The Asymptotically Safe Standard Model

from quantum gravity to dynamical chiral symmetry breaking

Álvaro Pastor Gutiérrez

[2207.09817] with Jan M. Pawlowski and Manuel Reichert

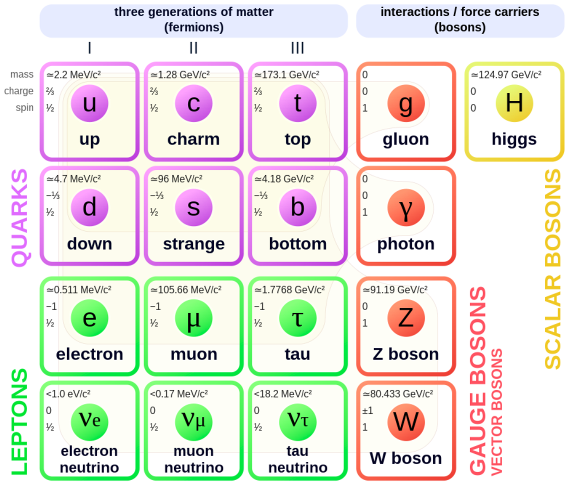
Max-Planck-Institut
für Kernphysik

Asymptotic Safety meets Particle Physics and friends

15.12.22

Where do we stand today? What is missing?

Standard Model of particle physics



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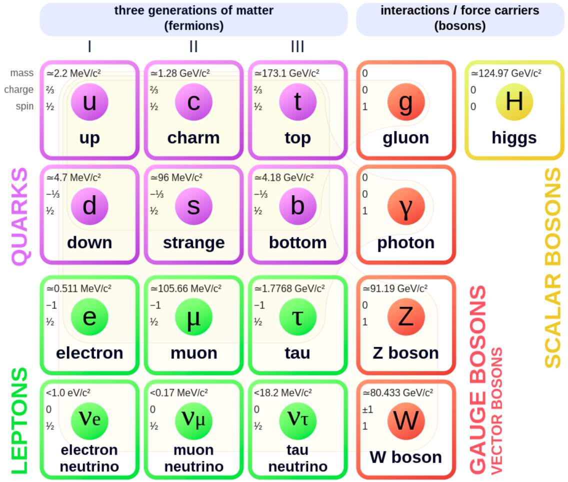

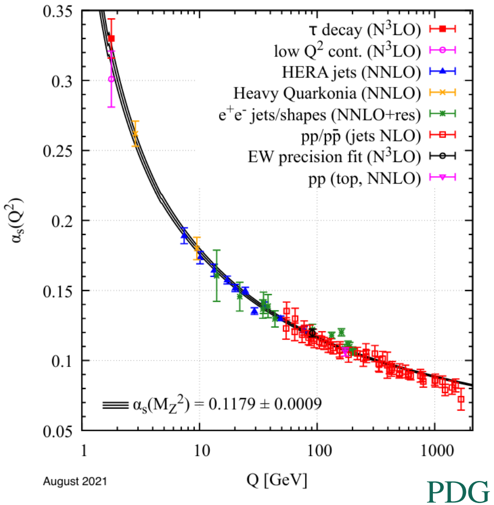

PDG‘22

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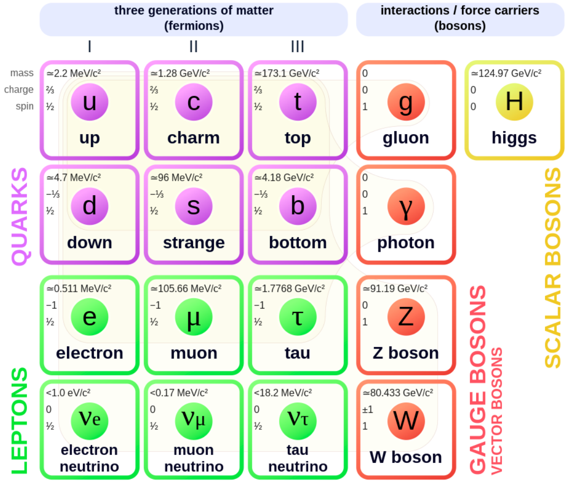

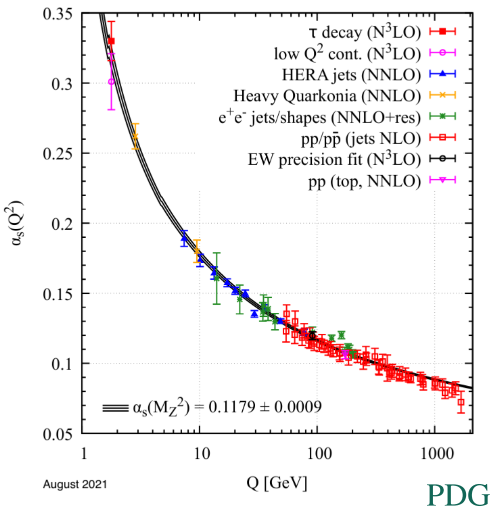

PDG'22

- Dark matter
- Neutrino masses

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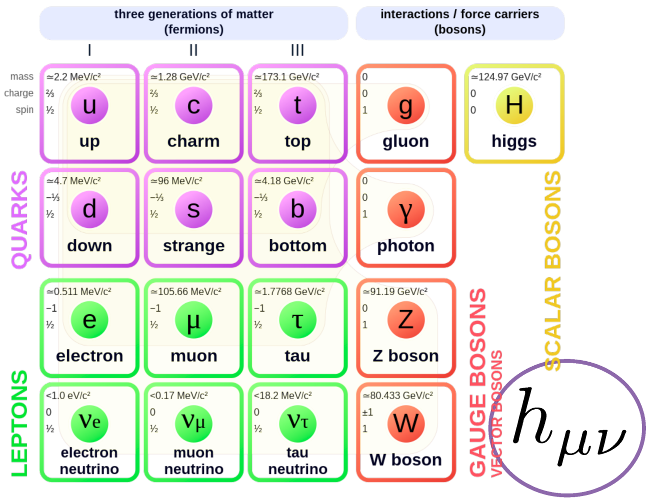

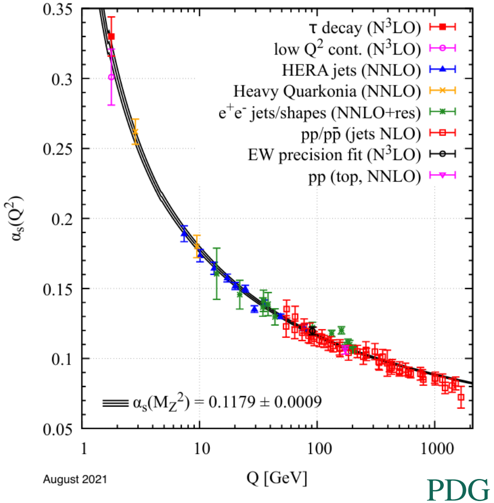

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➔ Quantum gravity

➔ Landau pole $U(1)_Y$

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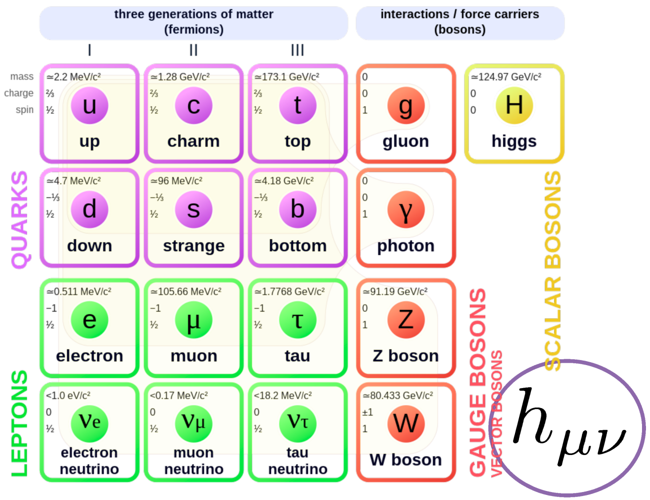

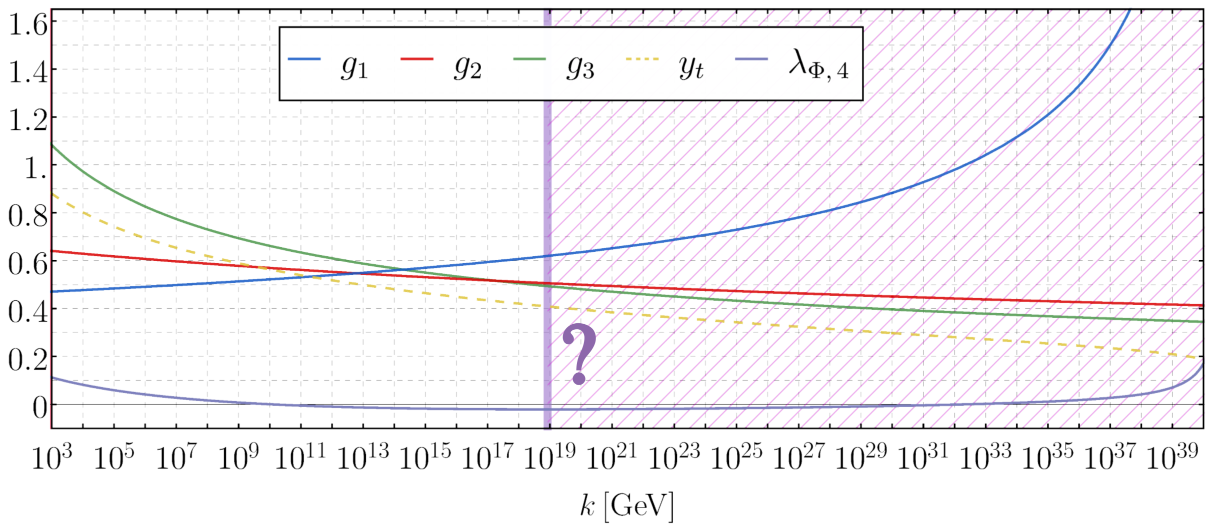

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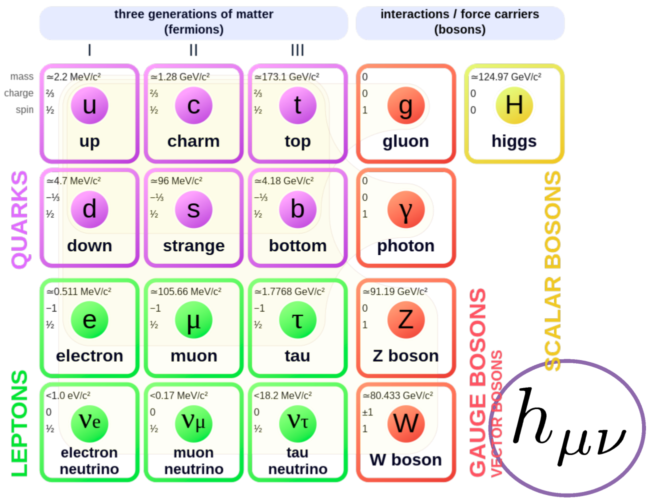

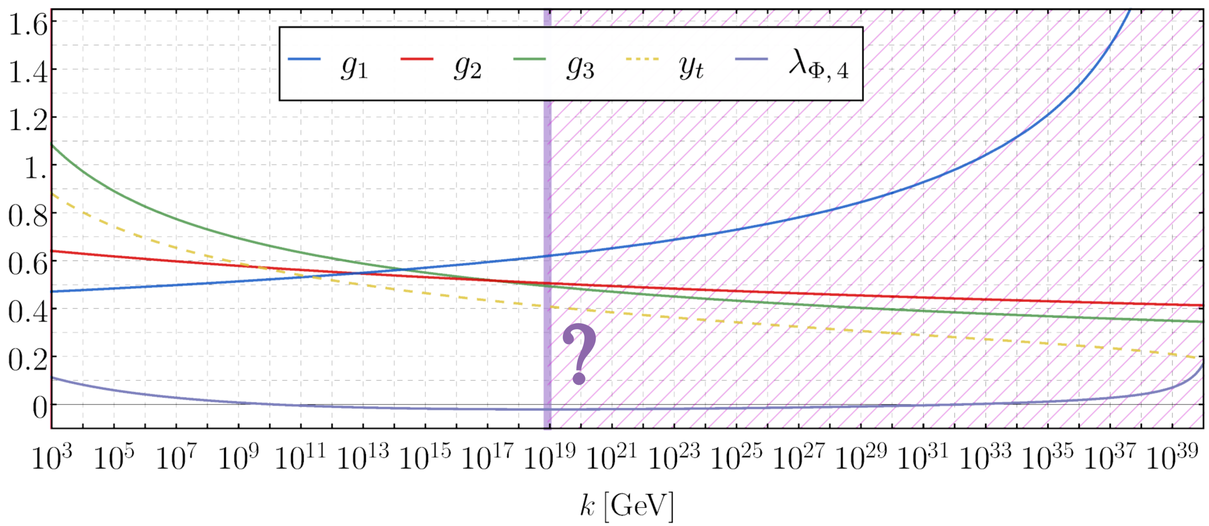

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◆ Unification of forces

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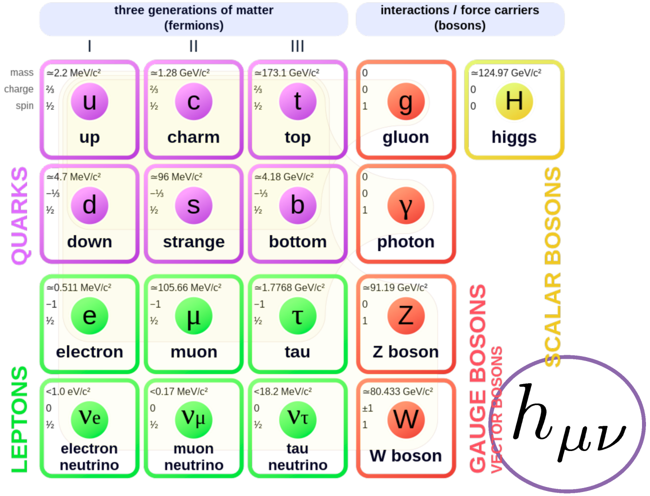

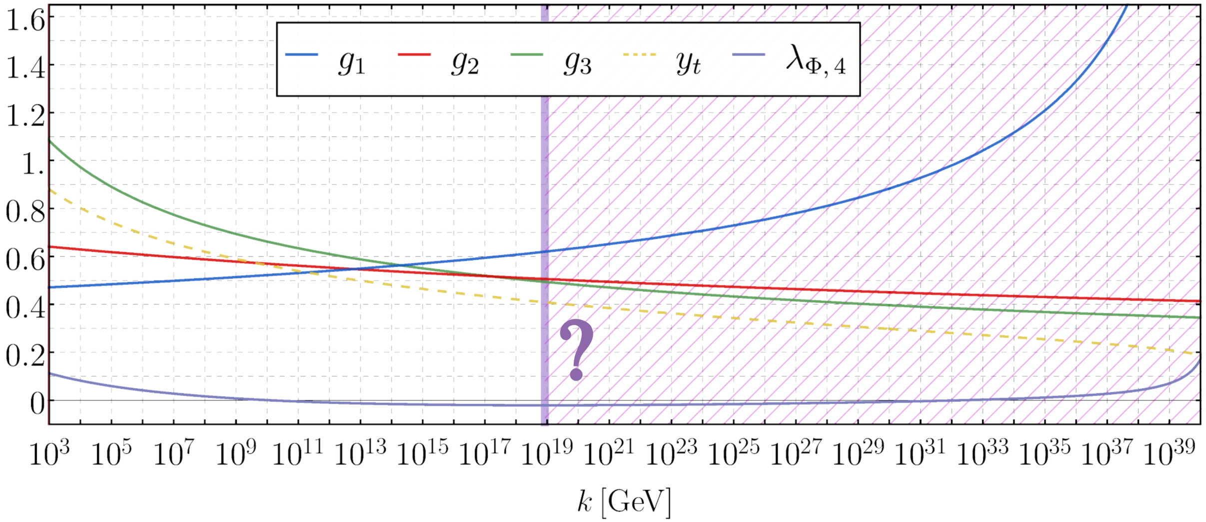

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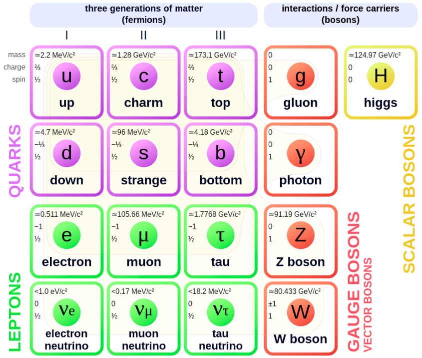
Asymptotic Safety Quantum Gravity

The Asymptotically Safe Standard Model



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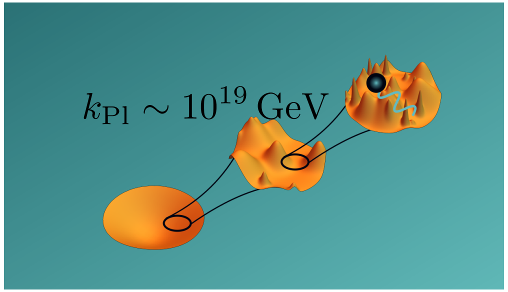
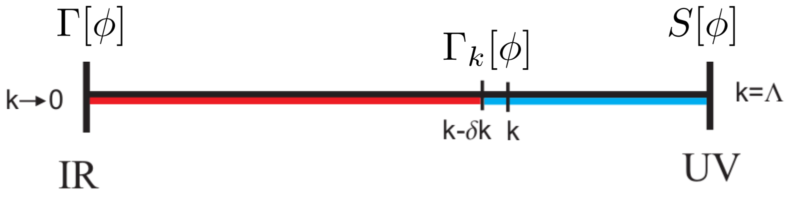


as-seminars.quantum-spacetime.net

- **Complete:** *from quantum gravity to dynamical chiral symmetry breaking*
- **Non-perturbative** framework
- **RG-consistent** implementation of gravity and matter at all scales
- Systematically **improvable computation**

Versatile for non-perturbative new physics extensions!

The functional renormalisation group



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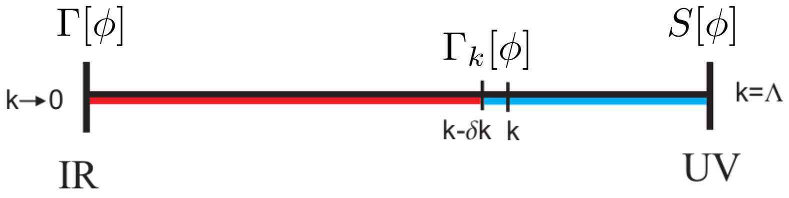

Wetterich'93

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k \right]$$

$\partial_t = k \partial_k$

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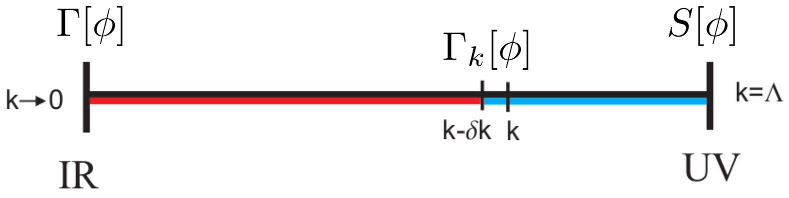

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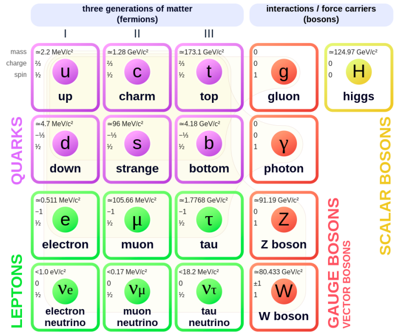
$\partial_t = k\partial_k$

- One-loop exact
- Non-perturbative
- Mass-thresholds accounted
- Analytic regulators: $R_k(p^2) = (k^2 - p^2)\theta(k^2 - p^2)$
- Systematically improvable truncations

The ASSM qualities



MATTER

- All SM matter content/symmetries
  - Flows in the broken phase $\langle H \rangle \neq 0$
  - Confinement and strong chiral symmetry breaking

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$
$$q = (d,\, u,\, s,\, c,\, b,\, t)$$
$$l = (e,\, \nu_e,\, \mu,\, \nu_\mu,\, \tau,\, \nu_\tau)$$

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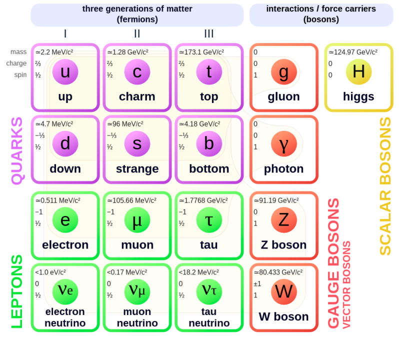

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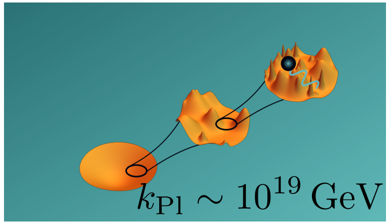

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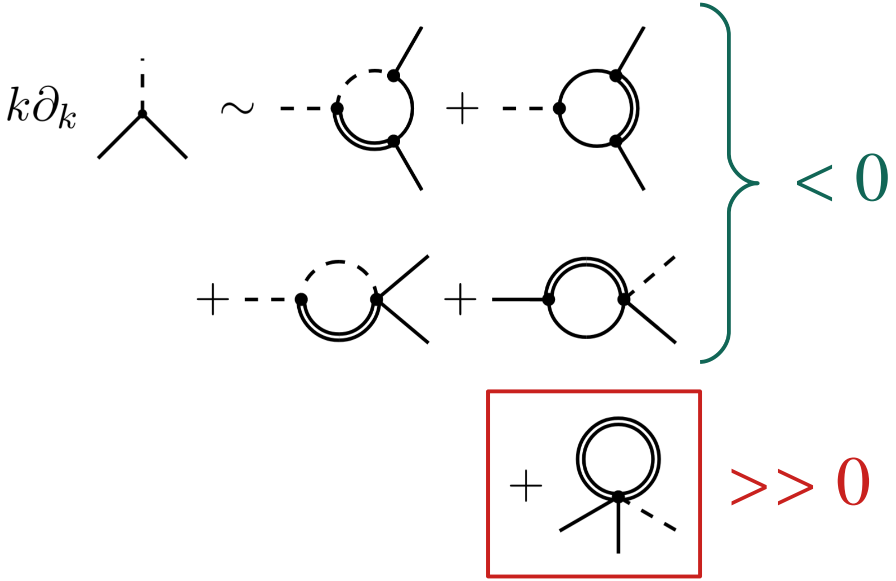
GRAVITY

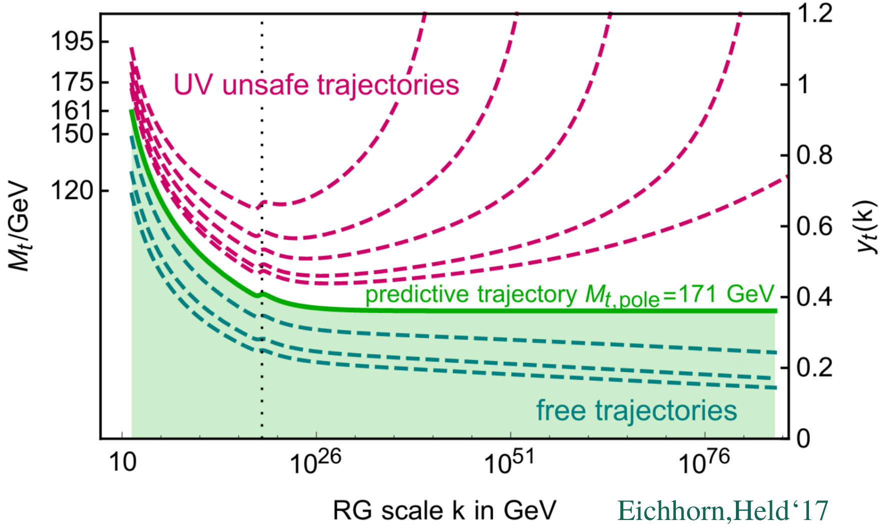
- Minimal gravity-matter coupling
- Fluctuation approach
  Meibohm,Pawlowski,Reichert‘15
  Christiansen,Litim,Pawlowski,Reichert‘17
  Eichhorn,Lippold,Schiffer‘18
  Eichhorn,Lippold,Pawlowski,Reichert,Schiffer‘19
- General Higgs potential coupled to gravity

$$V_{\Phi,\mathrm{eff}}(\rho) = \sum_{n=1}^{N_{\max}} \lambda_{\Phi,2n}\, Z_\Phi^n \rho^n \qquad \rho = \mathrm{tr}\, \Phi^\dagger \Phi \qquad \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathcal{G}_1 + i\mathcal{G}_2 \\ v + H + i\mathcal{G}_3 \end{pmatrix}$$

Gravity-matter beta functions

Zanusso,Zambelli,Vacca,Percacci‘10
Oda,Yamada‘15
Eichhorn,Held,Pawlowski‘16
Christiansen,Litim,Pawlowski,Reichert‘17
Eichhorn,Held‘17
Hamada,Yamada‘17





Eichhorn,Held‘17

Cutoff scale and regulator variation

$$r_{\text{flat}}(x) = \left(\frac{1}{x} - 1\right)\Theta(1-x) \qquad x = \frac{p^2}{k^2}$$

$$\mathrm{k}_{\text{mat}} = \gamma_{\text{mg}} k_{\text{grav}}$$

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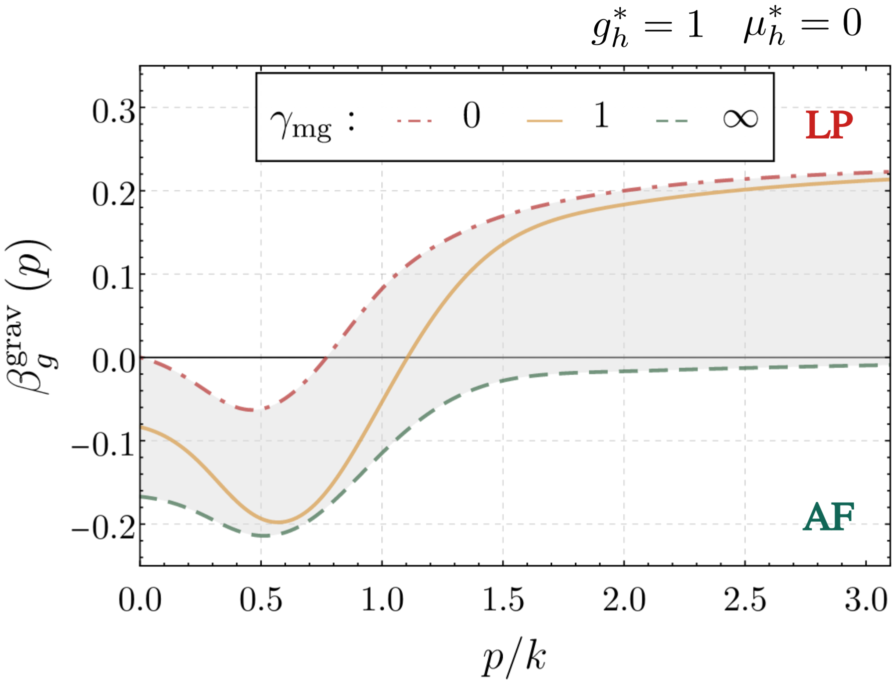
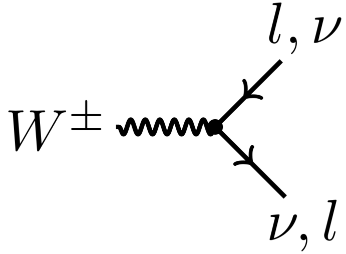
- *Matter matters:* $\gamma_{\text{mg}} \to 0, \quad \partial_t R_{\text{mat}} \to 0$
  - **Diffeomorphism** and **gauge** consistency in the matter sector
  - The **matter propagators enhanced** relative to the graviton propagator
- *Gravity rules:* $\gamma_{\text{mg}} \to \infty, \quad \partial_t R_{\text{grav}} \to 0$
  - **Diffeomorphism** consistency in the gravity sector
  - The **graviton propagator enhanced** relative to the matter propagators

Gauge beta function





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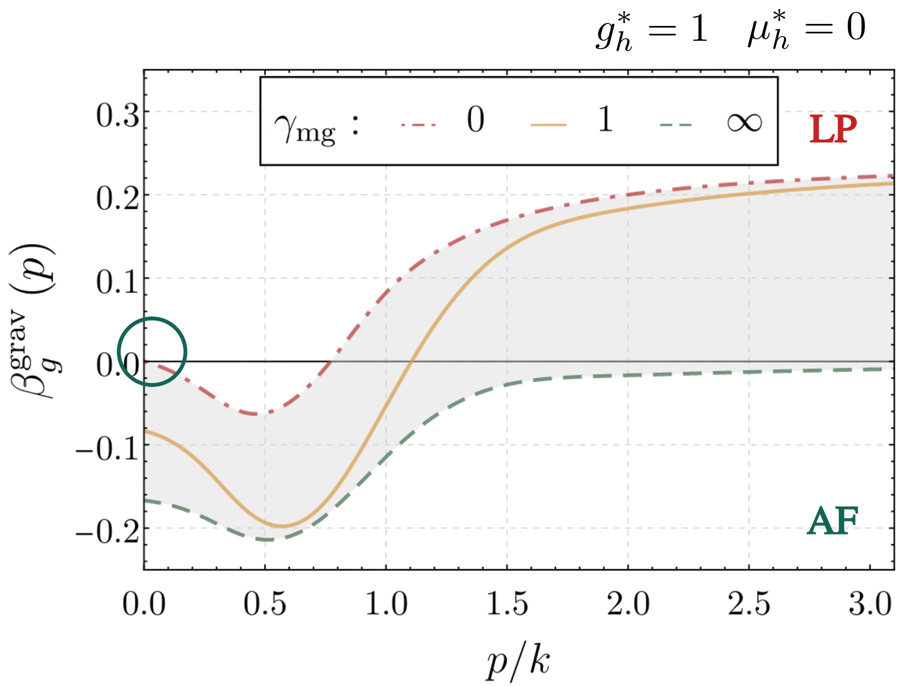

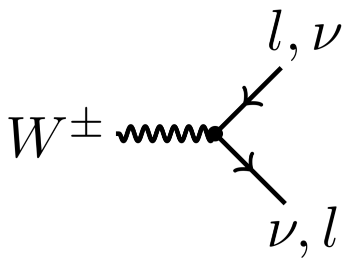

$$\beta_g^{\rm grav}(p=0) = 0\,, \qquad \text{for} \qquad \gamma_{\rm mg} = 0$$

- <u>Kinematic identity</u> for fermion-gauge-gravity systems

Folkerts,Litim,Pawlowski'12

Yukawa beta function and AS viability

Zanusso,Zambelli,Vacca,Percacci‘10
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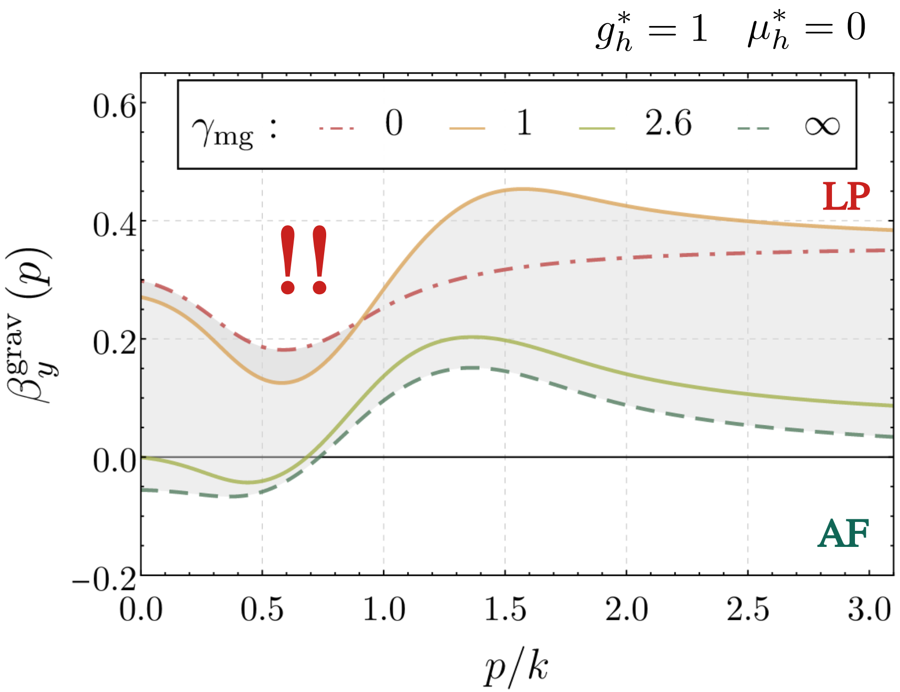

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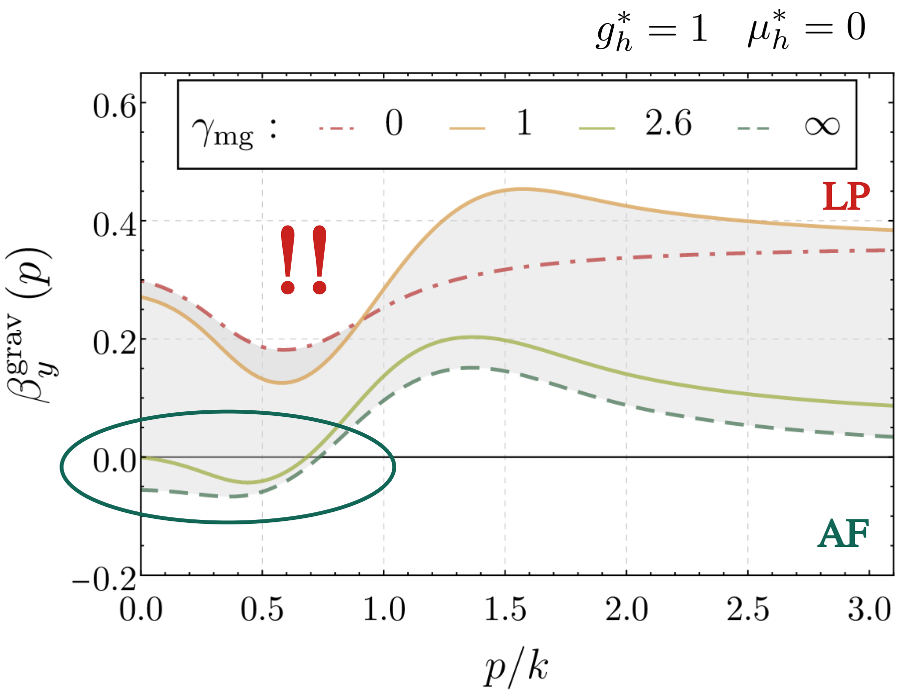

$$\gamma_{\rm mg}^{\rm stab} \approx 2.55 \sim \mathcal{O}(1)$$

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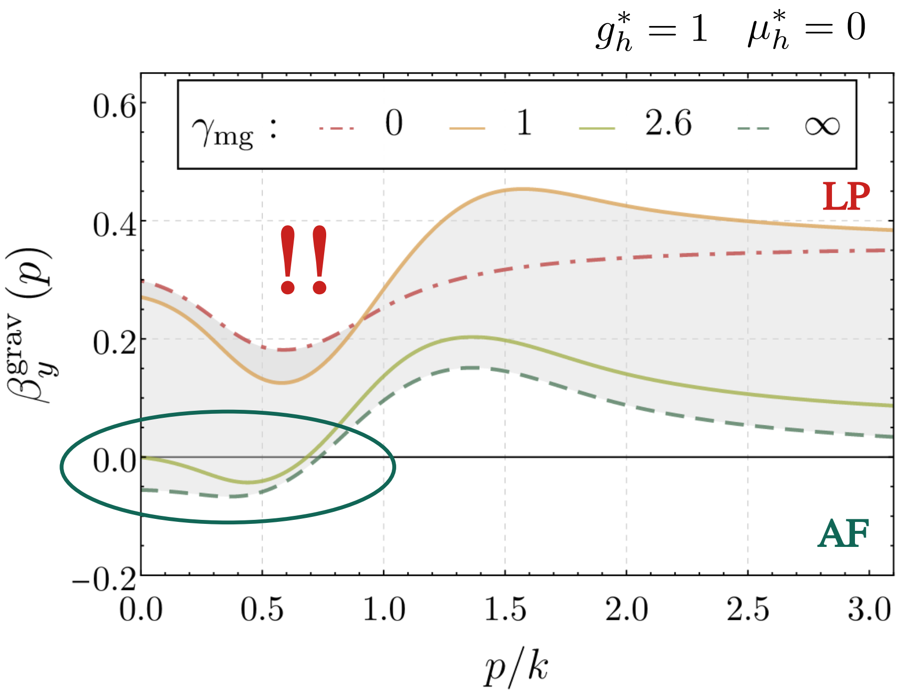

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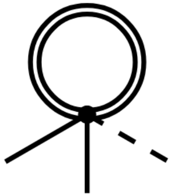
Approximated by:

$$\gamma_{mg} = 1$$



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$$\Rightarrow \beta_y^{grav}(p=0) < 0$$

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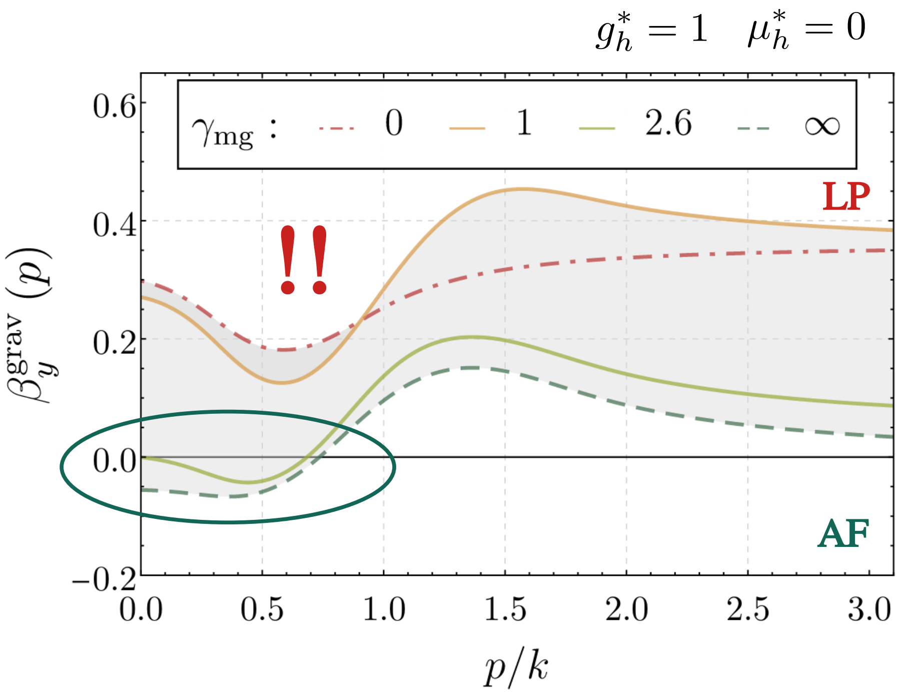

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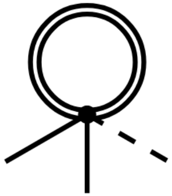

$$= 0$$

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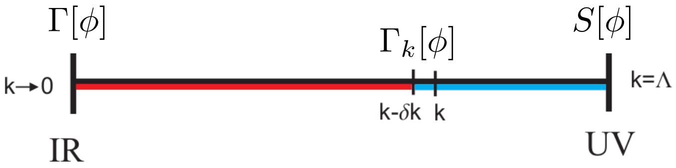
AS at stake! Need for better truncations!

The precise physical trajectory



- Recover physical quantities at **k→0 and p=0!**

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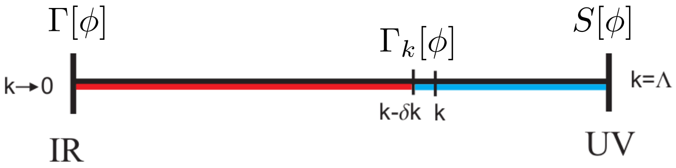

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Special treatment!

- Top Yukawa → <u>Pole mass</u>

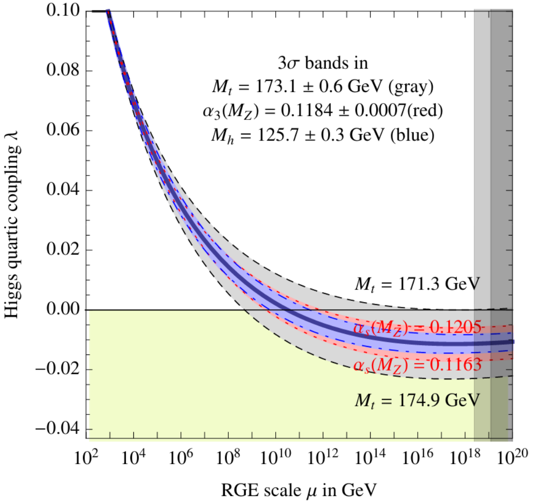
$$M_{t,\text{pole}}^{(\text{exp})} = 172.5 \pm 0.7\,\text{GeV}$$

$$\Gamma_{t,\text{pole}}^{(\text{exp})} = 1.42^{+0.19}_{-0.15}\,\text{GeV}$$

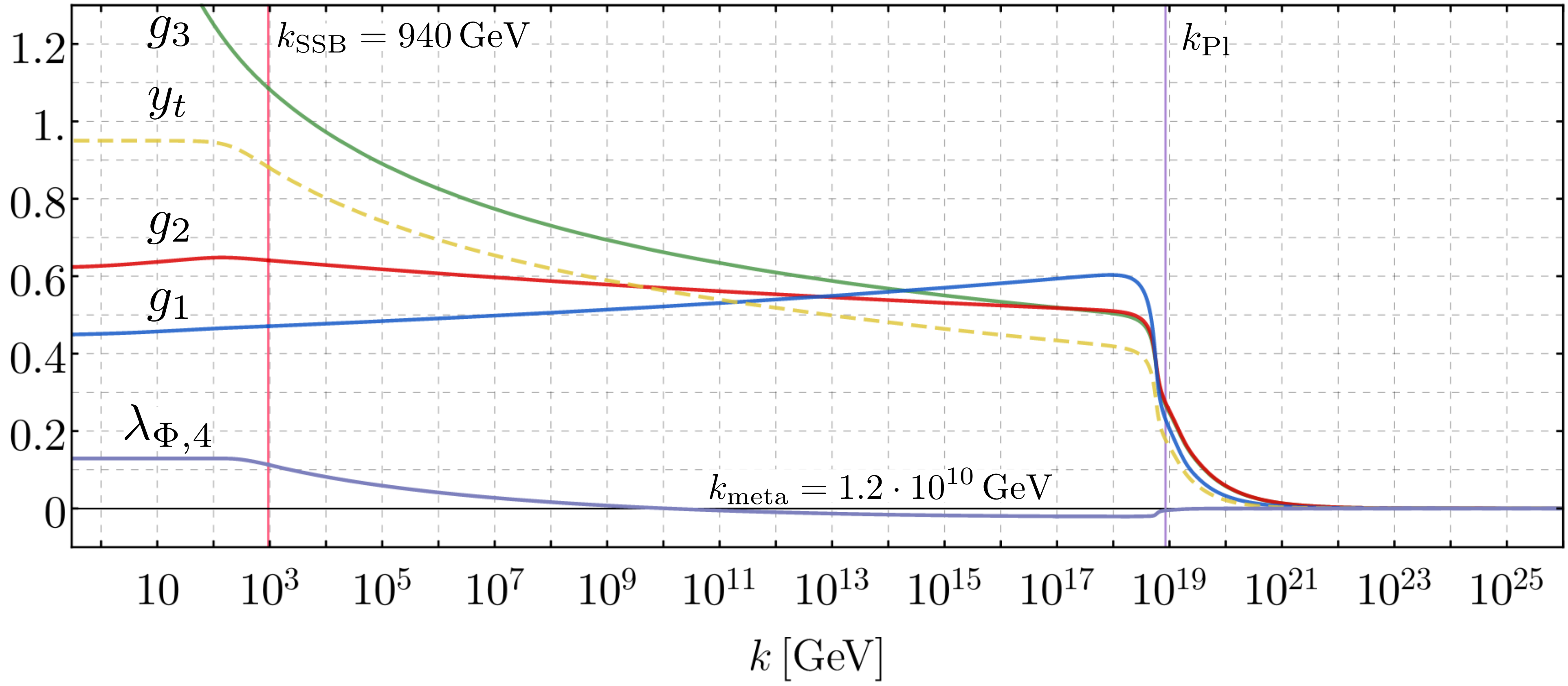
- <u>Top pole width prediction:</u>

$$\Gamma_{t,\text{pole}}^{(\text{theo})} = 1.72^{+0.09}_{-0.41}\,\text{GeV}$$

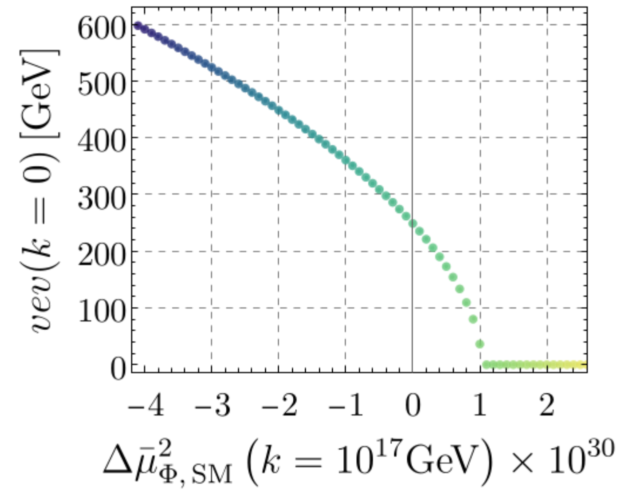
Degrassi,Di Vita,Elias-Miró, Espinosa,Giudice,Isidori,Strumia'12

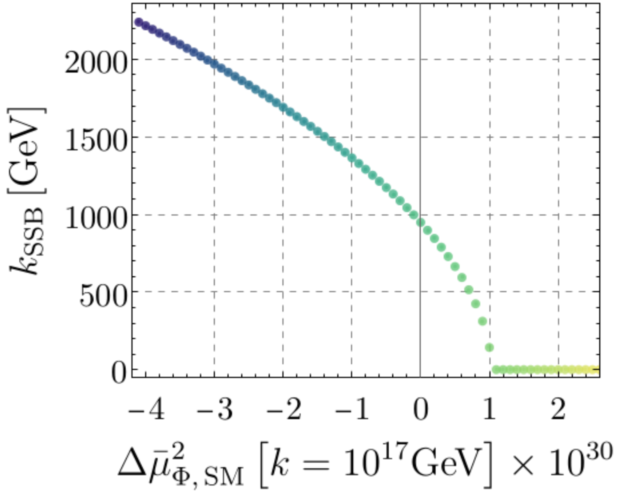


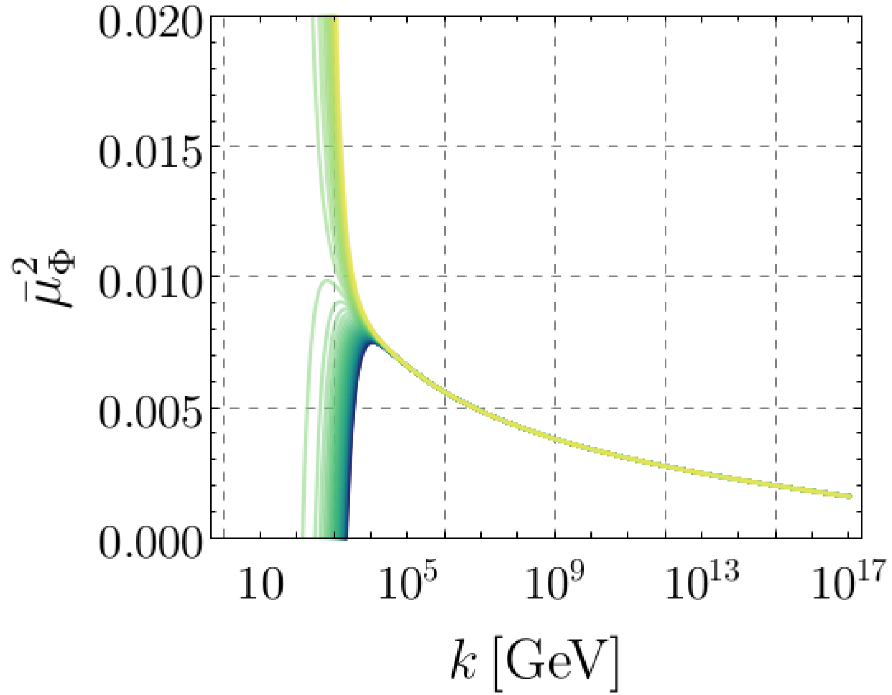
The ASSM trajectory



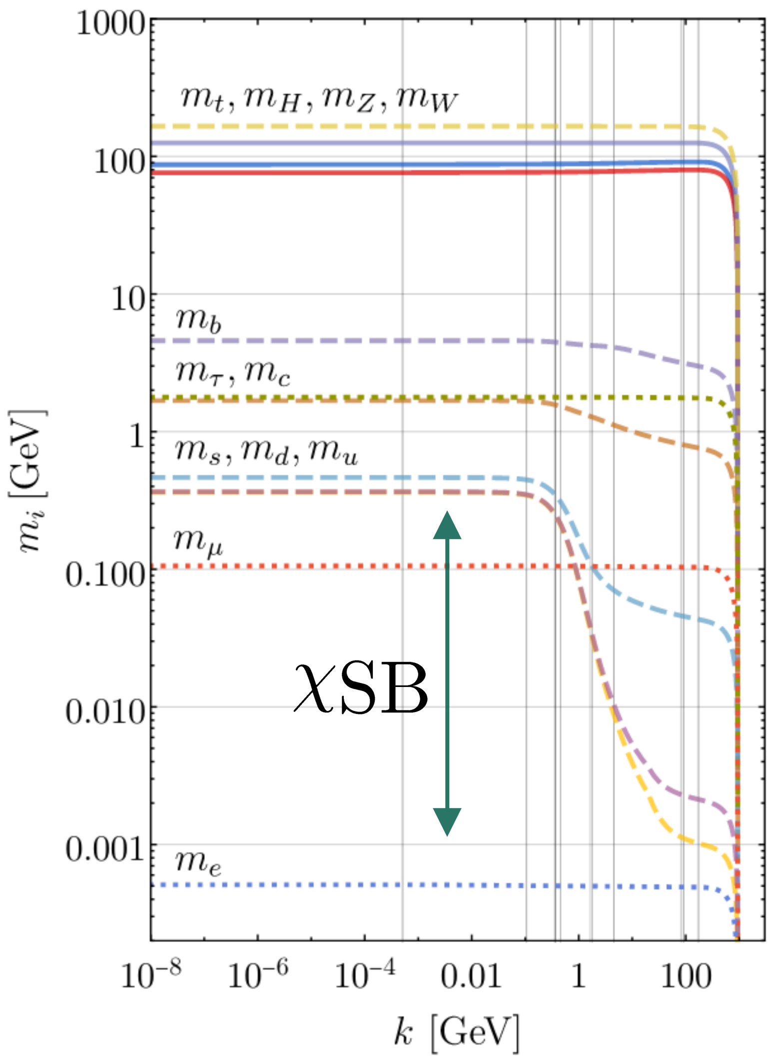
Higgs mass flow and fine-tuning







Broken phase flows



$$m_{W^\pm} = \frac{g_2\, v}{2}$$

$$m_Z = \frac{g_2\, v}{2\, \cos(\theta_W)}$$

$$m_H = \sqrt{2\, \lambda_{\Phi,4}}\, v$$

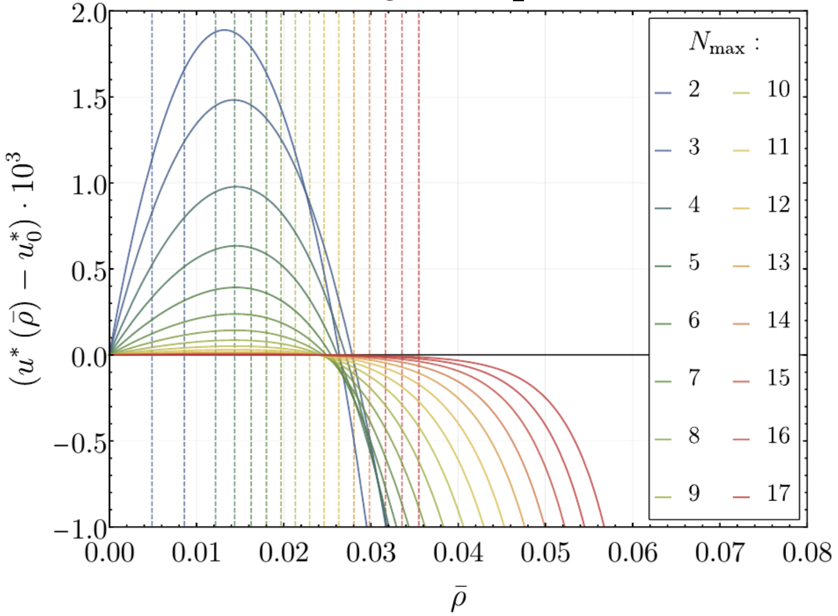
- Deep IR QCD: Dynamical chiral symmetry breaking

Braun,Fister,Pawlowski,Rennecke'14
Mitter,Pawlowski,Strodthoff'14
Rennecke'15
Cyrol,Mitter,Pawlowski,Strodthoff'17
Gao,Papavassiliou,Pawlowski'21

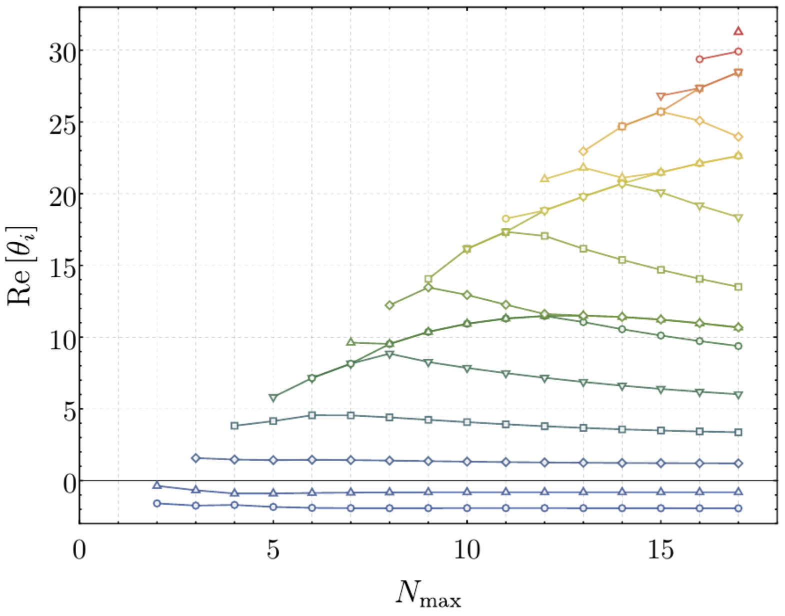
UV-Higgs potential

$$V_{\Phi,\text{eff}}(\rho) = \sum_{n=1}^{N_{\max}=17} \lambda_{\Phi,2n}\, Z_\Phi^n \rho^n$$

Converged **flat potential**



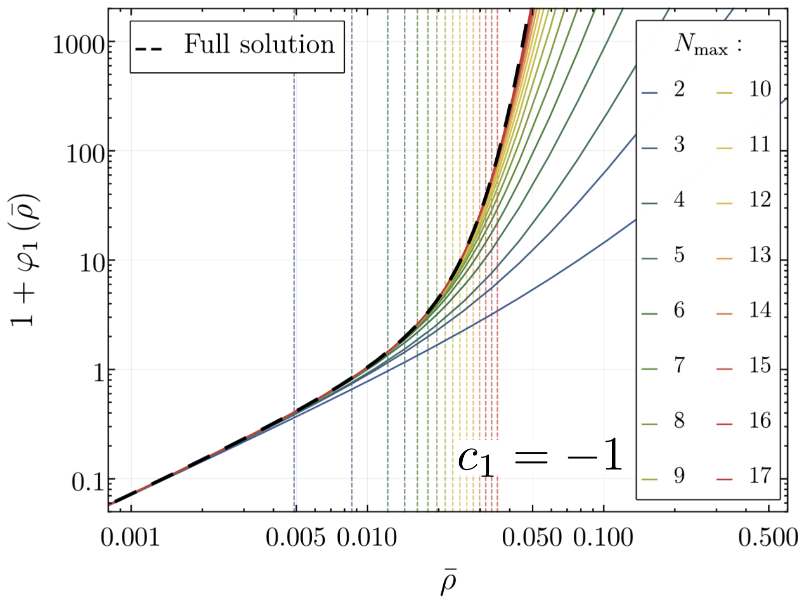
As many free parameters as the SM Higgs potential

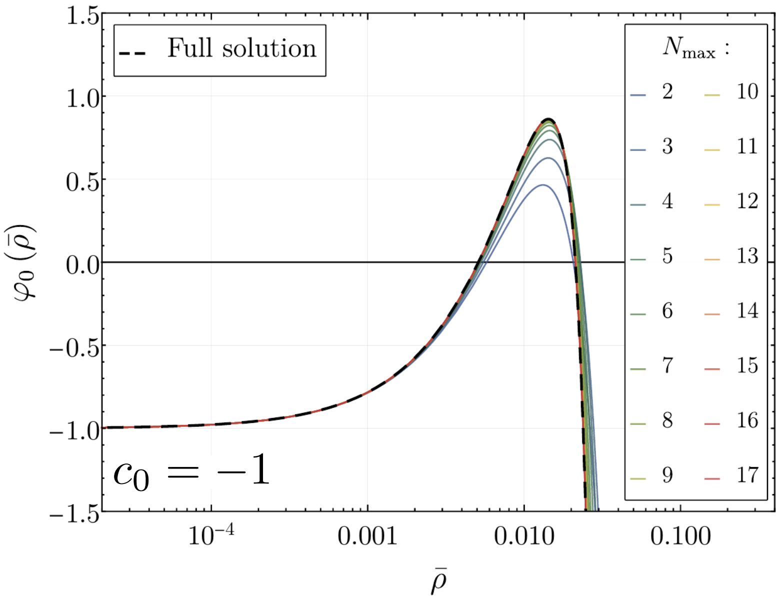


$$\theta_1 = -1.93 \pm 2 \cdot 10^{-3} \qquad \theta_2 = -0.811 \pm 7 \cdot 10^{-5}$$

Eigenperturbations and global solution

$$u(\bar{\rho}) = u^*(\bar{\rho}) + \sum_i b_i\, \varphi_i(\bar{\rho}) \qquad u^*(\bar{\rho}) \equiv 0$$



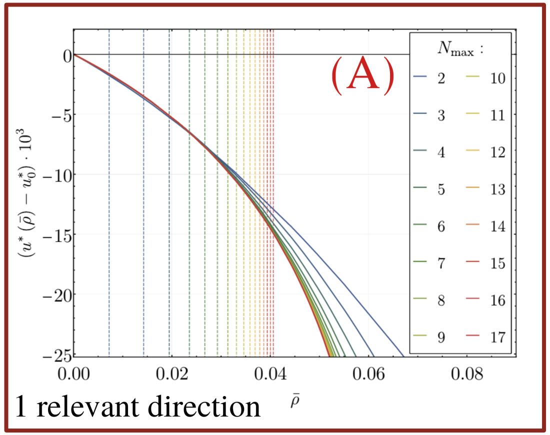


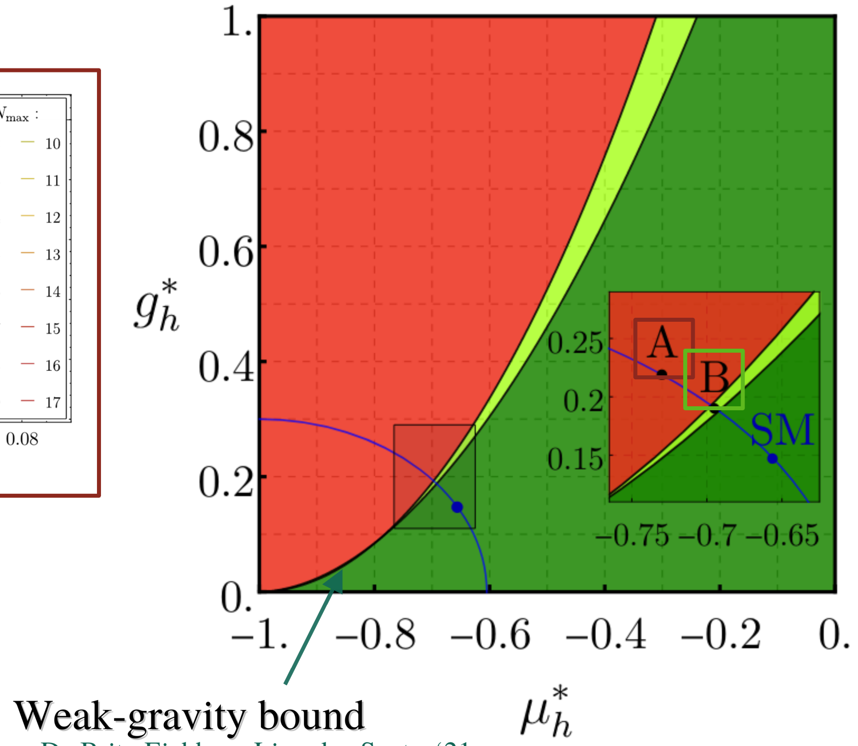
$$\varphi_i(\bar{\rho}) = c_i\, M\left(-\frac{4+\Theta_i}{2},\, 2,\, 32\pi^2\bar{\rho}\right) \qquad \Theta_i = \theta_i - \frac{3}{\pi}\frac{g_h^*}{(1+\mu_h^*)^2}$$

M( ):= Kummer function

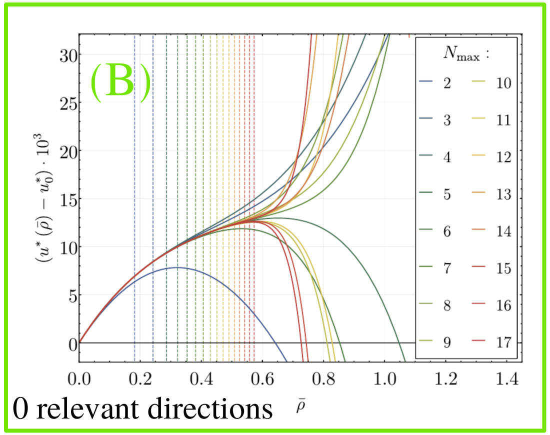
Halpern,Huang'94'95
Morris'22
Laporte,Locht,Pereira,Saueressig'22

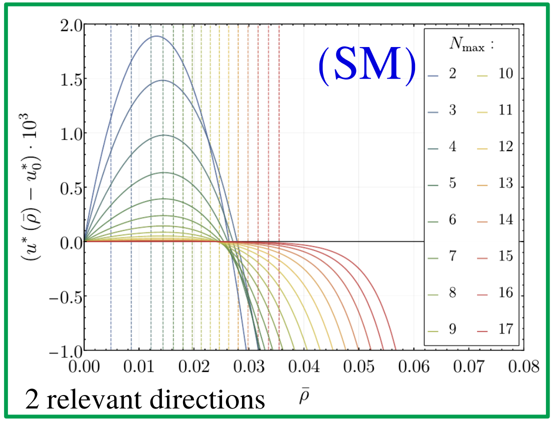
UV-Higgs landscape



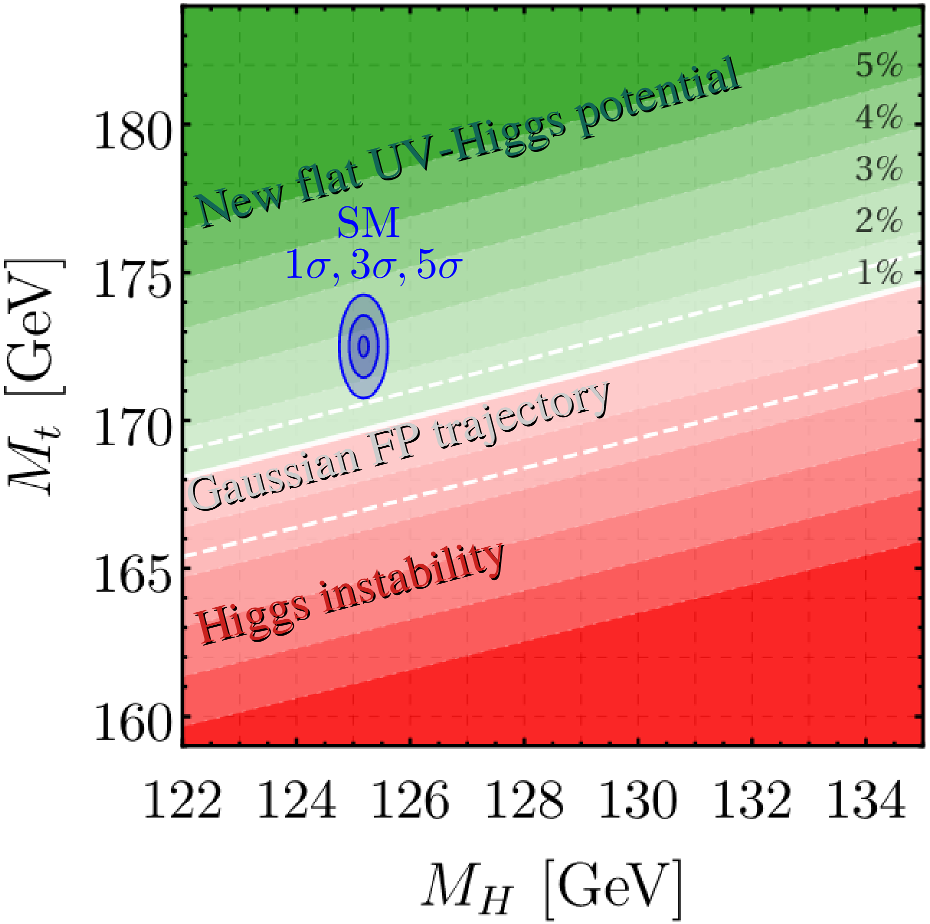


De Brito,Eichhorn,Lino dos Santos'21
Eichhorn,Kwaspisz,Schiffer,'22





Quantitative for *qualitative*



<u>Sources of systematic errors:</u>

- Strong coupling: $^{+0.9}_{-2.7}\,\mathrm{GeV}$

  $\alpha_{s,k=M_Z} \in \left[\alpha_s^{\overline{\mathrm{MS}}},\, 1.10\,\alpha_s^{\overline{\mathrm{MS}}}\right]$

- Top pole mass: $^{+0.9}_{-0.2}\,\mathrm{GeV}$

- Gravity FP: $\ll 1\mathrm{GeV}$

- FRG truncation: $< 1\mathrm{GeV}$

Conclusions

- **Unified** and **UV-complete** picture of SM and ASQG
  - **Consistent non-perturbative renormalisation** from *trans-planckian* to *sub-Fermi scales* with inclusion of **mass-thresholds** at all scales
- Thorough analysis of **systematics**
  - FRG top pole mass $\rightarrow$ **width prediction**

Conclusions

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  - FRG top pole mass→ **width prediction**

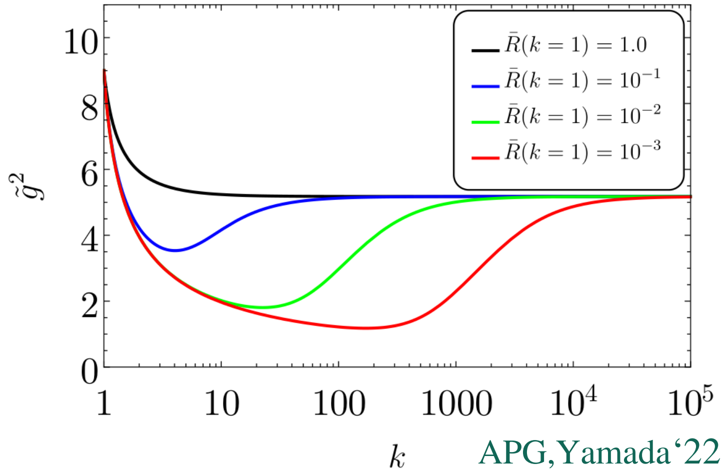
On the gravity side…

- Momentum dependent **gravity-matter beta functions**
  - **Kinematic identity** for gravity-gauge-fermion systems
  - Critical state of **Yukawa-gravity systems**

    → Asymptotic safety at stake!
- **ASSM FP and landscape**
  - **Flat UV Higgs potential** with **2 relevant directions**

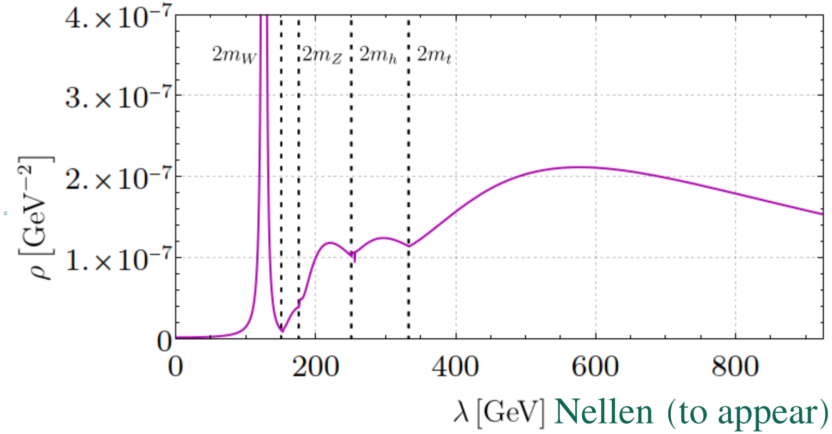
Beyond the ASSM

- Non-perturbative extensions:
  - Extra-dimensions and Gauge-Higgs unification
    in collaboration with Masatoshi Yamada
  - Composite Higgs
    in collaboration with Florian Goertz
  - …



APG,Yamada‘22

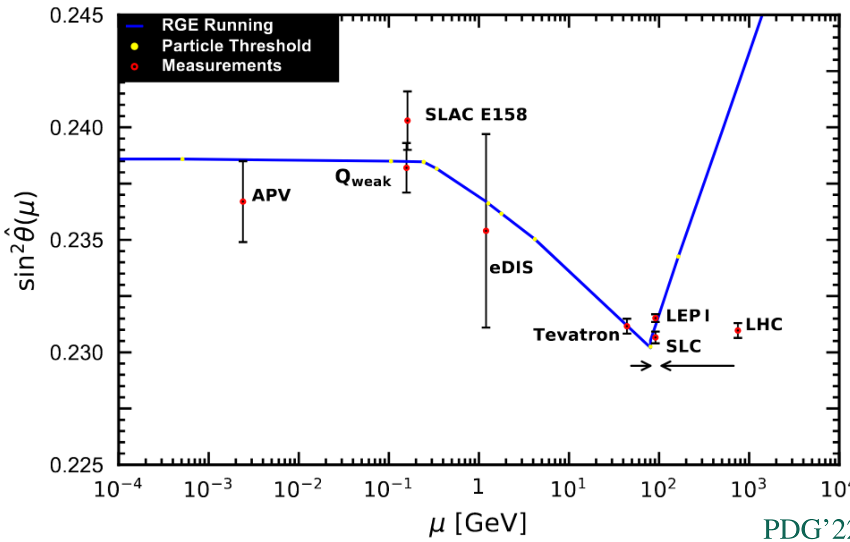
- Spectral functions: Higgs, top quark, …



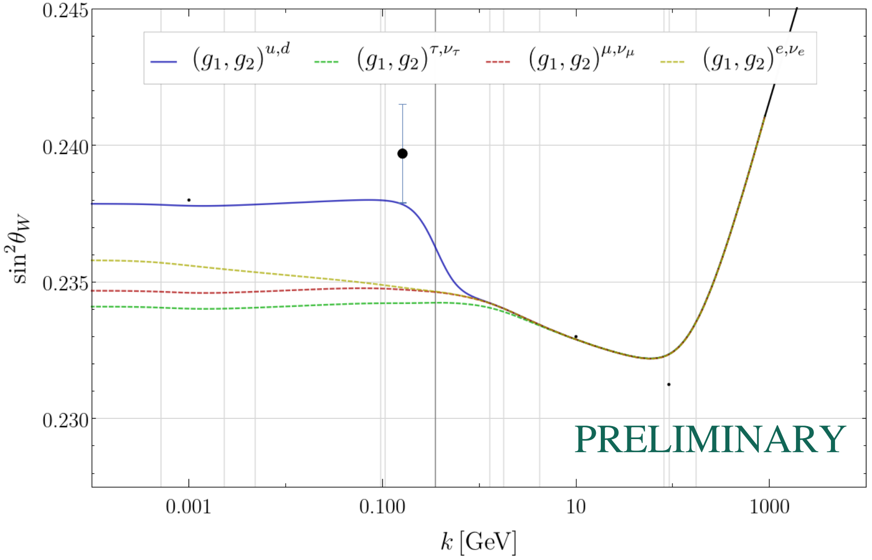
Nellen (to appear)

Thank you for your attention!

Low energy observables: Weinberg angle



PDG’22

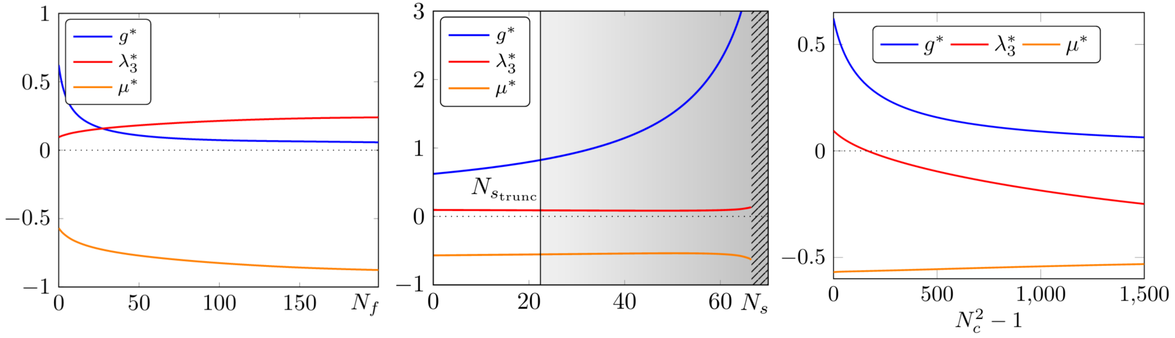


Gravity-matter systems

<u>Fluctuation approach</u>

Meibohm,Pawlowski,Reichert‘15
Christiansen,Litim,Pawlowski,Reichert‘17
Eichhorn,Lippold,Schiffer‘18
Eichhorn,Lippold,Pawlowski,Reichert,Schiffer‘19

$$(g_h^*, \mu_h^*)_{\text{SM}}^{\text{fluc}} = (0.14, -0.65)$$



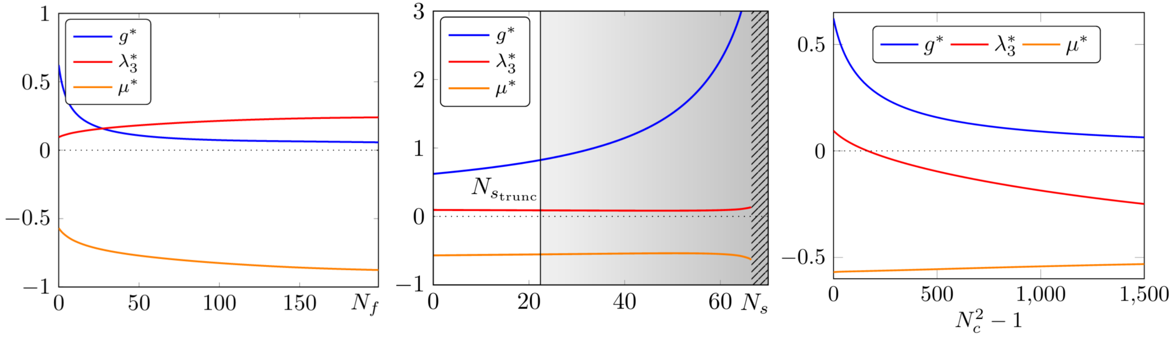
- Expansion over the flat background: $g_{\mu\nu} = \delta_{\mu\nu} + \sqrt{16\pi G_{\text{N}}}\, h_{\mu\nu}$

- Flows from 2 and 3-point functions: $Z_h,\ Z_c,\qquad g_h = g_{h,3},\qquad \mu_h = -2\lambda_{h,2}$

- Avatars: $g_{\Phi_1\cdots\Phi_i\,h\cdots h} = g_h$ *Effective universality*

Eichhorn,Lippold,Pawlowski,Reichert,Schiffer‘19

Gravity-matter systems



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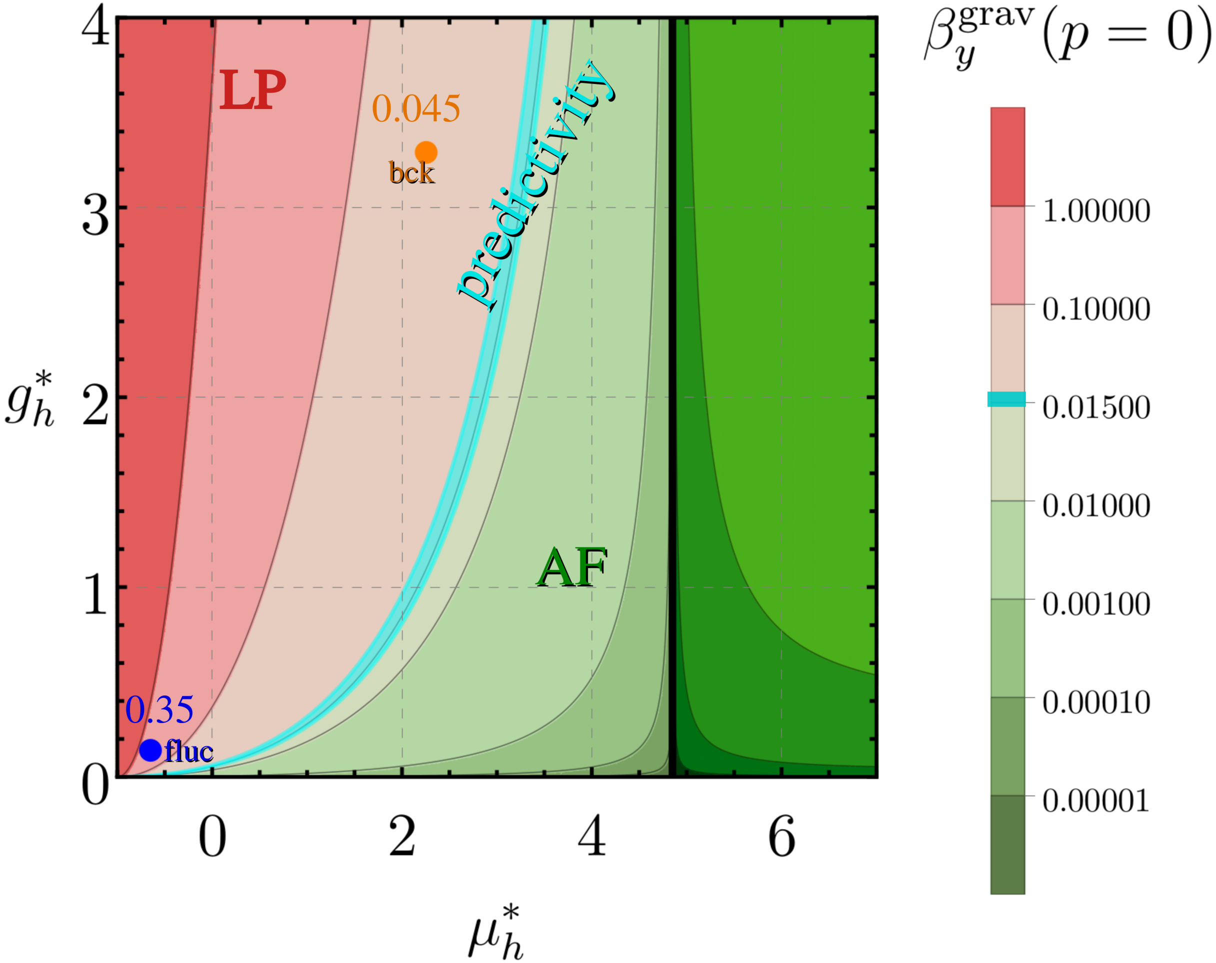
  Eichhorn,Lippold,Pawlowski,Reichert,Schiffer'19

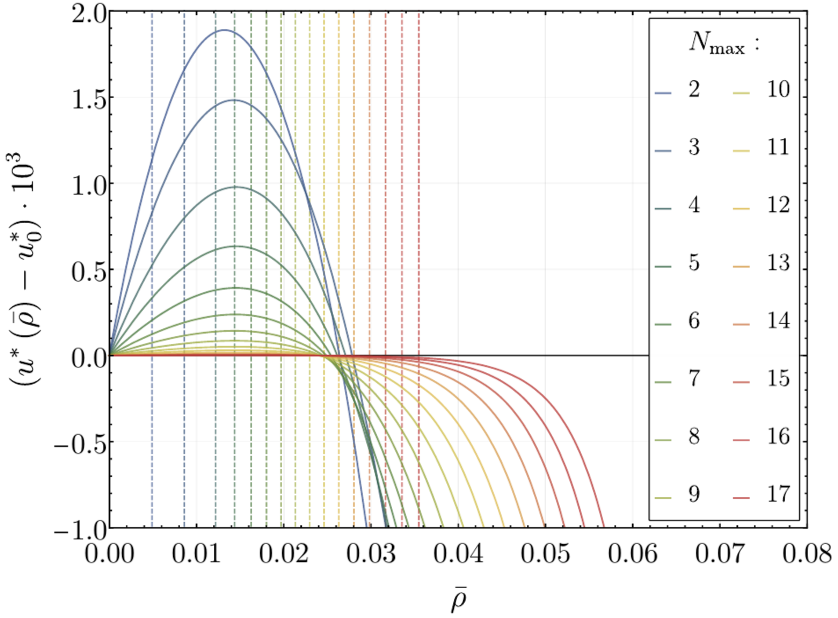
Background approximation

Dona,Eichhorn,Percacci'14
Oda,Yamada'15
Dona,Eichhorn,Labus,Percacci'16
Wetterich,Yamada'19

$$(g_h^*, \mu_h^*)_{\rm SM}^{\rm bck} = (3.29, 2.25)$$


LP
0.045
bck
predictivity
AF
0.35
fluc
g_h^*
μ_h^*
4
3
2
1
0
0
2
4
6
$\beta_y^{\text{grav}}(p=0)$
1.00000
0.10000
0.01500
0.01000
0.00100
0.00010
0.00001

Details of the UV-Higgs potential



$$V_{\Phi,\text{eff}}(\rho) = \sum_{n=1}^{N_{\max}=17} \lambda_{\Phi,2n}\, Z_\Phi^n \rho^n$$

$$\mathcal{D}\, u = \Big[4 - (2+\eta_\Phi)\,\bar\rho\,\partial_{\bar\rho}\Big] u(\bar\rho) \qquad \Delta u = u - u_0$$

$$\bar\rho_{\max} = \max_{\bar\rho} \left\{ \bar\rho \;\middle|\; \frac{|\mathrm{FP}_u(\Delta u^*)|}{\sqrt{(\mathcal{D}\,\Delta u^*)^2 + \mathrm{Flow}_u^2(\Delta u^*)}} \leq 1\% \right\}$$

- Converged **flat potential**
- As many free parameters as the SM Higgs potential (two relevant directions)

Eigenperturbations

$$\mathcal{D}\,\varphi_i(\bar\rho) = \Big[(4+\theta_i) - (2+\eta_\Phi)\,\bar\rho\,\partial_{\bar\rho}\Big]\varphi_i(\bar\rho)$$

$$\eta_\Phi = \frac{3g_h}{\pi}\frac{(2+\mu_h)}{(1+\mu_h)^2}\frac{u_0'}{(1+u_0')^2}$$

$$(4+\Theta_i)\,\varphi_i(\bar\rho) - (2+\eta_\Phi)\;\bar\rho\,\varphi_i'(\bar\rho) + \frac{1}{16\pi^2}\left[2\varphi_i'(\bar\rho) + \bar\rho\,\varphi_i''(\bar\rho)\right] = 0$$

$$\boxed{\varphi_i(\bar\rho) = c_i\,M\left(-\frac{4+\Theta_i}{2+\eta_\Phi},\,2,\,32\pi^2\left(1+\frac{\eta_\Phi}{2}\right)\bar\rho\right)}$$

Kummer function

$$\Theta_i = \theta_i - \frac{3}{\pi}\frac{g_h^*}{(1+\mu_h^*)^2}$$

Polynomial form for $\bar\rho\to\infty$

$$\begin{cases}\varphi_i \to \dfrac{1}{\Gamma\left(4+\frac{\Theta_i}{2}\right)}(32\pi^2\bar\rho)^{\frac{4+\Theta_i}{2+\eta_\Phi}} \\[2ex] \Theta = -4 + (2+\eta_\Phi)\,n \qquad \text{with} \qquad n\in\mathbb{N}\end{cases}$$

Exponential form for $\bar\rho\to\infty$

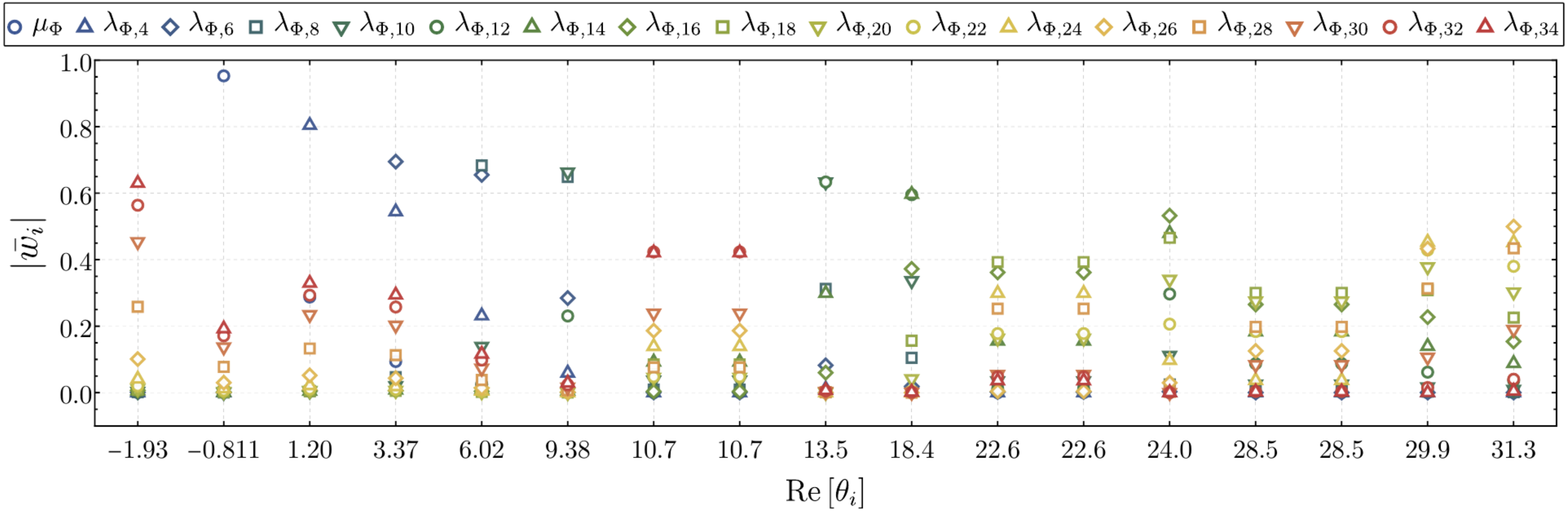
$$\begin{cases}\varphi_i \to \dfrac{1}{\Gamma\left(-2-\frac{\Theta_i}{2}\right)}\dfrac{1}{(32\pi^2\bar\rho)^{4-\frac{\Theta_i}{2}}}\,e^{32\pi^2\left(1+\frac{\eta_\Phi}{2}\right)\bar\rho} \\[2ex] \Theta \neq -4 + (2+\eta_\Phi)\,n \qquad \text{with} \qquad n\in\mathbb{N}\end{cases}$$

Expandable in Hermite polynomials (squared integrable)

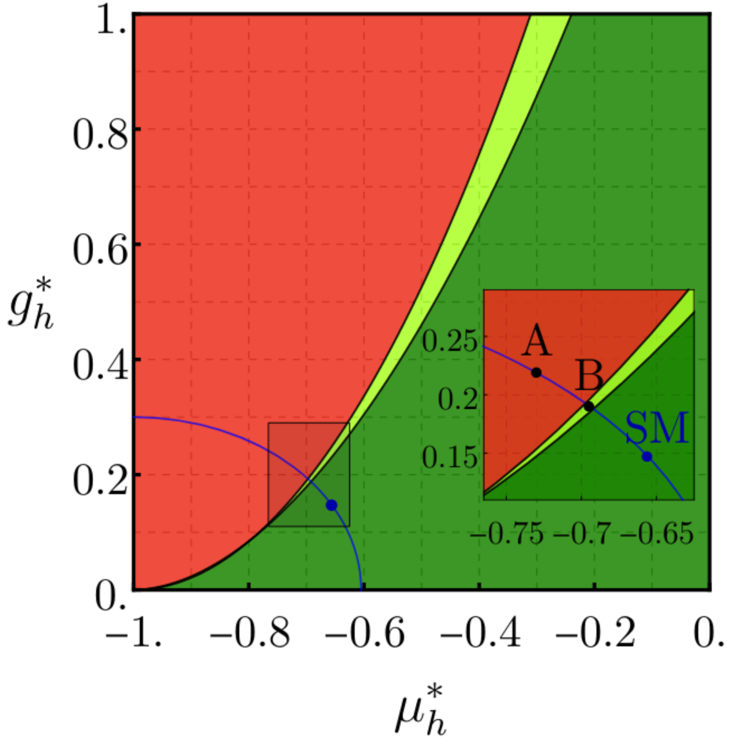
Normalised eigenvector

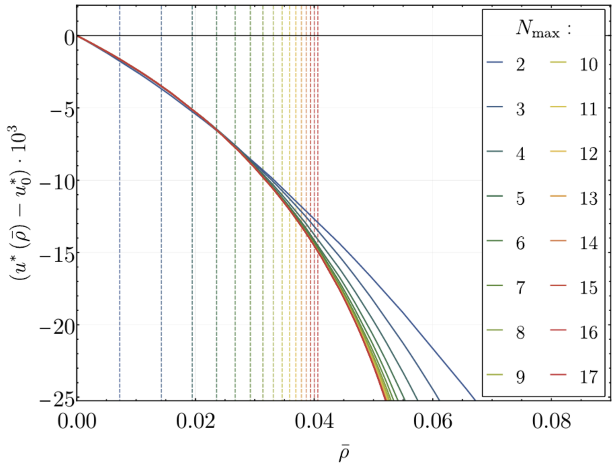
$$\bar{w}_l^{(i)} = \frac{\zeta_l\, v_l^{(i)}}{\sqrt{\sum_{m=0}^{N-1} \left|\zeta_m v_m^{(i)}\right|^2}}$$

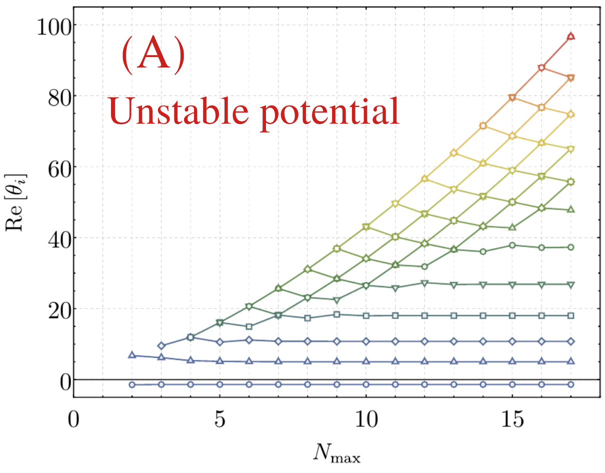
Kluth,Litim'20

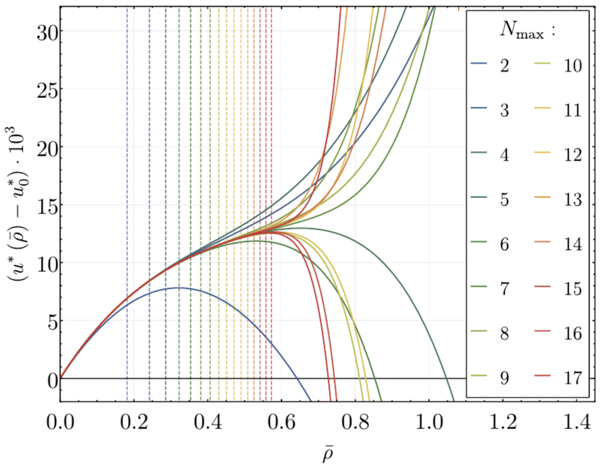


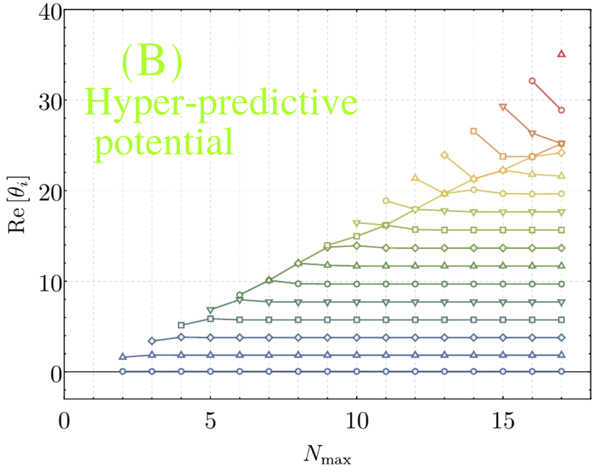
UV-Higgs landscape





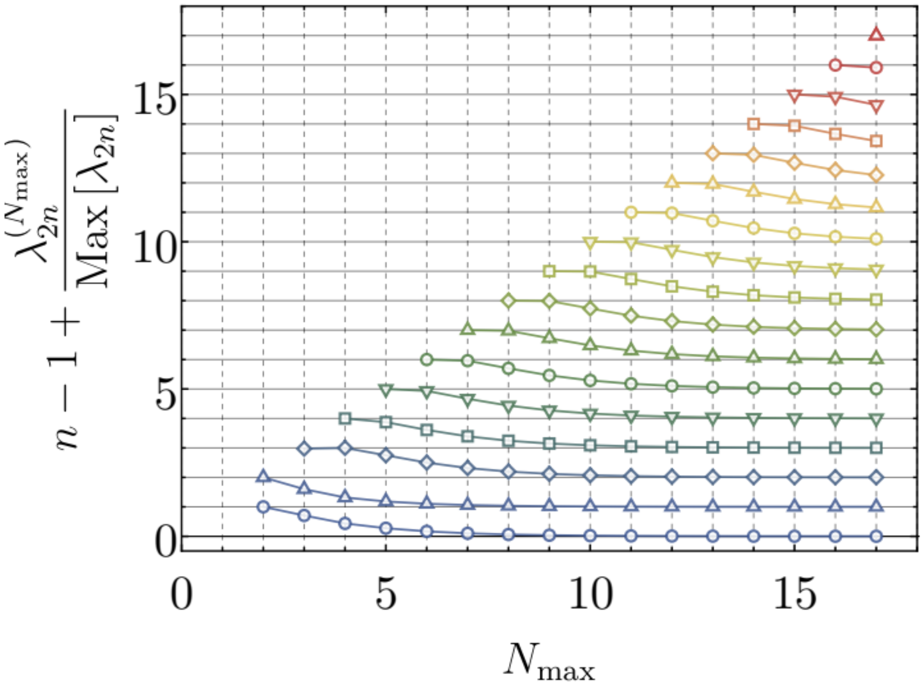






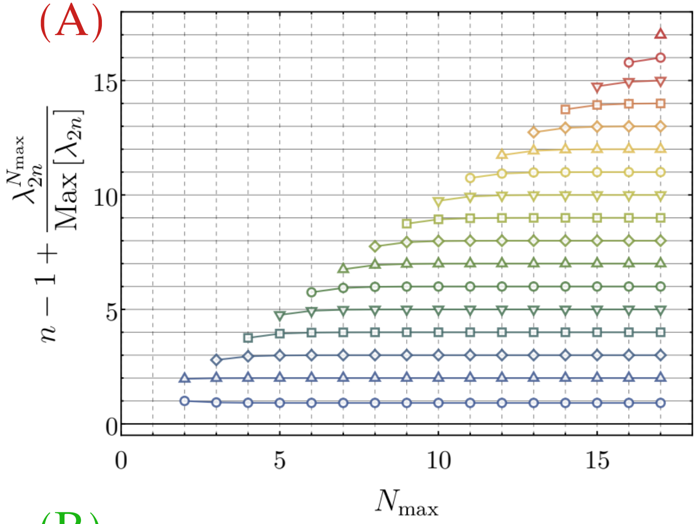
Flattening of the potential

(SM)

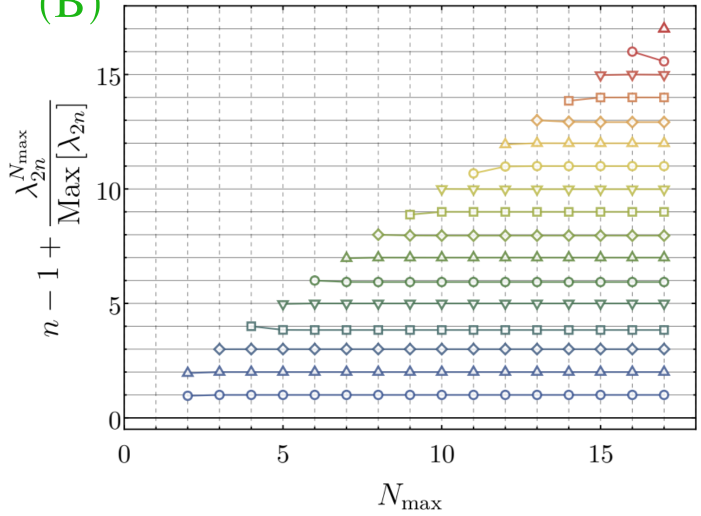


$\mu_{\Phi}^{*} = 0.0487 \qquad \lambda_{\Phi,4}^{*} = -0.0758$

(A)

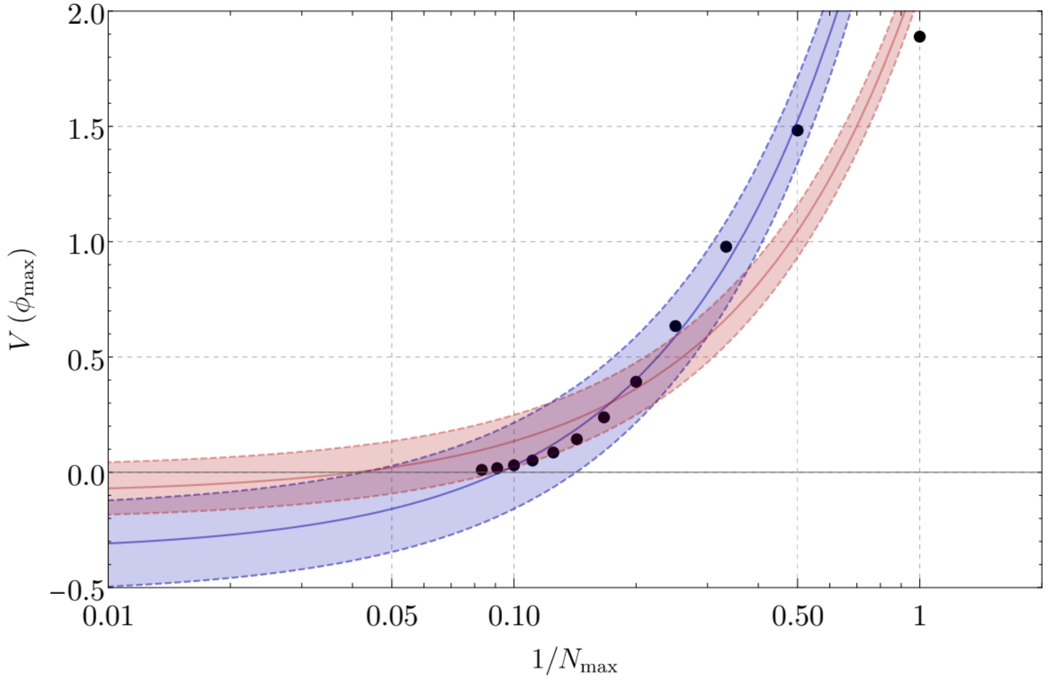


(B)



○ μ_{Φ}^{2} △ λ_4 ◇ λ_6 □ λ_8 ▽ λ_{10} ○ λ_{12} △ λ_{14} ◇ λ_{16} □ λ_{18} ▽ λ_{20} ○ λ_{22} △ λ_{24} ◇ λ_{26} □ λ_{28} ▽ λ_{30} ○ λ_{32} △ λ_{34}

Flattening of the potential



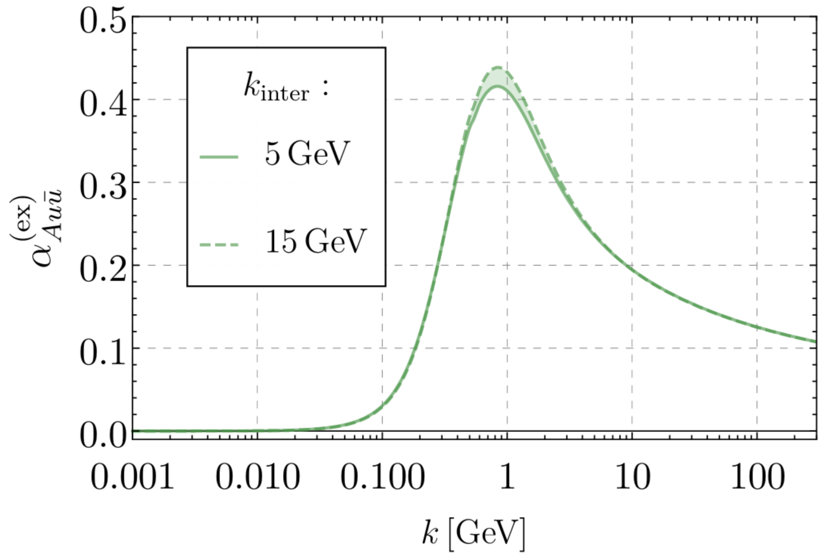
Strong QCD

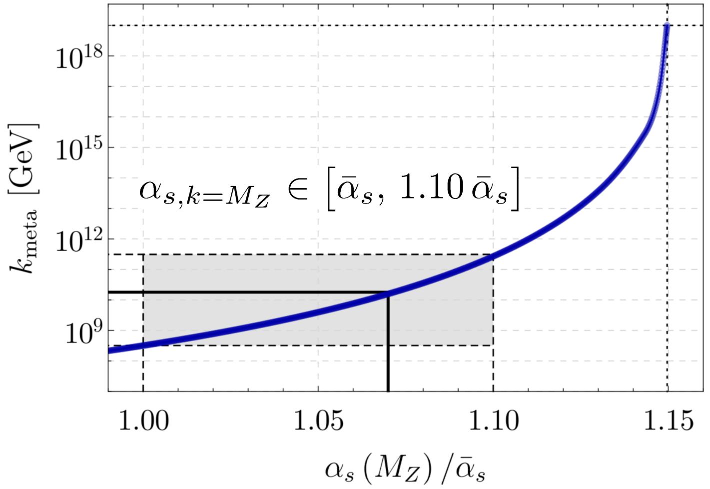
- Implementation of FRG results from 2+1 flavor QCD.
- QCD mass-gap accounted
- Dynamical chiral symmetry breaking
- Scale setting:

$$g^{\overline{\text{MS}}}_{3,k=M_Z} \approx 1.22$$

$$\bar{\alpha}_s := \frac{\left(g^{\overline{\text{MS}}}_{3,k=M_Z}\right)^2}{4\pi} \approx 0.118$$

$$g_{3,k=M_Z} = \sqrt{1.07\,(4\pi\bar{\alpha}_s)} = 1.26$$



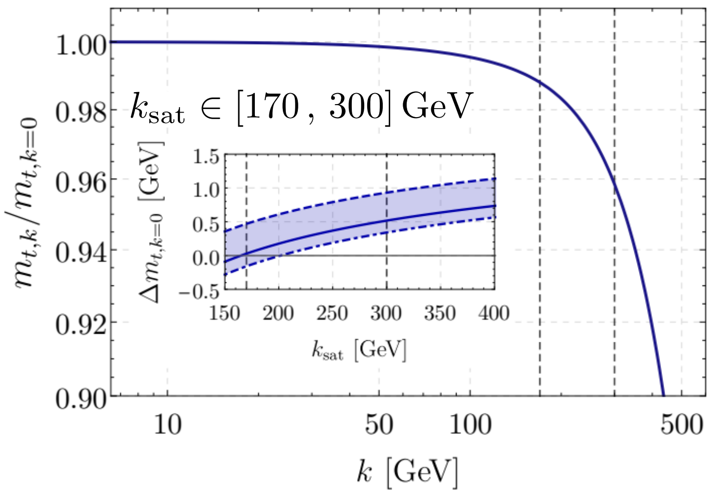


The top pole mass determination

$$\left[p^2 + M_t(p)^2\right]_{p^2=-\left[M_t^{(\mathrm{pole})}\right]^2} = 0 \qquad \partial_t[Z_t(p)M_t(p)] = \frac{1}{4}\mathrm{tr}_D\,\partial_t\Gamma^{(2)}_{t\bar t}(p)$$

$$[Z_tM_t]^{1\mathrm{loop}}(p \lesssim k_{\mathrm{SSB}}) = Z_tm_t + \Delta\,[Z_tM_t]\,(p)$$

$$\Delta[Z_t(p)M_t(p)] = \frac{m_t}{16\,\pi^2}\left[4\,g_3^2\,\mathcal{I}(p^2) + 3\left(\frac{2\,g_2\sin\theta_W}{3}\right)^2\mathcal{I}(p^2) - \frac{h_t^2}{2}\,\mathcal{I}(p^2,m_H) + \frac{h_t^2}{2}\,\mathcal{I}(p^2,m_Z) + h_b^2\,\mathcal{I}(p^2,m_W) - \frac{g_Y^2\,(1+2\cos 2\theta_W)}{9}\,\mathcal{I}(p^2,m_Z)\right]$$



$$M_{t,\mathrm{pole}}^{(\mathrm{exp})} = 172.5 \pm 0.7\,\mathrm{GeV}$$
$$\Gamma_{t,\mathrm{pole}}^{(\mathrm{exp})} = 1.42^{+0.19}_{-0.15}\,\mathrm{GeV}$$

$$m_t = 165.4^{+0.9}_{-0.2}\,\mathrm{GeV}$$
$$\Gamma_{t,\mathrm{pole}}^{(\mathrm{theo})} = 1.72^{+0.09}_{-0.41}\,\mathrm{GeV}$$