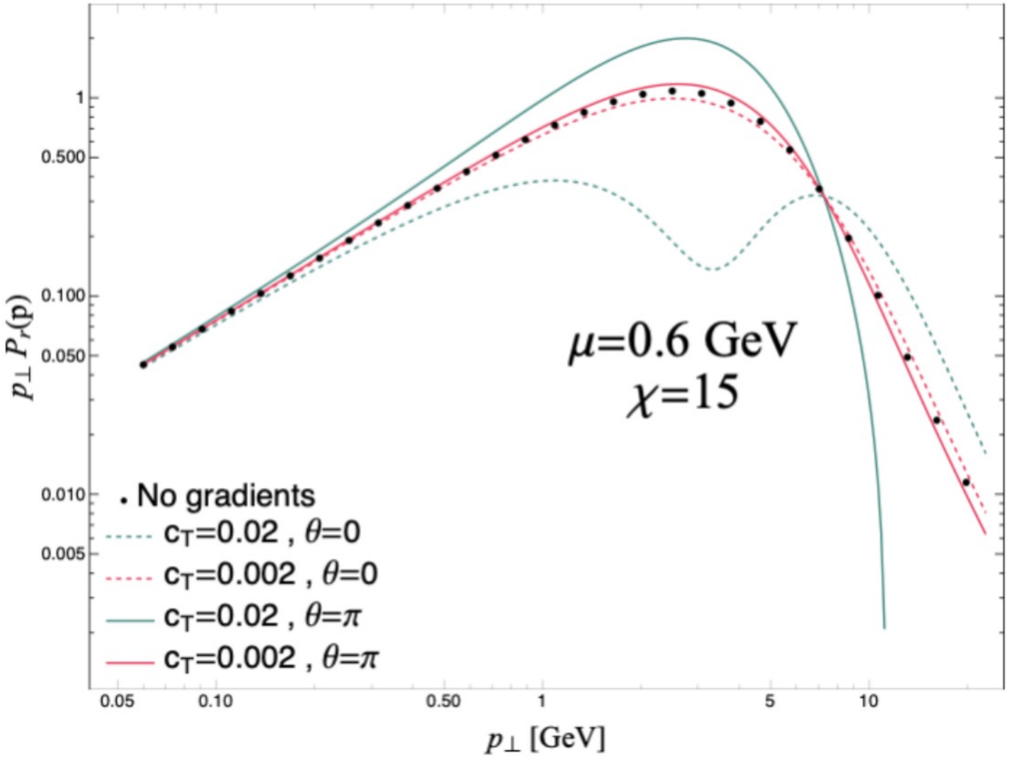
The two point function $S^{(2)}$ has been solved at first order in gradinets in 2202.08847; Barata et al

An operator that acts on the initial distribution appears due to the effect of gradients

$$\frac{d\mathcal{N}}{d^2\mathbf{x}dE} = \mathcal{P}(\mathbf{x})\hat{\mathcal{S}}(\mathbf{x})\frac{d\mathcal{N}^0}{d^2\mathbf{x}dE}$$

$$\mathcal{P}(\mathbf{p}) = \int d^2\mathbf{x}\, e^{-i\mathbf{p}\cdot\mathbf{x}}\, e^{-\mathcal{V}(\mathbf{x})L}\left[1 - \frac{iL^3}{6E}\nabla\mathcal{V}(\mathbf{x})\cdot\hat{\mathbf{g}}\mathcal{V}(\mathbf{x})\right]$$

$$\hat{\mathcal{S}}(\mathbf{x}) = e^{-\mathcal{V}(\mathbf{x})L}\left[1 + \frac{iL^2}{2E}\hat{\mathbf{g}}\mathcal{V}(\mathbf{x})\cdot\nabla\right]$$



The radiative kernel

The radiative kernel is a path integral

$$\mathcal{K}(\mathbf{y}, \mathbf{x}_{in}, \bar{z}; \mathbf{x}, \mathbf{x}_{in}, z) = \int_{\mathbf{x}}^{\mathbf{y}} \mathcal{D}\mathbf{r}\, e^{\frac{i\omega}{2}\int d\tau\, \dot{\mathbf{r}}^2 - \int_z^{\bar{z}} d\tau \left[1 + \frac{\mathbf{r}(\tau)+\mathbf{x}_{in}}{2}\cdot\hat{\mathbf{g}}\right]\mathcal{V}(\mathbf{r}(\tau)-\mathbf{x}_{in})}$$

