

# Gravitational Wave signature from QCD matter in Proto-Compact Stars

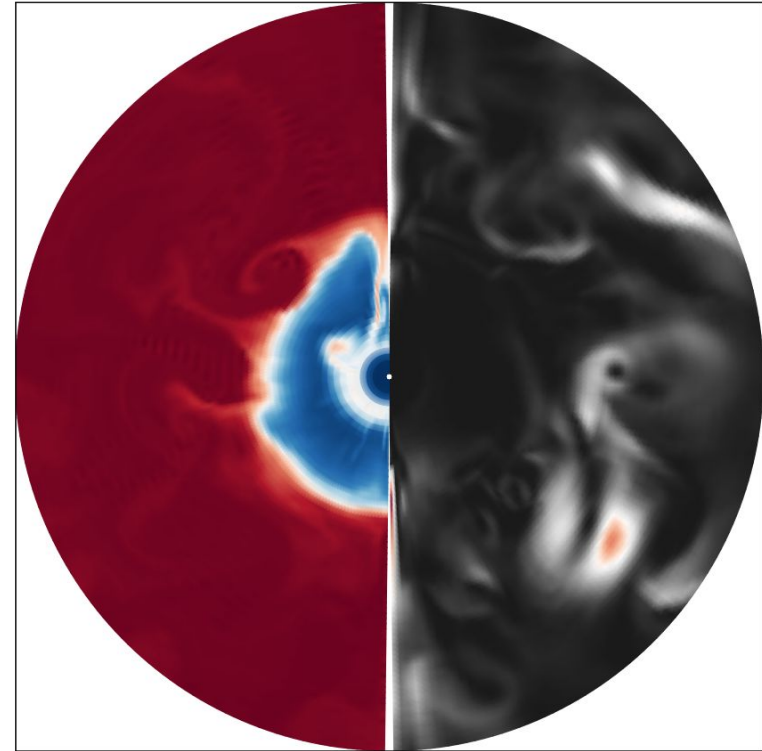


Workshop on the QCD Equation of State 2023

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MONASH  
University

ASTRO3D

- Stars with  $M > 8 M_s$  (H,He,C,Ne,O,Si)
  - Iron final product of nuclear fusion
  - Silicon shell burning accreting on PCS;  $M > 1.44 M_s$
- Core collapses



- Core bounces
  - Formation of shock
  - Neutrinos become trapped
  - Photodissociation of heavy nuclei into nucleons
- Accretion shock at  $\sim 100\text{-}200\text{km}$

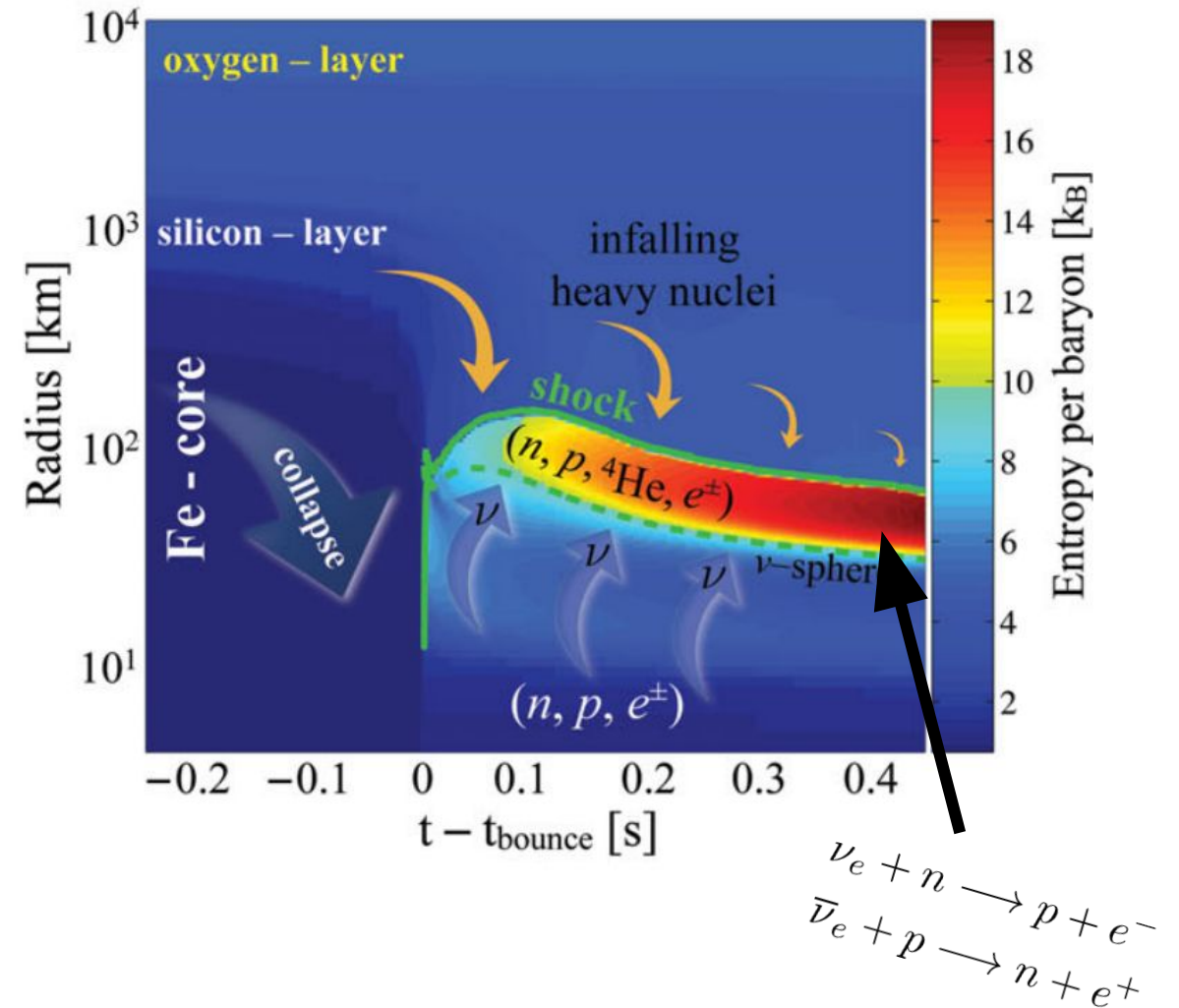


- Hot Proto-Compact Star in interior accreting material
  - Neutrinos decouple from Neutrinosphere
  - Transport energy into "hot bubble region"
- Shock revival?

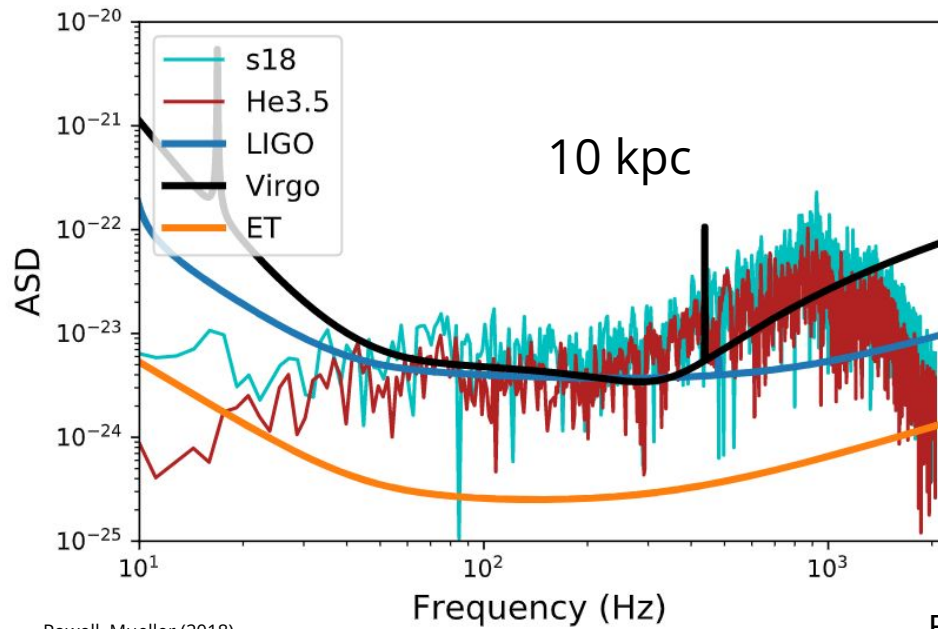


- Late Proto-Compact Star fully transparent to neutrinos

## Overview I: Core-Collapse Supernovae



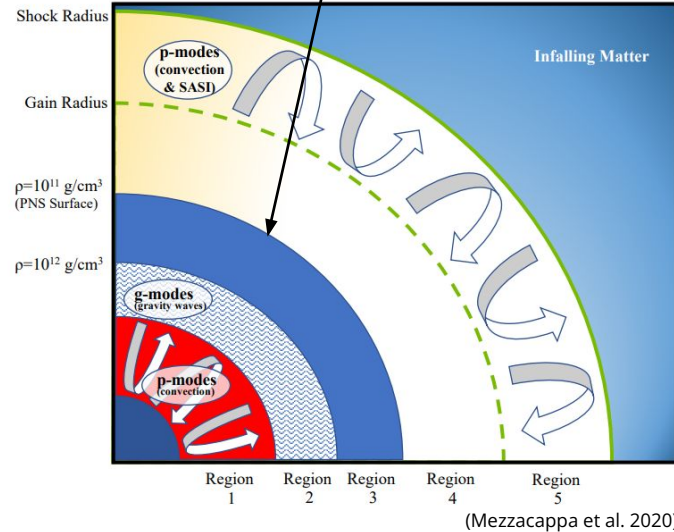
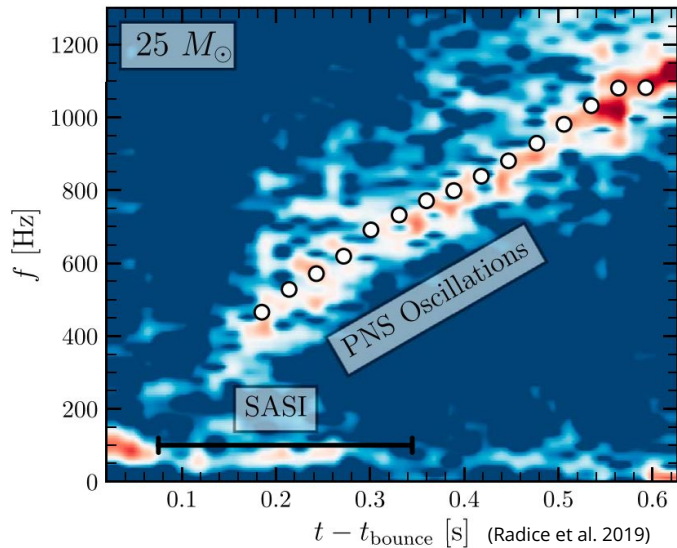
# Gravitational Waves from Core-Collapse Supernovae



Powell, Mueller (2018)

- CCSNe ~3 event per century in our Galaxy
- Observable distance ~10-20 kpc
- Time changing mass quadrupole moment
- GWs provide access to compact inner core

Radius of Proto-Compact Stars ~40km to ~10km  
 Mass accretion on PCS:  $\sim 1.4-2M_{\odot}$



(Mezzacappa et al. 2020)

- Different regions susceptible to different mode oscillations (p-,g- f-modes)
- Positive entropy gradient: g-mode oscillations
- Characteristic frequency of g-modes:

*Brunt Väisälä frequency:*

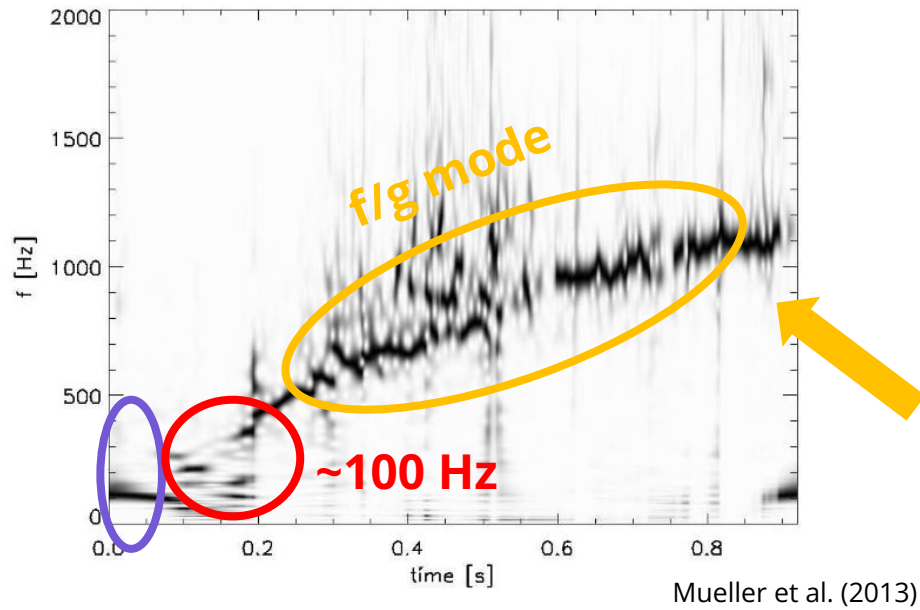
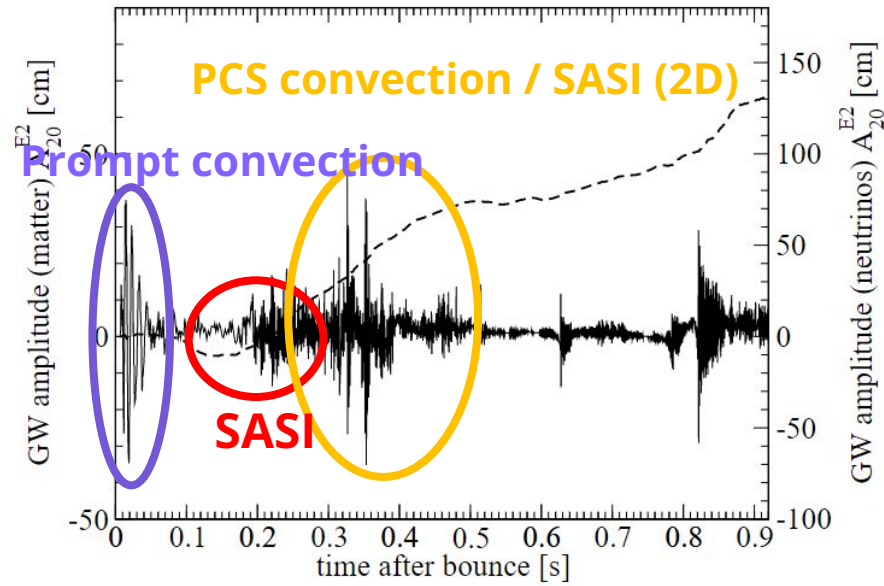
$$\omega_{\text{BV}}^2 = \frac{d\alpha}{dr} \frac{\alpha}{\rho h \Phi^4} \cdot \left( \frac{1}{c_s^2} \frac{dP}{dr} - \frac{d\rho}{dr} \right)$$

Stripy clouds  $\rightarrow$

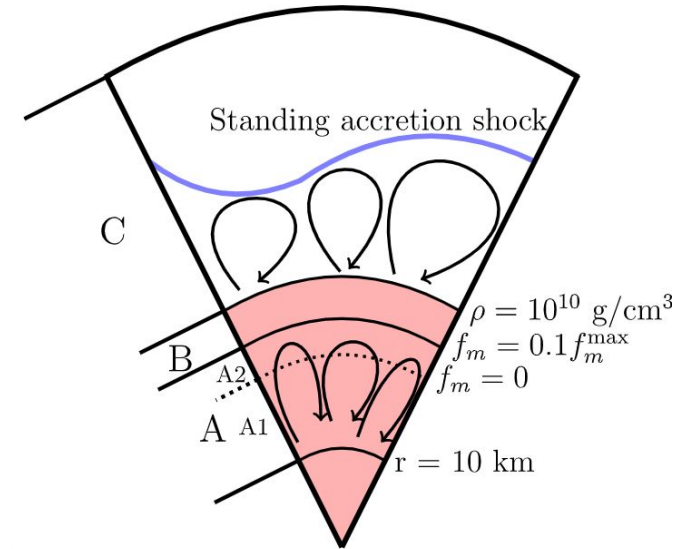
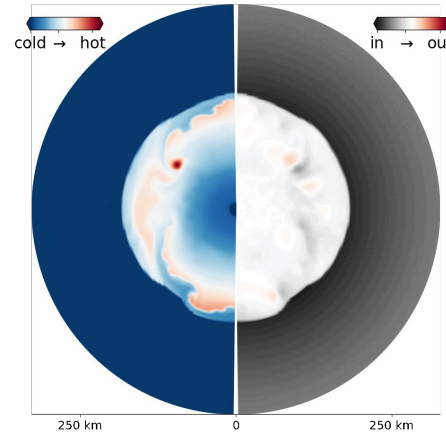


# Common features of the GW signal

- Turbulent structure by convective downflows stronger in 2D
- Frequency spectrum of SASI *narrower* in 3D
- Amplitudes in 2D overestimated (PCS amplitudes ~10x)



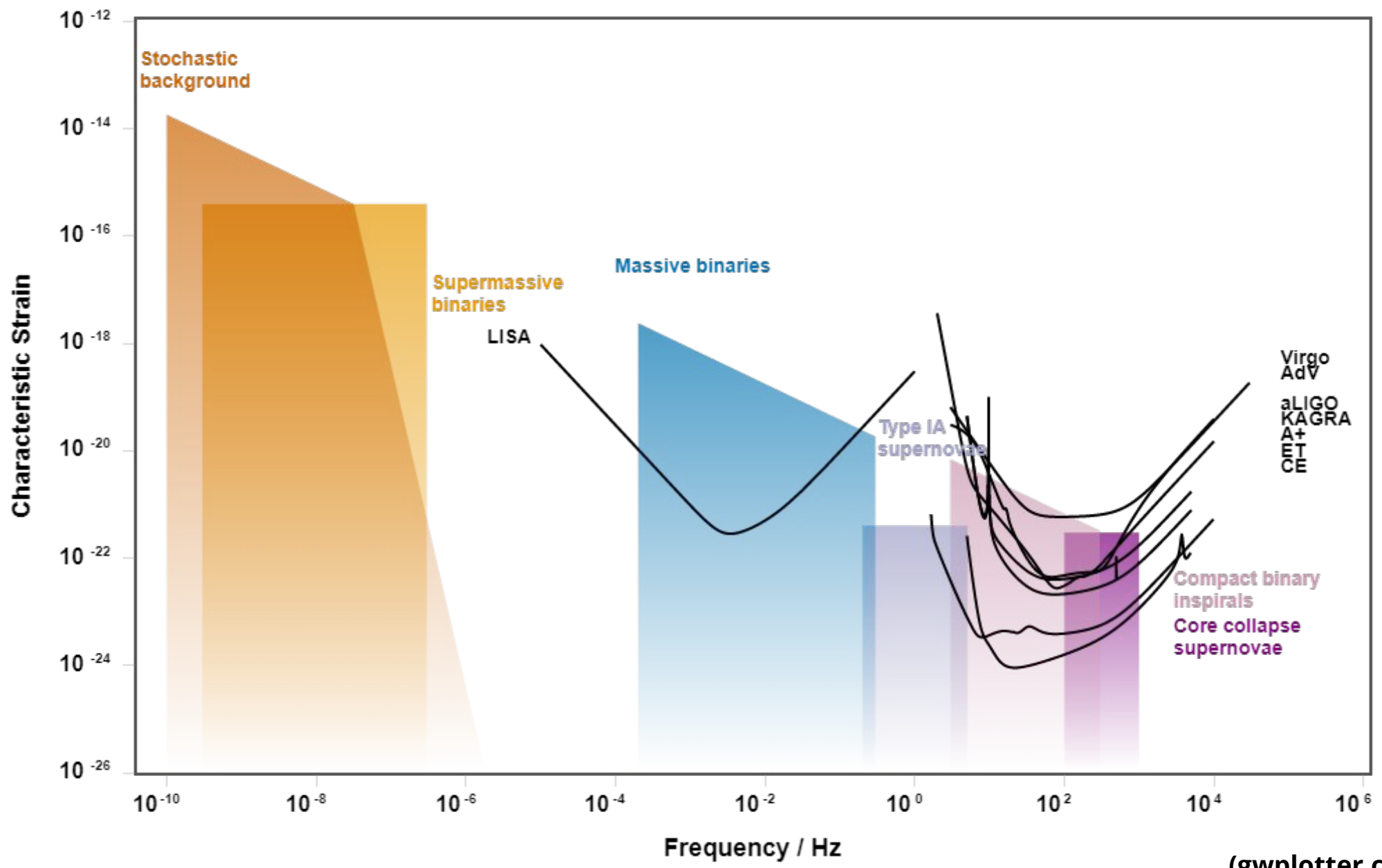
2D GW output for 15 solar mass progenitor



