SUPPORTED BY:
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The ReBB model at 8 TeV: Odderon exchange is a certainty

based on

Eur. Phys. J. C **81**, 611 (13 July 2021)

Eur. Phys. J. C **82**, 827 (19 September 2022)

I. Szanyi[1,2,3], **T. Csörgő**[2,3]

[1]Eötvös University, Budapest, Hungary
[2]Wigner RCP, Budapest, Hungary
[3]MATE KRC, Gyöngyös, Hungary

Zimányi School 2022
Budapest, Hungary

Bialas-Bzdak p=(q,d) model

$$\tilde{\sigma}_{in}(\vec{b}) = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} d^2\vec{s}_q d^2\vec{s}'_q d^2\vec{s}_d d^2\vec{s}'_d \, D(\vec{s}_q,\vec{s}_d)\, D(\vec{s}'_q,\vec{s}'_d)\, \sigma(\vec{s}_q,\vec{s}_d;\vec{s}'_q,\vec{s}'_d;\vec{b})$$

unitarity → $t_{el}(\vec{b})$

- **quark-diquark distribution inside the proton:**

$$D(\vec{s}_q,\vec{s}_d) = \frac{1+\lambda^2}{R_{qd}^2\pi} e^{-\frac{s_q^2+s_d^2}{R_{qd}^2}} \delta^2(\vec{s}_q + \lambda\vec{s}_d)$$

$$\lambda = \frac{m_q}{m_d}$$

$$\vec{s}_d = -\lambda\vec{s}_q$$

$$\vec{s}'_d = -\lambda\vec{s}'_q$$

- **inelastic interaction probability of the constituents:**

$$\sigma(\vec{s}_q,\vec{s}_d;\vec{s}'_q,\vec{s}'_d;\vec{b}) = 1 - \prod_a \prod_b [1 - \sigma_{ab}(\vec{b} + \vec{s}'_a - \vec{s}_b)]$$

$$\sigma_{ab}(\vec{s}) = A_{ab} e^{-|\vec{s}|^2/S_{ab}^2}$$

$$S_{ab}^2 = R_a^2 + R_b^2$$

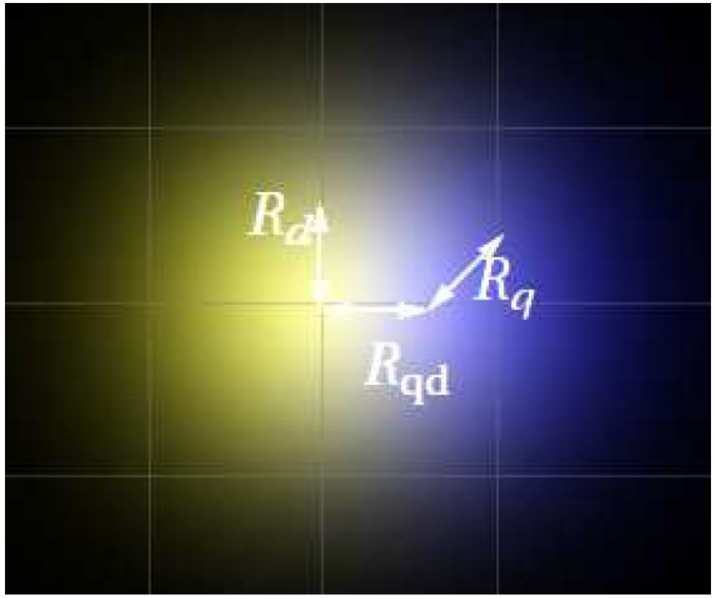
$$a, b \in \{q,d\}$$

- **free parameters** (assuming $\sigma_{qq}^{in}:\sigma_{qd}^{in}:\sigma_{dd}^{in} = 1:2:4$):

$$R_q, R_d, R_{qd}, A_{qq}, \lambda$$

$$(A_{qq} = 1 \text{ and } \lambda = 0.5 \text{ can be fixed})$$

A. Bialas, A. Bzdak, Acta Phys.Polon. B 38, 159-168 (2007), Phys.Lett.B 649: 263-268 (2007)



Proton-(anti)proton scattering in the quark-diquark model (Glauber style calculation).

Unitarily Real Extended Bialas-Bzdak (ReBB) model

- **elastic scattering amplitude in the impact parameter space:**

$$t_{el}(s,\vec{b}) = i\left[1 - e^{-\Omega(s,\vec{b})}\right]$$

arXiv:1505.01415

F. Nemes, T. Csörgő, M. Csanád, Int. J. Mod. Phys. A Vol. 30 (2015) 1550076

- **the opacity function:**

$$Re\Omega(s,\vec{b}) = -\frac{1}{2}ln[1 - \tilde{\sigma}_{in}(s,\vec{b})]$$

$$Im\Omega(s,\vec{b}) = -\alpha\, \tilde{\sigma}_{in}(s,\vec{b})$$

$Im\Omega \neq 0$ as the real part of the amplitude is not negligibly small

NEW FREE PARAMETER,
it has different values for pp and $\bar{p}p$

- **elastic scattering amplitude in momentum space:**

$$T(s,t) = 2\pi \int_0^\infty t_{el}(s,|\vec{b}|)\, J_0(|\vec{\Delta}||\vec{b}|)|\vec{b}|d|\vec{b}|$$

$$|\vec{\Delta}| \equiv \sqrt{-t} \quad as\ \sqrt{s} \to \infty$$

(t is the squared momentum transfer)

The pp and p$\bar{\text{p}}$ elastic scattering amplitude & the odderon

- **according to the Regge formalism the strong scattering amplitude for pp and $p\bar{p}$ scattering is written in terms of $C = +1$ and $C = -1$ exchange components**

$$T^{pp}(s,t) = T^{+}(s,t) - T^{-}(s,t)$$

$$T^{p\bar{p}}(s,t) = T^{+}(s,t) + T^{-}(s,t)$$

W. Broniowski, L. Jenkovszky, E. Ruiz Arriola, I. Szanyi: Phys. Rev. D 98, 074012 (2018)

- **for $\sqrt{s} \gtrsim 1\,\mathrm{TeV}$ the mesonic reggeon exchanges are negligible and essentially only the gluonic Pomeron and Odderon exchanges are present implying that**

$$T^{+}(s,t) \equiv T^{P}(s,t)$$

$$T^{-}(s,t) \equiv T^{O}(s,t)$$

$$\Rightarrow$$

$$T^{P}(s,t) = \frac{1}{2}\left(T^{pp}(s,t) + T^{p\bar{p}}(s,t)\right)$$

$$T^{O}(s,t) = \frac{1}{2}\left(T^{p\bar{p}}(s,t) - T^{pp}(s,t)\right)$$

- **a simple and model independent consequence:**

$$\text{if } \frac{d\sigma^{pp}}{dt}(s,t) \neq \frac{d\sigma^{p\bar{p}}}{dt}(s,t) \text{ for } \sqrt{s} \gtrsim 1\text{ TeV then } T^{O}(s,t) \neq 0$$

Fit method for Odderon search

- least squares fitting with the method developed by the PHENIX collaboration
- this method is **equivalent to the diagonalization of the covariance matrix** if the experimental errors are separated into three different types:
  - **type A: point-to-point varying uncorrelated statistical and systematic errors**
  - **type B: point-to-point varying 100% correlated systematic errors**
  - **type C: point-independent, overall systematic uncertainties**
- **i.e least squares fitting with:**

A. Adare *et al.* (PHENIX Collab.) Phys. Rev. C 77, 064907

$$\chi^2 = \left(\sum_{j=1}^{M} \left(\sum_{i=1}^{n_j} \frac{\left(d_{ij} + \epsilon_{bj}\tilde{\sigma}_{bij} + \epsilon_{cj} d_{ij}\sigma_{cj} - th_{ij}\right)^2}{\tilde{\sigma}_{ij}^2} \right) + \epsilon_{bj}^2 + \epsilon_{cj}^2 \right) + \left(\frac{d_{\sigma_{tot}} - th_{\sigma_{tot}}}{\delta\sigma_{tot}} \right)^2 + \left(\frac{d_{\rho_0} - th_{\rho_0}}{\delta\rho_0} \right)^2$$

$$\tilde{\sigma}_{ij}^2 = \tilde{\sigma}_{aij} \left(\frac{d_{ij} + \epsilon_{bj}\tilde{\sigma}_{bij} + \epsilon_{cj} d_{ij}\sigma_{cj}}{d_{ij}} \right)$$

$$\tilde{\sigma}_{kij} = \sqrt{\sigma_{kij}^2 + (d'_{ij}\delta_k t_{ij})^2}, \qquad k \in \{a, b\}$$

- **minimization with CERN Root MINUIT, parameter error estimation by MINOS.**

Fit method for Odderon search

- the method takes into account (in M separately measured t ranges):
  - the t-dependent statistical (type A) and systematic (type B) errors (both vertical σ_k and horizontal $\delta_k t$) → ϵ_b parameters;
  - the t-independent σ_c normalization uncertainties (type C) → ϵ_c parameters;
  - the measured total cross-section $d_{\sigma_{tot}}$ and ratio d_{ρ_0} and their total uncertainties $\delta\sigma_{tot}$ and $\delta\rho_0$.

