The early stage of heavy-ion collisions

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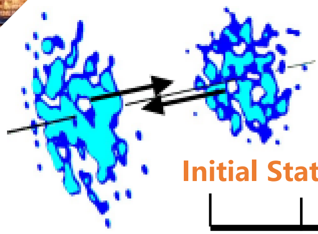
Outline

- **What is in the early stage of heavy-ion collisions (HICs)**
  - The gap between initial states and hydrodynamic states
- **Modelling the early stage dynamics**
  - Non-equilibrium dynamics and hydrodynamization
- **Probing the early stage dynamics**
  - Phenomenology on pre-equilibrium di-lepton in HICs
- **Propagating space-time fluctuation**
  - Towards a complete picture of pre-equilibrium stage in HICs
- **Conclusions**

[1] XD, Schlichting, PRL127(2021)122301, PRD104(2021)054011
[2] Coquet, XD, Ollitrault, Schlichting, Winn, PLB821(2021)136626, NPA1030(2023)122579
[3] Others, in preparation...

Pre-equilibrium stage in HICs



$\tau << 1$ fm/c | $\tau \sim 1$ fm/c | $\tau \sim 10$ fm/c

Initial State — Pre-equilibrium Plasma — Hydrodynamic Plasma — Hadronic Matter — Free-Streaming

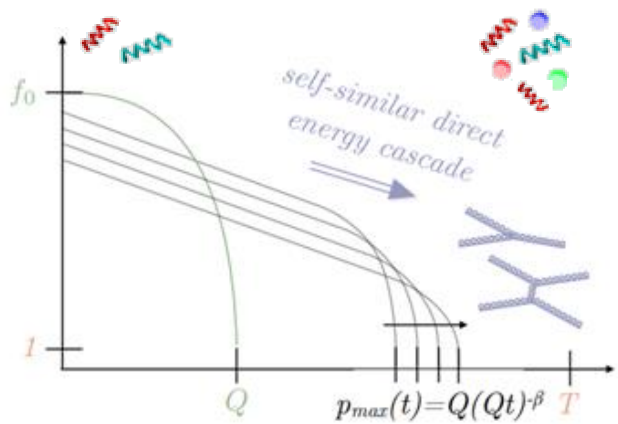
Saturation theory | Hydrodynamic theory

Gauge field (gluon) dynamics
Far from equilibrium
(off-thermal, anisotropic)

Classical Yang-Mills

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

Not for on-shell gluon
Not for fermionic dynamics
May not even thermalize



Quark gluon dynamics
Near-equilibrium (viscous hydro)

Hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0$$

Mesoscopic effective theory
Not for far from equilibrium

Connects initial condition to equilibrium states (Kinetic theory with QCD particles)

- Off-thermal initial states into near-thermal hydrodynamic states (kinetics equilibration)
- Saturated gluon fields into quark-gluon plasma (chemical equilibration)

Modelling the early stage dynamics

Non-equilibrium dynamics and hydrodynamization

QCD Effective Kinetic Theory

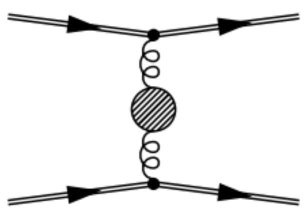
First principle QCD Effective Kinetic Theory (EKT)

$$\left(\frac{\partial}{\partial\tau}-\frac{p_\parallel}{\tau}\frac{\partial}{\partial p_\parallel}\right)f_a(\tau,p_T,p_\parallel)=-C_a^{2\leftrightarrow2}[f](\tau,p_T,p_\parallel)-C_a^{1\leftrightarrow2}[f](\tau,p_T,p_\parallel)$$

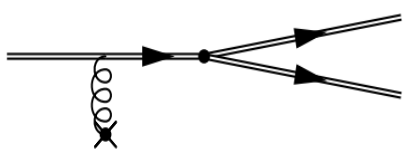
Arnold, Moore, Yaffe, JHEP01 (2003) 030
Arnold, Moore, Yaffe, JHEP0206 (2002) 030
Kurkela, Mazeliauskas, PRD99 (2019) 054018

Solving a set of coupled Boltzmann equations

- LO 2↔2 elastic scatterings & 1↔2 inelastic scatterings



2↔2: Color screening by Debye mass fit to Hard Thermal Loop (HTL) calculation



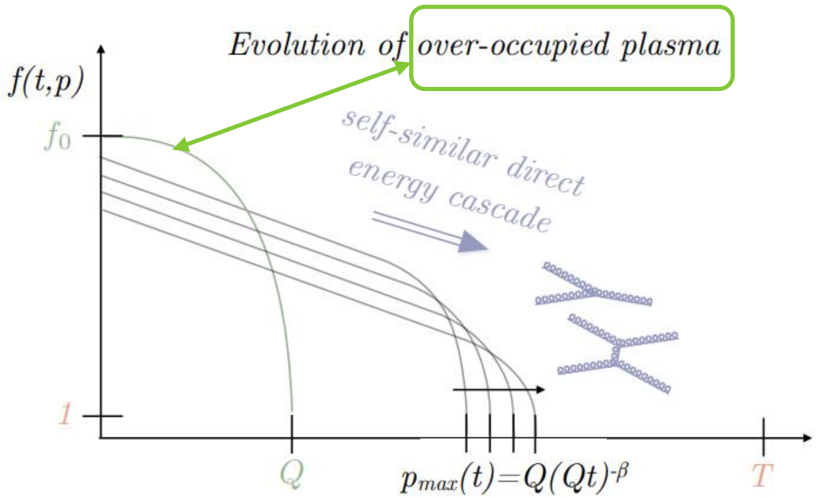
1↔2: Collinear radiation including Landau-Pomeranchuk-Migdal (LPM) effect via effective vertex resummation

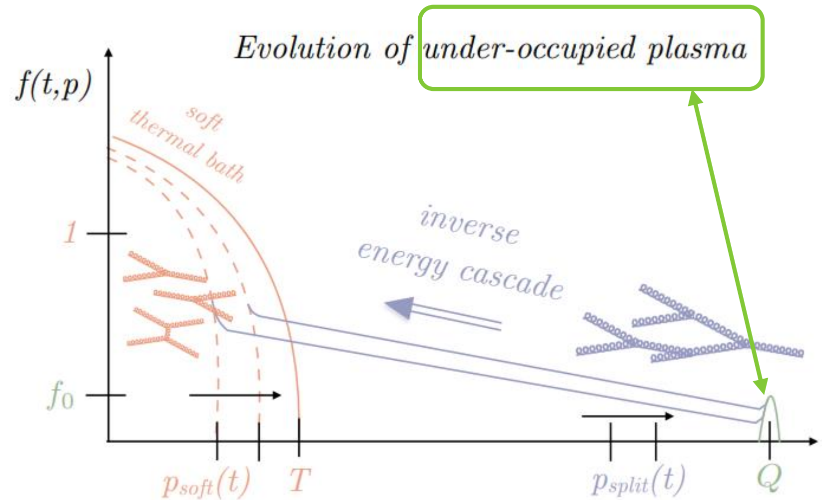
- Gluon + all light quarks/antiquarks (finite net-baryon density) $a=g,u,\bar{u},d,\bar{d},s,\bar{s}$

