Flow Studies with UrQMD

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Motivation

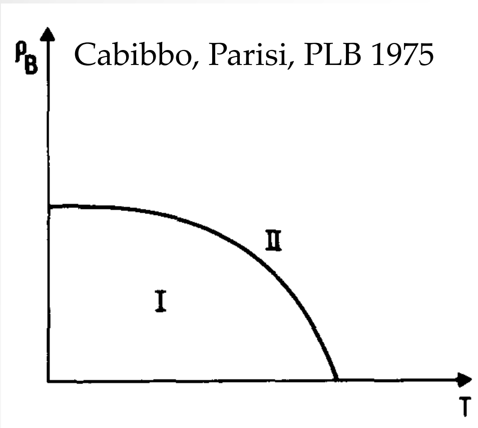
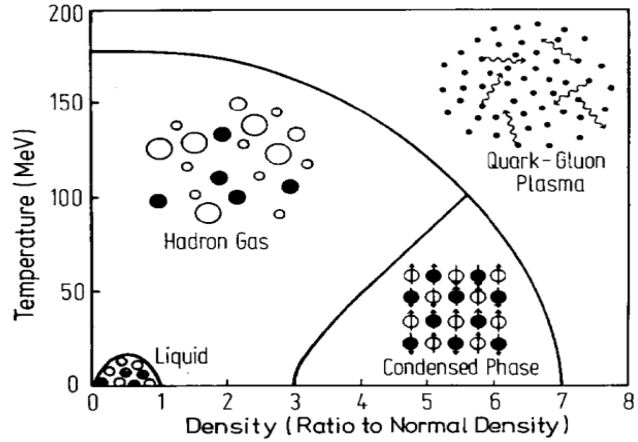
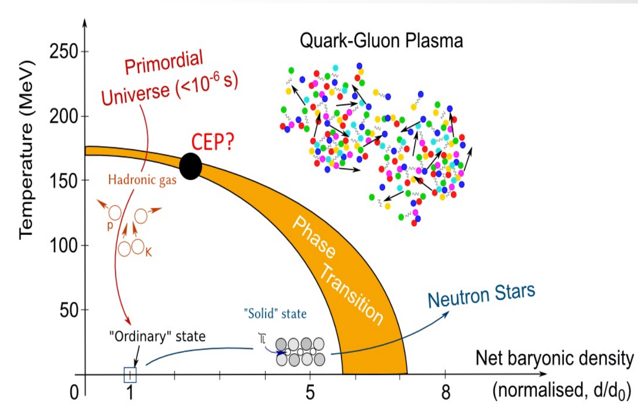


Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.



W. Greiner, PR 1986



A. Maire, CERN, 2015

- Learn about phase structure of QCD
- Explore strangeness, fluctuations, leptons, clusters, spectra,
  flow, fluctuations, correlations,…
- Unfortunately we do not have QCD in box → simulations

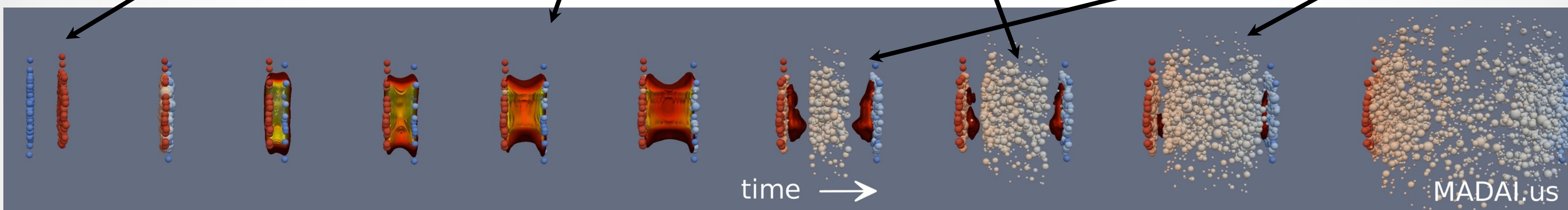
Time Evolution of Heavy Ion Collisions

Nuclei at 99 % speed of light

Quark Gluon Plasma

Hadronic Rescattering

Cluster emissions vs. formation



1x 10^{-23} s

10 x 10^{-23} s

30 x 10^{-23} s

Nonequilibrium initial state dynamics → Relativistic Hydrodynamics/ Particle dynamics → Hadron Transport

At high energies hybrid approaches are very successful for the description of the dynamics

Figure adapted from H. Elfner, arXiv:1404.1763

Ultra-relativistic Quantum Molecular Dynamics (UrQMD)

Hadron cascade (standard mode)

- Based on the propagation of hadrons
- Rescattering among hadrons is fully included
- String excitation/decay (LUND picture/PYTHIA) at higher energies
- Provides a solution of the relativistic n-body transport eq.:

$$p^{\mu} \cdot \partial_{\mu} f_i(x^{\nu}, p^{\nu}) = \mathcal{C}_i$$

The collision term C includes more than 100 hadrons

- "*Standard Reference*" for low and intermediate energy hadron and nucleus interactions

M. Bleicher et al, J.Phys. G25 (1999) 1859-1896

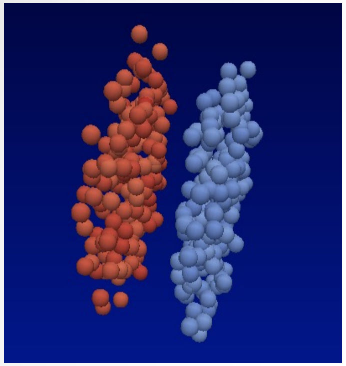
Ultra-relativistic Quantum Molecular Dynamics (UrQMD)

Hybrid mode calculations (RHIC and LHC energies)

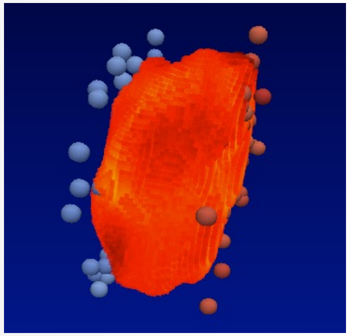
- At energies above 100 GeV (CM-energy) the early intermediate state should not be modeled by strings and particles alone
- To take the local equilibration and the phase transition to a QGP into account, a hydrodynamic phase is introduced
- This is known as hybrid model (Boltzmann+hydrodynamics), hybrid models have become the standard at RHIC and LHC energies

Petersen, Steinheimer, Burau, Bleicher et al, Phys.Rev. C78 (2008) 044901

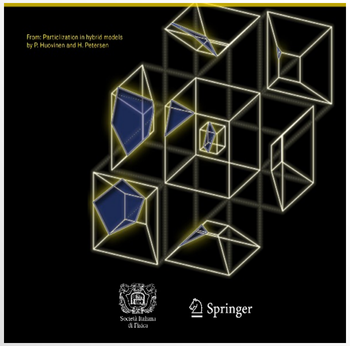
Option: Hybrid model



- Initial State:
  - Initialization of two nuclei
  - Non-equilibrium hadron-string dynamics
  - Initial state fluctuations are included naturally



- 3+1d Hydro +EoS:
  - **SHASTA** ideal relativistic fluid dynamics
  - Net baryon density is explicitly propagated
  - Equation of state at finit μ_B



- Final State:
  - Hypersurface at constant energy density
  - Hadronic rescattering and resonance decays within UrQMD

H.Petersen, M. Bleicher et al, PRC78 (2008) 044901

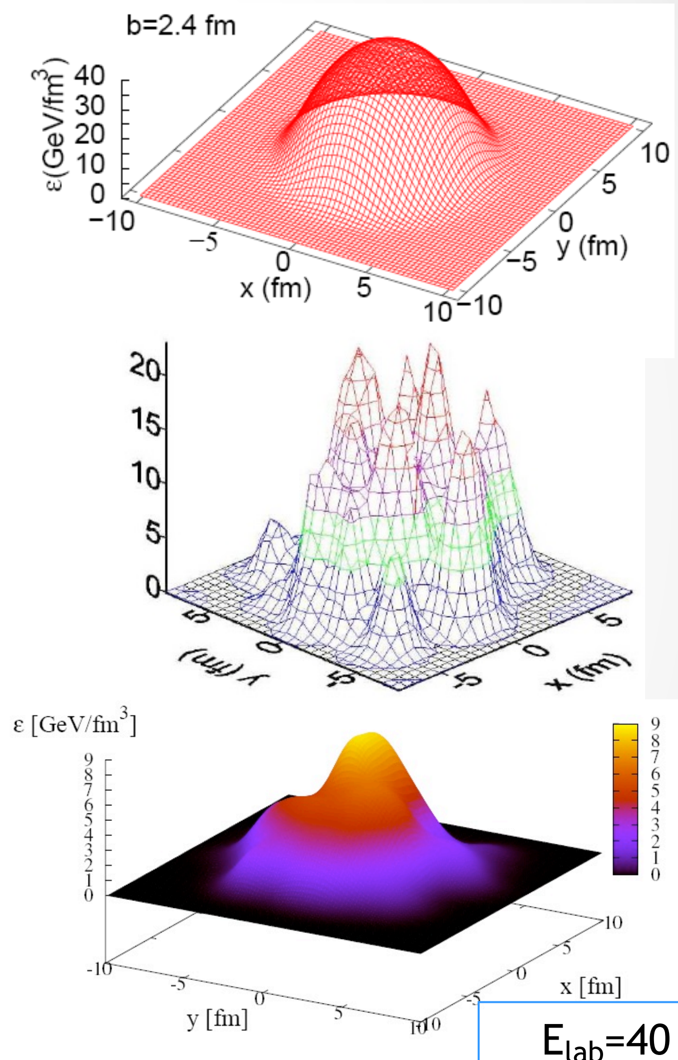
Initial State

- Contracted nuclei have passed through each other

$$t_{start} = \frac{2R}{\gamma v}$$

  - Energy is deposited
  - Baryon currents have separated
- Energy-, momentum- and baryon number densities are mapped onto the hydro grid
- **Event-by-event fluctuations** are taken into account
- Spectators are propagated separately in the cascade

(J.Steinheimer et al., PRC 77,034901,2008)



E_{lab}=40 AGeV
b=0 fm

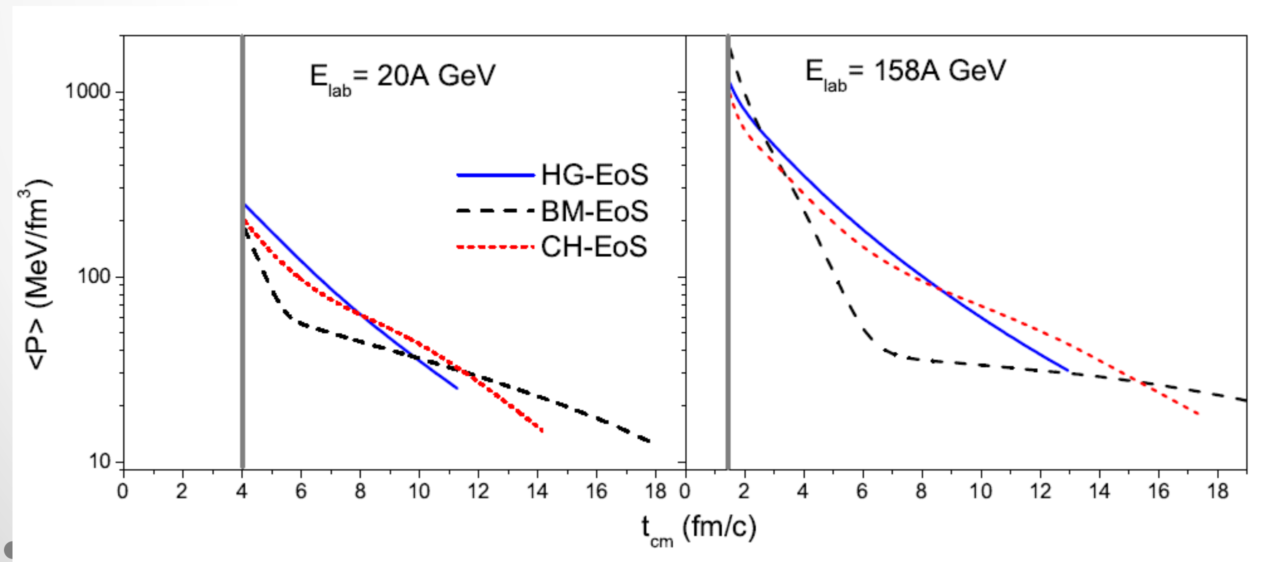
(nucl-th/0607018, nucl-th/0511021)

Equations of State

Ideal relativistic one fluid dynamics:

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{and} \quad \partial_\mu (n u^\mu) = 0$$

- HG: **Hadron gas** including the same degrees of freedom as in UrQMD (all hadrons with masses up to 2.2 GeV)
- CH: **Chiral EoS** from quark-meson model with first order transition and critical endpoint
- BM: **Bag Model EoS** with a strong first order phase transition between QGP and hadronic phase



D. Rischke et al., NPA 595, 346, 1995,

D. Zschiesche et al., PLB 547, 7, 2002

Papazoglou et al., PRC 59, 411, 1999

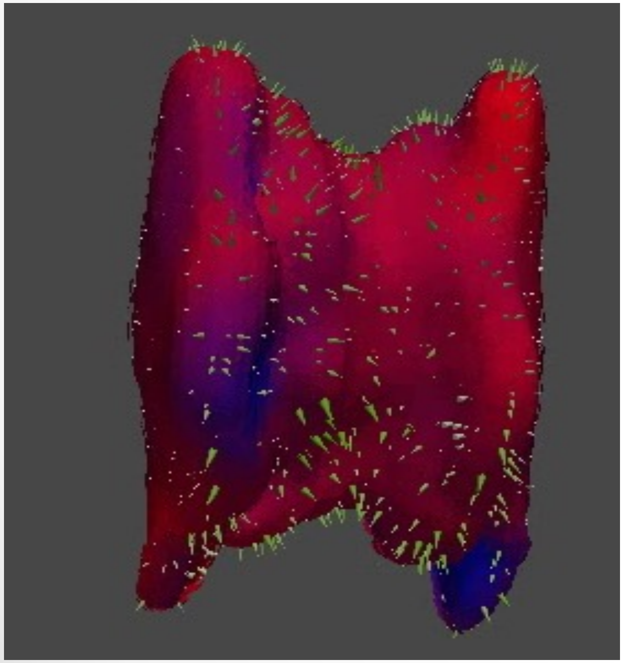
J. Steinheimer, et al., J. Phys. G38 (2011) 035001

