Asymptotically Free UV Completions of Higgs Models

Luca Zambelli (INFN-Bologna)

talk based on:
(Gies, LZ '15 and '16) (Ugolotti, Sondenheimer, Gies, LZ '18 and '19)
and work in progress

Asymptotic safety meets particle physics & friends
DESY, Hamburg 13-15/12/2022

Fellowship for Innovation at INFN

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Motivations

New Physics Beyond the Standard Model (SM)

Understanding built-in theoretical issues

- naturalness
  gauge hierarchy, flavor hierarchy, strong CP, cosmological constant
  ...
- effectiveness
  locating the scale of new physics, assessing compositeness, questioning locality, enhancing predictivity

General Understanding of Quantum Field Theories *per se*

$$\Downarrow$$

TRIVIALITY

Motivations

TRIVIALITY PURSUIT: CAN ELEMENTARY SCALAR PARTICLES EXIST?

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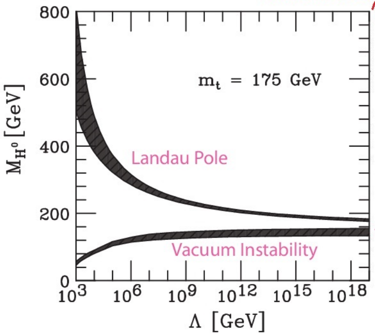
Received February 1988

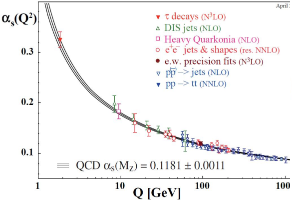


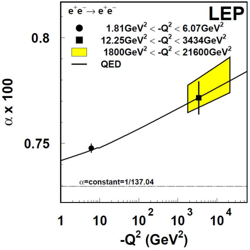
ASYMPTOTIC SAFETY

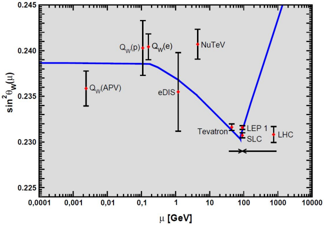
ASYMPTOTIC FREEDOM

NATURALNESS

Motivations

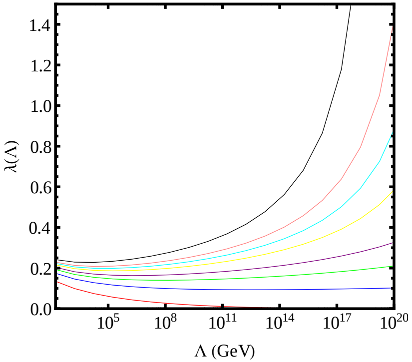




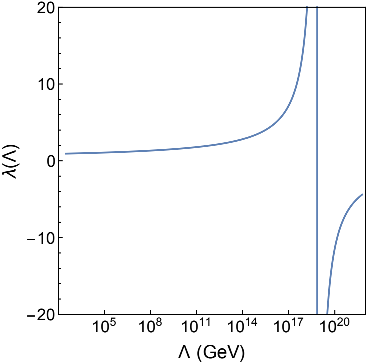


RG Flow of α_s, α and $\sin^2\theta_W = \frac{g'^2}{g'^2+g^2}$ in the SM (Particle Data Group)

RG Flow of λ in the Standard Model



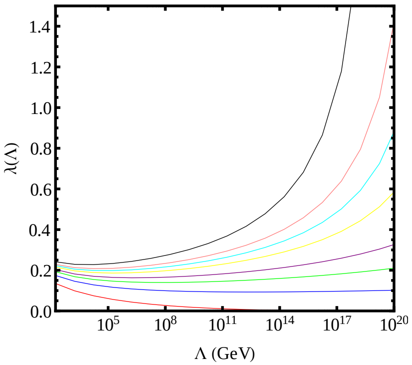
(Holthausen, Lim, Lindner '12)



Landau pole

Light Higgs = almost vanishing self-interaction to high scales

RG Flow of λ in the Standard Model



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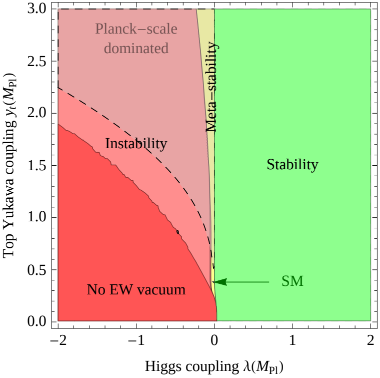


(Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia '13)

Light Higgs = almost vanishing self-interaction to high scales

Total Asymptotic Freedom (AF): known facts

E.g. one-loop RG of the non-Abelian Higgs model

$$\beta_{g^2} = -b_0 g^4 \ , \quad \beta_{\xi_2} = g^2(A\xi_2^2 + B\xi_2 + C), \quad \xi_2 = \frac{\lambda}{g^2}$$

AF condition: $\Delta = B^2 - 4AC > 0$
then UV asymptotics = RG fixed point ξ_{2*} (Gross, Wilczek '73)

Perturbatively renormalizable AF models have been classified:

(Cheng, Eichten, Li '74) (Chang '74) (Fradkin, Kalashnikov '75)
(Chang, Perez-Mercader '78) (Bais, Weldon '78) (Callaway '88)
(Giudice, Isidori, Salvio, Strumia, '15) (Holdom, Ren, Zhang '15)

strong constraints on matter content and symmetries!

no AF in the SM, guiding principle for BSM

Total Asymptotic Freedom (AF): new results

How close can AF be to the SM?

NEW! AF is possible already in the generic nonabelian Higgs model
&
in generic non-Abelian Higgs-Yukawa models

Where is the loophole?

NEW! Higher-dimensional operators are needed
&
AF is realized OUT of the Deep Euclidean Limit

The Higgs VEV scales like the RG scale in the UV: $v^2 \sim k^2$

Non-Abelian Higgs Model & Higher-Dimensional Operators

Functional RG accounts for generic local effective potential

EFT-like analysis straightforward = polynomial truncations

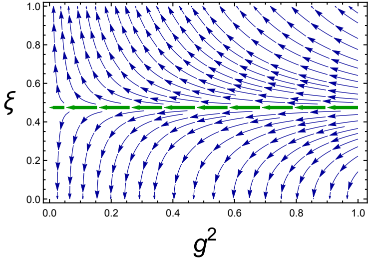
$$U = \frac{\lambda}{2}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2 + \frac{\lambda_3}{6k^2}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^3 + \dots$$

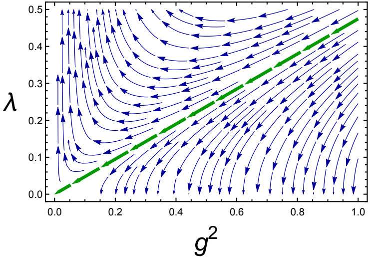
to lowest order, in a mass-dependent scheme

$$\beta_{\xi_2} = g^2\left(A\xi_2^2 + B\xi_2 + C\right) - \frac{1}{g^2}\left(\frac{\lambda_3}{16\pi^2} + \frac{9\lambda_3}{64\pi^2\xi_2}\right) + \dots$$

Fixed points at $\xi_2 > 0$ if $\lambda_3/g^4 = \chi$ is kept free and nonvanishing

Non-Abelian Higgs Model & Higher-Dimensional Operators





Fixed points at $\xi_2 > 0$ if $\lambda_3/g^4 = \chi$ is kept free and nonvanishing

Line of fixed points

$\lambda/g^2 = \xi_2, \quad \lambda_3/g^4 = \chi, \quad v^2/k^2 = 2\kappa$ finite and nonvanishing

Higher-dimensional operators suppressed by higher powers of g^2

Rescale: $x = (g Z_\phi/k^2)\, \phi^\dagger \phi\,, \quad f(x) = k^{-4} U$

Weak-g^2 expansion:

$$\partial_t f(x) = -4\, f(x) + 2x f'(x) - g \left(\frac{9x}{64\pi^2} + \frac{f'(x)}{8\pi^2} + \frac{x f''(x)}{16\pi^2} \right) + O(g^2)$$

Fixed-point condition = linear ODE

Line of fixed points

$\lambda/g^2 = \xi_2, \quad \lambda_3/g^4 = \chi, \quad v^2/k^2 = 2\kappa$ finite and nonvanishing

Higher-dimensional operators suppressed by higher powers of g^2

Rescale: $x = (g Z_\phi/k^2)\ \phi^\dagger\phi\ , \qquad f(x) = k^{-4} U$

Weak-g^2 expansion:

$$f(x) \sim \frac{\xi_2}{2}\ x^2 - \frac{3(3+4\xi_2)}{128\pi^2} g\ x + O(g^2)$$

Stable potential!

