The high-density equation of state in heavy-ion collisions: Constraints from microscopic collisions.

Jan Steinheimer
many thanks to
A. Motornenko, M. Omana Kuttan, O. Savchuk, E. Most, M Bleicher, H. Stöcker
and many more.

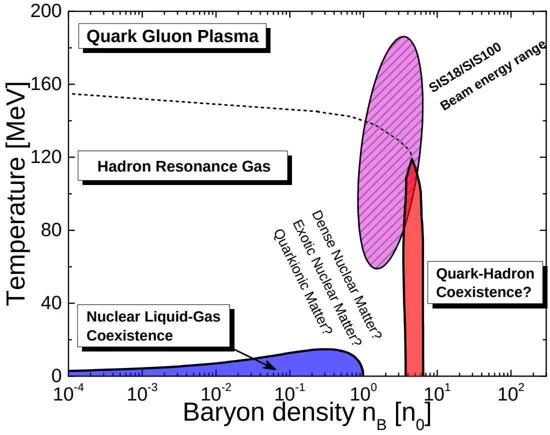
29.03.2023

FIAS Frankfurt Institute for Advanced Studies

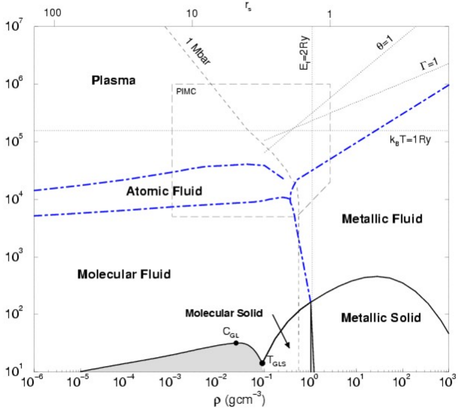
The 'holy grail' of high energy nuclear physics: The QCD phase diagram

Can we 'draw' something like this for the textbooks?

QCD

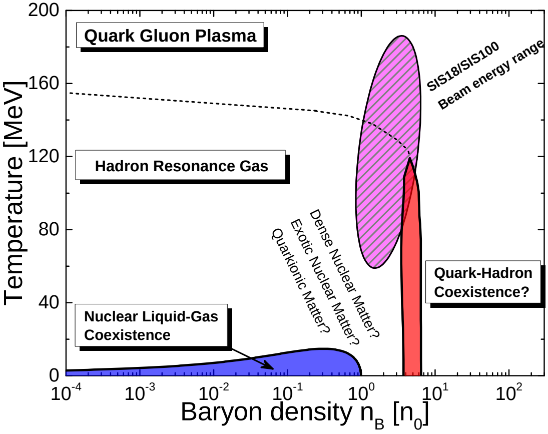


Hydrogen



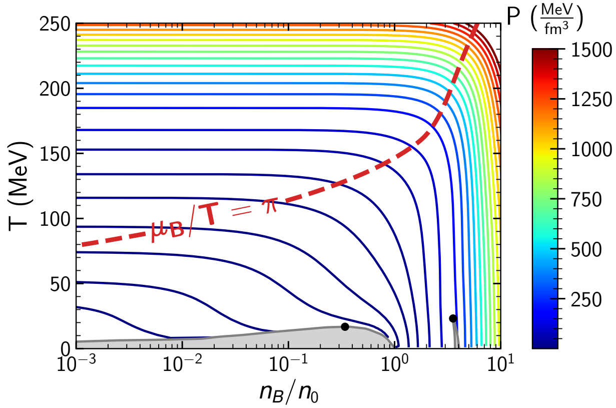
http://militzer.berkeley.edu/diss/node5.html

The 'holy grail' of high energy nuclear physics: The QCD phase diagram



- This is just a sketch.
- Direct QCD simulations face the sign problem and expansions break down for $\mu_B/T \gtrsim 3-4$.
- Results at low density: Crossover is now confirmed.
- Established $T_{cep} \lesssim 120$ MeV.
- High density: room for speculations.
- We have to rely on effective model descriptions of the EoS.

The baryonic problem



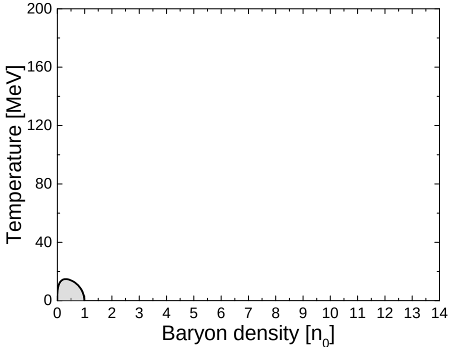
Why do the methods break down?

- Sudden change of isobaric lines at this point.
- From Boson (mesons/gluons) dominated matter to fermionic matter (nucleons/quarks).
- Calculations seem to fail for matter where (multi-) baryonic interactions become important.
- Positive: for the region of interest a density dependent EoS may be enough.

A. Motornenko, JS, V. Vovchenko, S. Schramm and H. Stoecker, Nucl. Phys. A 1005 (2021), 121836

Regions of access to the PD - NS

- Starting from the phase diagram in Temperature and density.



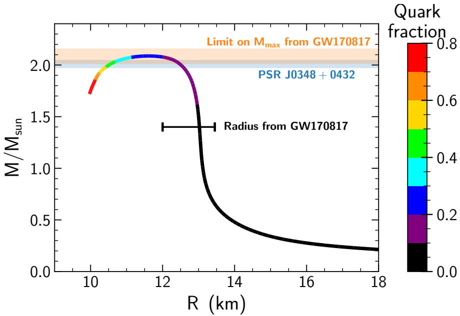
Disclaimer: For now we will ignore any isospin dependence, or assume it can be constraint by measurement.

