Opacity dependence of pre-equilibrium and applicability of hydrodynamics in heavy-ion collisions

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Strong-interaction matter under extreme conditions

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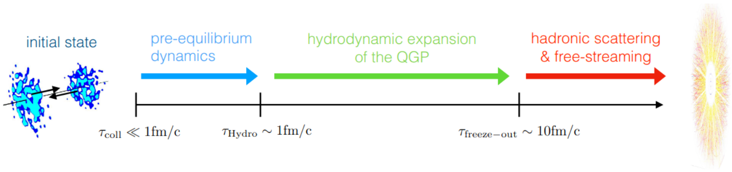
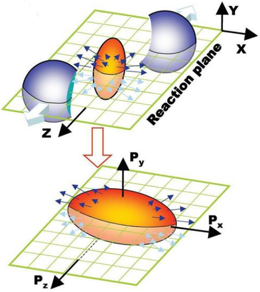
Standard modelling of heavy ion collisions



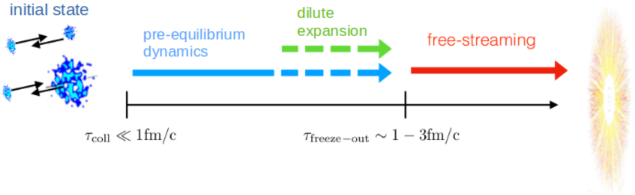
- early stage requires non-equilibrium description, but system quickly equilibrates
- strongly interacting QGP leaves imprints of thermalization and collectivity in final state observables
- transport description after hadronization



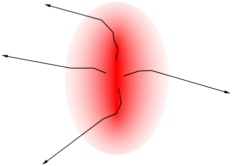
Hiroshi Masui (2008)

Small systems

Very dilute, hydrodynamics not necessarily applicable



- still collective behaviour is observed!
- collectivity can also be explained in kinetic theory, a microscopic description which does not rely on equilibration
- limit of large interaction rate is hydrodynamics!



Aim

Case study in simplified kinetic theory description on full range from small to large system size with comparison to hydrodynamics based on transverse flow

Model and Setup

- microscopic description in terms of averaged on-shell phase-space distribution of massless bosons:

$$f(\tau, \mathbf{x}_\perp, \eta, \mathbf{p}_\perp, y) = \frac{(2\pi)^3}{\nu_{\text{eff}}} \frac{\mathrm{d}N}{\mathrm{d}^3x\,\mathrm{d}^3p}(\tau, \mathbf{x}_\perp, \eta, \mathbf{p}_\perp, y)$$

  - boost invariance
  - initialized with vanishing longitudinal pressure and no transverse momentum anisotropies

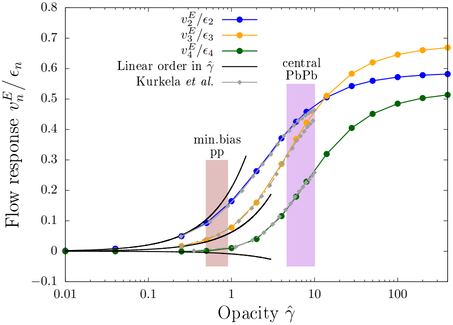
- time evolution: Boltzmann equation in conformal relaxation time approximation

$$p^\mu \partial_\mu f = C_{\text{RTA}}[f] = -\frac{p^\mu u_\mu}{\tau_R}(f - f_{\text{eq}}) \,, \quad \tau_R = 5\frac{\eta}{s}T^{-1}$$

  - time evolution of f depends only on opacity $\hat{\gamma} = \left(5\frac{\eta}{s}\right)^{-1} \left(\frac{1}{a\pi}\frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}R\right)^{1/4}$

Kurkela, Wiedemann, Wu EPJC 79 (2019) 965

Opacity Dependence of Anisotropic flow



- linear order results tangential to numerical curve at small opacities
- agreement with previous results in identical setup