One free parameter (here ξ_2) = boundary condition of a linear ODE

= ambiguity in resumming an infinite number of vertices

Generalization: $\lambda \sim g^{4P}$

Back to

$$\beta_\lambda = A\lambda^2 + B'\lambda g^2 + Cg^4 - \left(\frac{\lambda_3}{16\pi^2} + \frac{9\lambda_3 g^2}{64\pi^2\lambda}\right) + \ldots$$

look for scaling solutions: $\lambda = g^{4P}\xi_2$

if $0 < P < 1/2$ and $\lambda_3 = g^{8P}\chi$ then

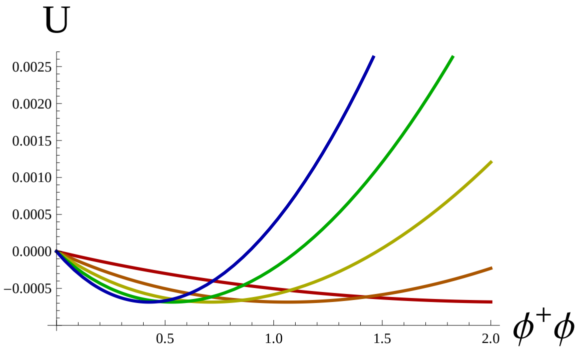
$$\beta_{\xi_2} = \left(A\xi_2^2 - \frac{\chi}{16\pi^2}\right) g^{4P} + O(g^2)$$

Fixed points at $\xi_2 > 0$ if χ is kept free and nonvanishing

Generalization: $\lambda \sim g^{4P}$

$P > 1/2$ is also possible but requires $v^2/k^2 = 2\kappa \to +\infty$ in the UV

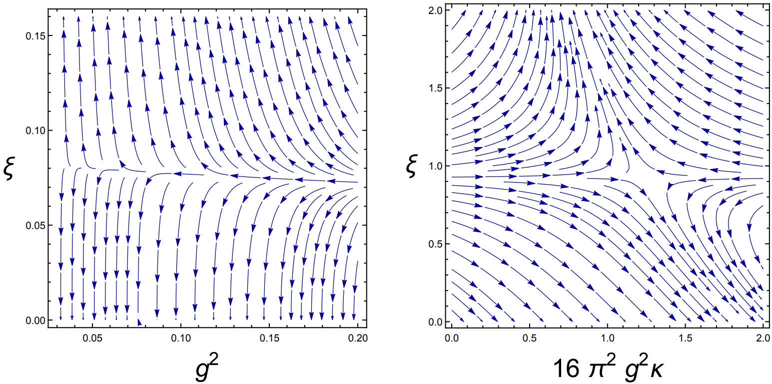
$P = 1$



Relevant – Irrelevant

For all solutions: $\kappa =$ relevant, $\lambda =$ marginally irrelevant

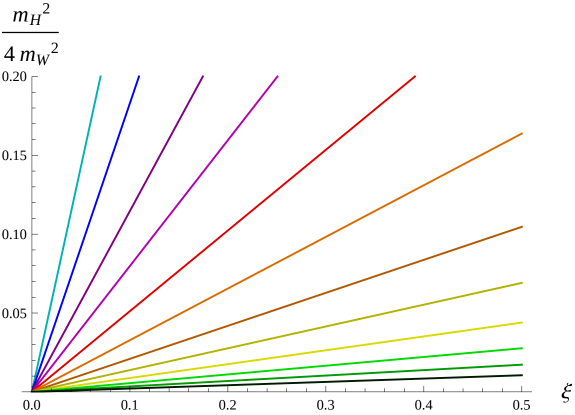
$P = 1$



Higgs Phase and Masses

Every AF scaling solution can be connected to the Higgs phase

- by choosing compatible scale-dependent boundary conditions
- by adding a suitable relevant component



Including the Yukawa(s)