Regions of access to the PD - NS

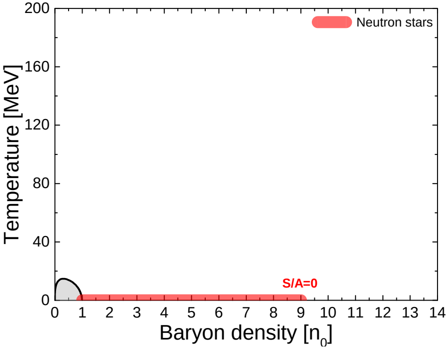
- Starting from the phase diagram in Temperature and density.
- For $T = 0$ we can use the mass-radius relation of observed stars.

TOV equation:

$$\frac{dP}{dr} = -(P+\rho)\frac{m + 4\pi r^3 P}{r(r-2m)}$$



A. Motornenko, JS, V. Vovchenko, S. Schramm and H. Stoecker, Phys. Rev. C **101** (2020) no.3, 034904



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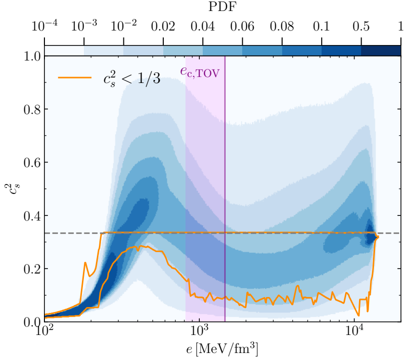
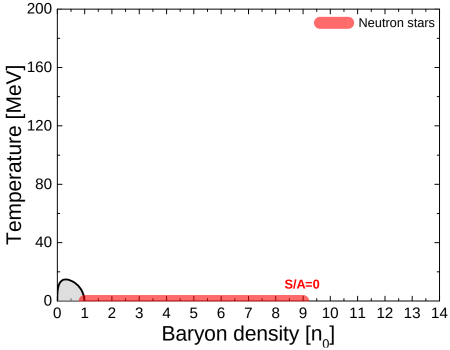


S. Altiparmak, C. Ecker and L. Rezzolla, [arXiv:2203.14974 [astro-ph.HE]].

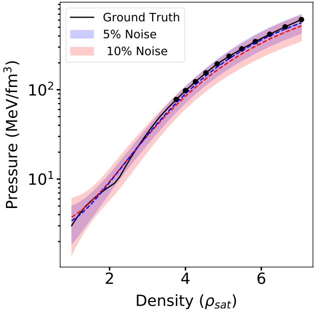


Regions of access to the PD - NS

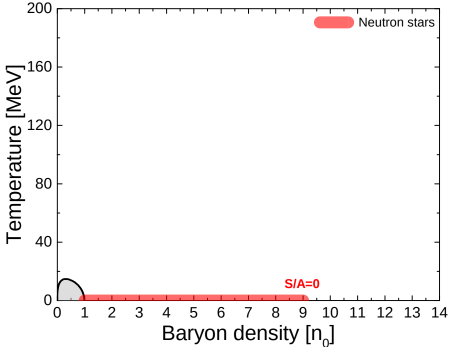
- Starting from the phase diagram in Temperature and density.
- For $T = 0$ we can use the mass-radius relation of observed stars.
- Constraints from neutron star mergers (pre-merger).
- New ML methods.

TOV equation:

$$\frac{dP}{dr} = -(P+\rho)\frac{m + 4\pi r^3 P}{r(r-2m)}$$

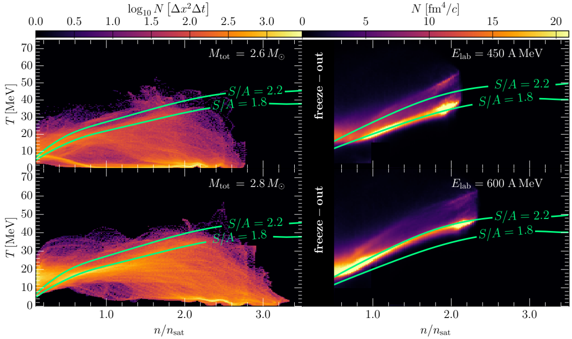


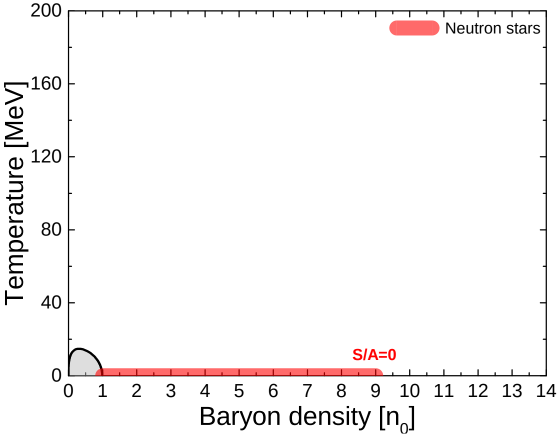
S. Soma, L. Wang, S. Shi, H. Stöcker and K. Zhou, JCAP **08**, 071 (2022).



Regions of access to the PD - BNSM

- Using BNSM we can also turn on the heat.
- During the post-merger $T < 40$ MeV is reached
- Observables?