A. Adare *et al.* (PHENIX Collab.) Phys. Rev. C 77, 064907

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ϵ_i-s must be considered as both measurements and fit parameters not effecting the NDF (since they have known central value of zero and known standard deviation of one)

[A. Adare *et al.* (PHENIX Collab.) Phys. Rev. C 77, 064907]

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Fit method for Odderon search

The PHENIX method is validated by evaluating the χ2 from a full covariance matrix fit of the √s = 13 TeV TOTEM differential cross-section data using the Lévy expansion method from T. Csörgő, R. Pasechnik, & A. Ster, Eur. Phys. J. C 79, 62 (2019).

- the t-independent σ_c normalization uncertainties → ϵ_c parameters;
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- the t-independent σ_c normalization uncertainties → ϵ_c parameters:

The PHENIX method and the fit with the full covariance matrix result in the same minimum within one standard deviation of the fit parameters.

$\delta\sigma_{tot}$ and $\delta\rho_0$.

A. Adare *et al.* (PHENIX Collab.) Phys. Rev. C 77, 064907

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ReBB model analysis of pp and p$\bar{\text{p}}$ data

T. Csörgő, I. Szanyi, *Eur. Phys. J. C* 81, 611 (2021)

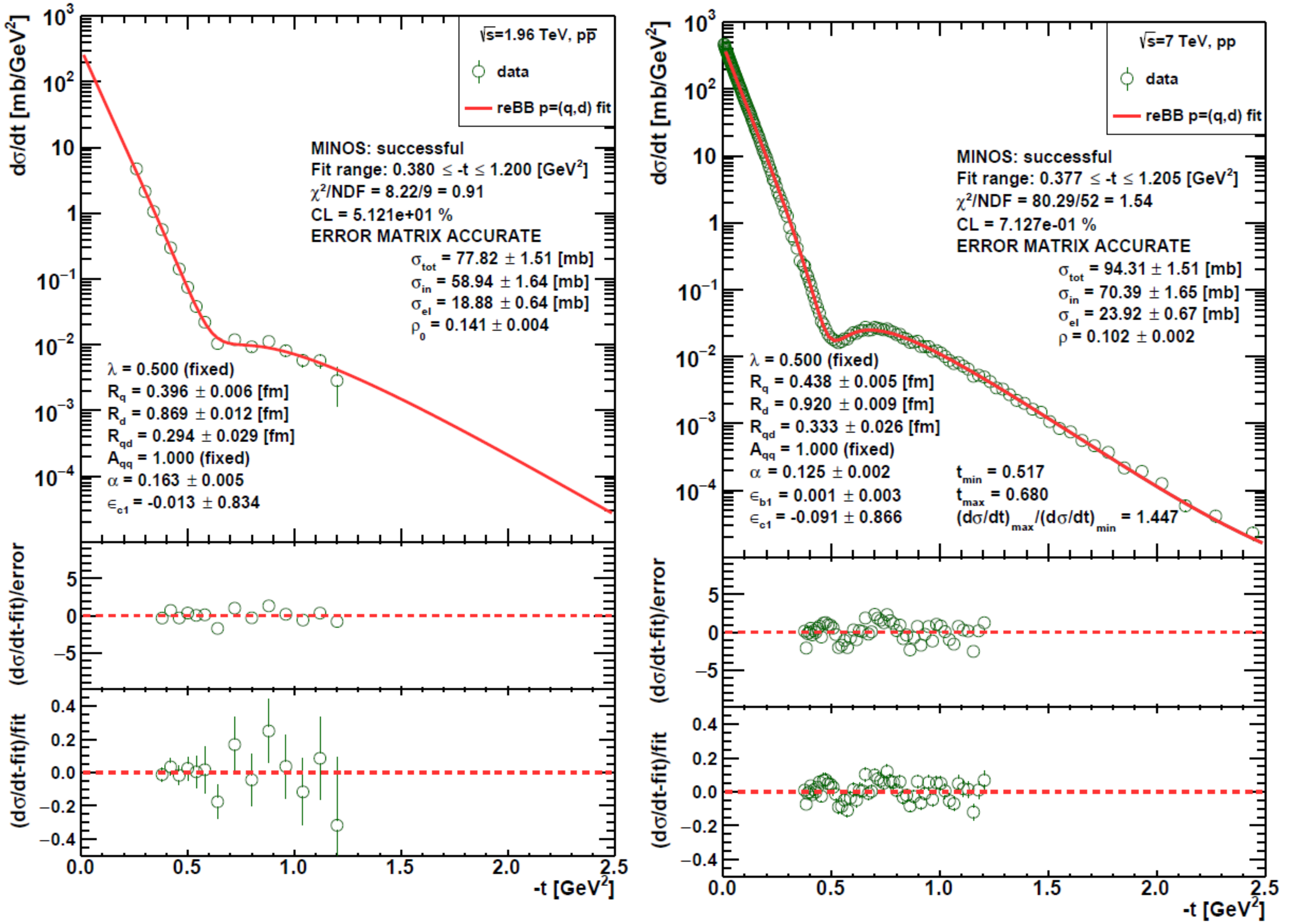
→ fits for pp dσ/dt data at 2.76 TeV and 7 TeV and for p$\bar{\text{p}}$ dσ/dt data at 0.546 TeV and 1.96 TeV

→ use of the χ^2 definition developed by PHENIX

→ determination of the energy dependences of the model parameters

→ satisfactory description in the kinematical range: $0.546 \leq \sqrt{s} \leq 7$ TeV & $0.37 \leq -t \leq 1.2$ GeV^2

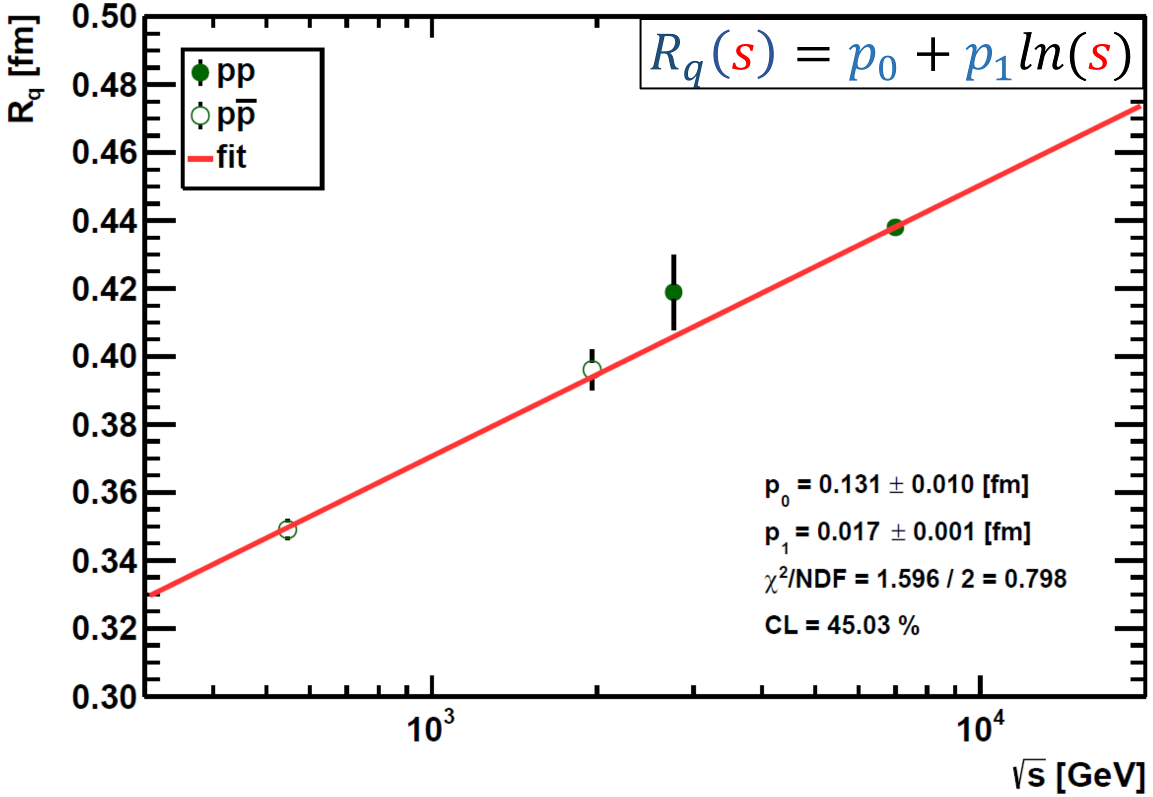
→ observation of a significant model-dependent Odderon signal