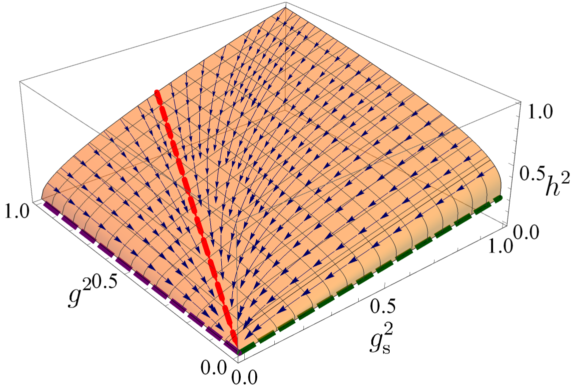
The $SU(2)_L \otimes SU(3)_c$ sector of the SM

$$S_{\text{cl}} = \int d^4x \Big[\frac{1}{4} F_{i\mu\nu} F_i^{\mu\nu} + \frac{1}{4} G_{I\mu\nu} G_I^{\mu\nu} + (D_\mu \phi)^{\dagger a} (D^\mu \phi)^a$$
$$+ \bar{m}^2 \phi^\dagger \phi + \frac{\bar{\lambda}}{2} \left(\phi^\dagger \phi\right)^2 + \bar{\psi}_{\mathrm{L}}^{aA} \mathrm{i} \not{D}^{abAB} \psi_{\mathrm{L}}^{bB} + \bar{\psi}_{\mathrm{R}}^{A} \mathrm{i} \not{D}^{AB} \psi_{\mathrm{R}}^{B}$$
$$+ \mathrm{i} \bar{h} (\bar{\psi}_{\mathrm{L}}^{aA} \phi^a \psi_{\mathrm{R}}^{A} + \bar{\psi}_{\mathrm{R}}^{A} \phi^{\dagger a} \psi_{\mathrm{L}}^{aA}) \Big]$$

Non-Abelian Higgs-Yukawa models

The $SU(2)_L \otimes SU(3)_c$ sector of the SM

Phase diagram of the perturbatively renormalizable model

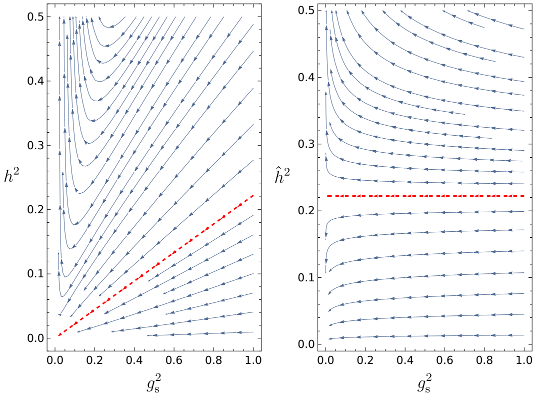


$\mathbb{Z}_2$-Yukawa-QCD (Perturbatively Renormalizable)

Set $g^2 = 0$ and define

$$\hat{h}^2 = \frac{h^2}{g_\mathrm{s}^2}$$

$$\partial_t \hat{h}^2 = \frac{3 + 2N_\mathrm{c}}{16\pi^2} g_\mathrm{s}^2 \hat{h}^2 \left(\hat{h}^2 - \chi_\mathrm{s}^2\right)$$



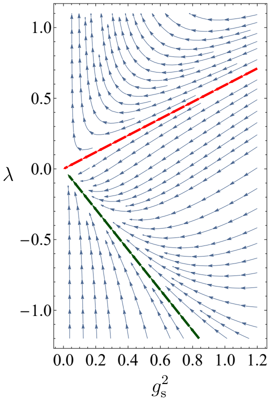
$\mathbb{Z}_2$-Yukawa-QCD (Perturbatively Renormalizable)

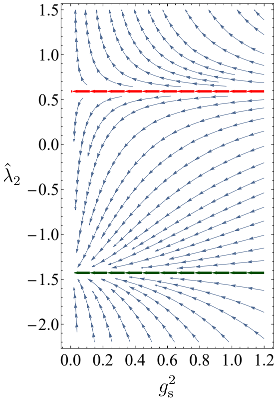
Set $g^2 = 0$ and define

$$\hat{\lambda}_2 = \frac{\lambda}{g_s^{4P}}, \quad P = 1/2$$

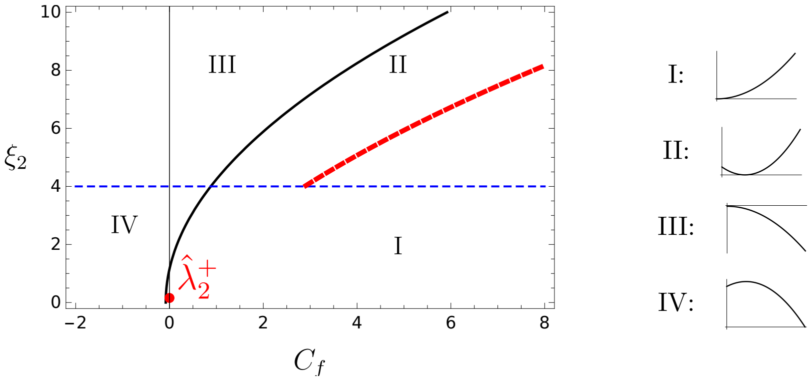
$$\hat{\lambda}_2^{\pm} = \frac{1}{6}\left(-25 \pm \sqrt{673}\right)$$

(Cheng, Eichten, Li '74)





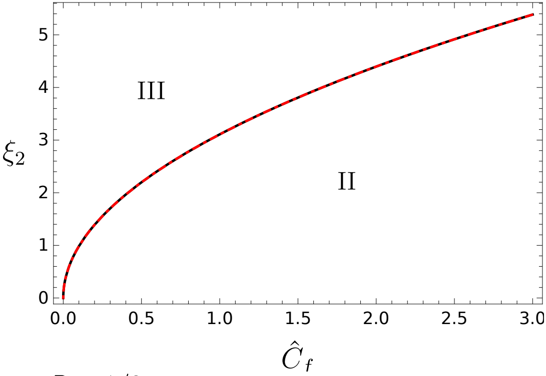
$\mathbb{Z}_2$-Yukawa-QCD (Functional Approach)

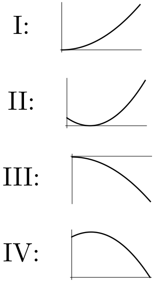


$P = 1/2$:

- The Cheng-Eichten-Li solution is stable
- Also a new line of fixed points, stable and with a nontrivial minimum

$\mathbb{Z}_2$-Yukawa-QCD (Functional Approach)





$1/4 \leq P < 1/2$:

- Only a new line of fixed points, stable and with a nontrivial minimum

Overview in Mass-Dependent Schemes (FRG)

	$SU(2)_L \times SU(3)_c$	$\mathbb{Z}_2$-Yukawa-QCD	Non-Abelian Higgs
FRG	$P \in \left[\frac{1}{4}, \frac{1}{2}\right]$, ξ_2	$P \in \left[\frac{1}{4}, \frac{1}{2}\right]$, ξ_2	$P \in (0, +\infty)$, ξ_2

Constructed by:

- EFT-like (polynomial) approximations
- ϕ^4-dominance approximation
- Weak-coupling functional expansions
- Large-N approximations
- Numerical integration of QFP equations

$\overline{\text{MS}}$ Scheme?

$$\beta_f = -4f + d_x x f' + \frac{1}{16\pi^2}\left\{ l_0^{(\mathrm{H})}(z_\mathrm{H}) + 3 l_0^{(\theta)}(z_\theta) + 9 l_0^{(\mathrm{W})}(z_\mathrm{W}) - 12 l_0^{(\mathrm{F})}(z_\mathrm{F}) \right\}$$

where

$$x = g_\mathrm{s}^{2P}\phi^\dagger\phi$$

$$z_\mathrm{H} = g_\mathrm{s}^{2P}(f' + 2xf''), \qquad z_\theta = g_\mathrm{s}^{2P} f'$$

$$z_\mathrm{F} = \hat{h}_*^2 g_\mathrm{s}^{2-2P} x, \qquad z_\mathrm{W} = \hat{g}_*^2 g_\mathrm{s}^{2-2P}\frac{x}{2}$$

For $\overline{\text{MS}}$: $l_0(\omega) = \frac{\omega^2}{2}$

$\overline{\text{MS}}$ Scheme?

