# The Collins-Soper kernel from lattice QCD at the physical pion mass

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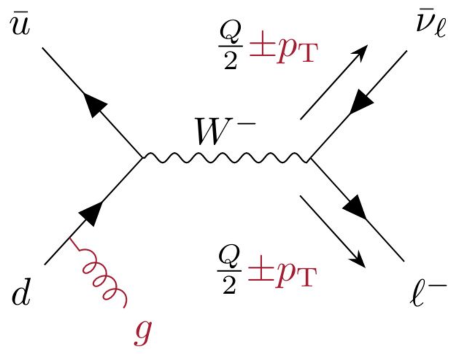
# Transverse momentum physics affects $m_W$

- Extract $m_W$ from lepton's transverse momentum in $p\bar{p} \to W^- \to \ell^- \bar{\nu}_\ell$.
- Transverse momentum physics in cross section calculations affects mass and uncertainty through template fits.



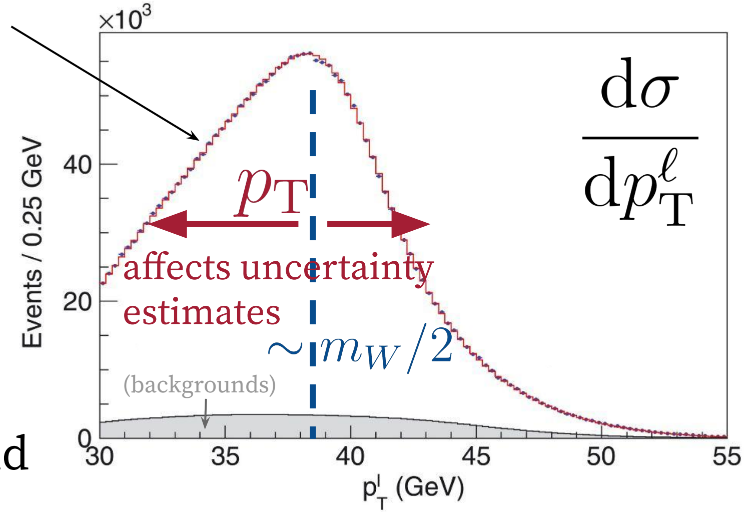




Fig. 4(B) from High-precision measurement of the W boson mass with the CDF II detector, CDF Collaboration *et al.*, *Science* **376**, 170–176 (2022) *(annotated for presentation)*

- $p_T$ due to both initial state radiation (ISR) and intrinsic $q_T$ of the quark pair (not shown).
- Intrinsic effects crucial at $q_T \sim \Lambda_{QCD}$!

# Cross-section sensitive to Collins-Soper (CS) kernel (I/III)

- Differential cross-section for $p\bar{p} \to W^- \to \ell^- \bar{\nu}_\ell$ computed by resumming large logs in

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}y\,\mathrm{d}^2\mathrm{q}_\mathrm{T}} \propto \sum_{i,\bar{j}} \underbrace{H_{i\bar{j}}(Q,\mu)}_{\text{Hard coefficient}} \underbrace{\int_0^\infty \mathrm{d}^2\mathrm{b}_\mathrm{T} e^{i\,\mathrm{q}_\mathrm{T}\cdot\mathrm{b}_\mathrm{T}}}_{\text{Fourier transform of } q_\mathrm{T} \text{ (large } b_\mathrm{T} \text{ non-perturbative!)}} \underbrace{f_{i/p}^{\mathrm{TMD}}(x_a,\mathrm{b}_\mathrm{T},\mu,\zeta_a) f_{\bar{j}/\bar{p}}^{\mathrm{TMD}}(x_b,\mathrm{b}_\mathrm{T},\mu,\zeta_b)}_{\text{Transverse-Momentum Dependent (TMD) PDFs for parton } i\,(\bar{j}) \text{ in } p\,(\bar{p})}$$

Hard coefficient

Fourier transform of $q_\mathrm{T}$ (**large** $b_\mathrm{T}$ **non-perturbative!**)

Transverse-Momentum Dependent (TMD) PDFs for parton $i\,(\bar{j})$ in $p\,(\bar{p})$

**Simple example of resummation with Renormalization Group (RG) equations:**

$\mu \sim Q$ good choice for $H_{i\bar{j}}(Q,\mu) \propto \ln Q/\mu$, but leads to $f_{i/p}^{\mathrm{TMD}} \propto \ln Q/q_\mathrm{T} \sim \ln Qb_\mathrm{T}$
$\Rightarrow$ use RG evolution $\frac{\mathrm{d}H(Q,\mu)}{\mathrm{d}\ln\mu} = \gamma_\mu^H(Q,\mu)$ to evolve $H$ from $H(Q,Q)$ to $\mu \sim q_\mathrm{T}$, resumming the large logs.