The case without the gradients has been solved in some limits. We are going to use the solution under the Gaussian approximation See for example 1205.5739, Mehtar-Tani et al

First order gradient correction obtained expanding the path integral

$$\delta\mathcal{K}(\mathbf{y}, \bar{z}; \mathbf{x}, z) = -\int_z^{\bar{z}} dt \int_{\mathbf{s}} \mathcal{K}_0(\mathbf{y}, \bar{z}; \mathbf{s}, t)\, \delta\mathcal{V}(\mathbf{s})\, \mathcal{K}_0(\mathbf{s}, t; \mathbf{x}, z)$$

MI radiation in inhomogeneus media

J. Barata, XML, A. Sadofyev, C. Salgado

The most general expression for the MI soft gluon spectrum at leading order in gradients is the following

We can separate the gradient corrections coming from different sources

No initial state contributions $\left|J(\mathbf{x}_{in})\right|^2 = \delta^{(2)}(\mathbf{x}_{in})$

The kernel and the effective potential are taken in the harmonic approximation

$$\mathcal{V}(\mathbf{y}) \approx \frac{\hat{q}}{4}\mathbf{y}^2\left(1+\frac{\hat{\mathbf{g}}}{2}\cdot\mathbf{y}\right) \equiv \mathcal{V}(\mathbf{y}) + \delta\mathcal{V}(\mathbf{y})$$

$$(2\pi)^2\omega\frac{dI}{d\omega d^2\mathbf{k}} = (2\pi)^2\omega\frac{dI_0}{d\omega d^2\mathbf{k}} + (2\pi)^2\omega\frac{dI_{\mathcal{P}}}{d\omega d^2\mathbf{k}} + (2\pi)^2\omega\frac{dI_{\mathcal{K}}}{d\omega d^2\mathbf{k}} + (2\pi)^2\omega\frac{dI_{\hat{\mathcal{S}}}}{d\omega d^2\mathbf{k}}$$

$$\equiv \frac{2\alpha_s C_F}{\omega^2}\Re\int_0^L d\bar{z}\int_0^{\bar{z}} dz\int_{\mathbf{y}} e^{-i\mathbf{k}\cdot\mathbf{y}}\mathcal{P}_{0;L-\bar{z}}(\mathbf{y})\nabla_{\mathbf{y}}\cdot\nabla_{\mathbf{x}}\mathcal{K}_0\left(\mathbf{y},\bar{z};\mathbf{x},z\right)\Big|_{\mathbf{x}=0}$$

$$+\frac{2\alpha_s C_F}{\omega^2}\Re\int_0^L d\bar{z}\int_0^{\bar{z}} dz\int_{\mathbf{y}} e^{-i\mathbf{k}\cdot\mathbf{y}}\delta\mathcal{P}_{L-\bar{z}}(\mathbf{y})\nabla_{\mathbf{y}}\cdot\nabla_{\mathbf{x}}\mathcal{K}_0\left(\mathbf{y},\bar{z};\mathbf{x},z\right)\Big|_{\mathbf{x}=0}$$

$$+\frac{2\alpha_s C_F}{\omega^2}\Re\int_0^L d\bar{z}\int_0^{\bar{z}} dz\int_{\mathbf{y}} e^{-i\mathbf{k}\cdot\mathbf{y}}\mathcal{P}_{0;L-\bar{z}}(\mathbf{y})\nabla_{\mathbf{y}}\cdot\nabla_{\mathbf{x}}\delta\mathcal{K}\left(\mathbf{y},\bar{z};\mathbf{x},z\right)\Big|_{\mathbf{x}=0}$$

$$+\frac{2\alpha_s C_F}{\omega^2}\Re\int_0^L d\bar{z}\int_0^{\bar{z}} dz\int_{\mathbf{y}} e^{-i\mathbf{k}\cdot\mathbf{y}}\mathcal{P}_{0;L-\bar{z}}(\mathbf{y})$$