XD, Schlichting, PRD104(2021)054011
XD, Schlichting, PRL127(2021)122301

Far-from-equilibrium plasma

Two typical far-from-equilibrium systems





Schlichting, Teaney, ARNPS 69 (2019) 447

Over-occupied plasma

- Separation of scale $\langle p \rangle_0 \leq T, f_0 \gg 1$
- Direct energy cascade

Self-similarity/non-thermal fixed point

- **Initial state in HICs**

See Talks by

Sergio 11:30 Monday,

Florian 10:25 Friday

Under-occupied plasma

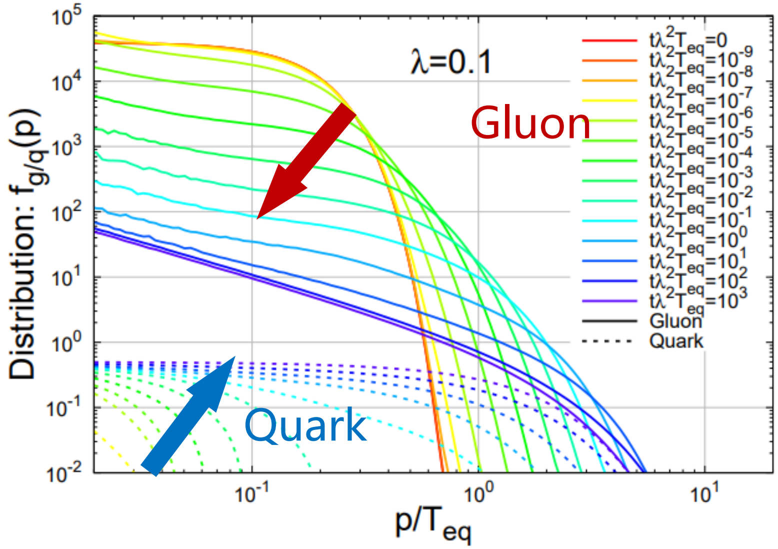
- Separation of scale $\langle p \rangle_0 \geq T, f_0 \ll 1$
- Inverse energy cascade

Bottom-up thermalization

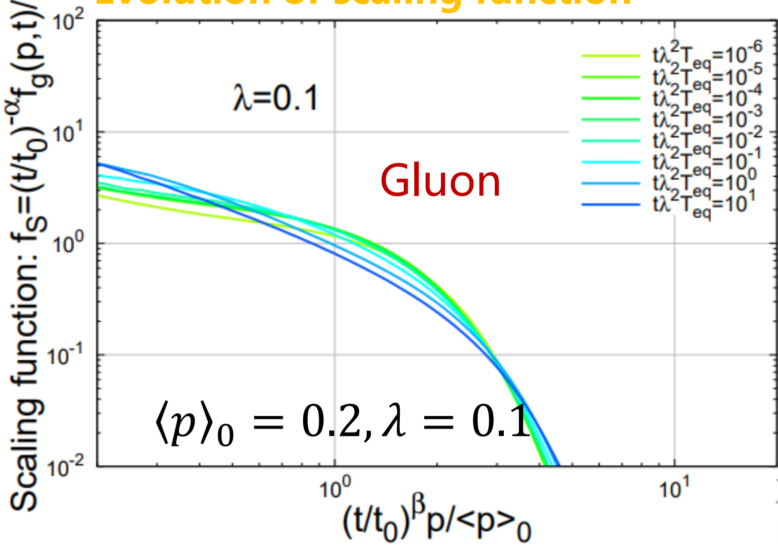
- **Jets in HICs**

Over-occupied plasma

Evolution of distribution



Evolution of scaling function



Self-similar energy cascade

- Self-similar scaling spectra

$$f_g(p,t) = (t/t_0)^{\alpha} f_0 f_S\left((t/t_0)^{\beta}\frac{p}{\langle p\rangle_0}\right)$$

Universal Scaling Function

$$f_S\left((t/t_0)^{\beta}\frac{p}{\langle p\rangle_0}\right)$$

Scaling Exponents from Yang-Mills plasma

$$\alpha = -\frac{4}{7}, \beta = -\frac{1}{7}$$

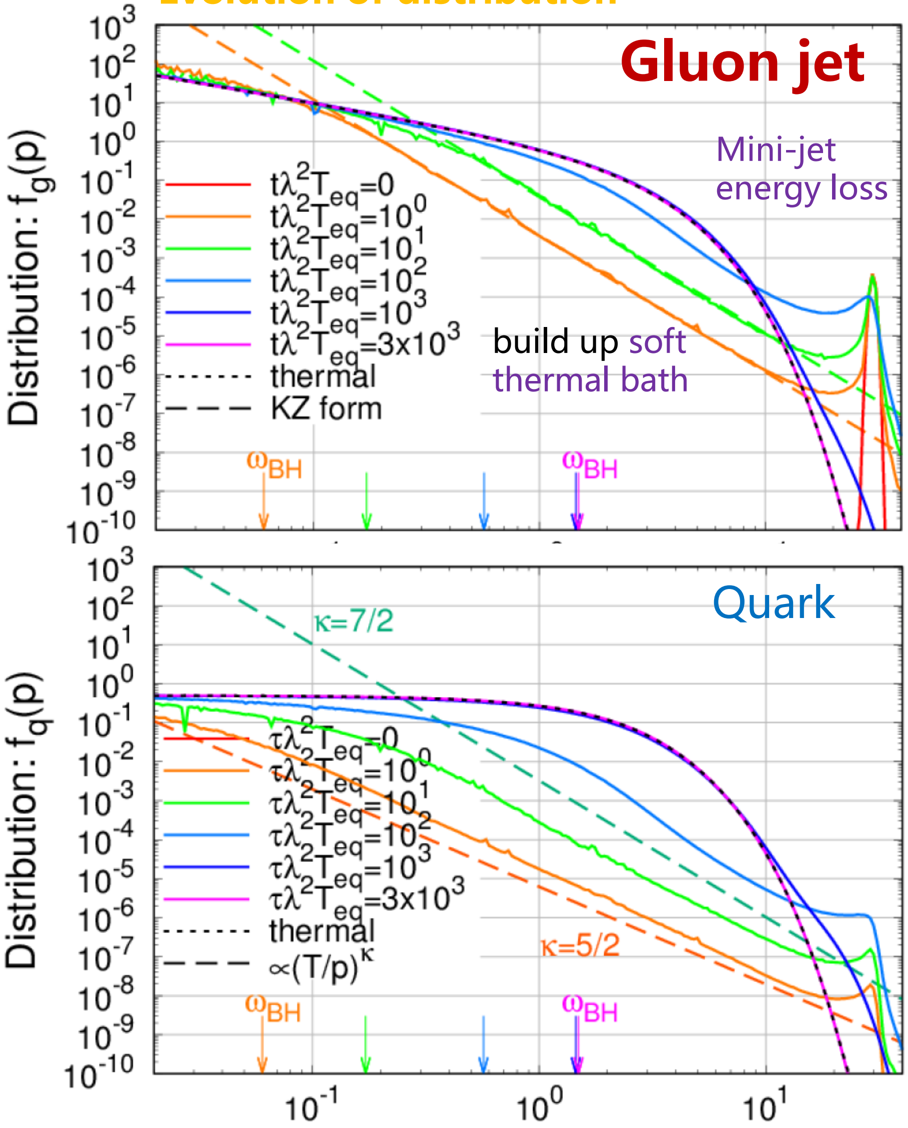
- Also work for quark-gluon plasma
  gluon dominated

Quark spectra following gluon spectrum

XD, Schlichting, PRD104(2021)054011
Berges, Boguslavski, Schlichting, Venugopalan, PRD89 (2014) 114007
Abraao York, Kurkela, Lu, Moore, PRD89(2014)074036

Under-occupied plasma

Evolution of distribution



$\langle p\rangle_0 = 30, \lambda = 1$

Wave turbulence

- Kolmogorov-Zakharov spectrum (exponent κ=7/2 for gluon)

$$f_{KZ}(p,t) = \eta(t)\left(\frac{\langle p\rangle_0}{p}\right)^{\kappa}$$

Blaizot, Iancu, Mehtar-Tani, PRL 111, 052001 (2013)
Mehtar-Tani, Schlichting, JHEP 09, 144 (2018)

Bottom-up thermalization

- Emission of (soft) quarks and gluon
- Radiative breakup by multiple branchings → build up soft thermal bath
- Mini-Jet energy loss → heating up thermal bath

Baier, et al. PLB 502 (2001) 51

XD, Schlichting, PRD104(2021)054011

Close-to-equilibrium plasma

Kinetic equilibration

- First-order hydrodynamics

$$\frac{p_L}{e} = \frac{1}{3} - \frac{4}{9\pi\tilde{\omega}} \qquad \tilde{\omega} = \frac{(e+p)\tau}{4\pi\eta}$$

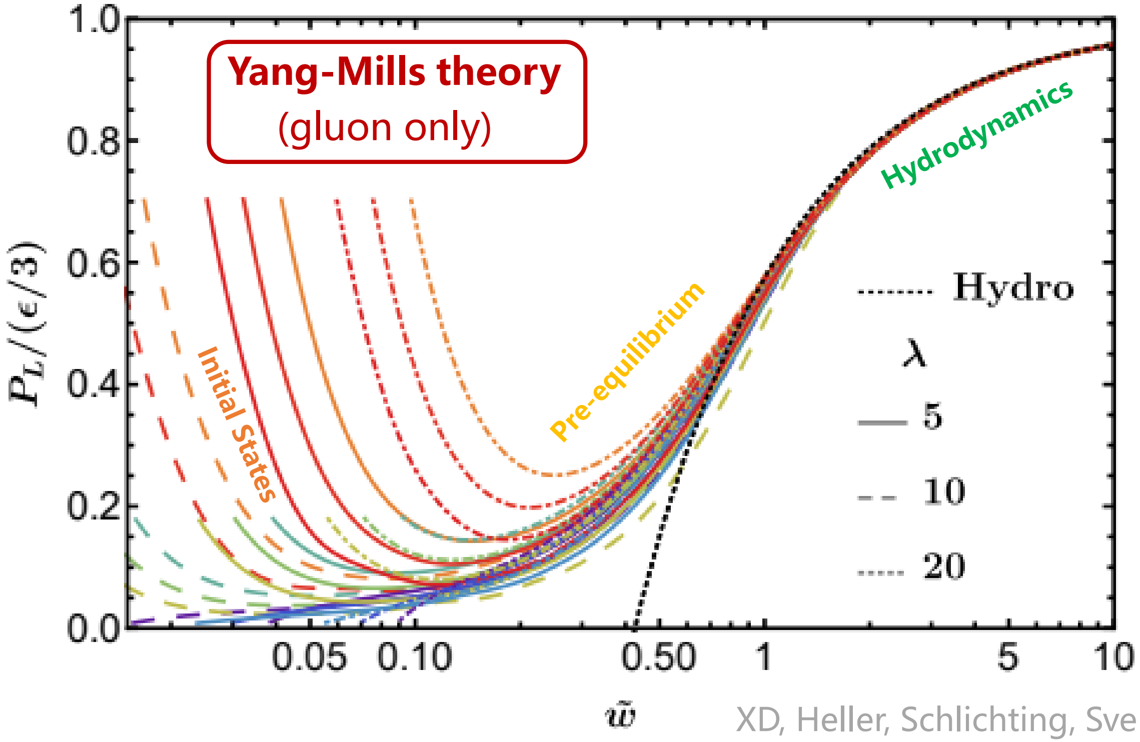
Universal time scale

From EKT

- Effective constitutive relation

$$\frac{p_L}{e} = f(\tilde{\omega})$$

Evolution of pressure



XD, Heller, Schlichting, Svensson, arXiv:2203.16549

Pressure attractor

- Quick memory loss regardless of **initial states**
- Universal attractor towards **hydrodynamics**

