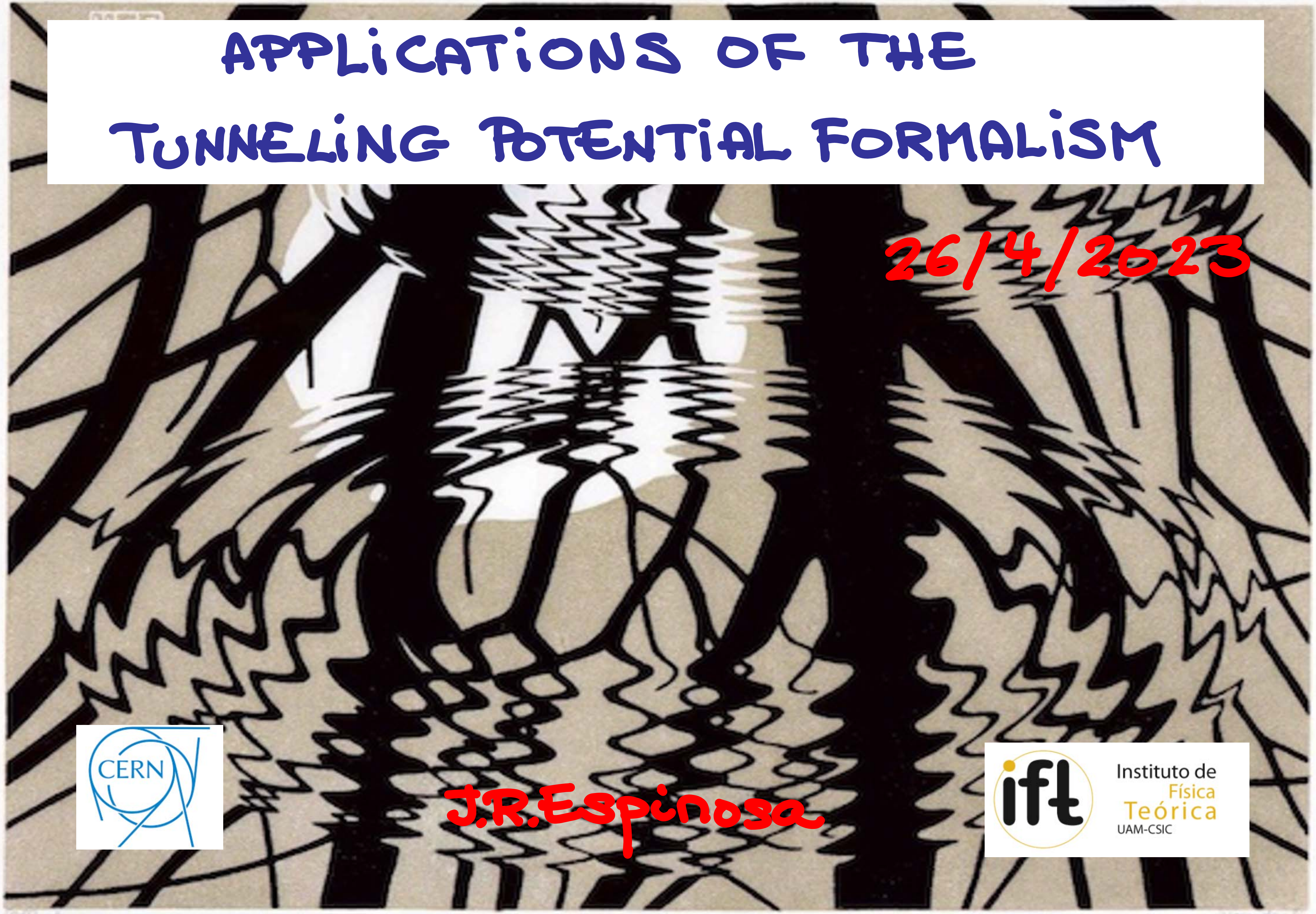
APPLICATIONS OF THE TUNNELING POTENTIAL FORMALISM



26/4/2023

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OUTLINE

0. Tunneling Potential Formalism

– Applications –

1. Exact tunneling sols. in multifield potentials
2. Gravitational corrections to vacuum decay
3. Stability of AdS maxima
4. Bubble of nothing decays
5. (Ungauged) Q-balls

DISCLAIMERS

- Work in progress

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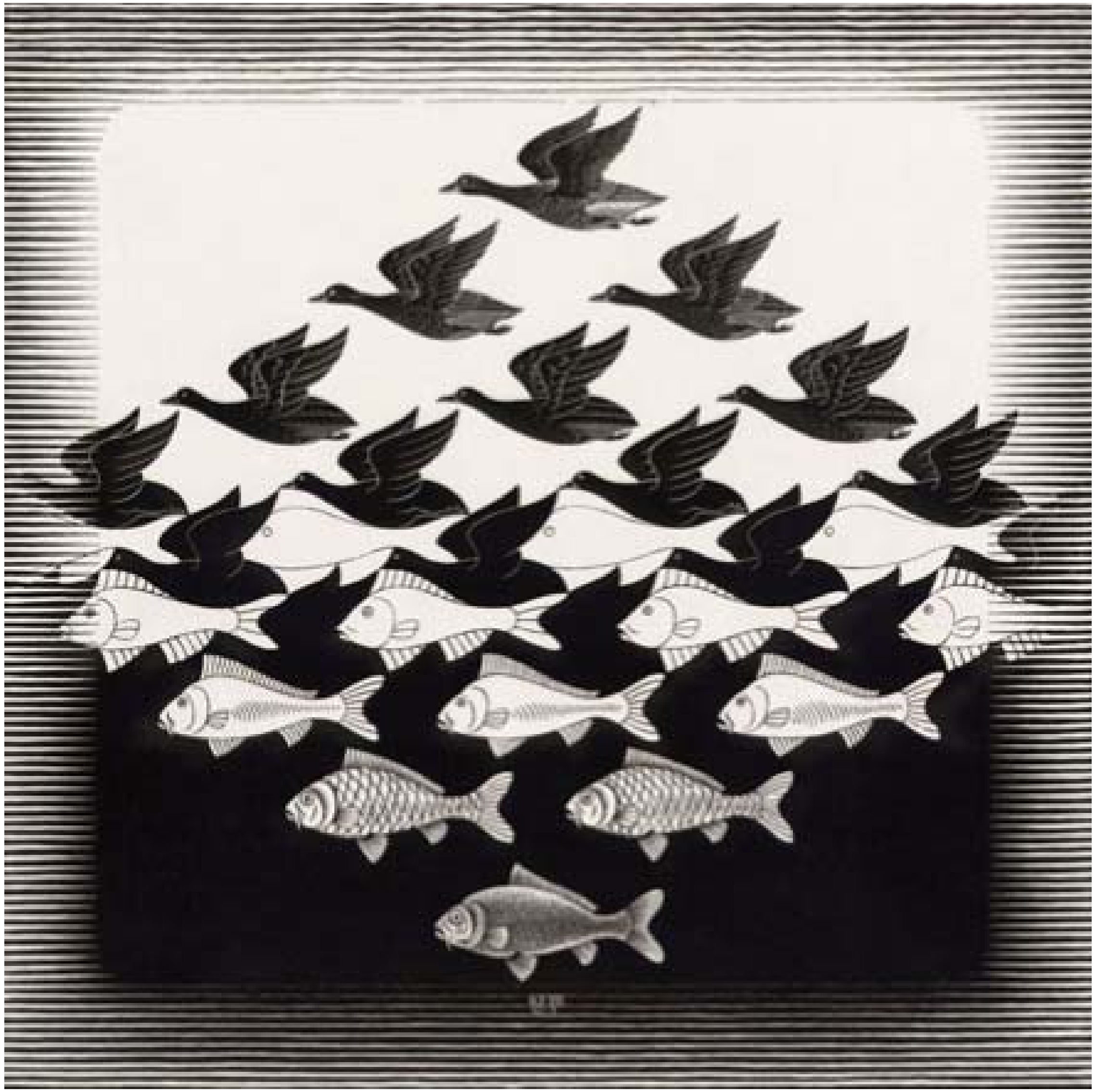
- Work in progress
- Some are incursions (of a hep-ph guy) into hep-th territory

0. TUNNELING POTENTIAL FORMALISM

JRE'1805

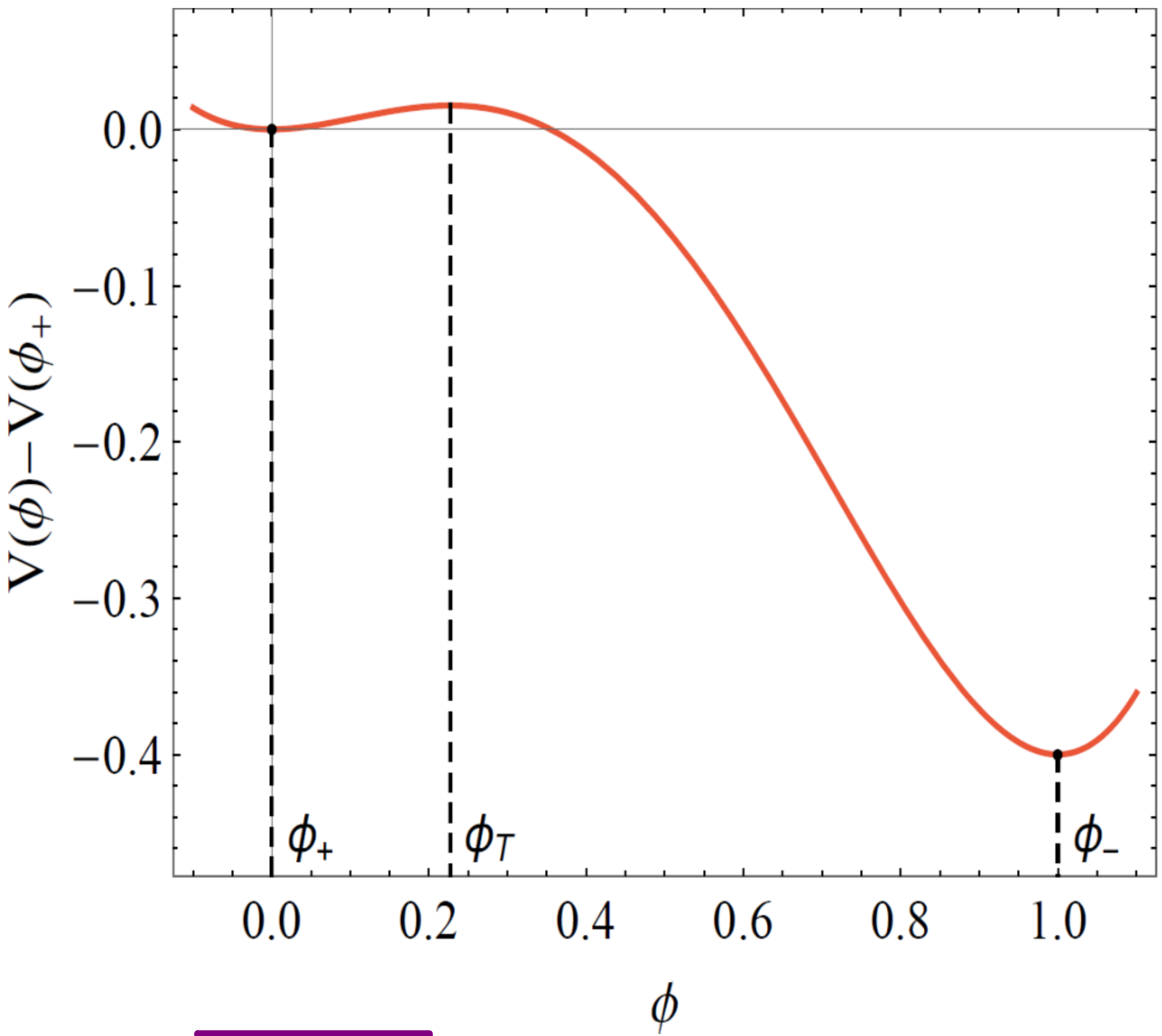


WHY VAC. DECAY ?

- BSM/SM in false vacuum
- Finite lifetime of the universe?
- Early universe phase transitions
- Relevant during inflation
- How the string vacuum landscape is populated

⋮

THE ORIGINAL PROBLEM



$$\Gamma/V = A\, e^{-S}$$

Calculate this

EUCLIDEAN FORMALISM

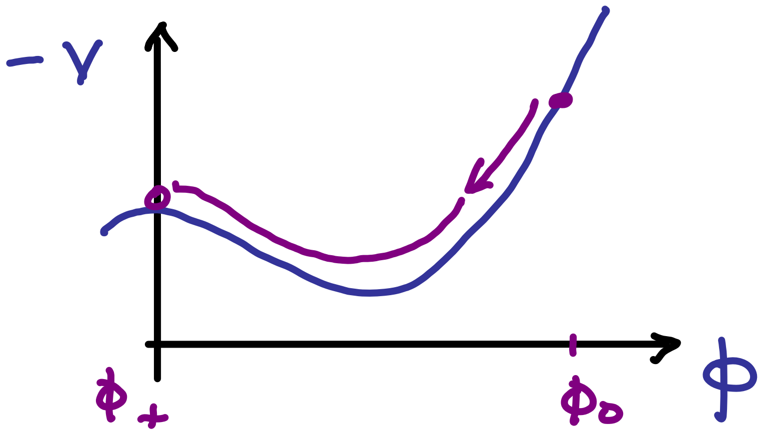
Coleman '77

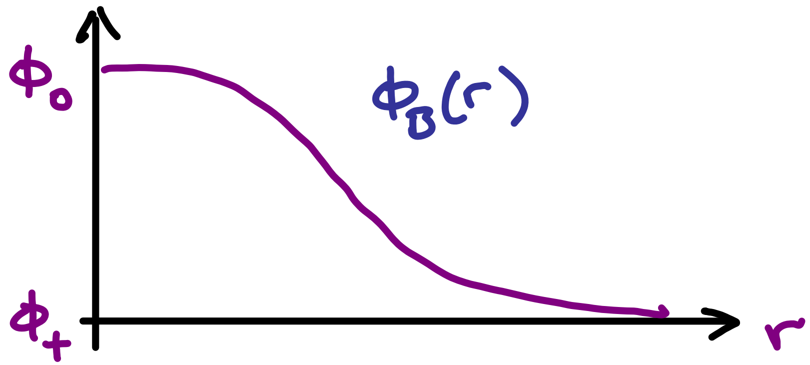
Euclidean bounce $\phi_B(r)$ extremizes

$$S_E = 2\pi^2 \int_0^\infty dr\, r^3 \left[\frac{1}{2} \dot{\phi}^2 + V(\phi) - V_+ \right]$$

$$\Rightarrow \qquad \ddot{\phi} + \frac{3}{r} \dot{\phi} = \frac{\partial V}{\partial \phi}$$

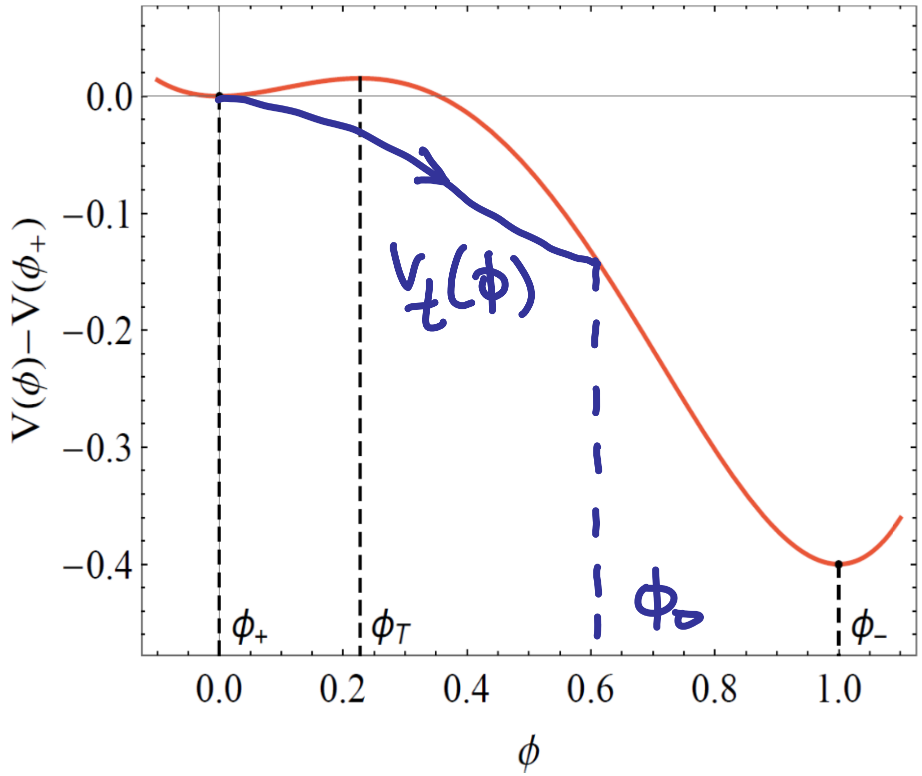
with $\phi(0) = \phi_0 \quad \dot{\phi}(0) = 0 \quad \phi(\infty) = V_+$





TUNNELING POTENTIAL FORMALISM

JRE'1805



$$S[V_t] = 54\pi^2 \int_{\phi_+}^{\phi_0} \frac{(V - V_t)^2}{(-V_t')^3}\, d\phi$$

$$S = \underset{V_t}{\mathrm{Min}}\, S[V_t] \qquad \phi_0 = \text{Euclidean } \phi(0)$$

DE-EUCLIDEANIZE

JRE '1805

⇨ Get rid of Euclidean quantities in terms of V & V_t ones

LINK : $V_t = V - \frac{1}{2}\dot{\phi}^2$

⇨ $\dot{\phi} = -\sqrt{2(V-V_t)}$ $\quad\quad$ $\ddot{\phi} = \frac{d}{d\phi}\left(-\sqrt{2(V-V_t)}\right)\dot{\phi} = V' - V_t'$

EoM : $\ddot{\phi} + \frac{3}{r}\dot{\phi} = V'$ $\quad$ ⇨ $r = \frac{3\sqrt{2(V-V_t)}}{-V_t'}$

$\frac{d}{dr}(\ldots)$ ⇨ $(4V_t' - 3V')V_t' + 6(V-V_t)V_t'' = 0$

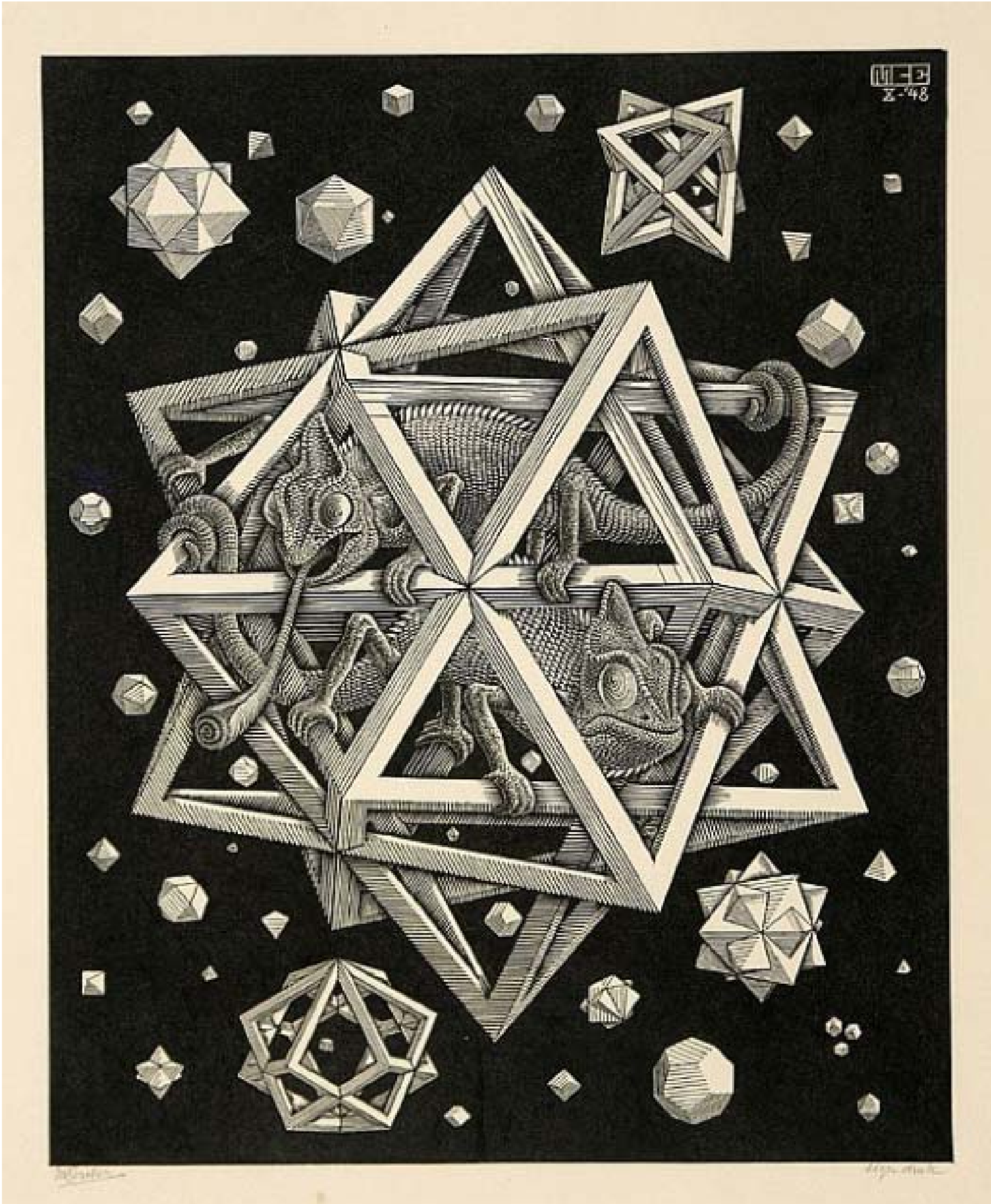
Action gives this EoM for V_t

PROPERTIES & APPS OF V_t-FORM.

- ★ V_t monotonic ⇨ Easy to approximate JRE'1805
- ★ $S[V_t]$ minimized ($\phi_B(r)$ saddle-point)
  - ⇨ Good for numerics
  - ⇨ Useful for multi-field case JRE, Konstandin'1811
- ★ Generalizes to any dimension
  - $d=3$ applicable to finite T phase transitions JRE'1805
- ★ New way of finding exact examples JRE'1805 JRE, Fortin'2211
- ★ Related to Euclidean form. by a canonical transf. JRE, Jinno, Konstandin'2209

1. EXACT SOLS. FOR $V(\phi_i)$

JRE, Konstandin



WHY ?

Solvable examples are always useful

- Crosscheck/discard general conjectures
- Use as approx. to more complicated cases
- Learn about parametric dependences
- Testbed for numerical codes

⋮

1-FIELD EXACT SOLS.

JRE'1805 ; JRE, Fortin'2211

Solve

$$(4V_t' - 3V')V_t' + 6(V - V_t)V_t'' = 0$$

for V as

$$V(\phi) = V_t + \frac{1}{3}(V_t')^2 \int_{\phi_0}^{\phi} \frac{d\bar{\phi}}{V_t'(\bar{\phi})}$$

Choose simple monotonic V_t

Simplest ex :

$$V_t = -\frac{\lambda}{4}\phi^3\phi_0 \quad \Rightarrow \quad V = -\frac{\lambda}{4}\phi^4$$

EXAMPLE

$$V_t = \phi^2(2\phi - 3)$$

$$\Rightarrow \quad V = \phi^2\left[2\phi - 3 + (1-\phi)^2 \log \frac{(1-\phi)^2\phi_0^2}{(1-\phi_0)^2\phi^2}\right]$$

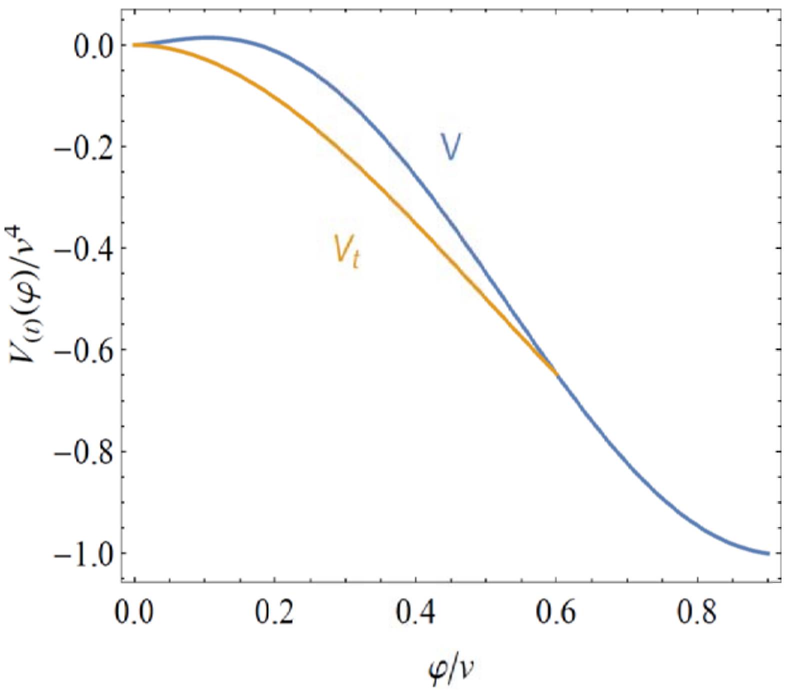
A 1-parameter family (ϕ_0)

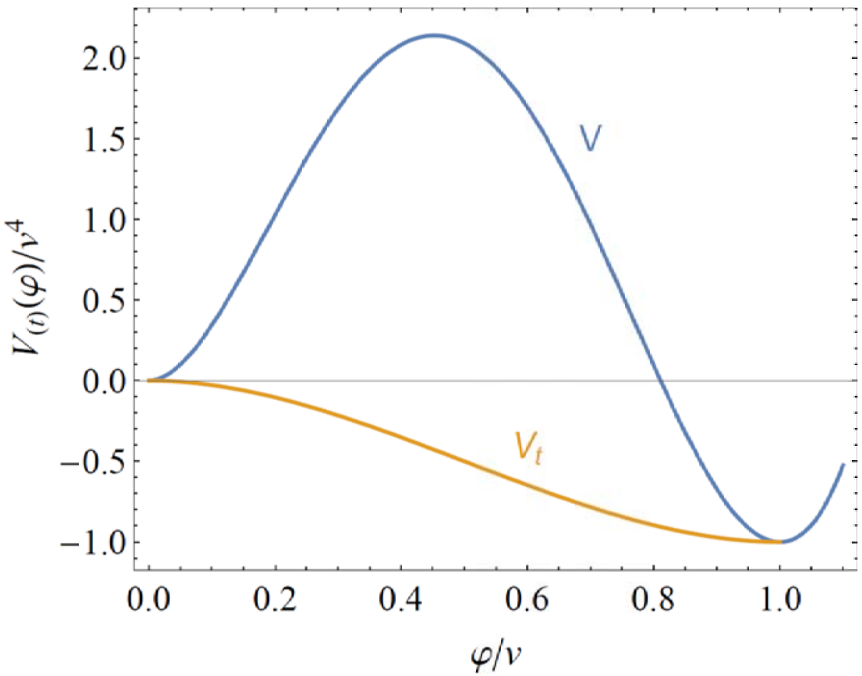
Action : $S = -\frac{\pi^2}{3}\left[\phi_0 + \mathrm{Li}_2\left(\frac{\phi_0}{\phi_0 - 1}\right)\right]$

Bounce : (from $\dot{\phi} = -\sqrt{2(V - V_t)}$)

$$\phi_B(r) = \frac{1}{2}\left[1 - \tanh\left(r^2/2 + \tanh^{-1}(1 - 2\phi_0)\right)\right]$$

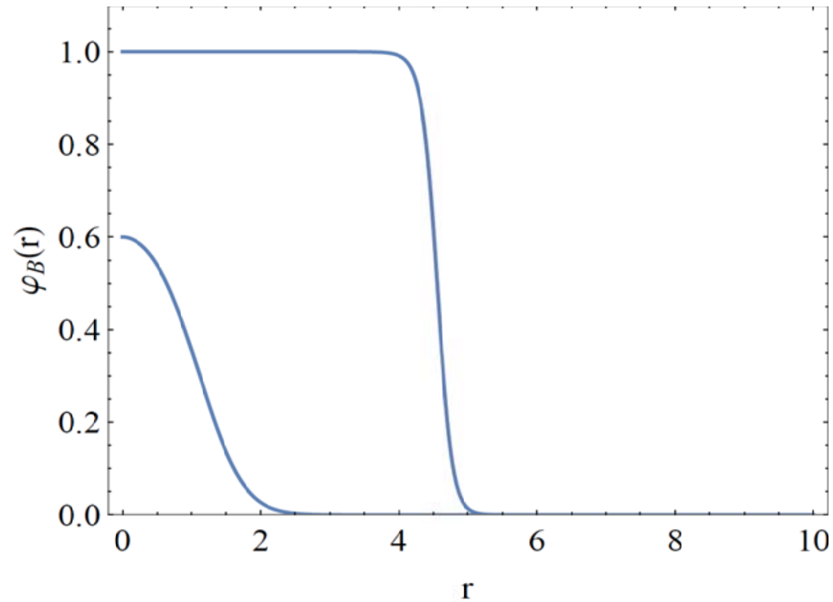
EXAMPLE





$\phi_0 = 0.6$

Thick-wall

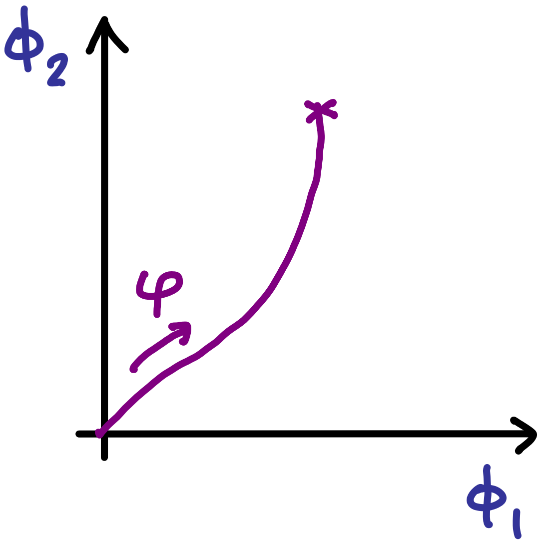


$\phi_0 \simeq 1$

Thin-wall

FROM 1-FIELD TO 2

V_t formalism for 2 fields



$$d\varphi^2 = d\phi_1^2 + d\phi_2^2 \qquad x' \equiv dx/d\varphi$$

$$S[V_t] = 54\pi^2 \int_0^{\varphi_0} \frac{(V-V_t)^2}{(-V_t')^3}\, d\varphi$$

EoMs

Along path: $\quad (4V_t' - 3V')\, V_t' + 6(V-V_t)\, V_t'' = 0$

Path curvature: $\quad 2(V-V_t)\, \vec{\Phi}'' = \vec{\nabla}_T V \equiv \vec{\nabla} V - V' \vec{\phi}'$

$\vec{\nabla}_T V$ TELLS PATH HOW TO CURVE

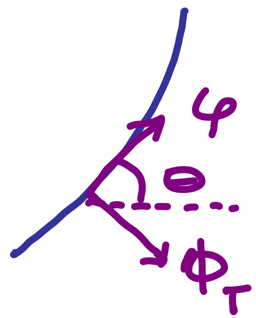
Path curvature : $$2(V - V_t)\,\vec{\Phi}' = \vec{\nabla}_T V \equiv \vec{\nabla} V - V'\vec{\Phi}'$$

FROM 1-FIELD TO 2

Step 1 1-field solution along path $V(\varphi)$ $V_t(\varphi)$

Step 2 Choose any path you like

Step 3 Integrate curvature EoM



$$2(V-V_t)\,\theta' = -\partial V/\partial \phi_T$$

in the transverse direction. Many solutions

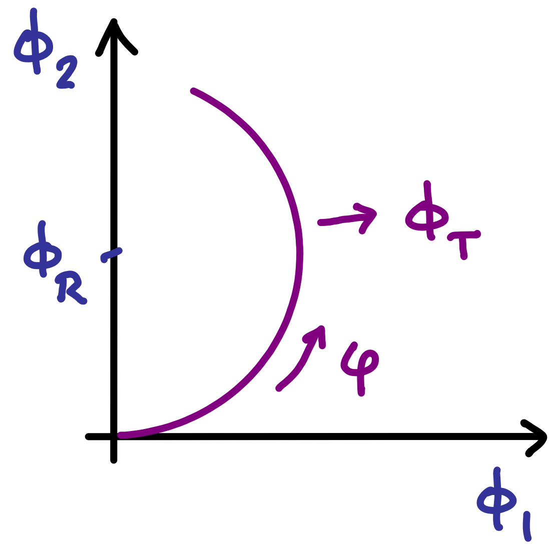
$$V(\varphi,\phi_T) = V(\varphi) - 2f(\phi_T)\left[V(\varphi)-V_t(\varphi)\right]\theta'(\varphi) + g(\phi_T,\varphi)$$

with $f(\phi_T) = \phi_T + O(\phi_T^2)$ $\quad g(\phi_T,\varphi) = \phi_T^2\, g_0(\varphi) + O(\phi_T^3)$

2-FIELD EXAMPLE

Use $V(\varphi)$, $V_t(\varphi)$ of 1-field example given ϕ_0

Take a circular path



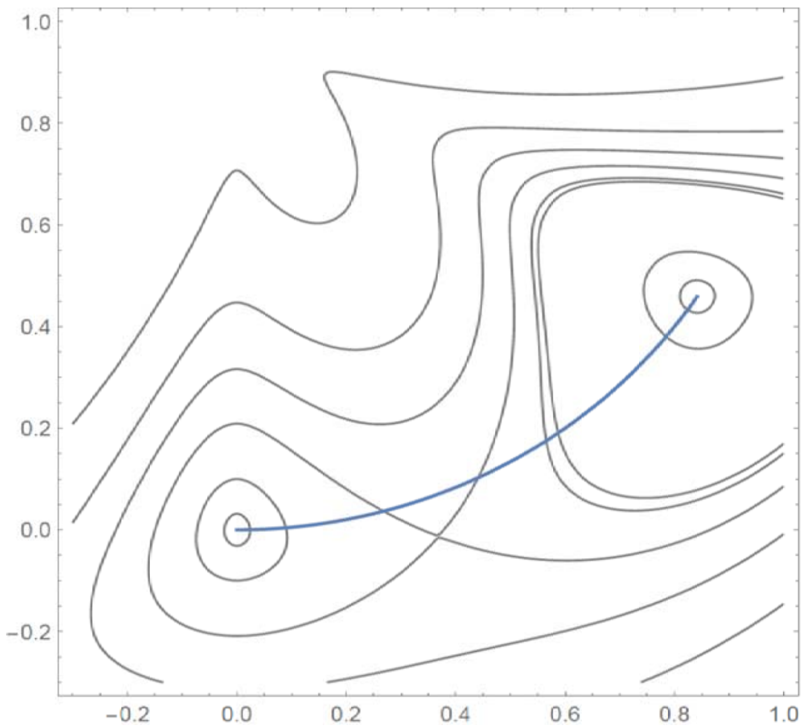
$(\phi_1, \phi_2) \leftrightarrow (\varphi, \phi_T)$

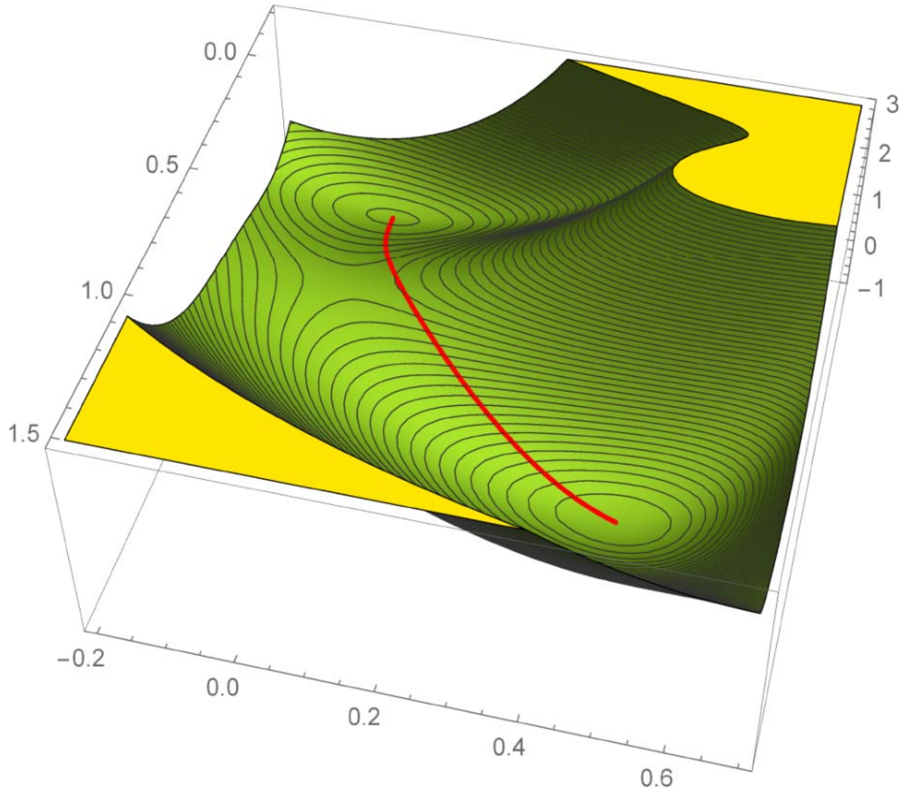
$$\phi_1 = (\phi_R + \phi_T)\sin\varphi$$

$$\phi_2 = \phi_R - (\phi_R + \phi_T)\cos\varphi$$

2-FIELD EXAMPLE

For $f(\phi_T) = \phi_T - 20\phi_T^2$ and $g(\varphi, \phi_T) = \phi_T^2$





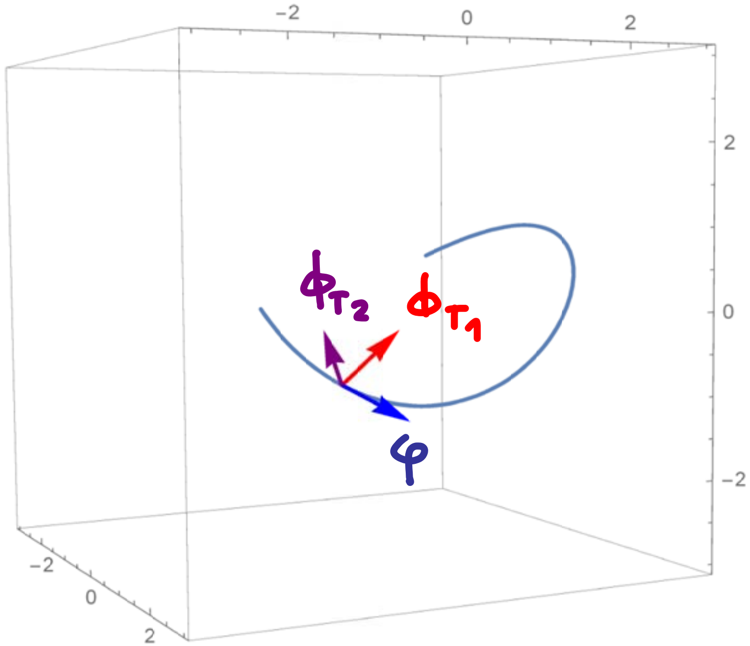
$S[V_t]$ as for 1-field

⇨ Standard to test numerical codes

FURTHER WORK

★ Extend to $n>2$ fields

Use Frenet-Serret basis

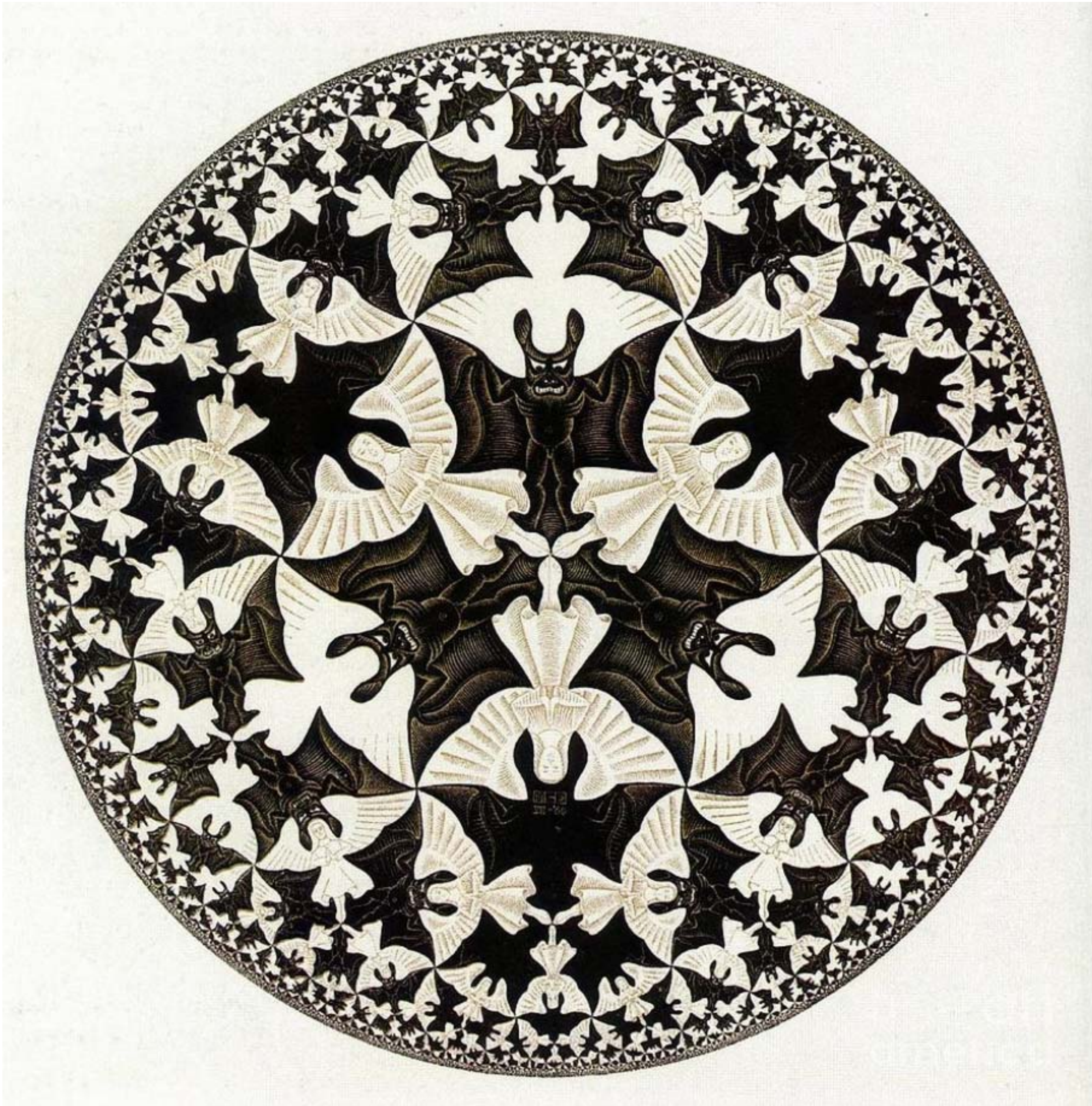


★ Include gravitational corrections

1-field problem solved

JRE, Fortin '22 11

2. GRAVITY CORRECTIONS



JRE '1808
JRE, Huertas, Fortin '2106
JRE, Fortin '2211

WHY ?

Gravity relevant if tunneling involves

$\Delta\phi \lesssim m_p$ or $\Delta V \lesssim m_p^4$ or $|V_+| \lesssim m_p^4$

Note $\Delta\phi \lesssim m_p$ already in SM...

Need to include reaction of the metric.

Gravity can cause qualitative changes

EUCLIDEAN FORMALISM W/GRAVITY

Coleman, De Luccia '80

Euclidean bounce $\phi_B(\xi)$ and metric function $\rho(\xi)$

$O(4)$ $$ds^2 = d\xi^2 + \rho^2(\xi)\, d\Omega_3^2$$

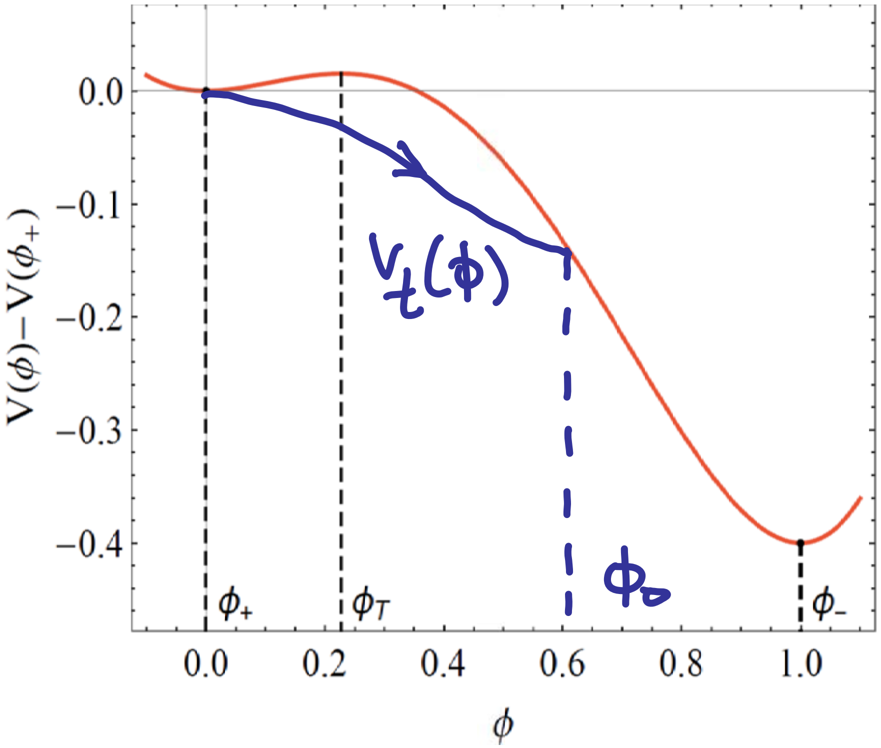
$$\Rightarrow \quad \ddot{\phi} + \frac{3\dot{\rho}}{\rho}\dot{\phi} = \frac{\partial V}{\partial \phi} \qquad \dot{\rho}^2 = 1 + \frac{\kappa \rho^2}{3}\left(\frac{1}{2}\dot{\phi}^2 - V\right)$$

$$S = \Delta S_E = S_E[\phi_B] - S_E[\phi_+]$$

$\kappa \equiv 1/m_P^2$

V_t FORMALISM w/ GRAVITY

JRE'1808



$$S[V_t] = \frac{6\pi^2}{\kappa^2}\int_{\phi_+}^{\phi_0} \frac{(D+V_t')^2}{D V_t^2}\, d\phi$$

with

$$D = \sqrt{(V_t')^2 + 6\kappa (V - V_t) V_t} \qquad \kappa \equiv 1/m_P^2$$

DE-EUCLIDEANIZE

JRE '1808

⇨ Get rid of Eucl. quant. in terms of V & V_t

$$V_t = V - \frac{1}{2}\dot{\phi}^2$$

⇨ $\dot{\phi} = -\sqrt{2(V-V_t)}$ $\qquad$ $\ddot{\phi} = \frac{d}{d\phi}\left(-\sqrt{2(V-V_t)}\right)\dot{\phi} = V' - V_t'$

EoMs: $\quad \ddot{\phi} + \frac{3\dot{\rho}}{\rho}\dot{\phi} = \frac{\partial V}{\partial\phi} \qquad \dot{\rho}^2 = 1 + \frac{\kappa\rho^2}{3}\left(\frac{1}{2}\dot{\phi}^2 - V\right)$

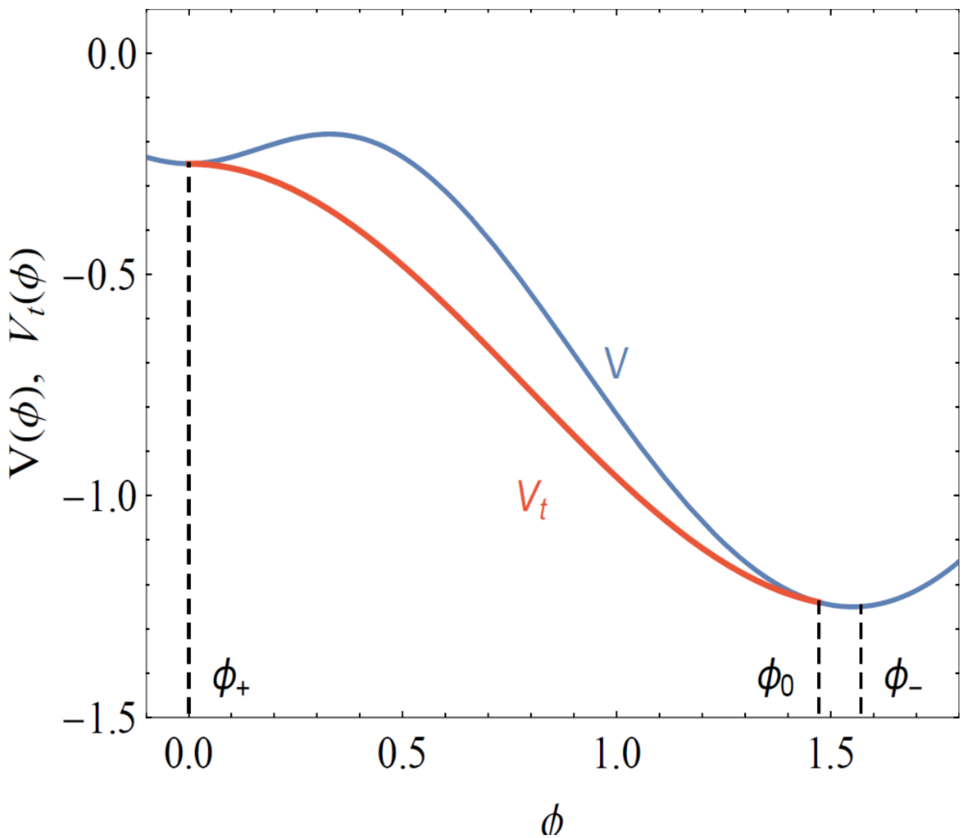
⇨ $\rho = \frac{3\sqrt{2(V-V_t)}}{D}$

$\frac{d}{d\xi}(\ldots)$ ⇨ $0 = (4V_t' - 3V')V_t' + 6(V-V_t)\left[V_t'' + \kappa(3V - 2V_t)\right]$

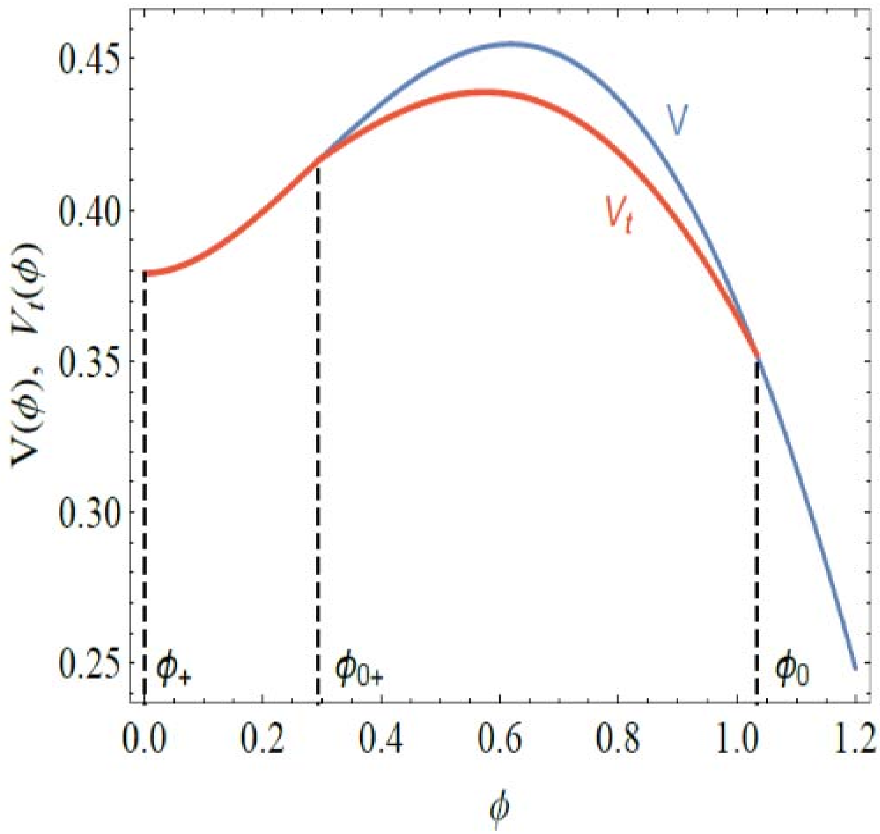
Action gives this EoM for V_t

PROPERTIES & APPS OF V_t-FORM.

★ Universal formula, valid for AdS, Minkowski or dS



Minkowski or AdS



dS

$$S[V_t] = \Delta S_E$$

PROPERTIES & APPS OF V_t-FORM.

- ★ Universal formula, valid for AdS, Minkowski or dS

- ★ For dS, $V_+ \uparrow \Rightarrow$ no CdL bounce

  Hawking-Moss decay with rate

$$S_{HM} = \frac{24\pi^2}{\kappa^2}\left(\frac{1}{V_+} - \frac{1}{V_{top}}\right)$$

$S[V_t]$ reproduces this ✓

PROPERTIES & APPS OF V_t-FORM.

- ★ Universal formula, valid for AdS, Minkowski or dS
- ★ For dS, $V_+ \uparrow$ ⇒ no CdL bounce
  Hawking–Moss decay rate ✓
- ★ For AdS, Minkowski $(V_+ \leq 0)$
  gravitational quenching of decay possible

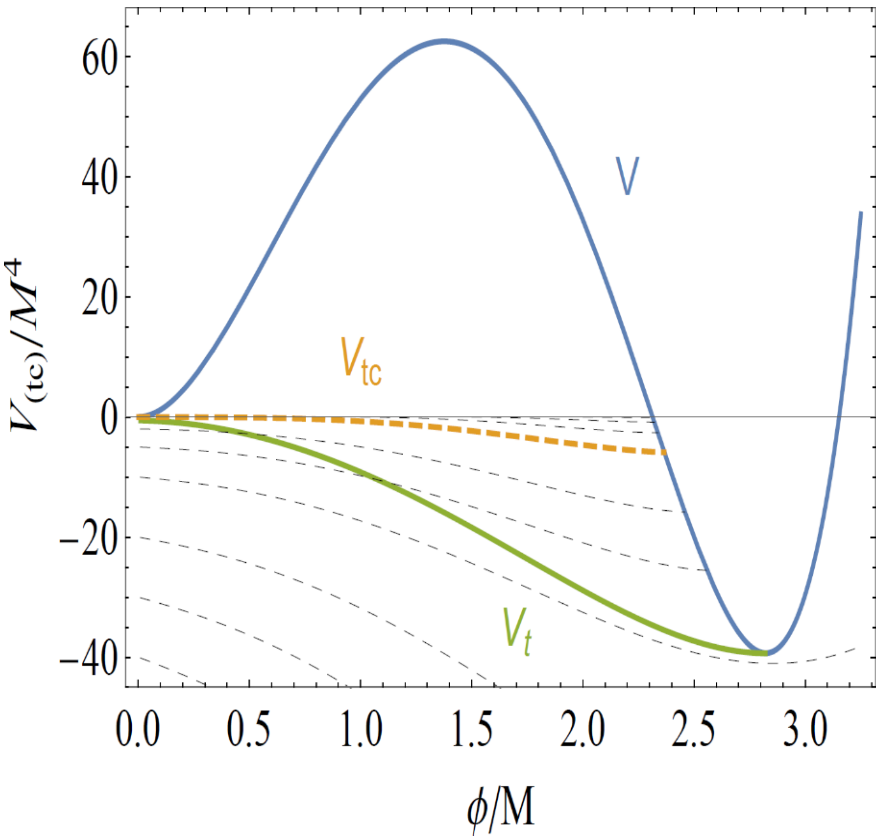
Coleman, De Luccia '80

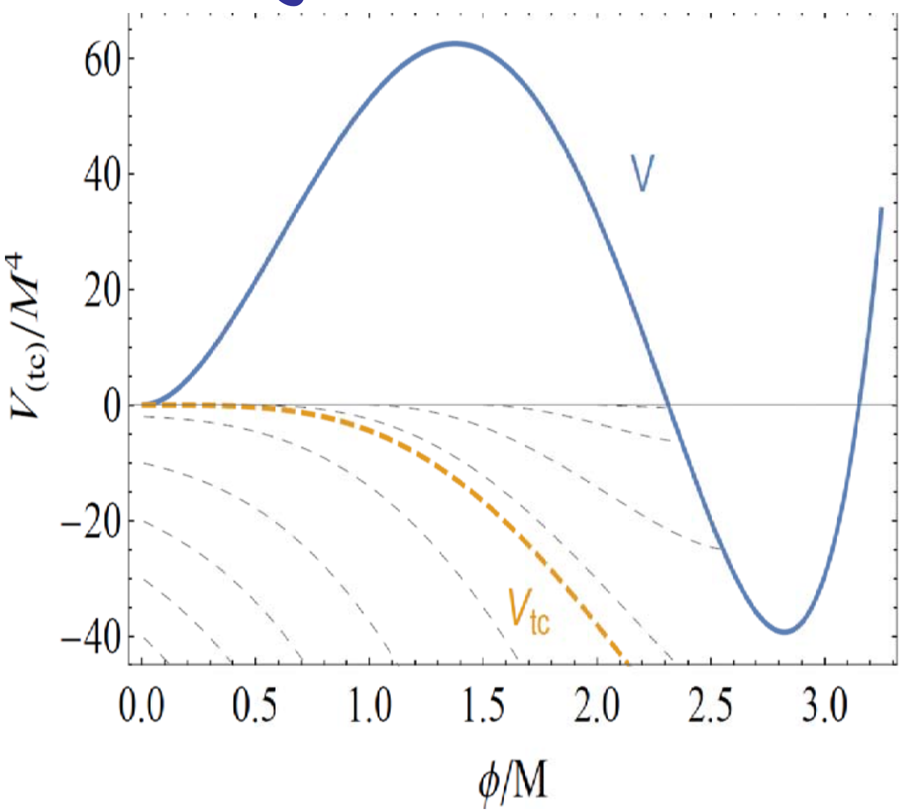
$$D = \sqrt{(V_t')^2 + 6\kappa(V - V_t)V_t} \quad \text{must be real}$$

$$|V_t'| > |V_{tc}'| \equiv \sqrt{6\kappa(V - V_t)(-V_t)}$$

GRAVITATIONAL QUENCHING

Weak grav. effects ⟶ Strong grav. effects





Decays

No decay

NO EXIT

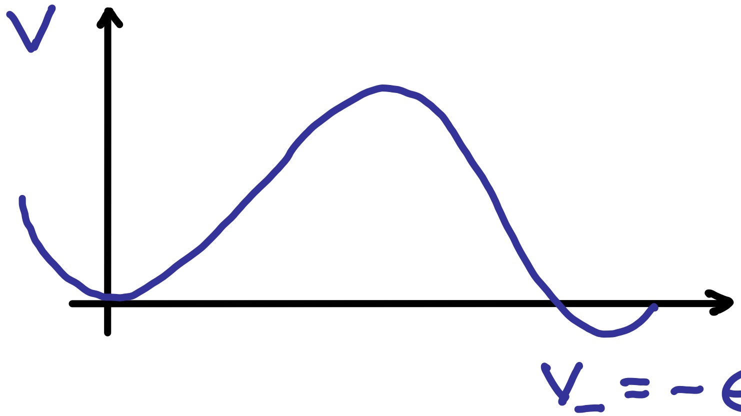
JRE '2005

Positive Energy theorem

V_{t_c} AS BRINICLE ("FINGER OF DEATH")

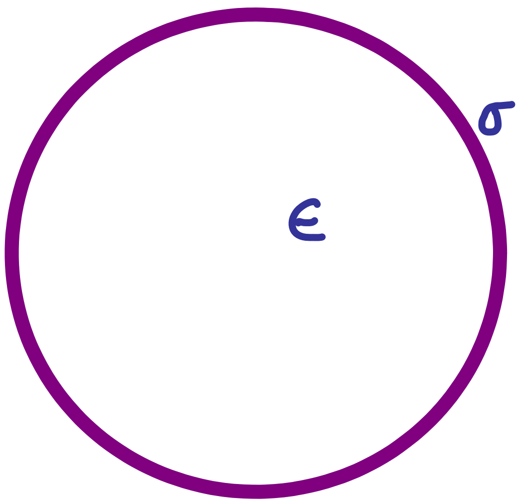
https://www.youtube.com/watch?v=lAupJzH31tc&t=100s
https://www.youtube.com/watch?v=SNr6k6zgJoA

GRAVITATIONAL QUENCHING



No gravity

$$E \sim -\epsilon R^3 + \sigma R^2 = 0$$



large $R \sim \sigma/\epsilon$

With gravity

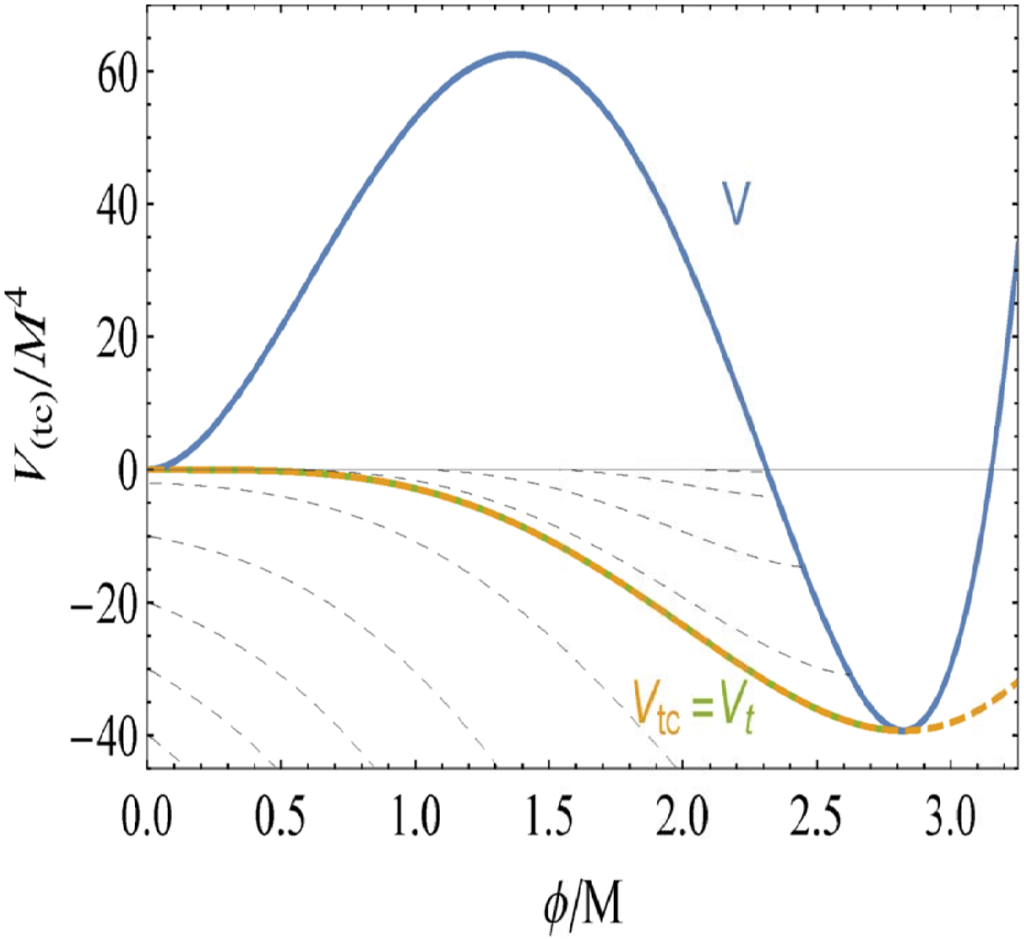
$$E \sim -\epsilon R^2 + \sigma R^2$$

$E=0$ not guaranteed

GRAVITATIONAL QUENCHING

JRE'2005

Critical case



Domain Wall

$$D = 0$$

$$\Downarrow$$

$$V = V_t' - \frac{(V_t')^2}{6\kappa V_t}$$

Critical Potentials

Boucher'84
Abbott, Park '85 '86

PROPERTIES & APPS OF V_t-FORM.

- ★ Universal formula, valid for AdS, Minkowski or dS
- ★ For dS, $V_+ \uparrow$ ⇨ no CdL bounce
  Hawking-Moss decay rate ✓
- ★ For AdS, Minkowski ($V_+ \leq 0$)
  gravitational quenching of decay ✓
- ★ New way of finding exact examples
  JRE, Fortin, Huertas '2106

3. STABILITY OF ADS MAXIMA

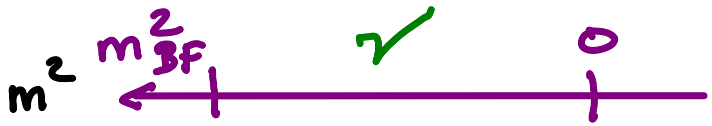
w/ Jinno

WHY !?

★ $V(\phi) = V_+ + \frac{1}{2} m^2 \phi^2 + \dots \qquad V_+ , m^2 < 0$

Stable if $\quad m^2 \geq m^2_{BF} \equiv \frac{3\kappa}{4} V_+$

Breitenlohner, Freedman '82



Heuristics:

(d+1) AdS:
$$ds^2 = -\underbrace{\left(1+\frac{\rho^2}{L^2}\right)}_{-1-2V_N} dt^2 + \frac{d\rho^2}{1+\rho^2/L^2} + \rho^2 d\Omega^2_{d-2}$$

$$L^2 = \frac{3}{\kappa |V_+|}$$

$$\Rightarrow V_N = \frac{1}{2}\frac{\rho^2}{L^2}$$

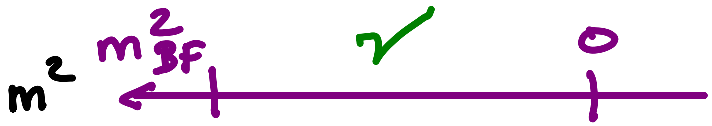
harmonic with $\omega^2 = 1/L^2$

WHY !?

★ $V(\phi) = V_+ + \frac{1}{2} m^2 \phi^2 + \dots \qquad V_+ ,\ m^2 < 0$

Stable if $\quad m^2 \geq m^2_{BF} \equiv \frac{3\kappa}{4} V_+$

Breitenlohner, Freedman '82



★ Many SUGRA theories from strings have tachyonic scalars with $m^2_{BF} \leq m^2 < 0$

★ 

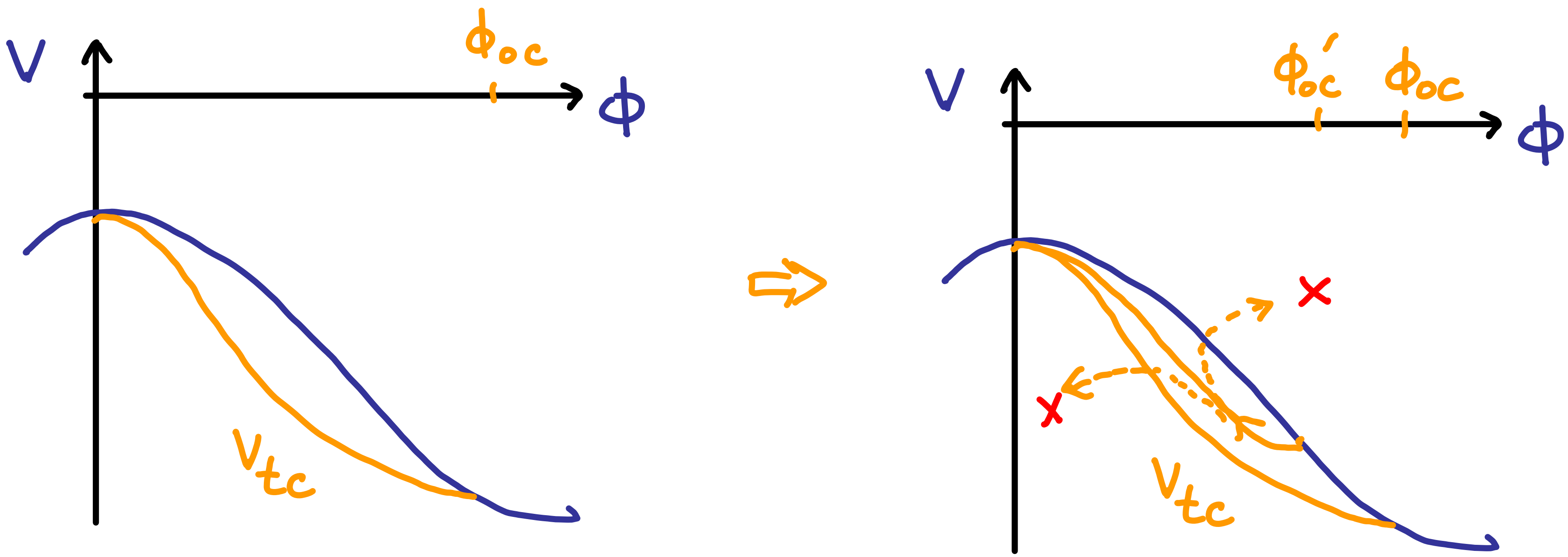
is particularly interesting

- AdS/CFT
- Black holes with scalar hair
- Designer gravity

CdL INSTABILITY ?

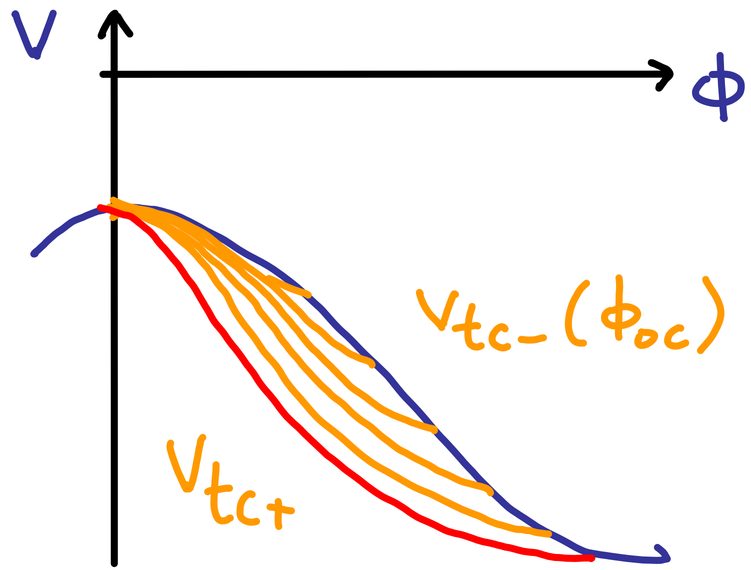
Apply V_{tc} technique

$D = 0$: $\quad V_{tc}' = -\sqrt{6\kappa(V - V_{tc})(-V_{tc})} \qquad V_{tc}(0) = V_+$



CdL INSTABILITY ?

⇨ 1-parameter family of V_{tc} solutions



The same happens for V_t solutions

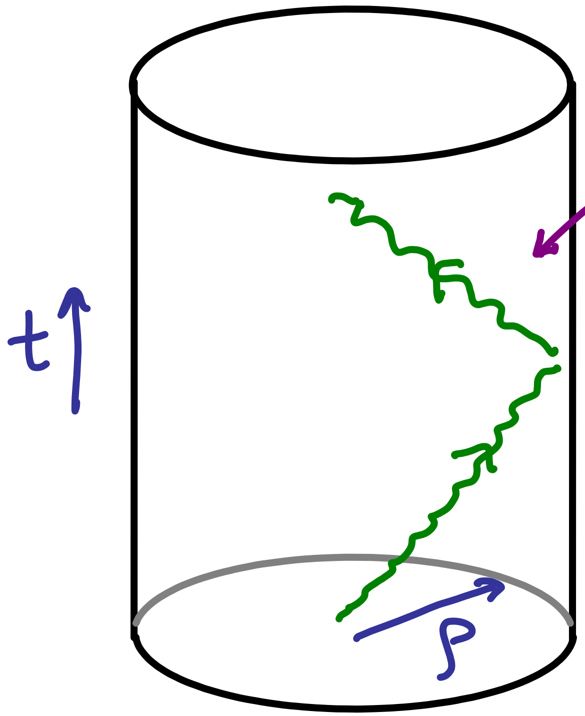
What does it mean !? Always unstable !?

Non-trivial asymptotic behaviour of $\phi(\rho \to \infty)$

AdS AS GRAVITY CAN

(d+1) AdS : $ds^2 = -\left(1+\frac{\rho^2}{L^2}\right)dt^2 + \frac{d\rho^2}{1+\rho^2/L^2} + \rho^2 d\Omega^2_{d-2}$

$\left(\text{Set} \quad L^2 = \frac{3}{\kappa|V_+|} = 1 \qquad V_+ = -3 \qquad m^2_{BF} = -\frac{d^2}{4}\right)$



conformal boundary
$\rho \to \infty$

Boundary conditions crucial

SCALAR FIELD ASYMPTOTICS

$$\phi(\rho\to\infty) \sim \frac{1}{\rho^{K}} \qquad K \text{ from } (\Box + m^2)\phi = 0$$

$$\Rightarrow \quad \phi(\rho\to\infty) \sim \frac{\alpha}{\rho^{\Delta_-}} + \frac{\beta}{\rho^{\Delta_+}} + \dots$$

with $\Delta_\pm \equiv \frac{d}{2}\left[1 \pm \sqrt{1 + \frac{m^2}{d^2/4}}\,\right]$ $\quad (\Delta \equiv \Delta_-)$

- For $m^2 > 0$ $\quad \Delta_- < 0 \Rightarrow \alpha = 0$
- For $m^2_{BF} < m^2 < m^2_{BF} + 1 < 0$

Normalizable solutions, both $\Delta_\pm$ ok in a nontrivial way...

SCALAR FIELD ASYMPTOTICS

$$\phi(\rho \to \infty) \sim \frac{\alpha}{\rho^{\Delta_-}} + \frac{\beta}{\rho^{\Delta_+}} + \dots \quad \left(\phi \sim \frac{\alpha}{\rho} + \frac{\beta}{\rho^2} \text{ for } \Delta = 1 \right)$$

- $\alpha \neq 0$ ⇨ slower fall-off of ϕ

⇨ feeds back in metric $\quad g_{\rho\rho} \sim \frac{1}{1+\rho^2} - \frac{\alpha^2}{2\rho^4} + \dots$

⇨ Can modify the asymptotic symmetries (and charges)

⇨ Gives scalar contributions to boundary terms

⇨ Causes divergences in energy, action, ...

SCALAR FIELD ASYMPTOTICS

Charges, energy, action can be modified by scalar boundary terms to get conserved charges, energy and finite action

Henneaux, Martínez, Troncoso, Zanelli '0603

Requires functional relation

$$\beta(\alpha) = W'(\alpha)$$

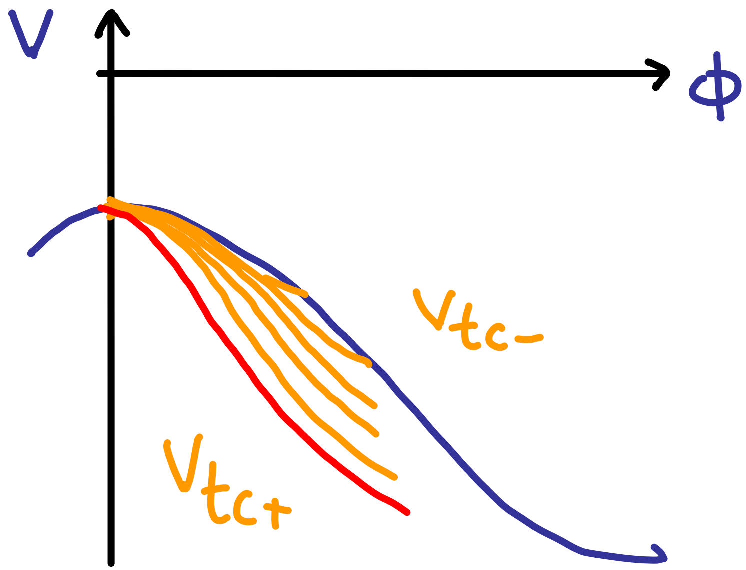
Choice of diff. function $W(\alpha)$ fixes the theory

- Designer gravity Hertog, Horowitz '0412

V_{tc} ASYMPTOTICS

$$\phi(\rho\to\infty) \Rightarrow V_{tc}(\phi\to 0)$$

Take $d=3$ $\quad m_{BF}^2 < m^2 < m_{BF}^2 + 1 \Rightarrow 1/2 < \Delta_- \leq 3/2$



$$V = -3 - \frac{1}{2}\Delta_-\Delta_+\phi^2 + \dots$$

$$V_{tc-} \simeq -3 - \frac{3}{2}\Delta_-\phi^2 + A_{c-}\phi^{3/\Delta_-}$$

$$V_{tc+} \simeq -3 - \frac{3}{2}\Delta_+\phi^2 + \dots$$

$A_{c-} \in (-\infty, \infty)$ free parameter

$\phi \leftrightarrow V_{tc}$ ASYMPTOTICS

Field associated to V_{tc} is of domain-wall type $D=0$

$$\dot{\phi} = -\sqrt{2(V-V_{tc})} \qquad \dot{\rho} = \rho\sqrt{-\kappa V_{tc}/3}$$

$$\Downarrow$$

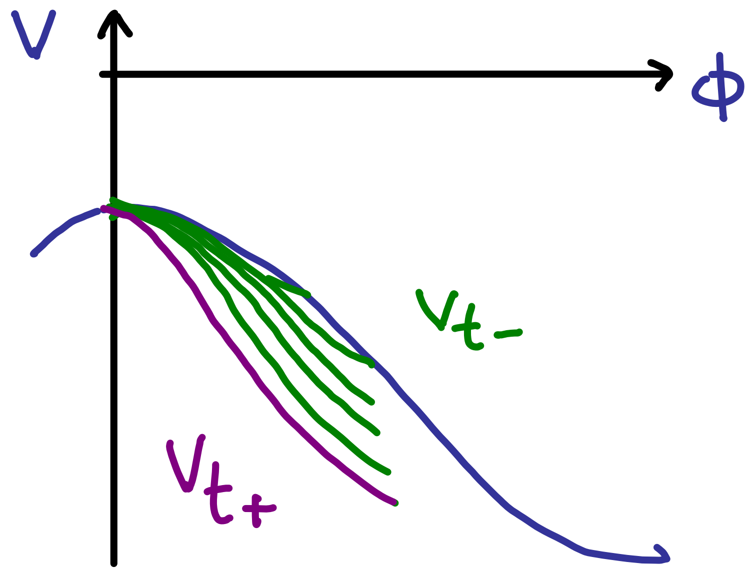
$$\frac{d\phi}{d\rho} = -\frac{1}{\rho}\sqrt{\frac{6(V-V_{tc})}{-\kappa V_{tc}}}$$

Use to match $\phi(\rho)$ with $V_{tc}(\phi)$ expansions

V_{tc-} : $\qquad \beta = A_{c-}\dfrac{\alpha^{\Delta_+/\Delta}}{\Delta(2\Delta-3)}$

V_{tc+} : $\qquad \alpha = 0$

V_t ASYMPTOTICS



$$V = -3 - \frac{1}{2}\Delta_-\Delta_+\phi^2 + \dots$$

$$V_{t-} \simeq \overbrace{-3 - \frac{3}{2}\Delta_-\phi^2 + A_{c-}\,\phi^{3/\Delta_-}}^{V_{tc-}} + A_-\,\phi^{2+2/\Delta_-} + \dots$$

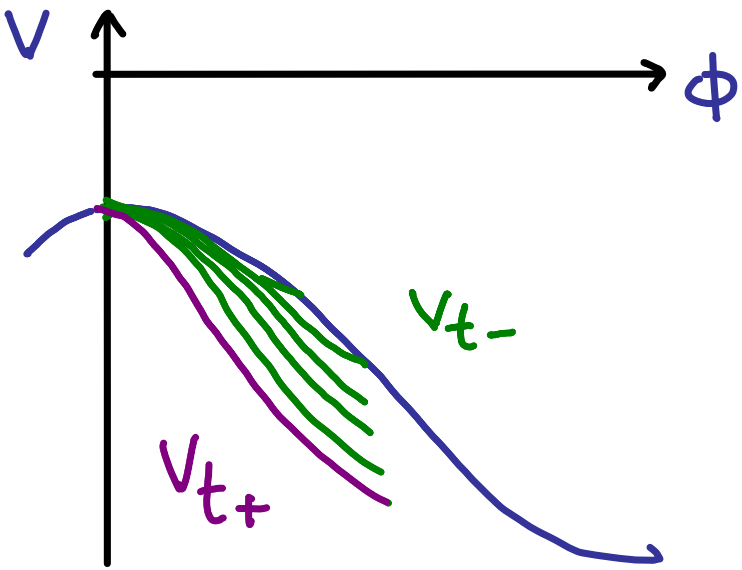
$$V_{t+} \simeq \underbrace{-3 - \frac{3}{2}\Delta_+\phi^2}_{V_{tc+}} + A_+\,\phi^{2+2/\Delta_+} + \dots$$

$A_\pm$ fixed by BC at $\rho = 0$.

And $$D_\pm^2 = -6(2\Delta_\pm - 1)A_\pm\,\phi^{2+2/\Delta_\pm} + \dots$$

$\Rightarrow$ $A_\pm < 0$ required for $D_\pm^2 > 0$

V_t ASYMPTOTICS



$$V = -3 - \frac{1}{2}\Delta_-\Delta_+\phi^2 + \ldots$$

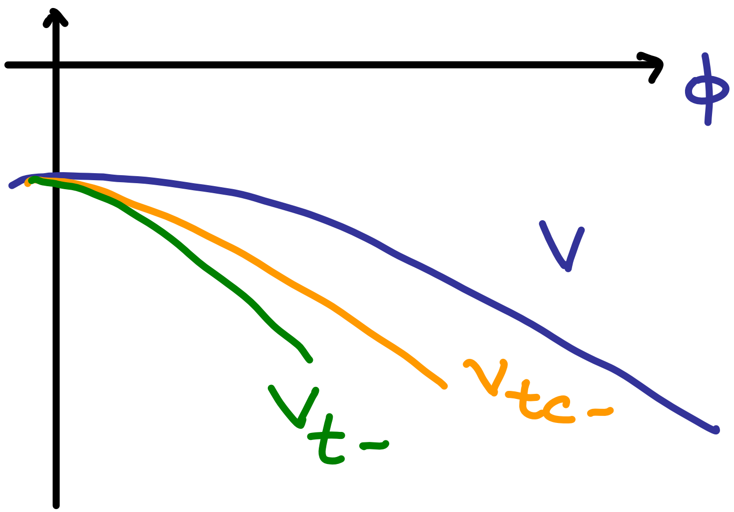
$$V_{t_-} \simeq \overbrace{-3 - \frac{3}{2}\Delta_-\phi^2 + A_{c-}\,\phi^{3/\Delta_-} +}^{V_{tc-}}$$

$$+ A_-\,\phi^{2+2/\Delta_-} + \ldots$$

$$V_{t_+} \simeq \underbrace{-3 - \frac{3}{2}\Delta_+\phi^2}_{V_{tc+}} + A_+\,\phi^{2+2/\Delta_+} + \ldots$$

Pairing

$$V_{t-} \leq V_{tc-}$$



$\phi \leftrightarrow V_t$ ASYMPTOTICS

Field associated to V_t is of CdL type $D^2 \geq 0$

$$\rho^2 = \frac{18(V - V_t)}{D^2}$$

Use to match $\phi(\rho)$ with $V_t(\phi)$ expansions

V_{t_-}:

$$\beta = A_{c-} \frac{\alpha^{\Delta_+/\Delta}}{\Delta(2\Delta - 3)} \qquad \alpha^{2/\Delta} = \frac{3\Delta^2}{2A_-(1-2\Delta)}$$

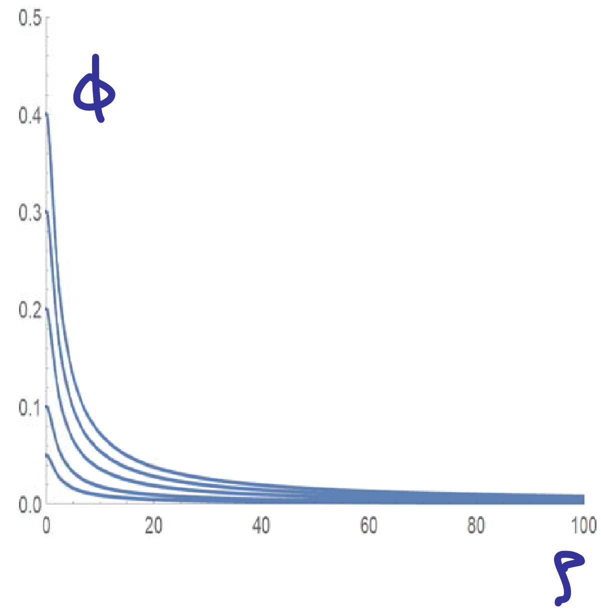
V_{t_+}:

$$\alpha = 0 \qquad \beta^{2/\Delta_+} = \frac{3\Delta_+^2}{2A_+(1-2\Delta_+)}$$

SMALL CdL INSTANTONS

Take $\Delta = 1$ $(\Delta_+ = 2)$ $V = -3 - \phi^2 + \dots$

Small CdL solutions $\phi(0) = \delta \ll 1$



$$\phi(\xi) = \delta\,\mathrm{sech}^2(\xi/2) + O(\delta^2)$$
$$\rho(\xi) = \sinh(\xi) + O(\delta^2)$$

$$\Rightarrow \quad \phi(\rho) = 2\delta\,\frac{\sqrt{1+\rho^2}-1}{\rho^2} + O(\delta^2) \simeq \frac{2\delta}{\rho} - \frac{2\delta}{\rho^2} + O\left(\frac{1}{\rho^3}\right)$$

$$\Rightarrow \quad \alpha = 2\delta \qquad \beta = -\alpha$$

EUCLIDEAN ACTION

$$\Delta S_E = S_G + S_{GHY} + S_B - S_{AdS}$$

$$S_G = -2\pi^2 \int_0^\infty \rho^3 V \, d\xi \;=\; 2\pi^2 \left[\rho_\infty^3 + \rho_\infty \left(-\frac{3}{2} + \delta^2 \right) + 2 - 2\delta^2 \right] + O(1/\rho_\infty)$$

$$S_{GHY} = -\frac{6\pi^2}{\kappa} \rho^2 \dot{\rho} \Big|_0^\infty \;=\; 3\pi^2 \left[-2\rho_\infty^3 - \rho_\infty (1 + 2\delta^2) + \frac{16}{3} \delta^2 \right] + O(1/\rho_\infty)$$

$$\Rightarrow \; S_B = \oint \left[(\nabla\phi)^2 - m^2 \phi^2 \right] = 4\pi^2 \left[\rho_\infty - \frac{8}{3} \right] \delta^2 + O(1/\rho_\infty)$$

$$S_{AdS} = S_G + S_{GHY} \Big|_{\phi \equiv 0} \;=\; 4\pi^2 \left[-\rho_\infty^2 - \frac{3}{2} \rho_\infty + 1 \right] + O(1/\rho_\infty)$$

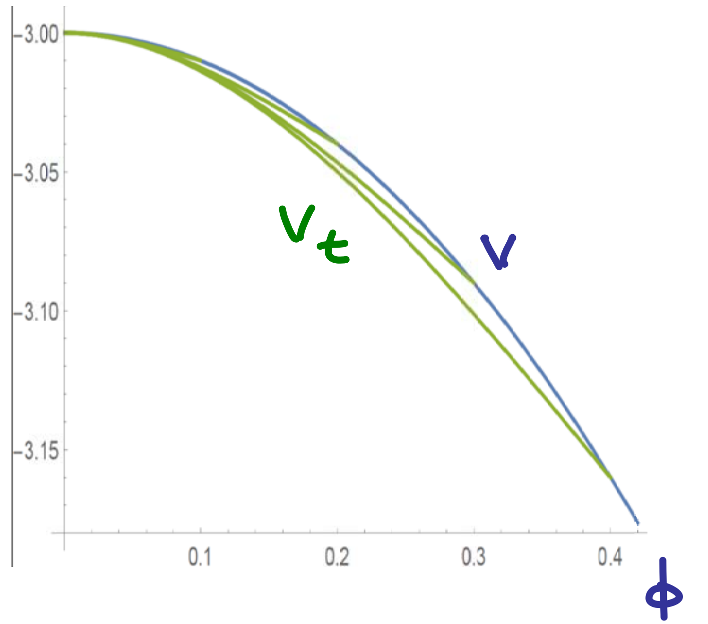
$$\Delta S_E = \frac{4\pi^2}{3} \delta^2 + O(\delta^3)$$

SMALL V_t INSTANTONS

$$V = -3 - \phi^2 \qquad V_t = -3 - \frac{3}{2}\phi^2 + \frac{1}{2\delta}\phi^3 + \dots$$

General exp

$$V_t = \underbrace{-3 - \frac{3}{2}\phi^2 + A_{c-}\phi^3 + \frac{3}{2}A_{c-}^2\phi^4}_{V_{tc-}} + A_-\phi^4 + \dots$$



$$\Rightarrow \quad A_{c-} = \frac{1}{2\delta} \qquad A_- = -\frac{3}{8\delta^2}$$

agree with $\alpha = -\beta = 2\delta$

Action

$$S[V_t] = \frac{6\pi^2}{\kappa^2}\int_0^\delta \frac{(D+V_t')^2}{D V_t^2}\,d\phi = \frac{4\pi^2}{3}\delta^2 + O(\delta^3) \checkmark$$

SIMPLE EXAMPLE OF P.E.T.

Consider $\beta(\alpha) = C \cdot \alpha^{\Delta_+/\Delta}$ (AdS preserved)

$\beta = A_{c-} \dfrac{\alpha^{\Delta_+/\Delta}}{\Delta(2\Delta - 3)} \Rightarrow A_{c-} = \Delta(2\Delta - 3) \cdot C$

For the pair A_c, A

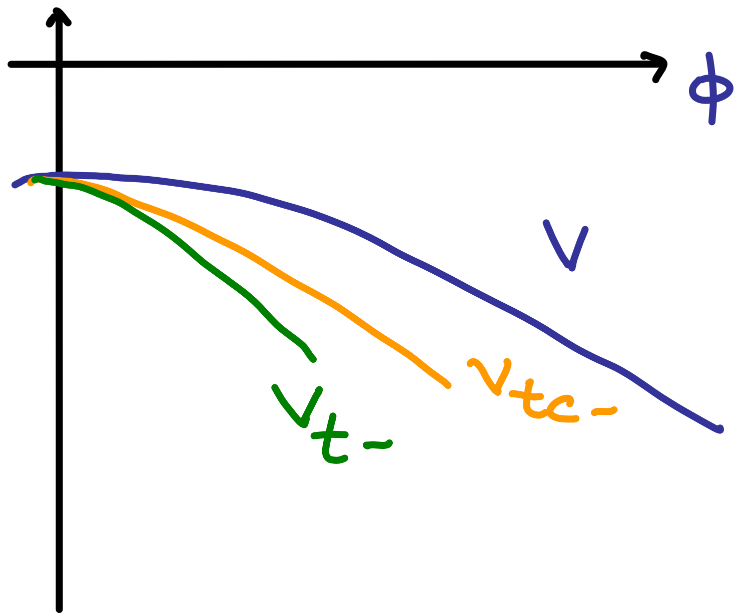
$D^2 > 0$

$\Downarrow$

$A < 0$

$\Downarrow$

$V_{t-} \leq V_{tc-}$



If V_{tc} does not intersect V $\Rightarrow$ No decay

FURTHER WORK

- Establish positive energy theorems for generic

$W(\alpha)$

extending (and simplifying) previous work

Amsel, Marolf '0701
Amsel, Hertog, Hollands, Marolf '0801
Faulkner, Horowitz, Roberts '1006

- CFT interpretation

- CFT flows between AdS vacua in this language ?

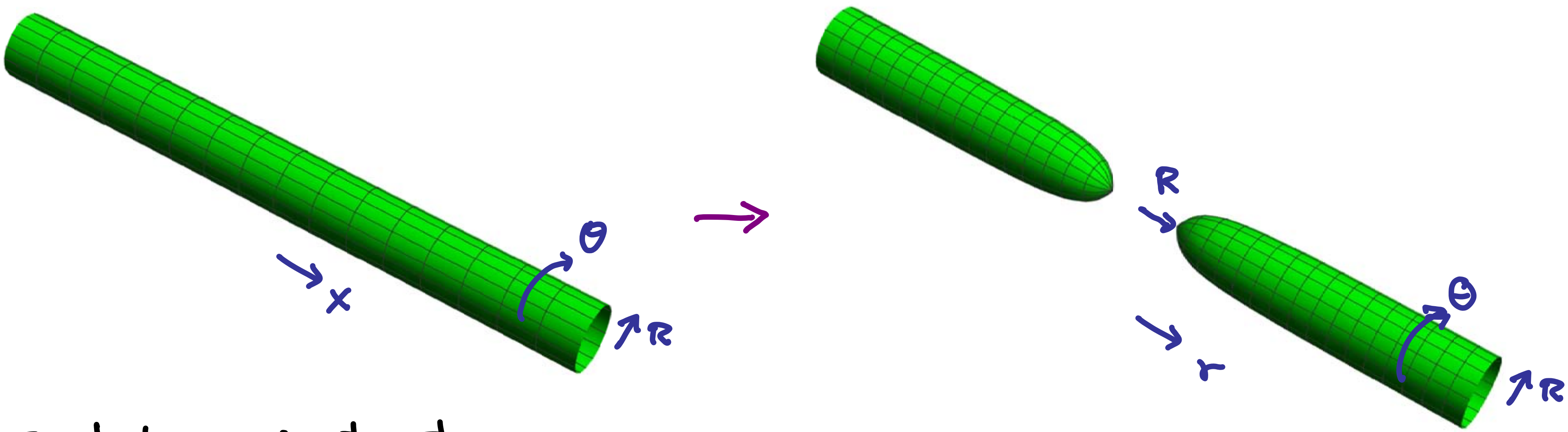
4. BUBBLE OF NOTHING DECAYS

w/ Blanco-Pillado, Huertas, Sousa

BUBBLE OF NOTHING DECAYS

Decays of spacetimes with compactified dim. like

5d KK ($M^4 \times S^1$) Witten' 82



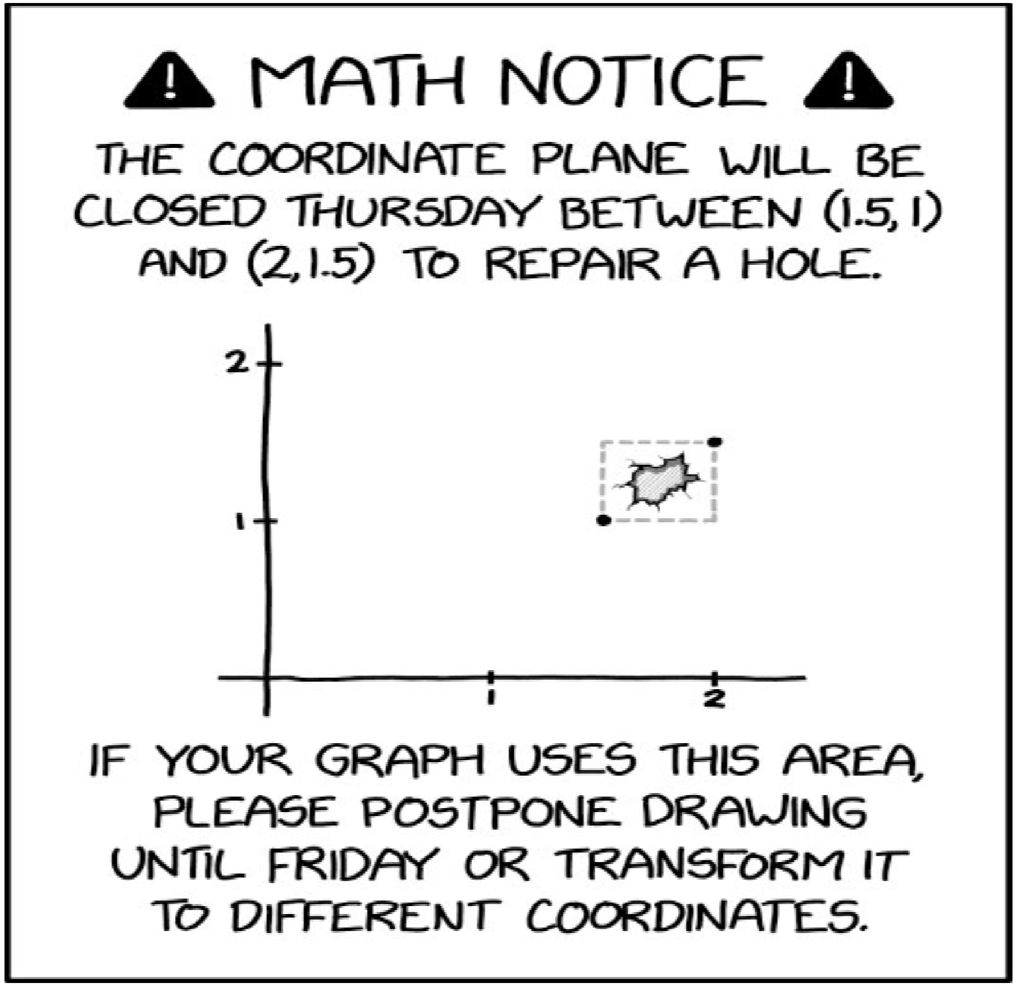
Euclidean instanton:

$$ds^2 = \frac{dr^2}{1-R^2/r^2} + r^2 d\Omega_3^2 + \underbrace{R^2\left(1-\frac{R^2}{r^2}\right)}_{R(r)^2 \to 0 \text{ at } r \to R} d\theta^2$$

with $g = (\pi R m_p)^2$

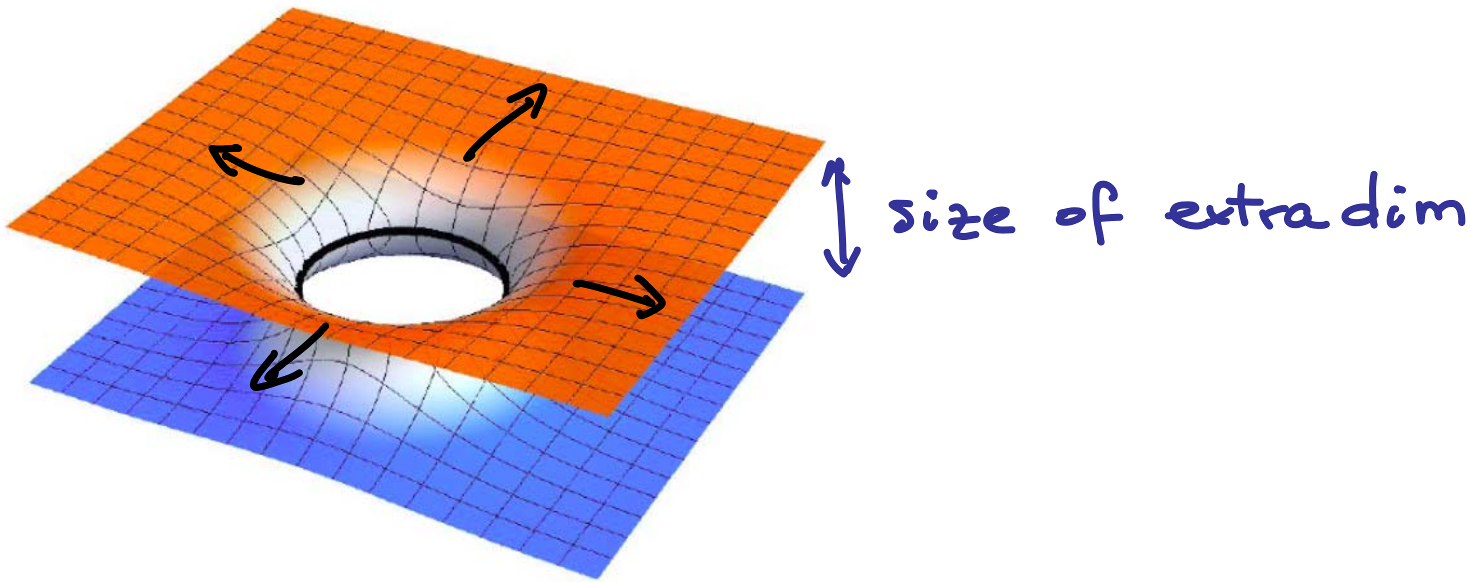
BUBBLE OF NOTHING DECAYS

End-product of tunneling process is a hole in space-time



BUBBLE OF NOTHING DECAYS

End-product of tunneling process is a hole in space-time



It's smooth, w/o singularities, curvature not large

But it grows, eating the whole space

WHY ?

- Relevant for string vacua landscape

- BoNs thought to be ultimate cause of decay of non-SUSY vacua in string compactifications

García-Etxebarría, Montero, Sousa, Valenzuela '2005

4d VIEW

Dine, Fox, Gorbatov '0405

5d → 4d + Scalar ϕ

$$R^2\left(1-\frac{R^2}{r^2}\right) = R^2 e^{-2\sqrt{2\kappa/3}\,\phi} \qquad \begin{cases} r\to R & \phi\to\infty \\ r\to\infty & \phi\to 0 \end{cases}$$

and
$$ds_4^2 = d\xi^2 + \rho(\xi)^2 d\Omega_3^2 \qquad \frac{d\xi}{dr} = \frac{1}{(1-R^2/r^2)^{1/4}}$$

BoN reduces to a CdL problem, with

$$\phi(0)=\infty \qquad \dot\phi(0) = -\infty \qquad \phi(\infty) = \phi_+ = 0$$

Action reproduced (paying attention to boundary terms)

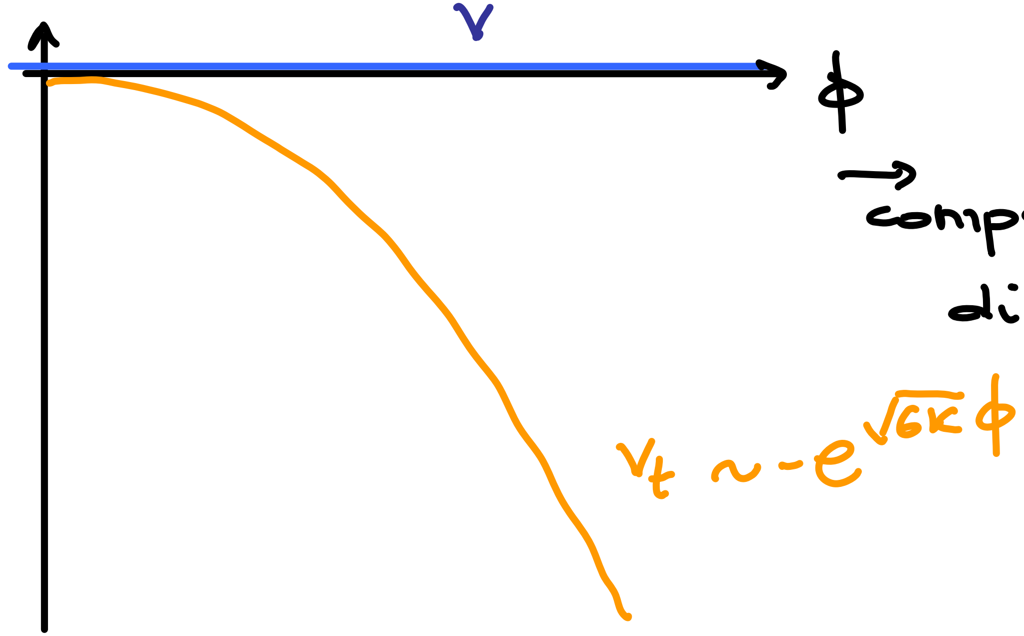
★ 4d approach useful to study effect of $V(\phi)$

Draper, García-García, Lillard '2105

V_t FOR BONS

Blanco-Pillado, JRE, Huertas, Sousa

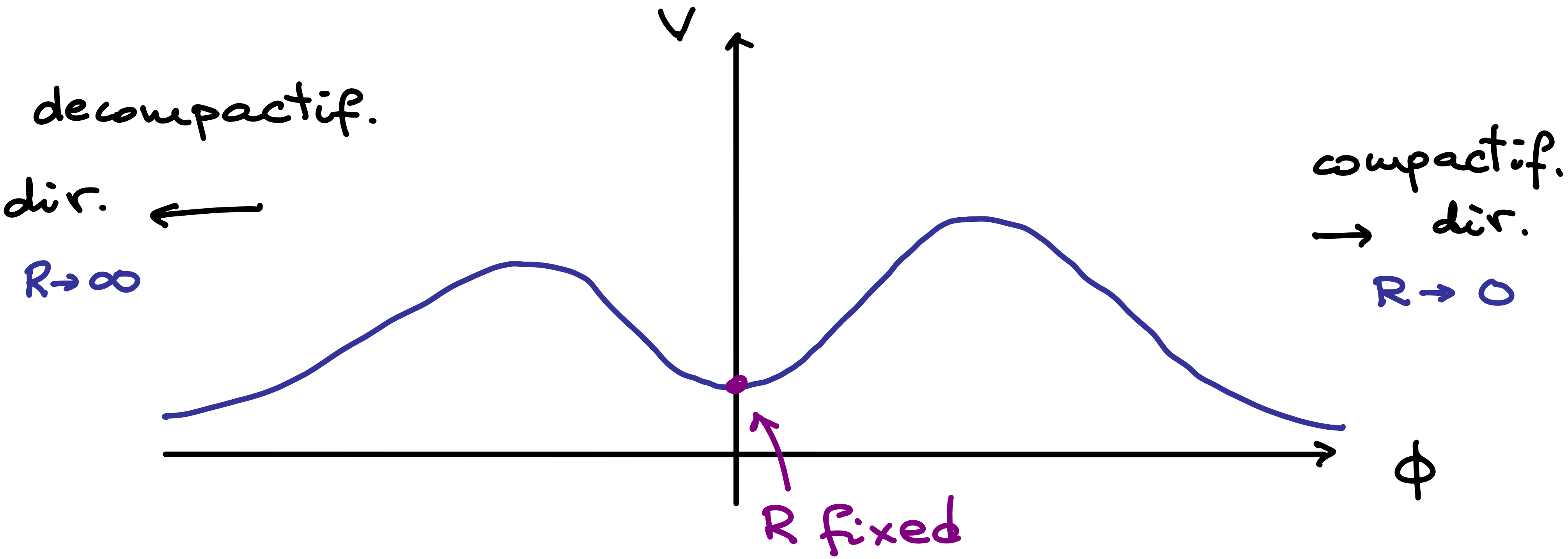
Witten's BoN $\quad V_t = -6 m_p^2 R^2 \sinh^3\left(\sqrt{2\kappa/3}\,\phi\right)$, $\quad V = 0$



BoN tunneling to $-\infty$ AdS

$S[V_t]$ ✓ without additional boundary terms

BONS WITH $V(\phi)$?



★ Universal asymptotic behaviours uncovered via

$$0 = (4V_t' - 3V')\,V_t' + 6(V - V_t)\left[V_t'' + \kappa(3V - 2V_t)\right]$$

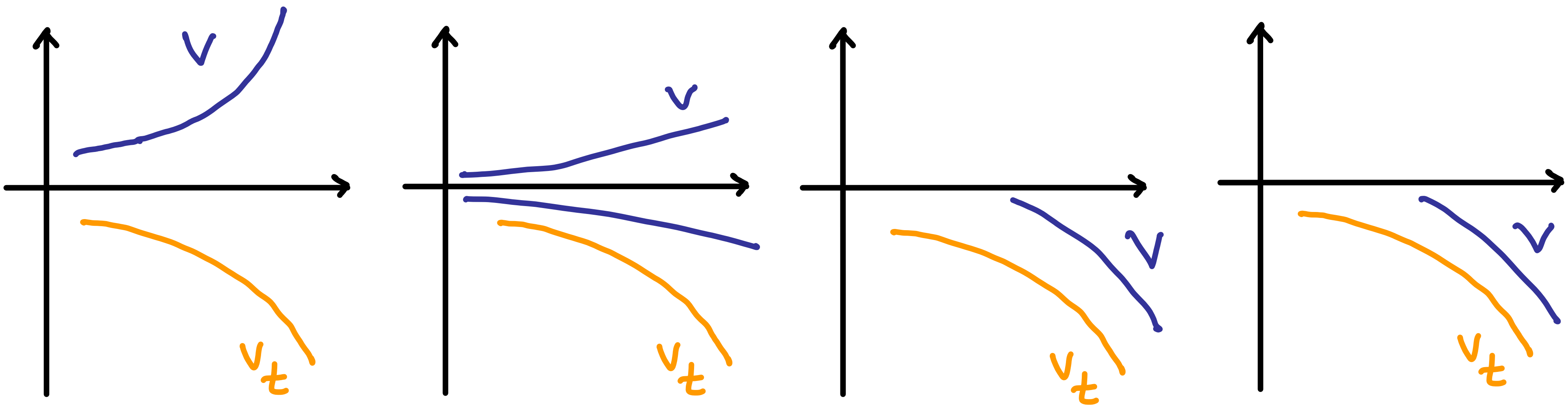
expanded at $\phi \to \infty$

BONS WITH $V(\phi)$?

Four types

$$V, V_t \sim e^{a\sqrt{6\kappa}\,\phi}$$

+   0   − $-^*$



$a = a > 1$ $a < 1 \quad a = 1$ $1/\sqrt{3} < a = a < 1$ $a = a > 1/\sqrt{3}$

(Witten's)

ASYMPTOTICS FOR BON DECAYS

Type	$V_t(\phi \to \infty)$	Constraints	β	$D(\phi \to \infty)$
$+$	$V_A/(1-a^2)e^{a\sqrt{6\kappa}\phi}$	$V_A > 0\ ,\ a > 1$	$1/(3a^2)$	$e^{\sqrt{\kappa/6}(3a+1/a)\phi}$
0	$V_{tA}e^{\sqrt{6\kappa}\phi}$	$V_{tA} < 0\ ,\ a < 1$	$1/3$	$e^{\sqrt{8\kappa/3}\phi}$
$-$	$V_A/(1-a^2)e^{a\sqrt{6\kappa}\phi}$	$V_A < 0\ ,\ 1/\sqrt{3} < a < 1$	$1/(3a^2)$	$e^{\sqrt{\kappa/6}(3a+1/a)\phi}$
$-^*$	$(3V_A/2)e^{a\sqrt{6\kappa}\phi}$	$V_A < 0\ ,\ a > 1/\sqrt{3}$	1	$e^{a\sqrt{6\kappa}\phi}$

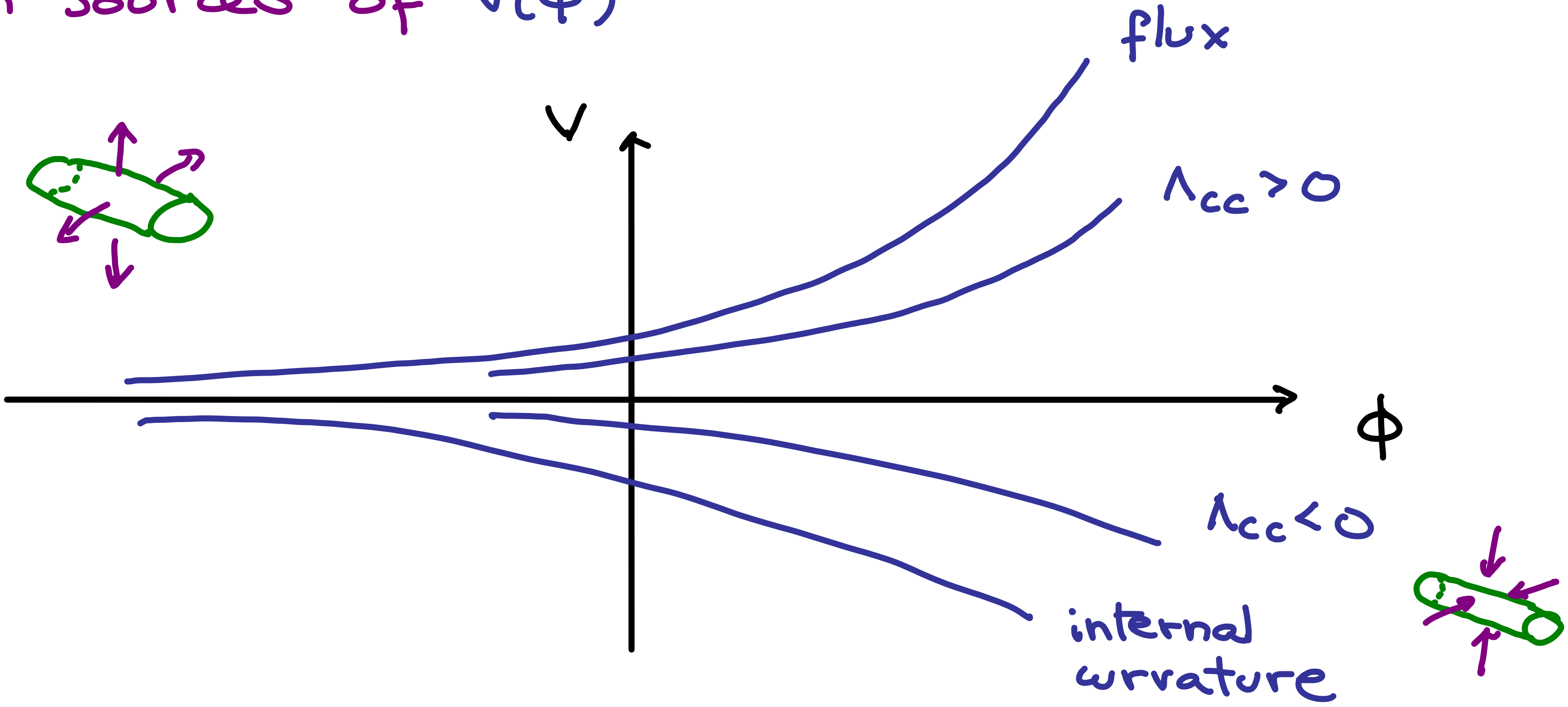
Table 1: *Taking* [illegible] *we show, for the four different types discussed in the text: the asymptotic behaviours at $\phi \to \infty$ of the tunneling potential, $V_t(\phi)$, and the quantity $D(\phi)$; several constraints on their parameters; and the exponent β entering* [illegible].

All give finite $S[V_t]$

TOP-DOWN INPUT

which $V(\phi)$ can be obtained from extra-d theories?

Main sources of $V(\phi)$



TOP-DOWN INPUT

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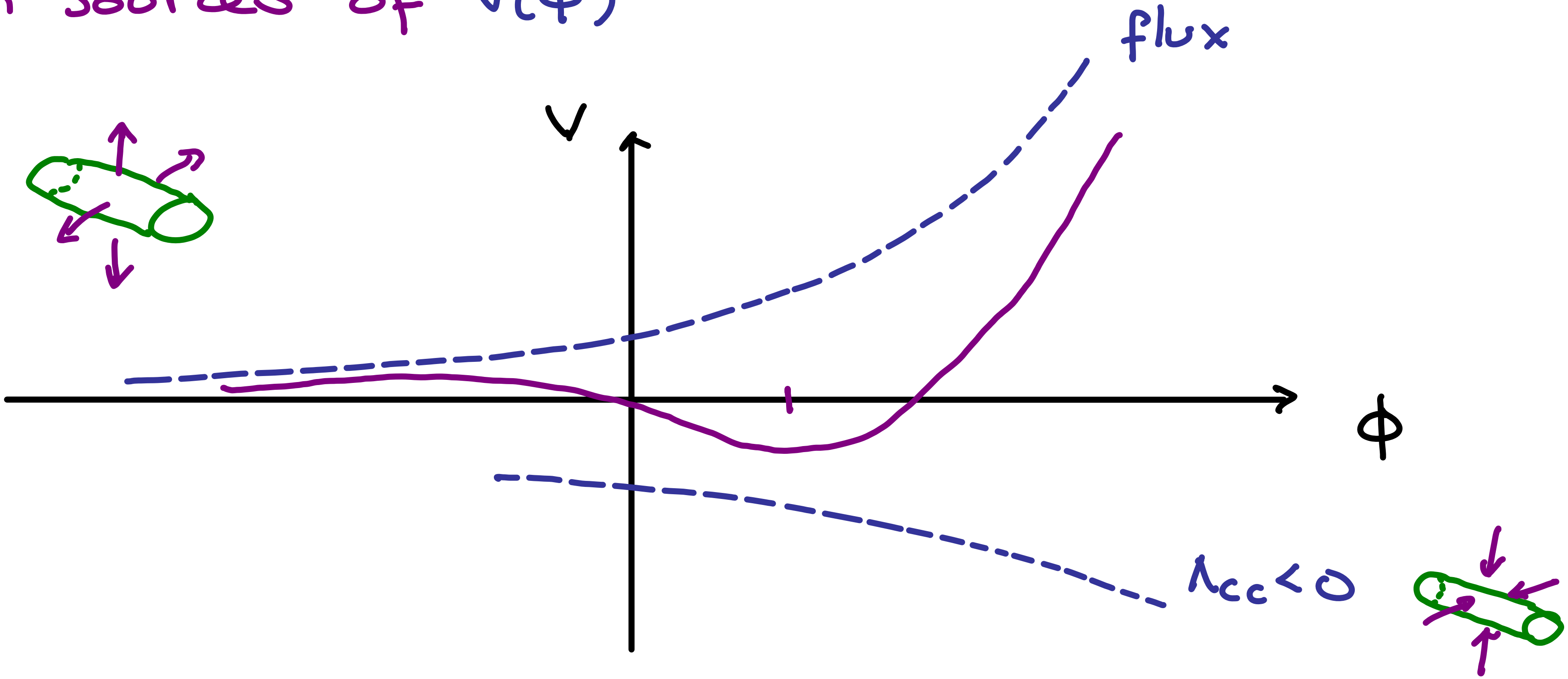


UV inputs for parameters in V and V_t

TOP-DOWN INPUT

$V_{\Lambda cc}$ with S^1 ⇒ Type 0 ✓

V_{curv} with S^n $(n>1)$ ⇒ Type − ✓

V_{flux} with S^n gives $a>1$ ⇒ Type + but,

Type +, −* ⇒ conical singularity ⇒ UV sensitive

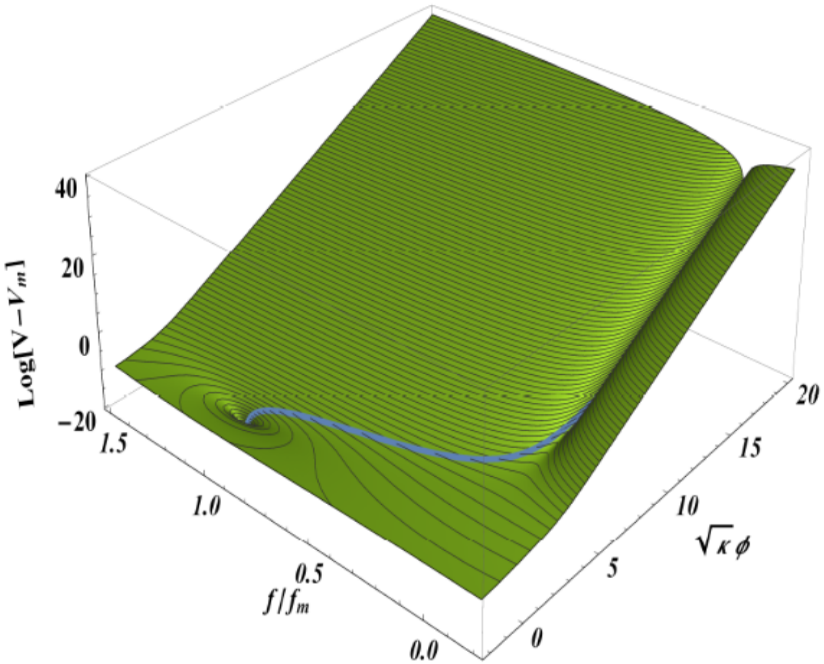
⇓

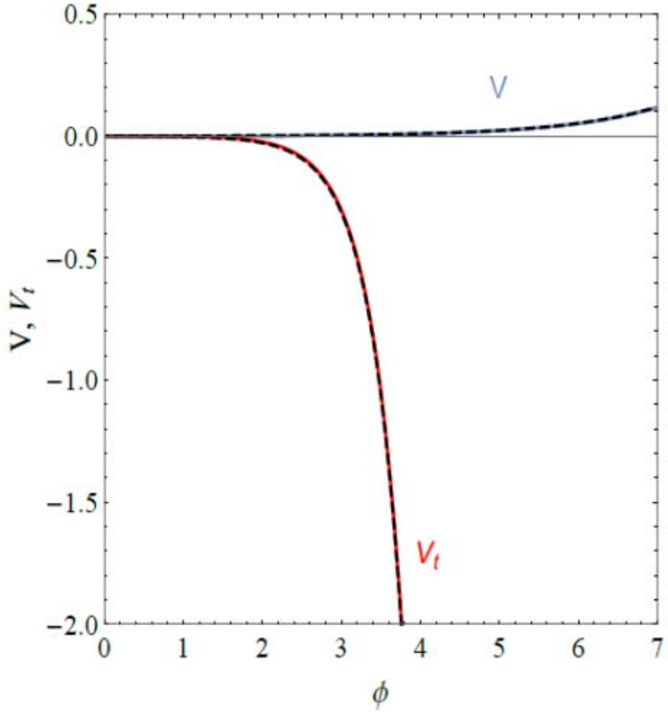
Can require UV object to fix or some additional fields

EXAMPLE WITH FLUX

Explicit 5d BoN example with flux realizing this

Blanco-Pillado, Shlaer '1002





V_t analysis confirms the expected scalings

FURTHER WORK

- Solvable examples of the 4 types
- Is BoN always the dominant decay?

  Have counterexample but of type $-^*$
- Can analyze gravitational quenching for BoNs too

  Related to end-of-the-world branes and cobordism

  Angius, Calderón-Infante, Delgado, Huertas, Uranga '2203

5. Q-BALLS

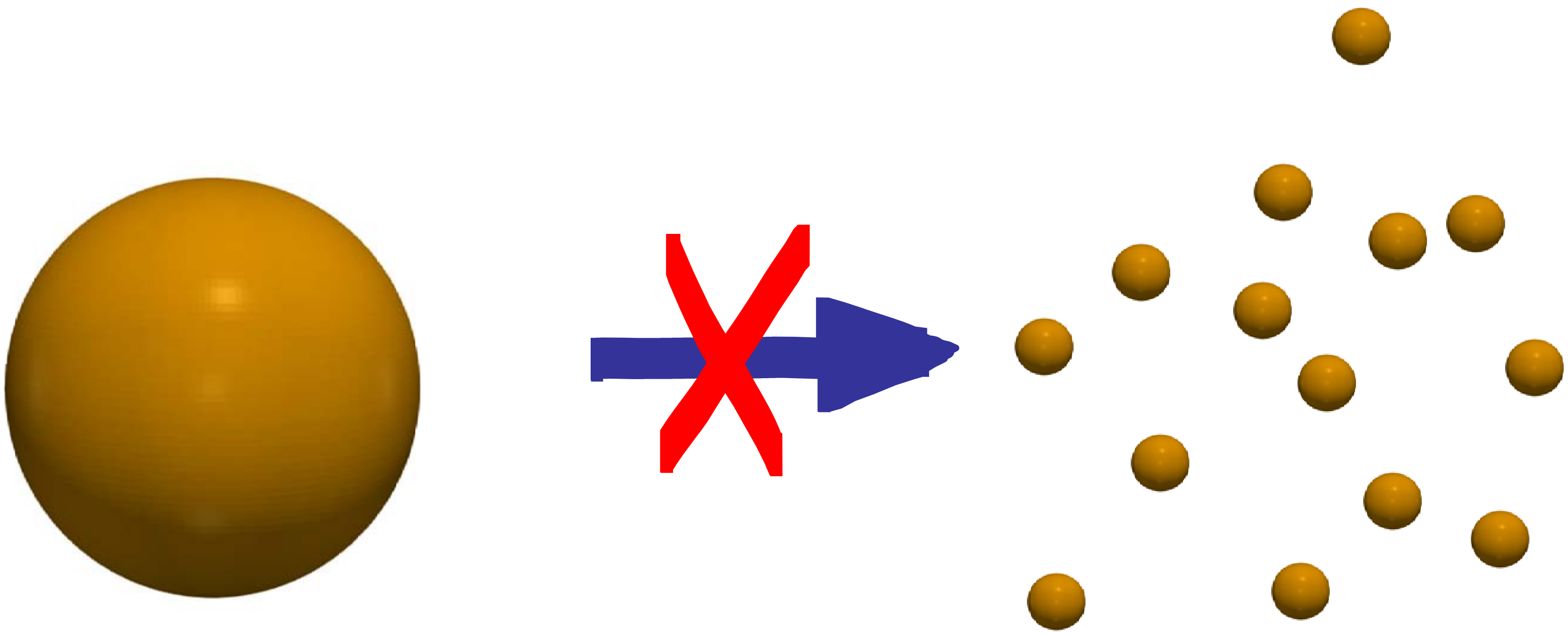
w/ Heeck, Sokhashvili

Q-BALLS

Coleman'85

Non-topological solitons stabilized by a (global) charge

They minimize energy for a fixed $Q \gg 1$.



WHY ?

★ Can be DM candidates

★ Natural in SUSY models (scalars with B, L)

★ Can play a role in baryogenesis and early universe phase transitions

★ Can produce GW signatures

Kusenko, Steinhardt, Shaposhnikov, Enquist, Rubakov, Dvali, ...

WHY ?

★ They feature in a movie

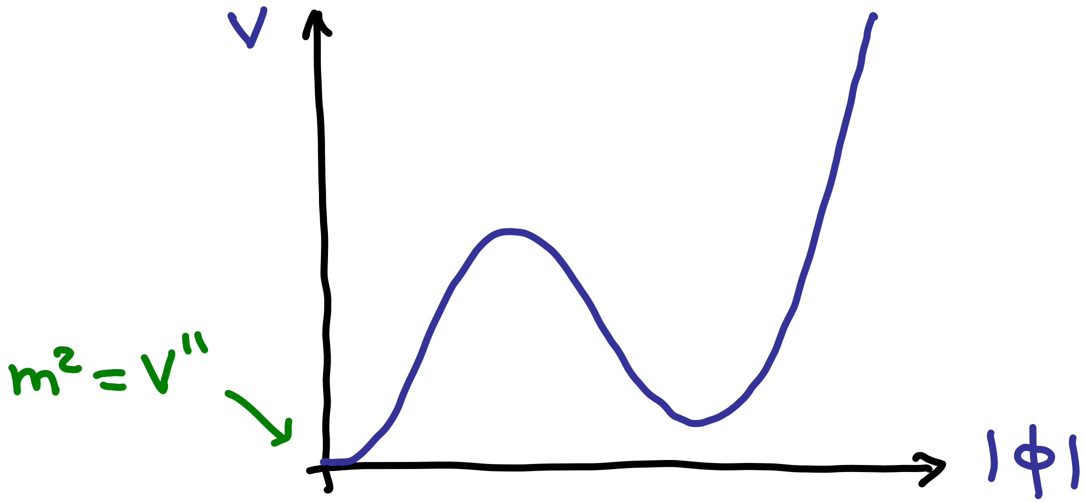
a SUSY Q-ball eating the sun...

Q-BALLS

Coleman '85

Example

Complex ϕ with global $U(1)$ and $V(|\phi|)$ like



$$\omega_0^2 \equiv \min \frac{2V}{\phi^2} < m^2$$

admits Q-ball solutions $\phi(r) = \frac{1}{\sqrt{2}} f(r) e^{i\omega t}$

with $0 < \omega_0^2 < \omega^2 < m^2$

Q-BALLS

Q-ball profile $\phi(r)$ extremal of (Lorentzian) action

$$S = 4\pi \int_0^\infty dr\, r^2 \left[-\frac{1}{2} \dot{f}^2 + \frac{1}{2} f^2 \omega^2 - V(f) \right]$$

$$\Rightarrow \qquad \ddot{f} + \frac{2}{r} \dot{f} = \frac{\partial V}{\partial f} - \omega^2 f^2$$

with charge $\qquad Q = 4\pi\omega \int_0^\infty dr\, r^2 f^2$

and energy $\qquad E = 4\pi \int_0^\infty dr\, r^2 \left[\frac{1}{2} \dot{f}^2 + \frac{1}{2} f^2 \omega^2 + V(f) \right]$

$$= \omega Q + 4\pi \int_0^\infty dr\, r^2 \left[\frac{1}{2} \dot{f}^2 + V(f) - \frac{1}{2} \omega^2 f^2 \right]$$

Q-BALLS

Q-ball profile $\phi(r)$ extremal of (Lorentzian) action

$$S = 4\pi \int_0^\infty dr\, r^2 \left[-\frac{1}{2}\dot{f}^2 + \frac{1}{2} f^2 \omega^2 - V(f) \right]$$

$$\Rightarrow \quad \ddot{f} + \frac{2}{r}\dot{f} = \frac{\partial V}{\partial f} - \omega^2 f^2 = \frac{\partial}{\partial f} \big(\underbrace{V - \tfrac{1}{2}\omega^2 f^2}_{\tilde{V}(f)} \big)$$

with charge $\qquad Q = 4\pi\omega \int_0^\infty dr\, r^2 f^2$

and energy $\qquad E = 4\pi \int_0^\infty dr\, r^2 \left[\frac{1}{2}\dot{f}^2 + \frac{1}{2} f^2 \omega^2 + V(f) \right]$

$$= \omega Q + 4\pi \int_0^\infty dr\, r^2 \left[\frac{1}{2}\dot{f}^2 + \underbrace{V(f) - \tfrac{1}{2}\omega^2 f^2}_{\tilde{V}} \right]$$

V_t FOR Q-BALLS

JRE, Heeck, Sokhashvili

$$\ddot{f} + \frac{2}{r}\dot{f} = \frac{\partial \tilde{V}}{\partial f} \qquad d=3 \text{ bounce eqn. !}$$

$$\delta E = 4\pi \int dr\, r^2 \left(\frac{1}{2}\dot{f}^2 + \tilde{V} \right) \qquad d=3 \text{ action} \equiv \text{energy}$$

Tunneling Potential approach:

$$E[V_t] = \omega Q + \frac{16\pi}{3} \int_{\phi_0}^{\phi_+} \frac{[2(\tilde{V} - V_t)]^{3/2}}{(V_t')^2}\, d\phi$$

with all the good properties of the V_t-formalism

⇨ Alternative to Euclidean analyses

Heeck, Rajaraman, Riley, Verhaaren '2009

CONCLUSIONS

Tunneling potential formalism simple and useful in a variety of problems

More cases ?

Euclidean formalism for instantons is powerful (and beautiful) but not the only way

Euclidean solutions $\Rightarrow$ V_t solutions

must be possible in other cases