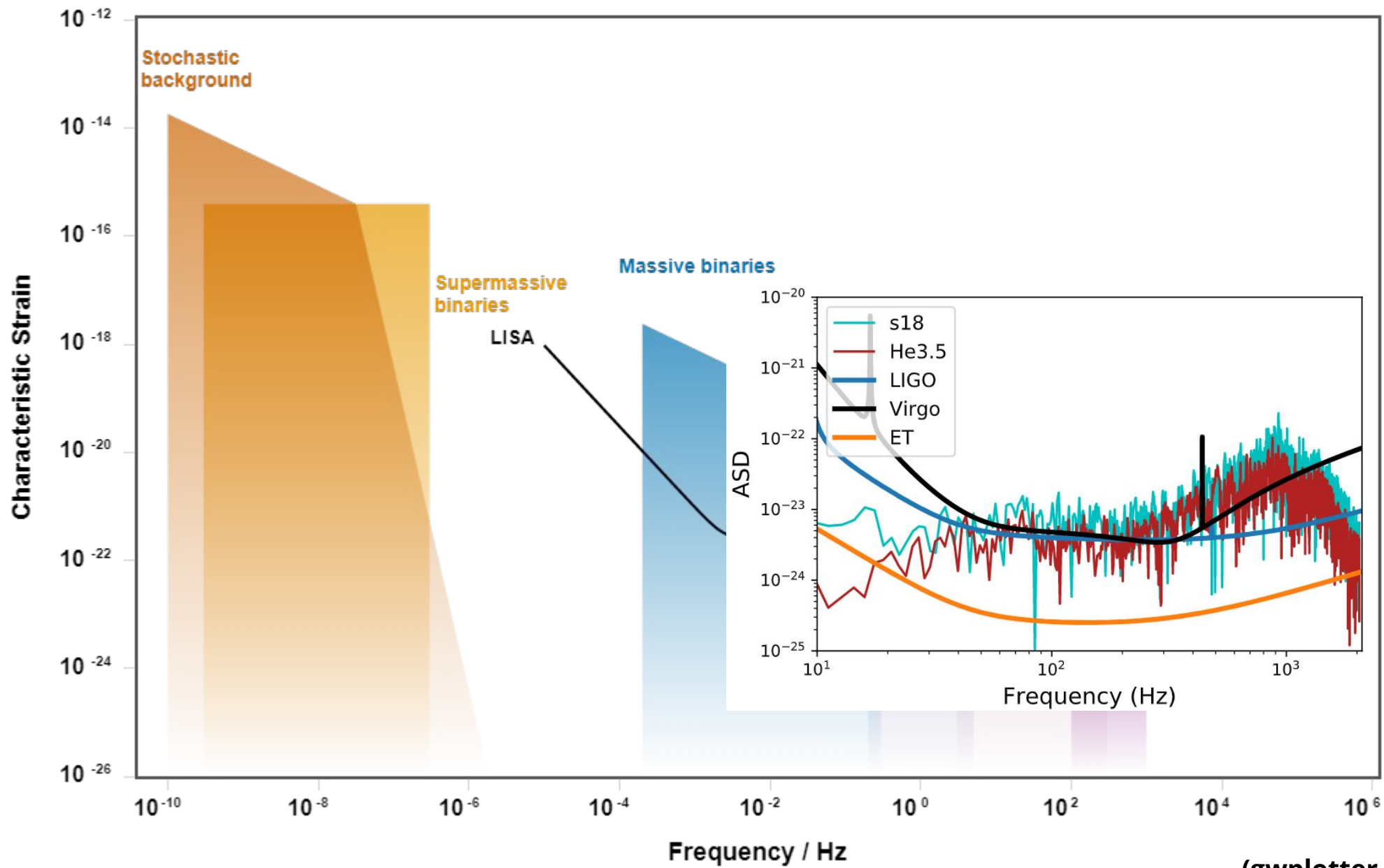
$$f_{\text{peak}} \approx \frac{1}{2\pi} \frac{GM}{R^2} \sqrt{1.1 \frac{m_n}{\langle E_\nu \rangle} \left(1 - \frac{GM}{Rc^2}\right)^{-2}}$$

Electron antineutrino mean energy





(gwplotter.com)



(gwplotter.com)

CCSN & NSM

Quark-gluon plasma

Hadronic matter

?  $T_{crit}$

$10^{12}$  K

Chiral restoration / Phase transition

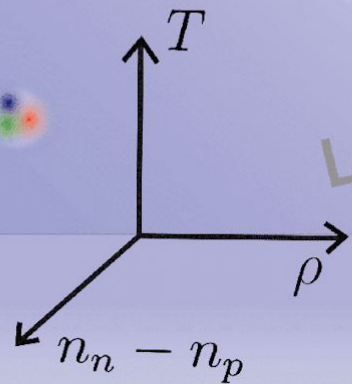
Liquid gas

Nuclear matter

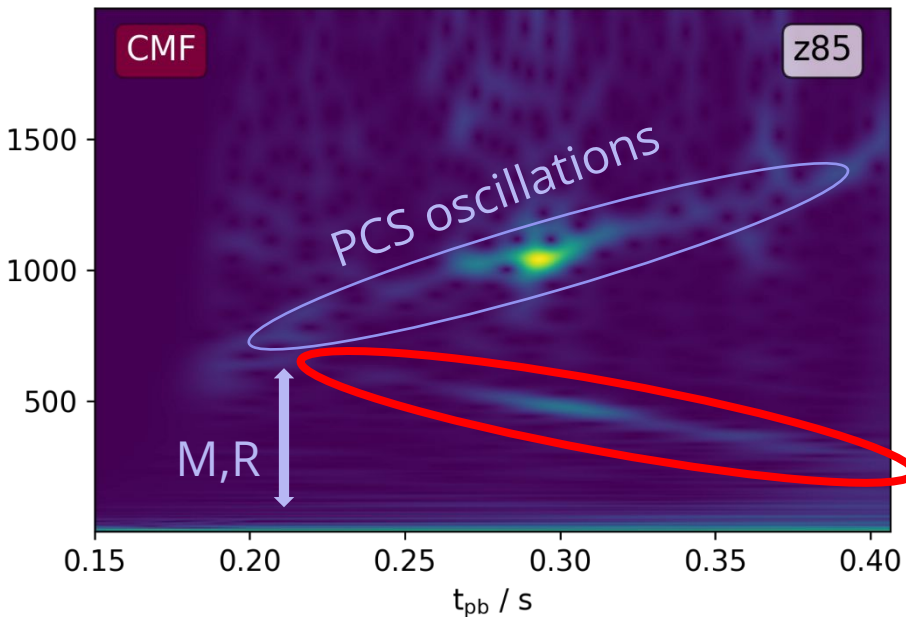
$2.3 \cdot 10^{14} \text{gcm}^{-3}$

Neutron stars

$10^{15} \text{gcm}^{-3}$



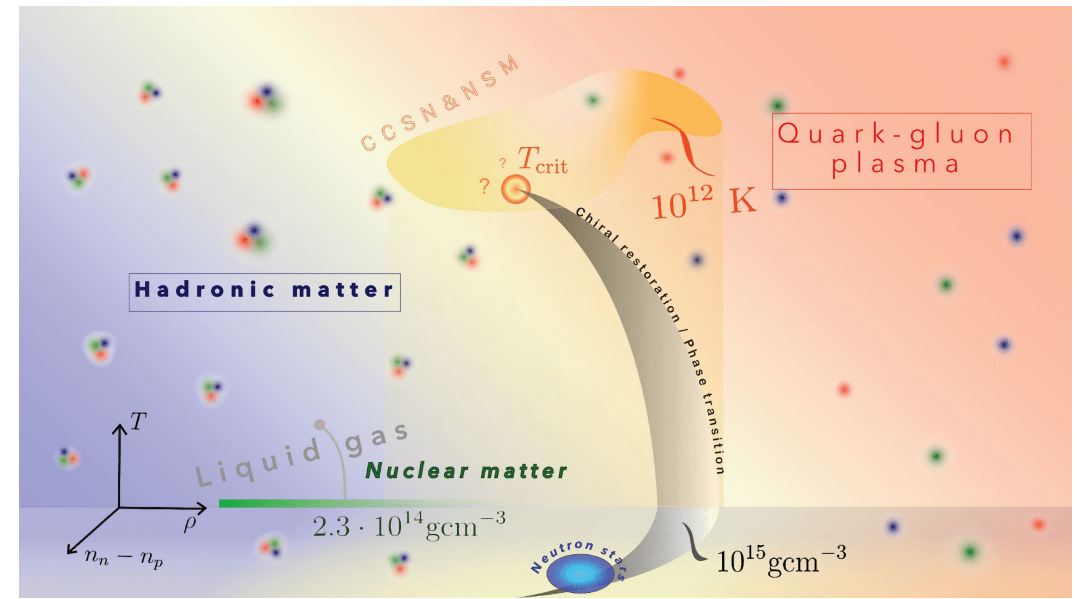




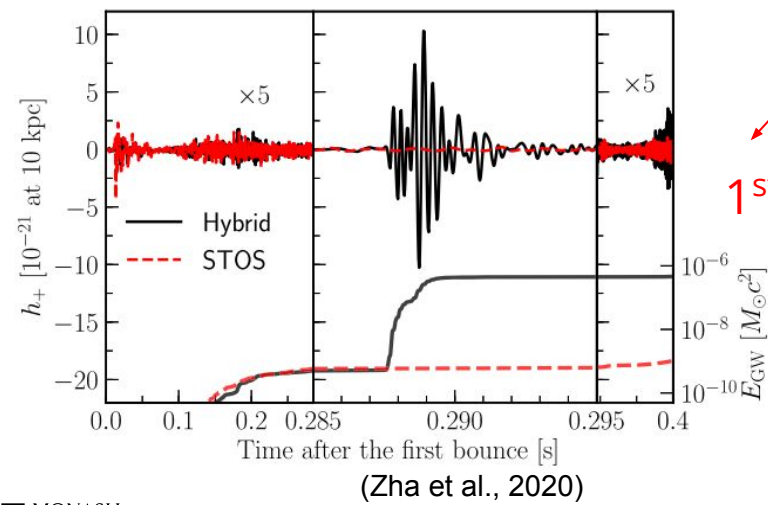
PCS evolution



Constrain EoS

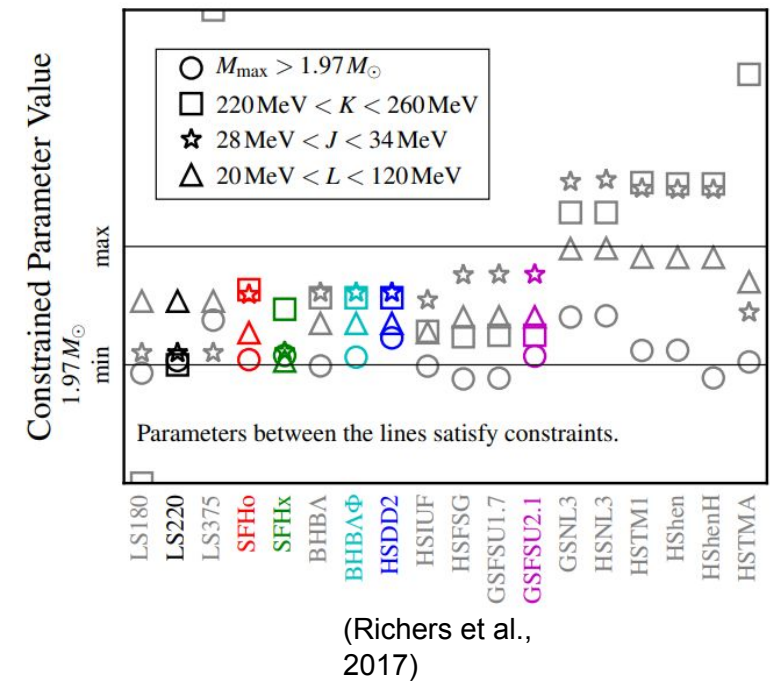
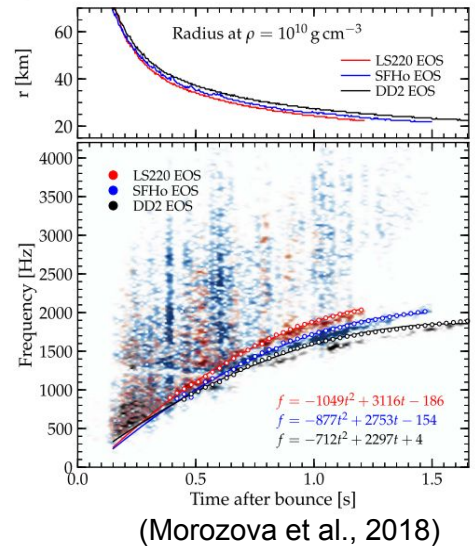


$$f_{\text{peak}} \approx \frac{1}{2\pi} \frac{GM}{R^2} \sqrt{1.1 \frac{m_n}{\langle E_\nu \rangle} \left(1 - \frac{GM}{Rc^2}\right)^2}$$



Free quarks?

1<sup>st</sup> order PT





EOS	$n_0$ [fm <sup>-3</sup> ]	$E_0$ [MeV]	$K$ [MeV]	$S$ [MeV]	$L$ [MeV]	$R_{1.4}$ [km]	$M_{\max}$ [ $M_{\odot}$ ]
SFHo	0.1583	16.19	245	31.57	47.10	11.89	2.06
SFHx	0.1602	16.16	238	28.67	23.18	11.99	2.13
HS(TM1)	0.1455	16.31	281	36.95	110.99	14.47	2.21
HS(TMA)	0.1472	16.03	318	30.66	90.14	13.85	2.02
HS(FSUgold)	0.1482	16.27	229	32.56	60.43	12.55	1.74
HS(DD2)	0.1491	16.02	243	31.67	55.04	13.22	2.42
HS(IUFSU)	0.1546	16.39	231	31.29	47.20	12.68	1.95
HS(NL3)	0.1482	16.24	272	37.39	118.49	14.77	2.79
STOS(TM1)	0.1452	16.26	281	36.89	110.79	14.50	2.22
LS (180)	0.1550	16.00	180	28.61	73.82	12.16	1.84
LS (220)	0.1550	16.00	220	28.61	73.82	12.67	2.05
Exp.	$\sim 0.15$	$\sim 16$	$240 \pm 10^1$	$29.0 - 32.7^2$	$40.5 - 61.9^2$	$10.4 - 12.9^3$	$\gtrsim 2.0^{4,5}$

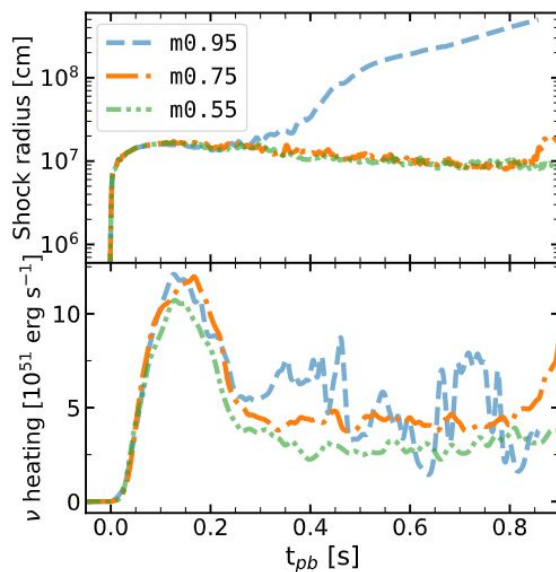
1. Lattimer-Swetsy EoS (1991)
2. Shen EoS (1998)

Lattimer, Swetsy: A generalized equation of state for hot, dense matter; 1991

Shen, Toki, Oyamatsu & Sumiyoshi: Relativistic equation of state of nuclear matter for supernova and neutron star; 1998

## EoS effects in CCSNe

- Pressure
  - Structure, core-bounce,...
- Entropy
  - neutrino energy
- Composition
  - Weak interactions



Andersen et al. 2021

- **Radius:** If PCS contracts faster an explosion can set in more easily (Janka, 2012, Suwa et al. (2013)); earlier (smaller BH mass)
- **Stiffness:** Neutrino luminosities increased; more efficient heating behind the shock
- **Symmetry energy:** impacts PCS convection, lower symmetry energy  $S$  leads to lower core  $Y_e$  (Fischer et al. 2014)
- **Effective nucleon masses:** larger  $m^*$  decrease thermal pressure in core (Schneider et al. 2019, Andresen et al. 2021)

# Equation of state in CCSNe

SFHx EoS (Steiner et al., 2013) ( $\rho > 10^{11} \text{ g cm}^{-3}$ )