Similarly, make use of small $b_\mathrm{T} = b^*$ to match TMDs onto collinear PDFs and use RG equations for TMDs to resum large logs (next slide)

# Cross-section sensitive to Collins-Soper (CS) kernel (II/III)

- Resummed differential cross-section for $p\bar{p} \to W^- \to \ell^- \bar{\nu}_\ell$ depends on TMD physics in particular through the Collins-Soper kernel:

Kernel defined by an **RG equation** for **TMD PDFs**:

$$\frac{\mathrm{d} f_{i/p}^{\mathrm{TMD}}(x, b_\mathrm{T}, \mu, \zeta)}{\mathrm{d} \ln \zeta} = \frac{1}{2} \gamma_\zeta^i (b_\mathrm{T}, \mu)$$

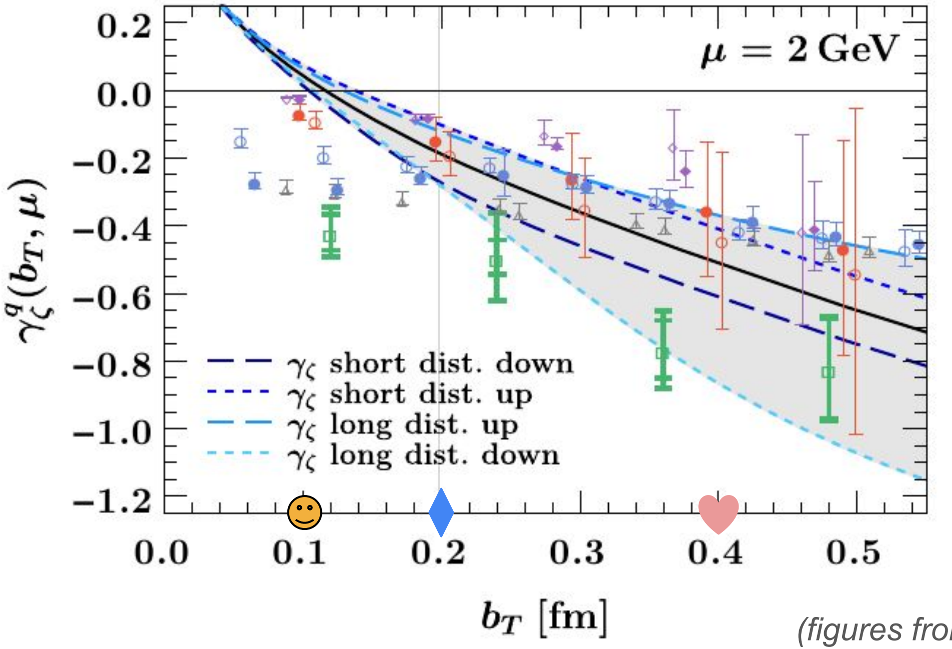
collinear PDFs matched at $b_\mathrm{T} = b^*$.

