Accessing non-perturbative TMDs from the lattice

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Collaborators:
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Support:

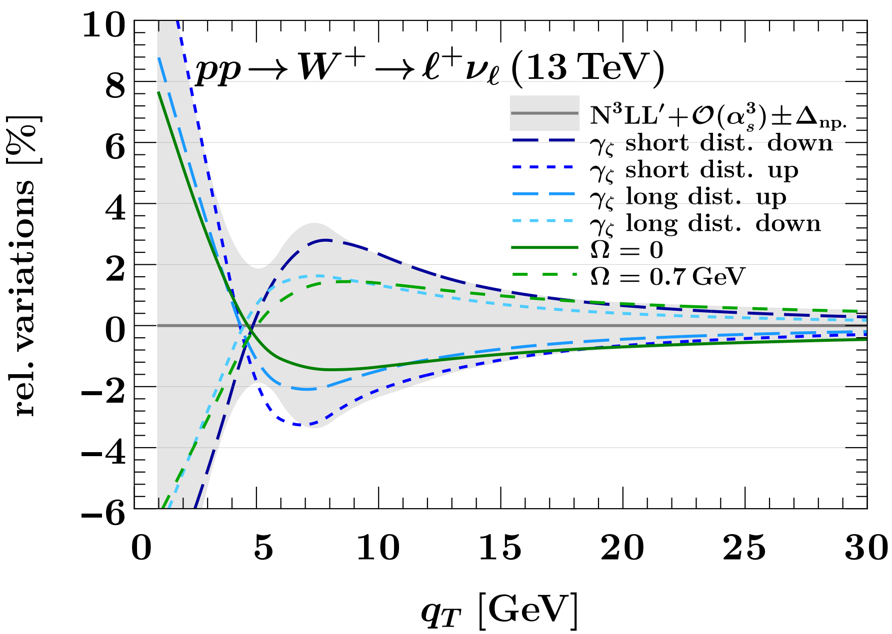
Based on:
2004.14831
2201.08401
2205.12369

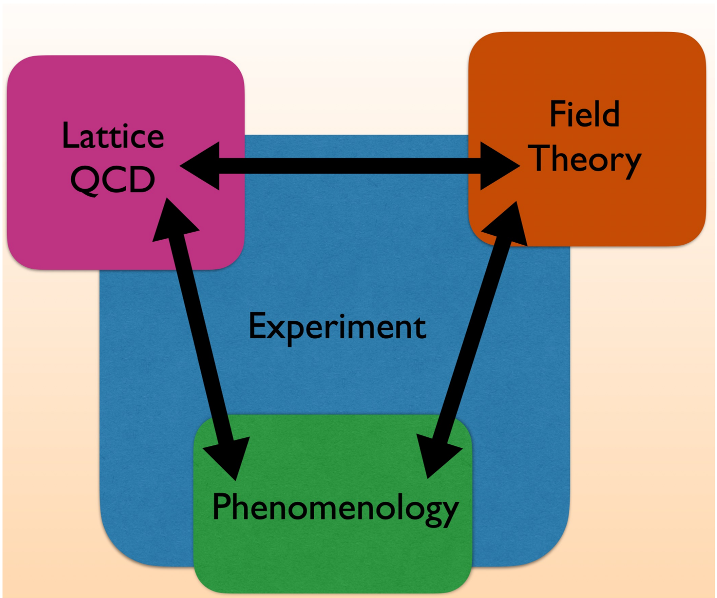
Motivation

ATLAS: $m_W = 80{,}370$ MeV $\pm$ 7 (stat) **$\pm$ 14 (mod syst)** $\pm$ 11 (exp syst)