Hadronization, Particlization, Decoupling

Experiments observe **finite number** of hadrons in detectors

Hadronization controlled by the equation of state

Sampling of particles according to **Cooper-Frye** equation:
-Respect **conservation laws**, maybe even locally?
-Introduces fluctuations on its own



$$E\frac{dN}{d^3p} = \int_\sigma f(x,p)p^\mu d\sigma_\mu$$

→ Yields 4-momenta, 4-positions of hadrons on the hypersurface

→ Final propagation
Relativistic transport equation $\left(p^\mu \partial_\mu\right) f = I_{coll}$

Sophisticated 3D hypersurface finder to resolve interesting structures in event-by-event simulations
Petersen, Huovinen, arXiv:1206.3371

List of included particles

nucleon	Δ	Λ	Σ	Ξ	Ω
N_{938}	Δ_{1232}	Λ_{1116}	Σ_{1192}	Ξ_{1317}	Ω_{1672}
N_{1440}	Δ_{1600}	Λ_{1405}	Σ_{1385}	Ξ_{1530}	
N_{1520}	Δ_{1620}	Λ_{1520}	Σ_{1660}	Ξ_{1690}	
N_{1535}	Δ_{1700}	Λ_{1600}	Σ_{1670}	Ξ_{1820}	
N_{1650}	Δ_{1900}	Λ_{1670}	Σ_{1775}	Ξ_{1950}	
N_{1675}	Δ_{1905}	Λ_{1690}	Σ_{1790}	Ξ_{2025}	
N_{1680}	Δ_{1910}	Λ_{1800}	Σ_{1915}		
N_{1700}	Δ_{1920}	Λ_{1810}	Σ_{1940}		
N_{1710}	Δ_{1930}	Λ_{1820}	Σ_{2030}		
N_{1720}	Δ_{1950}	Λ_{1830}			
N_{1900}		Λ_{1890}			
N_{1990}		Λ_{2100}			
N_{2080}		Λ_{2110}			
N_{2190}					
N_{2200}					
N_{2250}					

0^{-+}	1^{--}	0^{++}	1^{++}
π	ρ	a_0	a_1
K	K^*	K_0^*	K_1^*
η	ω	f_0	f_1
η'	ϕ	f_0^*	f_1'
1^{+-}	2^{++}	$(1^{--})^*$	$(1^{--})^{**}$
b_1	a_2	ρ_{1450}	ρ_{1700}
K_1	K_2^*	K_{1410}^*	K_{1680}^*
h_1	f_2	ω_{1420}	ω_{1662}
h_1'	f_2'	ϕ_{1680}	ϕ_{1900}

- Binary interactions between all implemented particles are treated
- Cross sections are taken from data or models
- Resonances are implemented in Breit-Wigner form
- No in-medium modifications

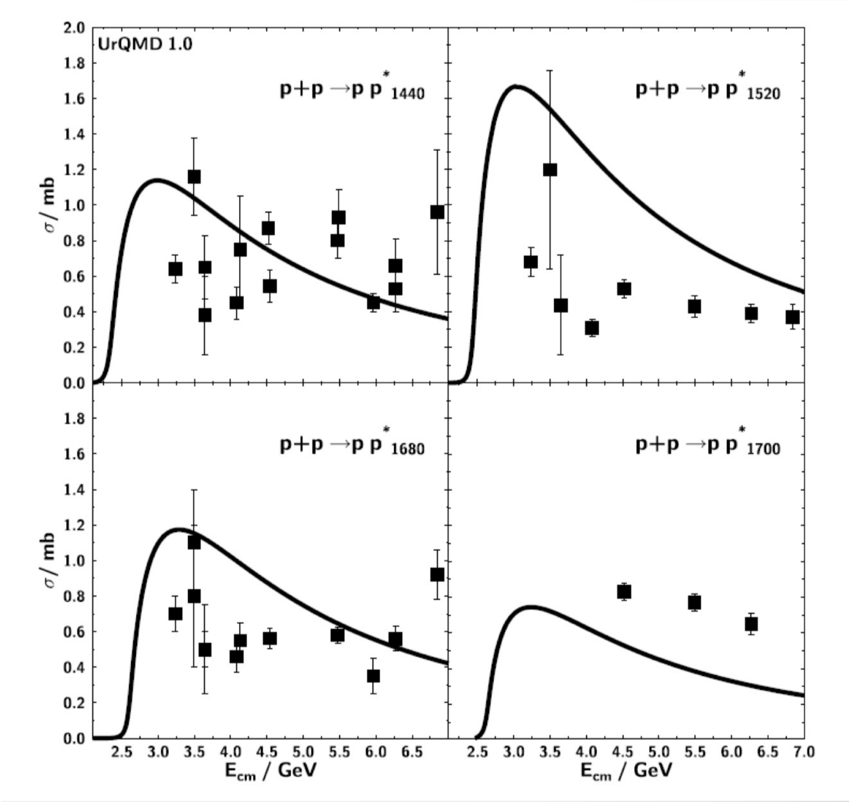
Baryon-baryon scattering cross section

- Phase space x matrix element:

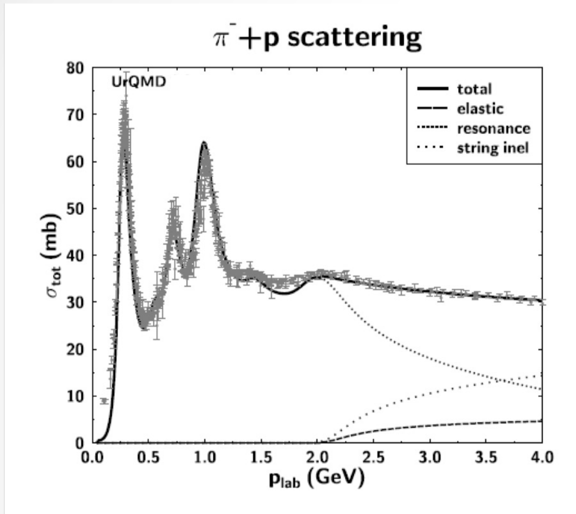
$$\sigma_{tot}^{BB}(\sqrt{s}) \propto (2S_D+1)(2S_E+1)\frac{\langle p_{D,E}\rangle}{\langle p_{A,C}\rangle}\frac{1}{s}|\mathcal{M}|^2$$

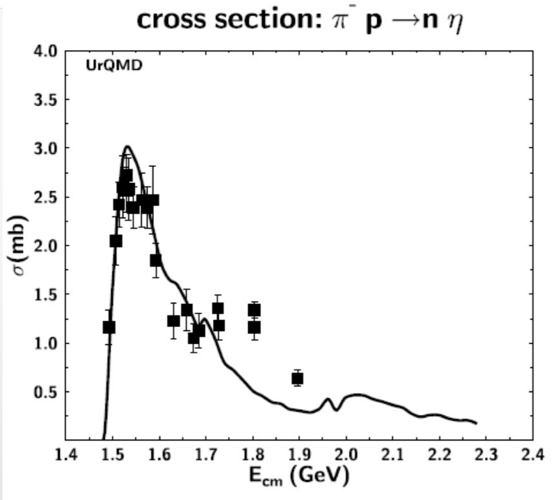
- Matrix element is fitted to data for groups of resonance channels
- Detailed balance is fulfilled for the inverse reaction:

$$\sigma(y \to x)\, p_y^2\, g_y = \sigma(x \to y)\, p_x^2\, g_x$$



Meson-baryon scattering cross section (resonances)

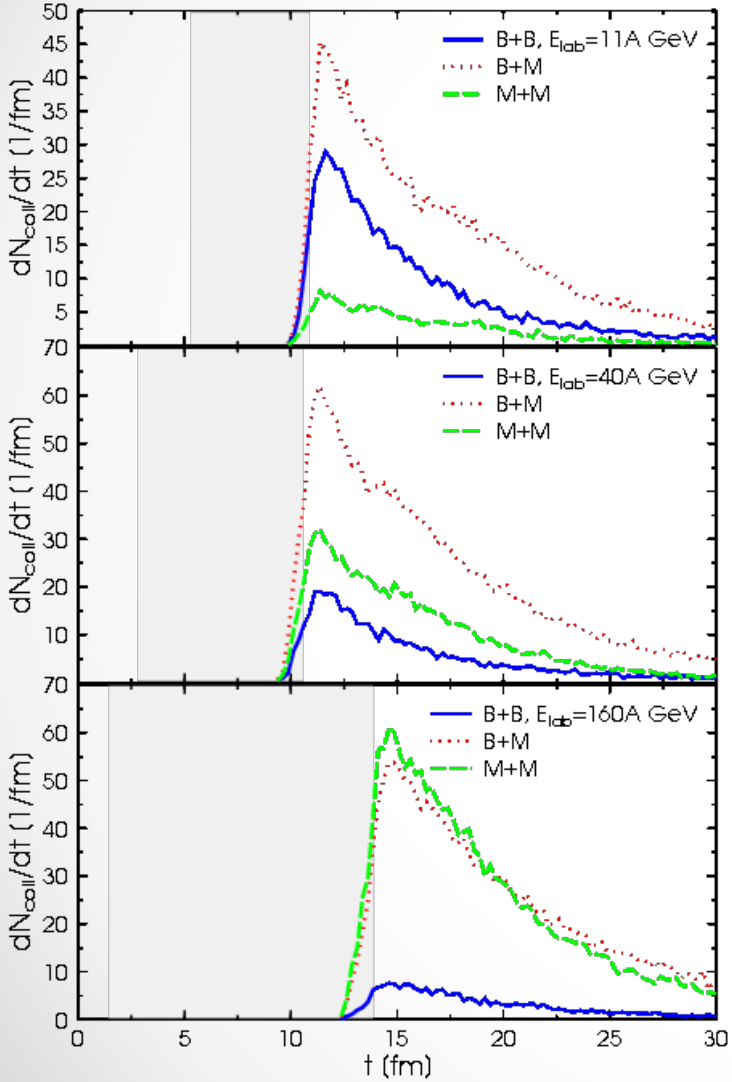


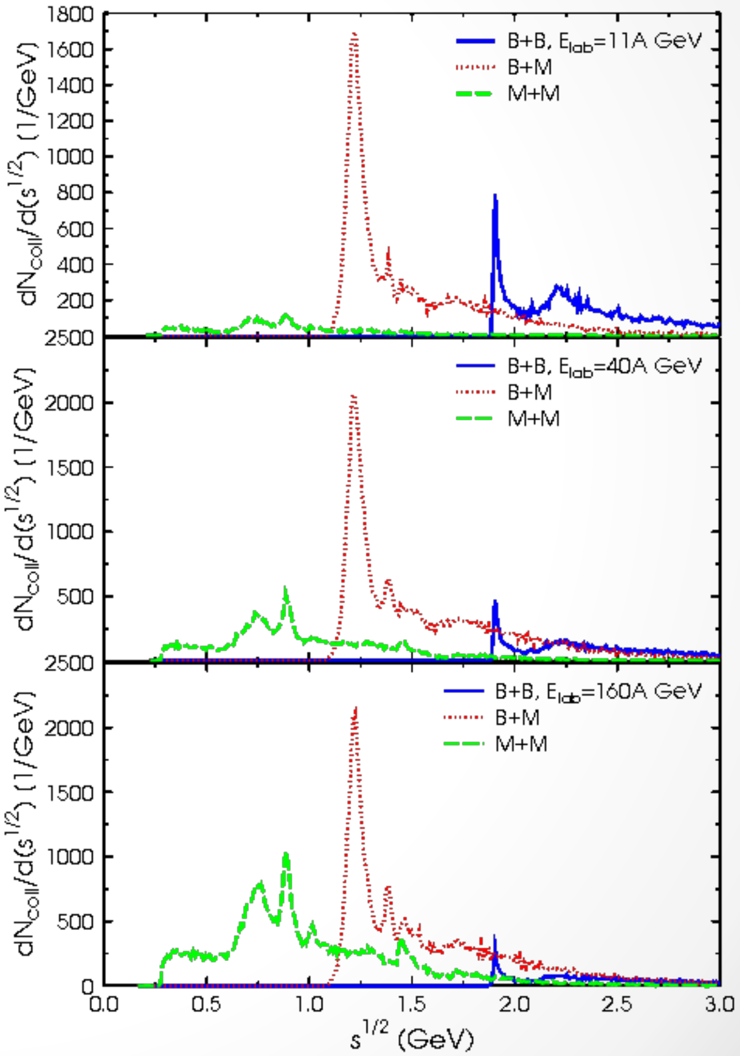


resonance	mass	width	$N\gamma$	$N\pi$	$N\eta$	$N\omega$	$N\varrho$	$N\pi\pi$	$\Delta_{1232}\pi$	$N^*_{1440}\pi$	ΛK
N^*_{1440}	1.440	200		0.70				0.05	0.25		
N^*_{1520}	1.520	125		0.60				0.15	0.25		
N^*_{1535}	1.535	150	0.001	0.55	0.35			0.05		0.05	
N^*_{1650}	1.650	150		0.65	0.05			0.05	0.10	0.05	0.10
N^*_{1675}	1.675	140		0.45					0.55		
N^*_{1680}	1.680	120		0.65				0.20	0.15		
N^*_{1700}	1.700	100		0.10	0.05		0.05	0.45	0.35		
N^*_{1710}	1.710	110		0.15	0.20		0.05	0.20	0.20	0.10	0.10
N^*_{1720}	1.720	150		0.15			0.25	0.45	0.10		0.05
N^*_{1900}	1.870	500		0.35		0.55	0.05		0.05		
N^*_{1990}	1.990	550		0.05			0.15	0.25	0.30	0.15	0.10
N^*_{2080}	2.040	250		0.60	0.05		0.25	0.05	0.05		
N^*_{2190}	2.190	550		0.35			0.30	0.15	0.15	0.05	
N^*_{2220}	2.220	550		0.35			0.25	0.20	0.20		
N^*_{2250}	2.250	470		0.30			0.25	0.20	0.20	0.05	
Δ_{1232}	1.232	115.	0.01	1.00							
Δ^*_{1600}	1.700	200		0.15					0.55	0.30	
Δ^*_{1620}	1.675	180		0.25					0.60	0.15	
Δ^*_{1700}	1.750	300		0.20			0.10		0.55	0.15	
Δ^*_{1900}	1.850	240		0.30			0.15		0.30	0.25	
Δ^*_{1905}	1.880	280		0.20			0.60		0.10	0.10	
Δ^*_{1910}	1.900	250		0.35			0.40		0.15	0.10	
Δ^*_{1920}	1.920	150		0.15			0.30		0.30	0.25	
Δ^*_{1930}	1.930	250		0.20			0.25		0.25	0.30	
Δ^*_{1950}	1.950	250	0.01	0.45			0.15		0.20	0.20	

$$\sigma^{MB}_{tot}(\sqrt{s}) = \sum_{R=\Delta,N^*} \langle j_B, m_B, j_M, m_M \| J_R, M_R \rangle \frac{2S_R+1}{(2S_B+1)(2S_M+1)} \times \frac{\pi}{p^2_{cm}} \frac{\Gamma_{R\rightarrow MB}\Gamma_{tot}}{(M_R-\sqrt{s})^2+\Gamma^2_{tot}/4} \quad ,$$