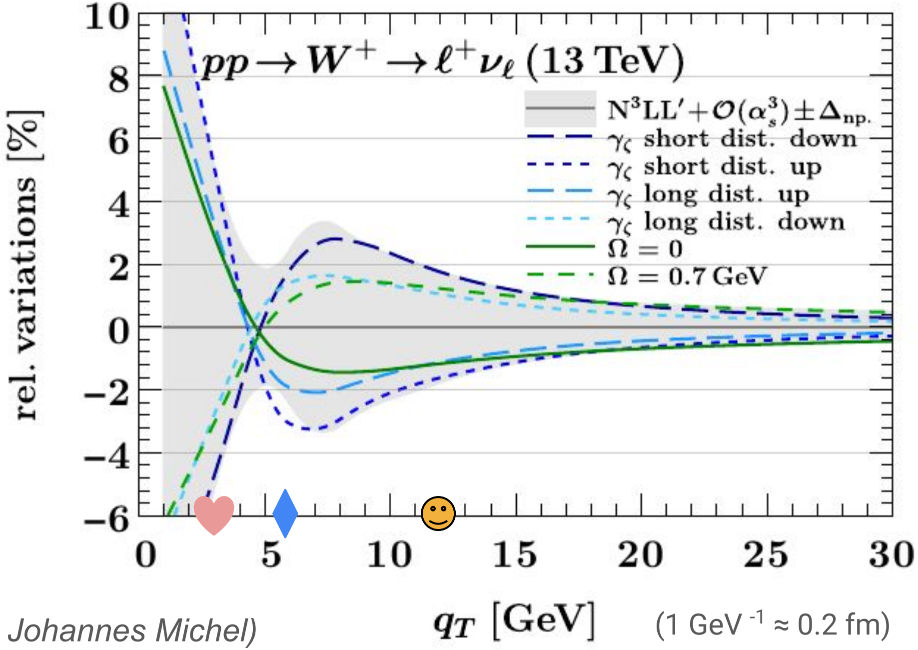
$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\, \mathrm{d}y\, \mathrm{d}^2 \mathrm{q_T}} \propto \sum_{i,\bar{j}} H_{ij} \int_0^\infty \mathrm{d}^2 \mathrm{b_T} e^{i\, \mathbf{q}_\mathrm{T} \cdot \mathbf{b}_\mathrm{T}} e^{-\mathcal{S}(Q, b_\mathrm{T})} \left( C_{ii'} \otimes f_{i'/p}^{\mathrm{coll.}} \right) \left( C_{\bar{j}\bar{j}'} \otimes f_{\bar{j}'/\bar{p}}^{\mathrm{coll.}} \right)$$

Non-perturbative piece of the Sudakov factor $\mathcal{S}(Q, b_\mathrm{T})$ sensitive to large $b_\mathrm{T}$ behavior of $\gamma_\zeta$ (from resummation)

# Cross-section sensitive to (CS) Collins-Soper kernel (III/III)

Short- and long-distance behavior in the model $\hat{\gamma}_\zeta^{q\,\mathrm{NP}}$ is varied by a reasonable amount (for the long-distance piece, to accommodate the spread in the lattice data) **⇒ <u>rel. variations in the cross section</u>**.



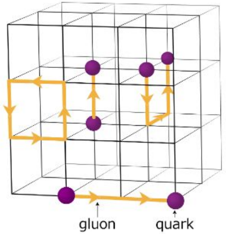




*(figures from Johannes Michel)*

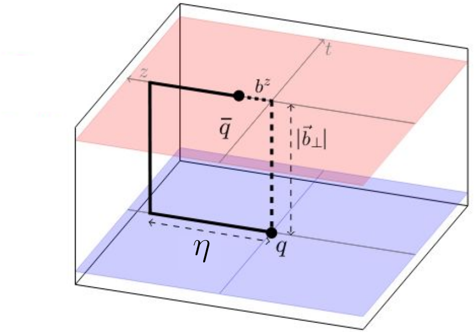
(1 GeV $^{-1}$ ≈ 0.2 fm)

# CS Kernel from Lattice QCD

Calculations on lattice field configurations with
- Different discretizations of QCD action
- Different lattice spacings
- Quark masses often larger than in nature




- QCD in discretized Euclidean space-time
  $\Rightarrow$ **can compute space-like QCD matrix elements (MEs) in position space.**
- CS kernel defined through ratios of **light-like MEs in momentum space** $\Rightarrow$ need Fourier transform and matching between space-like and light-like MEs.




Lorentz boost and $\eta \to \infty$

$$\frac{1}{P} \ll b_\mathrm{T} \ll \eta$$

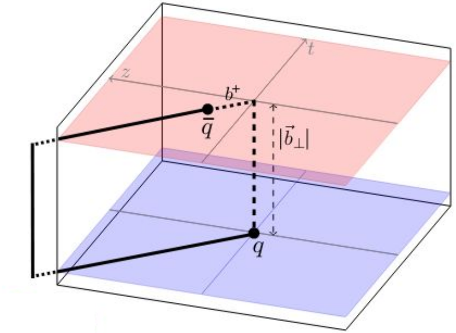



Fig. by Ebert, Stewart, Zhao, JHEP 1909 (2019) (notation changed).

Ji, Sun, Xiong and Yuan, PRD91 (2015);
Ji, Jin, Yuan, Zhang and Zhao, PRD99 (2019);
Ebert, Stewart, Zhao, PRD99 (2019), JHEP09 (2019) 037;
Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
Vladimirov and Schäfer, PRD 101 (2020);
Ebert, Schindler, Stewart and Zhao, JHEP04 (2022) 178.