CDF: $m_W = 80{,}435$ MeV $\pm$ 6 (stat) **$\pm$ 7 (syst)**

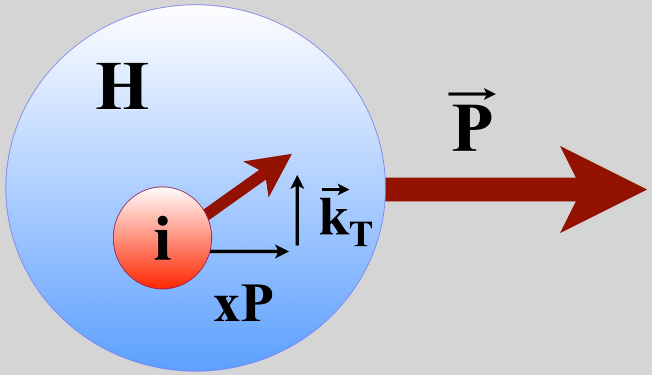
Effect of non-perturbative TMDs? Flavor dependence?





Bacchetta, Bozzi, et al., 1807.02101. Figure: J. Michel.

Transverse momentum distributions (TMDs)



- 3D distribution: quark & gluon momenta in proton
- Universal: SIDIS, DY, weak boson production, …

Collins-Soper evolution kernel

- Bands: Global fits
- Dots: Early lattice data

See Arthur's talk @ 1:45

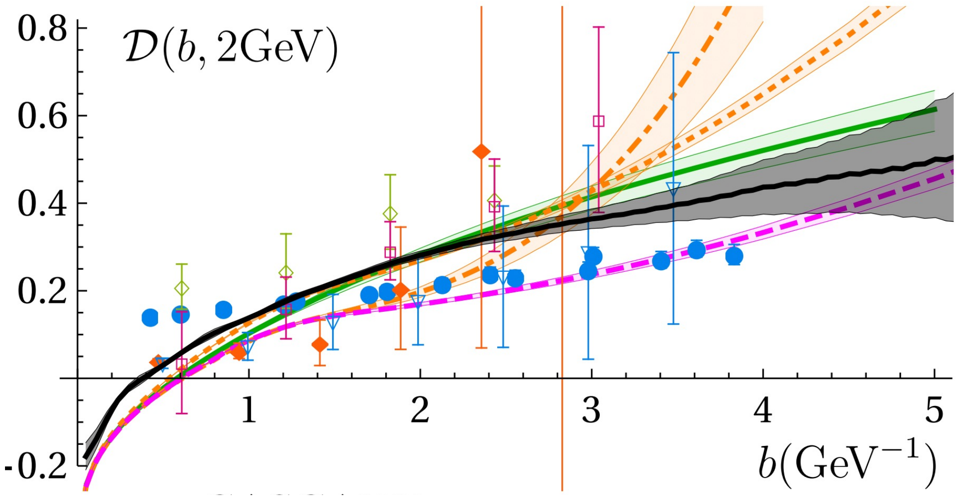
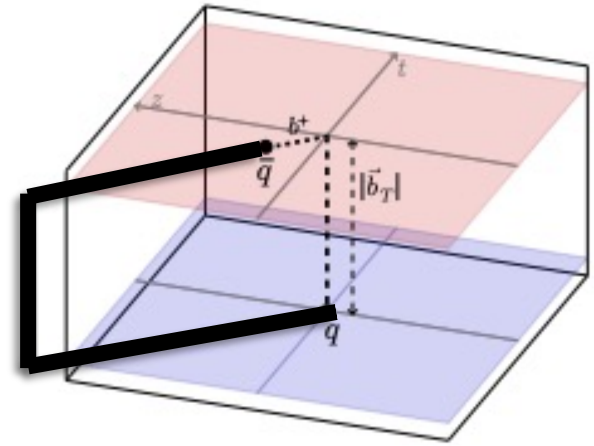


Figure: Martinez & Vladimirov, 2206.01105.