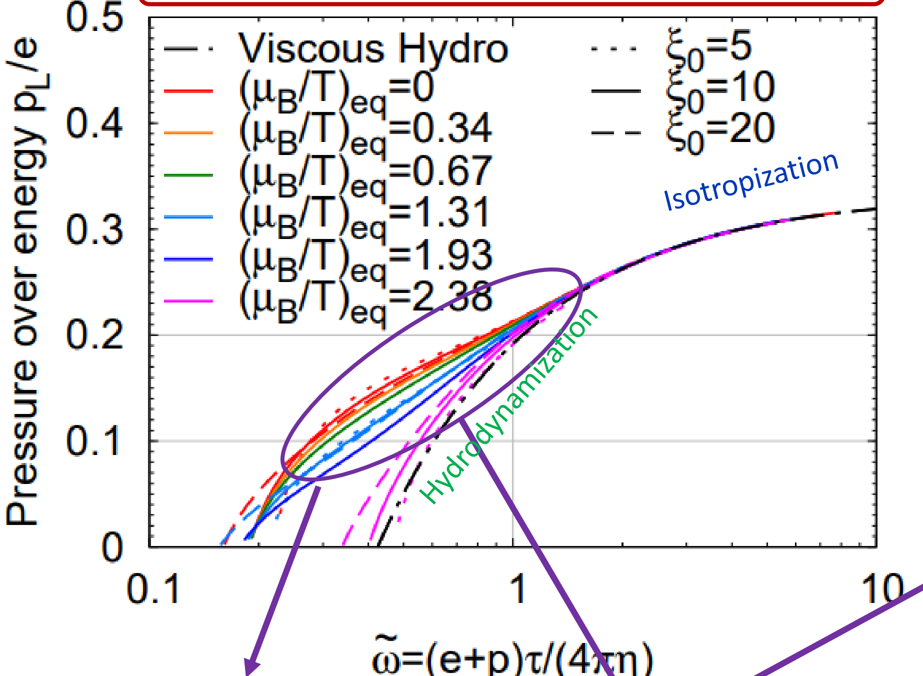
Hydrodynamization of QGP in HICs

Kinetic equilibration

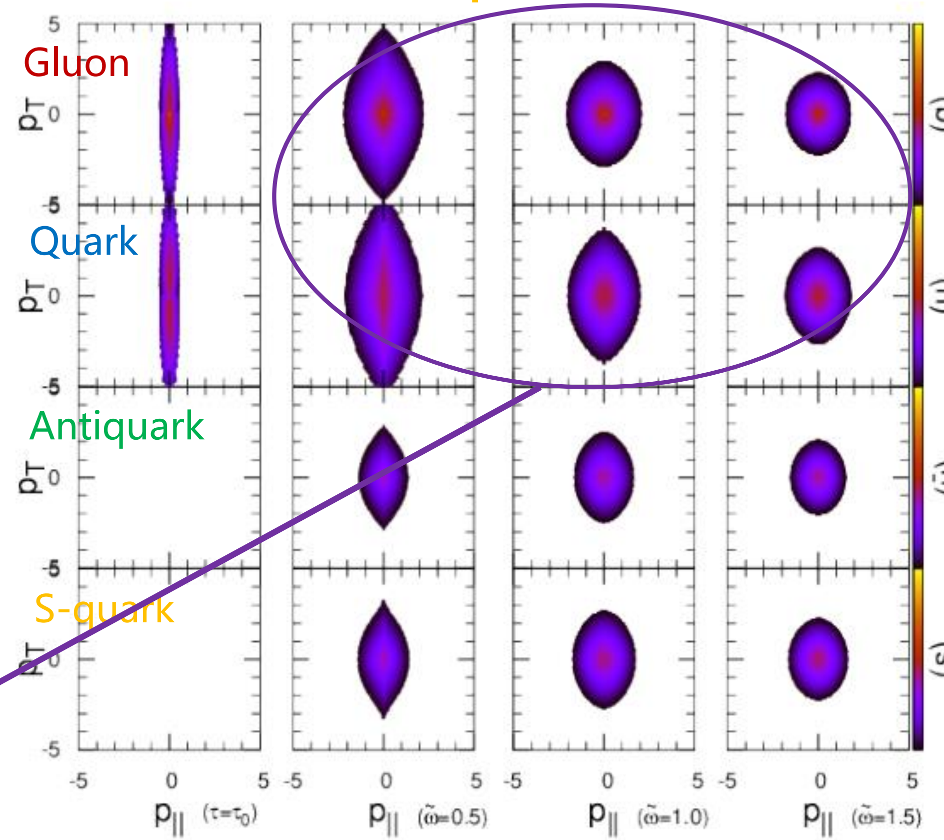
- Pressure anisotropy of QCD plasma still evolves to equilibrium
- Pressure anisotropy no longer guarantees an universal attractor

Evolution of pressure

QCD theory (gluon + quark/antiquark)



Evolution of anisotropic distribution



Non-universal

Quarks slow down equilibration
(Gluons isotropy fast while quarks persist anisotropy)

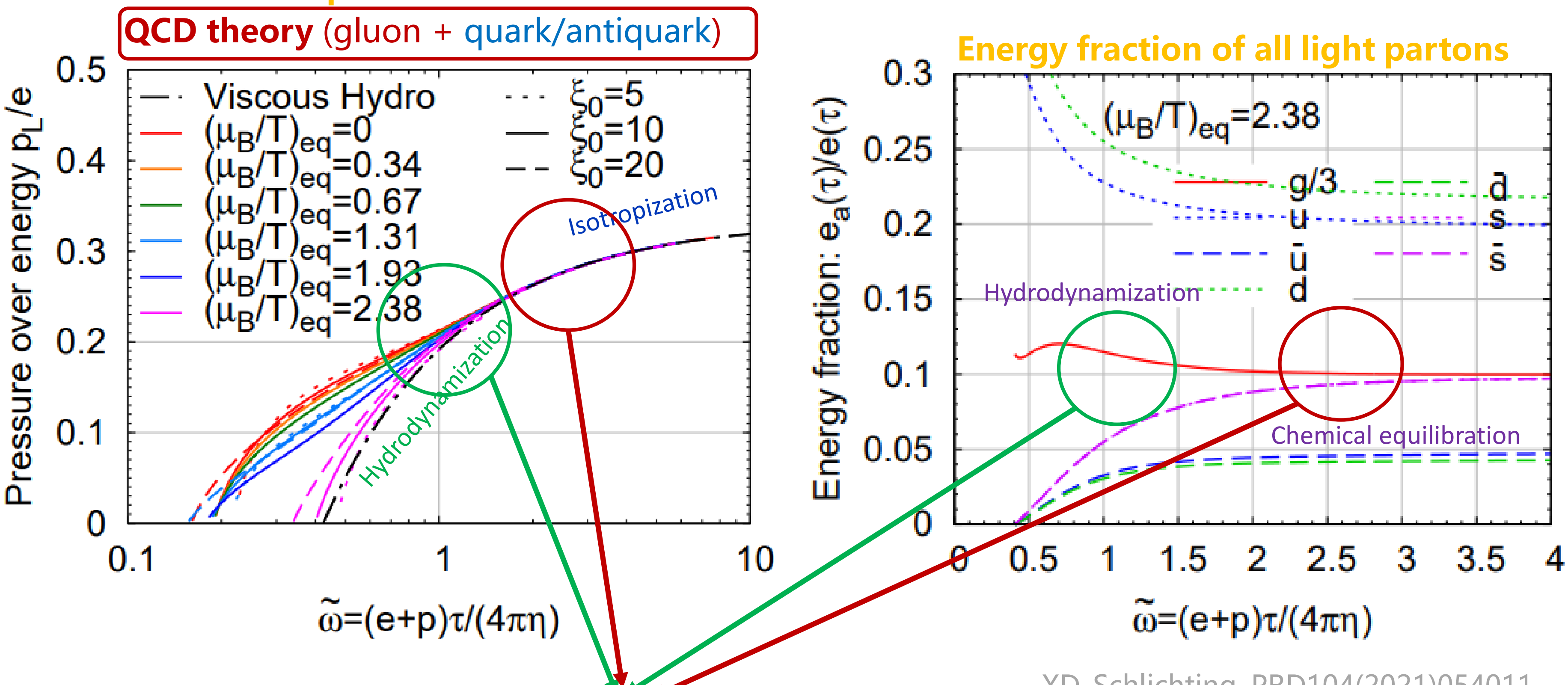
XD, Schlichting, PRD104(2021)054011
XD, Schlichting, PRL127(2021)122301