Final State Interactions (after Hydro)





Using flow to learn about the initial stage

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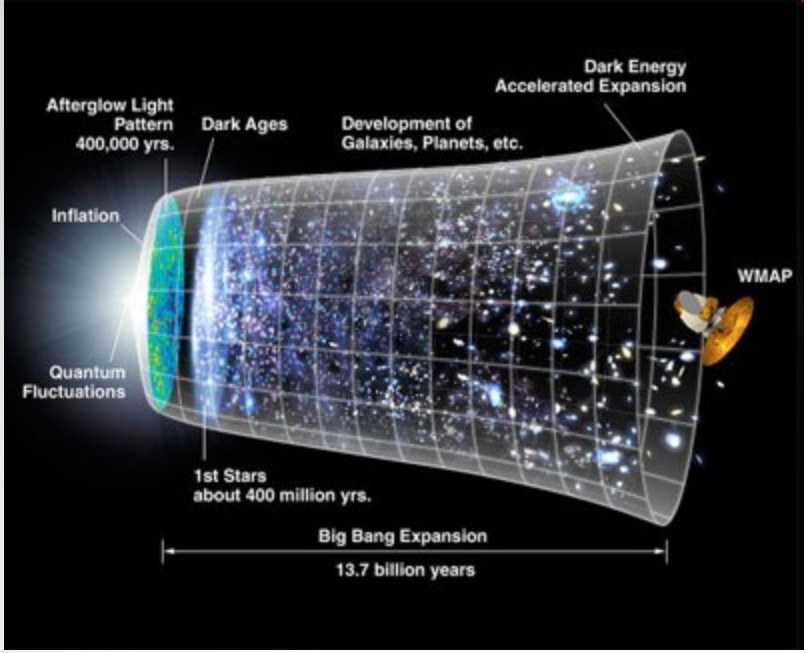
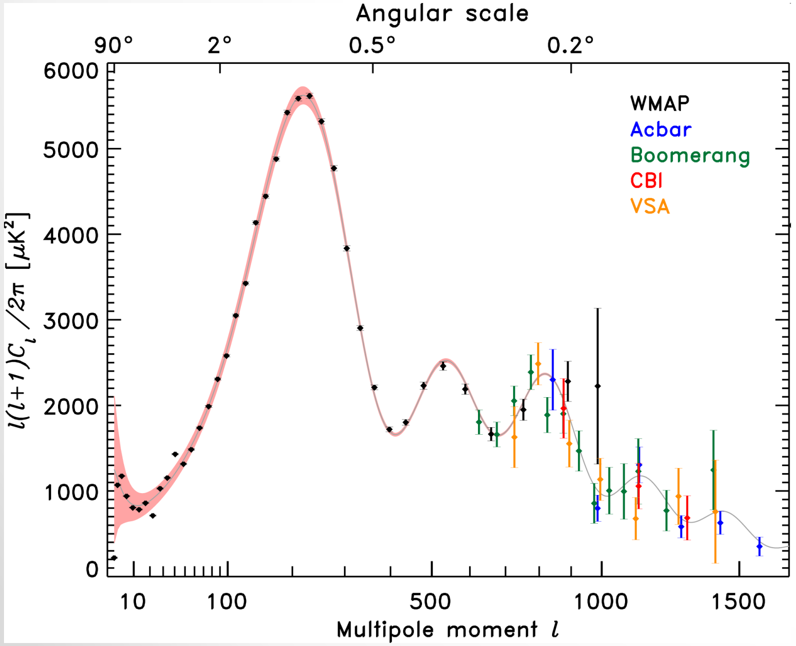
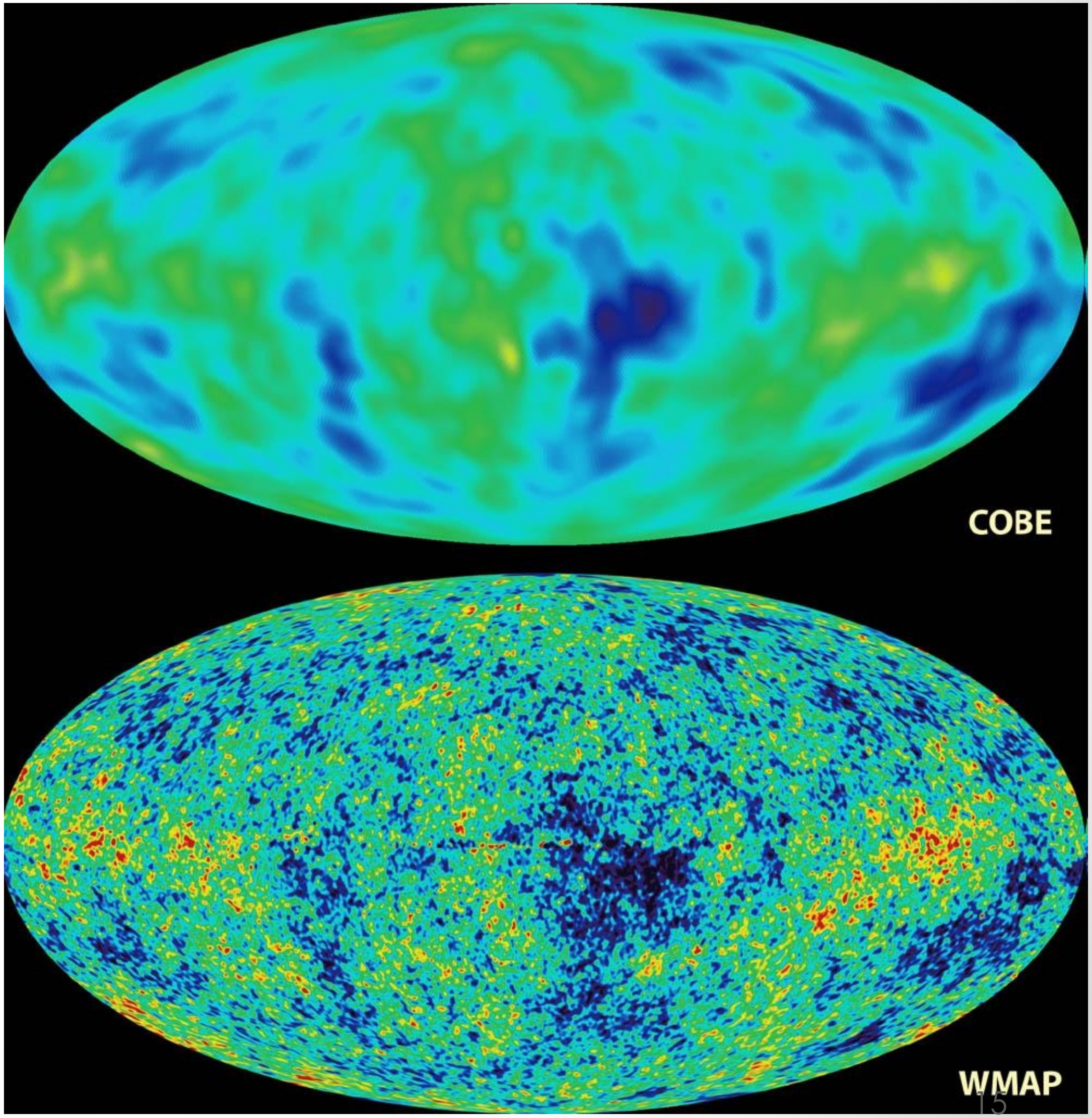
High energy