$$\times\left(\left\{i\frac{(L-\bar{z})^2}{2\omega}\hat{\mathbf{g}}\mathcal{V}(\mathbf{y})\cdot\nabla_{\mathbf{y}} - \frac{\mathbf{y}}{2}\cdot\hat{\mathbf{g}}\mathcal{V}(\mathbf{y})(L-\bar{z})\right\}\nabla_{\mathbf{y}} - \hat{\mathbf{g}}\mathcal{V}(\mathbf{y})(L-\bar{z})\right)$$

$$\cdot\nabla_{\mathbf{x}}\mathcal{K}_0\left(\mathbf{y},\bar{z};\mathbf{x},z\right)\Big|_{\mathbf{x}=0}$$

The ultra-soft limit

J. Barata, XML, A. Sadofyev, C. Salgado

In the ultra-soft limit $\omega \ll \omega_c \equiv \hat{q} L^2$ the spectrum can be written as

$$(2\pi)^2 \omega \frac{dI_0}{d\omega d^2\mathbf{k}} = \frac{2\alpha_s C_F}{\omega^2} \Re \int_0^L d\bar{z}\, \mathcal{P}_{0;L-\bar{z}}(\mathbf{k})\, \omega \frac{dI_0}{d\omega d\bar{z}}$$

See:
1205.5739, Mehtar-Tani et al
1209.4585, Blaizot et al

No momentum transfer from $\mathcal{K}$ to $\mathcal{P}$ Collinear splitting + broadening

$\hat{S}$ could break this factorization, but it doesn't

$$(2\pi)^2 \omega \frac{dI}{d\omega d^2\mathbf{k}} = \frac{2\alpha_s C_F}{\omega^2} \Re \int_0^L d\bar{z}\, \mathcal{P}_{L-\bar{z}}(\mathbf{k})\, \omega \frac{dI_0}{d\omega d\bar{z}}$$

Moments of the gluon's final momenta

J. Barata, XML, A. Sadofyev, C. Salgado

We define the leading averaged moments as

$$\langle k^\alpha f(\mathbf{k})\rangle = \frac{\int d^2\mathbf{k}\, k^\alpha\, f(\mathbf{k})\,(2\pi)^2\,\omega\,\frac{dI}{d\omega d^2\mathbf{k}}}{\int d^2\mathbf{k}\,(2\pi)^2\,\omega\,\frac{dI^0}{d\omega d^2\mathbf{k}}} \xrightarrow{\text{Ultra-soft limit}} \langle k^\alpha f(\mathbf{k})\rangle = \frac{\int d^2\mathbf{k}\, k^\alpha\, f(\mathbf{k})\,\int_0^L d\bar{z}\,\mathcal{P}_{L-\bar{z}}(\mathbf{k})}{\int d^2\mathbf{k}\,\int_0^L d\bar{z}\,\mathcal{P}^0_{L-\bar{z}}(\mathbf{k})}$$

Even powers of the momenta

$$\langle \mathbf{k}^{2n}\rangle = \frac{n!}{n+1}(\hat{q}L)^n$$

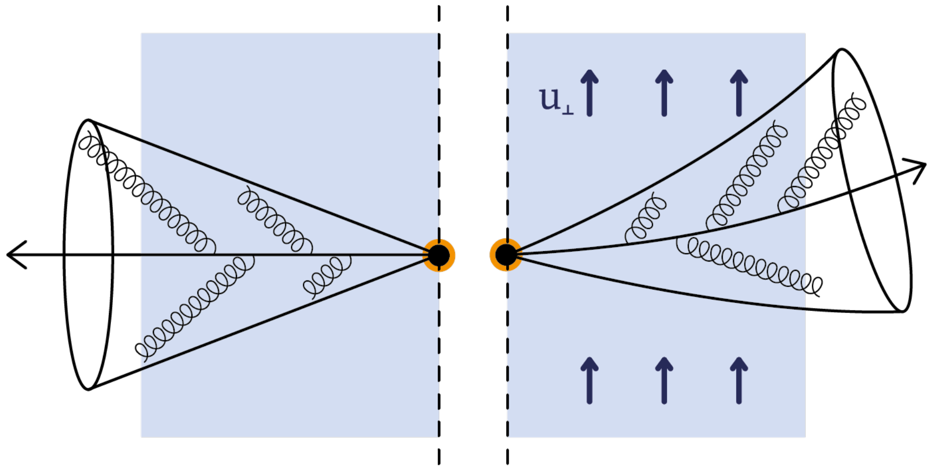
Odd powers of the momenta

$$\langle k^\alpha \mathbf{k}^{2n}\rangle = g^\alpha \frac{L\,n\,(n+1)!}{12(n+3)\omega}(\hat{q}L)^{n+1}$$