# CS kernel from lattice QCD

With bare matrix elements computed, their ratio yields the CS kernel after:

1. Renormalization;
2. Fourier Transform;
3. Perturbative Matching.

**Systematic uncertainties associated with each step.**

$$\gamma_\zeta^{q\,\overline{\mathrm{MS}}}(b_T,\mu) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{\int db^z e^{ixP_1^z} P_1^z \lim_{\eta\to\infty} \tilde{\phi}^{[\Gamma],\overline{\mathrm{MS}}}(b^z, b_T, P_1^z, \eta, a)}{\int db^z e^{ixP_2^z} P_2^z \lim_{\eta\to\infty} \tilde{\phi}^{[\Gamma],\overline{\mathrm{MS}}}(b^z, b_T, P_2^z, \eta, a)}\right] - \frac{d}{d\ln P^z} H_q^{\overline{\mathrm{MS}}}(xP^z,\mu)\Big|_{P^z=\sqrt{P_1^z P_2^z}} + \mathcal{O}\left(\frac{1}{(xP^z)^2 b_T^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(xP^z)^2}\right)$$

Matching to light-cone MEs up to power corrections

# Systematic uncertainties in analyses are significant

Same lattice QCD data with different treatments of matching, Fourier transform and renormalization leads to significant systematic effects and changes in uncertainty estimates.

Lattice calculations quickly evolving from proof-of-concept to increasing precision and control over systematic uncertainties.

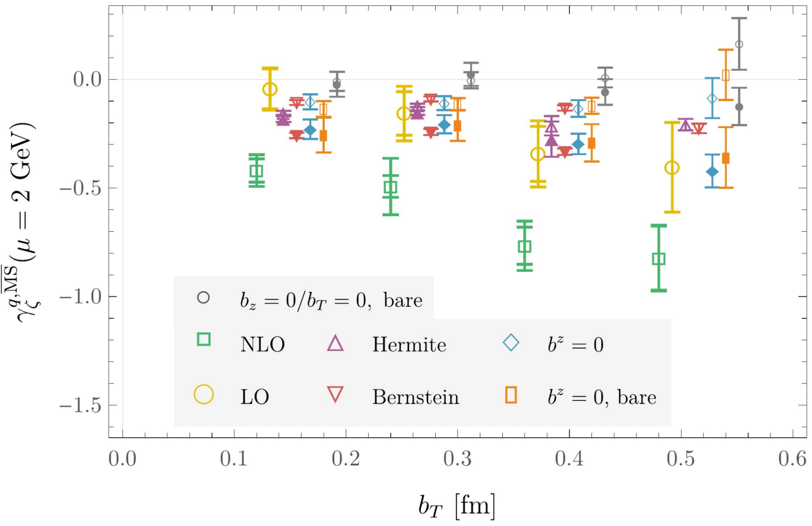



Fig. from Shanahan, Wagman, Zhao, Phys.Rev.D 104 (2021), 2107.11930

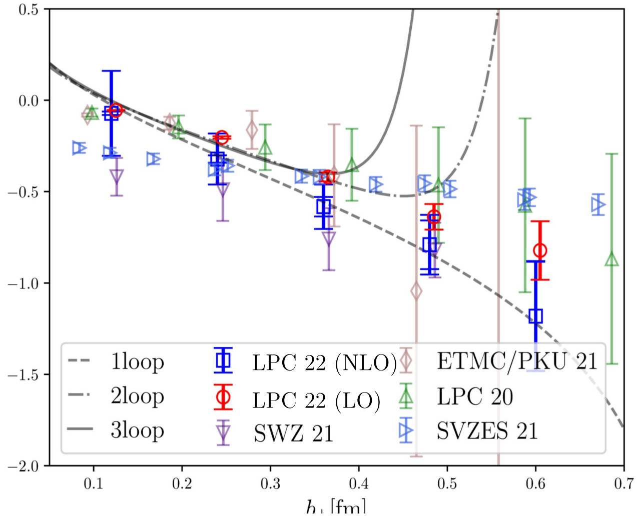
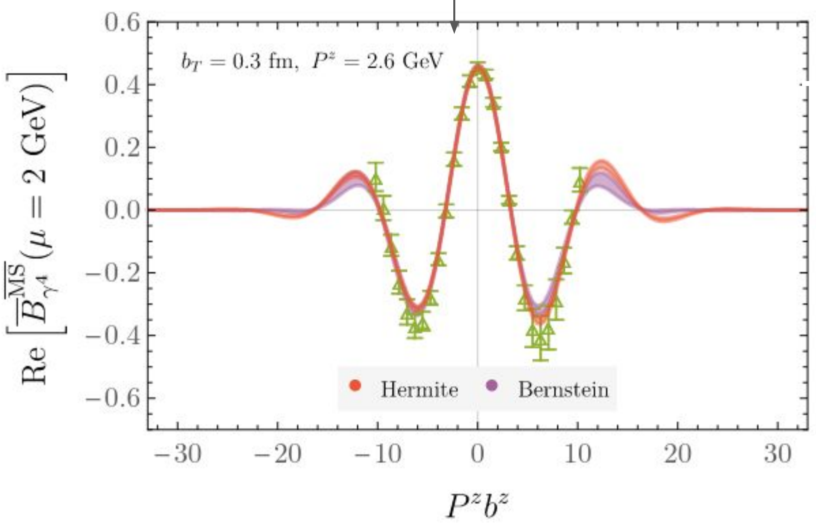