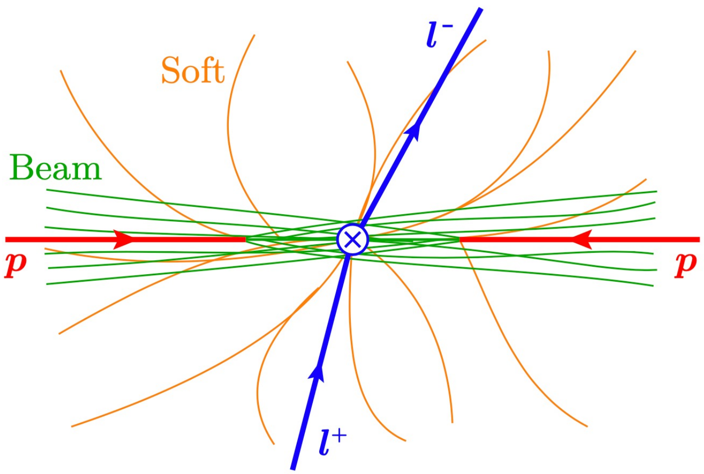
Defining TMDs in QFT

$$f = \lim_{\substack{\text{lightcone,} \\ \text{renormalization}}} Z_{UV} \frac{B^{[\Gamma]}_{q_i/H}}{\sqrt{S^R}}$$

$\langle P|$  $|P\rangle$

Beam function: hadronic matrix element

Soft factor: vacuum matrix element.



QCD analogue of a parton:

Quark field attached to *lightcone* Wilson line

Collins, *Foundations of Perturbative QCD* (2011).

TMDs from lattice QCD

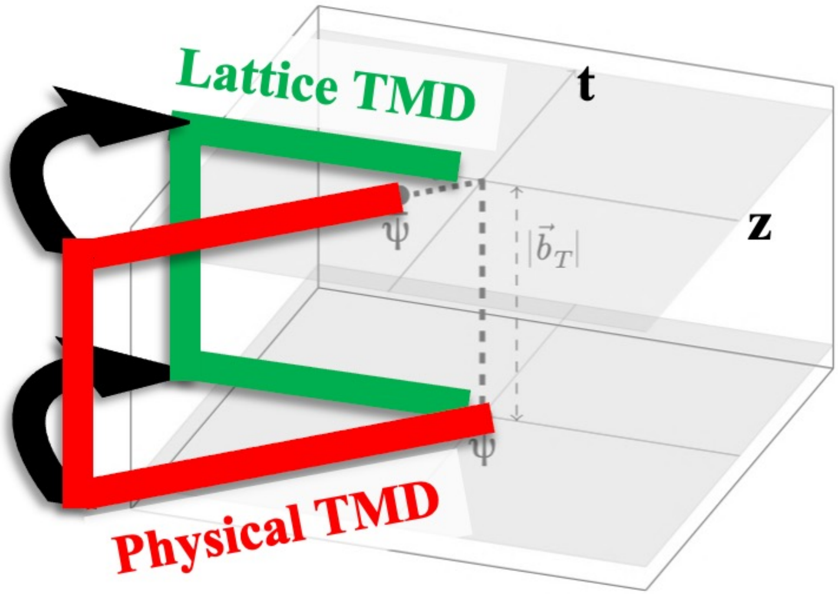
General premise:

- Discretize QFT to regulate divergences
- Use Monte Carlo to compute correlators
- Only known systematically improvable numerical approach to QCD