1st order in opacity

$$\left\langle \frac{\mathbf{k}_\perp}{k_\perp^2}\right\rangle \propto \frac{\mathbf{u}_\perp}{(1-u_z)xE}$$

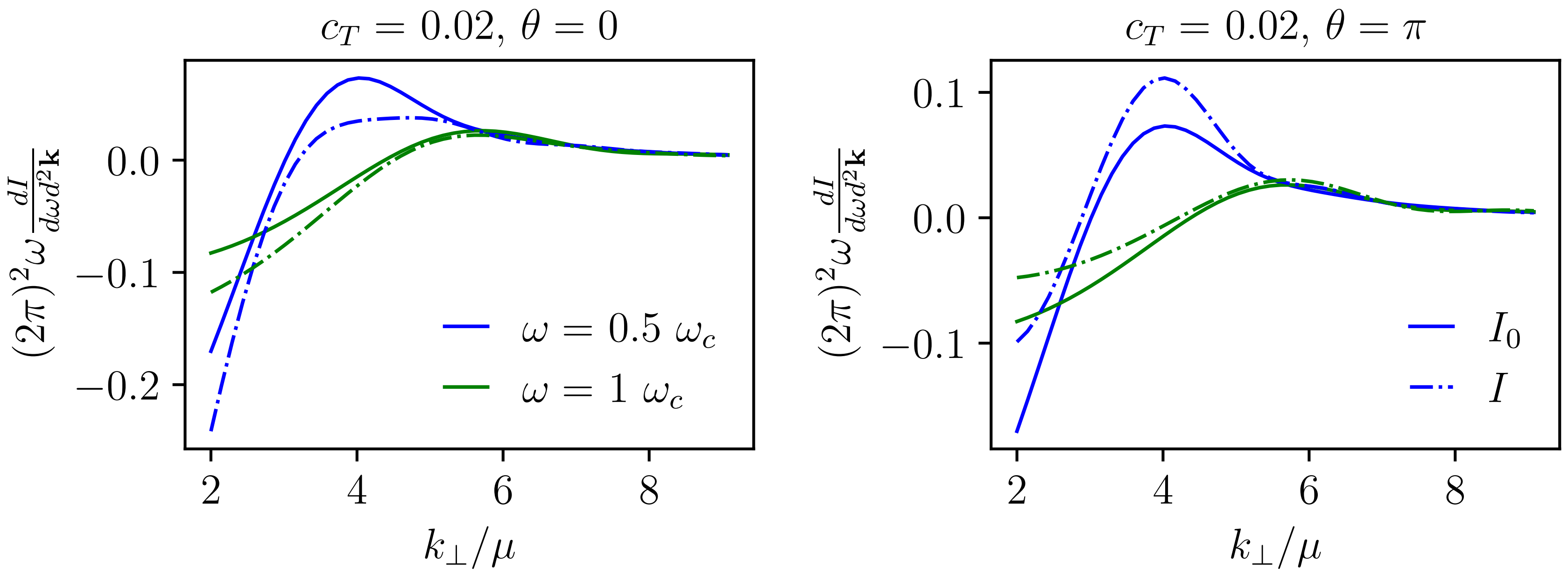
See 2104.09513, Sadofyev et al



Full MI spectrum

J. Barata, XML, A. Sadofyev, C. Salgado

Numerical representation of the full spectrum



Picture for the evolution of the jet in the medium:

- Jet core dominated by hard collinear radiation no sensitive to gradients
- Soft radiation from the core is sensitive to gradients modify the pattern around it

To take home:

- We have resummed to all order the leading gradient correction to the medium induced soft gluon radiation for a general potential
- We have computed the spectrum to LO gradients under the harmonic aproximation with GW model
- We have computed averaged values of the final momentum of the radiated gluon in the ultra-soft limit
- We can see that the radiation bends along the gradients direction and a picture of the jet substructure evolution

For the future:

- Include transverse flow of the matter in the formalism
- Search for a substructure observable sensitive to these effects

Thanks for your attention

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Back up

Broadening in inhomogeneus media

For more information: 2202.08847, Barata et al

Only the 2-point correlator of external field is non-zero

$$\langle t_i^a t_j^b \rangle = \frac{1}{2C_{\bar{R}}} \delta_{ij}\delta^{ab}$$

Color neutrality is enforced: probe interacts with the same scattering center both in amplitude and conjugated amplitude

The non-zero average is

$$\langle t^a_{proj} v^a(\mathbf{r},\tau) t^b_{proj} v^b(\bar{\mathbf{r}},\bar{\tau})\rangle \simeq \left(1 + \frac{\mathbf{r}(\tau)+\bar{\mathbf{r}}(\tau)}{2} \cdot \left(\nabla\rho \frac{\delta}{\delta\rho} + \nabla\mu^2 \frac{\delta}{\delta\mu^2}\right)\right) \mathcal{C}\delta(\tau-\bar{\tau})\rho g^4 \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{e^{i\mathbf{q}\cdot(\mathbf{r}-\bar{\mathbf{r}})}}{(\mathbf{q}^2+\mu^2)^2}$$

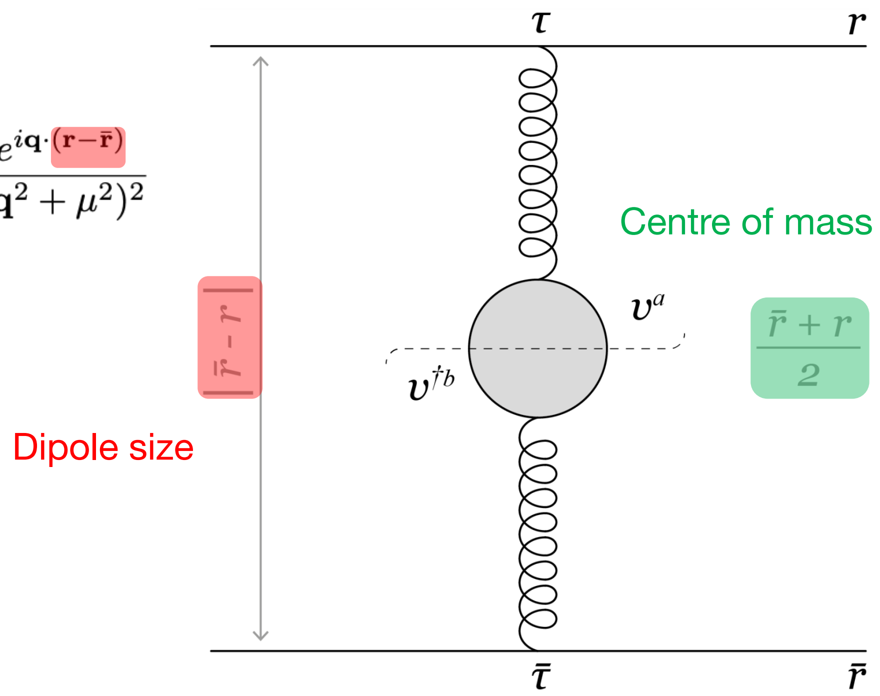
$\hat{\mathbf{g}}$

Introducing the effective scattering potential

$$\mathcal{V}(\mathbf{q},z) \equiv -\mathcal{C}\rho(z)\left(\left|v\left(\mathbf{q}^2\right)\right|^2 - \delta^{(2)}(\mathbf{q})\int d^2\mathbf{l}\,\left|v\left(\mathbf{l}^2\right)\right|^2\right)$$