Hydrodynamization of QGP in HICs

Chemical equilibration

- Fractions of gluon/quark/antiquark change and evolve to equilibrium value
- Chemical equilibration persists after kinetic equilibration (hydrodynamization)

Evolution of pressure



(Chemical equilibration continues after hydrodynamization)

XD, Schlichting, PRD104(2021)054011
XD, Schlichting, PRL127(2021)122301

Hydrodynamization of QGP in HICs

Energy attractor and particle production

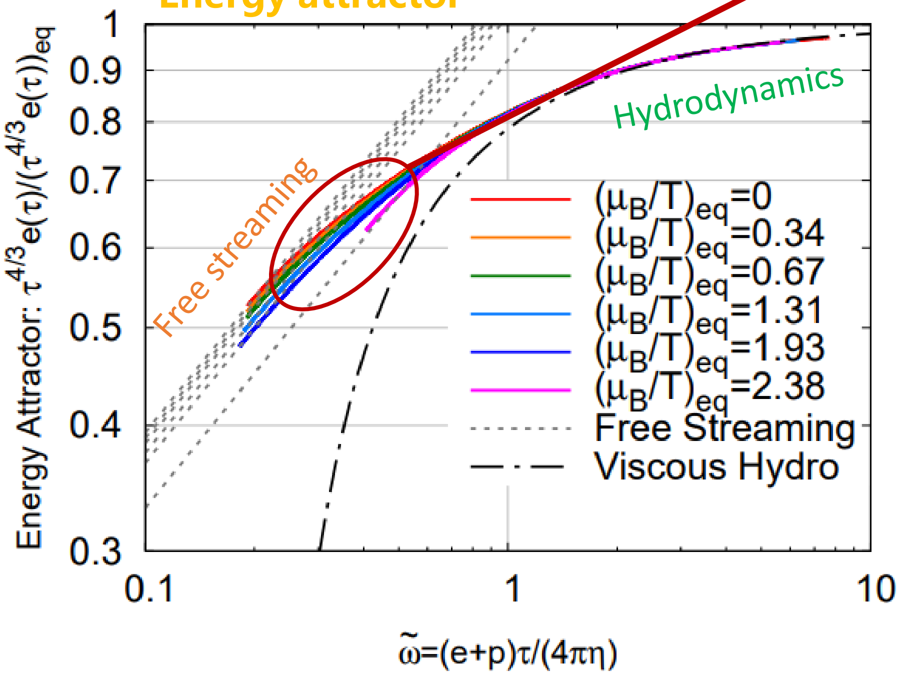
- **Pre-equilibrium** description connects initial states to hydrodynamics in HICs

$$(\tau^{4/3}e)_{\tilde{\omega}} = \left(4\pi\frac{\eta T_{eff}}{e+p}\right)^{\frac{4}{9}}\left(\frac{\pi^2}{30}\nu_{eff}\right)^{\frac{1}{9}}(\tau e)_0^{\frac{8}{9}}C_\infty\mathcal{E}(\tilde{\omega})$$

$$(\tau\Delta n_f)_{\tilde{\omega}} = (\tau\Delta n_f)_0$$

Vary by different η/s
(Quarks slow down equilibration, increase η/s at larger density)

Energy attractor



Energy attractor

$$\mathcal{E}\left(\tilde{\omega} = \frac{(e+p)\tau}{4\pi\eta}\right) = \frac{\tau^{4/3}e}{(\tau^{4/3}e)_{eq}}$$

Two-way application

- Provide input for hydrodynamics
- Learn the past !(**pre-eq**, initial…)

Giacalone, Mazeliauskas, Schlichting PRL123(2019)262301
XD, Schlichting, PRL127(2021)122301

Pre-equilibrium QGP trajectory

Fix the final equilibrium quantities

- From EKT: entropy

$$(\tau s)_{eq} = \frac{\tau\left(e + p - \sum_f \mu_f \Delta n_f\right)}{T}$$

- From data: charged particle multiplicity

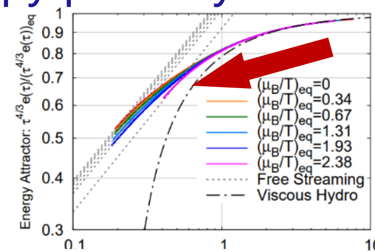
$$\frac{dN_{ch}}{d\eta} = \frac{N_{ch}}{JS}(\tau s)_{eq} S_T \approx 0.12 (\tau s)_{eq} S_T$$

- From EKT: net baryon number

$$\Delta n_B = \frac{1}{3}\Delta n_u + \frac{1}{3}\Delta n_d$$

- From data: entropy per baryon

$$\frac{S}{N_B} = \left(\frac{\tau s}{\tau \Delta n_B}\right)_{eq}$$



Pre-equilibrium QGP

- Apply non-equilibrium attractor

$$(\tau^{4/3}e)_{\tilde{\omega}} = \mathcal{E}(\tilde{\omega})(\tau^{4/3}e)_{eq}$$

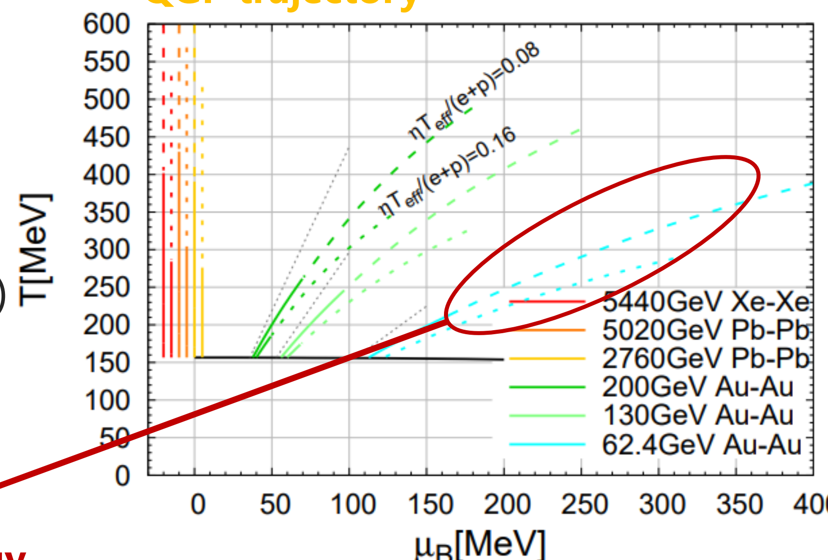
$$(\tau \Delta n_f)_{\tilde{\omega}} = (\tau \Delta n_f)_{eq}$$

- Define effective T and μ_B (Landau matching)

Non-equilibrium QGP trajectory

(at large baryon density)

QGP trajectory



Pre-equilibrium stage important at RHIC energy

XD, Schlichting, PRL127(2021)122301

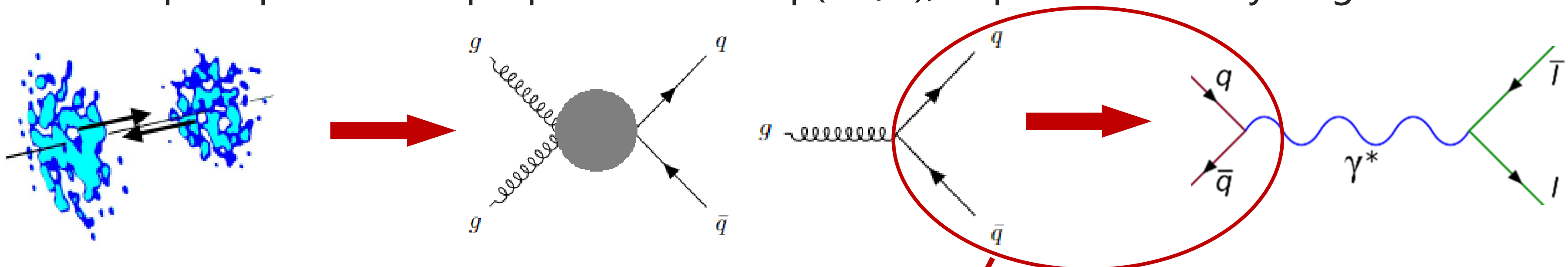
Probing the early stage dynamics

Phenomenology on pre-equilibrium di-lepton in HICs

Pre-equilibrium di-lepton production

Electromagnetic probes

- Photon, di-lepton are produced through-out HICs, not interacting with QGP
- Di-lepton production proportional to exp(-M/T), important at early stage of HICs



Gluon saturated **Gluon → quark/antiquark** **Di-lepton production**

$$\frac{dN^{l+l-}}{d^4x d^4K} = \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3} 4N_c \sum_f f_q(x,p_1) f_{\bar{q}}(x,p_1) v_{q\bar{q}} \sigma_{q\bar{q}}^{l+l-} \delta^{(4)}(K - P_1 - P_2)$$

Pre-equilibrium quark/antiquark distribution $f_{q/\bar{q}}(x,p)$

- Anisotropic distribution
- Quark/antiquark chemical production

Realistic space-time distribution complicated
(Parameterized distribution with fixed scale in equilibrium)

Coquet, XD, Ollitrault, Schlichting, Winn, PLB821(2021)136626

Pre-equilibrium di-lepton production

Quark/antiquark distribution

- Kinetic equilibration

Gluon

$$f_g(\tau, p_t, p_z) = f_{BE}\left(\frac{\sqrt{p_t^2 + \xi^2(\tau) p_z^2}}{\Lambda(\tau)}\right)$$

Quark/antiquark

$$f_{q/\bar{q}}(\tau, p_t, p_z) = q(\tau) f_{FD}\left(\frac{\sqrt{p_t^2 + \xi^2(\tau) p_z^2}}{\Lambda(\tau)}\right)$$

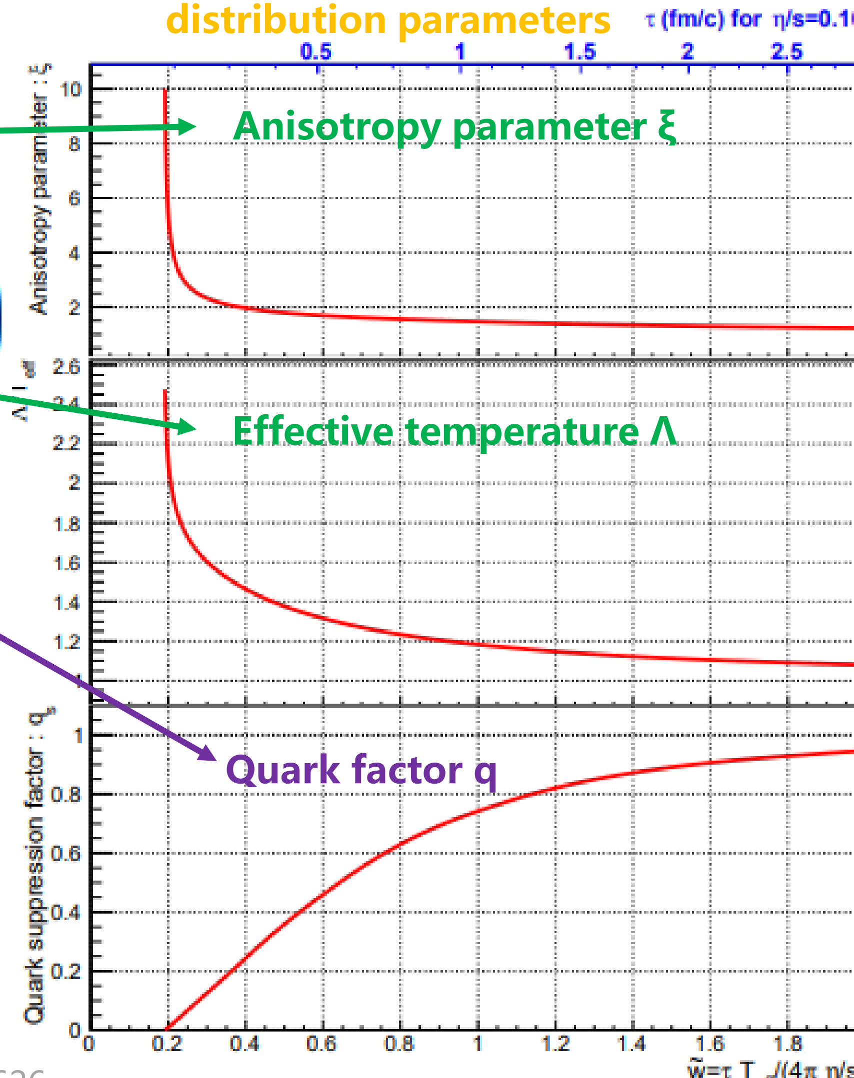
- Chemical equilibration

Quark factor

$$q(\tau) = \frac{e_g^{eq}}{e_q^{eq}} \frac{e_q(\widetilde{\omega})}{e_g(\widetilde{\omega})}$$