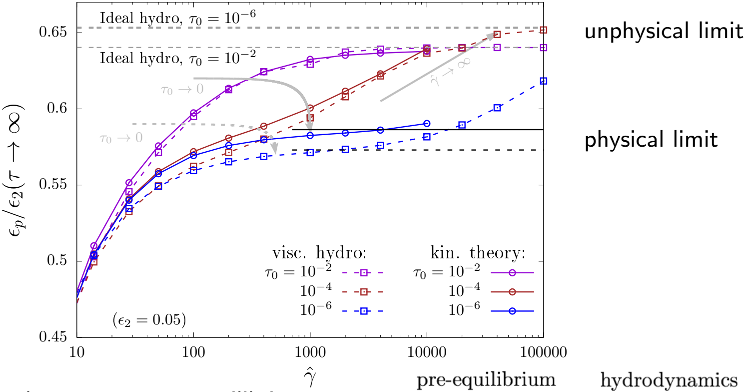
  Kurkela, Taghavi, Wiedemann, Wu PLB 811 (2020) 135901

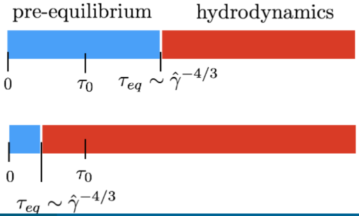
- saturation at higher $\hat{\gamma}$

  $\Rightarrow$ expectation: hydrodynamic behaviour at large opacities

Comparison to Hydrodynamics

Hydro: vHLLE Karpenko, Huovinen, Bleicher Comput. Phys. Commun. 185, 3016 (2014)



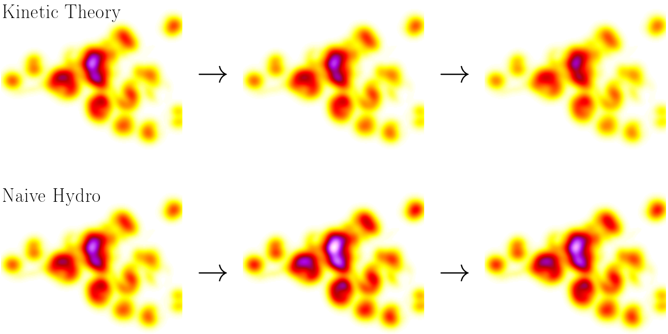


- agreement at large τ_0: no pre-equilibrium
- small τ_0: pre-equilibrium causes discrepancies
- convergence only in unphysical order of limits

$\Rightarrow$ Why is pre-equilibrium important for observables that develop at $\tau \sim R$?

- $\tau \ll R$: only longitudinal expansion $\Rightarrow$ local Bjorken flow cooling
  - follows universal attractor curve
    early times: $\tau^{4/3} e \propto (T\tau)^\gamma$, late times: $\tau^{4/3} e = \text{const.}$

evolution of $\tau e \sim \frac{dE_\perp}{d^2\mathbf{x}_\perp d\eta}$:



$\tau = 3 \cdot 10^{-6}$fm $\tau = 8 \cdot 10^{-4}$fm $\tau = 3 \cdot 10^{-3}$fm

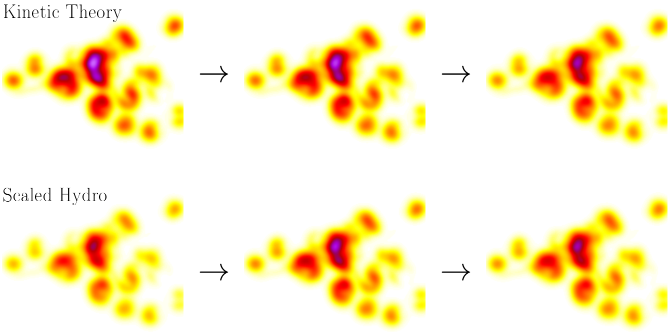
(times for $4\pi\eta/s = 0.05$)

- dynamics depend on local energy density $\Rightarrow$ inhomogeneous cooling
  - decrease of eccentricity before transverse flow develops

How to "fix" Hydro?