A crucial role is played by

$$\mathcal{A}_\Phi = -\frac{1}{16\pi^2}\left[\partial_z l_0^{(\Phi)}(z)\right]_{z=0}$$

with $\Phi \in \{\mathrm{H}, \theta, \mathrm{F}, \mathrm{W}\}$

In an FRG scheme

$$\mathcal{A}_\Phi = \frac{1}{2k^2}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\frac{\tilde{\partial}_t P_\Phi(p^2)}{\left[P_\Phi(p^2)\right]^2} > 0$$

while in $\overline{\text{MS}}$

$$\mathcal{A}_\Phi = 0$$

Does this rule out AF solutions in $\overline{\text{MS}}$?

$\overline{\text{MS}}$ Scheme?

No! In presence of a nontrivial minimum the relevant actor is

$$\mathcal{A}_\Phi(x_0) = -\frac{1}{16\pi^2} \lim_{x\to x_0} \left[\partial_z l_0^{(\Phi)}(z)\right]_{z=z_\Phi}$$

And in $\overline{\text{MS}}$

$$\mathcal{A}_\Phi^{(\overline{\text{MS}})}(x_0) = -\frac{1}{16\pi^2} \lim_{x\to x_0} z_\Phi < 0, \quad \forall\Phi$$

AF solutions in $\overline{\text{MS}}$ exist!

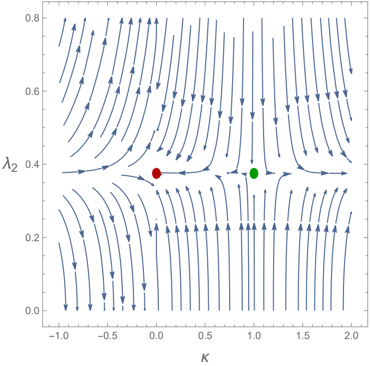
E.g. in the $\mathbb{Z}_2$-Yukawa-QCD model $P = 1$

$$f(x) = -C_f x^{4/d_x} + \frac{3\,x^2}{256\pi^2\eta_x}\left(16\hat{h}_*^4 - 3\hat{g}_*^4\right)$$

$\overline{\text{MS}}$ Scheme?

Non-Abelian Higgs model in $\overline{\text{MS}}$

$$P = 1/2, \qquad \kappa = \frac{\hat{\kappa}}{g^{2Q}}, \quad 0 < Q < 1$$



Summary

	$SU(2)_L \times SU(3)_c$	$\mathbb{Z}_2$-Yukawa-QCD	Non-Abelian Higgs
$\overline{\text{MS}}$	$P = 1,\ Q, \hat{\kappa}$	$P = 1,\ Q, \hat{\kappa}$	$P = \frac{1}{2},\ Q, \hat{\kappa}$
FRG	$P \in \left[\frac{1}{4}, \frac{1}{2}\right],\ \xi_2$	$P \in \left[\frac{1}{4}, \frac{1}{2}\right],\ \xi_2$	$P \in (0, +\infty),\ \xi_2$

- Two-parameter families of new AF models
- Compatible with any wanted Higgs/W mass ratio
- Small number of free UV parameters (g^2, v^2, λ, P)
- Infinitely many higher-dimensional operators predicted

Asymptotically Free U(1)?

The mechanism that cures λ might cure α?

- Couple the trivial sector to an asymptotically free sector
  U(1)$_Y \times$*SU(2)*$_L \times$*SU(3)*$_c$ ✓
- Add higher dimensional operators that change the β
  SMEFT, HEFT, ... ✓
- Allow for large masses in the UV
  no asymptotic symmetry: $\kappa > 0$ ✓

Asymptotically Free U(1)?

In QED higher dimensional operators, e.g. Pauli-Fierz, can change the running of α (Djukanovic, Gegelia, Meißner '17)

Even induce asymptotic safety (Gies, Ziebell '20 & '22)

Asymptotic freedom: a proof of concept: quarks+photons+gluons

$$S_{\text{cl}} = \int d^4x \left[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{4} G_{I\mu\nu} G_I^{\mu\nu} + \bar{\psi}^A \mathrm{i} \not{D}^{AB} \psi^B + \mathrm{i} n \, \bar{\psi}^A \sigma_{\mu\nu} F^{\mu\nu} \psi^A \right]$$

Treating the coupling n as a free parameter supports fixed-point solutions

$$\hat{\alpha}_* = \frac{\alpha}{g_{\mathrm{s}}^{2S}} > 0, \qquad 0 < S \leq 1$$

Functional analysis is needed for a fully consistent picture.