From QCD effective kinetic theory

Evolution of distribution parameters



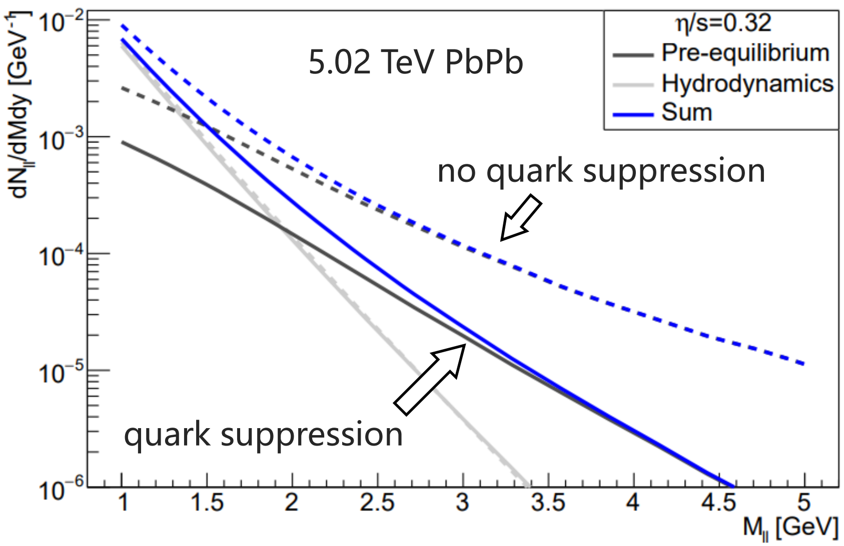
XD, Schlichting, PRL127(2021)122301
Coquet, XD, Ollitrault, Schlichting, Winn, PLB821(2021)136626

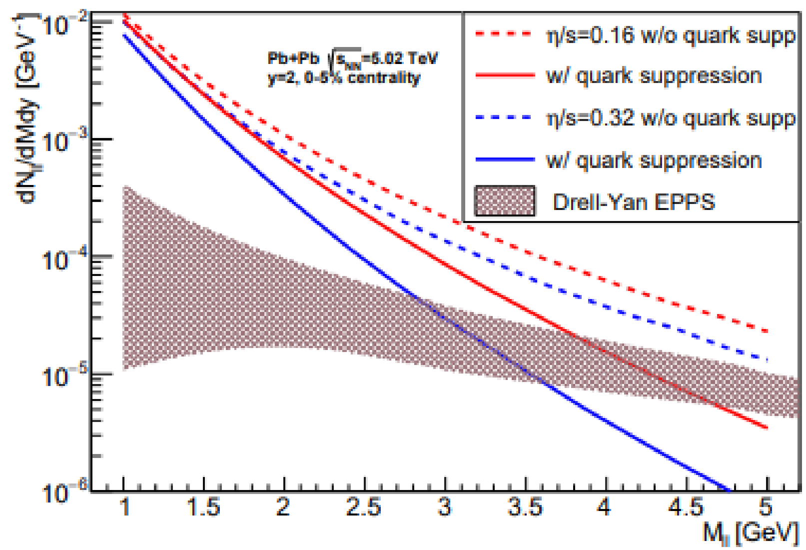
Pre-equilibrium di-lepton production

Di-lepton spectra in pre-equilibrium QGP

- Suppression compared to thermal quark (quark production)
- Suppression at weaker coupling medium (larger η/s, slower quark production)

Di-lepton spectra





- Pre-equilibrium production dominates at smaller invariant mass of di-lepton pair

Coquet, XD, Ollitrault, Schlichting, Winn, PLB821(2021)136626

Pre-equilibrium di-lepton production

Di-lepton spectra and scale violation

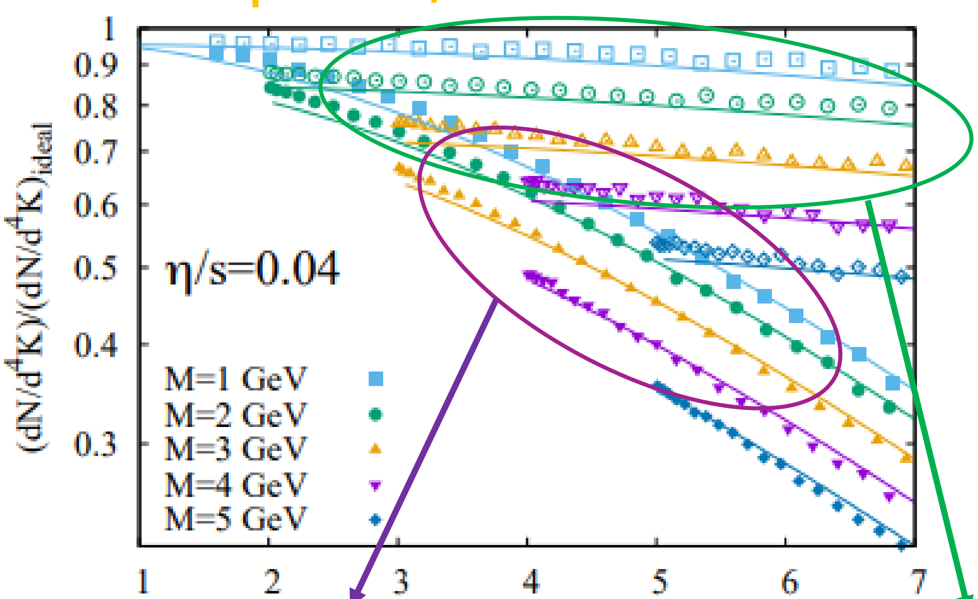
- Thermal and uniform QGP: McLerran-Toimela M_t^{-6} spectrum (ideal hydro)

$$\left(\frac{dN^{l^+l^-}}{d^4K}\right)_{\text{ideal}} = \frac{32N_c\alpha^2\sum_f q_f^2}{\pi^4}\frac{A_\perp(\tau T^3)^2}{M_t^6}$$

McLerran, Toimela, PRD31(1985)545

- Violate in pre-equilibrium QGP

Pre-equilibrium/thermal ratio



quark suppression (kinetic theory)

Viscous correction (N-S hydro)

Sensitivity of violation

- Depend on viscosity η/s, transverse mass M_t, invariant mass M

$$\frac{dN^{l^+l^-}}{d^4K} \simeq \left(\frac{dN^{l^+l^-}}{d^4K}\right)_{\text{ideal}} \frac{\left(1 + a\frac{\eta}{s}M_t^2/n\right)^{-n}}{\sqrt{1 + b\frac{\eta}{s}M^2}}$$