E. R. Most, A. Motornenko, JS, V. Dexheimer, M. Hanauske, L. Rezzolla and H. Stoecker, [arXiv:2201.13150 [nucl-th]].

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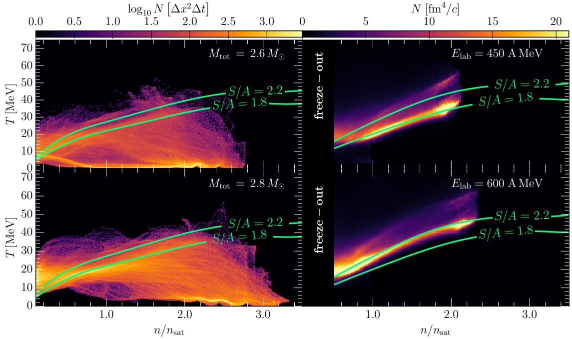

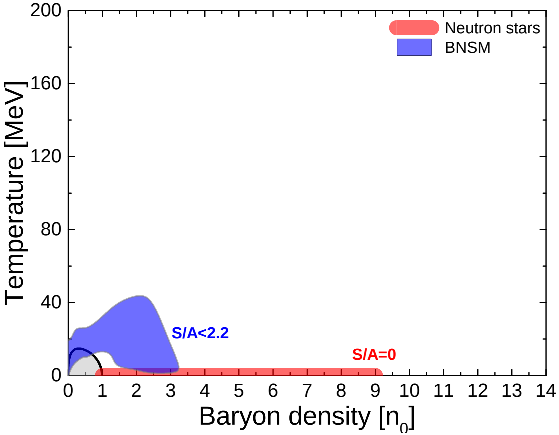

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Regions of access to the PD - CCSN

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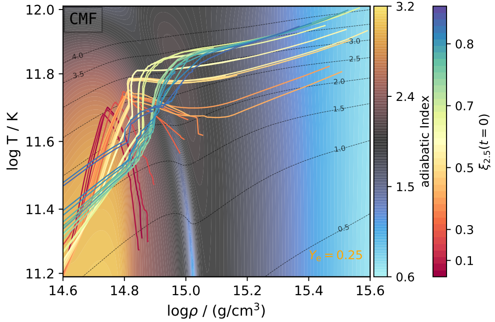
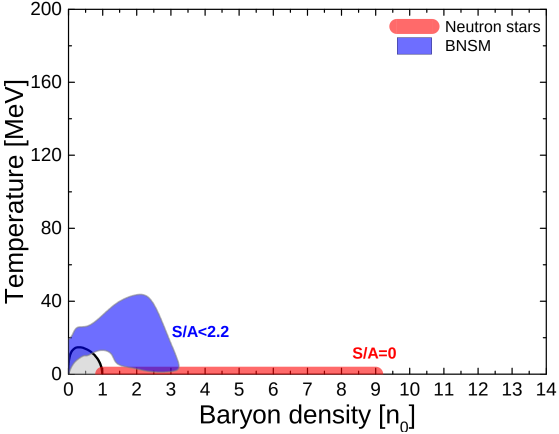
- GR Hydro simulation with same EoS (CMF model):





P. Jakobus, B. Mueller, A. Heger, A. Motornenko, JS and H. Stoecker, [arXiv:2204.10397 [astro-ph.HE]].

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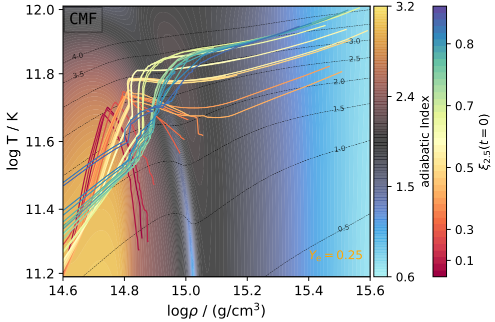
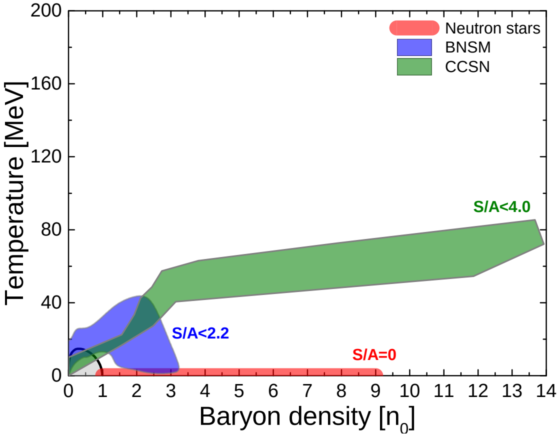
- Observables: Neutrinos, GW?





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The Strategy for Heavy Ion Collisions

Relying on experimental observations?

We want to understand QCD matter, not neutron star matter or heavy ion collision matter.

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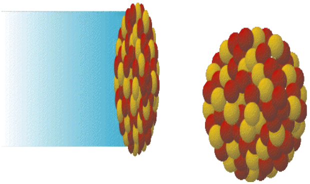
4. Cross check with astrophysical observations.
5. Reject unlikely EoS.