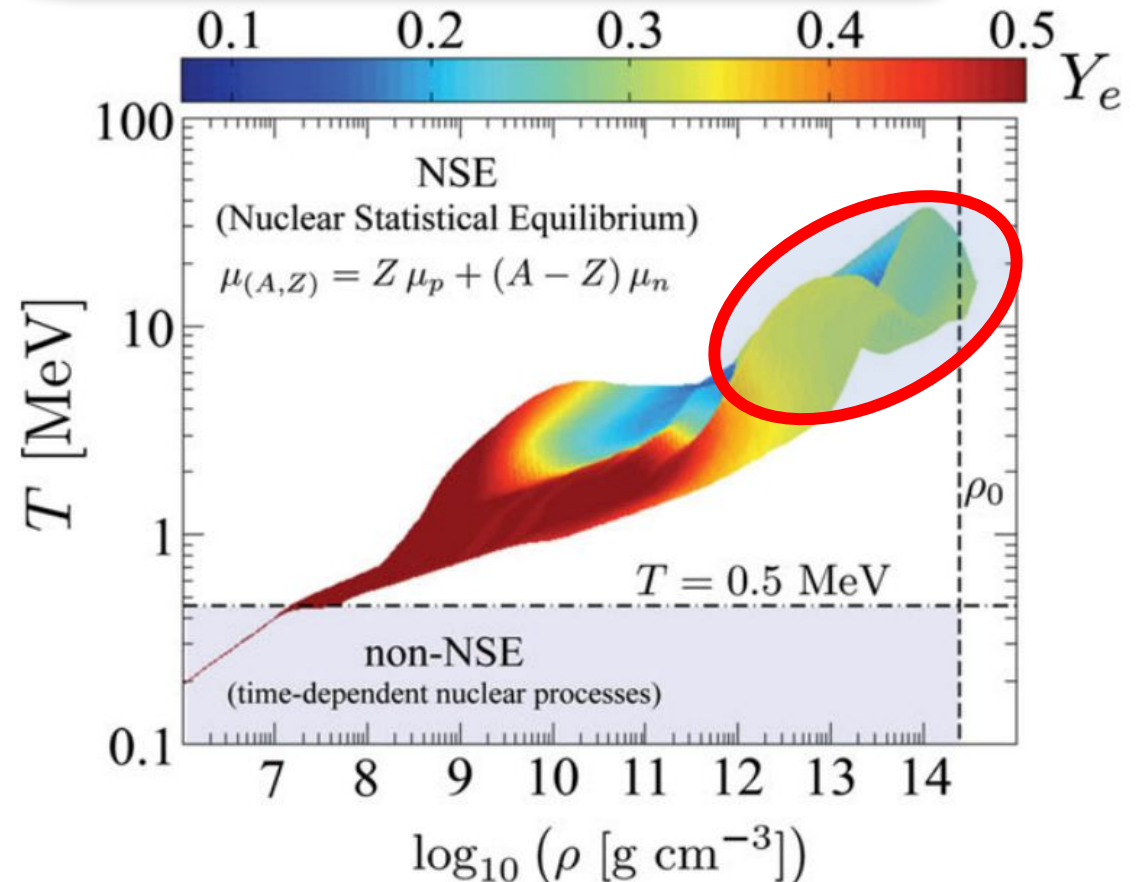
- **Purely hadronic**
- Class of 'HS' (Hempel & Schaffner-Bielich)
- Nuclei (several 1000) are treated as Boltzmann particles
- Nucleons described in **RMF approach (non-linear Walecka model)**
- Fitted to NS properties ( **$2.13 M_s$** )
- Rather low slope parameter 23.2 MeV

CMF EoS (Motornenko et al., 2020) ( $\rho > 10^{11} \text{ g cm}^{-3}$ )

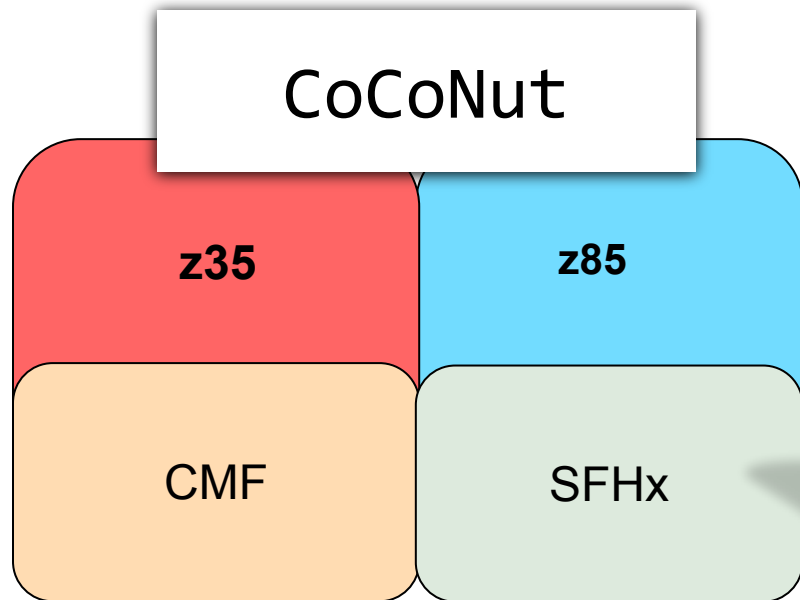
- **Hadron-Quark** Chiral mean field model (CMF)
- Fitted to lattice QCD data
- Excluded volume effects suppress Baryons at high  $\mu_B$
- **1<sup>st</sup> order nuclear liquid vapor phase transition**
- Weak **1<sup>st</sup> order chiral phase transition** at about  $4 \times \rho_{sat}$
- **Smooth deconfinement** to quarks (Polyakov-loop as order parameter)
- Maximum TOV Mass  **$2.10 M_s$**

Low density regime

- ideal gas EoS of iron group nuclei;  $e, e^+, \gamma$
- 5 GK < NSE with 470 different nuclei
- Nuclear reaction network below 5 GK



# Simulation CCSNe explosions

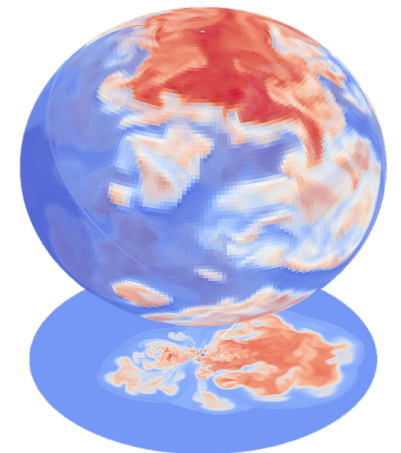


- GR neutrino hydrodynamic code (Dimmelmeier et al. 2002; Mueller et al. 2013)
- Fast three flavor multigroup transport (FMT) for neutrinos (Mueller et al., 2010; Mueller and Janka 2015)
- Inner 1.4 km calculated in spherical symmetry

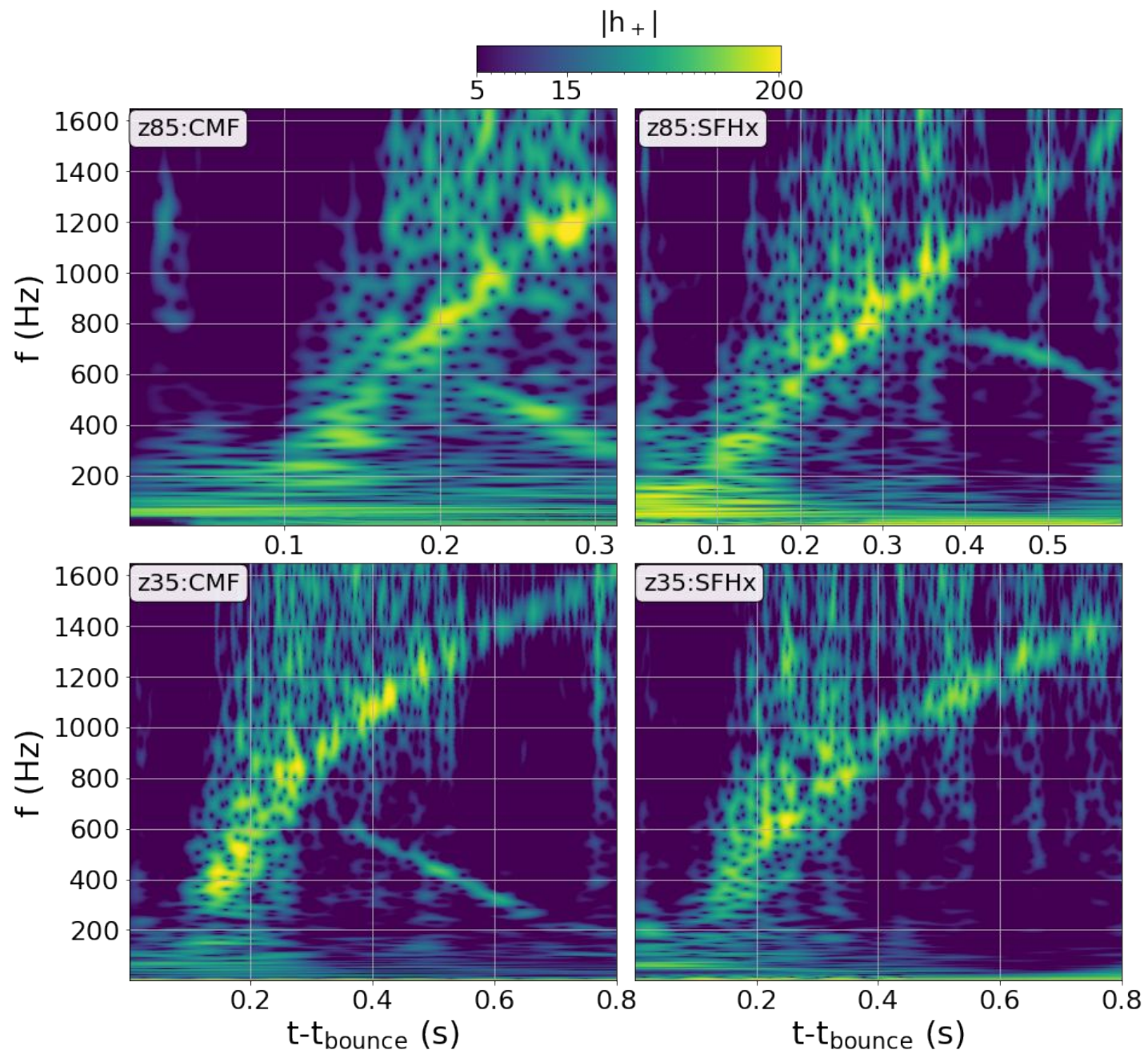
- 4 simulations in axisymmetry

## Neutrinos in CCSNe:

*Physical, numerical, and computational challenges of modeling neutrino transport in core-collapse supernovae (Mezzacappa et al. 2020)*







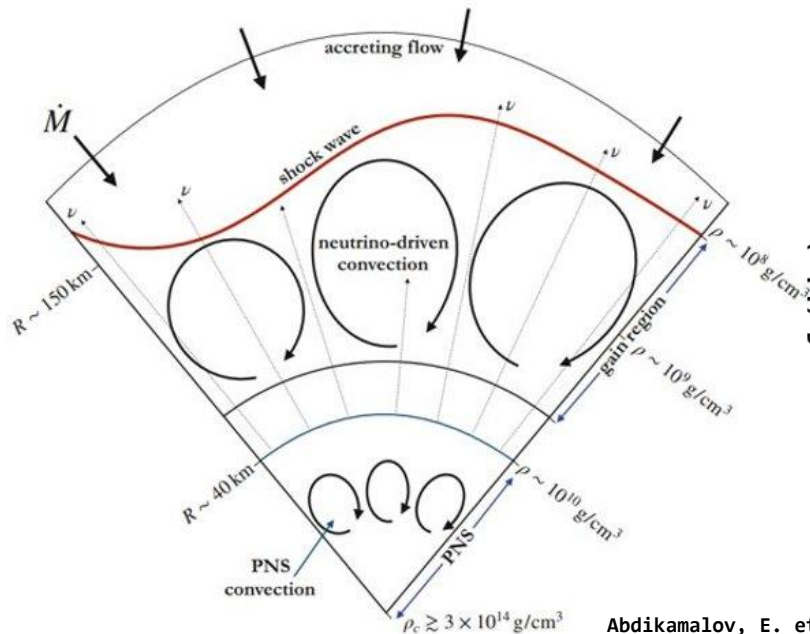


- Gravitational wave signals show lower frequency g-modes
- Frequency depends on Equation of State
  - Where is signal originating from?
  - No signal in z35:SFhx?
  - Lower frequencies in CMF?

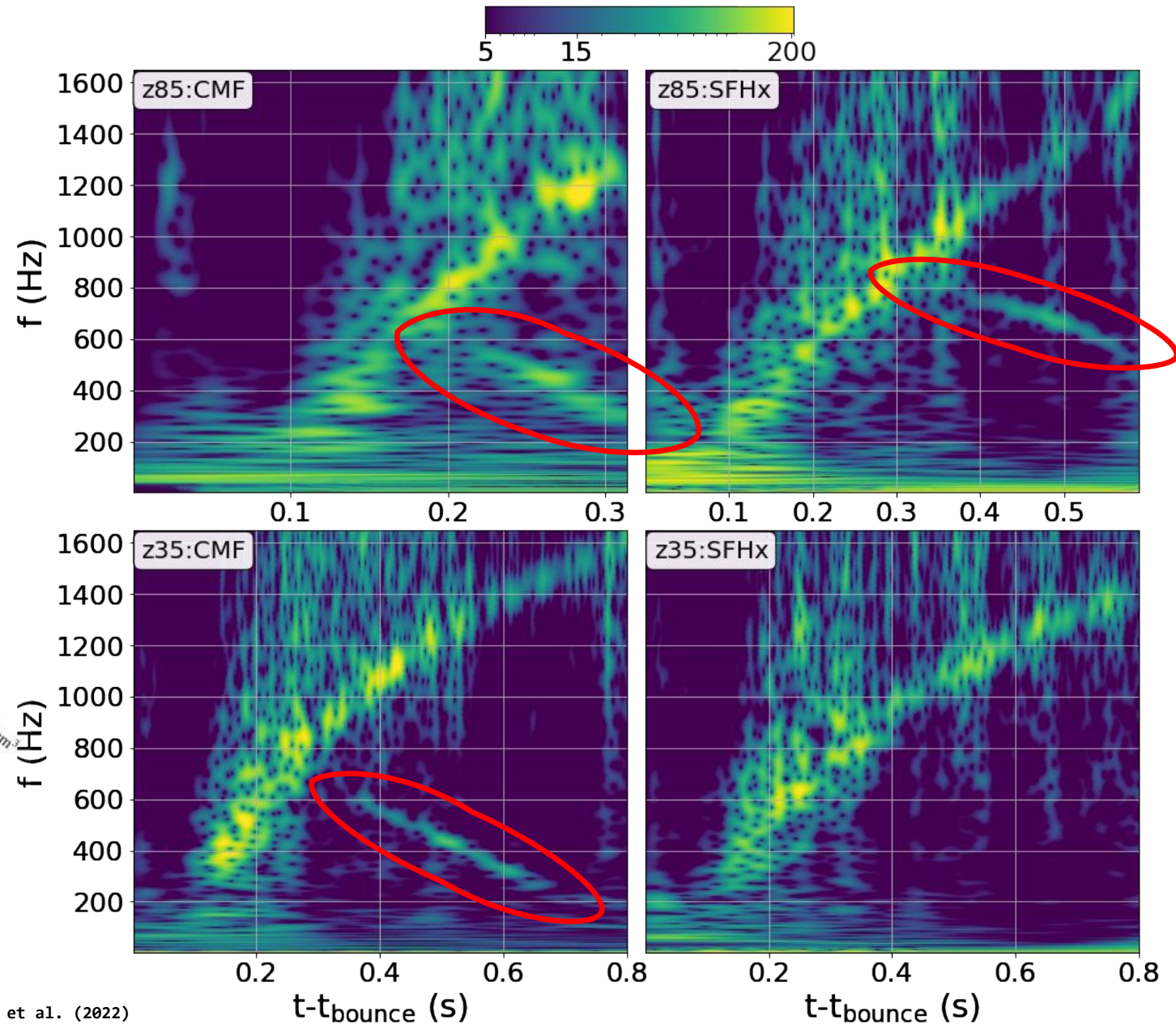
3. Lower frequencies in CMF?



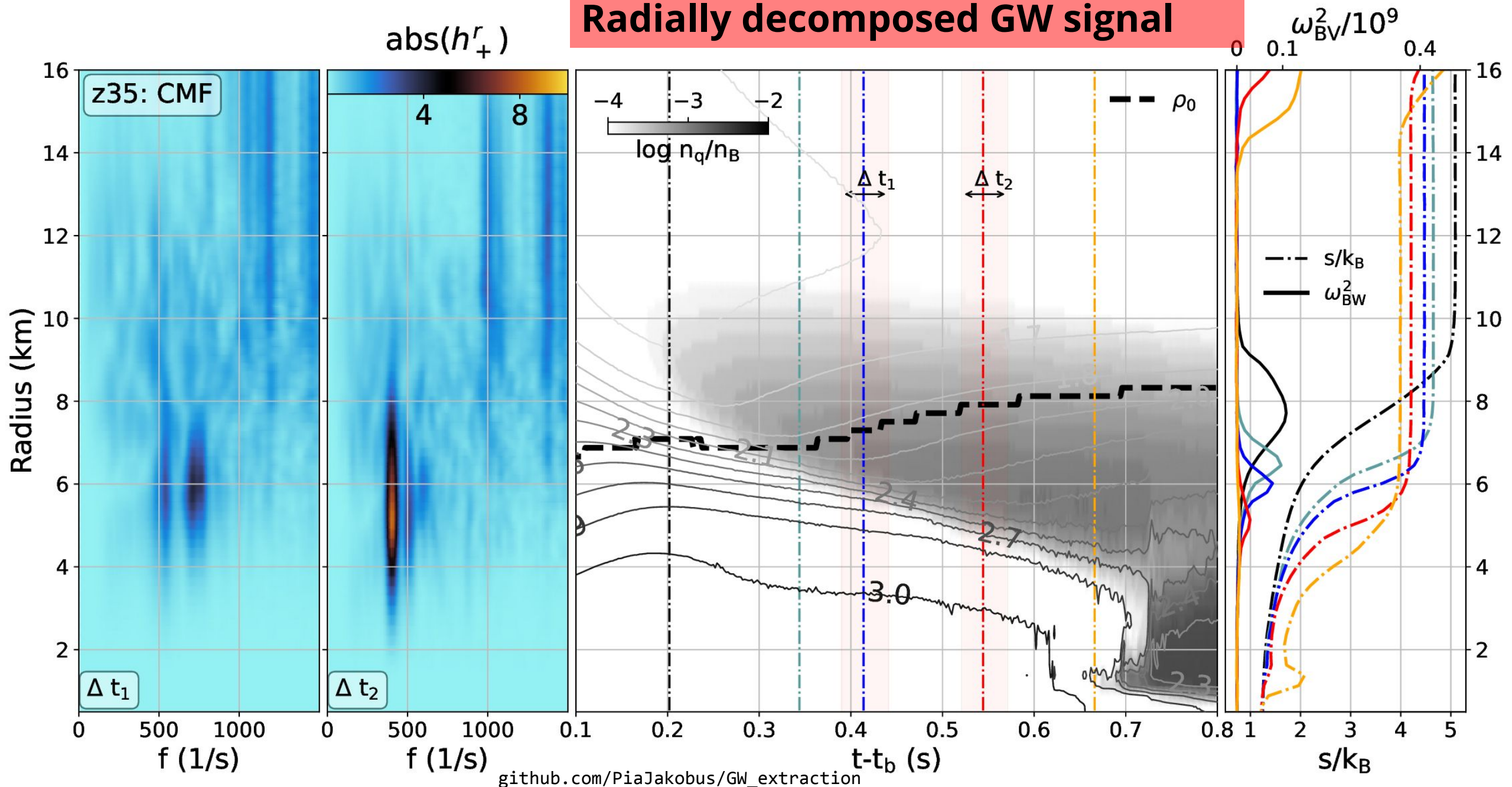
1) Where is the signal originating from?