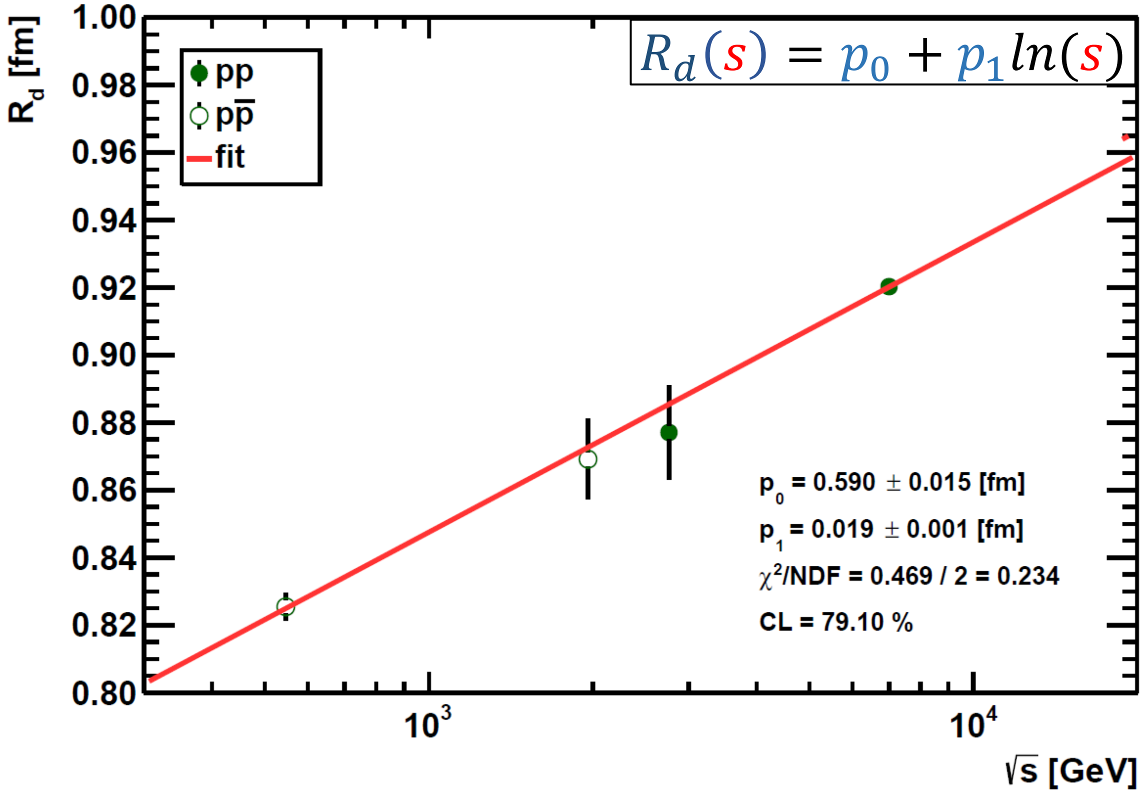


Examples of ReBB model fits for pp and p$\bar{\text{p}}$ differential cross section data.

Energy dependences of the ReBB model parameters

T. Csörgő, I. Szanyi, *Eur. Phys. J. C* 81, 611 (2021)



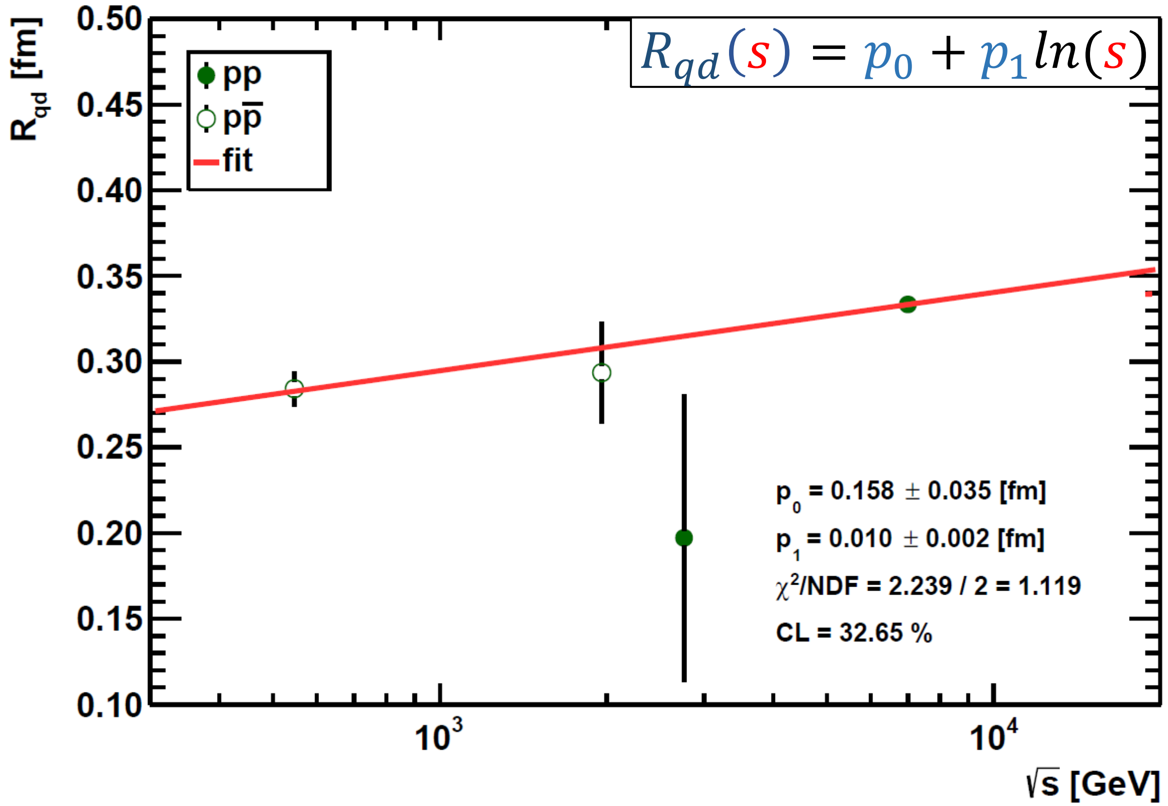


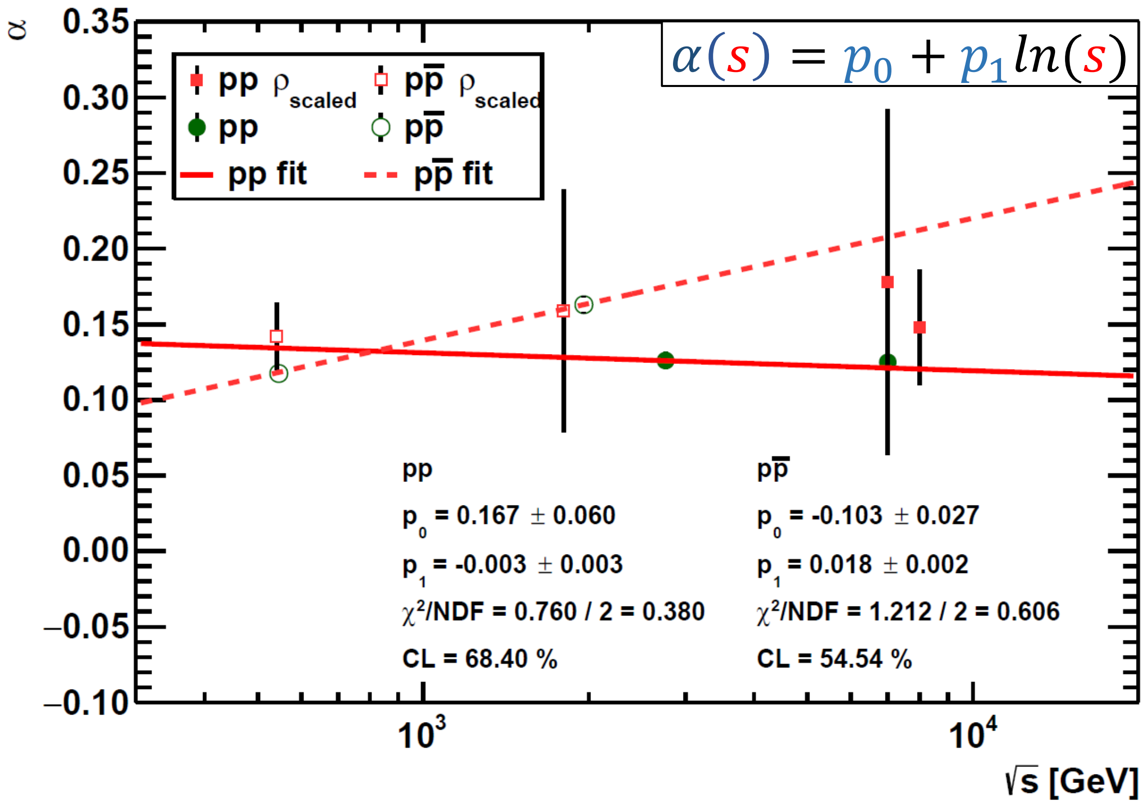
The energy dependences of the scale parameters, $R_q(s)$, $R_d(s)$, and $R_{qd}(s)$ are linear logarithmic and the same for pp and p$\bar{p}$ processes!

The energy dependence of the α parameter, $\alpha(s)$ is linear logarithmic too, but not the same for pp and p$\bar{p}$ processes!