How to study the equation of state using heavy ion collisions

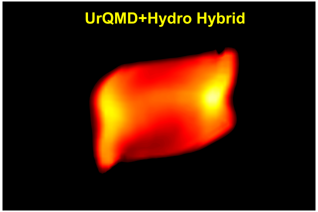
Much of we today think about heavy ion dynamics is motivated by the fluid dynamic picture of HIC:

Pre-equilibrium phase



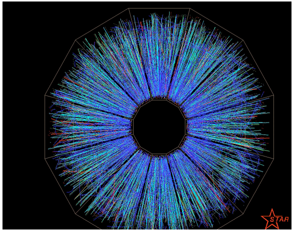
Non-equilibrium initial state

Equilibrated? phase



Fluid dynamic evolution

Final stage and particle freeze-out



Freeze-out: chemical and thermal

H. Petersen, **JS**, G. Burau, M. Bleicher and H. Stöcker, Phys. Rev. C **78** (2008) 044901

QCD properties from hydro

Fluid dynamics

- Fluid dynamics offers a convenient way to study the EoS
- In addition viscosities can be included

$$\partial_\mu T^{\mu\nu} = 0 \ ,$$

as well as the conservation of the baryon four-current

$$\partial_\mu j^\mu = 0 \ .$$

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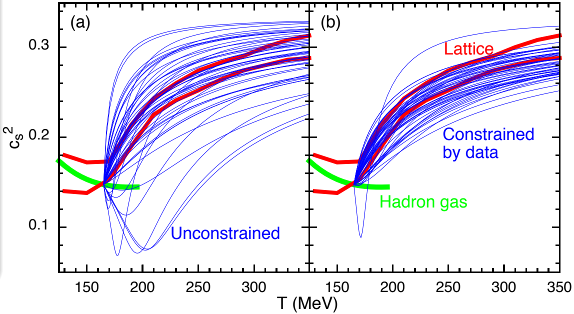
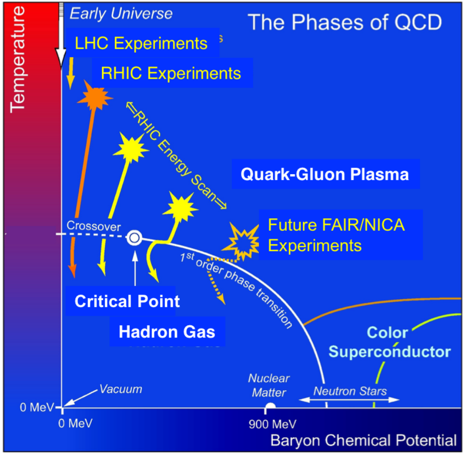
Statistical analysis for LHC and RHIC confirmed the QCD EoS.
Similar analysis was done for transport properties.



S. Pratt, E. Sangaline, P. Sorensen and H. Wang, Phys. Rev. Lett. **114** (2015), 202301

High density through lower beam energy

This is a well known propaganda plot depicting the idea:
Lower beam energy = higher density.



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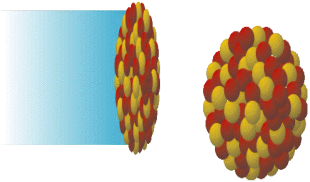

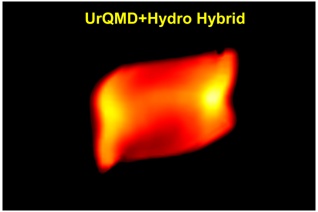

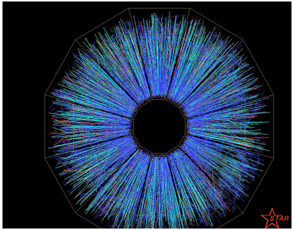

Non-equilibrium initial state

Fluid dynamic evolution

Freeze-out: chemical and thermal

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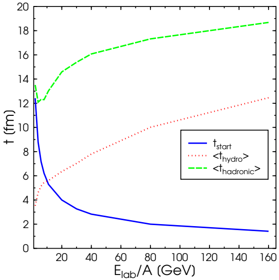
How to study the equation of state using heavy ion collisions

Much of we today think about heavy ion dynamics is motivated by the fluid dynamic picture of HIC:

At low beam energies the initial compression is most relevant.

Pre-equilibrium phase	Equilibrated? phase	Final stage and particle freeze-out



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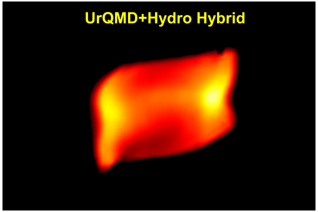

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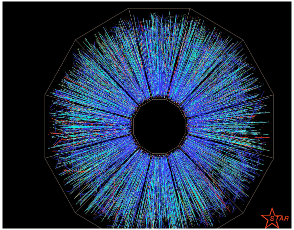

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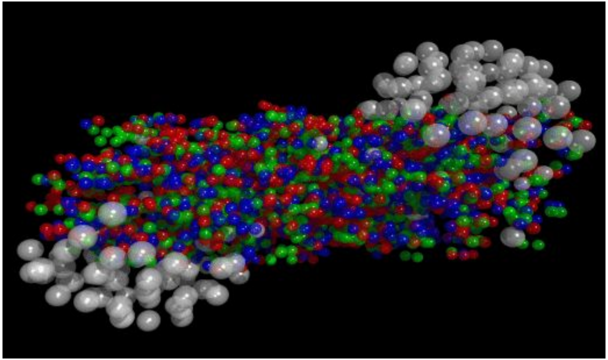
UrQMD for the description

UrQMD is a microscopic transport model

- In cascade mode: Particles follow a straight line until they scatter.
- EoS resembles a hadron resonance gas.