$$A_{20}^{E2} = \frac{32\pi^{3/2}G}{\sqrt{15}c^4} \int_0^r \int_0^\pi dr d\theta \Phi^6 r^3 \sin\theta \times \left\{ \frac{\partial}{\partial t} (S_r(3\cos^2\theta - 1)) + \frac{3}{r} S_\theta \sin\theta \cos\theta \right\} |h_+|$$

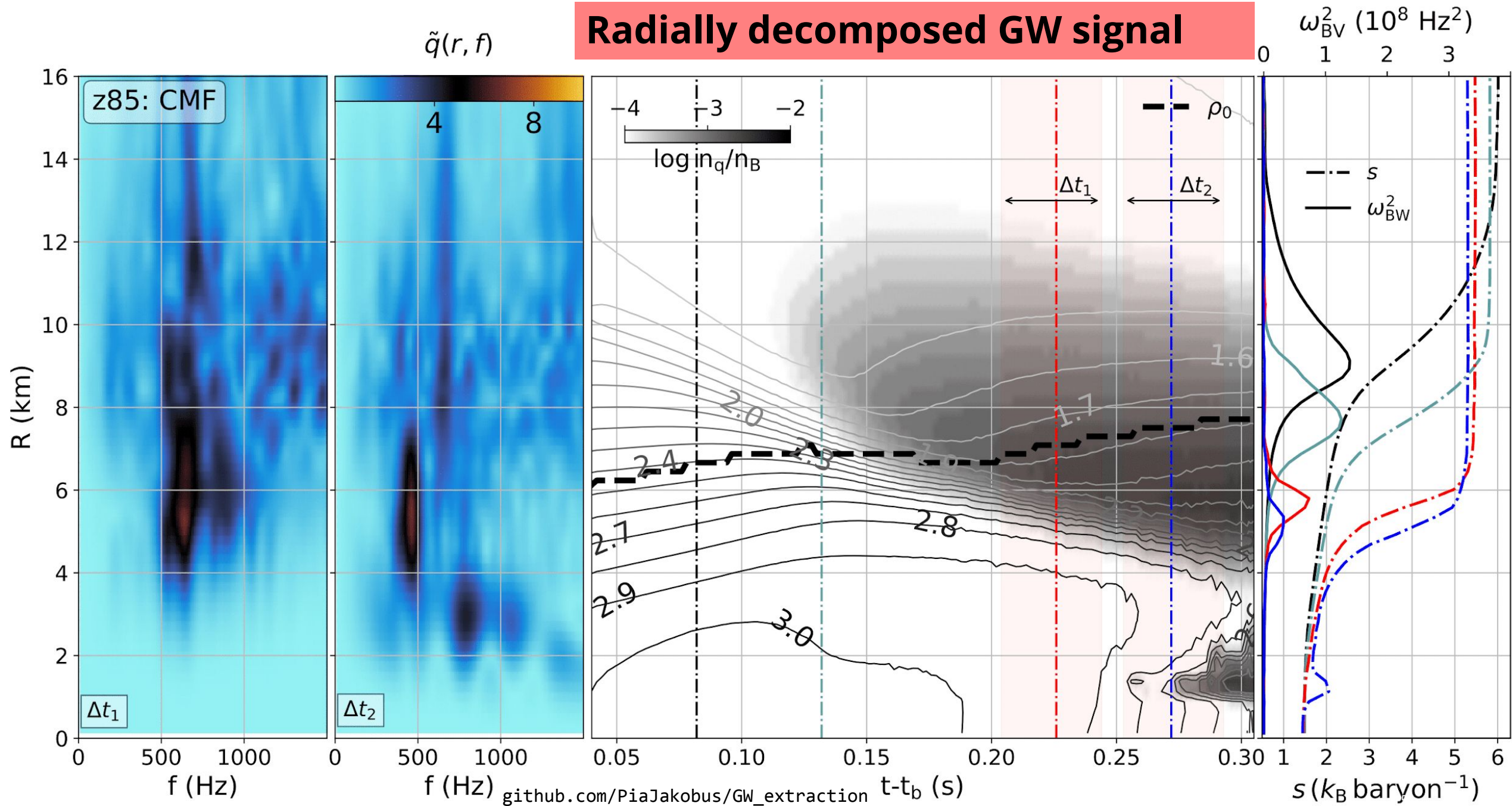


# Radially decomposed GW signal

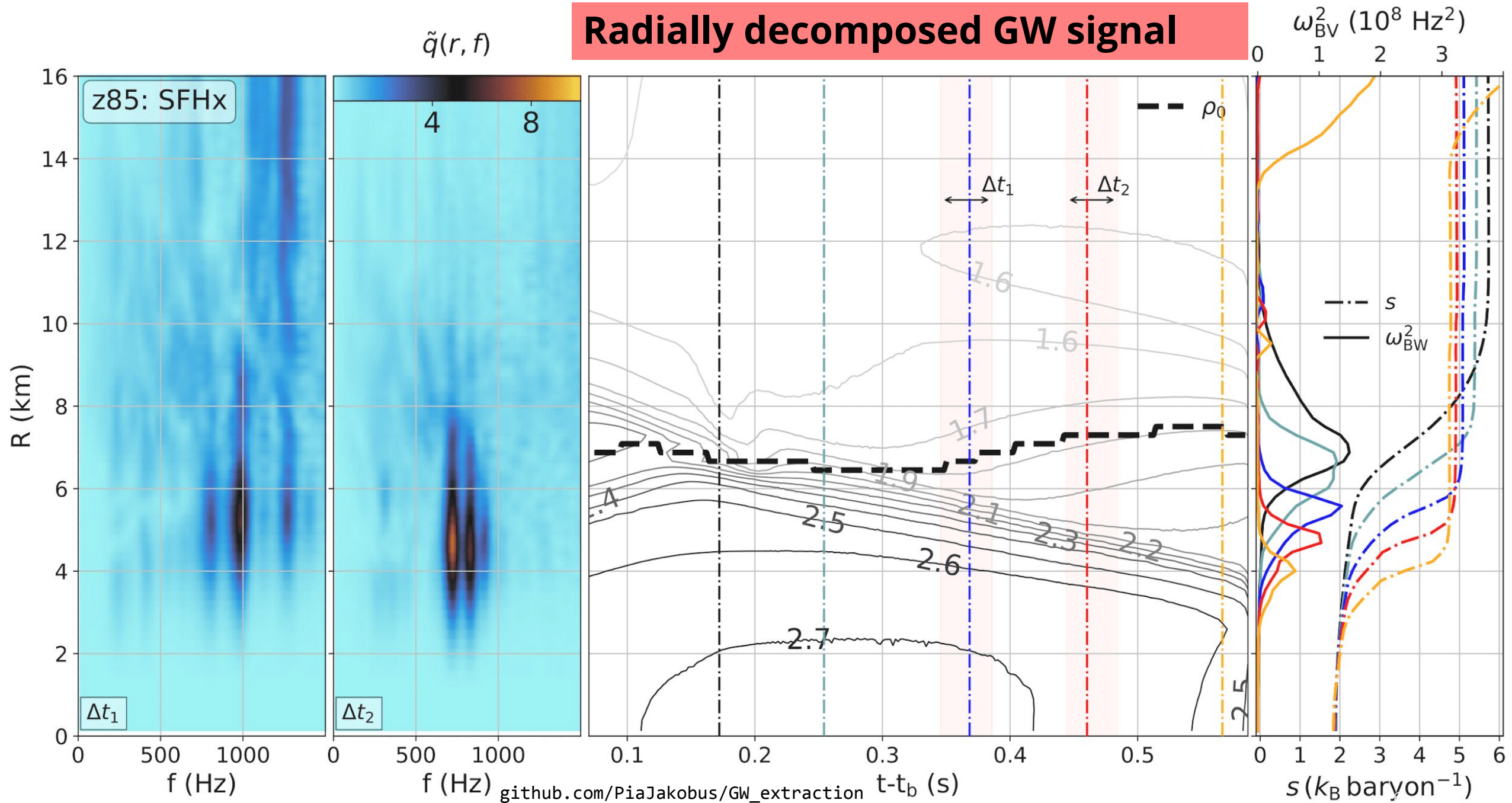




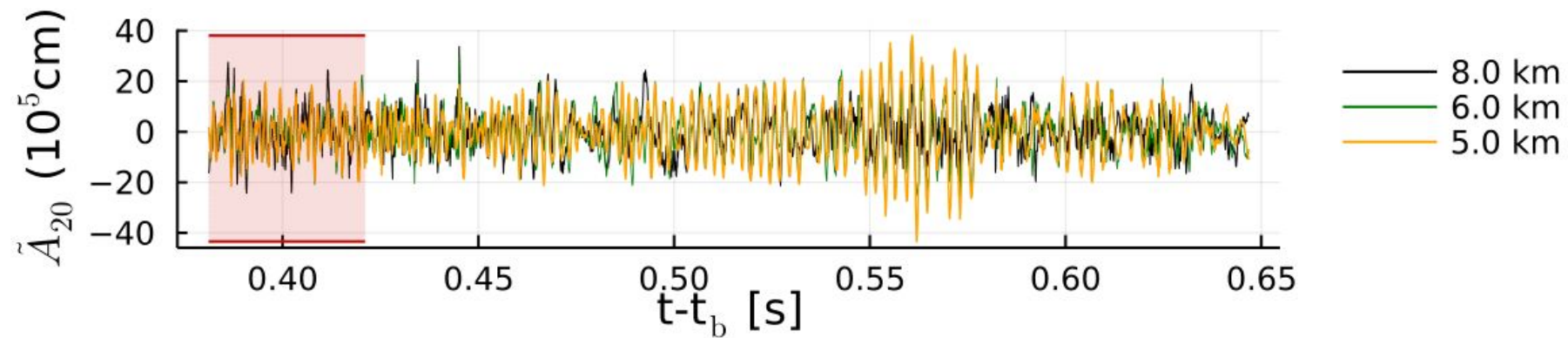
# Radially decomposed GW signal



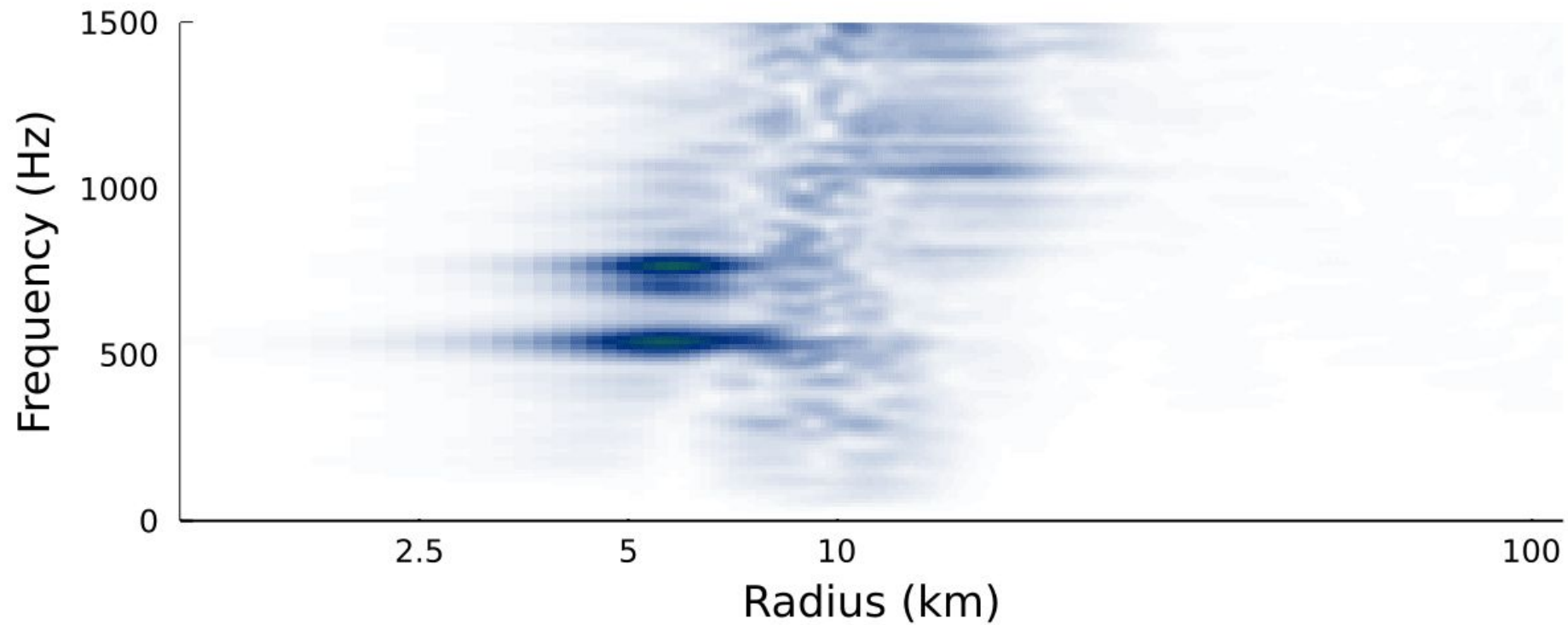
# Radially decomposed GW signal



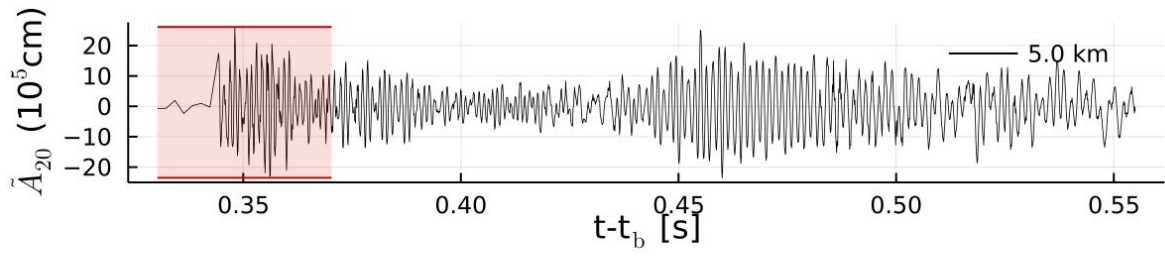




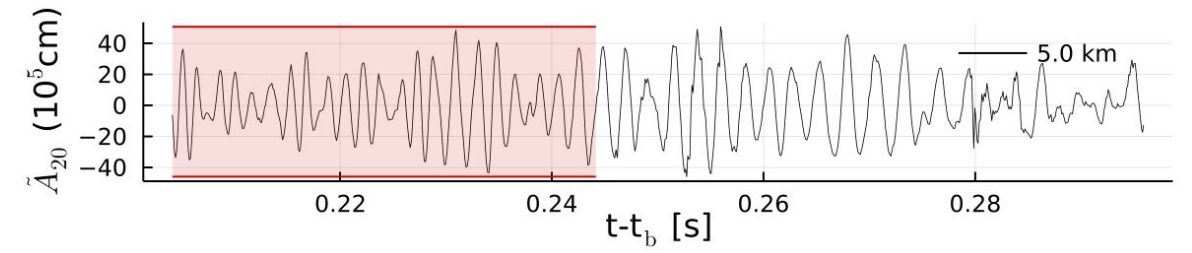
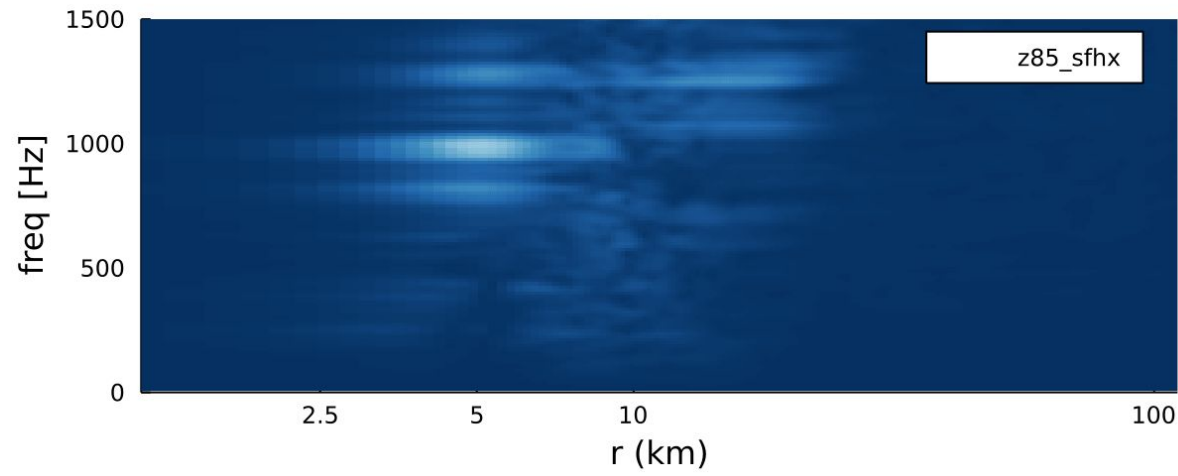
z35 cmf:  $\Delta t=40$  ms



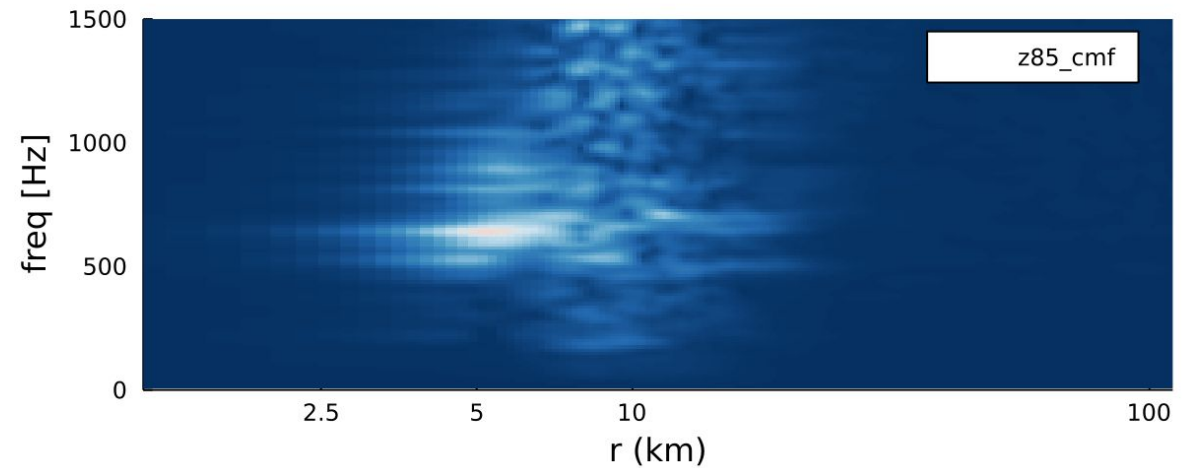
[github.com/PiaJakobus/GW\\_extraction](https://github.com/PiaJakobus/GW_extraction)