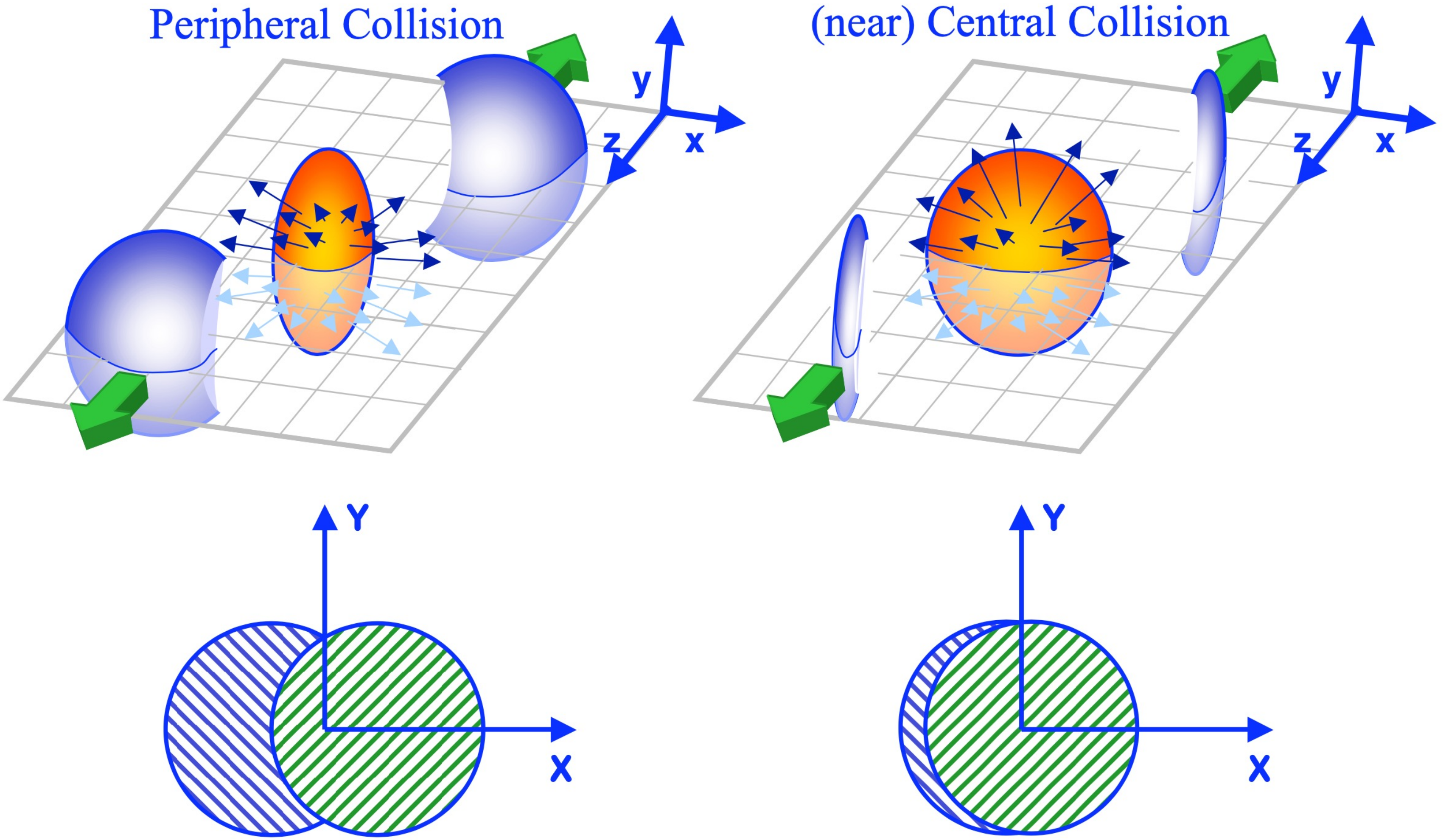
Idea: Angular correlation





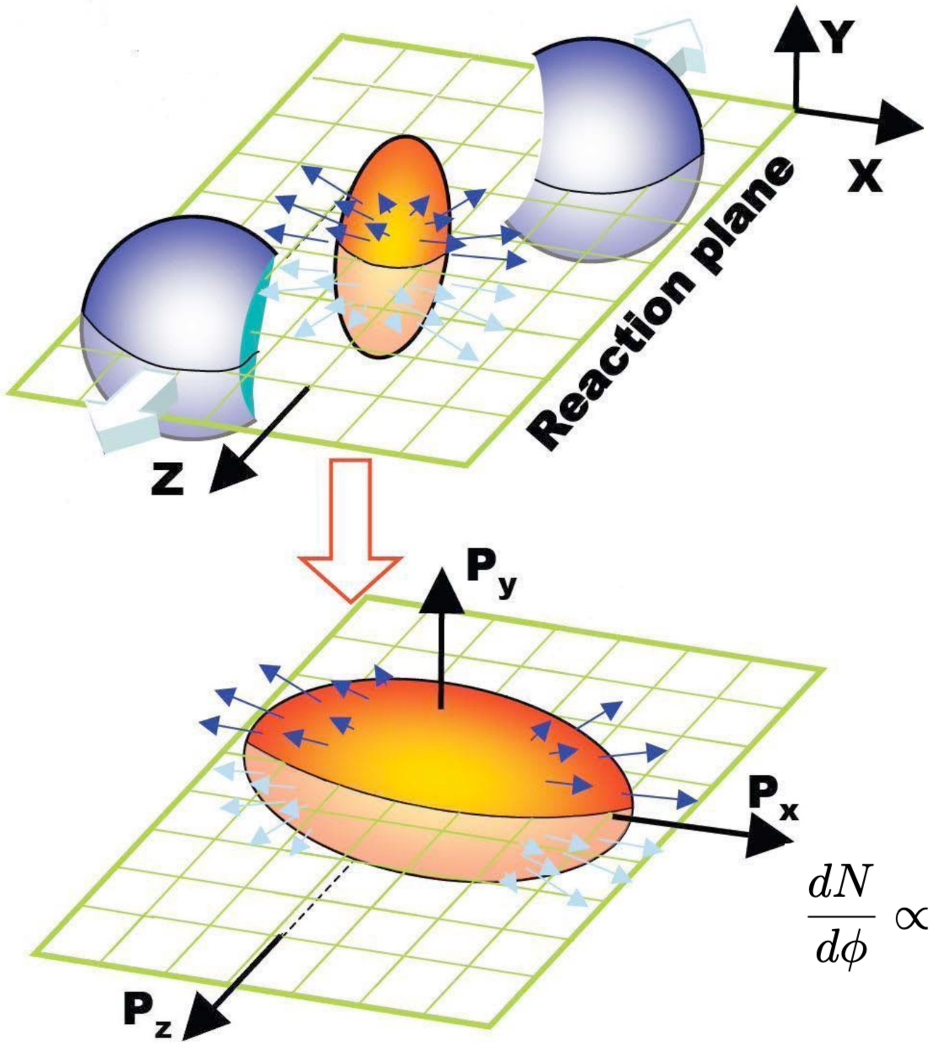


Where do they come from?



From Masashi Kaneta

How are they connected to the initial state?



spatial anisotropy ε_2

↓ ↓

momentum anisotropy v_2

$$\frac{dN}{d\phi} \propto 1 + 2\sum_{n=1}^{\infty} v_n^a \cos(n(\phi - \Phi_n))$$

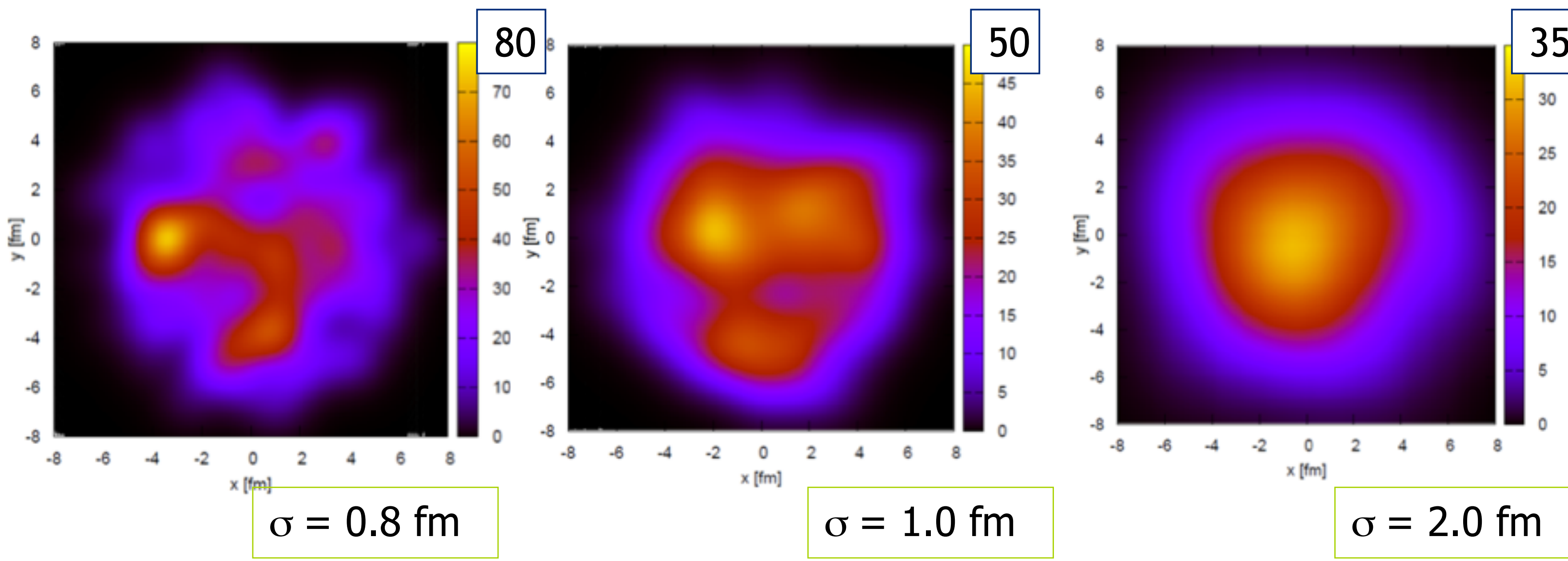
From Masashi Kaneta

Learning about the initial state at RHIC and LHC

- Energy-, momentum- and baryon number densities are mapped onto the hydro grid using for each particle

$$\epsilon(x,y,z) = \left(\frac{1}{2\pi}\right)^{\frac{3}{2}} \frac{\gamma_z}{\sigma^3} E_p \exp - \frac{(x-x_p)^2 + (y-y_p)^2 + (\gamma_z(z-z_p))^2}{2\sigma^2}$$

- Changing σ leads to different granularities, but also changes in the overall profile

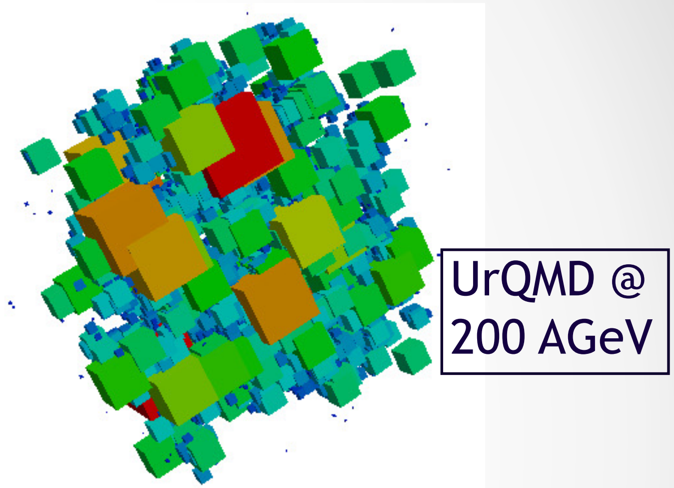


H.P. et al, PRC 83, 014916, 2011 and JPG 38, 045102, 2011

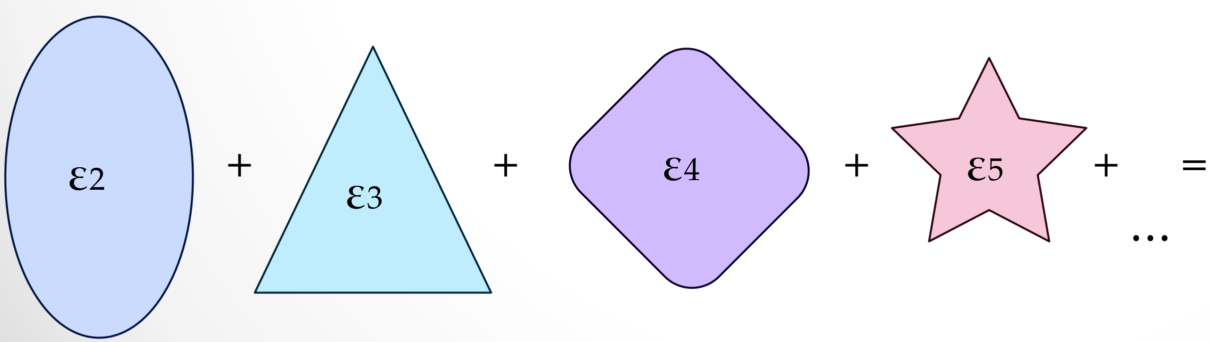
- How does changing the starting time affect the picture?

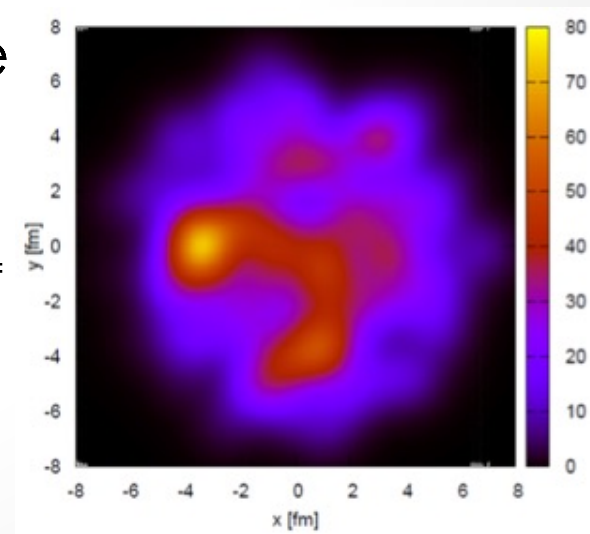
Sources of Fluctuations

- Granularity is driven by
    - position of nucleons
    - distribution of collisions
    - type of interaction
    - degree of thermalization



- How to quantify the fluctuating shape of the initial state?
  → Fourier-expansion in position space

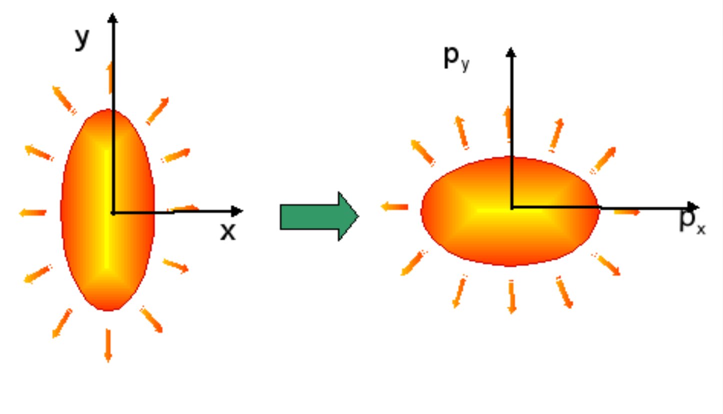




Anisotropic Flow – Higher order Fourier coefficients in momentum space

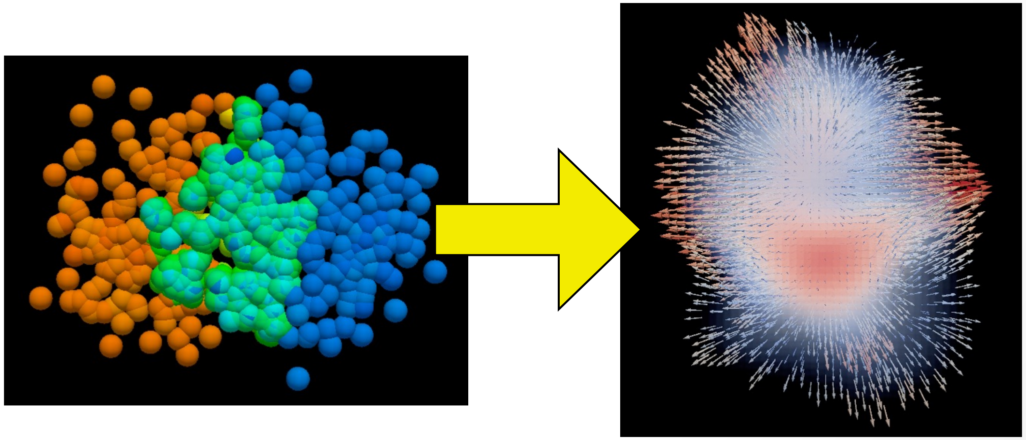
Simplified picture:

Position-space anisotropy
→ Momentum-space anisotropy



$$\frac{dN}{d\phi} \propto 1 + 2\sum_{n=1}^{\infty} v_n^a \cos(n(\phi - \Phi_n))$$

Real picture:
Complicated state,
mean free paths,…

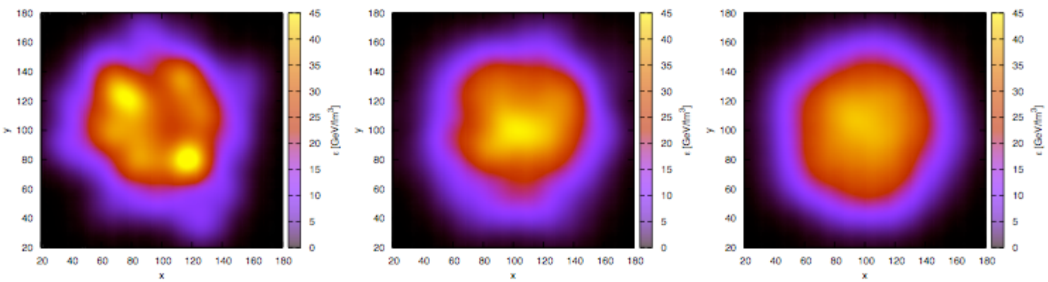


by MADAI.us

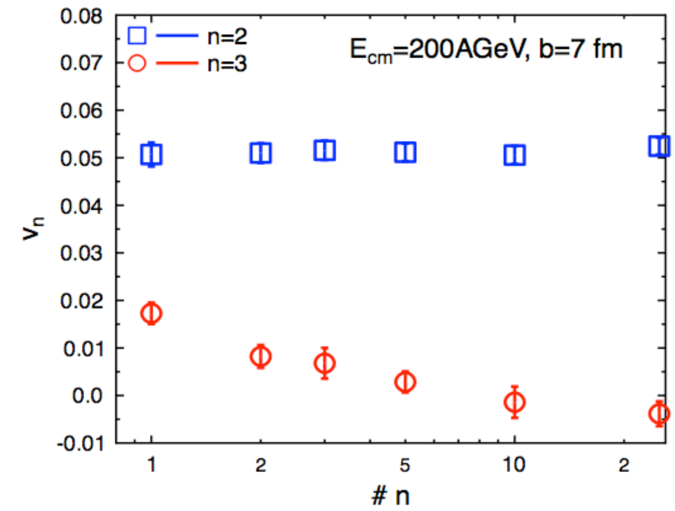
Use the v_n coefficients to learn about the initial state