Challenge: Lattice sign problem

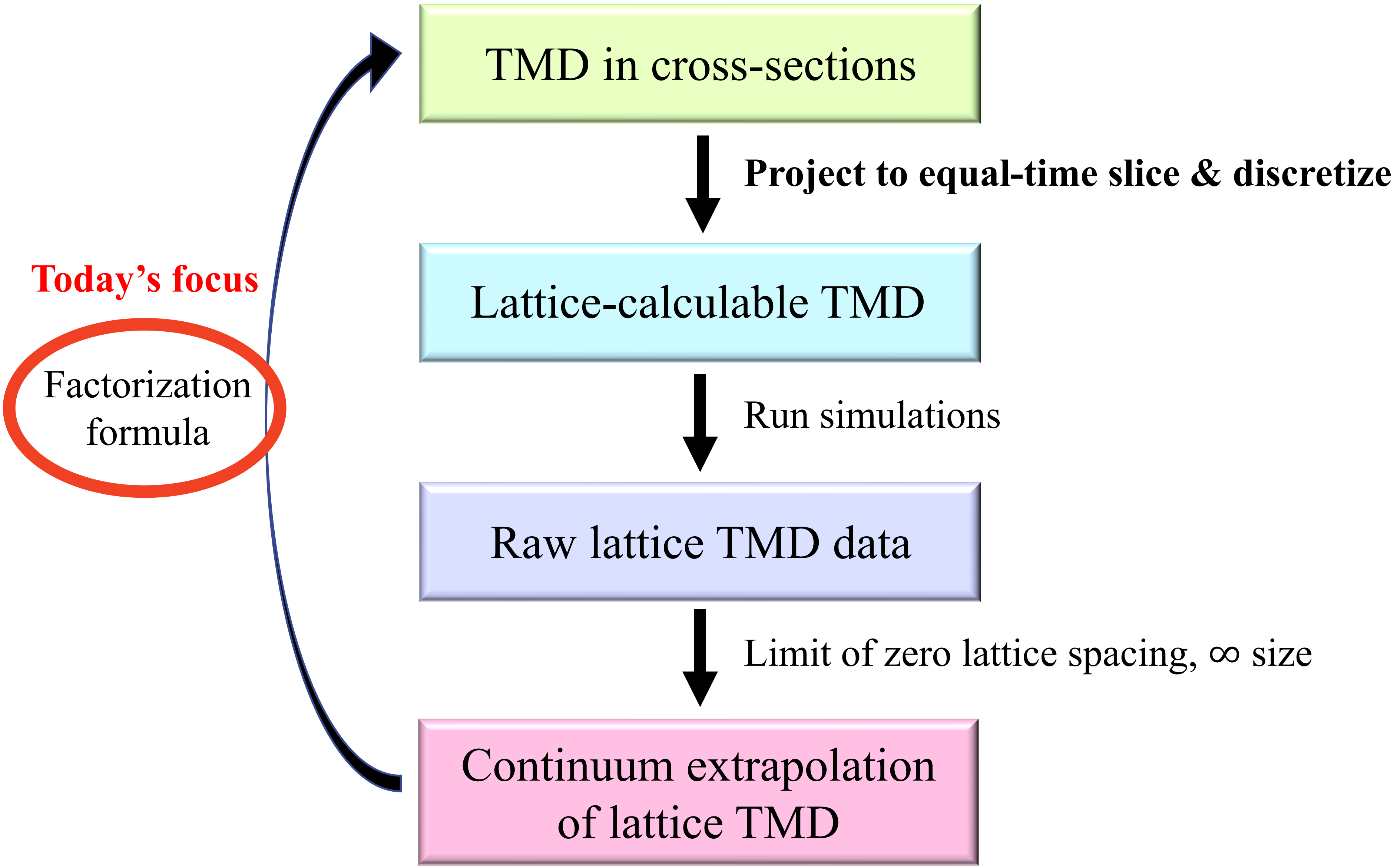
Lightcone Wilson lines: Hard!