And we can exponentiate the 2-point correlator

$$\left\langle \mathcal{P}\exp\left(-i\int_0^L d\tau t^a_{proj} v^a(\mathbf{r}(\tau),\tau)\right)\mathcal{P}\exp\left(-i\int_0^L d\bar{\tau} t^b_{proj} v^b(\bar{\mathbf{r}}(\bar{\tau}),\bar{\tau})\right)\right\rangle = \exp\left\{-\int_0^L d\tau \left(1+\frac{\mathbf{r}(\tau)+\bar{\mathbf{r}}(\tau)}{2}\cdot\left(\nabla\rho\frac{\delta}{\delta\rho}+\nabla\mu^2\frac{\delta}{\delta\mu^2}\right)\right)\mathcal{V}(\mathbf{r}(\tau)-\bar{\mathbf{r}}(\tau))\right\}$$



Broadening in inhomogeneus media

For more information: 2202.08847, Barata et al

The final result reads   for a real source

$$\frac{d\mathcal{N}}{d^2\mathbf{x}dE} \simeq \exp\{-\mathcal{V}(\mathbf{x})L\}\left\{\left[1 - i\frac{L^3}{6E}\nabla\mathcal{V}(\mathbf{x})\cdot\hat{\mathbf{g}}\mathcal{V}(\mathbf{x})\right]\frac{d\mathcal{N}^0}{d^2\mathbf{x}dE} + i\frac{L^2}{2E}\hat{\mathbf{g}}\mathcal{V}(\mathbf{x})\cdot\nabla\frac{d\mathcal{N}^0}{d^2\mathbf{x}dE}\right\} \longrightarrow \frac{d\mathcal{N}}{d^2\mathbf{x}dE} = \mathcal{P}(\mathbf{x})\hat{\mathcal{S}}(\mathbf{x})\frac{d\mathcal{N}^0}{d^2\mathbf{x}dE}$$

In the literature is known as the single particle broadening distribution

Operator that acts on the initial distribution

Opacity expansion can be resumed and leads to the same result

$$\langle|M|^2\rangle = \underbrace{\langle|M_0|^2\rangle}_{N=0} + \underbrace{\langle|M_1|^2\rangle + \langle M_2M_0^*\rangle + \langle M_0M_2^*\rangle}_{N=1} + \underbrace{\langle|M_2|^2\rangle + \langle M_3M_1^*\rangle + \langle M_1M_3^*\rangle + \langle M_4M_0^*\rangle + \langle M_0M_4^*\rangle}_{N=2} + \ldots$$

N is the order in the opacity expansion

$$\frac{d\mathcal{N}^{(N)}}{d^2\boldsymbol{x}dE} = \int\frac{d^2\boldsymbol{p}\,d^2\boldsymbol{r}}{(2\pi)^2}e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{r})}(-1)^N\left[\mathcal{V}(\boldsymbol{r})L\right]^N\left\{\frac{1}{N!} + \frac{L}{E(N+1)!}\times\sum_{m=1}^{N}\left[(N+1-m)\boldsymbol{p}\cdot\left(\frac{\mathcal{V}'(\boldsymbol{r})}{\mathcal{V}(\boldsymbol{r})}\boldsymbol{\nabla}\mu^2 + \frac{1}{\rho}\boldsymbol{\nabla}\rho\right) + i(N+1-m)^2\frac{\boldsymbol{\nabla}\mathcal{V}(\boldsymbol{r})}{\rho\,\mathcal{V}(\boldsymbol{r})}\cdot\boldsymbol{\nabla}\rho\right.\right.$$

$$\left.\left. + i(N+1-m)\left(\frac{\boldsymbol{\nabla}\mathcal{V}'(\boldsymbol{r})}{\mathcal{V}(\boldsymbol{r})} + (N-m)\frac{\mathcal{V}'(\boldsymbol{r})}{\mathcal{V}(\boldsymbol{r})}\frac{\boldsymbol{\nabla}\mathcal{V}(\boldsymbol{r})}{\mathcal{V}(\boldsymbol{r})}\right)\cdot\boldsymbol{\nabla}\mu^2\right]\right\}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{r}dE}$$