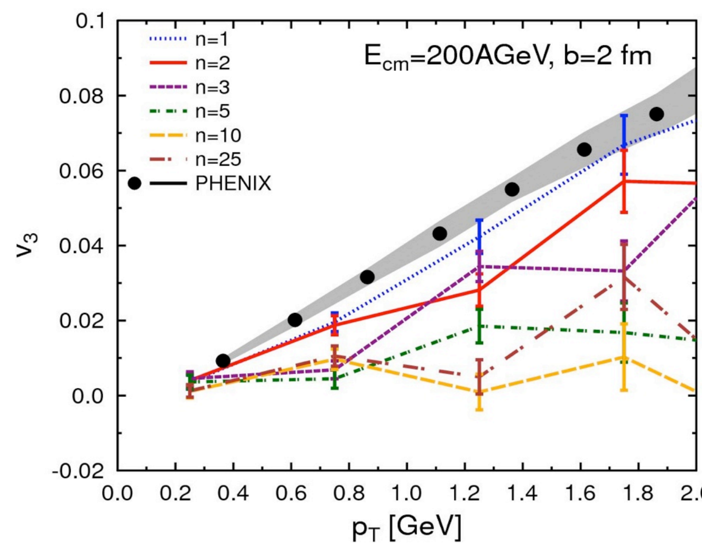
Constraining initial state granularity

H.P. et al, J.Phys.G G39 (2012) 055102



- Triangular flow is **very sensitive** to amount of initial state fluctuations
- It is important to have final state particle distributions to apply **same analysis** as in experiment
- Single-event initial condition provides best agreement with PHENIX data
- Does that imply that the initial state is well-described by binary nucleon interactions +PYTHIA?
- Lower bound for fluctuations!





Using flow to learn about the intermediate stage

Low energy

Ultra-relativistic Quantum Molecular Dynamics (UrQMD)

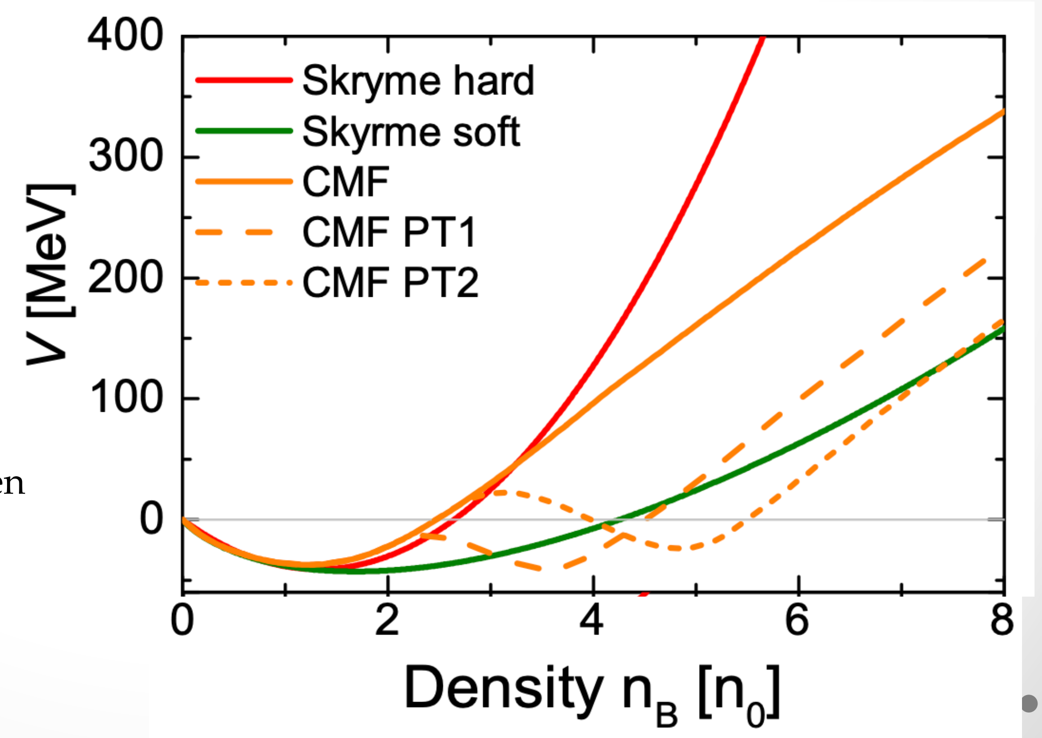
Remember talk by Jan Steinheimer

Potential mode calculations (RHIC-BES energies):

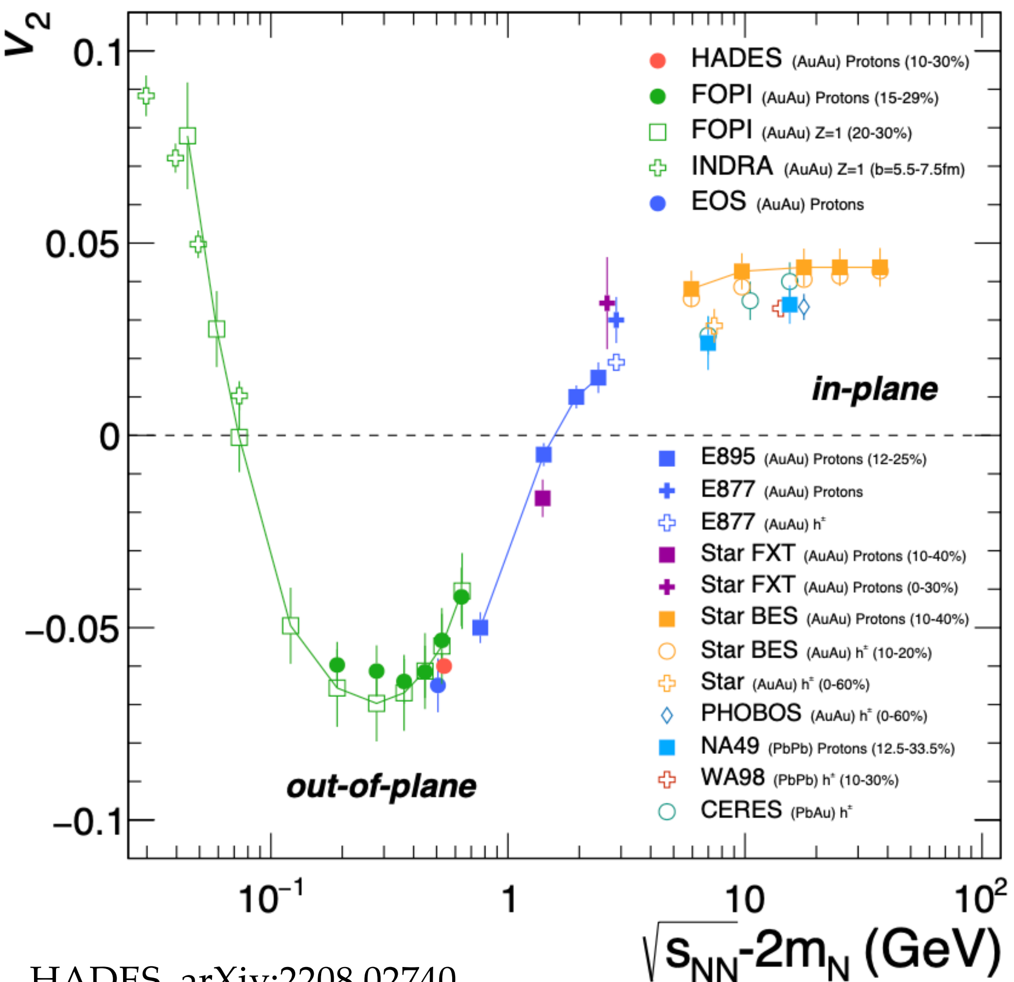
- Cascade calculation can be supplemented by hadronic potentials – standard: hard/soft Skyrme type

- Other potentials mimicking a phase transition are also possible.

Steinheimer, Motornenko, Sorensen
Nara, Koch, Bleicher, Stöcker,
Eur.Phys.J.C 82 (2022) 911



Elliptic flow versus energy

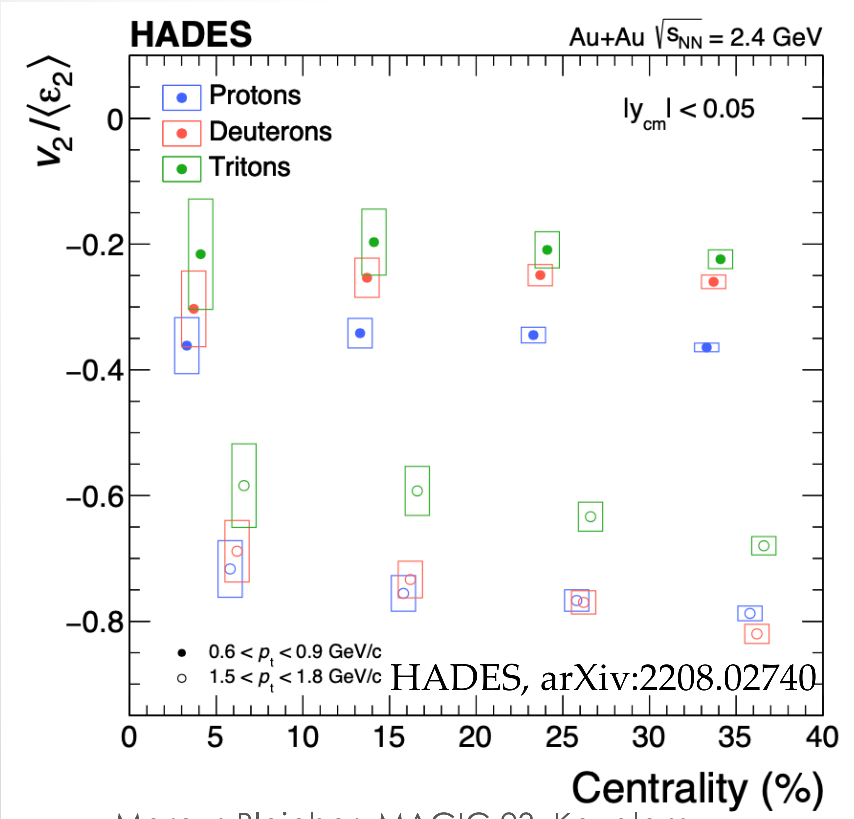


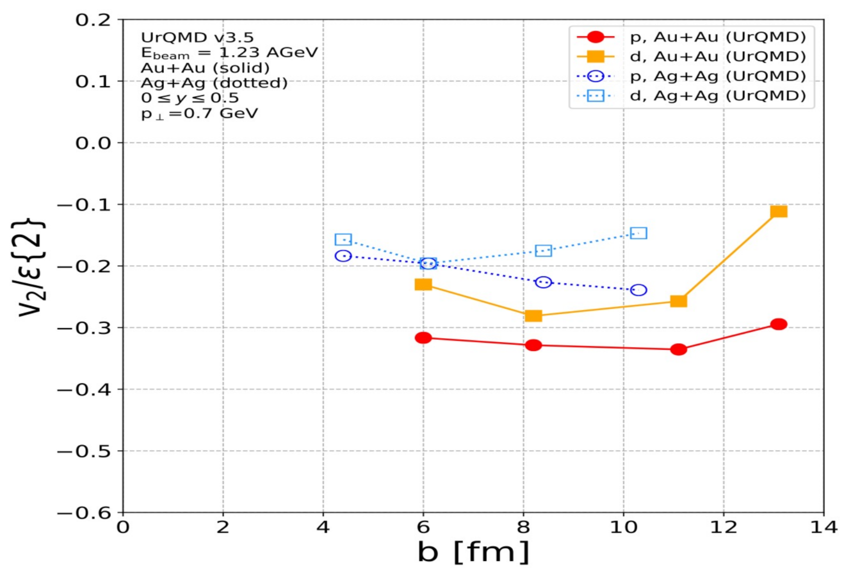
HADES, arXiv:2208.02740

- Elliptic flow is **negative** from $\sqrt{s_{NN}} = 2 - 4$ GeV
- Positive at higher energies
- Out-of-plane emission: Shadowing
- In-plane emission: Pressure gradient, transverse expansion

Elliptic flow scaling with eccentricity

- LHC & RHIC: initial ε_2 → $-\nabla P$ → final v_2
- GSI/FAIR: Negative scaling observed by HADES