- idea: counteract difference in pre-equilibrium by different hydro initialization

evolution of $\tau e \sim \frac{dE_\perp}{d^2\mathbf{x}_\perp d\eta}$:



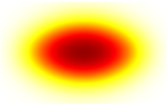
$\tau = 3 \cdot 10^{-6}$fm $\qquad$ $\tau = 8 \cdot 10^{-4}$fm $\qquad$ $\tau = 3 \cdot 10^{-3}$fm

(times for $4\pi\eta/s = 0.05$)

- our actual initial condition: average profile (30-40% Pb+Pb 5.02 TeV)

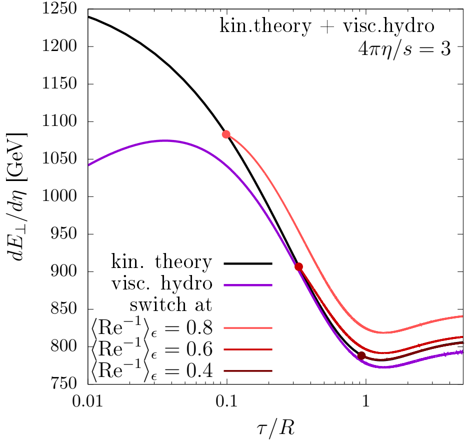
  Borghini, Borrell, Feld, Roch, Schlichting, Werthmann arXiv:2209.01176

  - fixed profile: vary $\hat{\gamma}$ via η/s: $\hat{\gamma} \approx 11 \cdot (4\pi\eta/s)^{-1}$

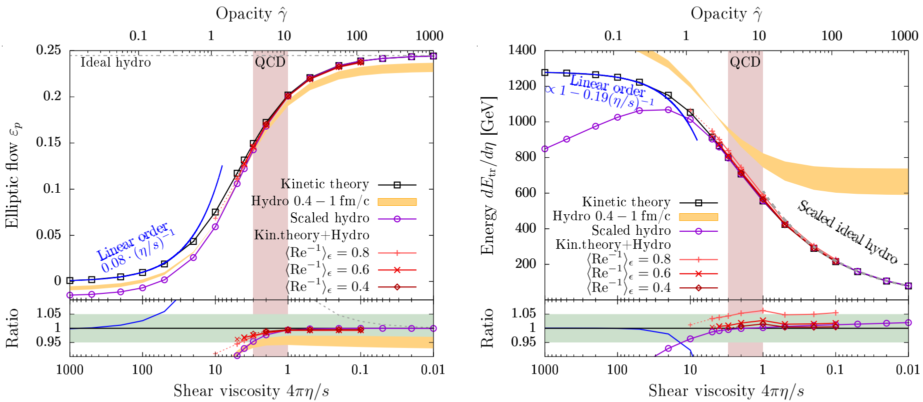


Hybrid schemes

- idea: evolve system in kinetic theory until $\langle \mathrm{Re}^{-1} \rangle$ drops to specific value, then match $T^{\mu\nu}$ to hydro code
- system immediately starts following similar evolution to a pure hydro run
  - switching too early causes errors in pre-equilibrium
  - results from late switching times more accurate than rescaled hydro

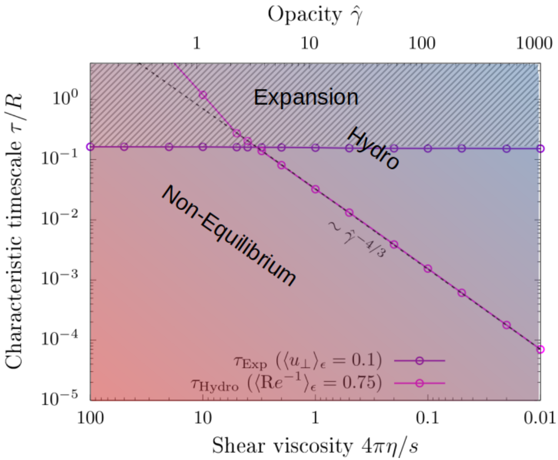


Comparison of improved hydro schemes with kin. theory



- naive hydro is off; improved schemes in perfect agreement at large $\hat{\gamma}$
- rescaled hydro accurate if $\hat{\gamma} \gtrsim 3$
- Hybrid kin. theory scheme can improve on scaled hydro at intermediate opacities