MI radiation in inhomogeneus media

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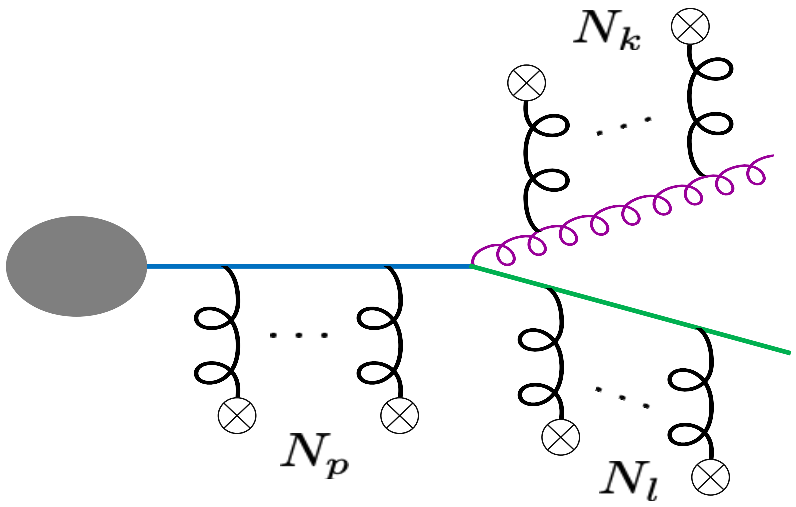
The relevant diagram is



$$iR = \sum_{N_p}\prod_{n=1}^{N_p}\left[(-1)\int\frac{d^3p_n}{(2\pi)^3}\,g t^a_{proj} v^a(p_{n+1}-p_n)\frac{2E}{p_n^2+i\epsilon}\right]$$

$$\times\sum_{N_l}\prod_{m=1}^{N_l}\left[(-1)\int\frac{d^3l_m}{(2\pi)^3}\,g t^a_{proj} v^a(l_{m+1}-l_m)\frac{2(1-x)E}{l_m^2+i\epsilon}\right]$$

$$\times\sum_{N_k}\prod_{r=1}^{N_k}\left[\left(-\frac{i}{g}\right)\int\frac{d^3k_r}{(2\pi)^3}\frac{N^{\mu_r\nu_r}}{k_r^2+i\epsilon}\Gamma^{a_r a_{r+1} c}_{\nu_r\mu_{r+1}0}(k_r,-k_{r+1},q_r)v^c(k_{r+1}-k_r)\right]$$

$$\times\int\frac{d^4p_s}{(2\pi)^4}\frac{i}{p_s^2+i\epsilon}\,i g t^a_{proj}(p_s+l_{in})_{\mu_1}(2\pi)^4\delta^{(4)}(p_s-l_{in}-k_{in})J(p_{in})\epsilon^{*\mu_{N_k+1}}(k_f)$$

The amplitude can be ressumed to the much simpler form

$$iR \simeq -\frac{g}{\omega}\int_0^\infty dz_s\int d^2x_{in}\, e^{-i\mathbf{x}_{in}\cdot\mathbf{l}_f}\, J(\mathbf{x}_{in})\,\mathcal{W}(\mathbf{x}_{in};L,z_s)\, t^a_{proj}\,\mathcal{W}(\mathbf{x}_{in};z_s,0)\, e^{i\frac{\mathbf{k}_{f\perp}^2}{2\omega}L}\left[\epsilon\cdot\nabla_{\mathbf{x}_{in}}\mathcal{G}^{ba}(\mathbf{k}_f,L;\mathbf{x}_{in},z_s)\right]$$

The two point function

$$\frac{d\mathcal{N}}{d^2\mathbf{x}dE} = \mathcal{P}(\mathbf{x})\hat{\mathcal{S}}(\mathbf{x})\frac{d\mathcal{N}^0}{d^2\mathbf{x}dE}$$

$$\mathcal{P}(\mathbf{p}) = \int d^2\mathbf{x}\, e^{-i\mathbf{p}\cdot\mathbf{x}}\, e^{-\mathcal{V}(\mathbf{x})L}\left[1 - \frac{iL^3}{6E}\nabla\mathcal{V}(\mathbf{x})\cdot\hat{\mathbf{g}}\mathcal{V}(\mathbf{x})\right]$$