$\Delta t = 40$  ms

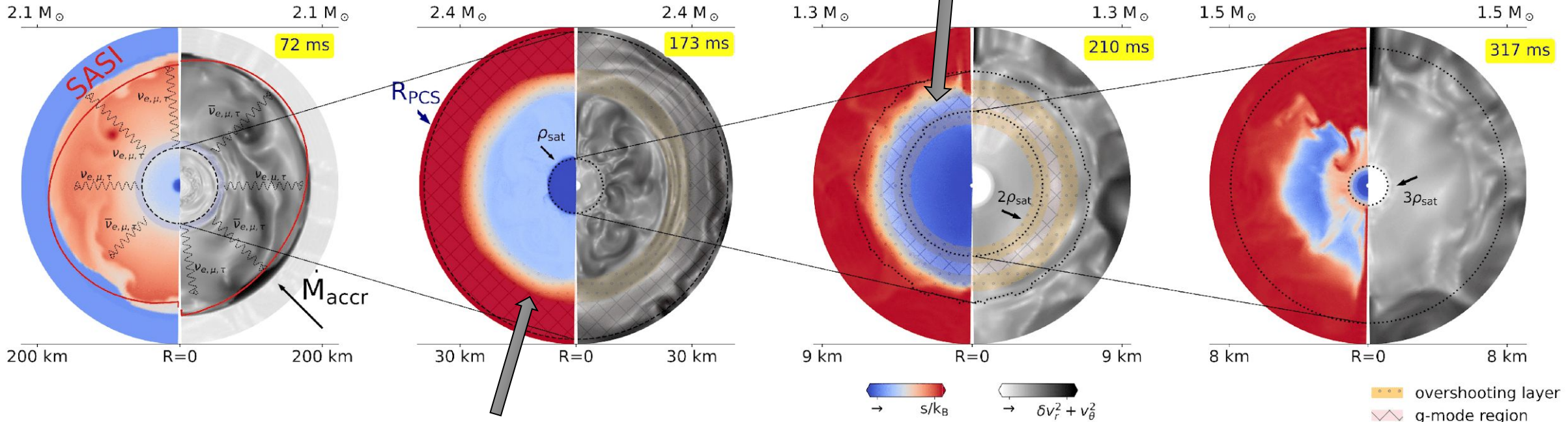
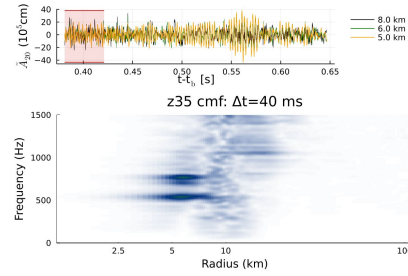


$\Delta t = 40$  ms

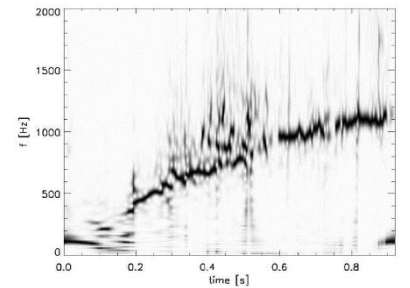


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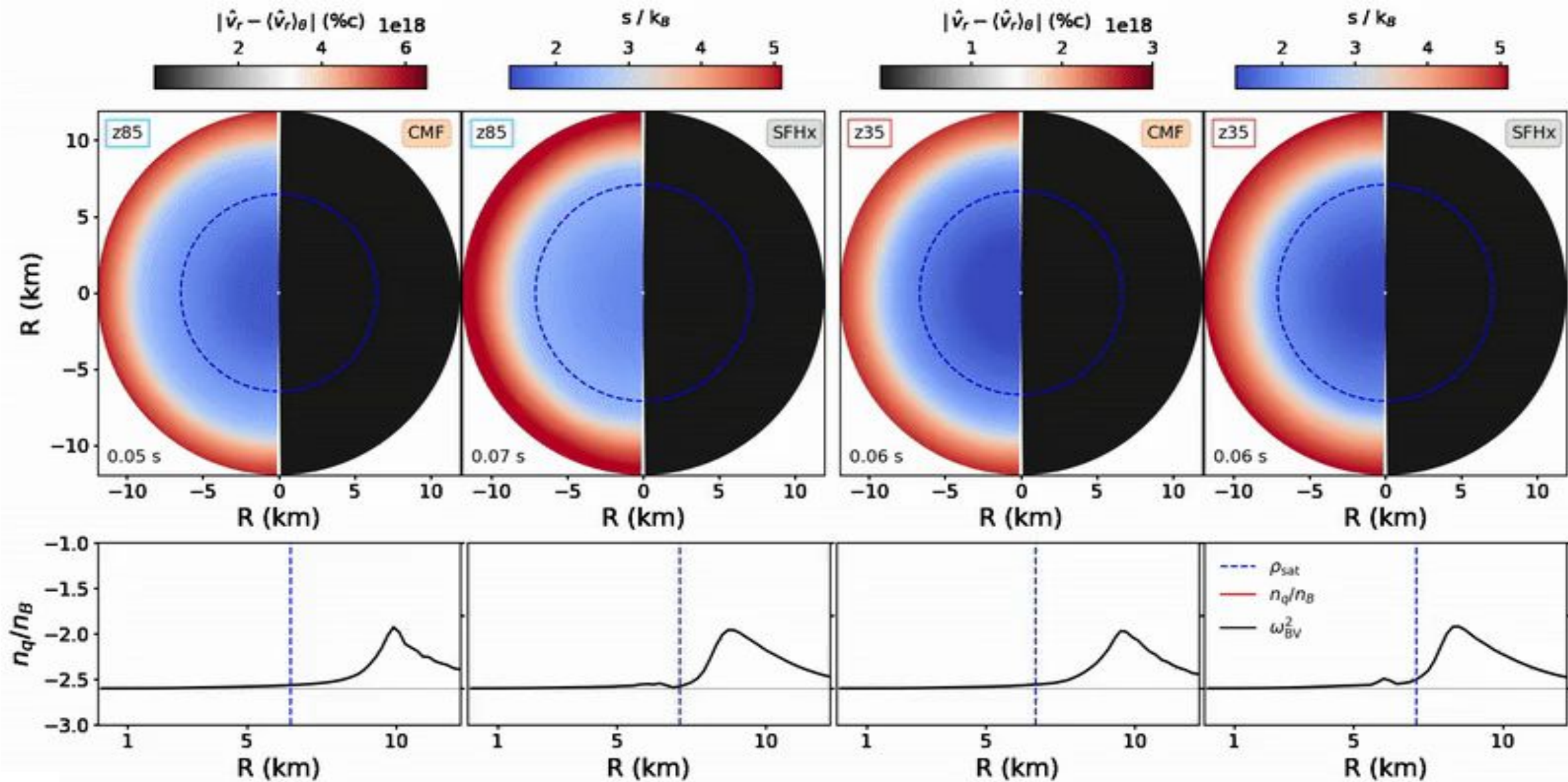
# Core g-mode



# dominant f-mode

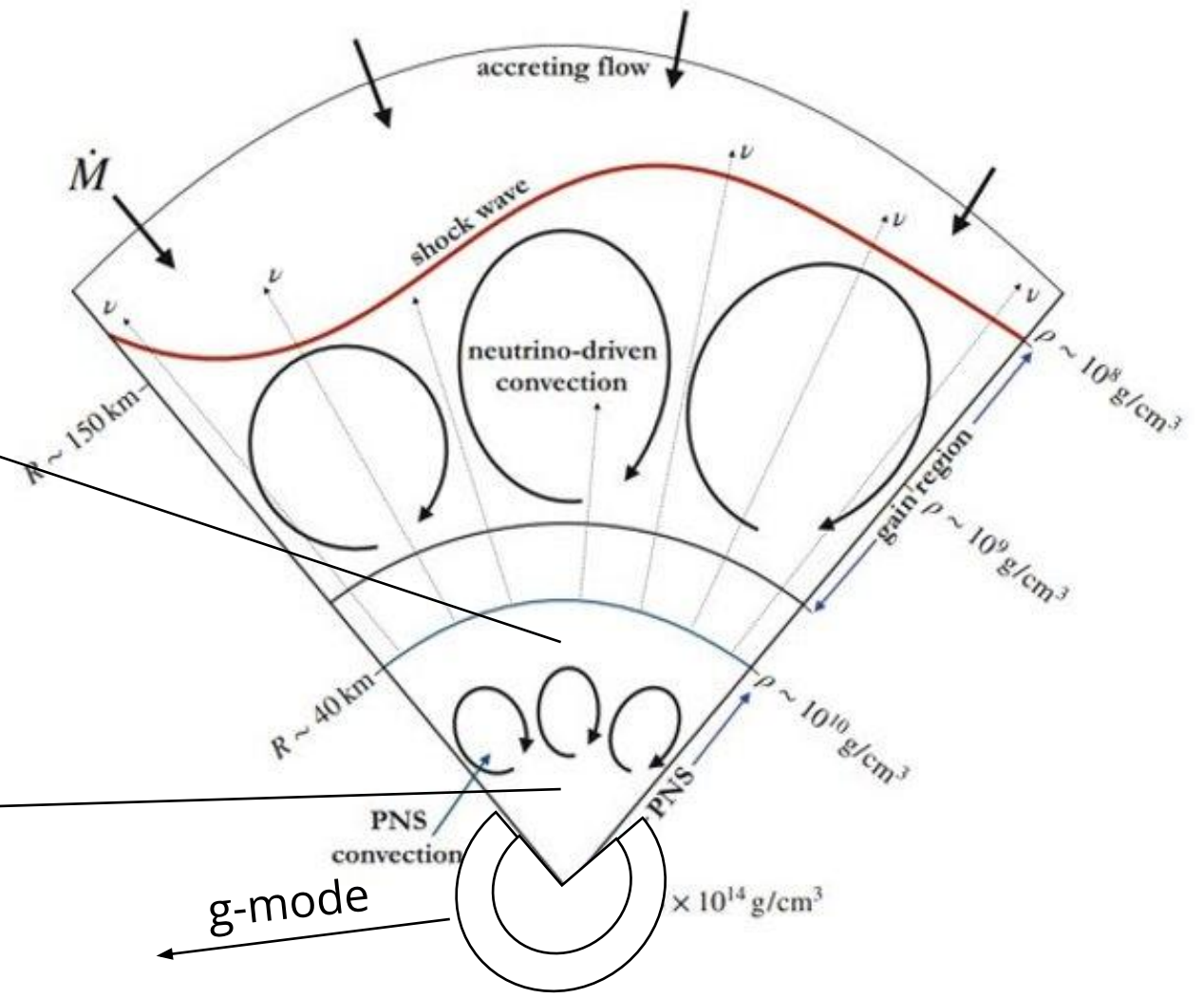
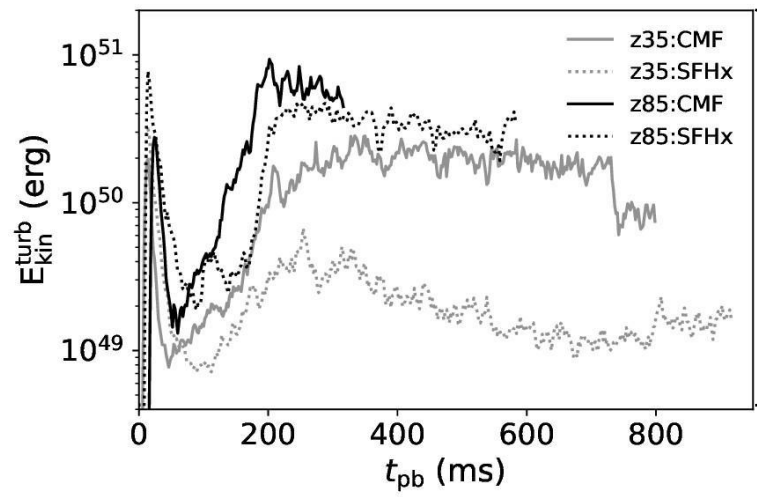








1. g-mode “lives” in convectively stable PCS region beneath the PCS convection zone
2. Higher turbulent convective energies seen in CMF, particularly z35

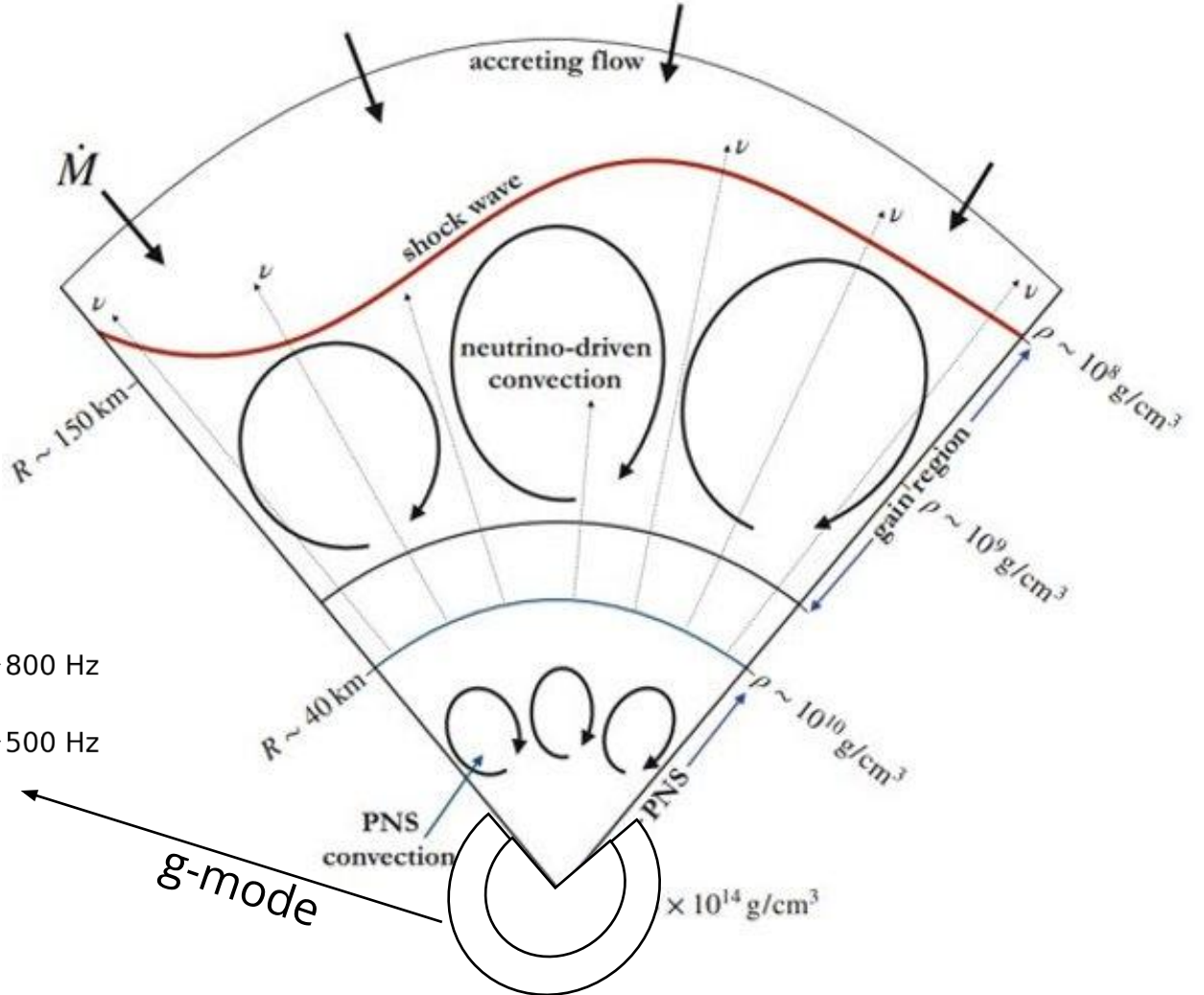
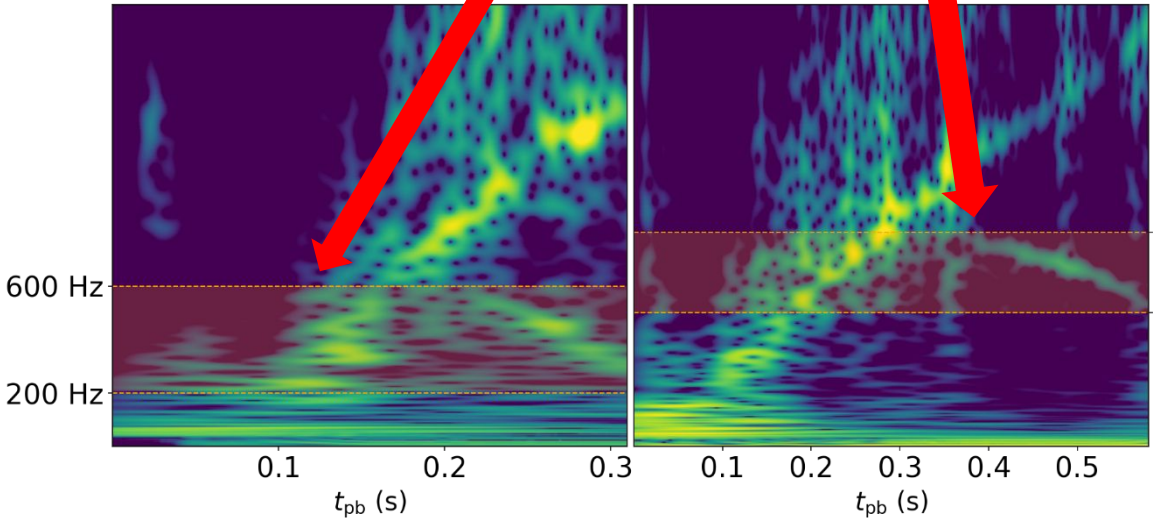


$$E_{\text{kin}}^{\text{turb}} = \frac{1}{2} \int_{\text{PCS}} \delta v^2 \rho r^2 dV$$

(Abdikamalov et al. 2022)

3. What EoS properties cause ~20% lower g-mode frequencies in CMF?

?