Energy dependences of the ReBB model parameters

T. Csörgő, I. Szanyi, *Eur. Phys. J. C* 81, 611 (2021)





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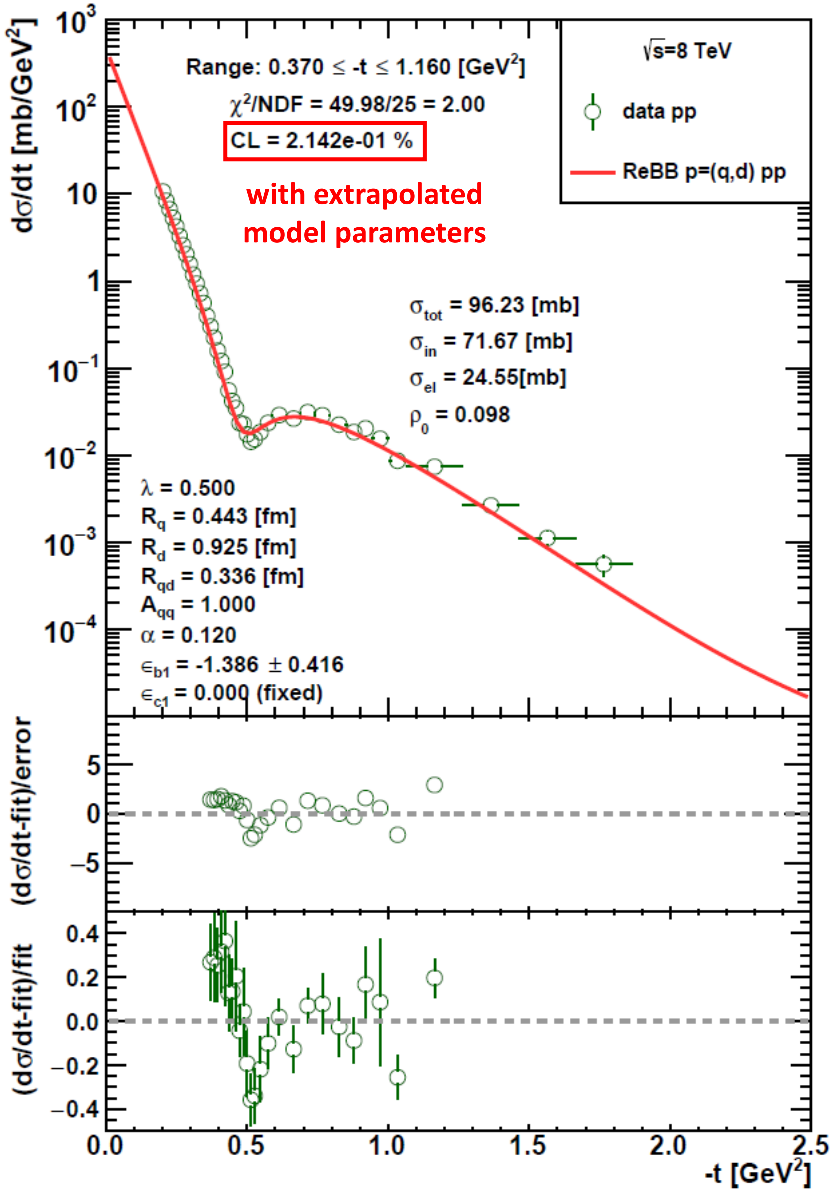
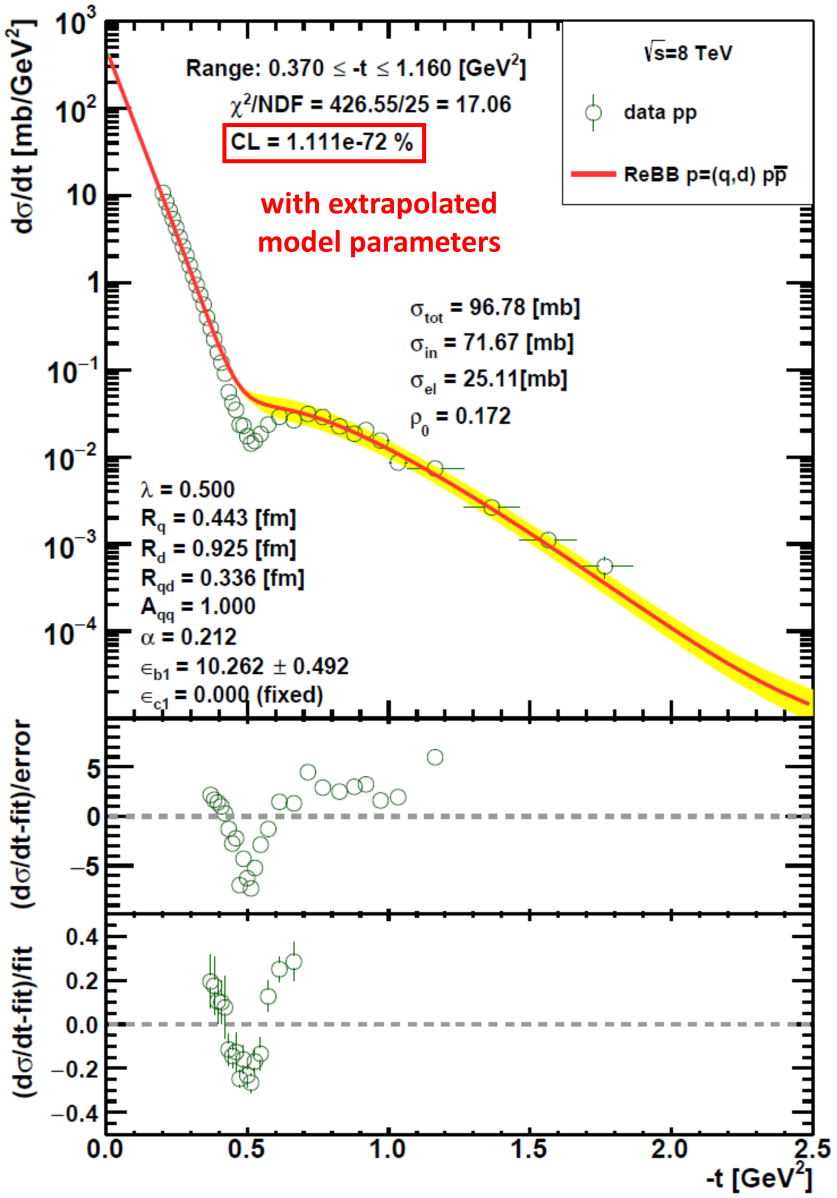
ReBB model & Odderon @ 8 TeV

I. Szanyi, T. Csörgő, Eur. Phys. J. C 82, 827 (2022)

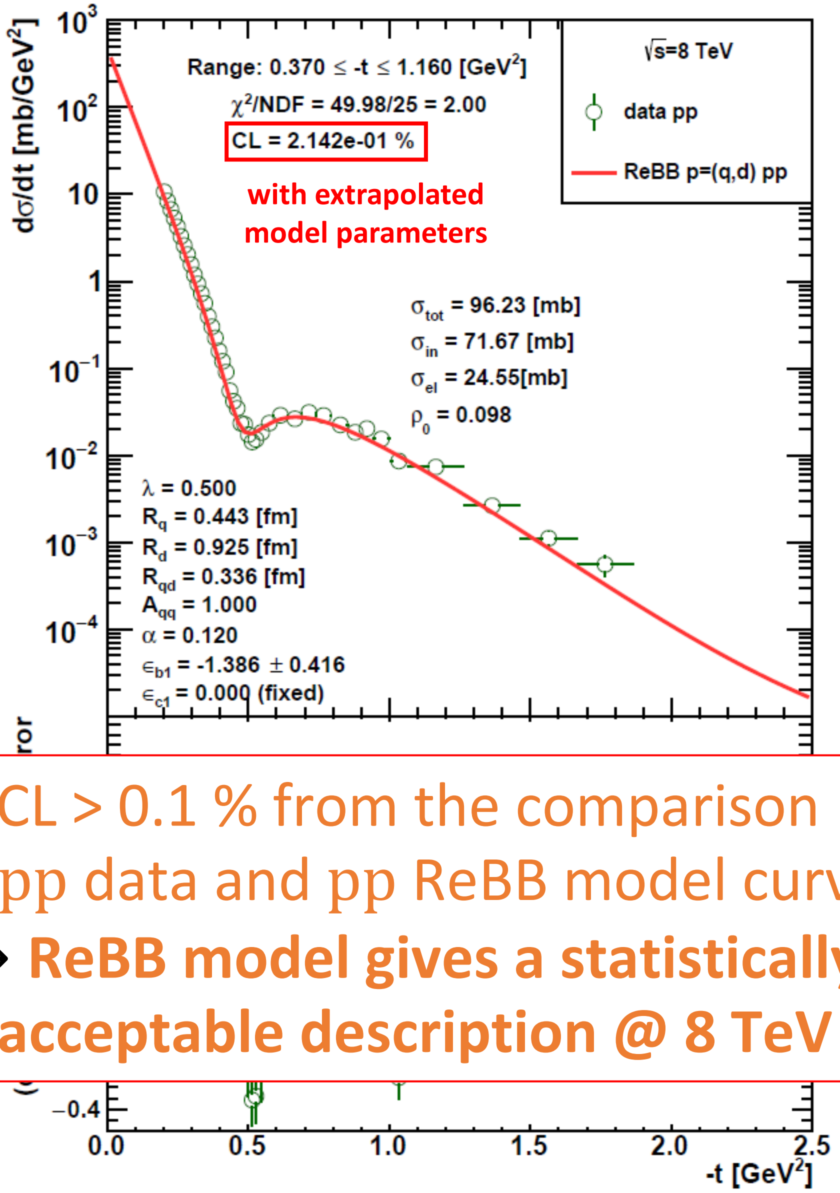




data: TOTEM Collab., *Eur. Phys. J.* C 82, 263 (2022)