Dynamics in UrQMD

UrQMD is a microscopic transport model

- Only $2 \leftrightarrow 2$, $2 \leftrightarrow 1$, $2 \to N$ and $1 \to N$ interactions allowed.
- Resonance decays according to PDG values + guesstimates.
- Detailed balance. (Violated in string excitations, annihilations and some dacays)

The Skyrme EoS in UrQMD

To implement any density dependent EoS in UrQMD:
In UrQMD the real part of the interaction is implemented by a density dependent potential energy $V(n_B)$.
Once the potential energy is known, the change of momentum of each baryon is calculated as:

$$\dot{\mathbf{p}}_i = -\frac{\partial \langle H \rangle}{\partial \mathbf{r}_i} = -\left(\frac{\partial V_i}{\partial n_i} \cdot \frac{\partial n_i}{\partial \mathbf{r}_i}\right) - \left(\sum_{j \neq i} \frac{\partial V_j}{\partial n_j} \cdot \frac{\partial n_j}{\partial \mathbf{r}_i}\right) , \tag{1}$$

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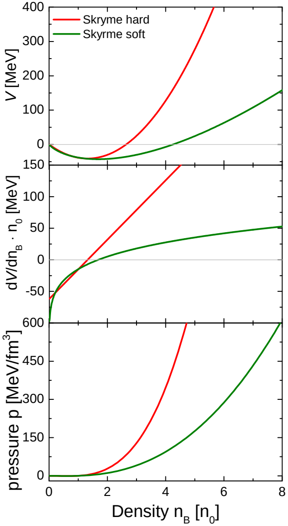
For the potential energy V often a Skyrme model was used that is based on a 2-term expansion in density:

$$U(n_B) = \alpha \cdot n_B + \beta \cdot n_B^\gamma \quad \text{with} \quad U(n_B) = \frac{\partial (n_B \cdot V(n_B))}{\partial n_B} \quad (2)$$

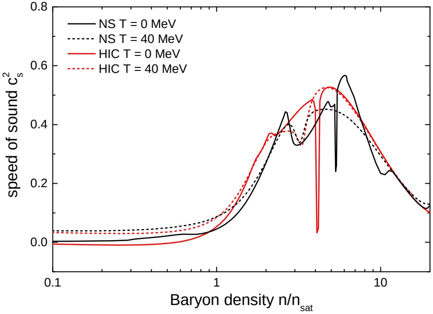
Problem: Once saturation density and binding energy is fixed, only 1 d.o.f. left and EoS likely becomes unphysical. No phase transition possible.

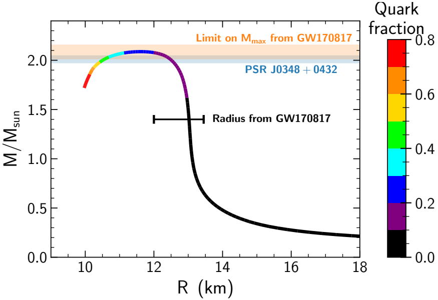


A different effective model: the CMF

Application for cold compact stars

- Compressibility of the CMF EoS is $\kappa_0 = 267$ MeV and the symmetry energy is $S_0 = 31.9$ MeV.
- Speed of sound for neutron star matter.
- Mass radius diagram consistent with astrophysical constraints.





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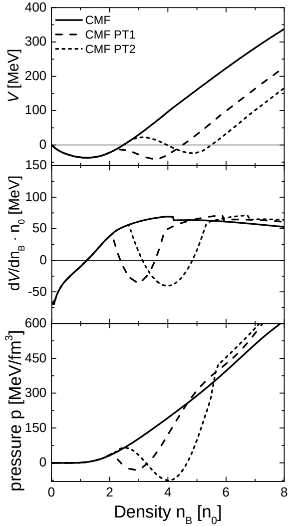
In CMF we can simply use the effective field energy per baryon E_{field}/A calculated from the CMF model:

$$V_{CMF} = E_{\text{field}}/A = E_{\text{CMF}}/A - E_{\text{FFG}}/A \,, \qquad (4)$$

A phase transition can be simply included by adding another minimum in the potential energy: leading to (meta-)stable solutions at high density.



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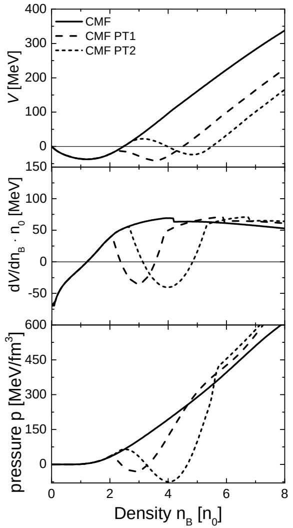
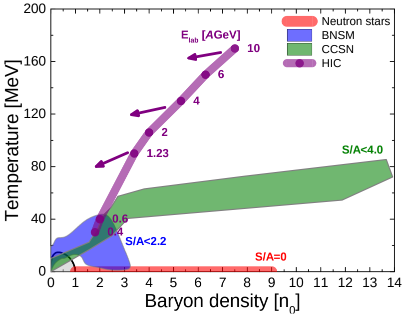
Disadvantage: Only density dependence + no change in d.o.f.
Advantage: Consistent description throughout, i.e. no change of model or d.o.f. required.
⟶ Focus on the effects of the equation of state and dynamic phase separation.