Negative(!) v_2 scaling is observed
For protons and deuterons

T. Reichert et al., J.Phys.G 50 (2023) 2, 025104

Elliptic flow scaling with eccentricity

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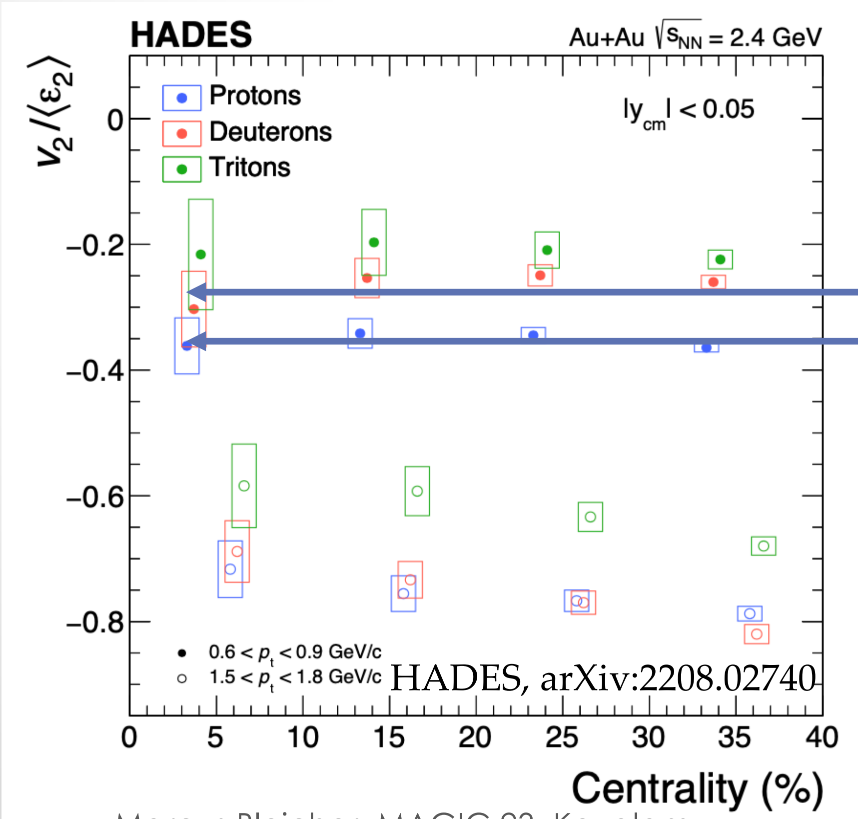

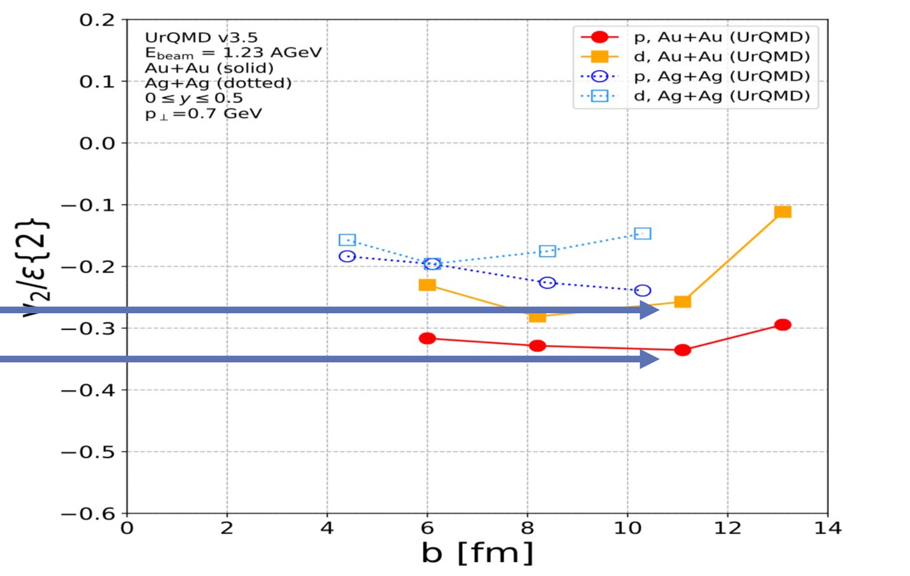

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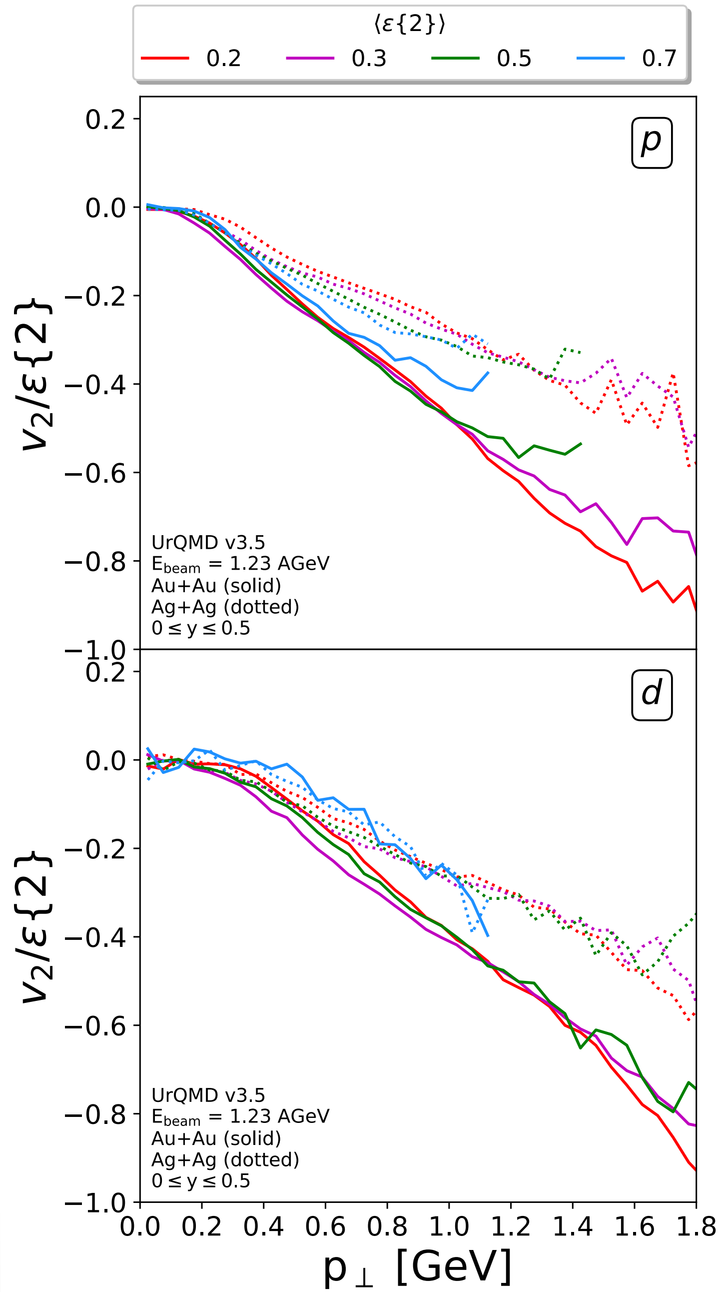
T. Reichert et al., J.Phys.G 50 (2023) 2, 025104

Flow scaling: p_T

T. Reichert et al., J.Phys.G 50 (2023) 2, 025104

- v_2 scaling with ε_2 is negative
- p_T dependence also scales
- Au+Au collisions and Ag+Ag collisions behave similarly
- Similar shadowing strength at equal eccentricity

→ Probe hot and dense phase



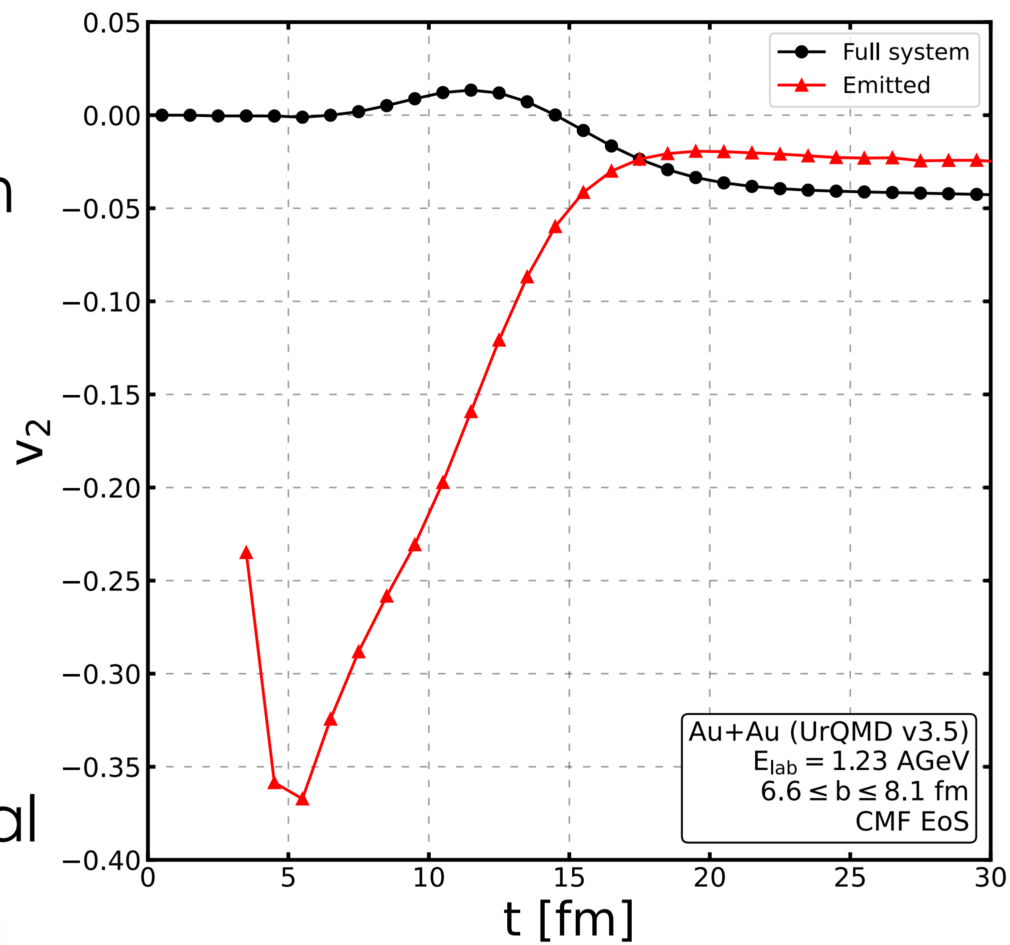
Is v2 always negative?
Time development of v_2

Full system:

- Zero until 7 fm
- Positive from 7 to 15 fm due to pressure gradient
- Momentum transfer to (semi-) spectators
- Turns negative

Emitted:

- First highly negative
- Increasing towards final value



T. Reichert et al., e-Print: 2302.13919

Time development of v_1 and v_2

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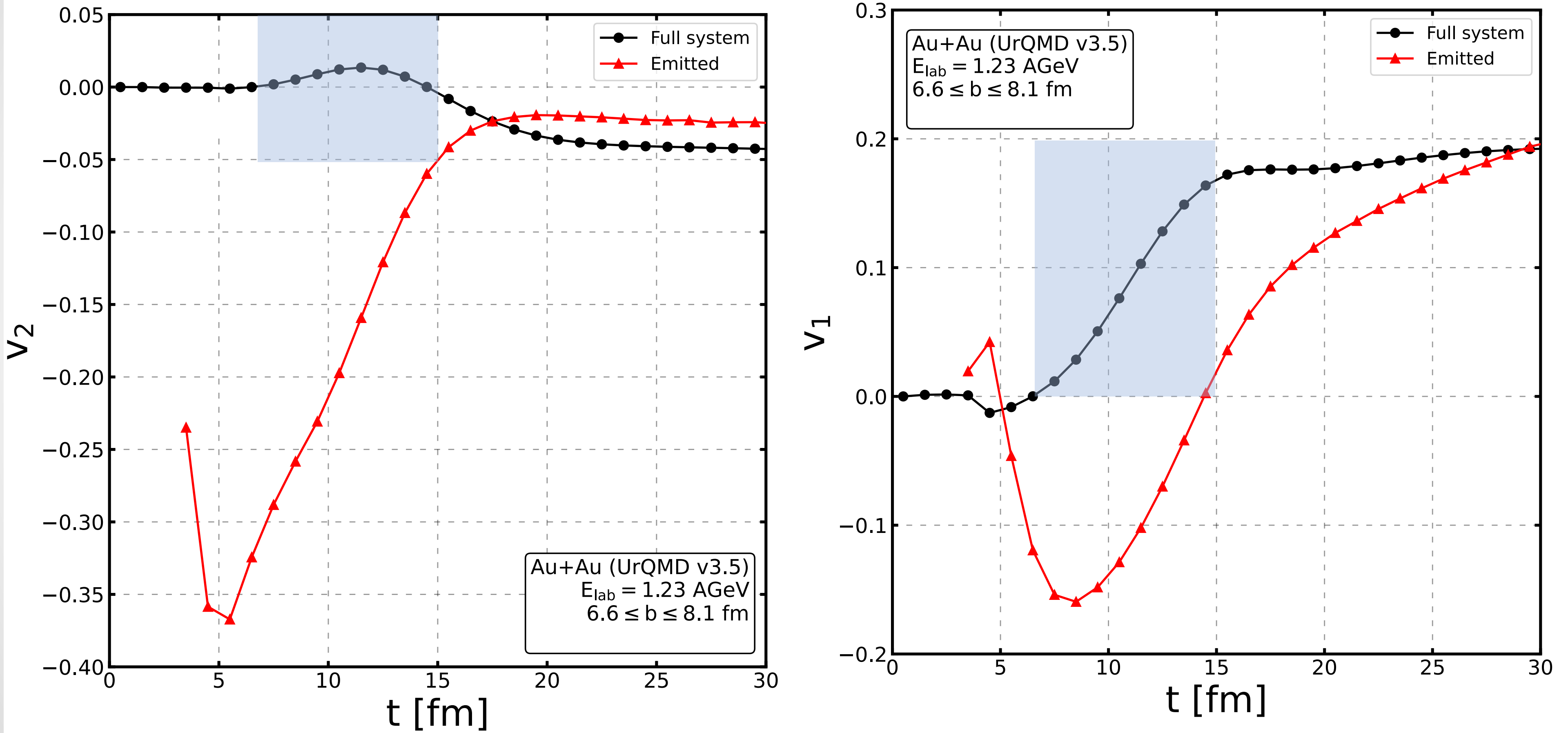
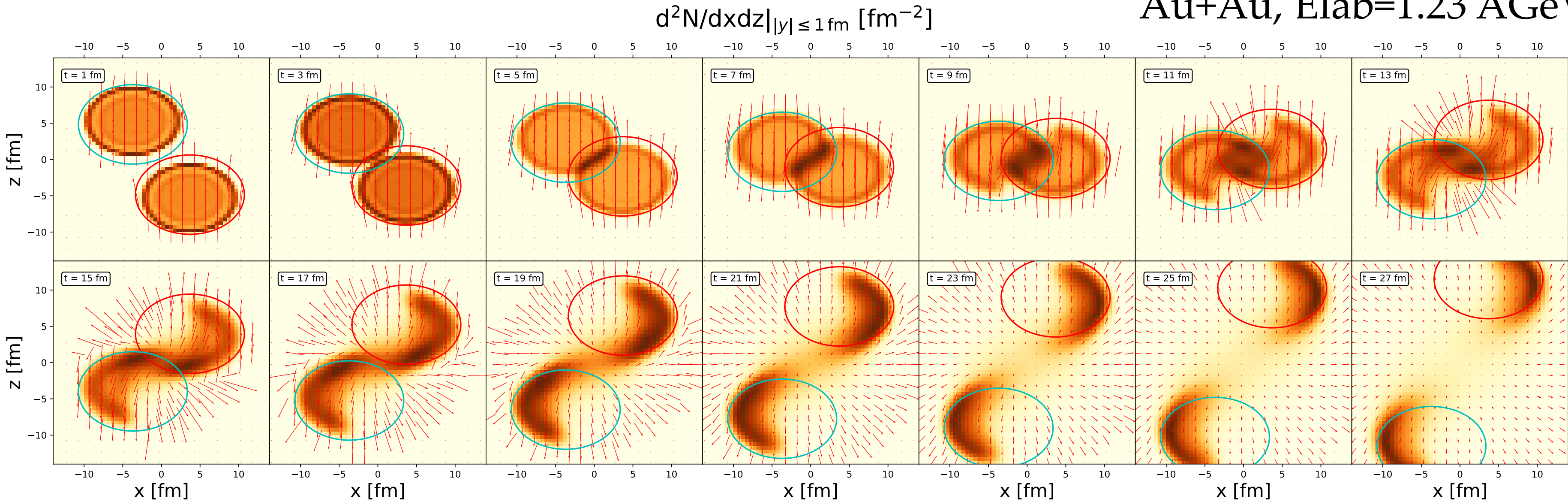