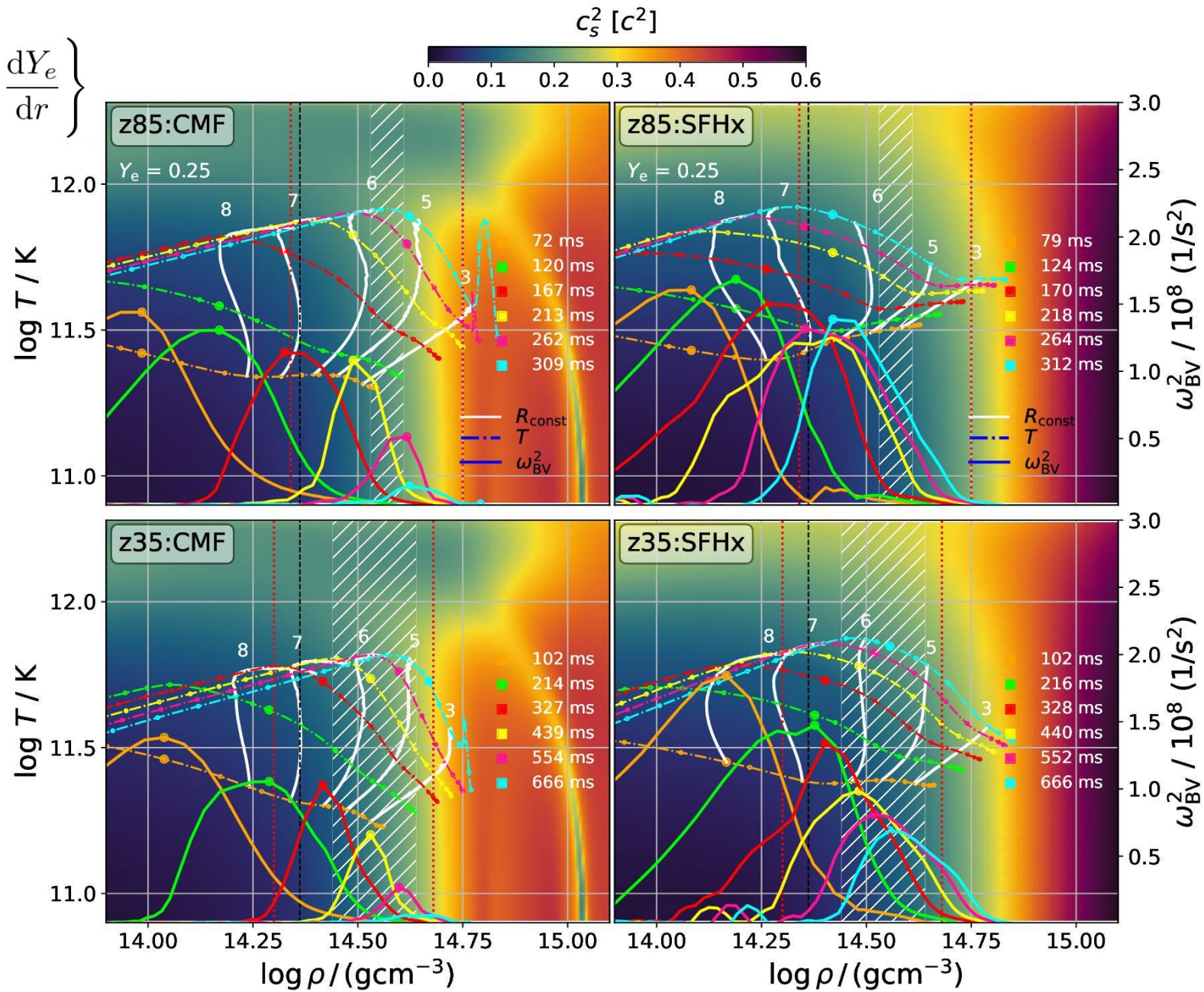
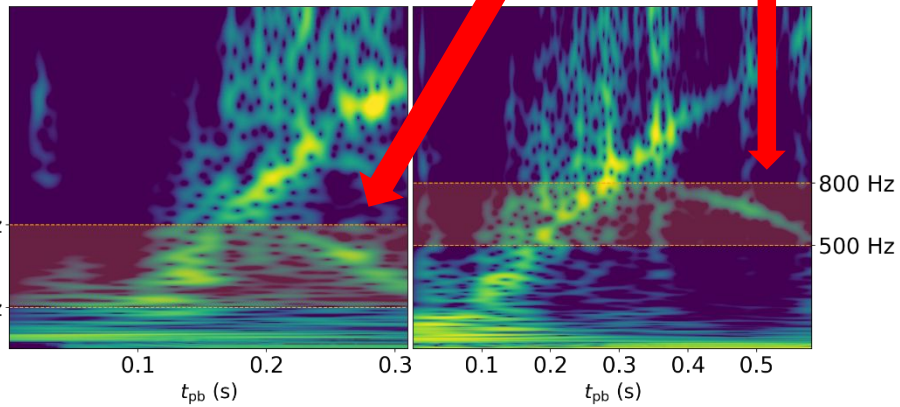


(Abdikamalov et al. 2022)



$$\omega_{\text{BV}}^2 = \frac{d\alpha}{dr} \frac{\alpha}{\rho h \Phi^4} \cdot \frac{1}{c_s^2} \left\{ \left( \frac{\partial P}{\partial s} \right)_{\tilde{\rho}, Y_e} \frac{ds}{dr} + \left( \frac{\partial P}{\partial Y_e} \right)_{\tilde{\rho}, s} \frac{dY_e}{dr} \right\}$$

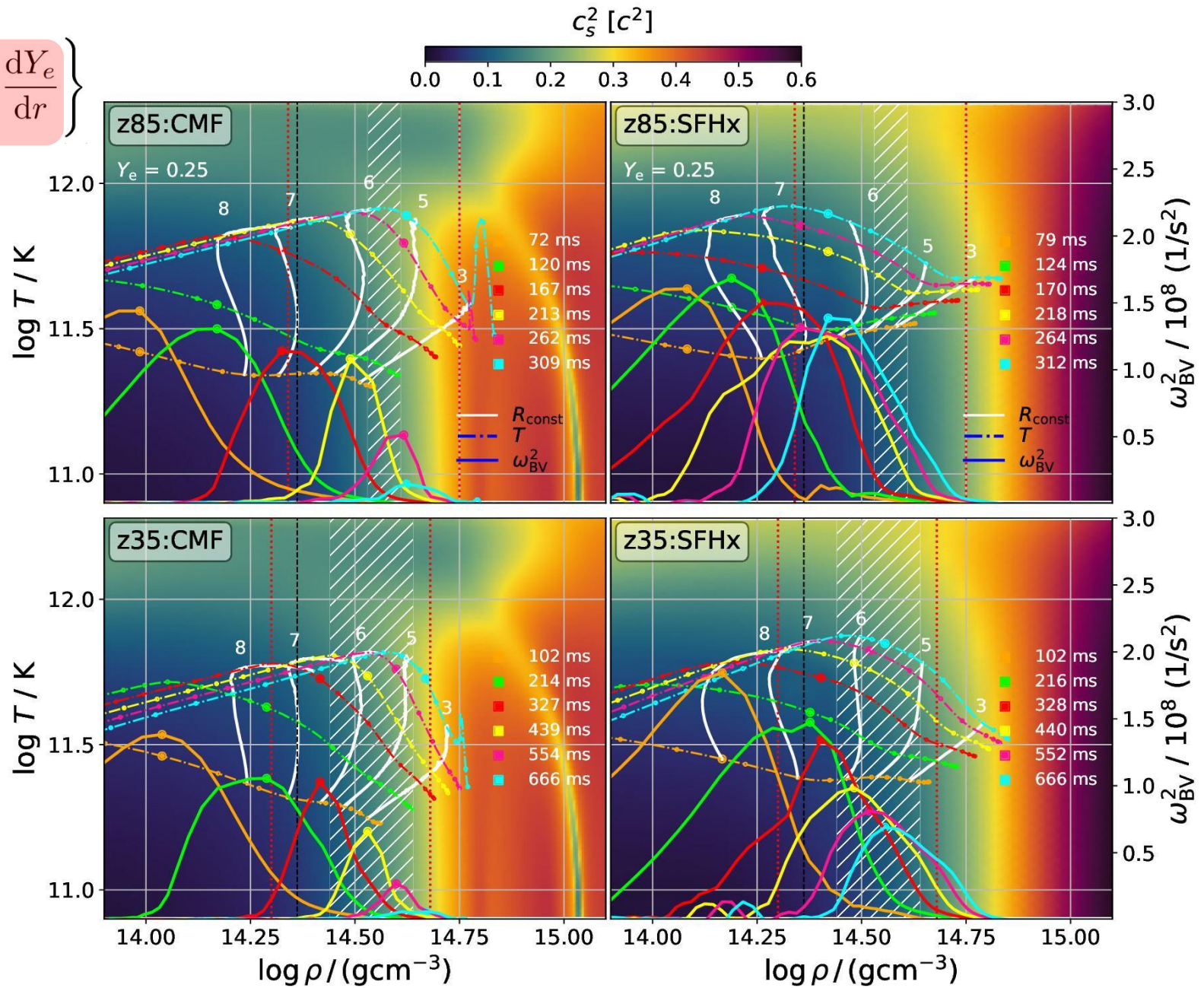
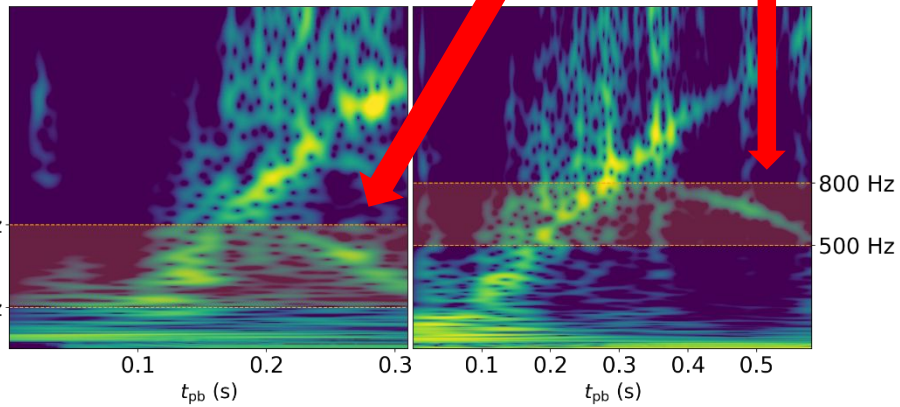
What EoS properties cause a ~20% lower g-mode frequency in CMF?





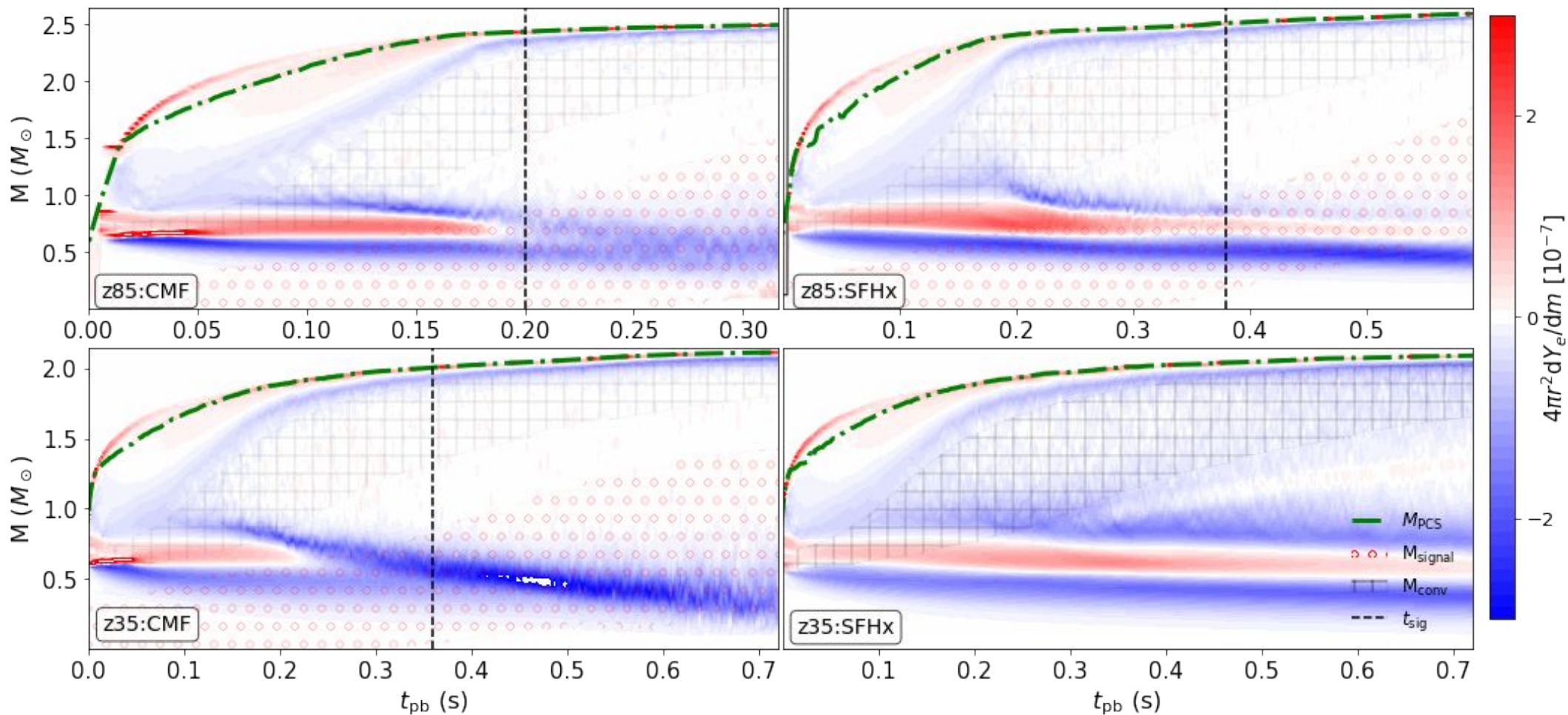
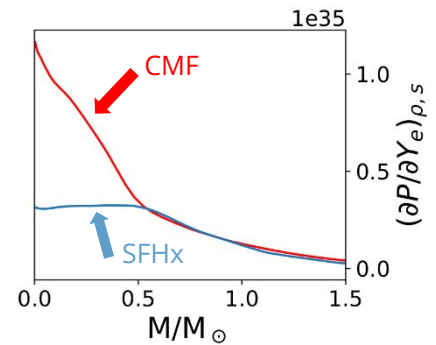
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What EoS properties cause a ~20% lower g-mode frequency in CMF?





$$\omega_{\text{BV}}^2 = \frac{d\alpha}{dr} \frac{\alpha}{\rho h \Phi^4} \cdot \frac{1}{c_s^2} \left\{ \left( \frac{\partial P}{\partial s} \right)_{\tilde{\rho}, Y_e} \frac{ds}{dr} + \left( \frac{\partial P}{\partial Y_e} \right)_{\tilde{\rho}, s} \frac{dY_e}{dr} \right\}$$



**z35 progenitor**  
**550 ms**

$$\omega_{\text{BV}}^2 = \frac{d\alpha}{dr} \frac{\alpha}{\rho h \Phi^4} \cdot \frac{1}{c_s^2} \left\{ \left( \frac{\partial P}{\partial s} \right)_{\tilde{\rho}, Y_e} \frac{ds}{dr} + \left( \frac{\partial P}{\partial Y_e} \right)_{\tilde{\rho}, s} \frac{dY_e}{dr} \right\}$$



Speed of sound larger  
(CMF)

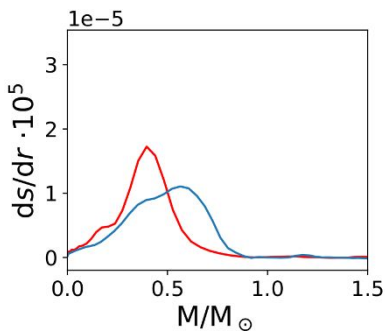
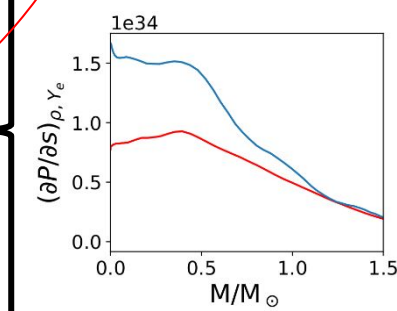
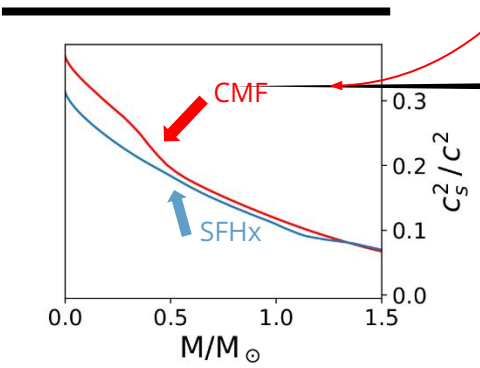


Increase  
s  
over time  
(CMF)

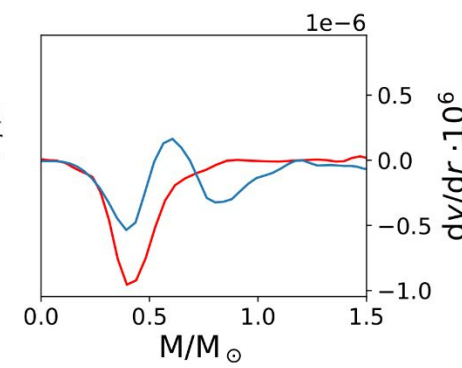
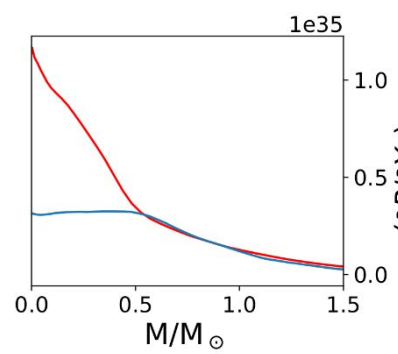


Changes sign  
Absolute becomes  
larger (CMF)