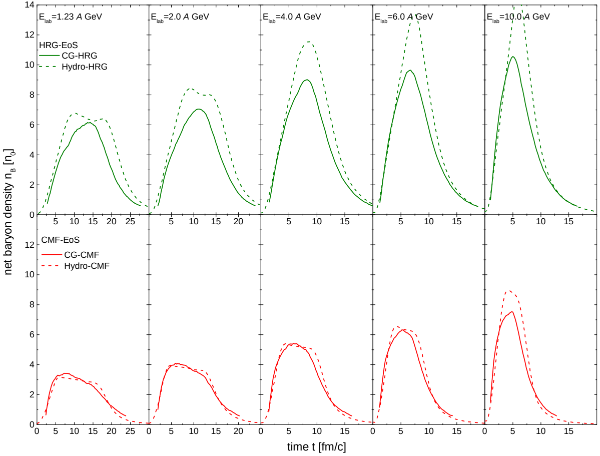


J. Steinheimer, A. Motornenko, A. Sorensen, Y. Nara, V. Koch and M. Bleicher, [arXiv:2208.12091 [nucl-th]].

1. HIC UrQMD vs. hydro, regions of access

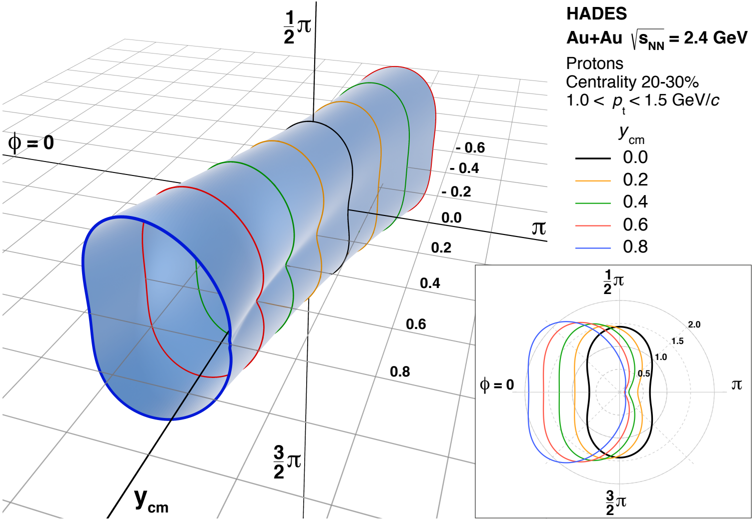
- Including the CMF EoS in UrQMD vs. a hadron resonance gas baseline.
- Bulk evolution consistent with 3+1D hydro + CMF
- Initial compression from CMF model in UrQMD





M. Omana Kuttan, A. Motornenko, **JS**, H. Stoecker, Y. Nara and M. Bleicher, Eur. Phys. J. C **82** (2022) no.5, 427

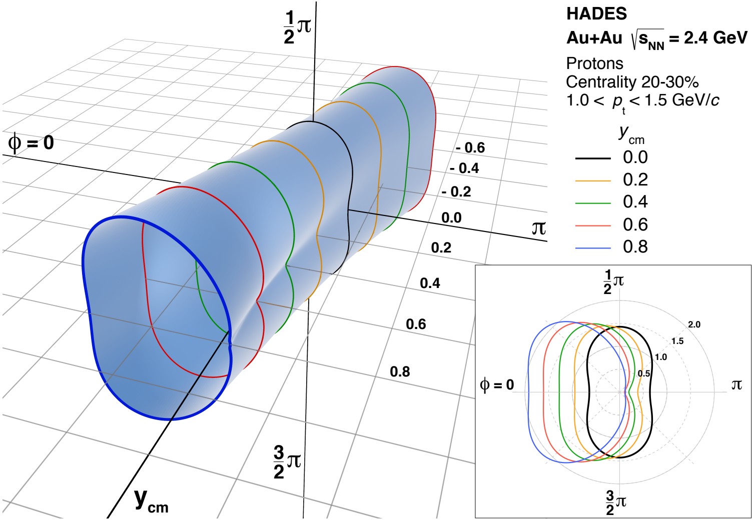
- Heavy ion collisions are rarely head-on.
- The complex 3D structure of the system gives rise to a complex shape in momentum space.



J. Adamczewski Musch *et al.* [HADES], Phys. Rev. Lett. **125** (2020), 262301

Results on flow - Why is flow sensitive to the EoS?

- Heavy ion collisions are rarely head-on.
- The complex 3D structure of the system gives rise to a complex shape in momentum space.
- Since this shape is a result of pressure gradients its sensitive to the EoS.
- Usually Fourier coefficients of the azimuth distributions are analyzed.

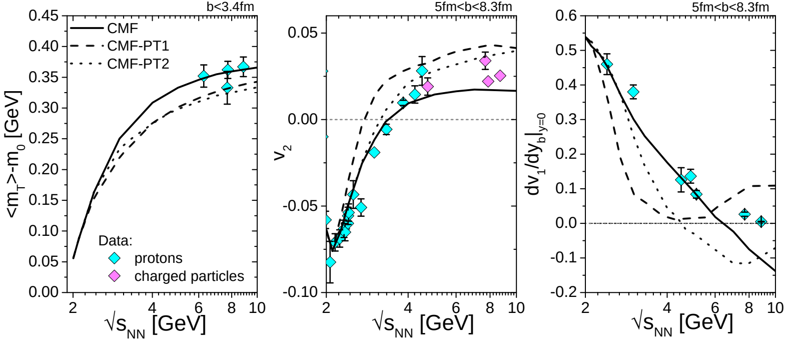


J. Adamczewski Musch *et al.* [HADES], Phys. Rev. Lett. **125** (2020), 262301

Results on flow

- The CMF EoS gives good results on all flow coefficients.
- Significant effects of a phase transition on all flow observables.
- Minimum in the slope of the directed flow confirmed.
- Sensitivity only up to $\approx 4n_0$.