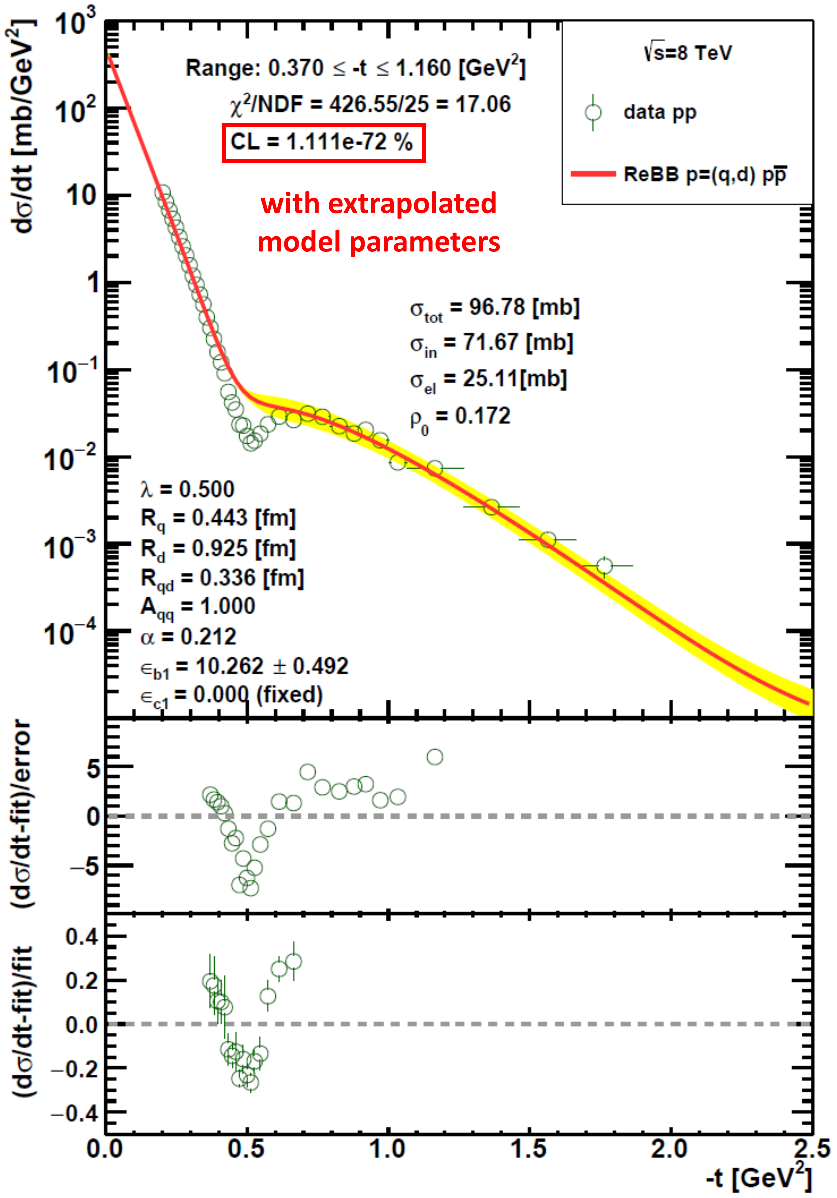
ReBB model & Odderon @ 8 TeV

I. Szanyi, T. Csörgő, Eur. Phys. J. C 82, 827 (2022)



CL > 0.1 % from the comparison of pp data and pp ReBB model curve

→ **ReBB model gives a statistically acceptable description @ 8 TeV**



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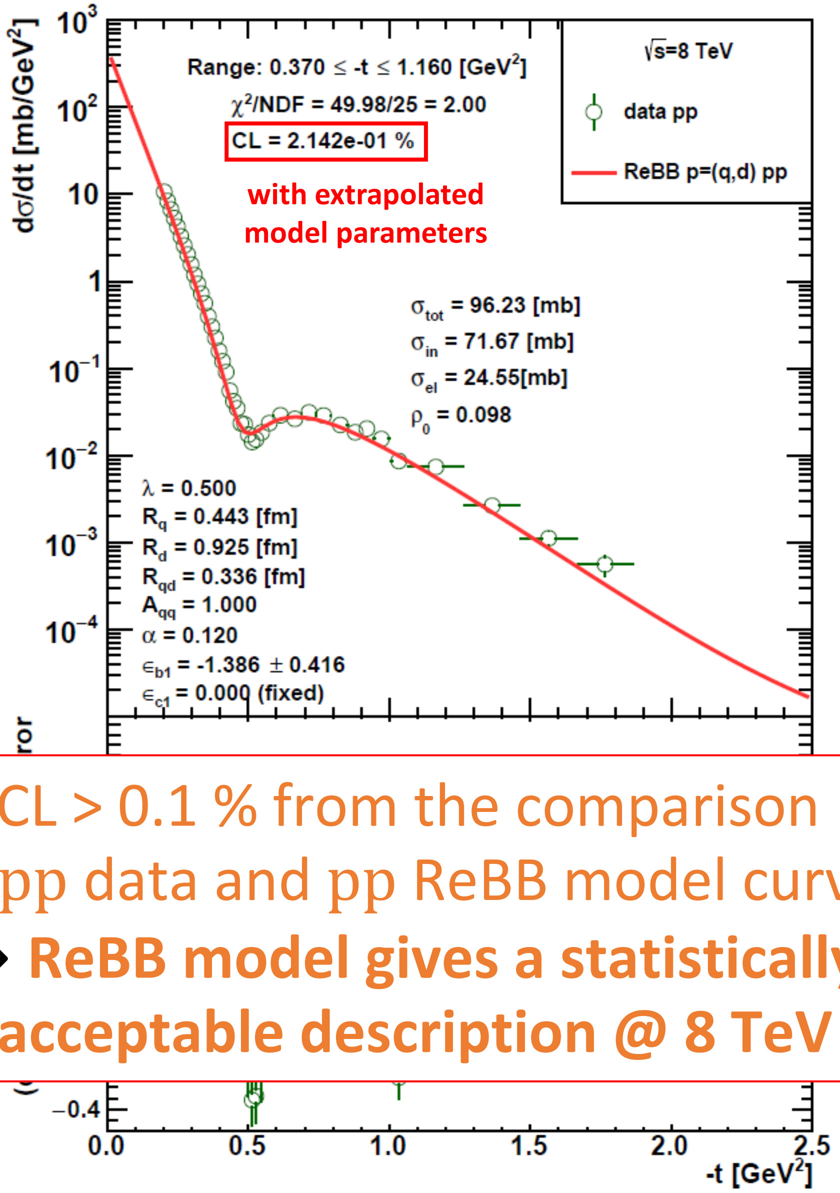

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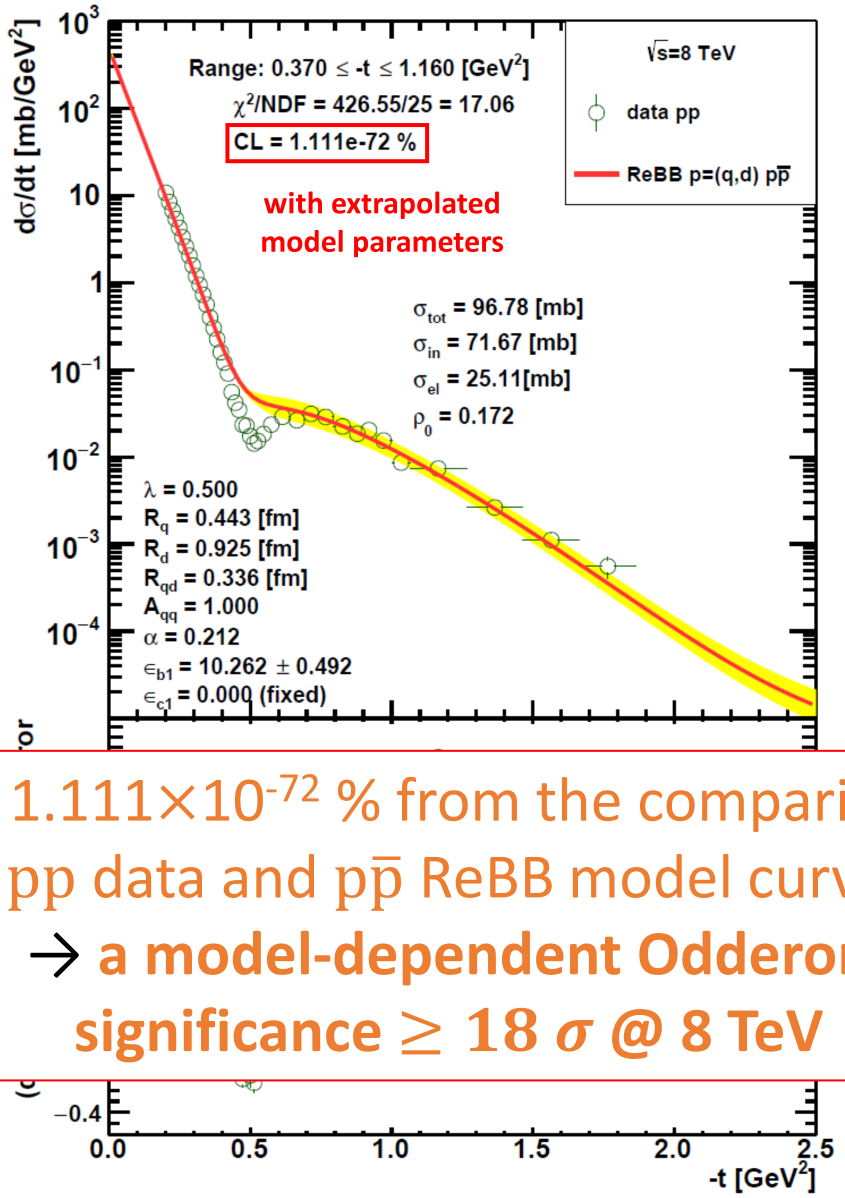