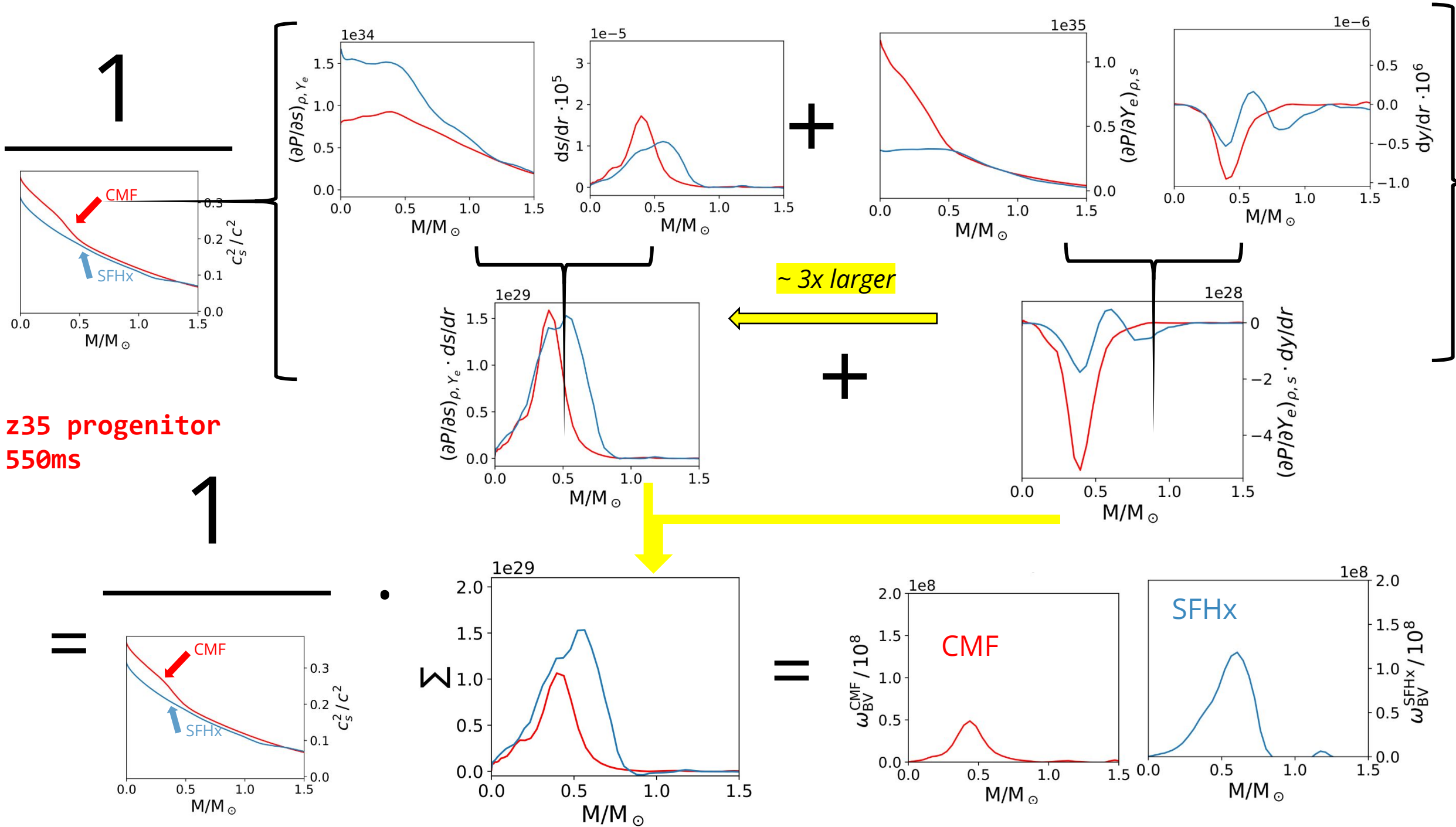
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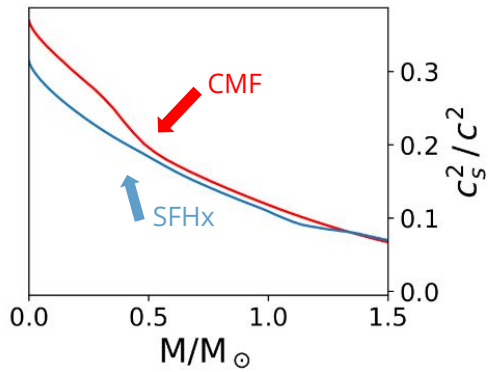


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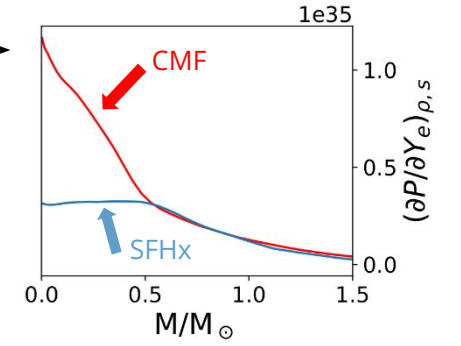




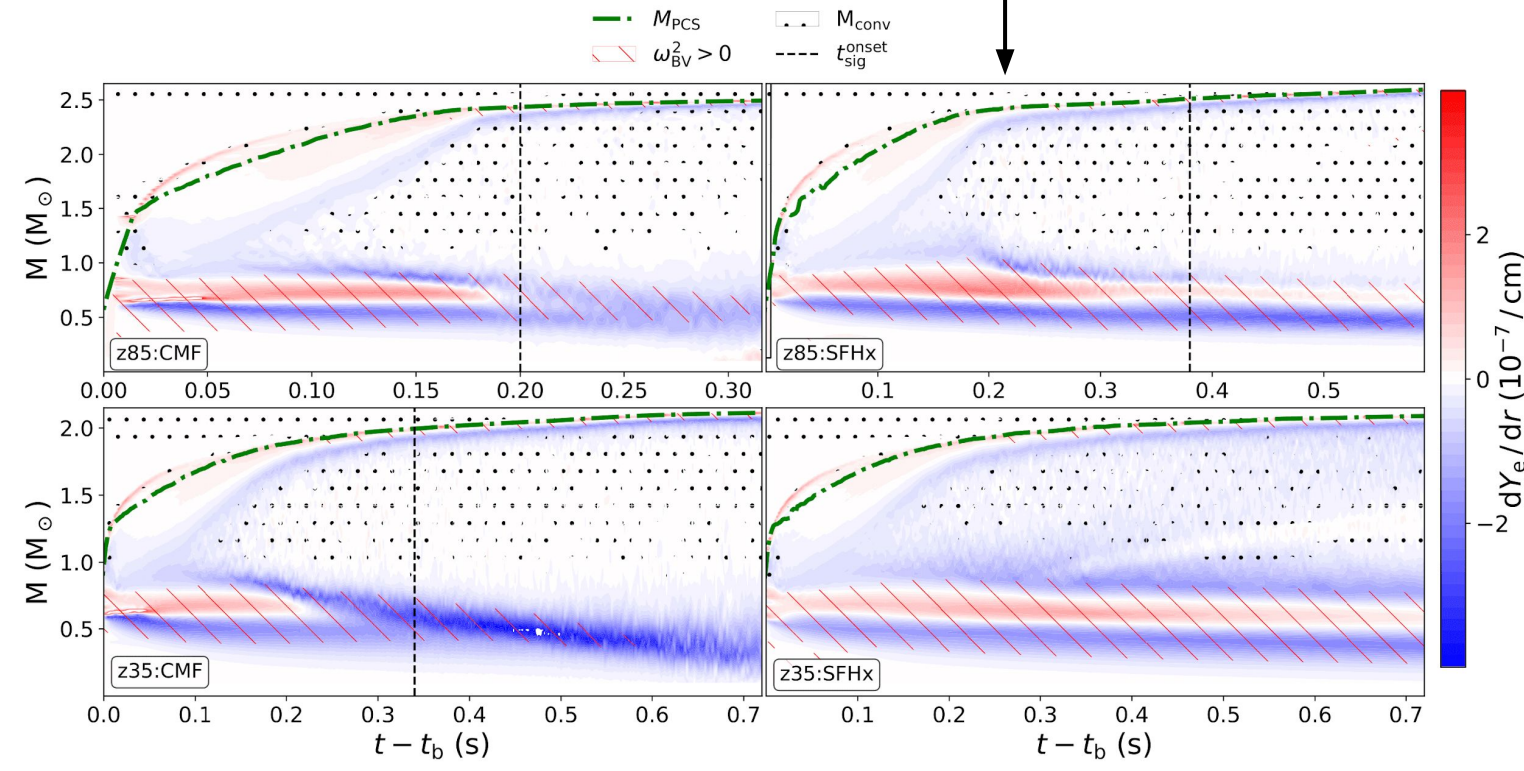
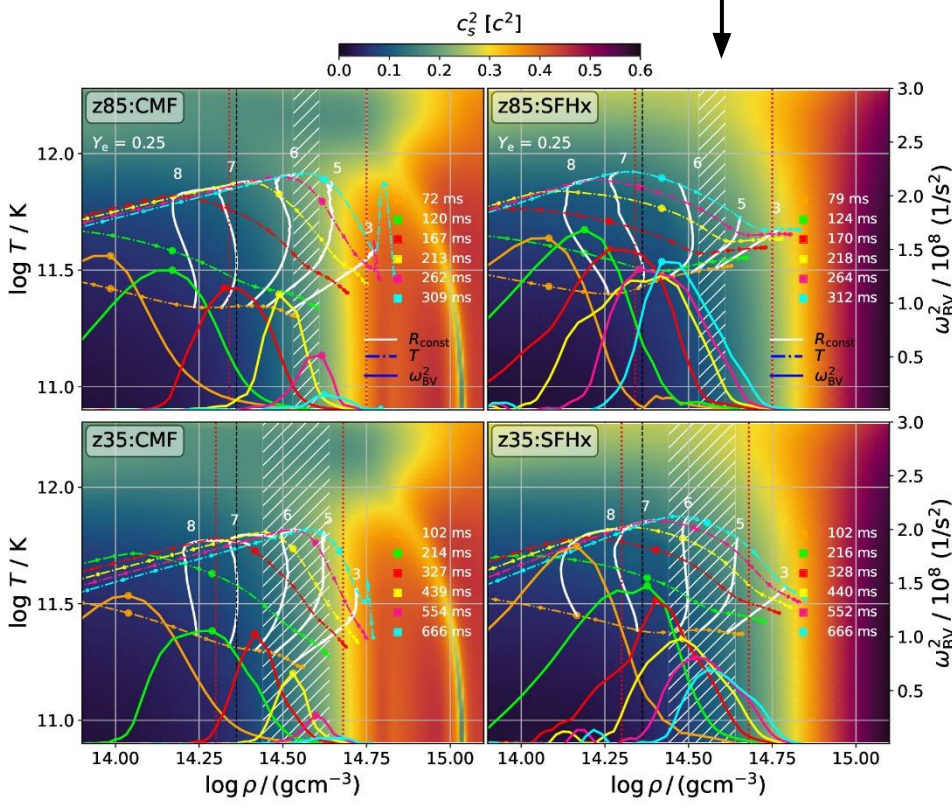




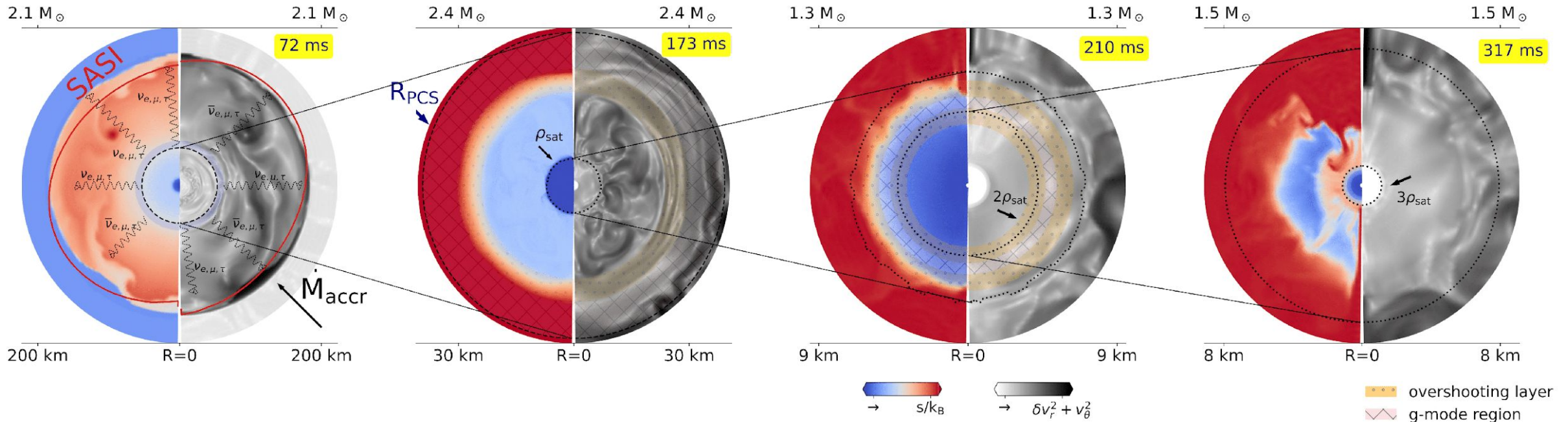
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What EoS properties cause ~20% lower g-mode frequencies in CMF?







# Summary

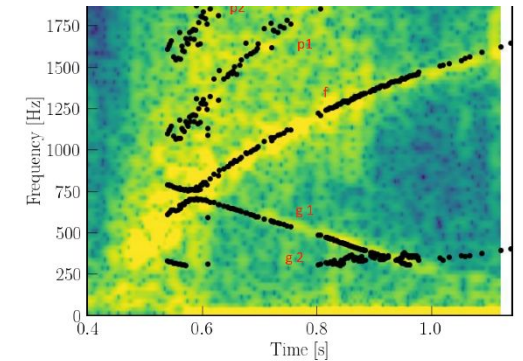
❖ Signal corresponds to g-mode oscillations in the PCS

→ ~5-10 km (200-500 ms post-bounce)

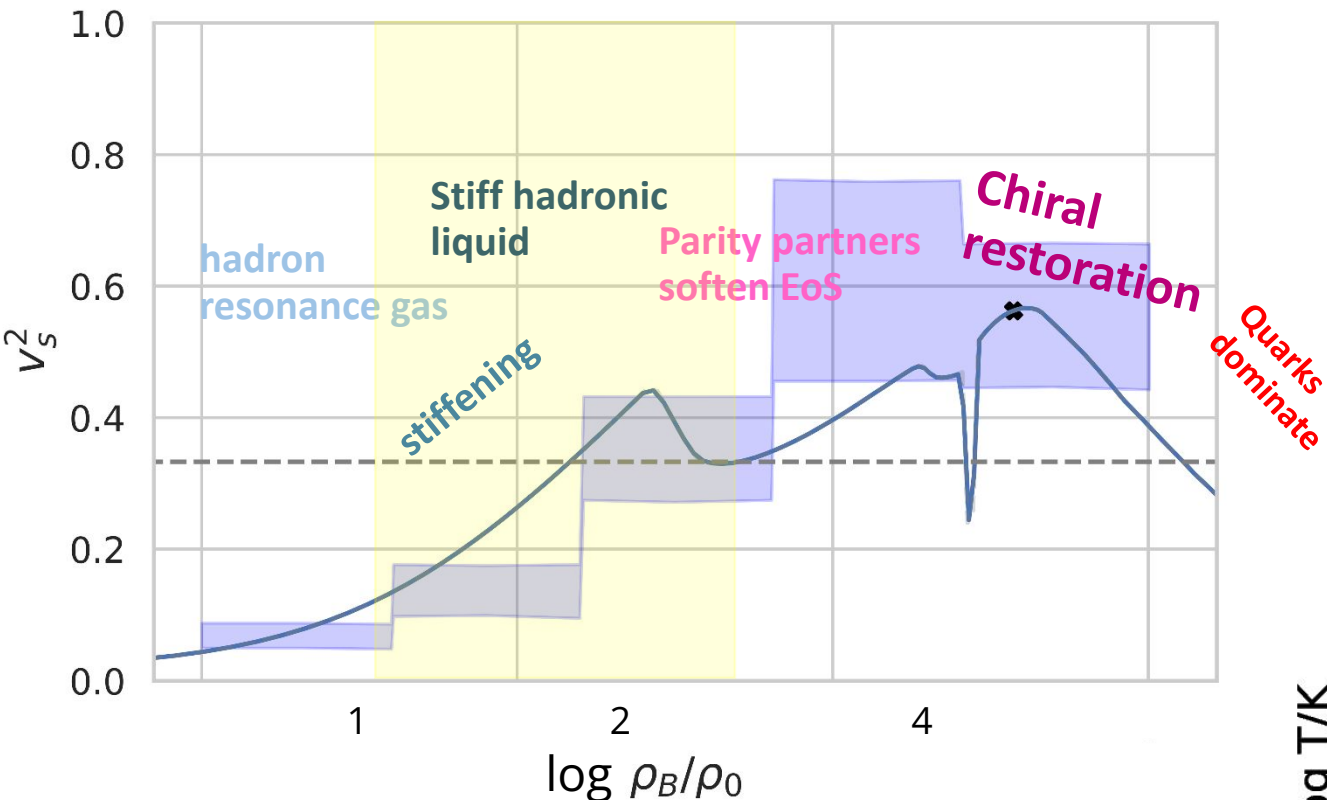
→  $l=2$   $g_1$ -mode [Zha et al., paper in prep]

❖ Mode frequency very sensitive to speed of sound at around twice saturation density

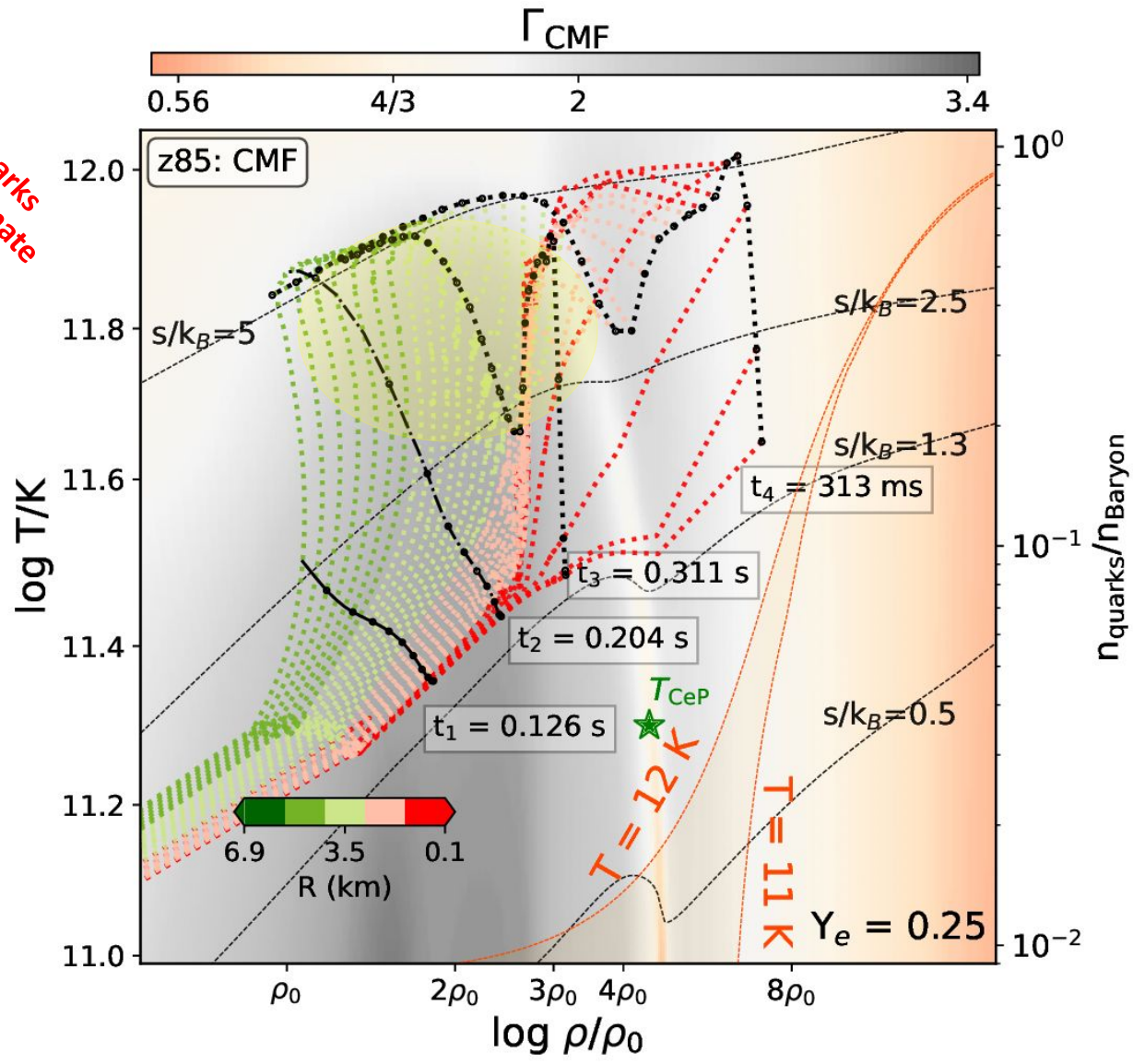
❖ Low frequency feature of mode lies within the sensitivity range of current GW detectors