- Flow is sensitive to the EoS (build-up during most dense phase)
- Tight connection between v_1 and v_2

Time evolution

Au+Au, Elab=1.23 AGeV



- Positive v_2 from 7 to 15 fm due to pressure gradient and larger surface in x-direction
- During that time span Momentum transfer to spectators

- Emitted hadrons always negative v_2 due to shadowing

T. Reichert et al., e-Print: 2302.13919

Messenger from the hot and dense state

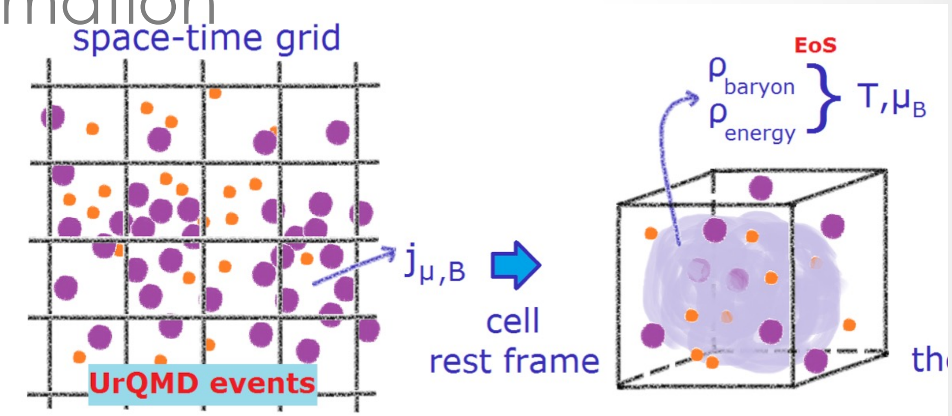
Testing the expansion scenario by Dileptons

Dileptons via coarse graining

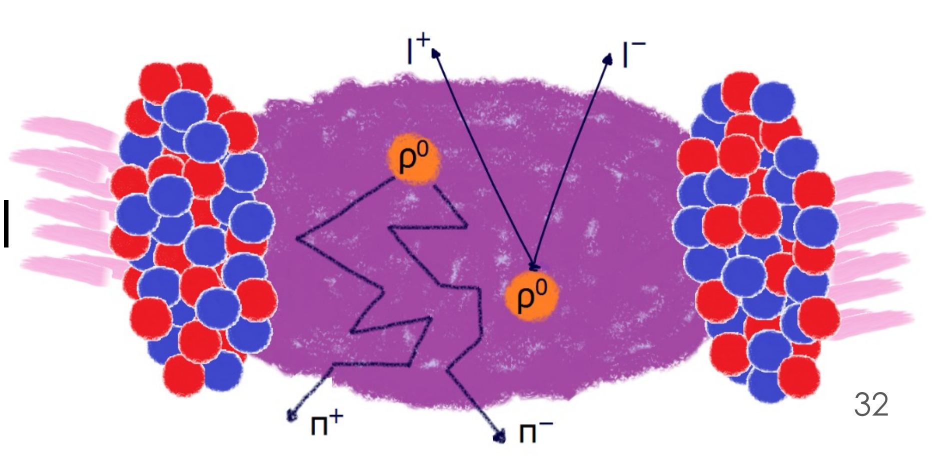
S. Endres et al. Phys.Rev.C 91 (2015) 5, 054911

C. Gale et al. Nucl. Phys. B357 (1991) 65

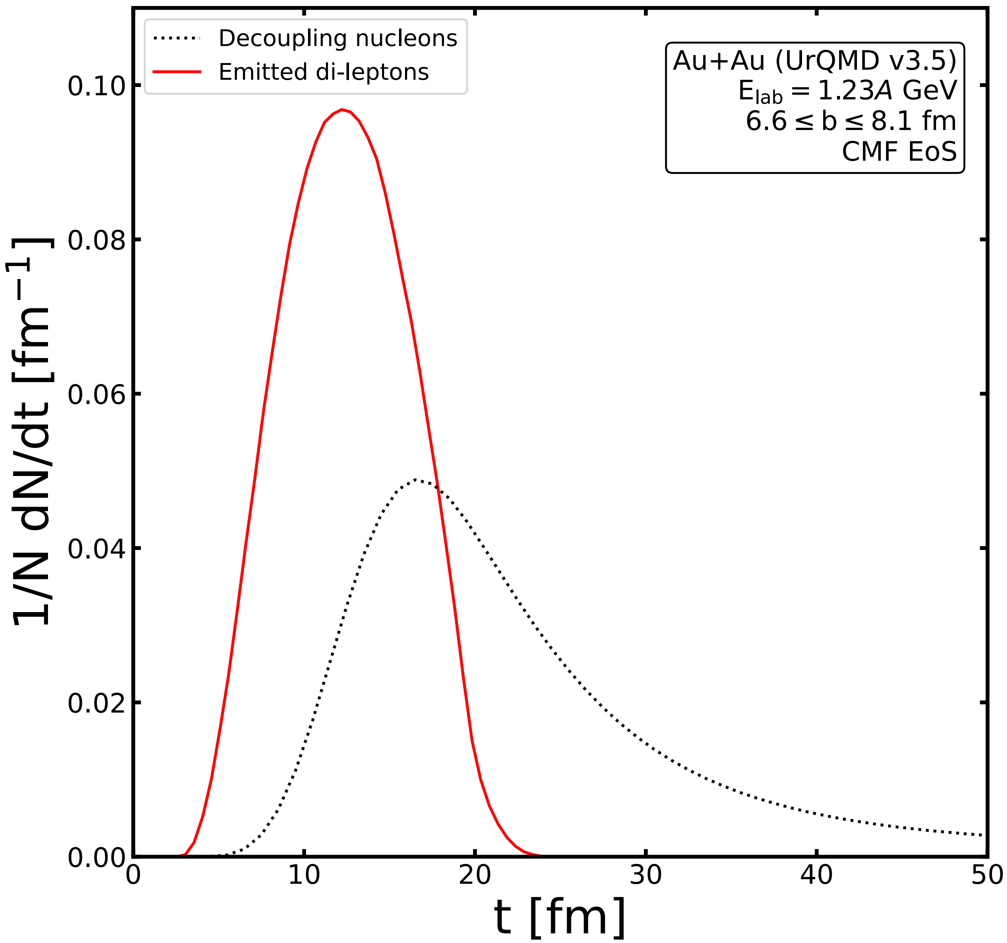
- $$\frac{dN_{\ell^+\ell^-}}{d^4x\,d^4q} = -\frac{\alpha^2}{3\pi^3}\frac{q^2+2m_\ell^2}{(k^2)^2}\sqrt{1-\frac{4m_\ell^2}{k^2}}\,\eta_{\mu\nu}\,\mathrm{Im}\,\Pi^{\mu\nu}_{\mathrm{ret}}(M,\vec{q})\,n_B(u\cdot q)$$
- Spectral and thermal information



- UrQMD + coarse-graining
- Evaluate $\langle T^{\mu\nu}\rangle$ and $\langle j^{\mu}_{B}\rangle$ in each cell and obtain T, μ_B
- Calculate dileptons using Rapp spectral functions
- Shining method (collisional broadening included)



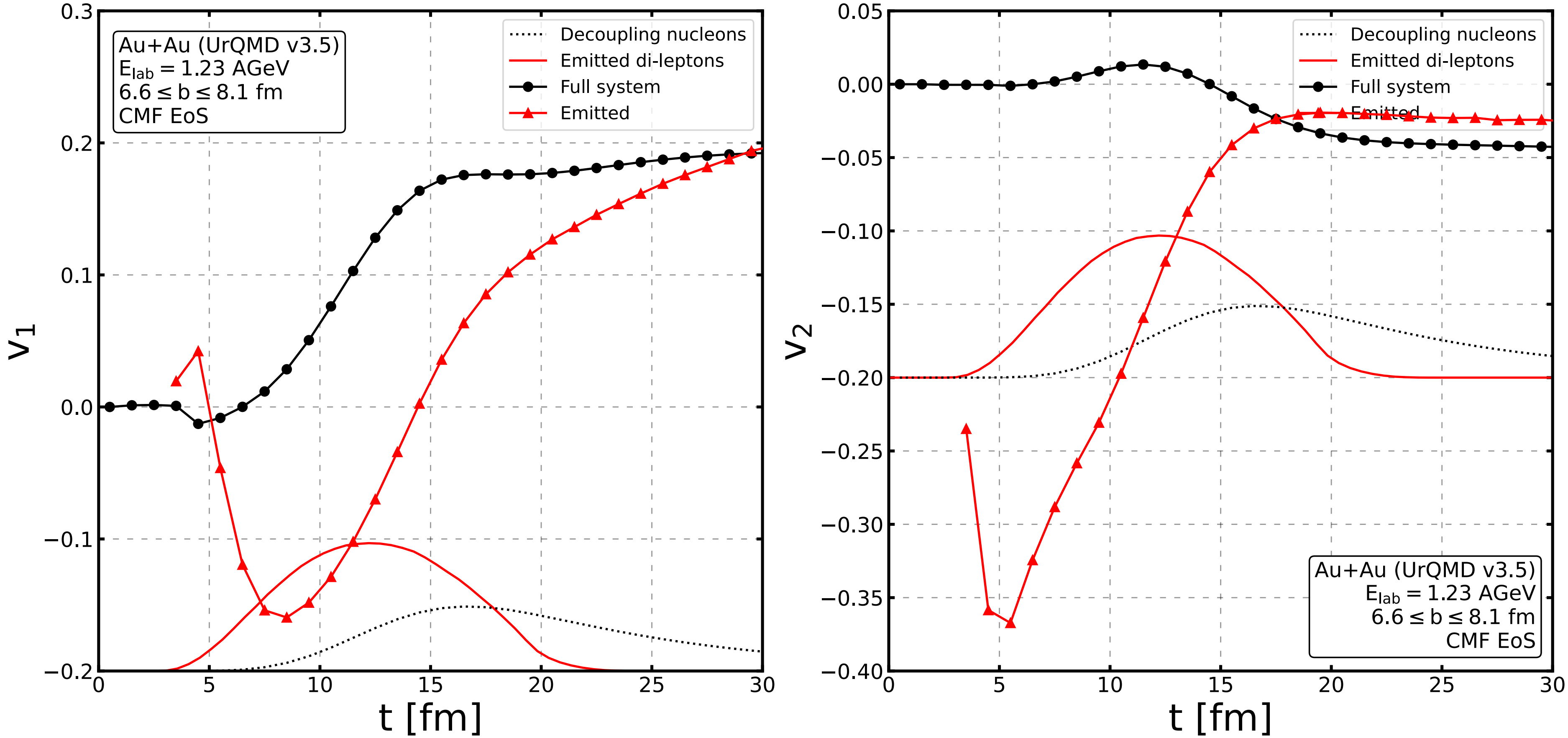
Decoupling time distribution



- Dileptons:
  - Decouple mainly from 5 to 15 fm
  - Narrow distribution
  - Time span when elliptic flow is **positive**
- Nucleons:
  - Decouple from 10 to 35 fm when $v_2<0$