- Strong coupling limit η/s → 0, smoothly converges to McLerran-Toimela

Reynolds number

$$Re^{-1}(M_t) \equiv \frac{\eta}{s}\frac{M_t^2}{\tau T^3}$$

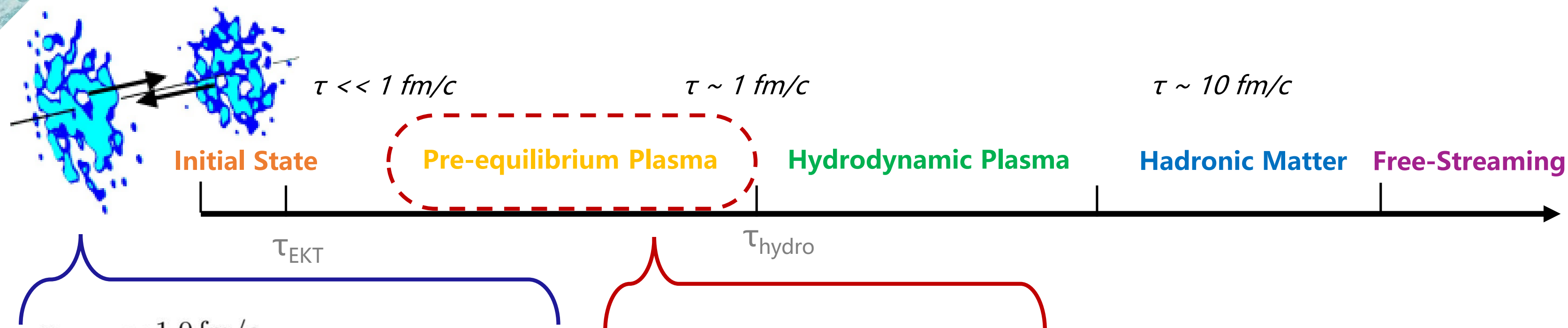
Coquet, XD, Ollitrault, Schlichting, Winn, arXiv:2112.13876

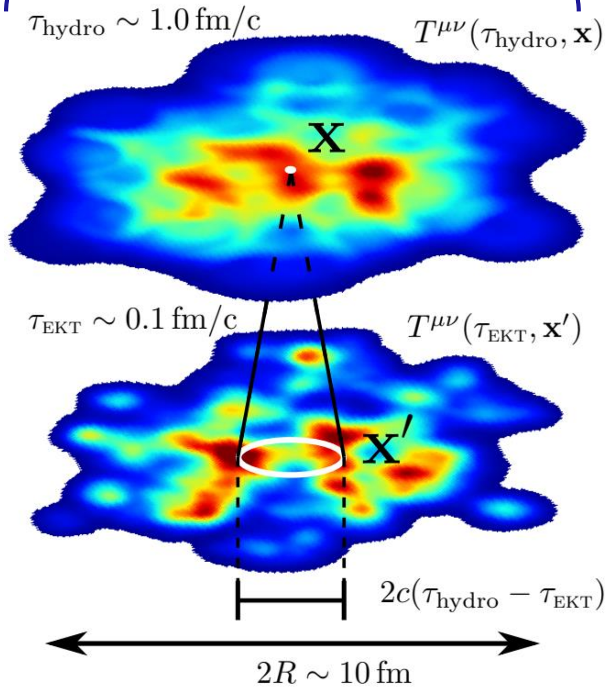
Sensitive to both kinetic (η/s) & chemical (quark factor) equilibration

Propagating space-time fluctuation

Towards a complete picture of pre-equilibrium stage in HICs

Propagation of space perturbation





Kinetic Theory

- Thermalization (in momentum)
- Chemical reaction (in chemical composition)

- Propagating initial fluctuation (in position space)

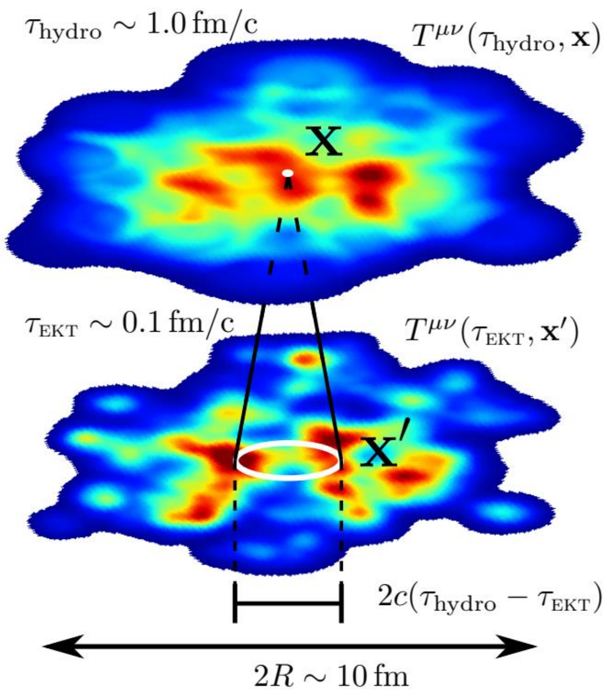
Linear response theory

KøMPøST framework (Yang-Mills):
Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney,
PRL122 (2019) 12, 122302, PRC99 (2019) 3, 034910

Propagation of space perturbation

Linearized QCD Effective Kinetic Theory

- Energy momentum tensor for inhomogeneous and anisotropic plasma



$$T^{\mu\nu}(\tau_{\rm EKT}, \mathbf{x}') = \overline{T}^{\mu\nu}_{\mathbf{x}}(\tau_{\rm EKT}) + \delta T^{\mu\nu}_{\mathbf{x}}(\tau_{\rm EKT}, \mathbf{x}')$$

Non-equilibrium background — Effective kinetic theory (EKT)

Perturbation — Linearized EKT

Linear response/Green's function

- Propagate energy-momentum fluctuation in both position and momentum space (with Fourier transform)

$$\delta T^{\mu\nu}_{\mathbf{x}}(\tau_{\rm hydro}, \mathbf{x}) = \int d^2\mathbf{x}'\; G^{\mu\nu}_{\alpha\beta}(\mathbf{x}, \mathbf{x}', \tau_{\rm hydro}, \tau_{\rm EKT})\, \delta T^{\alpha\beta}_{\mathbf{x}}(\tau_{\rm EKT}, \mathbf{x}') \frac{\overline{T}^{\tau\tau}_{\mathbf{x}}(\tau_{\rm hydro})}{\overline{T}^{\tau\tau}_{\mathbf{x}}(\tau_{\rm EKT})}$$

Extended from RTA: Kamata, Martinez, Plaschke, Ochsenfeld, Schlichting, PRD102(2020)056003

Extended from Boltzmann for QCD:
XD, Schlichting, in preparation (full QCD theory)
Software development done, large amount of numerical running in progress...

Conclusions

Summary and outlook

Conclusions

- **Early stage of heavy-ion collisions (HICs)**
  - Non-equilibrium dynamics from kinetic theory
- **Modelling the early stage dynamics**
  - Non-equilibrium dynamics and hydrodynamization

Importance of quarks

- **Probing the early stage dynamics**
  - Pre-equilibrium di-lepton production in HICs
- **Propagating space-time fluctuation**
  - ❑ **Outlook: Towards a complete picture of pre-equilibrium stage in HICs:**
  - Space-time fluctuation of Full QCD theory

[1] XD, Schlichting, PRL127(2021)122301, PRD104(2021)054011
[2] Coquet, XD, Ollitrault, Schlichting, Winn, PLB821(2021)136626, arXiv:2112.13876
[3] Others, in preparation…