Unphysical equal-time paths: OK

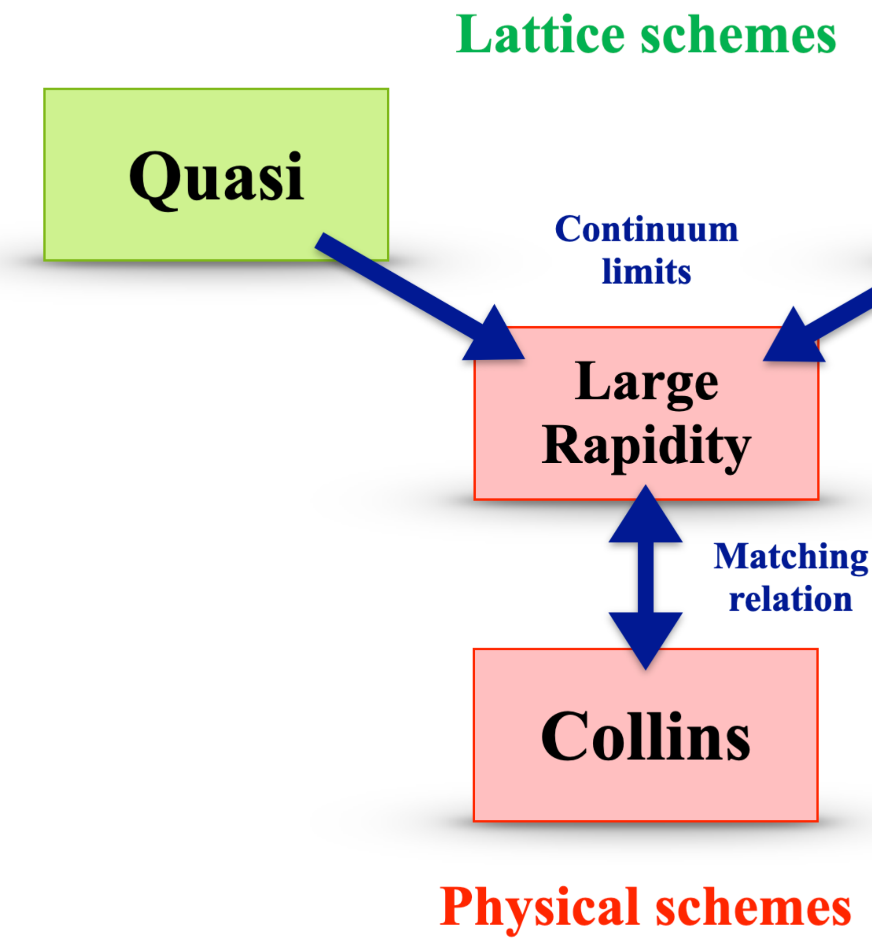


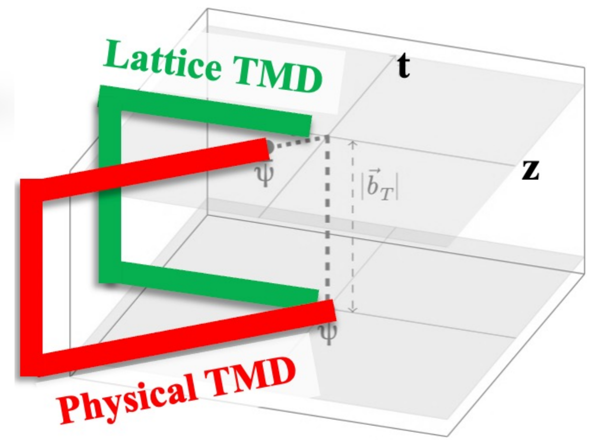
Musch, Hägler, Engelhardt, Negele, Schafer, 1111.4249. Ji, 1305.1539.

Recipe for TMDs on the lattice



Relating TMD schemes





Differentiate TMD schemes by:

- Path of Wilson lines / boost
- Infinite vs. finite length
- Renormalization

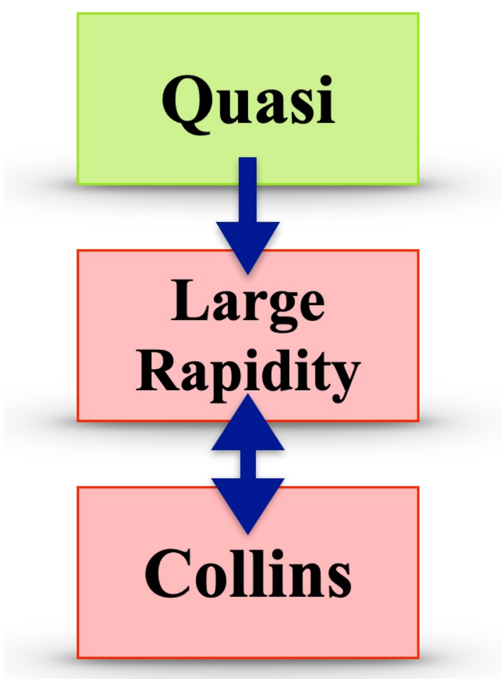
Ebert, **Schindler**, Stewart, and Zhao, 2201.08401.

Proving factorization at all orders in α_s

Collins-Soper (CS) kernel

$$\tilde{f}^{[s]}_{i/H}\left(x, \vec{b}_T, \mu, \tilde{\zeta}, xP^z\right) = C_i\left(xP^z, \mu\right) \exp\left[\frac{1}{2}\gamma^i_\zeta(\mu, b_T) \ln\frac{\tilde{\zeta}}{\zeta}\right] f^{[s]}_{i/H}\left(x, \vec{b}_T, \mu, \zeta\right)$$

Lattice



Continuum

Step 1: same at large rapidity $P^z \gg \Lambda_{QCD}$

- Expand & relate their parameters
- Take Wilson line length $|\eta| \to \infty$

Step 2: need a matching coefficient

- Different UV renormalizations
- Nontrivial relationship

Ebert, **Schindler**, Stewart, and Zhao, 2201.08401.

What is the matching coefficient?

$$\tilde{f}_{i/H}^{[s]}\left(x, \vec{b}_T, \mu, \tilde{\zeta}, \mathrm{x}\tilde{P}^z\right) = \boldsymbol{C_i\left(x\tilde{P}^z, \mu\right)} \exp\left[\frac{\gamma_\zeta^i}{2} \ln\frac{\tilde{\zeta}}{\zeta}\right] f_{i/H}^{[s]}\left(x, \vec{b}_T, \mu, \zeta\right)$$

Convenient properties:

- Independent of spin
- No quark-gluon or flavor mixing
- Known at one-loop & logarithmic terms

Ebert, **Schindler**, Stewart, and Zhao, 2004.14831 & 2201.08401. **Schindler**, Stewart, and Zhao, 2205.12369.

Straightforward access to flavor ratios

Ratios of different TMD spins, flavors, or hadrons can be calculated directly from lattice beam functions:

$$\lim_{\tilde{\eta}\to\infty} \frac{f_{q_i/h}^{[\tilde{\Gamma}_1]}}{f_{q_j/h'}^{[\tilde{\Gamma}_2]}} = \lim_{\tilde{\eta}\to\infty} \frac{\tilde{B}_{q_i/h}^{[\tilde{\Gamma}_1]}}{\tilde{B}_{q_j/h'}^{[\tilde{\Gamma}_2]}}$$

This follows from the factorization formulas:

$$C_i \exp\left[\frac{1}{2}\gamma_\zeta^i \ln\frac{\tilde{\zeta}}{\zeta}\right] f_{q_i/H}^{[\Gamma]} \;=\; \tilde{f}_{q_i/H}^{[\Gamma]} \;=\; \lim Z_{UV} \frac{\tilde{B}_{q_i/H}^{[\Gamma]}}{\sqrt{S^R}}$$

Lattice-to-continuum TMD factorization

Factorization of a lattice TMD into matrix elements

Physics impact

Factorization formula enables many **lattice calculations**:

- **CS kernel**
- Spin-dependent TMD ratios
- **Flavor ratios**
- Gluon TMDs
- Proton-pion TMD ratios
- Normalized TMD
- Etc.

The upshot: strong synergy between experiment, lattice, and EFT.