Propagation properties of spin degrees of freedom within the framework of relativistic hydrodynamics with spin

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Spin tensor in the GLW pseudogauge

- $P^\mu = \int d\Sigma_\nu T^{\mu\nu}$ and $J^{\mu\nu} = \int d\Sigma_\lambda J^{\lambda,\mu\nu}$ are invariant under the **pseudogauge transformations**. F. W. Hehl, Rept. Math. Phys. **9** (1976) 55
- We use the **GLW** (Groot-van Leeuwen-van Weert) pseudogauge for the Dirac field.
- LTE for a polarized fluid can be described by

$$f^\sigma_{\rm eq} = \left[\exp\left(\beta p\cdot U - \sigma\xi - \frac{1}{2}\omega^{\mu\nu}s_{\mu\nu}\right) + 1\right], \quad s^{\mu\nu} = \frac{1}{m}\epsilon^{\mu\nu\alpha\beta}p_\alpha s_\beta, \tag{1}$$

where $\beta = T^{-1}$, $\xi = \mu/T$, $\sigma = \pm 1$ and $\omega^{\mu\nu}$ is the spin potential.
- The spin tensor comes out as:

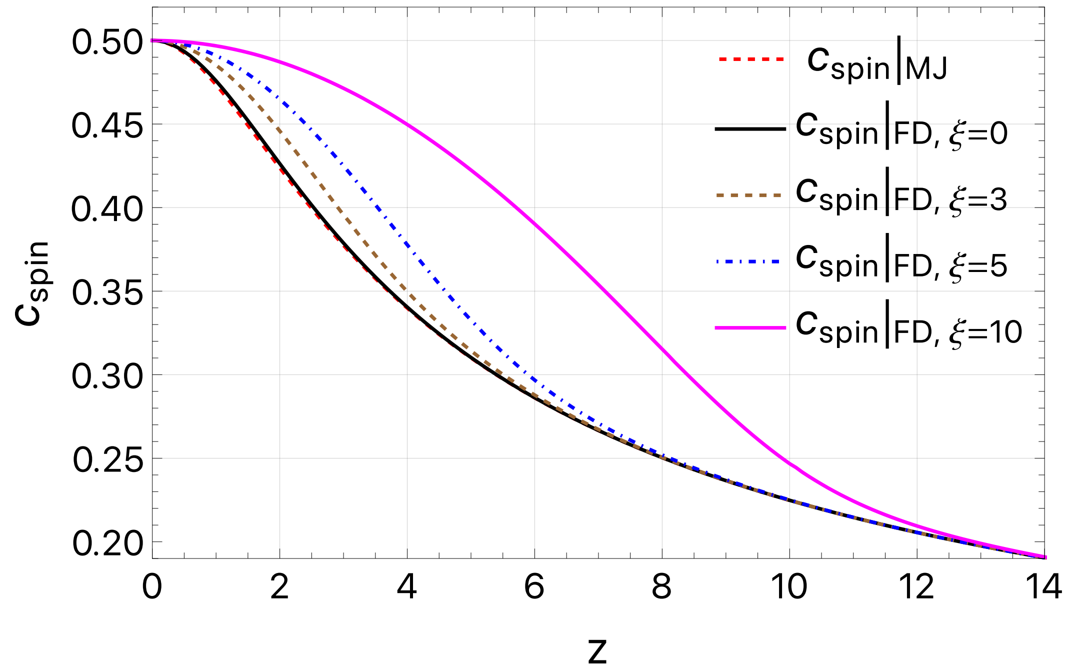
$$S^{\lambda,\mu\nu} = (\mathcal{A}_1 + \mathcal{A}_3)U^\lambda\omega^{\mu\nu} + (2\mathcal{A}_1 - \mathcal{A}_3)U^\lambda U^\alpha U^{[\mu}\omega^{\nu]}{}_\alpha + \mathcal{A}_3(\Delta^{\lambda\alpha}U^{[\mu}\omega^{\nu]}{}_\alpha + U^\lambda\Delta^{\alpha[\mu}\omega^{\nu]}{}_\alpha + U^\alpha\Delta^{\lambda[\mu}\omega^{\nu]}{}_\alpha), \tag{2}$$

where

$$\mathcal{A}_1 = \frac{\mathfrak{s}^2}{9}\left[\left(\frac{\partial\mathcal{N}}{\partial\xi}\right)_\beta - \frac{2}{m^2}\left(\frac{\partial\mathcal{E}}{\partial\beta}\right)_\xi\right],$$

$$\mathcal{A}_3 = \frac{2\mathfrak{s}^2}{9}\left[\left(\frac{\partial\mathcal{N}}{\partial\xi}\right)_\beta + \frac{1}{m^2}\left(\frac{\partial\mathcal{E}}{\partial\beta}\right)_\xi\right]. \tag{3}$$

Spin waves as transverse waves



- Decomposing $\omega^{\mu\nu}$ into its electric C_κ and magnetic C_ω components,

$$\omega^{\mu\nu} = \begin{pmatrix} 0 & C_{\kappa X} & C_{\kappa Y} & C_{\kappa Z} \\ -C_{\kappa X} & 0 & -C_{\omega Z} & C_{\omega Y} \\ -C_{\kappa Y} & C_{\omega Z} & 0 & -C_{\omega X} \\ -C_{\kappa Z} & -C_{\omega Y} & C_{\omega X} & 0 \end{pmatrix},$$

we find $\mathcal{C}_{\kappa Z} = \mathcal{C}_{\omega Z} = 0$, while

$$\left(\frac{\partial^2}{\partial t^2} - c^2_{\rm spin}\frac{\partial^2}{\partial z^2}\right)\begin{pmatrix} \mathcal{C}_{\kappa X} \\ \mathcal{C}_{\kappa Y} \\ \mathcal{C}_{\omega X} \\ \mathcal{C}_{\omega Y} \end{pmatrix} = 0, \quad c^2_{\rm spin} = -\frac{1}{4}\frac{\mathcal{A}_3}{\mathcal{A}_1} \to \begin{cases} \frac{1}{2}, & \frac{m}{T} \to 0, \\ \sqrt{\frac{T}{2m}}, & \frac{m}{T} \to \infty. \end{cases} \tag{4}$$