Regime of applicability of hydrodynamics



- transverse expansion sets in at $\tau_\perp \sim 0.2R$, independent of opacity
- Hydro appicable when $\text{Re}^{-1} < \text{Re}_c^{-1} \sim 0.75$ after timescale

$$\tau_{\text{Hydro}}/R \approx 1.53\ \hat{\gamma}^{-4/3}\ \left[(\text{Re}_c^{-1})^{-3/2} - 1.21(\text{Re}_c^{-1})^{0.7}\right]$$

- $\hat{\gamma} \lesssim 3$ hydrodynamization only after transv. Expansion (if at all)

Summary

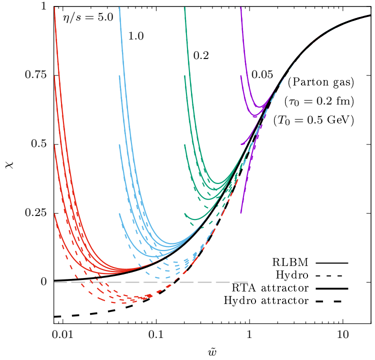
- kinetic theory description covers full range in opacity from small to large systems
- naive comparison to hydrodynamics: disagreement even at large opacities!
  - difference during pre-equilibrium
  - eccentricity decreases before onset of transverse expansion
- different setup of hydrodynamic simulations can bring agreement at large opacities
  - initializing hydrodynamics on its early-time attractor
  - hybrid models with kinetic theory for pre-equilibrium

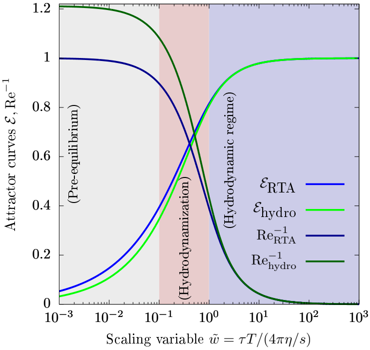
$\Rightarrow$ applicability of hydro vs τ, $\hat{\gamma}$

Backup

- longitudinal boost-invariant Bjorken flow exhibits universal behaviour
- time evolution curves converge to an attractor curve when expressed via the scaling variable $\tilde{w} = \frac{T\tau}{4\pi\eta/s}$
  $\Rightarrow$ expressed via universal scaling functions
  $\chi(\tilde{w}) = p_L/p_T, \quad \mathcal{E}(\tilde{w}) \propto \tau^{4/3}e, \quad f_{E_\perp}(\tilde{w}) \propto \tau^{1/3}\frac{dE_\perp}{dy}, \ldots$

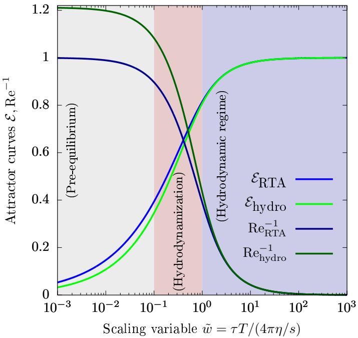
Giacalone, Mazeliauskas, Schlichting, PRL 123 (2019) 262301