T. Reichert et al., e-Print: 2302.13919

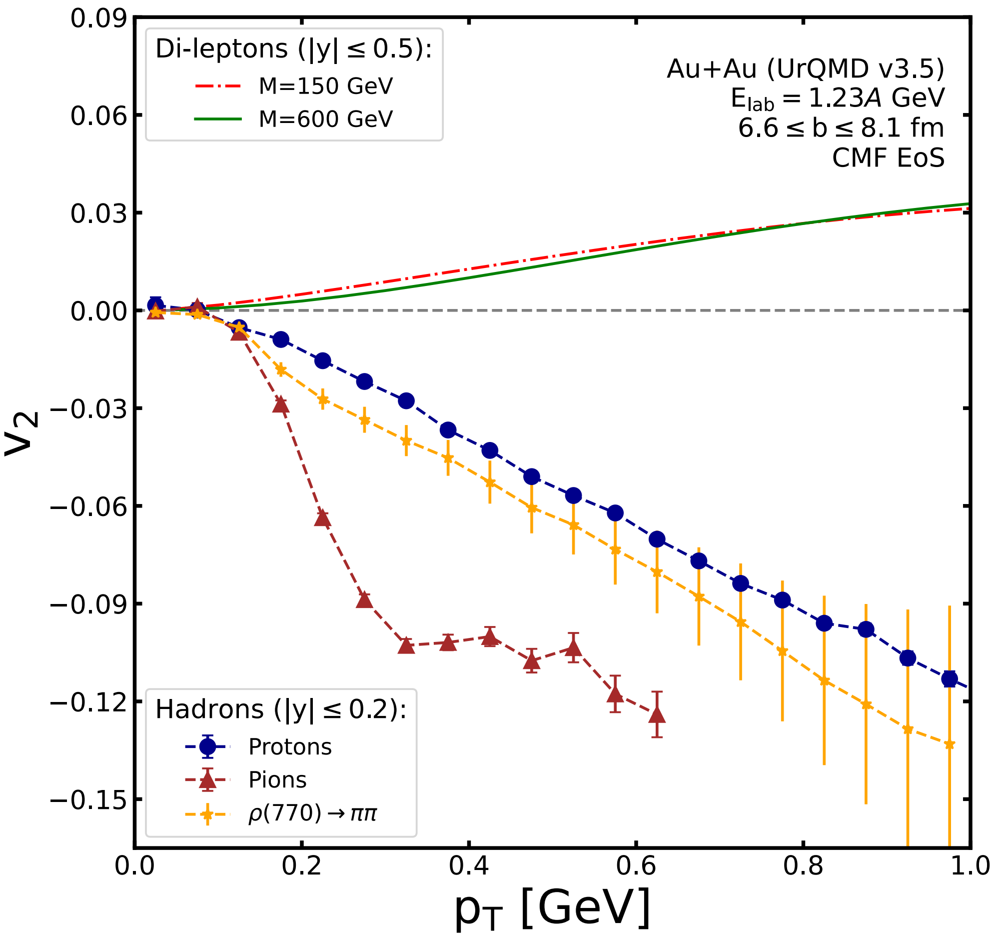
Emission time vs. flow



- Dileptons probe positive v_2 in hot and dense phase
- Hadrons probe negative v_2 at kinetic decoupling

T. Reichert et al., e-Print: 2302.13919

Elliptic flow: p_T dependence



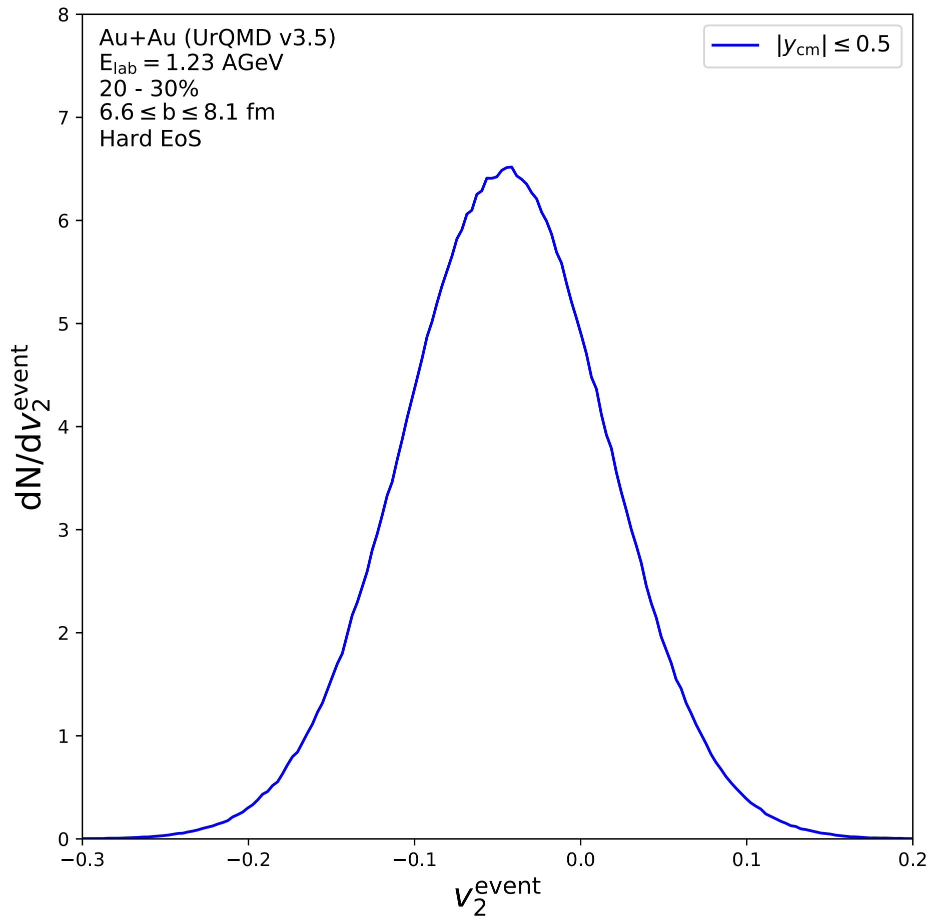
- Hadrons show negative v_2
- Simulation in line with HADES data
- Dileptons have positive v_2
- Dileptons show hydro-mass scaling
- Direct measurement of EoS at highest density!

T. Reichert et al., e-Print: [2302.13919](2302.13919)

Messenger from the hot and dense state

Testing the expansion scenario by Flow correlations

Elliptic flow fluctuation



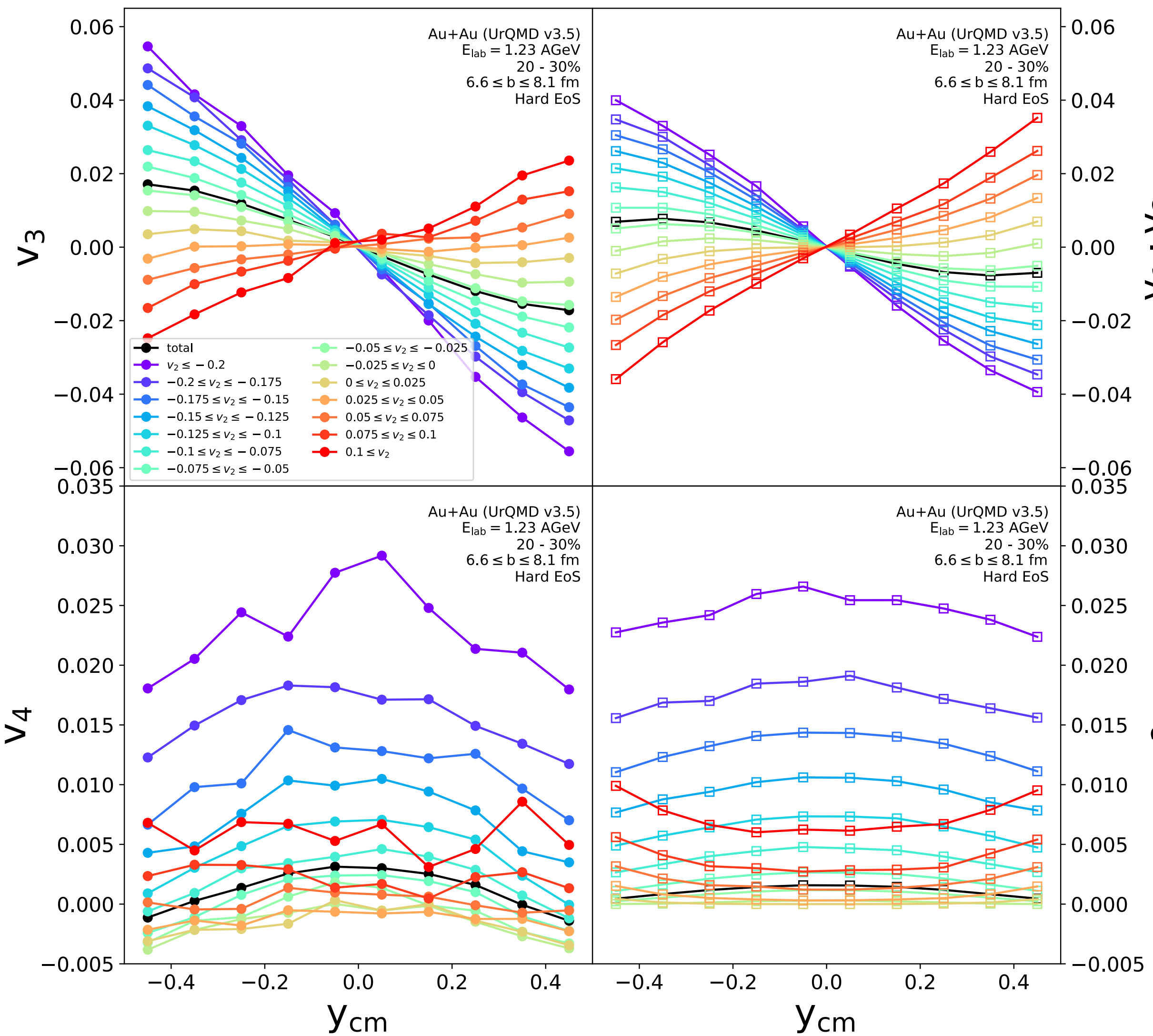
- Final v_2 fluctuates from -0.2 to 0.1
- Average $\langle v_2 \rangle \approx -0.05$ consistent with HADES data
- Where does the fluctuation come from?
- Connection to eccentricity?

→ Investigate how flow develops during time evolution

T. Reichert et al, Eur. Phys. J. C 82, no.6, 510 (2022)

Flow scaling with v_2 trigger

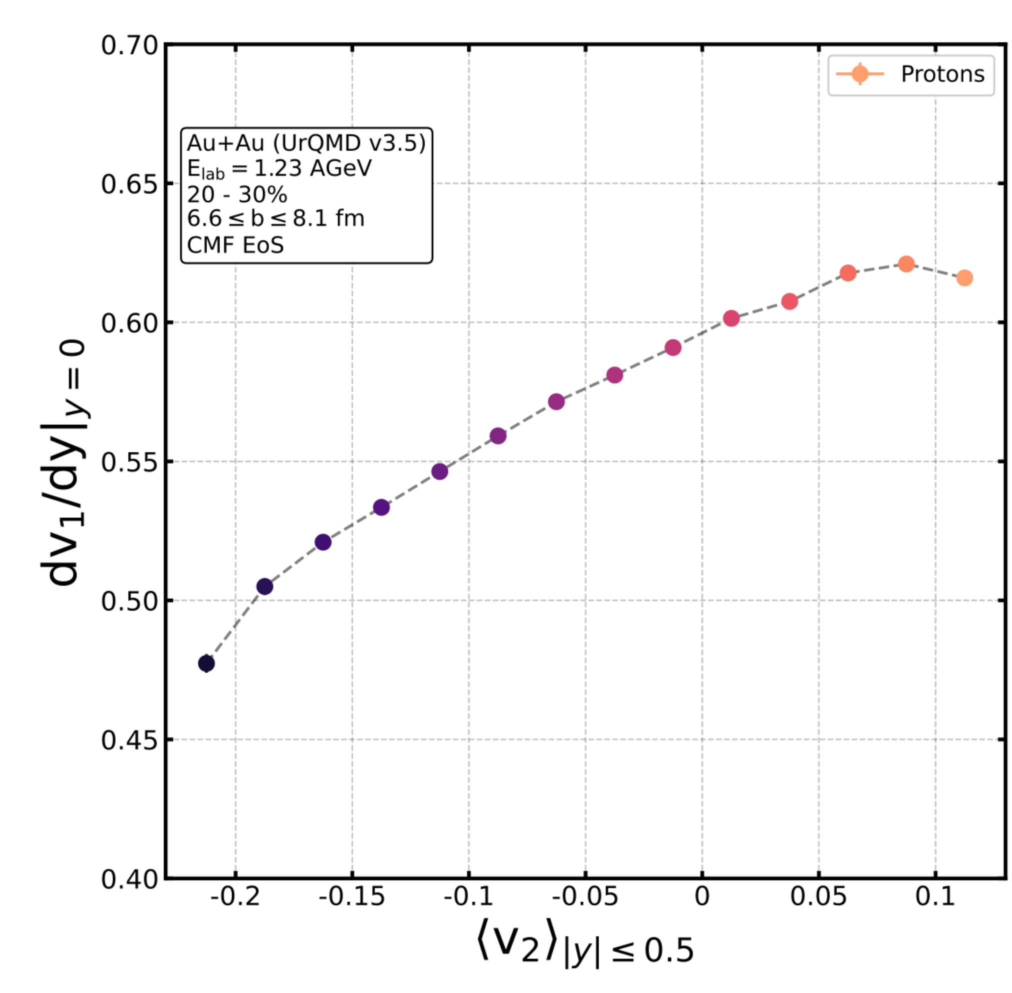
T. Reichert, Eur. Phys. J. C 82, no.6, 510 (2022)



- We understand flow development
- Thus scaling can be explained
- Initial ε_2 fluctuation drives built-up of v_1 and v_2
- Pressure gradient creates correlation:

  $v_3 \propto v_1 \cdot v_2$

v_2 defines v_1 !



- Selecting a specific v2 directly translates into a final state v1
- This demonstrates again that the initial v2 is the source of the momentum transfer to the spectators

T. Reichert et al., e-Print: 2302.13919

Summary

- Transport models are excellent tools to describe and explore the dynamics of matter in heavy ion collisions
- Directed, elliptic, triangular and higher order flows allow to probe the initial and intermediate stages of heavy ion reactions
- The flows are sensitive to different equations of state and allow to pin down the density dependence precisely