CL ≤ 1.111×10^{-72} % from the comparison of pp data and $p\bar{p}$ ReBB model curve
→ **a model-dependent Odderon significance ≥ 18 σ @ 8 TeV**

data: TOTEM Collab., *Eur. Phys. J.* C 82, 263 (2022)

Odderon observation within the ReBB model analysis

I. Szanyi, T. Csörgő, Eur. Phys. J. C 82, 827 (2022)

$\sqrt{s}$ (TeV)	χ^2	NDF	CL	significance (σ)
1.96	24.283	14	0.0423	2.0
2.76	100.347	22	5.6093×10^{-12}	6.89
7	2811.46	58	$< 7.2853 \times 10^{-312}$	>37.7
8	426.553	25	1.1111×10^{-74}	≥18.2

$\sqrt{s}$ of combined data (TeV)	χ^2	NDF	CL	combined significance (σ) χ^2/NDF method	combined significance (σ) Stouffner method
1.96 & 2.76	124.63	36	1.0688×10^{-11}	6.79	6.3
1.96 & 2.76 & 7	2936.09	94	$< 9.1328 \times 10^{-312}$	>37.7	>26.9
1.96 & 2.76 & 8	551.183	61	4.6307×10^{-80}	>18.9	>15.7
1.96 & 2.76 & 7 & 8	3362.64	119	$< 8.0654 \times 10^{-312}$	>37.7	>32.4

- **combination of significances by summing the individual χ^2 and NDF values:**

$$\chi^2 = \sum_i \chi_i^2$$

$$NDF = \sum_i NDF_i$$

- **combination of significances s_i by Stouffner method:**

$$s = \frac{\sum_{i=1}^{N} s_i}{\sqrt{N}}$$

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I. Szanyi, T. Csörgő, Eur. Phys. J. C 82, 827 (2022)

$> 6\ \sigma$ combined Odderon signal from the data-model comparison at the two lowest energies, 1.96 & 2.76 TeV

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$>6\ \sigma$ combined Odderon signal from the data-model comparison at the two lowest energies, 1.96 & 2.76 TeV

$>30\ \sigma$ combined Odderon signal from the data-model comparison at all the four energies, 1.96, 2.76, 7 & 8 TeV

$\sqrt{s}$ of combined data (TeV)	χ^2	NDF	CL	combined significance (σ) χ^2/NDF method	combined significance (σ) Stouffner method
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Summary and conclusions

- the ReBB model represents all available pp and $\mathrm{p\bar{p}}$ $d\sigma/dt$ data in the kinematical ranges $0.546 \leq \sqrt{s} \leq 8$ TeV and $0.37 \leq -t \leq 1.2$ GeV^2 in a statistically acceptable manner
- a comparative study of pp and $\mathrm{p\bar{p}}$ differential cross sections is done with ReBB model by interpolations & extrapolations to the same kinematical regions
- model-dependent evidence for Odderon exchange in t-channel is observed
  - the combined significance is > 6 σ for the two lowest energies at TeV scale, i.e. 1.96 and 2.76 TeV
  - the combined significance is > 30 σ for four energies at the TeV scale i.e. 1.96, 2.76, 7 and 8 TeV
- Conclusion: the Odderon exchange is a certainty in the ReBB model study

I. Szanyi, T. Csörgő, Eur. Phys. J. C 82, 827 (19 September 2022)

Thank you for your attention!

Backup slides

Analytical approximation for significance calculation

I. Szanyi, T. Csörgő, Eur. Phys. J. C 82, 827 (2022)

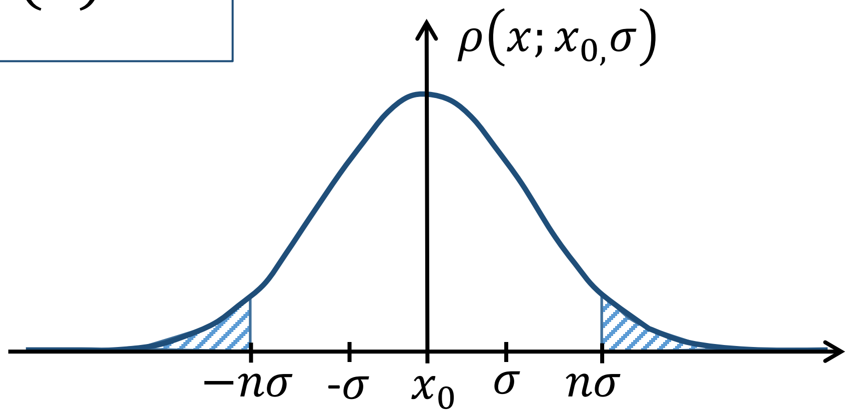
- **the Gaussian probability density function with mean x_0 and and variance σ^2 :**

$$\rho(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-x_0)^2}{2\sigma^2}}$$

$$\int_{-\infty}^{\infty} dx\, \rho(x) = 1$$

- **the confidence level corresponding to $n\sigma$ significance:**

$$CL = 2\int_{x_0+n\sigma}^{\infty} dx\, \rho(x)$$



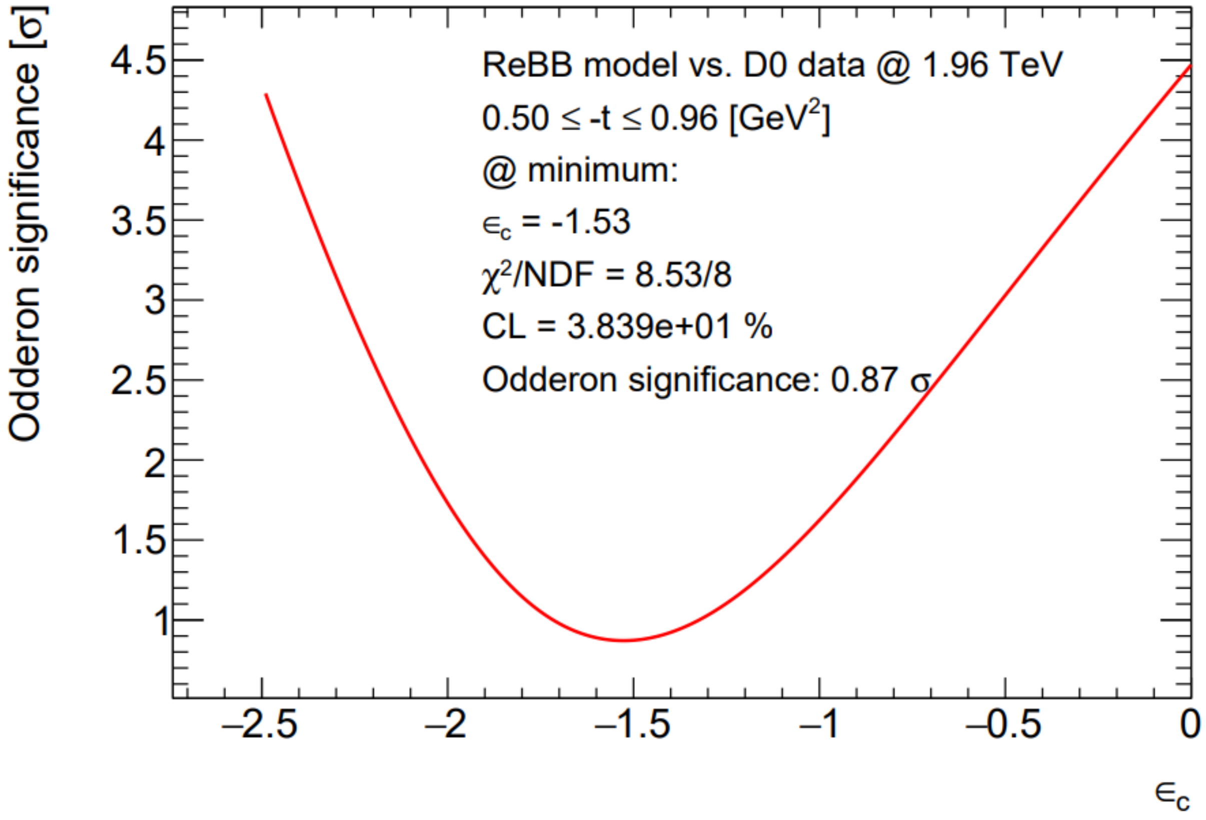
- **applying a variable change, $x \to x' = x - (x_0 + n\sigma)$:**

