Isospin breaking in the ELσM model

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How well one can describe isospin breaking in a chiral meson model?

We use a Lagrangian that has global $U(3)_L \times U(3)_R$ chiral symmetry – like in QCD if the quark masses are zero – plus explicit symmetry breaking terms

$U(3)_L \times U(3)_R \simeq U(3)_V \times U(3)_A = SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$

$U(1)_V \longrightarrow$ baryon number conservation (exact symmetry of nature)

$U(1)_A \longrightarrow$ connected to axial anomaly

$SU(3)_A \longrightarrow$ broken down by any quark mass

$U(3)_L \times U(3)_R \longrightarrow$ broken to $U(1)_V \times SU(2)_V$ if $m_u = m_d \neq m_s$ (isospin symm.)
$\longrightarrow$ or to $U(1)_V$ if $m_u \neq m_d \neq m_s$ (realized in nature)

Particle content

- Vector and Axial-vector meson nonets

$$V^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star +} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star -} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu \quad A^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

$\rho \to \rho(770), K^\star \to K^\star(894)$
$\omega_N \to \omega(782), \omega_S \to \phi(1020)$

$a_1 \to a_1(1230), K_1 \to K_1(1270)$
$f_{1N} \to f_1(1280), f_{1S} \to f_1(1426)$

- Scalar ($\sim \bar{q}_i q_j$) and pseudoscalar ($\sim \bar{q}_i \gamma_5 q_j$) meson nonets

$$S = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_0^{\star +} \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_0^{\star 0} \\ K_0^{\star -} & \bar{K}_0^{\star 0} & \sigma_S \end{pmatrix} \quad P = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

multiple possible assignments
mixing in the $\sigma_N - \sigma_S$ sector

$\pi \to \pi(138), K \to K(495)$
mixing: $\eta_N, \eta_S \to \eta(548), \eta'(958)$

If $\zeta_{N/S/3} \neq 0 \longrightarrow$ chiral symmetry is explicitly broken,
especially if $\zeta_3 \neq 0$ – and also $\delta_3 \equiv \delta_u - \delta_d \neq 0$ – the isospin symmetry is violated
Consequently nonzero vev for scalar-isoscalar fields: $\phi_{N/S} \equiv \langle \sigma_{N/S} \rangle$ and $\phi_3 \equiv \langle a_0^0 \rangle$

$$\sigma_{N/S} \quad \to \quad \phi_{N/S} + \sigma_{N/S}, \; a_0^0 \to \phi_3 + a_0^0.$$

Different particle mixings appear:

- mixings between nonets $V^\mu \longleftrightarrow S$ and $A^\mu \longleftrightarrow P$
- $N-3$ sectors of V^μ and A^μ
- $N-3-S$ sectors of S and P

Determination of the parameters

There are 21 unknown parameters:
m_0^2, m_1^2, c_1, δ_S, δ_3, g_1, g_2, ϕ_N, ϕ_S, ϕ_3, λ_1, λ_2, h_1, h_2, h_3, $m^2_{\mathrm{em},S}$, $m^2_{\mathrm{em},P}$, $m^2_{\mathrm{em},V}$, $m^2_{\mathrm{em},A}$, δm_V^2, δm_A^2 $\longrightarrow$ determined by the min. of χ^2:

$$\chi^2(x_1,\ldots,x_N) = \sum_{i=1}^{M}\left[\frac{Q_i(x_1,\ldots,x_N) - Q_i^{\mathrm{exp}}}{\delta Q_i}\right]^2 ,$$

$(x_1,\ldots,x_N) = (m_0,\lambda_1,\lambda_2,\ldots)$, $Q_i(x_1,\ldots,x_N) \to$ calc. in the model, $Q_i^{\mathrm{exp}} \to$ PDG value,
$\delta Q_i =$ error (e.g. $\max\{5\%, \mathrm{PDG\ value}\}$) multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\to$ 2 physical quantities: f_π, f_K
- Charged and neutral masses $\to$ 24 physical quantities:
  m_{a_0}, $m_{K_0^\star}$, $m_{f_0^L}$, $m_{f_0^H}$, m_π, m_K, m_η, $m_{\eta'}$, m_ρ, $m_{K^\star}$, m_ω, m_Φ, m_{a_1}, m_{K_1}, $m_{f_1^H}$
- Charged and neutral decay widths $\to$ 21 physical quantities:
  $\Gamma_{\rho\to\pi\pi}$, $\Gamma_{\omega\to\pi\pi}$, $\Gamma_{K^\star\to K\pi}$, $\Gamma_{\Phi\to KK}$, $\Gamma_{a_1\to\rho\pi}$, $\Gamma_{a_1\to\pi\gamma}$, $\Gamma_{f_1\to K^\star K}$, $\Gamma_{K_0^\star\to K\pi}$, $\Gamma_{a_0\to KK}$, $\Gamma_{a_0\to\pi\eta}$, $\Gamma_{a_0\to\pi\eta'}$, $\Gamma_{f_0^{L/H}\to\pi\pi}$, $\Gamma_{f_0^{L/H}\to KK}$,