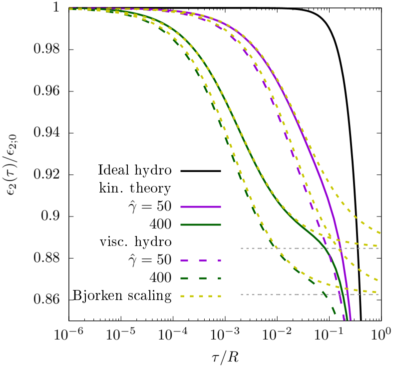




Ambruș, Bazzanini, Gabbana, Simeoni, Succi, Tripicione, arXiv:2201.09277

- $\tau \ll R$: no transverse expansion, system locally behaves like 0+1D Bjorken flow
  - universal attractor curve scaling in the variable $\tilde{w}(\tau, \mathbf{x}_\perp) = \frac{T(\tau,\mathbf{x}_\perp)\tau}{4\pi\eta/s}$
    Giacalone, Mazeliauskas, Schlichting, PRL 123 (2019) 262301
  - $\tilde{w} \gg 1$: $\tau^{4/3}e = \text{const.}$, $\tau^{1/3}\frac{dE_\perp}{dy} = \text{const.}$
  - $\tilde{w} \ll 1$: model dependent power law $\tau^{4/3}e \sim \tilde{w}^\gamma$





- inhomogeneous cooling changes energy density profile

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Bjorken flow universal attractor curve in scaling variable $\tilde{w}(\tau, \mathbf{x}_\perp) = \frac{T(\tau,\mathbf{x}_\perp)\tau}{4\pi\eta/s}$:

$$\epsilon(\tau)\tau^{4/3} = (4\pi\eta/s)^{4/9} a^{1/9} (\epsilon\tau)_0^{8/9}\ C_\infty\ \mathcal{E}(\tilde{w})\ ,$$

$$\tau^{1/3} \frac{\mathrm{d}E_\perp}{d^2\mathbf{x}_\perp \mathrm{d}\eta} = (4\pi\eta/s)^{4/9} a^{1/9} (\epsilon\tau)_0^{8/9}\ C_\infty\ f_{E_\perp}(\tilde{w})$$

- using $\epsilon = aT^4$, recast first eq. into self consistency eq. for $\tilde{w}$
- use this togehter with initial cond. for $\epsilon\tau$ to relate differentials of $\mathrm{d}\tilde{w}$ and $\mathrm{d}x_\perp$
- integrate second equation to find scaling of $\mathrm{d}E_\perp/\mathrm{d}\eta$
- use $\frac{(4\pi\eta/s)^4 a}{dE_\perp^0/d\eta\ R} = \frac{1}{\pi}\left(\frac{4\pi}{5\hat{\gamma}}\right)^4$ to identify $\hat{\gamma}$

$$\frac{dE_\perp/d\eta}{dE_\perp^0/d\eta} = \frac{9}{2}\ \left(\frac{4\pi}{5\hat{\gamma}}\right)^4\ \left(\frac{R}{\tau}\right)^3 \int_0^{\tilde{w}(\tau,\mathbf{x}_\perp=0)} \frac{\tilde{w}^3 d\tilde{w}}{\mathcal{E}(\tilde{w})}\left[1 - \frac{\tilde{w}}{4}\frac{\mathcal{E}'(\tilde{w})}{\mathcal{E}(\tilde{w})}\right]\ f_{E_\perp}(\tilde{w})\ ,$$

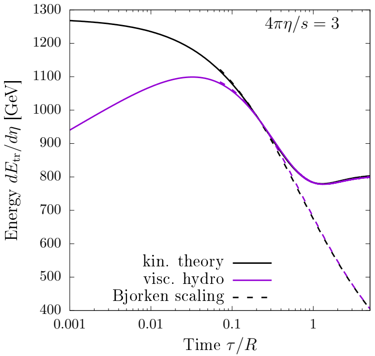
$$\tilde{w}(\tau, \mathbf{x}_\perp = 0) = \left(\frac{5\hat{\gamma}}{4\pi}\right)^{8/9} \left(\frac{\tau}{R}\right)^{2/3} [C_\infty \mathcal{E}(\tilde{w})]^{1/4}$$

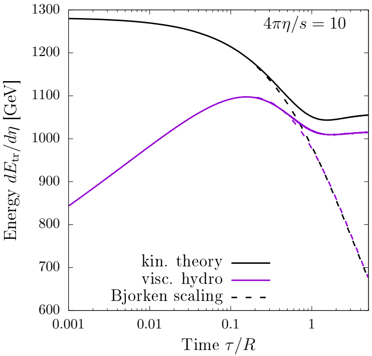
Limits of this scaling law:

- $\hat{\gamma}\left(\frac{\tau}{R}\right)^{3/4} \ll 1 \Rightarrow \tilde{w} \ll 1\ \Rightarrow\ \mathcal{E}(\tilde{w}) \approx f_{E_\perp}(\tilde{w}) \approx C_\infty^{-1}\tilde{w}^{4/9}\ \Rightarrow\ \frac{dE_\perp/d\eta}{dE_\perp^0/d\eta} = 1$
- $\hat{\gamma}^{3/4}\left(\frac{\tau}{R}\right) \gg 1\ \Rightarrow\ \tilde{w} \gg 1\ \Rightarrow\ \mathcal{E}(\tilde{w}) \approx 1, f_{E_\perp} \approx \frac{\pi}{4}$

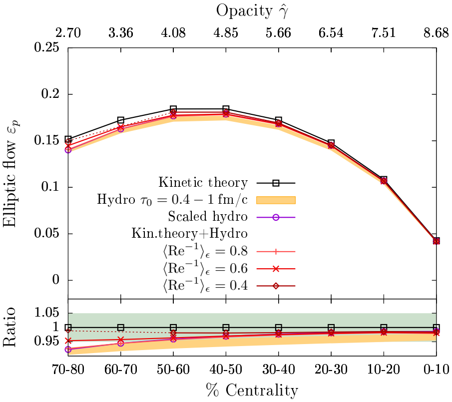
  $\Rightarrow\ \frac{dE_\perp/d\eta}{dE_\perp^0/d\eta} = \frac{9\pi}{32}\left(\frac{4\pi}{5\hat{\gamma}}\right)^{4/9}\left(\frac{R}{\tau}\right)^{1/3} C_\infty$

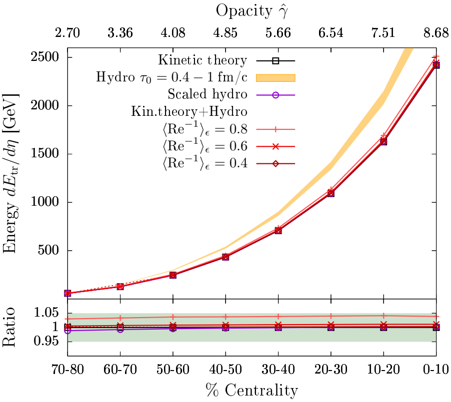
- accuracy depends on timescale separation of pre-equilibrium and transv. expansion

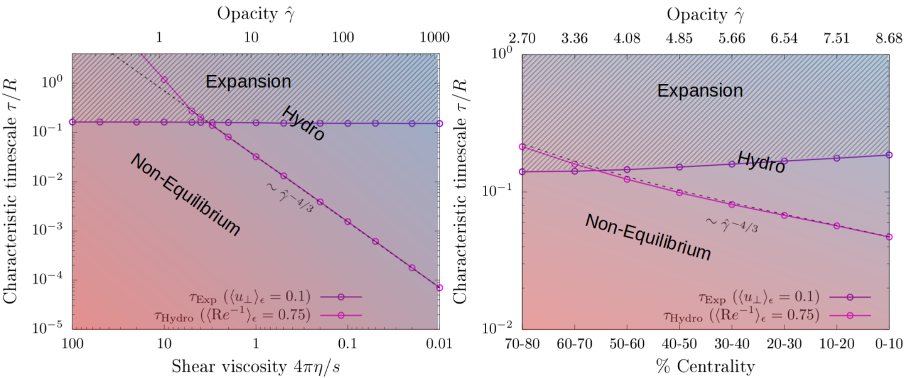




Centrality dependence







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- hydrodynamization before transv. Expansion for $\hat\gamma \gtrsim 3$