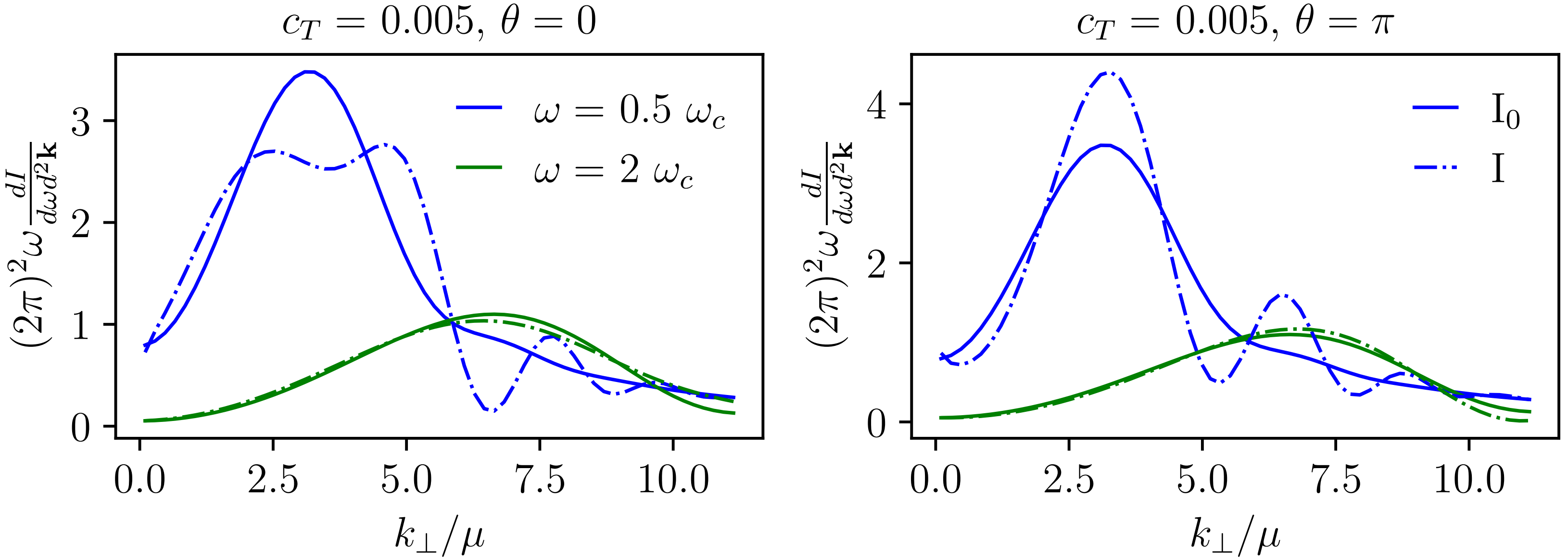
$$\hat{S}(\mathbf{x}) = e^{-\mathcal{V}(\mathbf{x})L}\left[1 + \frac{iL^2}{2E}\hat{\mathbf{g}}\mathcal{V}(\mathbf{x})\cdot\nabla\right]$$

$$\langle p^\alpha\, p_\perp^2\rangle = \frac{w^2L^2\mu^2}{E\,\lambda}\frac{\nabla^\alpha\rho}{\rho}\ln\frac{E}{\mu} + \frac{L^3\mu^4}{6E\,\lambda^2}\frac{\nabla^\alpha\rho}{\rho}\left(\ln\frac{E}{\mu}\right)^2$$

Full MI spectrum

J. Barata, XML, A. Sadofyev, C. Salgado

Numerical representation of the full spectrum



$$\chi = 15 \quad \mu = 0.6\,\mathrm{GeV} \quad L = 6\,\mathrm{fm} \quad \alpha_s = 0.28 \quad \omega_c = 53.53\,\mathrm{GeV} \quad \hat{q} = \hat{q}_0 \log\frac{\hat{q}_0 L}{\mu^2} \quad c_T = \left|\frac{\nabla T}{T\omega_c}\right|$$

$$\mathbf{g} = \frac{\nabla T}{T}\left(3 - \frac{2}{\log\frac{Q_s^2(L,0)}{\mu^2}}\right) \equiv \omega_c\, c_T \left(3 - \frac{2}{\log\frac{Q_s^2(L,0)}{\mu^2}}\right)$$