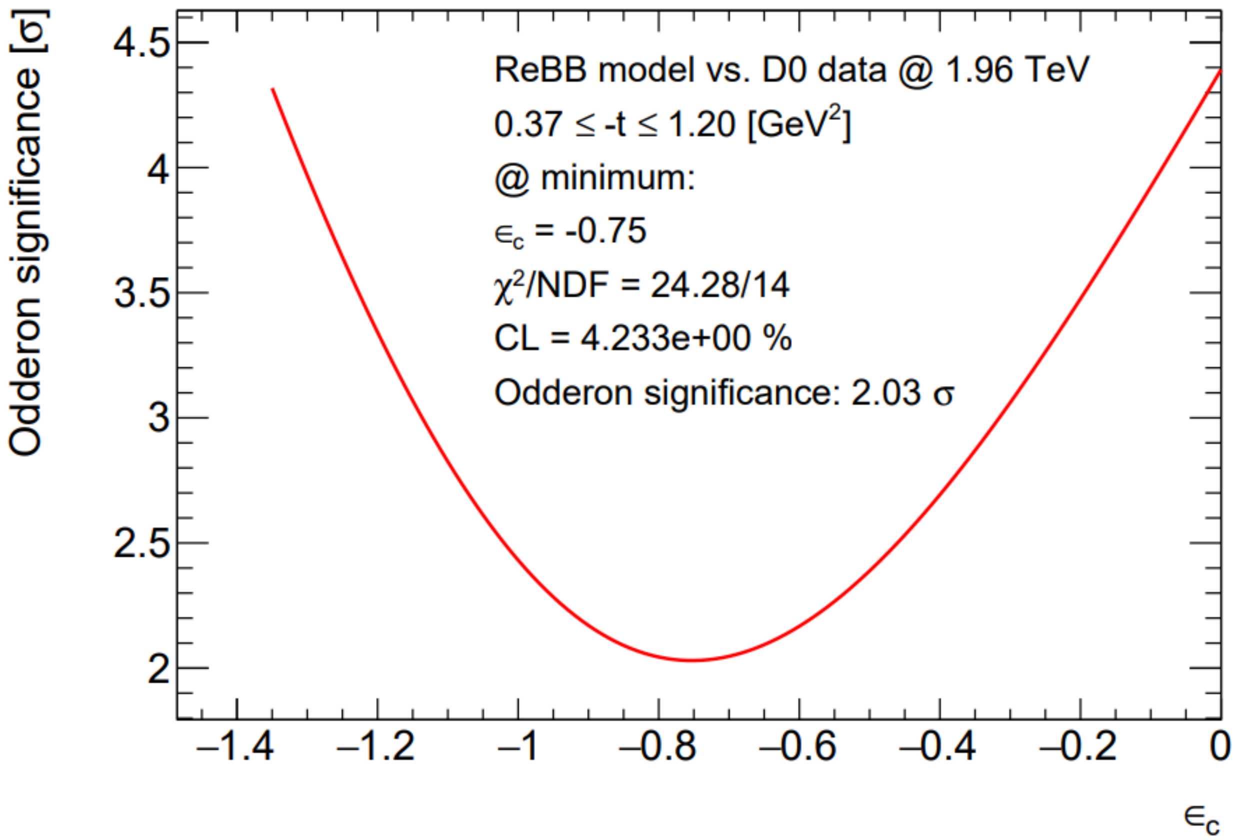
$$CL = \sqrt{\frac{2}{\pi\sigma^2}} \int_0^{\infty} dx'\, e^{-\frac{(x'+n\sigma)^2}{2\sigma^2}} = \sqrt{\frac{2}{\pi\sigma^2}}\, e^{-\frac{n^2}{2}} \int_0^{\infty} dx'\, e^{-\frac{x'^2+2x'n\sigma}{2\sigma^2}} \leq \sqrt{\frac{2}{\pi\sigma^2}}\, e^{-\frac{n^2}{2}} \int_0^{\infty} dx'\, e^{-\frac{x'n}{\sigma}}$$

$$CL \leq \sqrt{\frac{2}{\pi n^2}}\, e^{-\frac{n^2}{2}}$$

this formula gives the lower limit for the significance n in σ-s corresponding to a CL value

ϵ_c dependence of the significance





the significance of the odderon signal evidently increases if ϵ_c is not optimized to get minimal χ^2

Proportionality between $\rho_0(s)$ and $\alpha(s)$

$$t_{el}(s,b) = i\left(1 - e^{i\,\alpha\,\tilde{\sigma}_{in}(s,b)}\sqrt{1-\tilde{\sigma}_{in}(s,b)}\right)$$

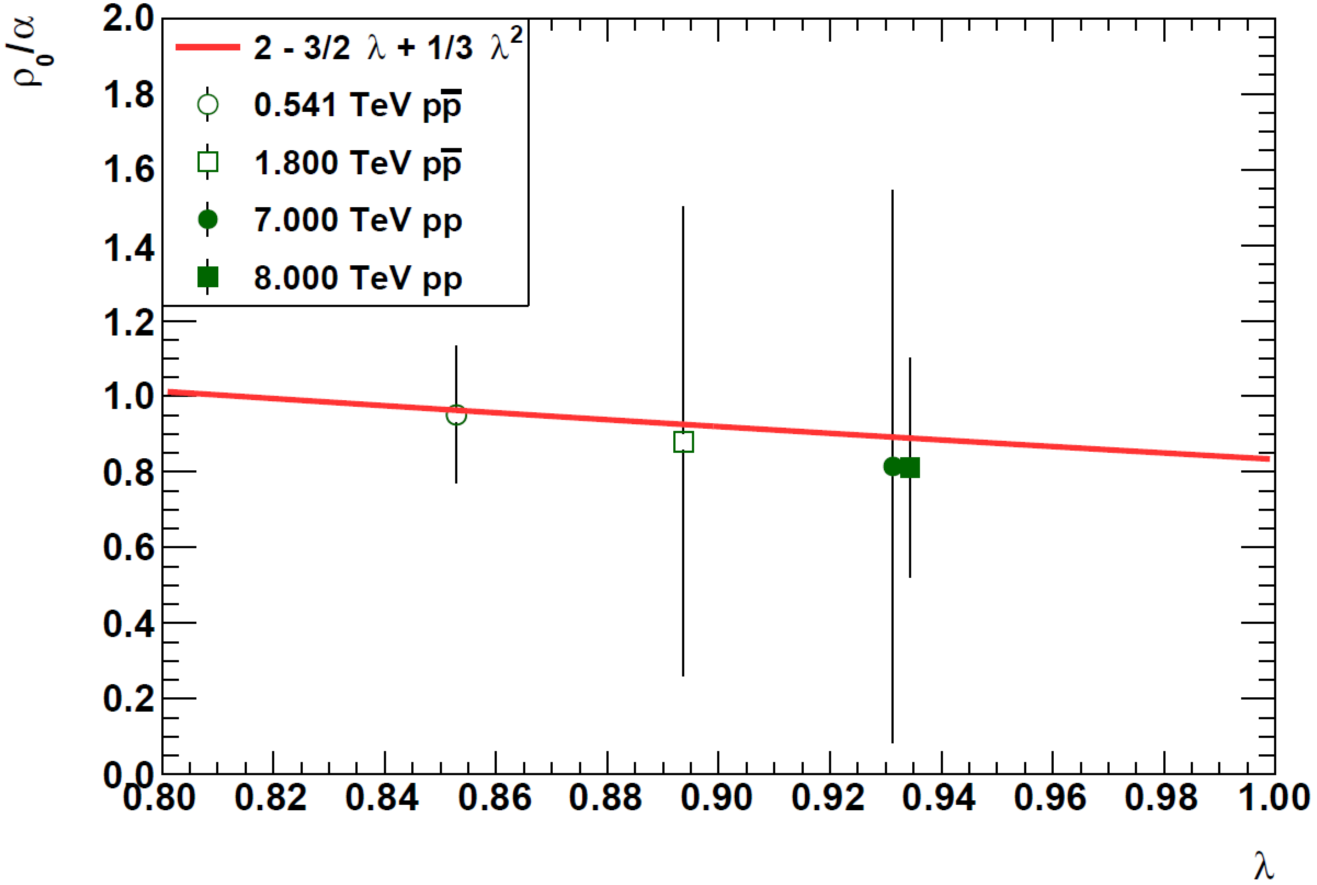
$$\alpha\tilde{\sigma}_{in} \ll 1$$

$$\mathrm{Im}\; t_{el}(s,b) \simeq \lambda(s)\exp\left(-\frac{b^2}{2R^2(s)}\right)$$

$$\rho_0(s) = \alpha(s)\left(2 - \frac{3}{2}\lambda(s) + \frac{1}{3}\lambda^2(s)\right)$$

$$\lambda(s) = \mathrm{Im}\; t_{el}(s, b=0)$$

→ by rescaling one can get additional α parameter values at energies where ρ_0 is measured (and vice versa)



The dependence of ρ_0/α on $\lambda = \mathrm{Im}\; t_{el}(s, b=0)$ in the TeV energy range. The data points are generated numerically by using the trends of the ReBB model scale parameters and the experimentally measured ρ-parameter values.