Fig. from LPC Collaboration, 2204.00200
[axes and legend labels modified]

# From proof-of-principle toward controlled calculations

Proof-of-concept calculations have had:

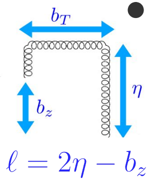
- larger-than-physical pion mass
- no attempt to control for lattice spacing effects
- limited data range ⇒ truncation errors + models used to enable fourier transform

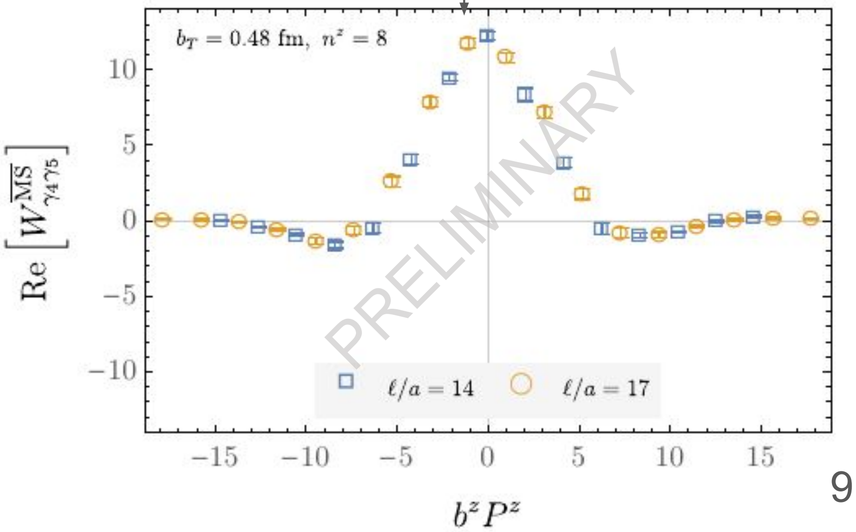



Shanahan, Wagman, Zhao, Phys.Rev.D 102 (2020), 2003.06063

State-of-the-art calculation in progress to enable **first controlled lattice QCD calculation of CS kernel** with

- physical pion mass
- multiple calculations with different lattice spacings to enable uncertainty quantification (to be done...)
- data rage that enables discrete fourier transform



# Preliminary results

Previous proof-of-principle (systematics not quantified):

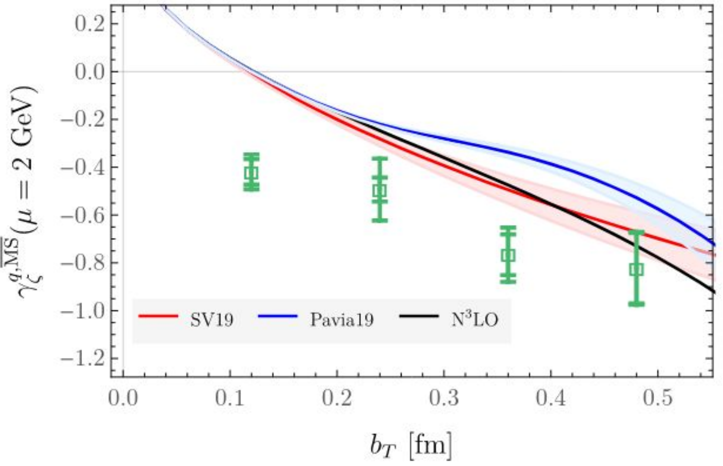



Fig. from Shanahan, Wagman, Zhao, Phys.Rev.D 104 (2021), 2107.11930

Preliminary first calculation with systematics controlled and quantified: