Nuclear Structure studies using high-resolution techniques

Caterina Michelagnoli

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    - Angular correlations, polarization and lifetimes
- Advanced γ-ray tracking
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  - The AGATA project

(Main) bibliography of today's lecture

P. Ring and P. Schuck "The Nuclear Many Body problem"

P. J. Nolan and J. F. Sharpey-Schafer, Rep. Prog. Phys. 42, 1 (1979)
"The measurement of the lifetimes of excited nuclear states"

J. Suhonen "From Nucleons to Nucleus: Concepts of Microscopic Nuclear Theory"

L.W. Fagg and SS. Hanna, Rew. Mod. Phys. 31, 711 (1959)
"Polarization measurements on nuclear γ rays"

J.K. Smith et al., Nucl. Instr. Meth. A922 (2019) 47
"$\gamma\gamma$ angular correlation analysis techniques with the GRIFFIN spectrometer"

D. Reygadas, PhD Thesis, Univ. Grenoble-Alps, 2021

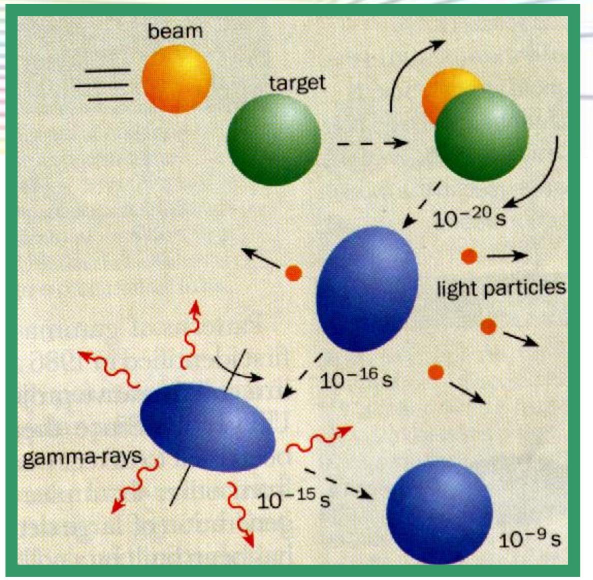
G. Bocchi, PhD Thesis, Univ. Milano, 2015

Introducing my lectures…

Scope : showing you (main) challenges and technologies in high-resolution Nuclear Structure, from and **experimentalist** point of view...

… through a “personal taste and experience”-biased **selection** of examples

In-beam γ-ray spectroscopy



The "check-list" of detection requirements, Eg ~ 10keV – 10MeV

Energy resolution,
to disentangle complex spectra → germanium detectors

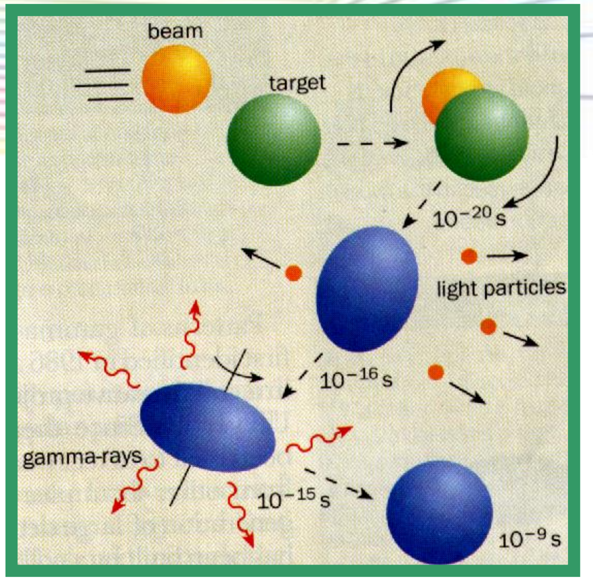
Peak-to-Total ratio,
to maximize "good events" → Compton background suppression

Doppler-correction capabilities,
to compensate the Doppler broadening (emitting nucleus in motion)

Good solid angle coverage,
to maximize the efficiency

Good granularity,
to reduce multiple hits on the detectors, measure angular distributions/correlations

In-beam γ-ray spectroscopy



The “check-list” of detection requirements, Eg ~ 10keV – 10MeV

Avoid dead materials,
to avoid radiation absorbtion and preserve low energies

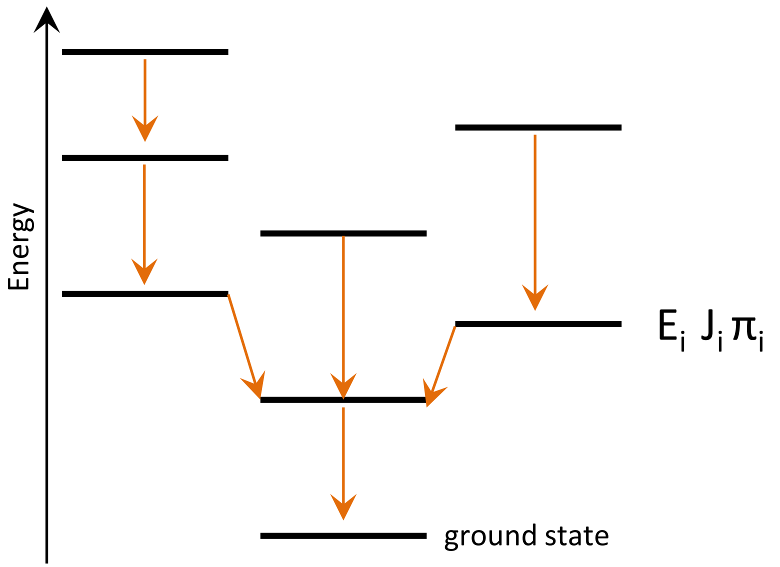
High counting rate capability,
often the channel of interest is (much!) less strong than the bakground

Time resolution,
for prompt events selection, lifetime measurements

Possibility to couple with ancillary devices,
to improve selectivity, determine velocity vector, …

The structure of the nucleus

Low-energy (<~10-15MeV) nuclear states



populate in excited state(s) and observe γ-ray de-excitation radiation

γ-ray spectroscopy is an approach for the study of nuclear structure

Observables:

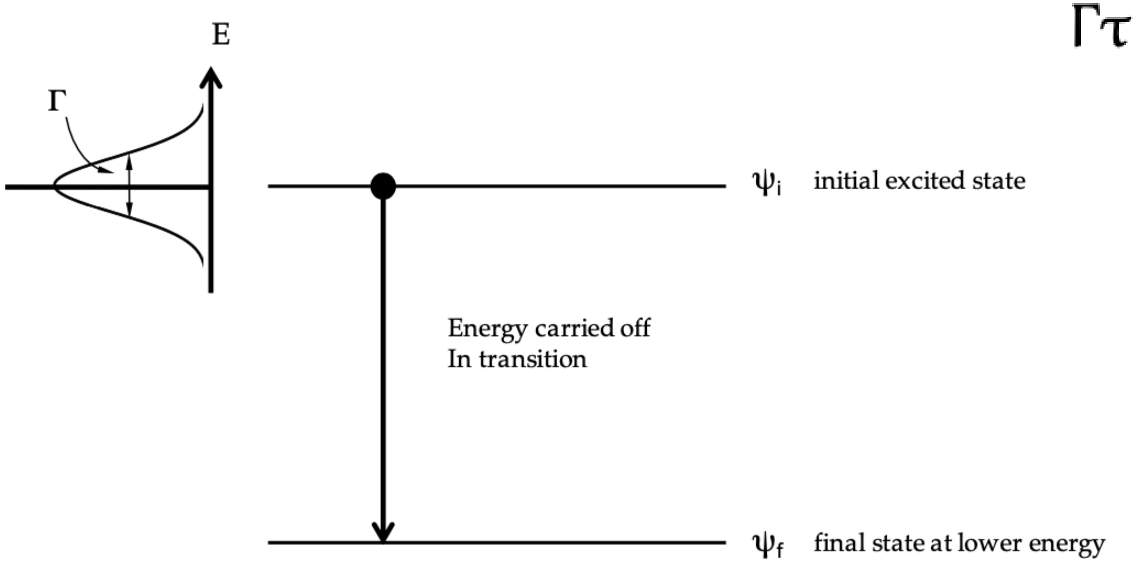
- ✓energy levels (E_i)
- ✓spin (J_i)
- ✓parity (π_i)
- ✓lifetime (transition probabilities) (τ_i)
- ✓nuclear moments (g-factors)

Measuring γ-ray:

- ✓energy
- ✓angular distribution
- ✓linear polarization
- ✓energy Doppler shift (for example)
- ✓angular distribution vs t

- ✓ systematics (e.g. shape transitions)
- ✓ benchmarks for nuclear models (e.g. Nuclear Shell Model)

Transition probability and multipolarity



$$\Gamma\tau = \hbar \qquad \Gamma \propto |\langle \psi_f | \hat{O}_{decay} | \psi_i \rangle|^2$$

If different decay modes (j): $\Gamma_{total} = \sum_j \Gamma_j$

transition probability (per unit time):

$$\tau_\gamma = \frac{T_{1/2}}{\ln 2} = \frac{1}{T_{if}} \qquad \tau = \tau_\gamma \frac{I_\gamma}{I_{tot}}$$

$I\gamma$= intensity of the transition with given multipolarity λ

E or M transition probability ("golden rule"):

$$T_{fi}^{(\sigma\lambda\mu)} = \frac{2}{\epsilon_0 \hbar} \frac{\lambda+1}{\lambda[(2\lambda+1)!!]^2} \left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1} |\langle \xi_f J_f m_f | \mathcal{M}_{\sigma\lambda\mu} | \xi_i J_i m_i \rangle|^2$$

σ = E, M $\quad$ μ = magnetic substate

$|J_f-J_i| < \lambda < |J_i+J_f|$

$-\lambda < \mu < \lambda$

$\pi_f = (-1)^{\lambda}\, \pi_i \qquad \sigma$ = E

$\pi_f = (-1)^{\lambda+1}\, \pi_i \qquad \sigma$ = M

$J_f=J_i$ ⍰ E0 (internal conversion)

Reduced transition probabilities

Averaging on initial magnetic substates and summing over all final ones :

$$T_{fi}^{(\sigma\lambda)} = \frac{1}{2J_i+1}\sum_{m_i \mu m_f} T_{fi}^{(\sigma\lambda\mu)}$$
$$= \frac{2}{\epsilon_0\hbar}\frac{\lambda+1}{\lambda[(2\lambda+1)!!]^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1} B(\sigma\lambda;\xi_i J_i \to \xi_f J_f)$$

Reduced transition probability

$$B({}^{M}_{E}\lambda; I_f \to I_i) = \frac{2I_i+1}{2I_f+1} B({}^{M}_{E}\lambda; I_i \to I_f)$$

$$[B(E\lambda)] = e^2 fm^{2\lambda}, \quad [B(M\lambda)] = (\mu_N/c)^2 fm^{2\lambda-2}$$

Weisskopf estimates ($J_i > J_f$) – single particle probabilities (*Energies in MeV*):

$E\lambda$	$T(E\lambda)(s^{-1})$	$B_W(E\lambda)(e^2 fm^{2\lambda})$	$T_W(E\lambda)(s^{-1})$
E1	$1.587 \times 10^{15} E^3 B(E1)$	$6.446 \times 10^{-2} A^{2/3}$	$1.023 \times 10^{14} E^3 A^{2/3}$
E2	$1.223 \times 10^{9} E^5 B(E2)$	$5.940 \times 10^{-2} A^{4/3}$	$7.265 \times 10^{7} E^5 A^{4/3}$
E3	$5.698 \times 10^{2} E^7 B(E3)$	$5.940 \times 10^{-2} A^{2}$	$3.385 \times 10^{1} E^7 A^{2}$
E4	$1.694 \times 10^{-4} E^9 B(E4)$	$6.285 \times 10^{-2} A^{8/3}$	$1.065 \times 10^{-5} E^9 A^{8/3}$
E5	$3.451 \times 10^{-11} E^{11} B(E5)$	$6.928 \times 10^{-2} A^{10/3}$	$2.391 \times 10^{-12} E^{11} A^{10/3}$

$M\lambda$	$T(M\lambda)(s^{-1})$	$B_W(M\lambda)((\mu_N/c)^2 fm^{2\lambda-2})$	$T_W(M\lambda)(s^{-1})$
M1	$1.779 \times 10^{13} E^3 B(M1)$	1.790	$3.184 \times 10^{13} E^3$
M2	$1.371 \times 10^{7} E^5 B(M2)$	$1.650 A^{2/3}$	$2.262 \times 10^{7} E^5 A^{2/3}$
M3	$6.387 \times 10^{0} E^7 B(M3)$	$1.650 A^{4/3}$	$1.054 \times 10^{1} E^7 A^{4/3}$
M4	$1.899 \times 10^{-6} E^9 B(M4)$	$1.746 A^{2}$	$3.316 \times 10^{-6} E^9 A^{2}$
M5	$3.868 \times 10^{-13} E^{11} B(M5)$	$1.924 A^{8/3}$	$7.442 \times 10^{-13} E^{11} A^{8/3}$

Weisskopf estimates of transition rates

When the details of the wave function are not known – single particle probabilities

Eλ	T(Eλ)(s^{-1})	B_W(Eλ)($e^2fm^{2\lambda}$)	T_W(Eλ)(s^{-1})
E1	$1.587 \times 10^{15} E^3 B(E1)$	$6.446 \times 10^{-2} A^{2/3}$	$1.023 \times 10^{14} E^3 A^{2/3}$
E2	$1.223 \times 10^{9} E^5 B(E2)$	$5.940 \times 10^{-2} A^{4/3}$	$7.265 \times 10^{7} E^5 A^{4/3}$
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E5	$3.451 \times 10^{-11} E^{11} B(E5)$	$6.928 \times 10^{-2} A^{10/3}$	$2.391 \times 10^{-12} E^{11} A^{10/3}$

Mλ	T(Mλ)(s^{-1})	B_W(Mλ)($(\mu_N/c)^2 fm^{2\lambda-2}$)	T_W(Mλ)(s^{-1})
M1	$1.779 \times 10^{13} E^3 B(M1)$	1.790	$3.184 \times 10^{13} E^3$
M2	$1.371 \times 10^{7} E^5 B(M2)$	$1.650 A^{2/3}$	$2.262 \times 10^{7} E^5 A^{2/3}$
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M5	$3.868 \times 10^{-13} E^{11} B(M5)$	$1.924 A^{8/3}$	$7.442 \times 10^{-13} E^{11} A^{8/3}$

T(Eλ)/T(Mλ) $\approx 2A^{2/3}$ for a given λ, electric transition always dominates

T(Eλ+1)/T(Mλ) $\approx 10^{-6} A^{4/3} E^2$ for large A and Eγ, E2 can compete with M1, E3 with M2, …

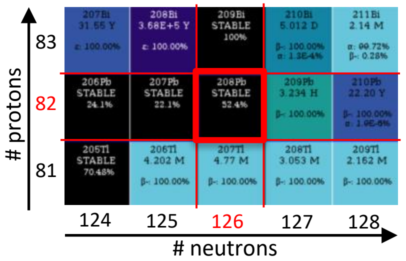
T(σλ+1)/T(σλ) $\approx 10^{-8} A^{2/3} E^2$ as L increases, T decreases quite "dramatically"

B(σλ) Weisskopf units → $\lesssim 1$ ***Single Particle***

B(σλ) Weisskopf units → >>1 ***Collective***

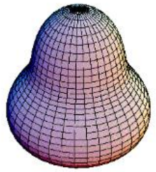
Of course not for same transition in same nucleus :) just to have a general idea in given mass region

Collectivity and nuclear shapes



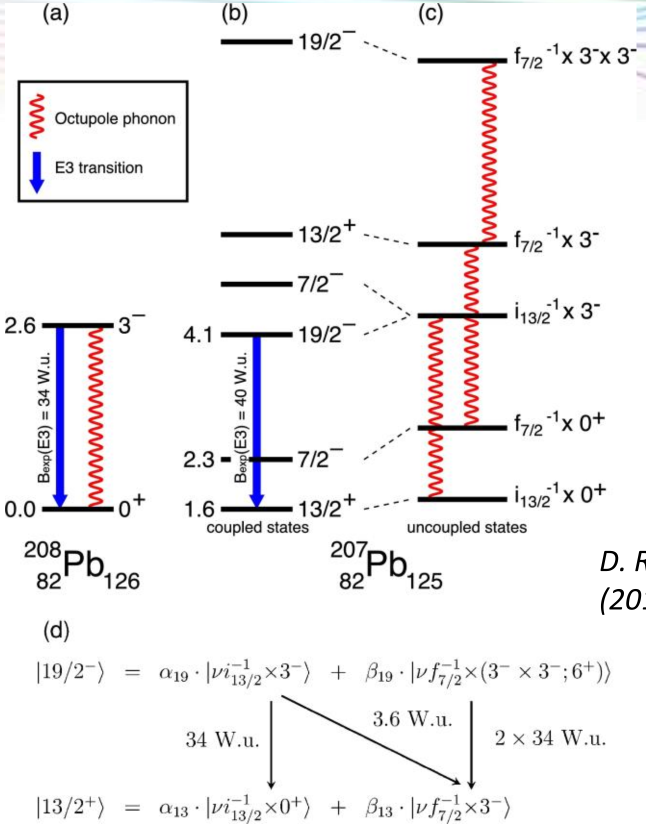
E(3$^-$) = 2.6 MeV

B(E3, 3$^-$ → 0$^+$) = 34.0(5)



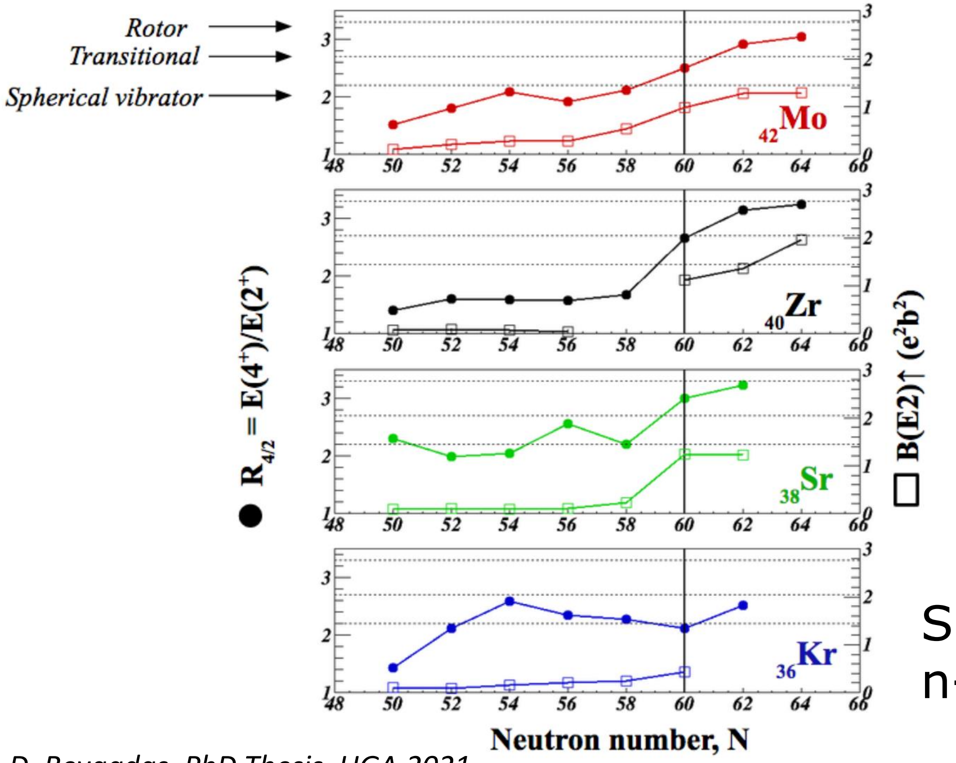
R.H. Spear et al. PLB128 (1983) 29

Octupole (collective) vibration



D. Ralet et al, PLB 797 (2019) 134797

Single particle (hole) + phonon states



D. Reygadas, PhD Thesis, UGA 2021

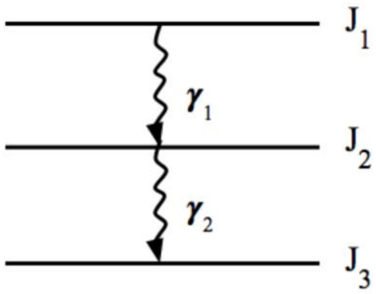
Shape transitions in n-rich nuclei A~100

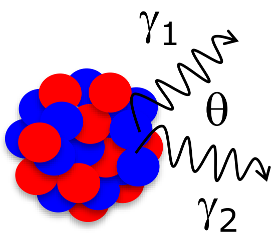
$B(\sigma\lambda;\ \xi_i J_i \longrightarrow \xi_f J_f)$ □ $1/\tau$

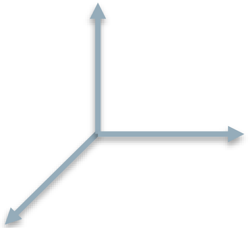
➡ need to measure λ, σ, τ

Angular distributions (correlations)

Measurement of the multipole order of the transition (λ, eventually mixed)







$I_\gamma(\theta)$ □ $W(\theta) = \sum_k A_k P_k(\cos(\theta))$

A_k □ Theory
$P_k(\cos(\theta))$ □ Legendre polinomials
k integer up to min$\{2J_2, \lambda_1, \lambda_2\}$

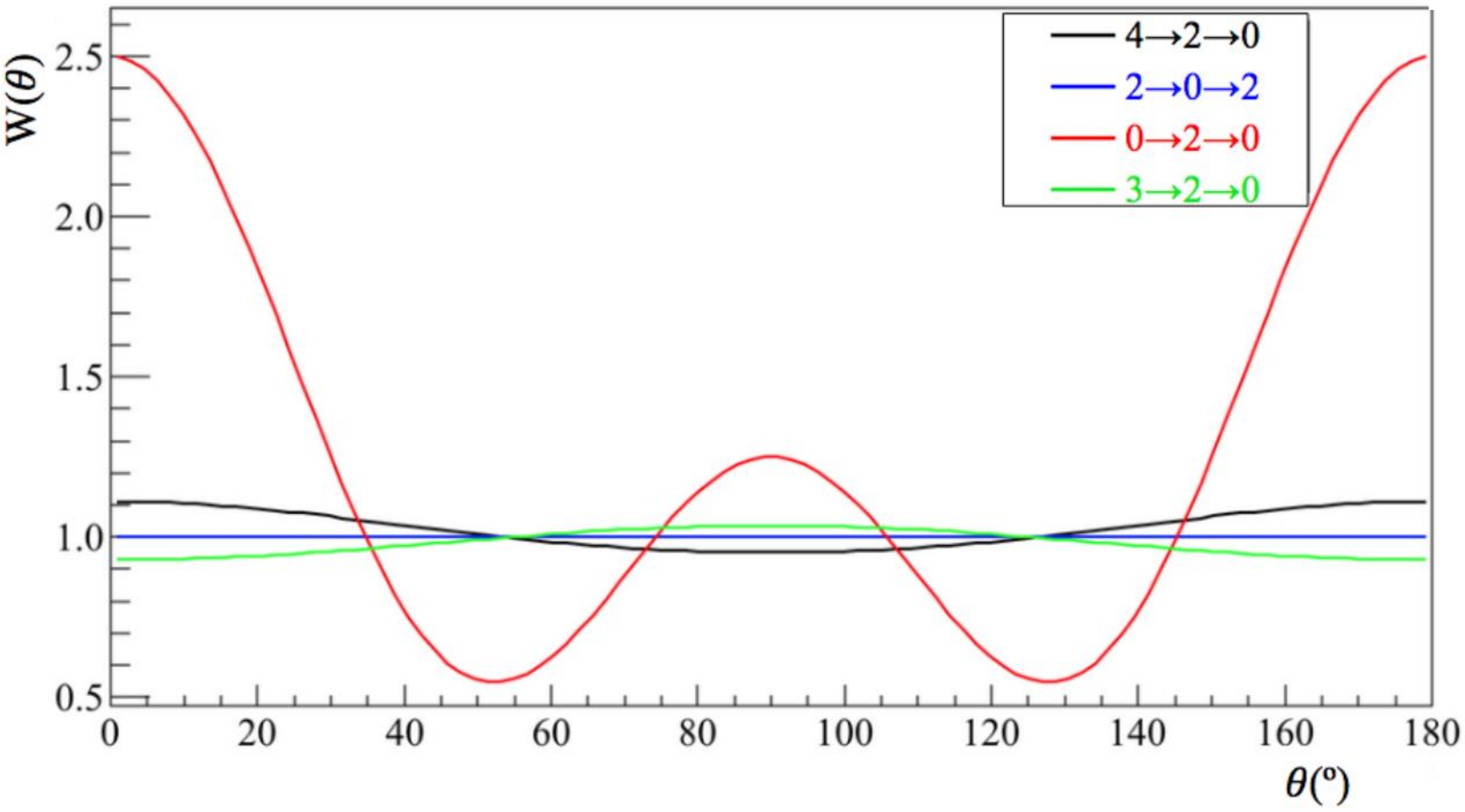
"sources" of alignement : magnetic field
nuclear reactions (heavy ion reactions)

Or set a direction by coincidence on a gamma on the cascade : *angular correlations*

If max multipolarity = 2 : $W(\theta) = A_0[1 + a_2 P_2(\cos(\theta)) + a_4 P_4(\cos(\theta))]$ $a_k = A_k/A_0$

$$P_2(\cos(\theta)) = \frac{1}{2}(3\cos^2(\theta) - 1), \quad P_4(\cos(\theta)) = \frac{1}{8}(35\cos^4(\theta) - 30\cos^2(\theta) + 3).$$

Examples of theoretical angular correlation functions



$$W(\theta) = A_0[1 + a_2 P_2(\cos(\theta)) + a_4 P_4(\cos(\theta))]$$

Cascade	a_2	a_4
$4 \to 2 \to 0$	0.10204	0.00907
$2 \to 0 \to 2$	0	0
$0 \to 2 \to 0$	0.35714	1.14286
$3 \to 2 \to 0$	-0.07143	0

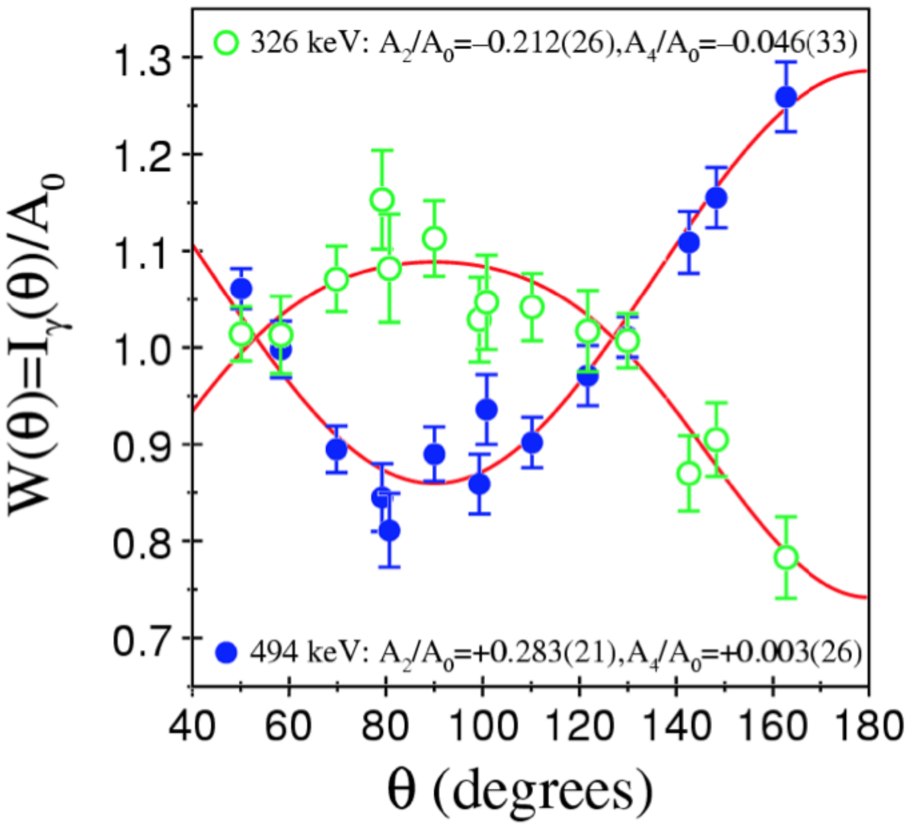
Real life:

$a_k = q_k\ a_{th}$

q_k □□ response function of the gamma array used

Data from https://griffincollaboration.github.io/AngularCorrelationUtility/

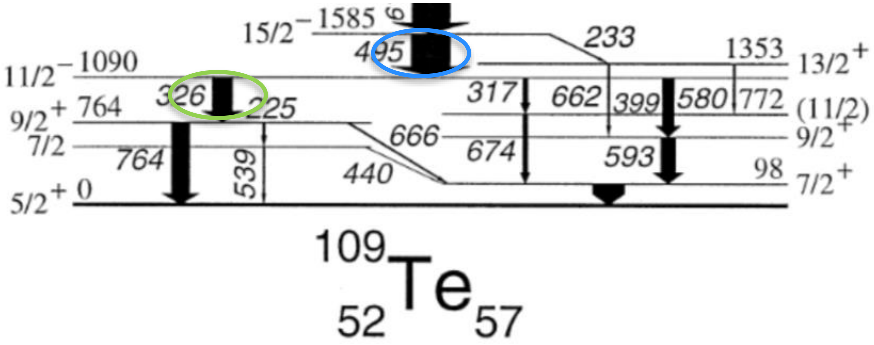
^{109}Te from fusion-evaporation reactions

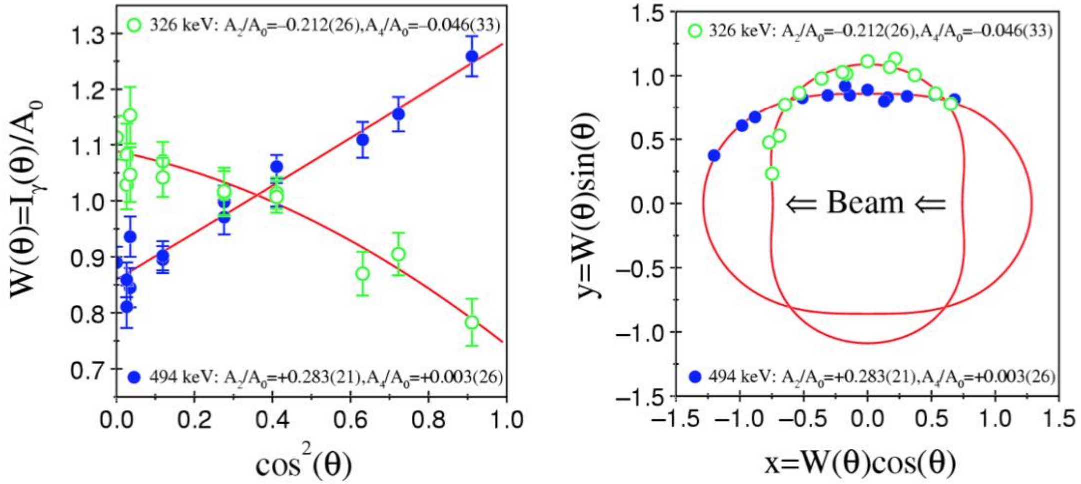


$A_4/A_0 \sim 0$

A2/A0 ~ +0.3 pure quadrupole transition

A2/A0 ~ -0.3 pure dipole transition

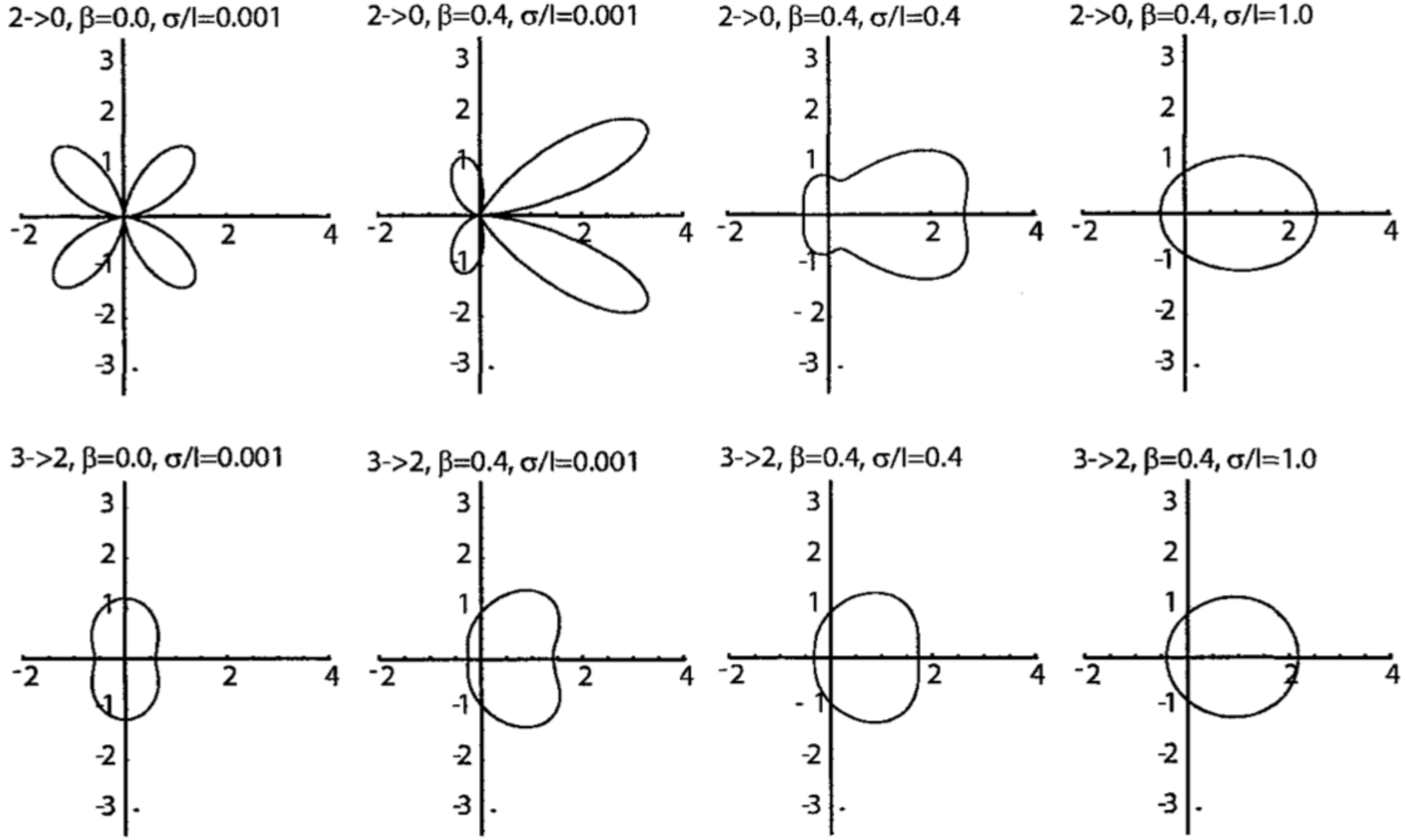




$A_4/A_0 \sim 0$ □ $W(\theta)$ linear in $\cos^2\theta$

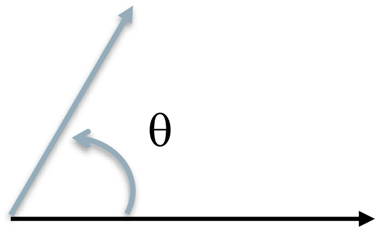
https://ns.ph.liv.ac.uk/~ajb/summerschool/files/ESP-Lecture4.pdf

Polar plots as a function of beam velocity β



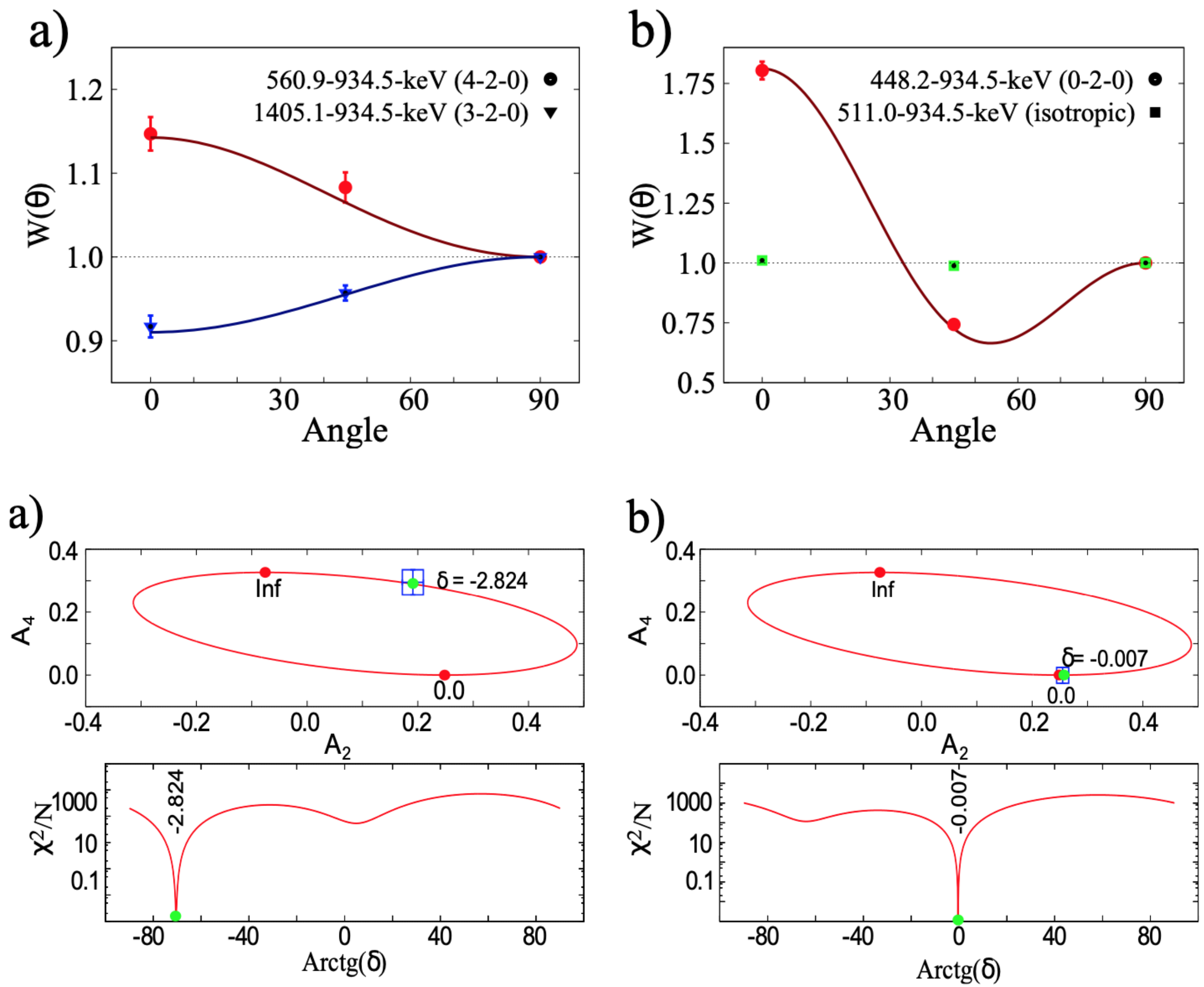
In the lab. frame

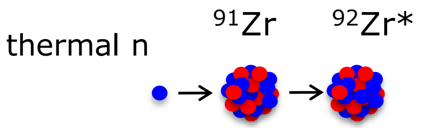
σ/l = width of the gaussian m-state distribution

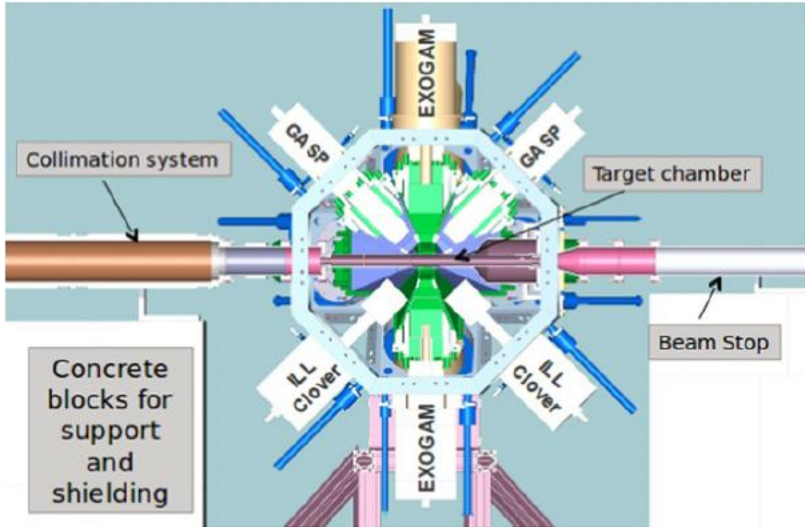


Mueller al, Nucl. Instrum. Methods 466,492 (2001).

Examples of angular correlations

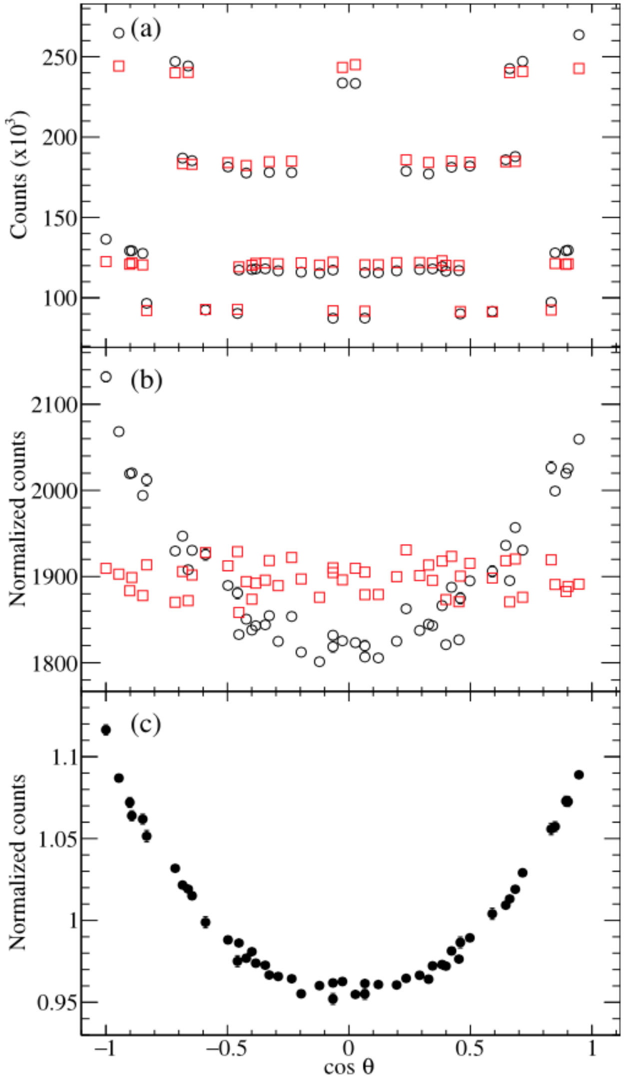






M. Jentschel et al., JINST 12 (2017) P11003

Angular Correlations: normalization



$4^+ \rightarrow 2^+ \rightarrow 0^+$

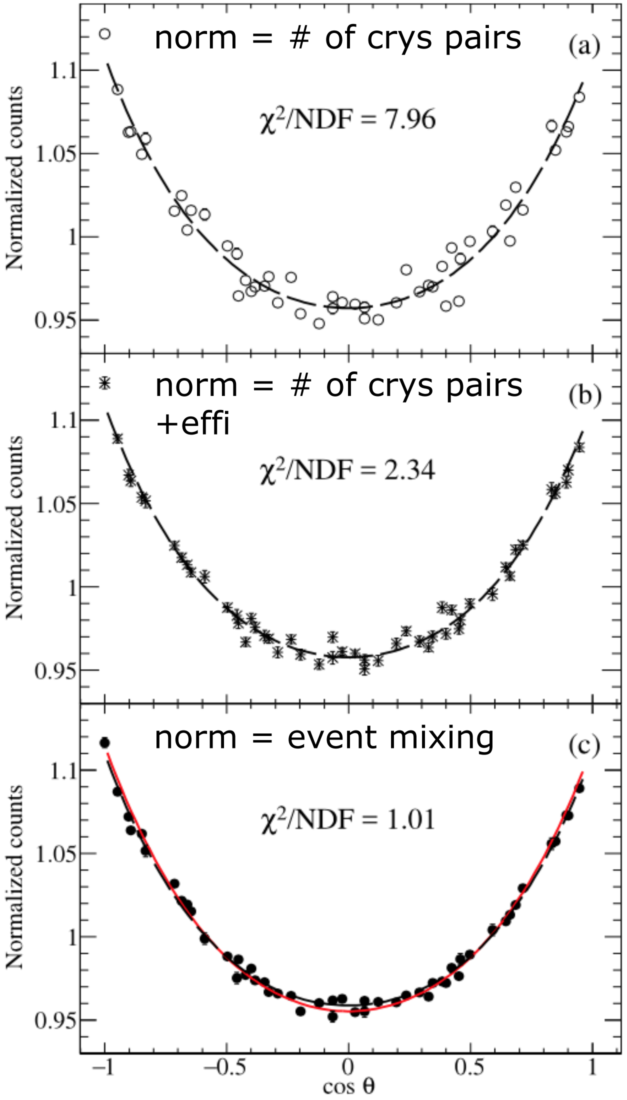
^{60}Ni

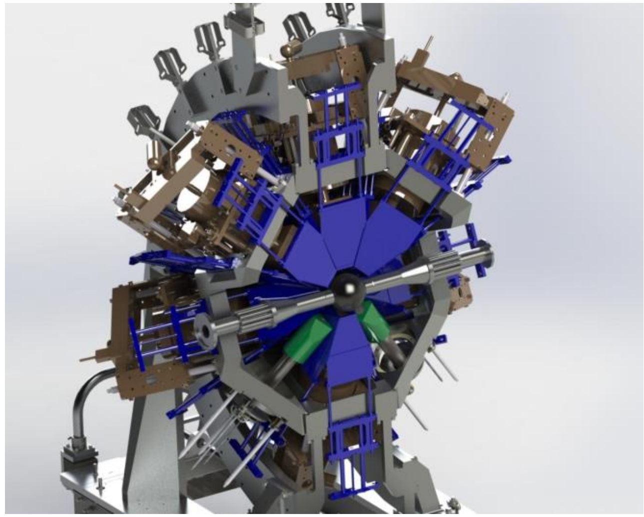
coincidences

event-mixed coincidences

divided by # of crystal pairs at each angle

final ang corr



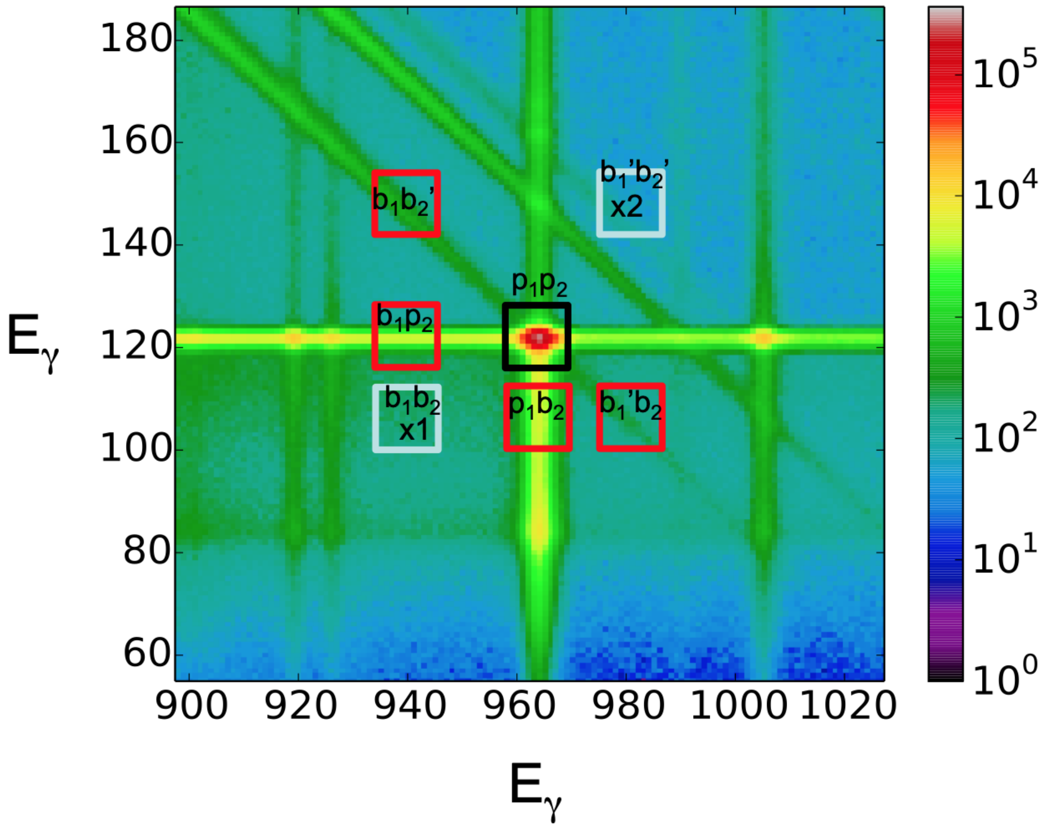


GRIFFIN @ TRIUMF

The intensities at each angle are normalized using the *EVENT MIXING* technique

J.K. Smith et al., Nucl. Instr. Meth. A922 (2019) 47

Angular Correlations: bg subtraction



Eq. (1):

$p_1p_2 - p_1b_2 - b_1p_2 + b_1b_2$

Eq. (2):*

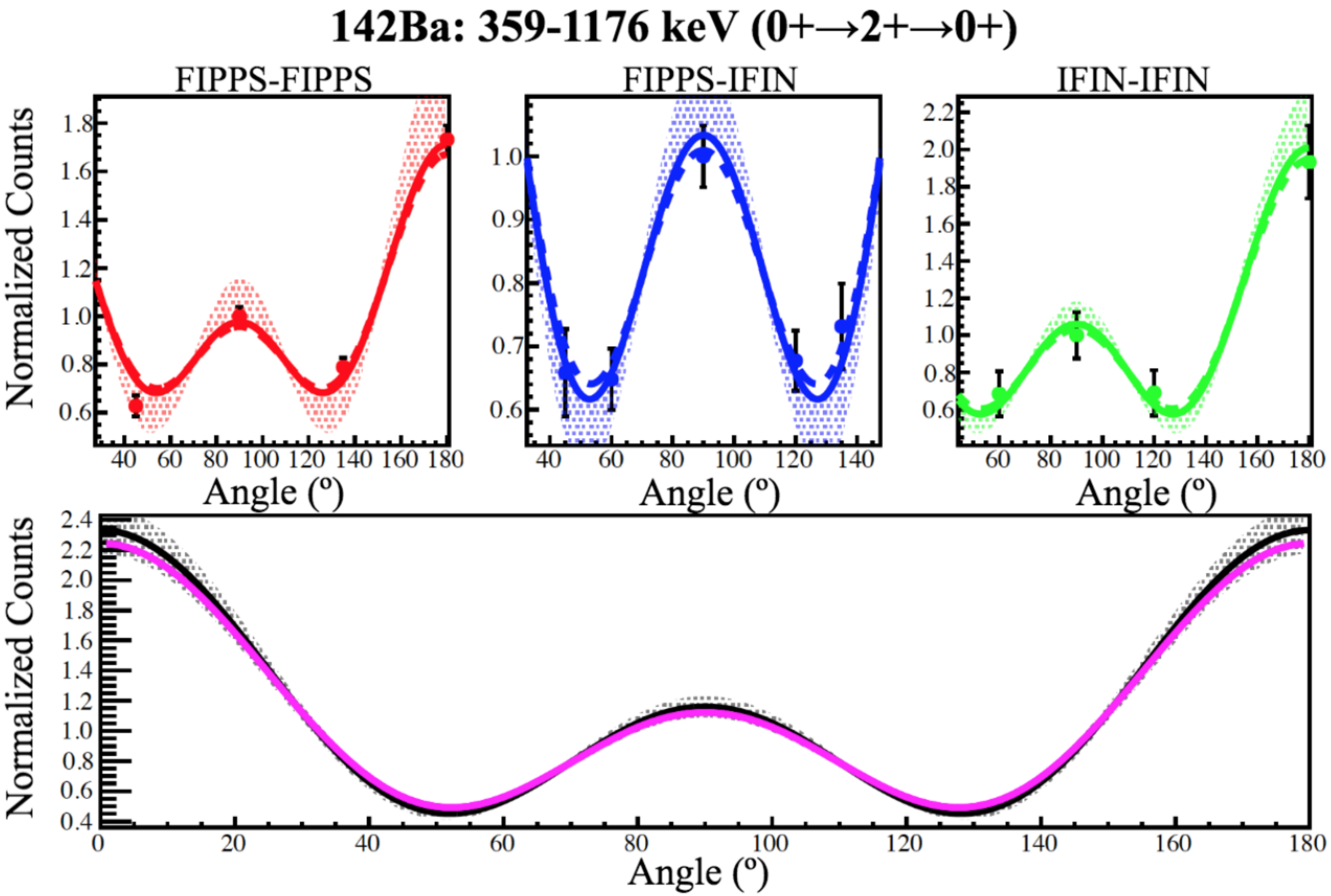
$p_1p_2 - p_1b_2 - b_1p_2 + b_1b_2 + 0.5*[- b_1b_2' - b_1'b_2 + 2b_1'b_2']$

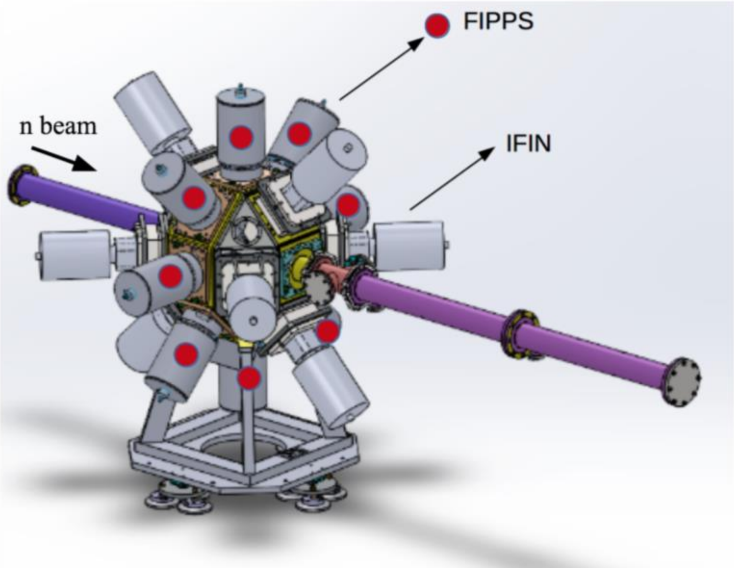
Correlated background must be properly subctrated (often ignored in a "simple" 1D analysis… !)

**Correlated background mostly impacts small $\theta_{\gamma\gamma}$ and weak $I(\gamma\gamma)$*

Courtesy of JM Allmond (GRETA collaboration)

Angular correlations with hybrid detector setup

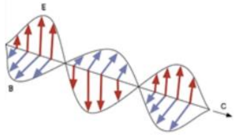




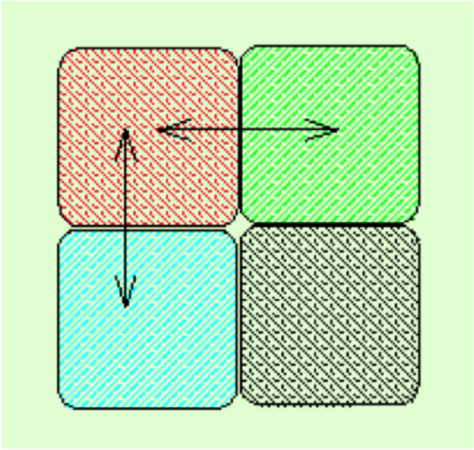
Hybrid detector setup!
Simultaneous fit of all type of combinations

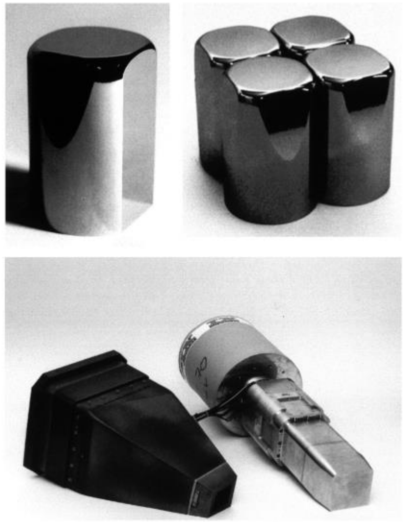
E/M character of γ transitions

Parity of nuclear states

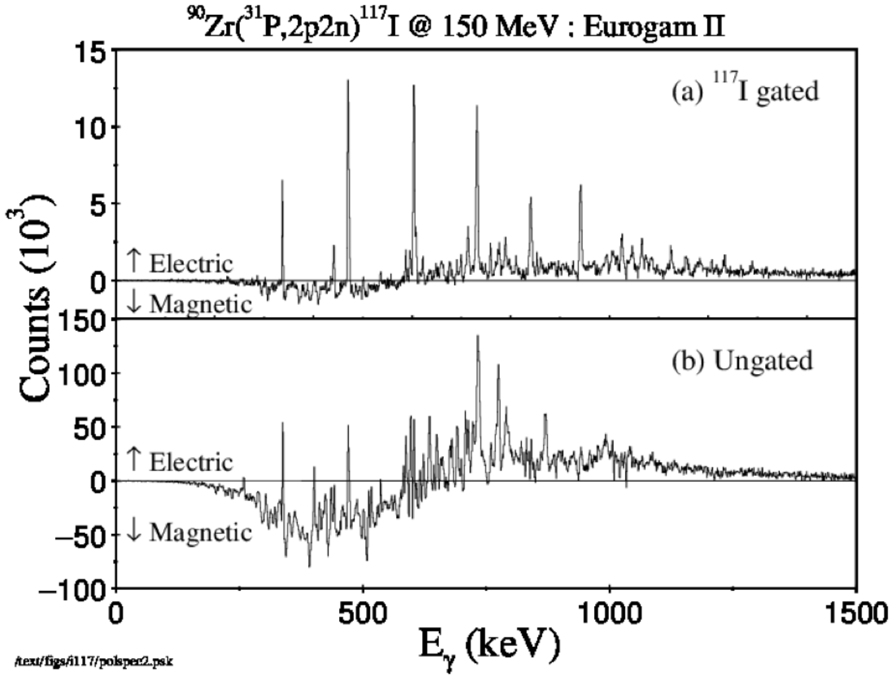


Compton scattering can be used to measure the γ-ray linear polarisation – the direction of the electric vector with respect to the beam-detector plane





Clover detectors are ideal



$$A = \frac{N_\perp - N_\parallel}{N_\perp + N_\parallel}$$

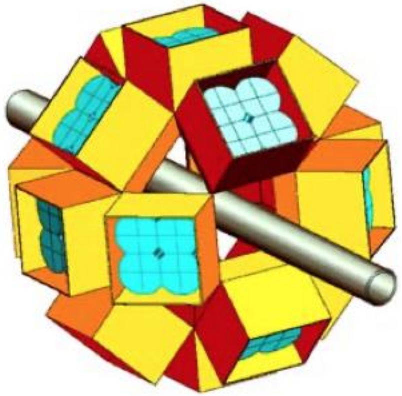
P = A/Q

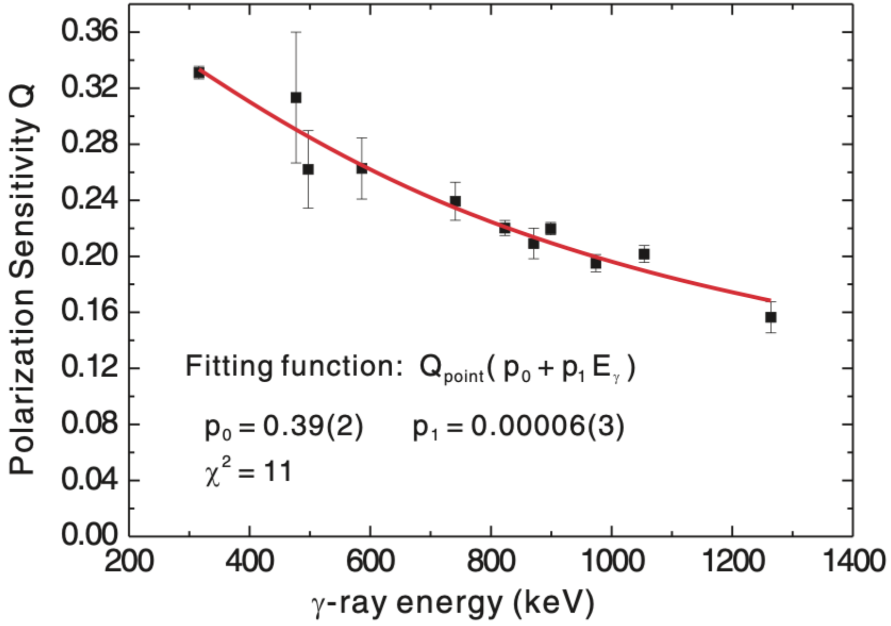
Q =polarization sensitivity (as a function of the energy)

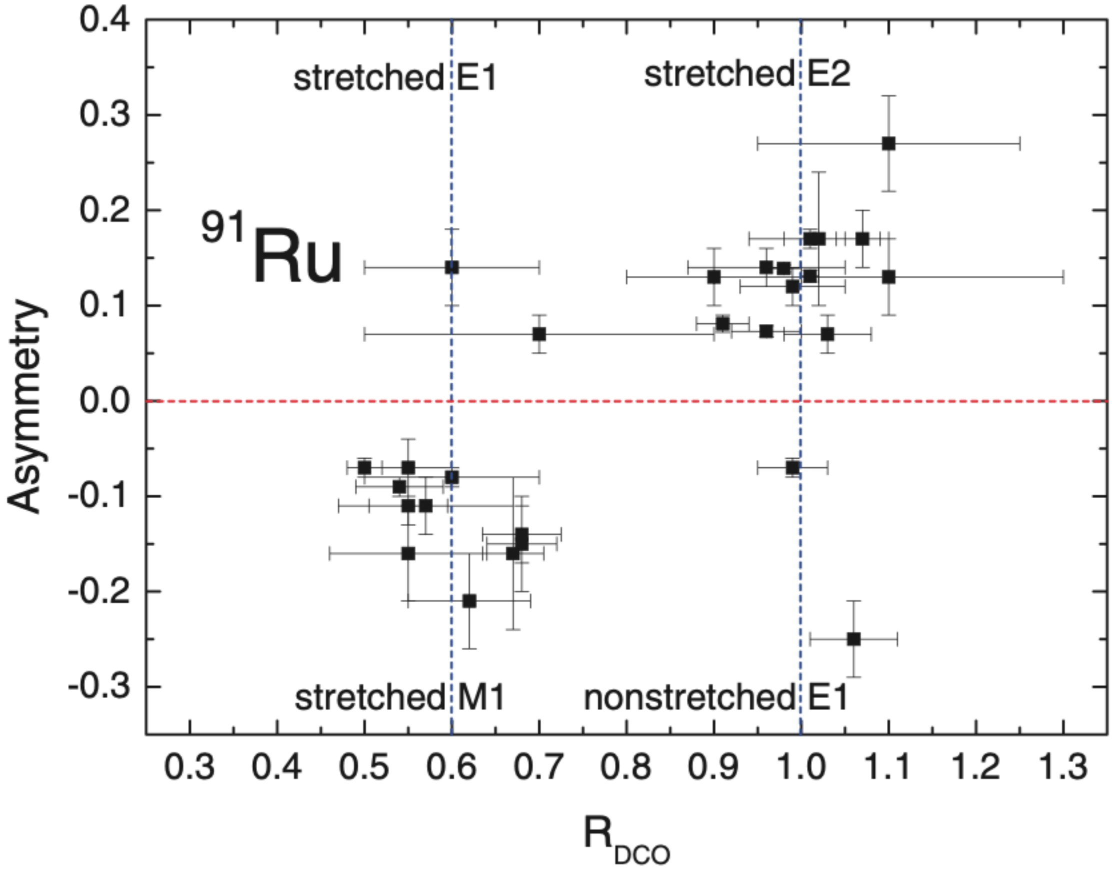
Streched E2 : P>0; Streched M1 : P<0

An example

$$A = \frac{[a(E_\gamma)N_\perp] - N_\parallel}{[a(E_\gamma)N_\perp] + N_\parallel}, \qquad a(E_\gamma) = \frac{N_\parallel(\text{unpolarized})}{N_\perp(\text{unpolarized})}$$







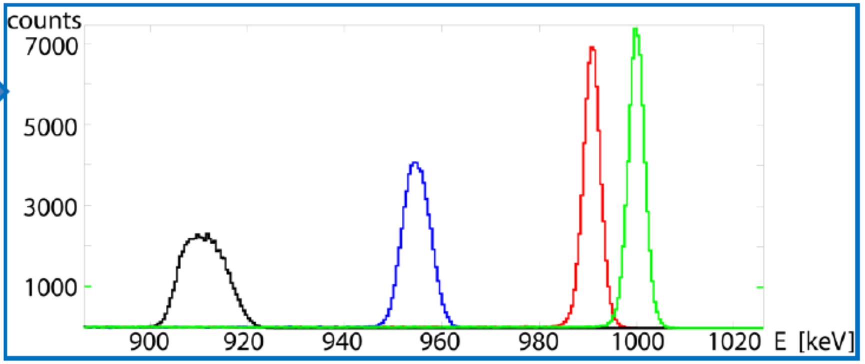
Y. Zheng, PHYSICAL REVIEW C 87, 044328 (2013)

Lifetime measurements using Doppler techniques

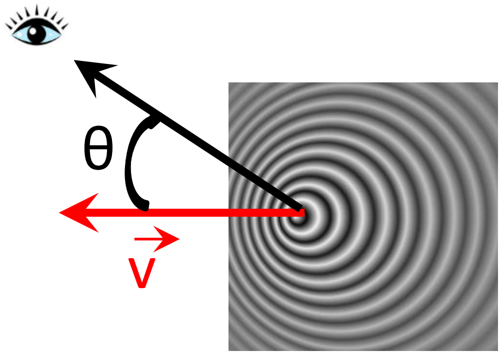
Reminder from last Lecture:

E_0=1MeV
β=0, 0.01, 0.05, 0.10
(fixed direction)
$\theta = 158\, deg$

$\vec{\beta}$ is a key info.

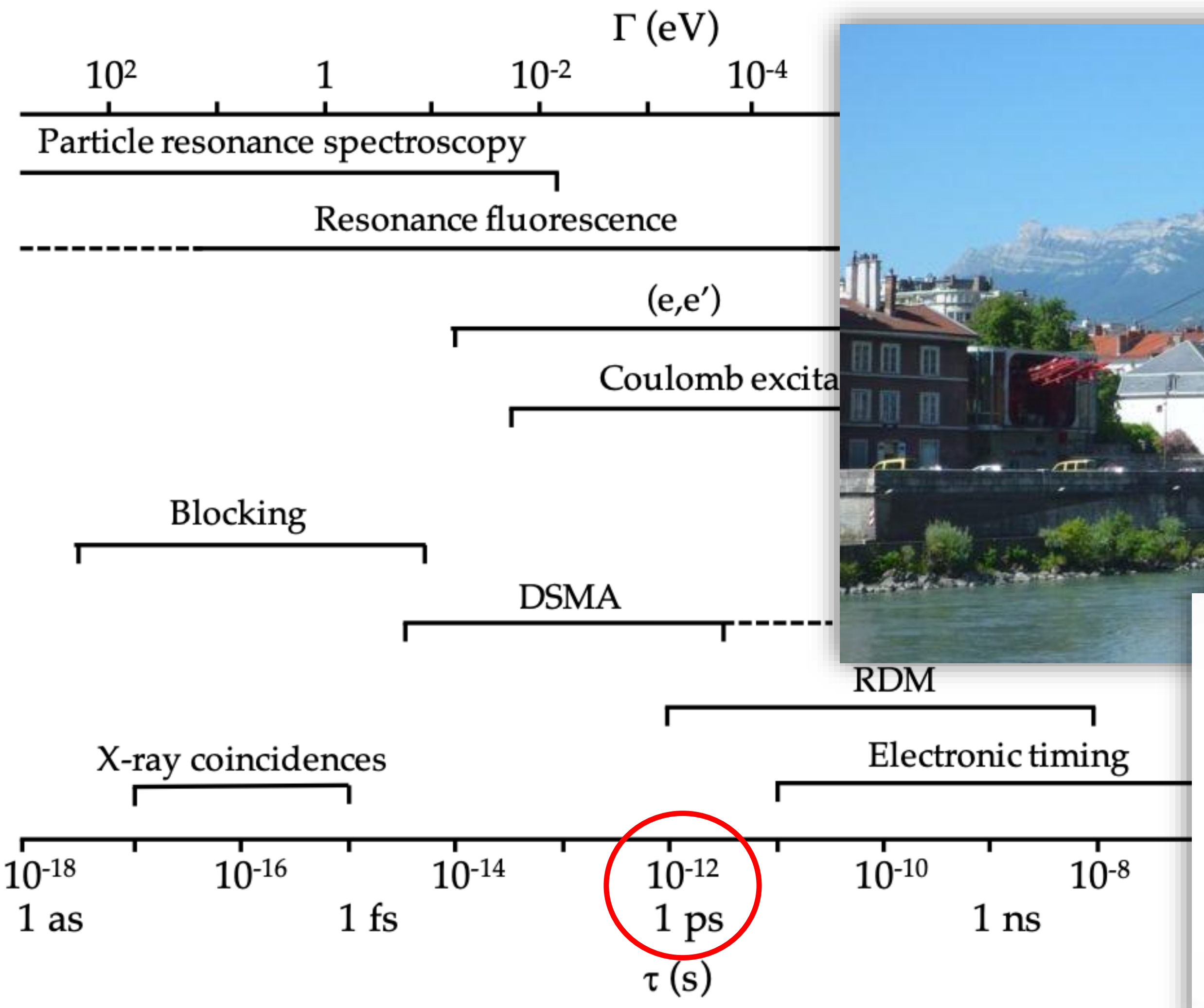


Source at rest → intrinsic energy resolution



$$E_\gamma^{Lab}(\theta) = E_\gamma^{CM} \frac{\sqrt{1-\beta^2}}{1-\beta\cdot\cos\theta}$$

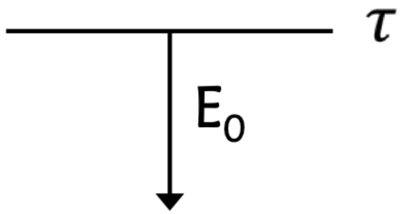
Direct vs Indirect methods for measuring lifetimes



water flow in Isere in the month of June / bottle of wine $\sim 10^{-12}$

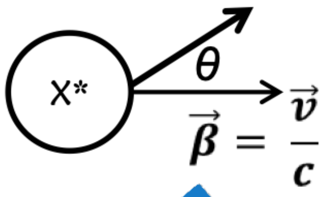
A very simple starting point…

a "simple" case: N_0 nuclei populated at t_0=0 in an isolated level with lifetime τ

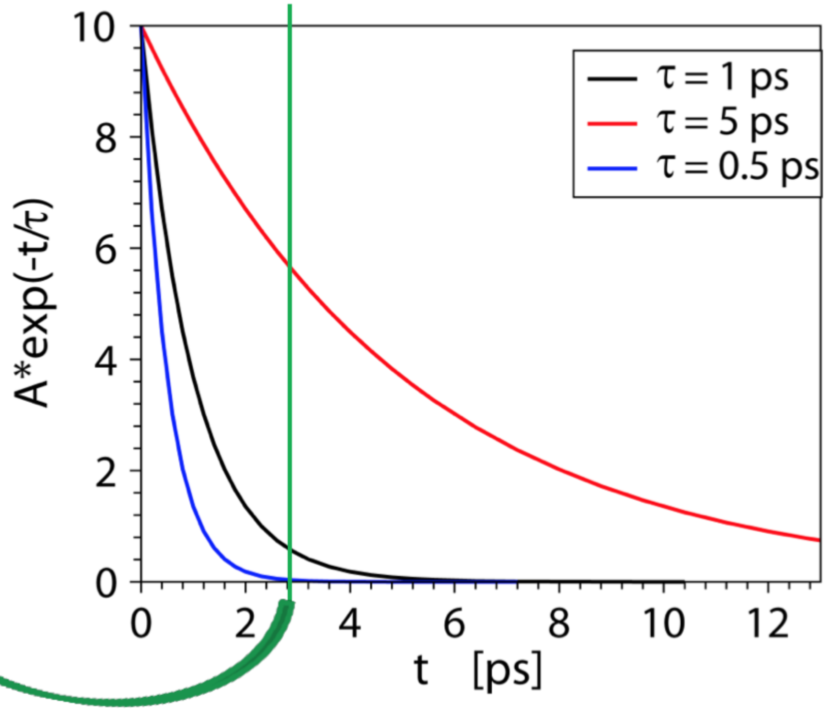


$$N(t) = \frac{N_0}{\tau}\, e^{-t/\tau}$$

$$I(t) = \int_0^t \frac{N_0}{\tau}\, e^{-t/\tau}$$



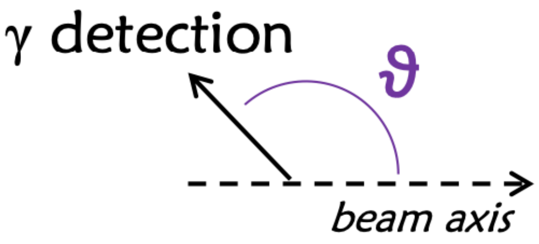
The intensity "observed" at the time t is sensitive to the lifetime of the de-exciting level



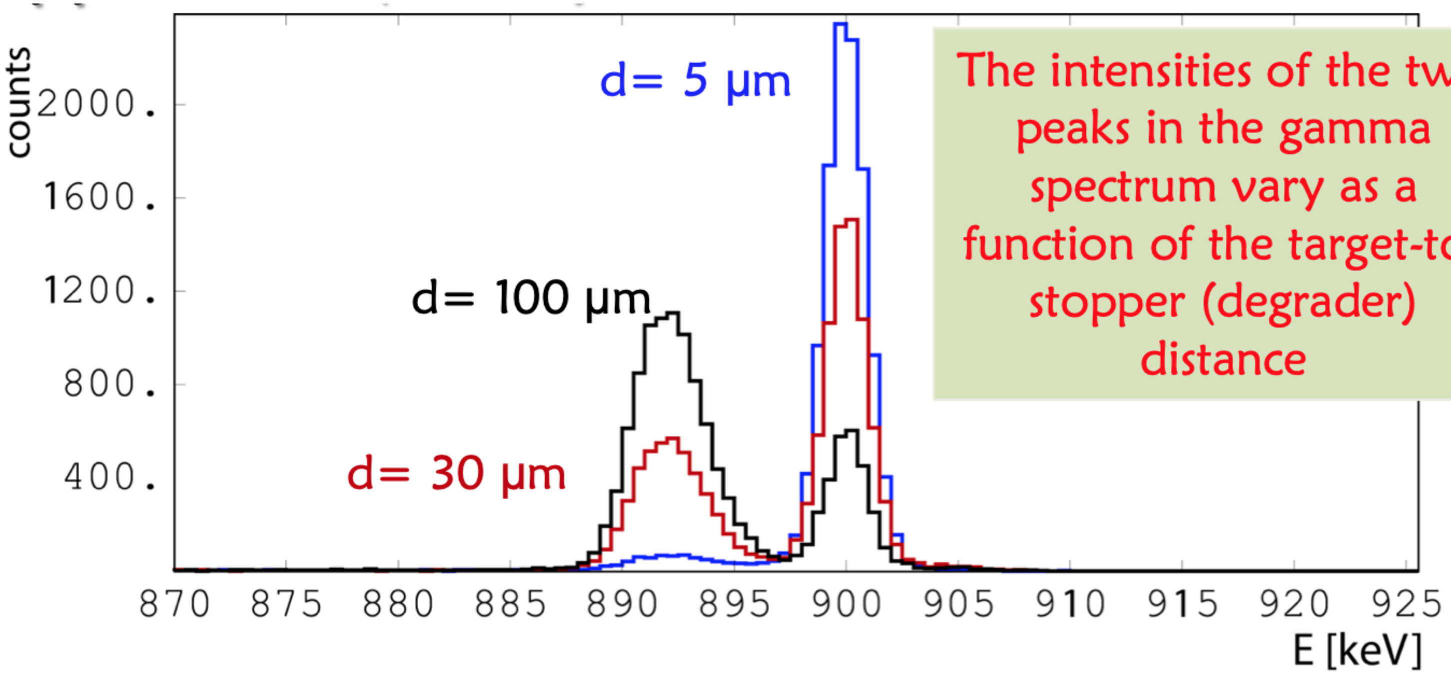
t = d/v can be taken as a reference time

("easy" in case of constant v…)

Recoil Distance Doppler Shift Techniques

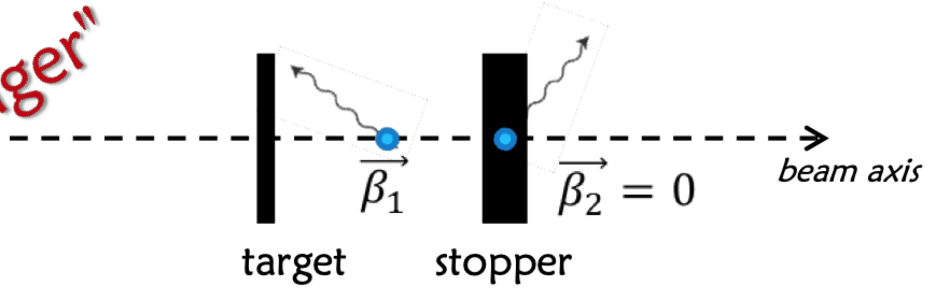


$t_{1/2}$ = 10 ps $\beta_1 \sim 1\%$
E_g = 900 keV $\beta_2 = 0$

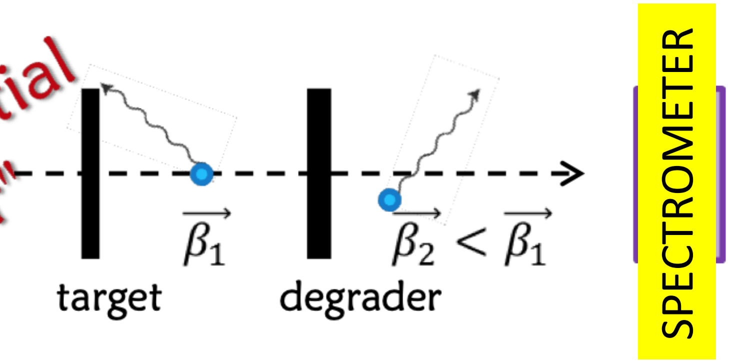


The intensities of the two peaks in the gamma spectrum vary as a function of the target-to-stopper (degrader) distance

"plunger"

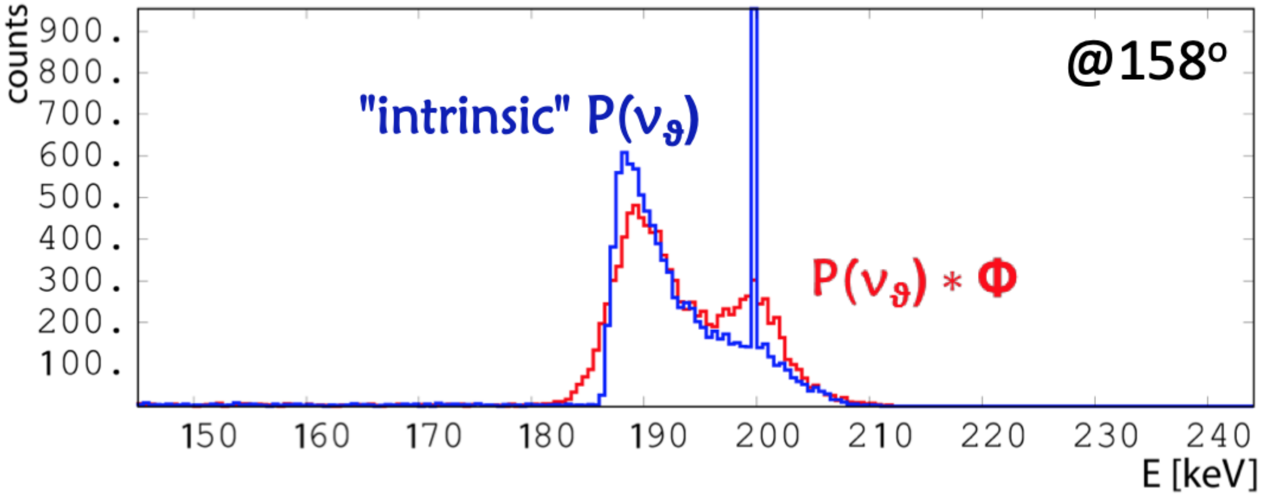


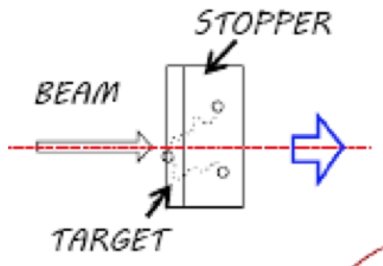
"differential plunger"

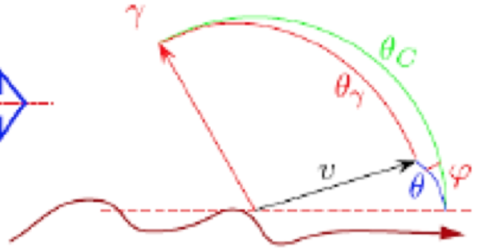


Doppler Shift Attenuation Method (DSAM)

when the lifetime is ≈ slowing down time of the emitting ion in the substrate material a continuous energy distribution is observed in the gamma spectrum, from the "nominal" energy to the one corresponding to the max. Doppler shift







$$P(\nu_\vartheta)=\int_0^\infty dt\, S(t,\nu_\theta)\,\frac{n(t)}{\tau}$$

n(t) = decay function
(exp(-t/τ) in "our simple case")
$S(t,\nu_\vartheta)$ = slowing down matrix
Φ = detector response function

Monte Carlo simulations of the stopping process allow for the determination of the lifetime from a *line-shape analysis* *(W.M.Currie, NIM 73 (1969) 173)*

Nowadays: full Geant4 simulations

Doppler shift techniques

Some remarks

Main systematic errors on the determination of τ:

- multiple (side) feeding → "true" lifetime ≠ effective lifetime
- nuclear de-orientation (plunger) → variation of the intensities for different distances travelled by the emitting ion not related to the lifetime (S.Harissopulos *et al.*, NP A467 (1987) 528 and ref. therein)
- uncertainties on the (nuclear) stopping powers (DSAM→10-15%)
- in "special cases" a lineshape analysis is advantageous also for RDDS data (P.Petkov *et al.*, NIM A431 (1999) 208)

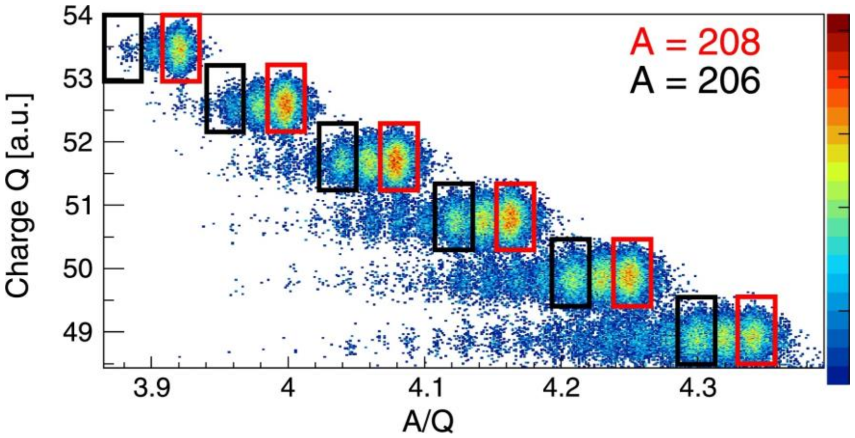
Analysis of coincidence data: RDDS → A.Dewald *et al.*, Z.Phys. A344 (1989) 163
DSAM → F.Brandolini *et al.*, NIM A417 (1998) 150

↓

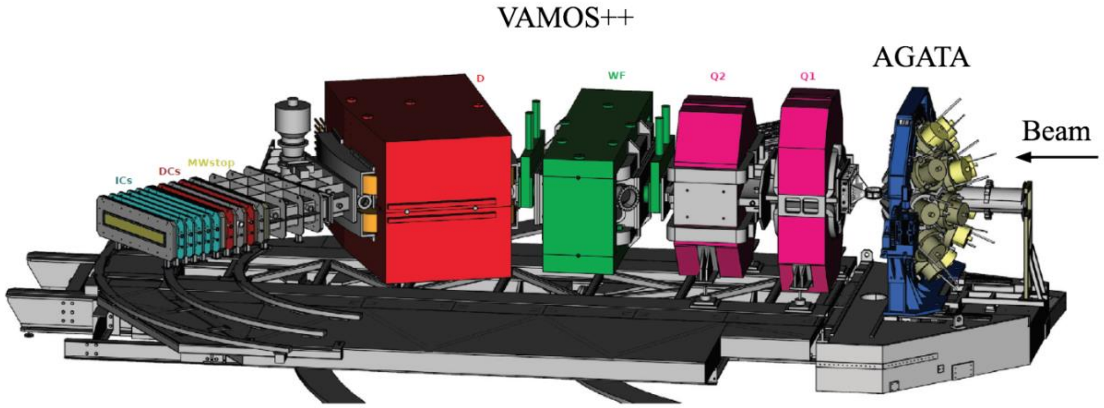
This is not the case when the statistics is sufficient only for singles gamma spectra analysis!!!

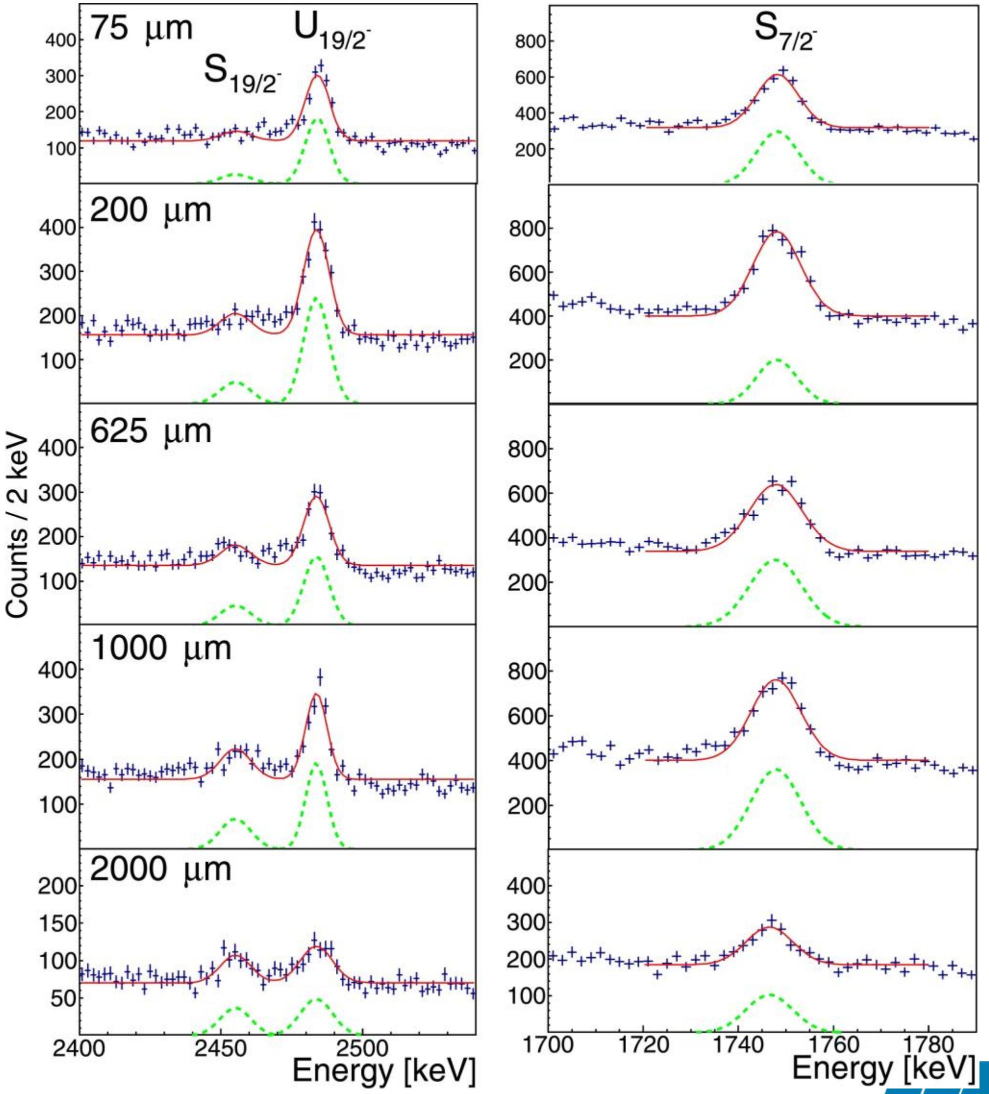
Lifetime measurements in ^{207}Pb

^{208}Pb beam @ 6.25*A* MeV on enriched 1.9 mg/cm^2-thick ^{100}Mo target 2 mg/cm^2-thick Ni degrader



Beam-like reaction products detected and identified on an event-by-event basis in the large-acceptance VAMOS++ spectrometer





D. Ralet et al, PLB 797 (2019) 134797

There's not a nuclear model that explains everything!

A challenge for both theory and nuclear instrumentation