Measurable quantities

- **differential cross section:**

$$\frac{d\sigma}{dt}(s,t) = \frac{1}{4\pi}|T(s,t)|^2$$

- **total, elastic and inelastic cross sections:**

$$\sigma_{tot}(s) = 2ImT(s,t=0)$$

$$\sigma_{el}(s) = \int_{-\infty}^{0} \frac{d\sigma(s,t)}{dt} dt$$

$$\sigma_{in}(s) = \sigma_{tot}(s) - \sigma_{el}(s)$$

- **ratio ρ_0:**

$$\rho_0(s) = \lim_{t\to 0} \rho(s,t) \equiv \frac{ReT(s,t\to 0)}{ImT(s,t\to 0)}$$

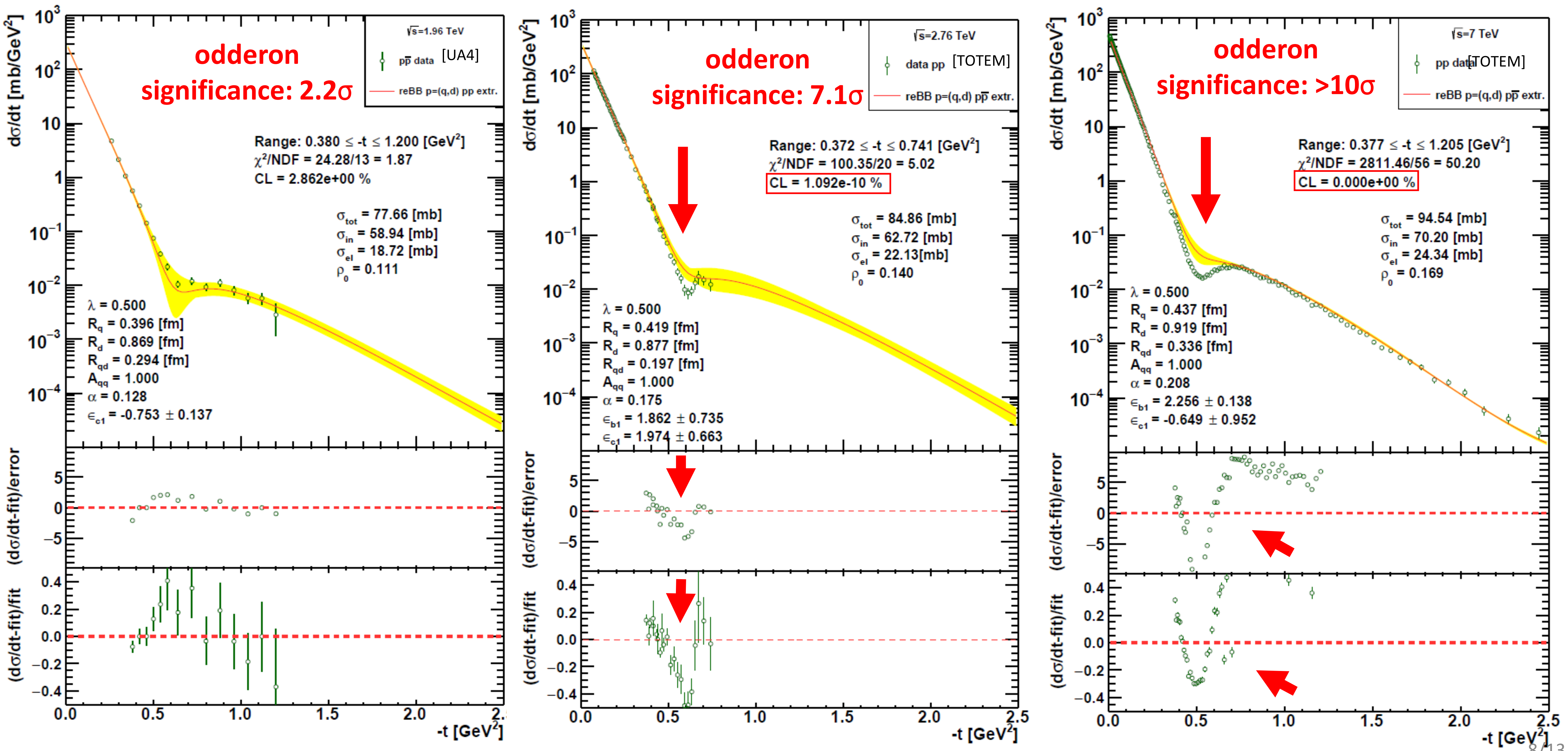
- **slope of dσ/dt:**

$$\mathrm{B}(\mathrm{s},\mathrm{t}) = \frac{\mathrm{d}}{\mathrm{dt}}\left(\ln \frac{d\sigma}{\mathrm{dt}}(\mathrm{s},\mathrm{t})\right)$$

$$B_0(s) = \lim_{t\to 0} \mathrm{B}(\mathrm{s},\mathrm{t})$$

Extrapolations: ODDERON

T. Csörgő, I. Szanyi, *Eur. Phys. J. C* 81, 611 (2021)



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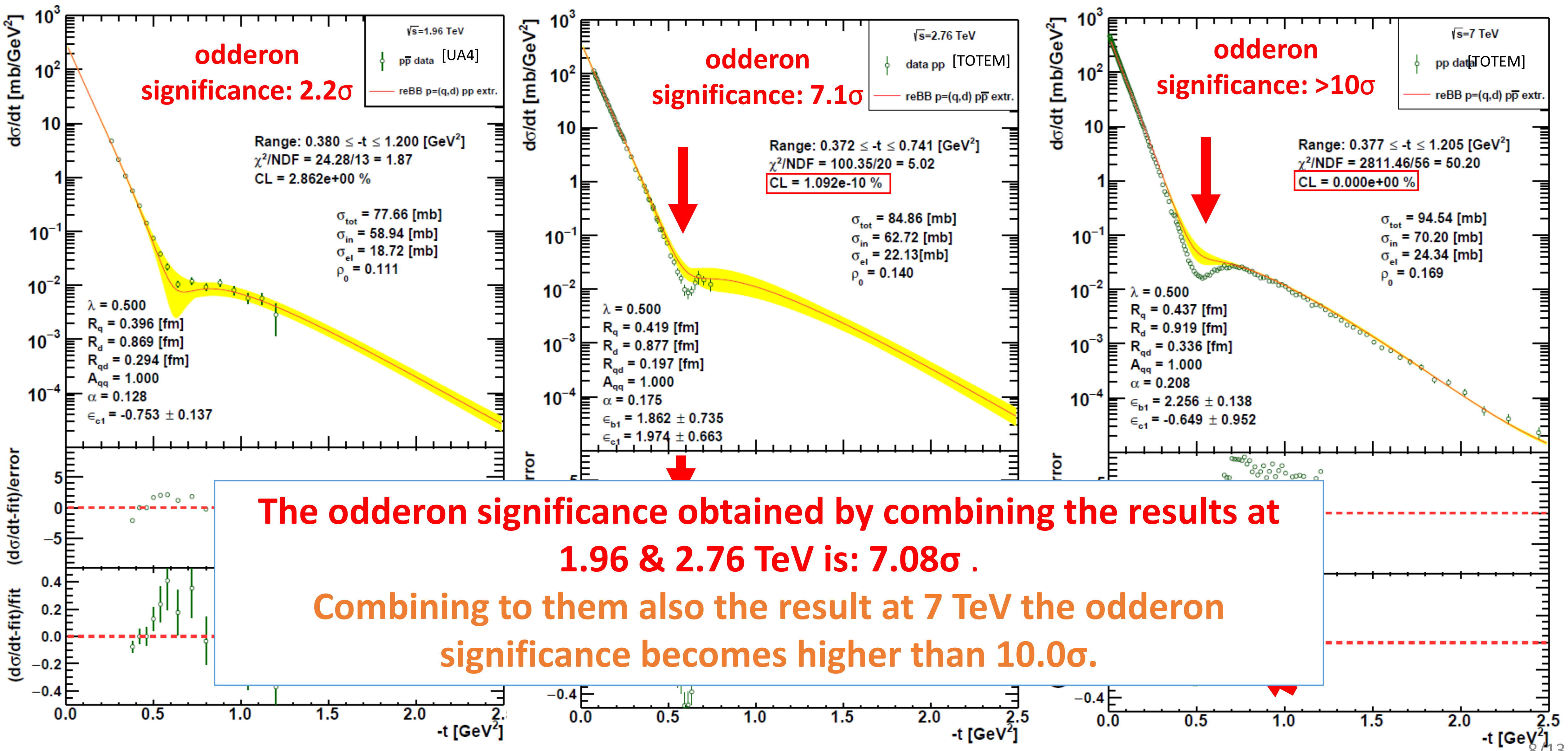

The odderon significance obtained by combining the results at 1.96 & 2.76 TeV is: 7.08σ .

Combining to them also the result at 7 TeV the odderon significance becomes higher than 10.0σ.

Unitarity and the elastic amplitude

- **unitarity of the scattering matrix S:**

$$SS^{\dagger} = I \qquad S = I + iT \qquad T - T^{\dagger} = iTT^{\dagger}$$

- **unitarity equation in impact parameter b representation:**

$$2\,\mathcal{Im}\, t_{el}(s,\vec{b}) = |t_{el}(s,\vec{b})|^2 + \tilde{\sigma}_{in}(s,\vec{b})$$

(s is the squared CM energy)

- **the elastic amplitude can be written as a solution of the unitarity equation in terms of $\tilde{\sigma}_{in}$**

- $0 \leq \tilde{\sigma}_{in}(s,\vec{b}) \leq 1$ **and $\tilde{\sigma}_{in}$ can be calculated using the rules of the probability calculus based on Glauber's multiple scattering theory**