- $m_T = p_T^2 + m^2$
- $v_1 = p_x/p_T$
- $v_2 = (p_x^2 - p_y^2)/p_T^2$



JS, A. Motornenko, A. Sorensen, Y. Nara, V. Koch and M. Bleicher, [arXiv:2208.12091 [nucl-th]].

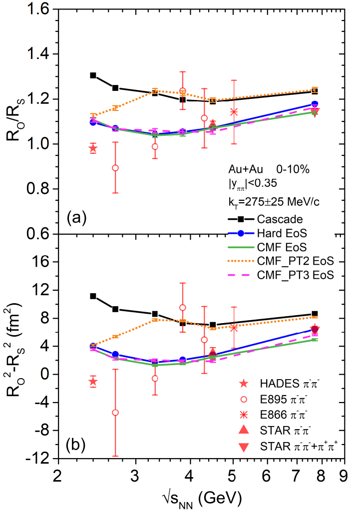
Other observables

The advantage of using an event generator like UrQMD: we can now study a multitude of observables:
All observables indicate a rather stiff EoS.

HBT

- Hanbury-Brown-Twiss (HBT) correlations for charged pions are a tool to measure the freezeout volume and time.
- Pions that are emitted close in coordinate space are correlated in momentum space.
- Simulation with a PT show a clear maximum.
- 'Old' data seem inconclusive, newest STAR data have much smaller error and favor the no-PT scenario.
- Sensitivity only up to $\approx 4n_0$.

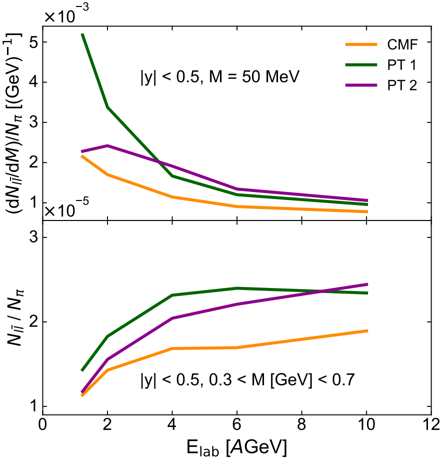
P. Li, T. Reichert, A. Kittiratpattana, JS, M. Bleicher, Q. Li, Sci. China Phys. Mech. Astron. 66, no.3, 232011 (2023)



Dileptons

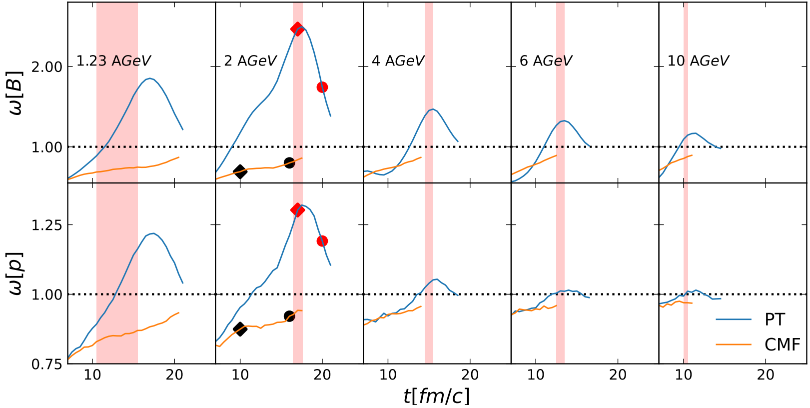
- Hydro simulations have suggested a strong increase (of factor 2) of the dilepton yield for a phase transition:
  F. Seck, T. Galatyuk, A. Mukherjee, R. Rapp, JS and J. Stroth, [arXiv:2010.04614 [nucl-th]].
- A significant increase of the low mass dilepton yield is observed when a phase transition is included in the UrQMD-CMF model.

O. Savchuk, A Motornenko, JS, V. Vovchenko, M. Bleicher, M. Gorenstein, T. Galatyuk, Phys. Rev. C 107, no.2, 024913 (2023).



Fluctuations

- As we employ a QMD approach local clumping in the unstable phase can occur.
- This leads to enhanced fluctuations of the baryon number in coordinate space, already observed in the scaled variance.



O. Savchuk, et.al., in preparation

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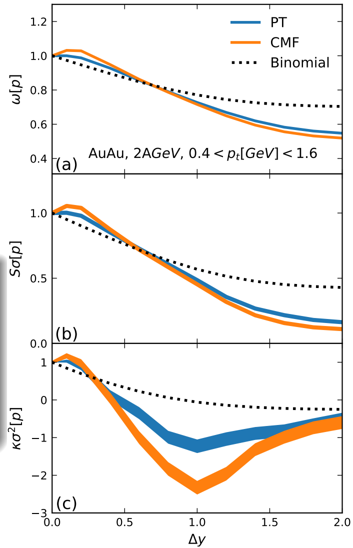
- While in coordinate space the fluctuations/correlations are enhanced due to the phase transition.
- In momentum space no enhancement is observed.
- The crossover scenario even shows an increased scaled variance. This is due to the larger radial flow pushing into the spectators leading to larger volume fluctuations.