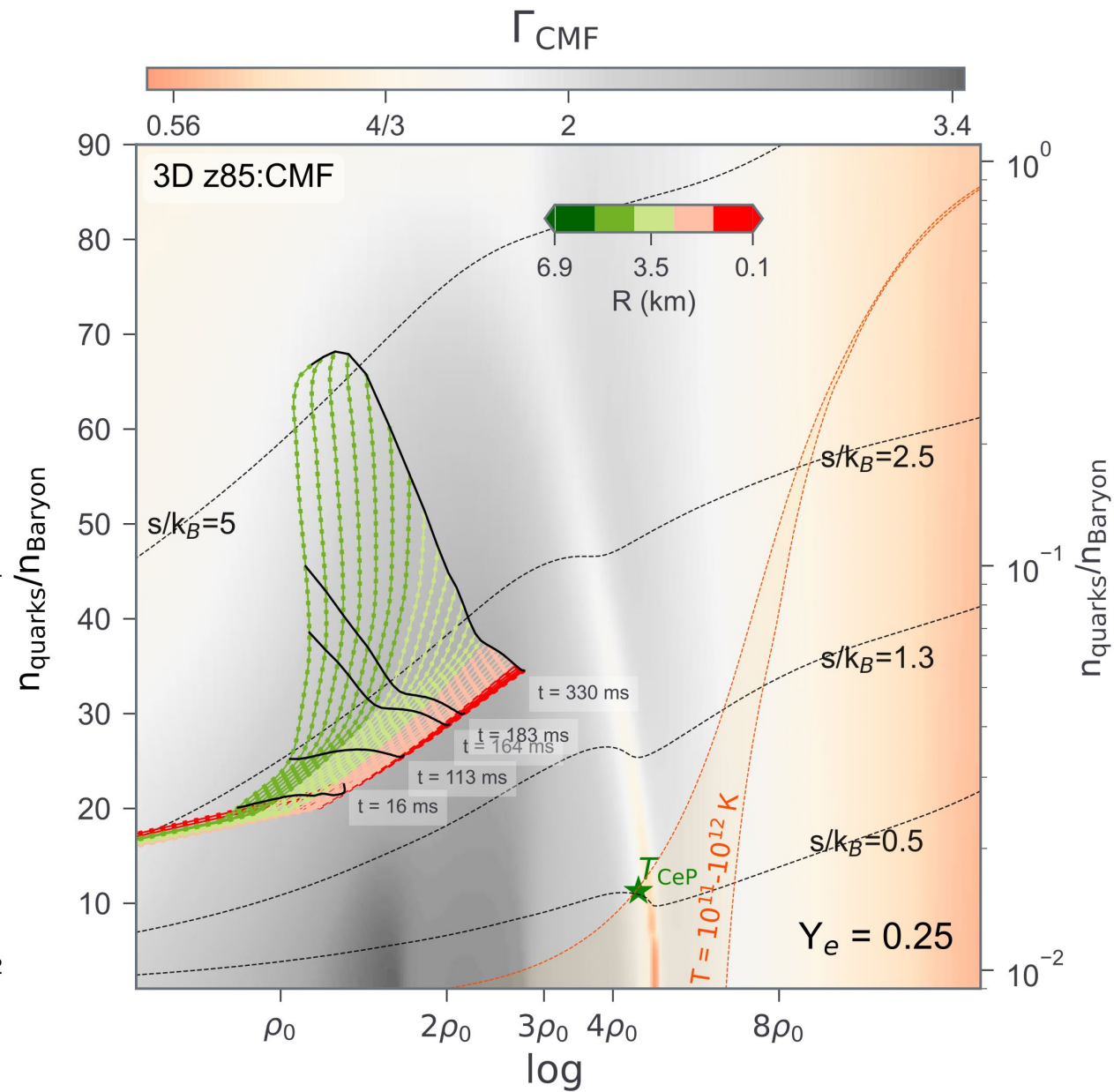
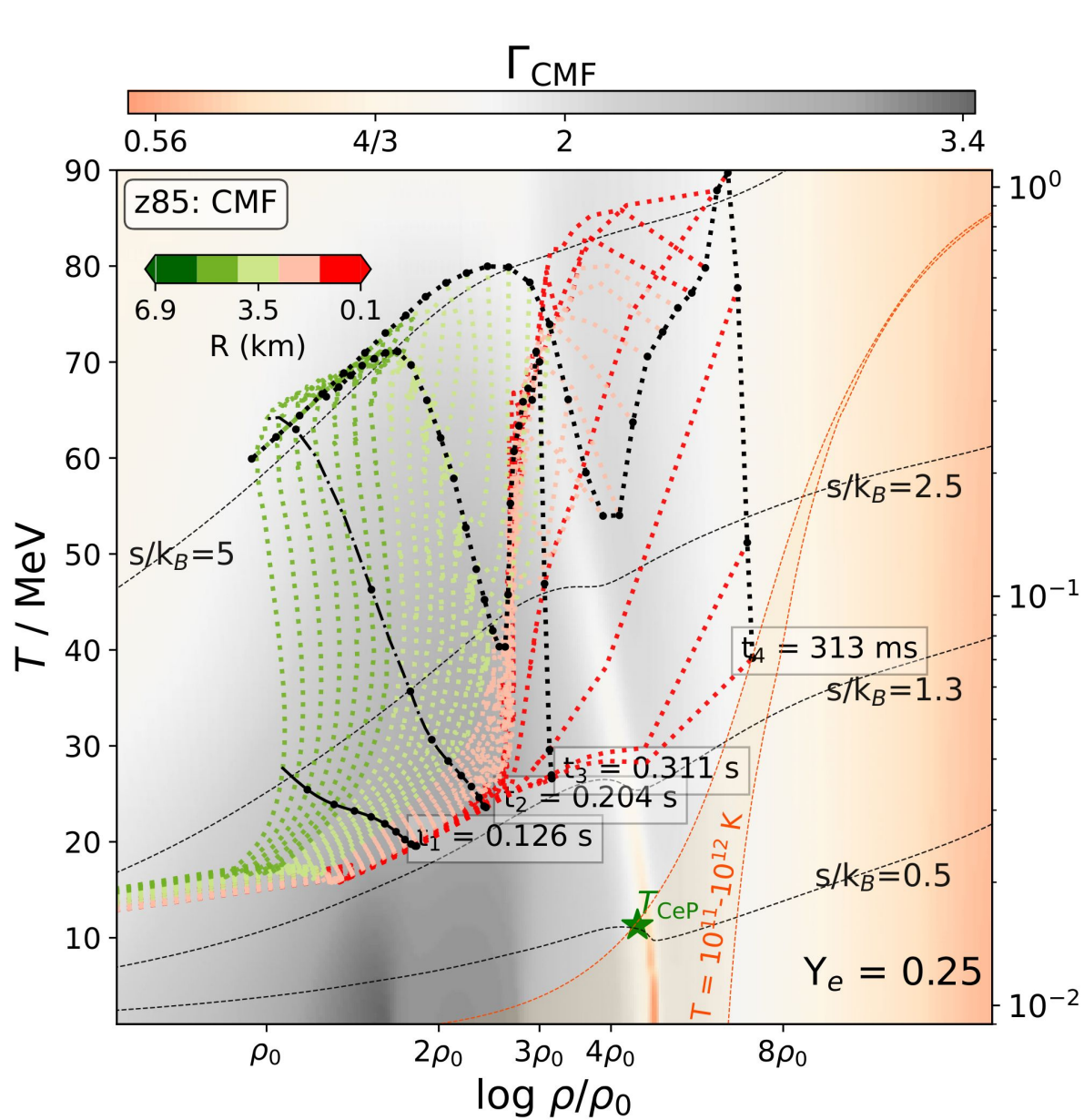




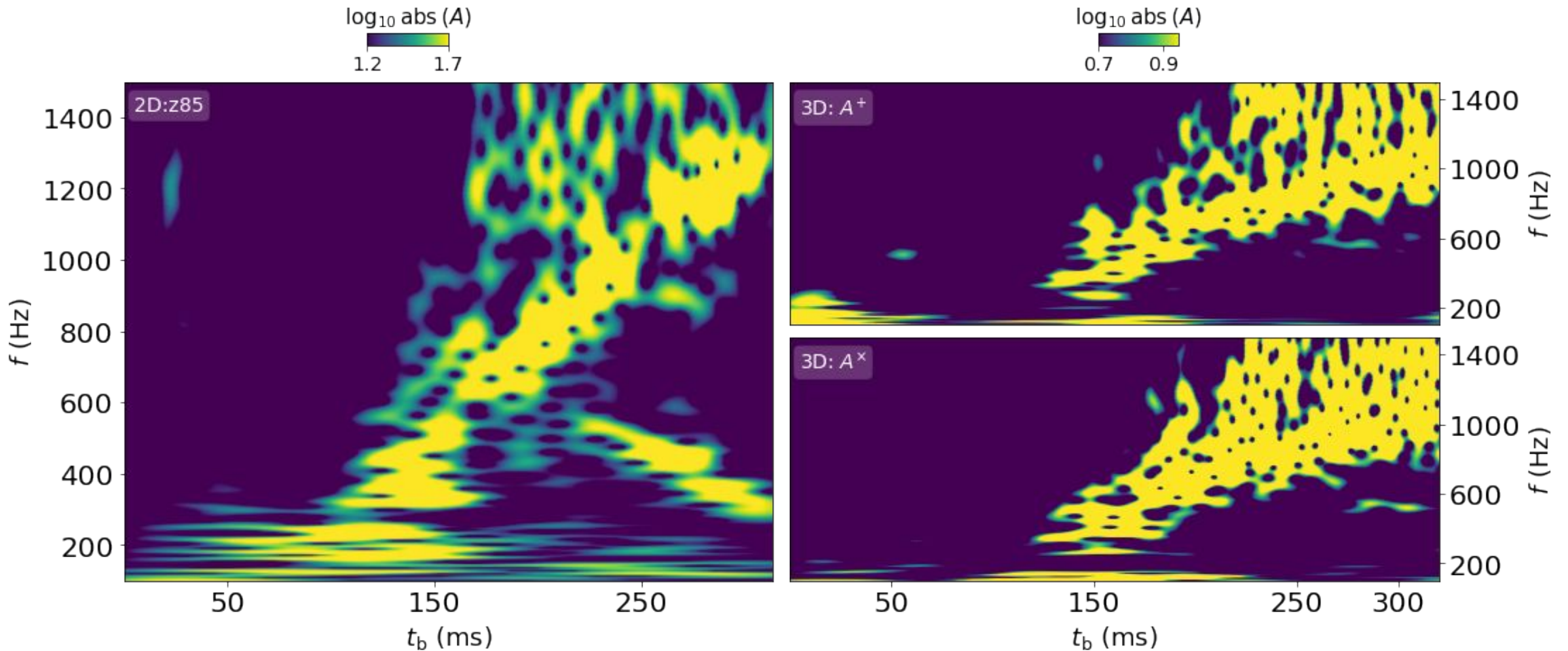


- Quarks are abundant in very low numbers
- Nucleon-nucleon interactions make EoS stiff
- Interplay of electron gradient in combination with a large  $(\partial P/\partial Y_e)_{\rho,s}$  drives down B.V.f.



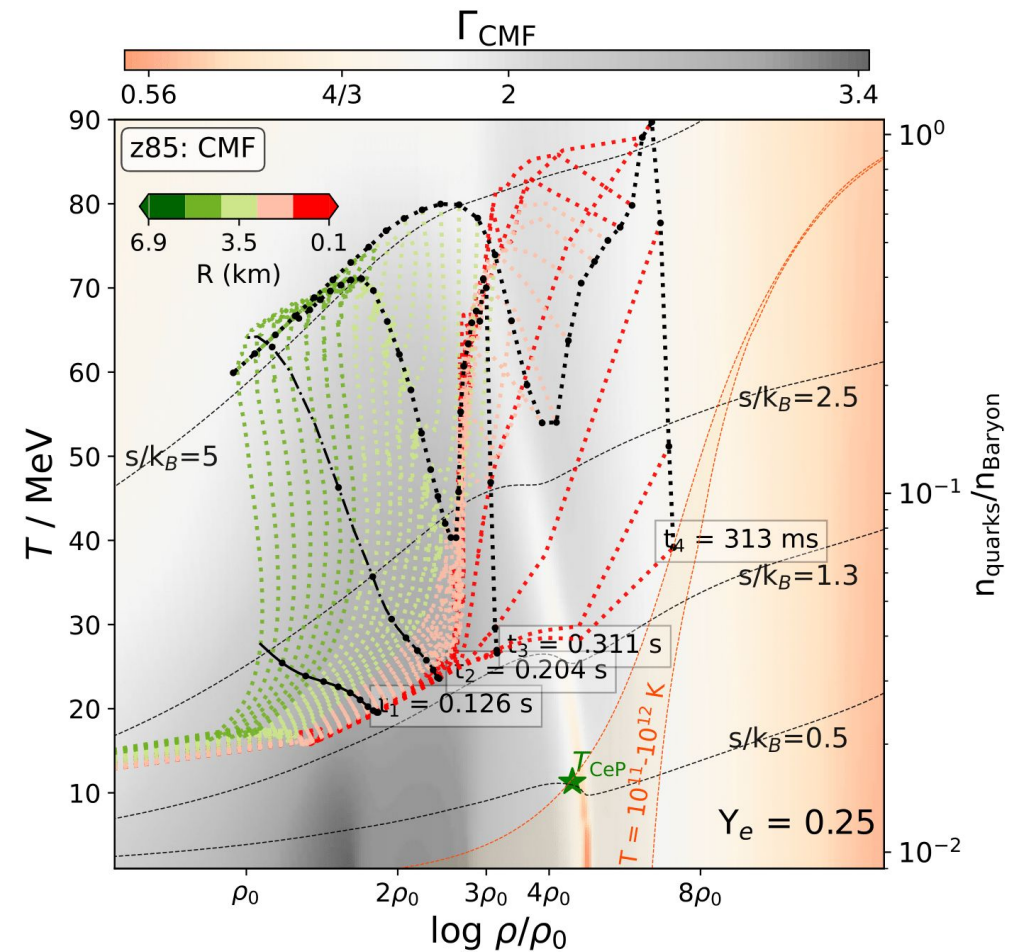
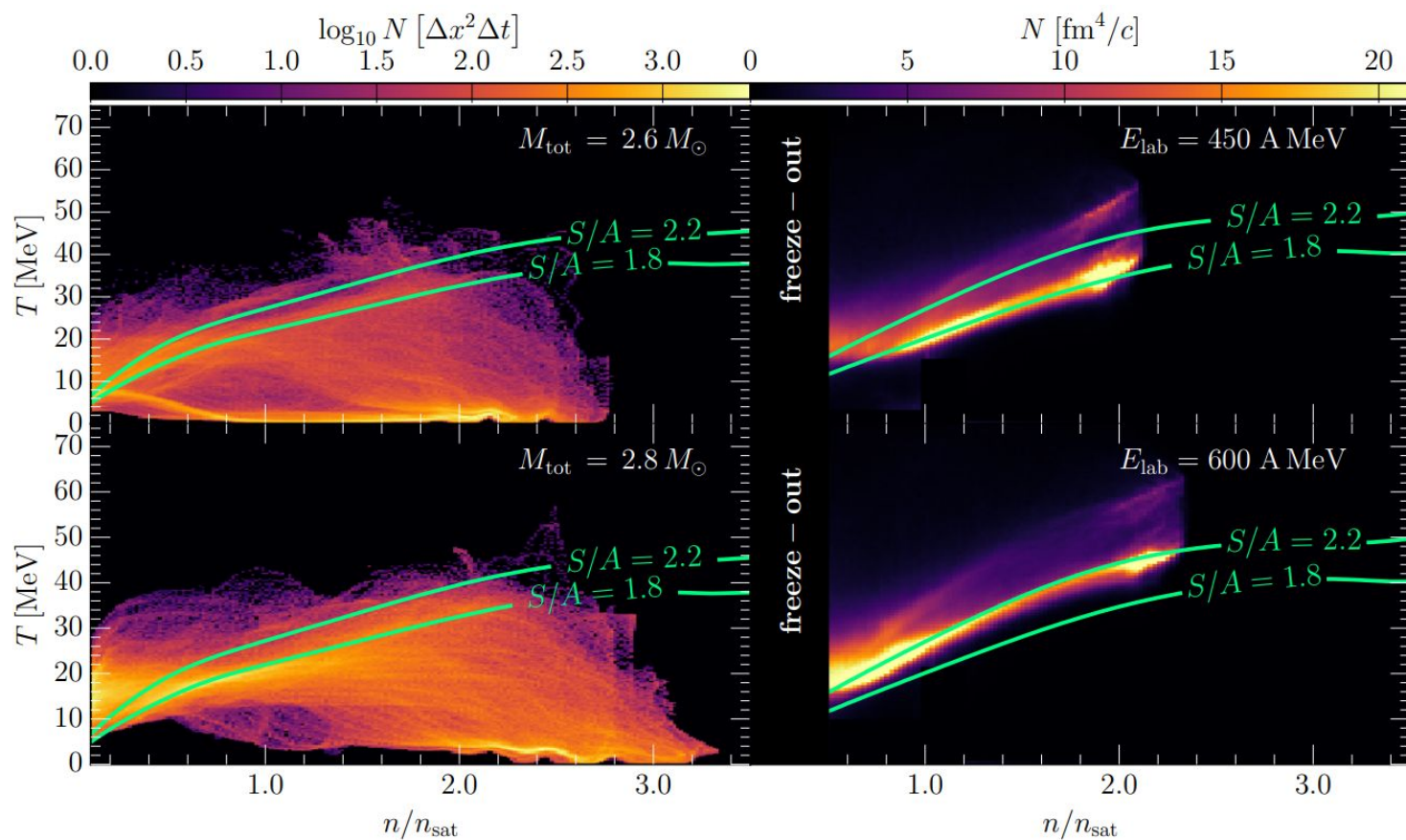


# How do 2D and 3D compare?





# Temperature and density comparison to NS-merger



(Most et al. 2022:arXiv-2201)

*Thank you for your attention!*