O. Savchuk, et.al., in preparation

Statistical analysis of available flow data

- Using Bayesian inference methods we can try to constrain the EoS from flow data
- See talk by Manjunath Omana Kuttan.

M. Omana Kuttan, JS, K. Zhou and H. Stöcker, [arXiv:2211.11670 [nucl-th]].

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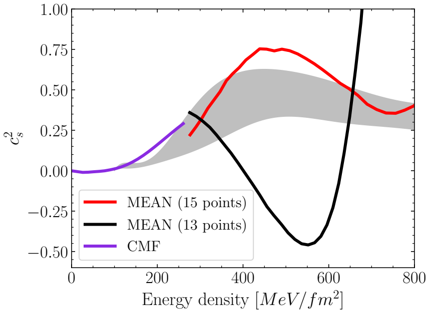
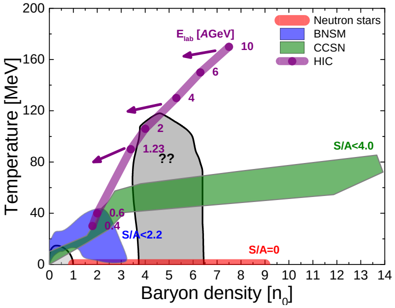
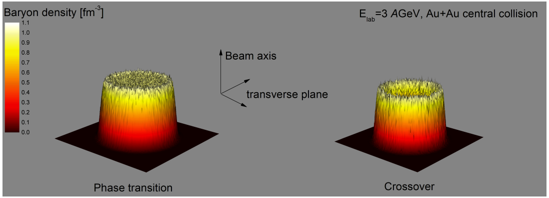
- Results depend strongly on the data used.



Summary and conclusions

- Can use HIC and BNSM to scan the high density QCD PD.
- Especially for HIC in the FAIR-regime new ideas/methods for old and new models are necessary.
- This work: Phase transitions in transport shown to influence observables.
- Best results obtained for model w/o phase transition, consistent with astrophysical observations (sensitivity only up to $\approx 4n_0$).
- Only consistent models can be used for statistical analyses of large datasets available now and in the future.
- Still room for development of critical phenomena, relativistic treatment of transport models...





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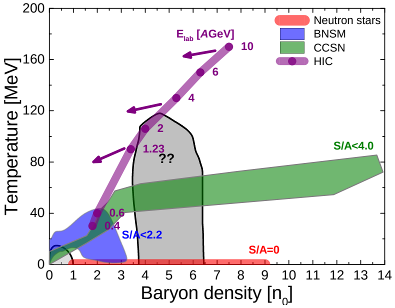

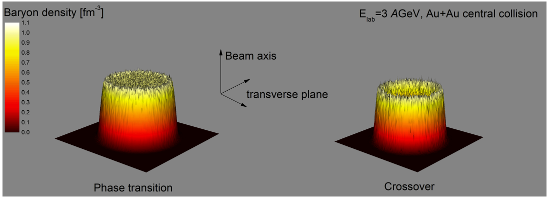

- The minimum of v_1 coincides with the maximum of the dilepton emission.
- The effect on HBT and maximum of the fluctuation enhancement seems to occur at even lower beam energies.
- Effects don't occur at the same beam energy: **Need consistent modeling!!**

Open challenges

What is missing/unclear in our description

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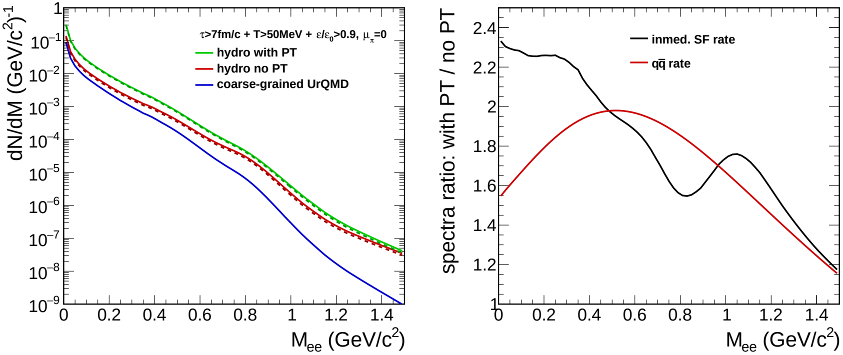
- Can we even think about changing d.o.f. at the phase transition?
- Fortunately we have so many experiments and observables to come.

Dileptons

Indeed di-lepton emission shows a significant effect

- A simulation for Au+Au at the current SIS18 beam energy.
- A factor 2 enhancement of di-lepton emission due to extended 'cooking'.



- Dilepton emission is sensitive to the time-integrated bulk evolution properties.

4. Light nuclei production

- The double ratio $t \cdot p/(d^2)$ is thought to be sensitive to spatial baryon fluctuations at freeze-out.
  K. J. Sun, L. W. Chen, C. M. Ko, J. Pu and Z. Xu, Phys. Lett. B **781** (2018), 499-504
- Can be studies by coalescence in UrQMD.
  P. Hillmann, K. Käfer, JS, V. Vovchenko and M. Bleicher,' J. Phys. G **49**, no.5, 055107 (2022)
- We see a very small enhancement in the scenario with a phase transition.
- Important to use realistic EoS with proper hadronic/nuclear matter.