Gravitational wave: A probe for towards high density Phase transition

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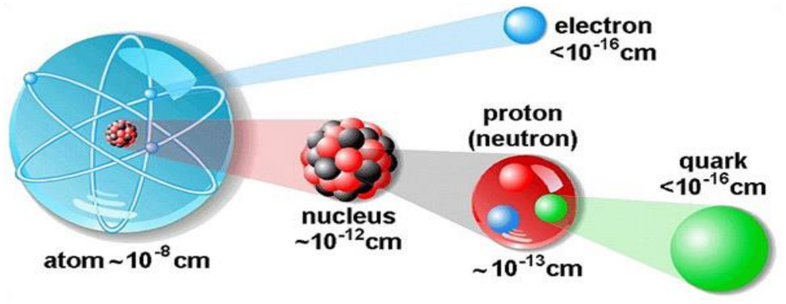
Astrophysical Phase Transition

Importance of Astrophysical PT:

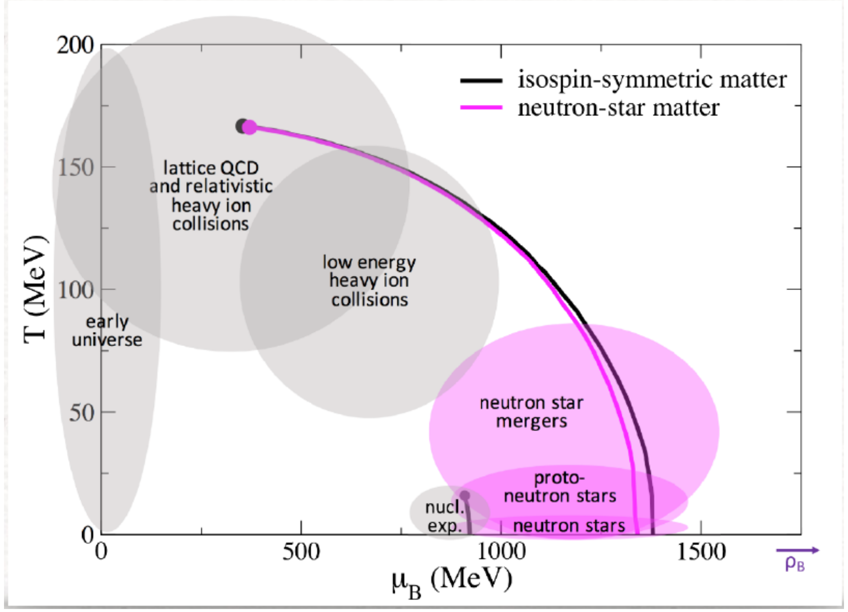
Quark-gluon plasma is confirmed at high temperature (collider experiments).
What about high density?
Still No earth-based experiments.



Natural laboratory are Neutron stars.
Renewed interest after detection of NS-NS mergers.

What about phase transition in neutron stars.

Neutron stars ⟶ **Quark stars**



A two-step process

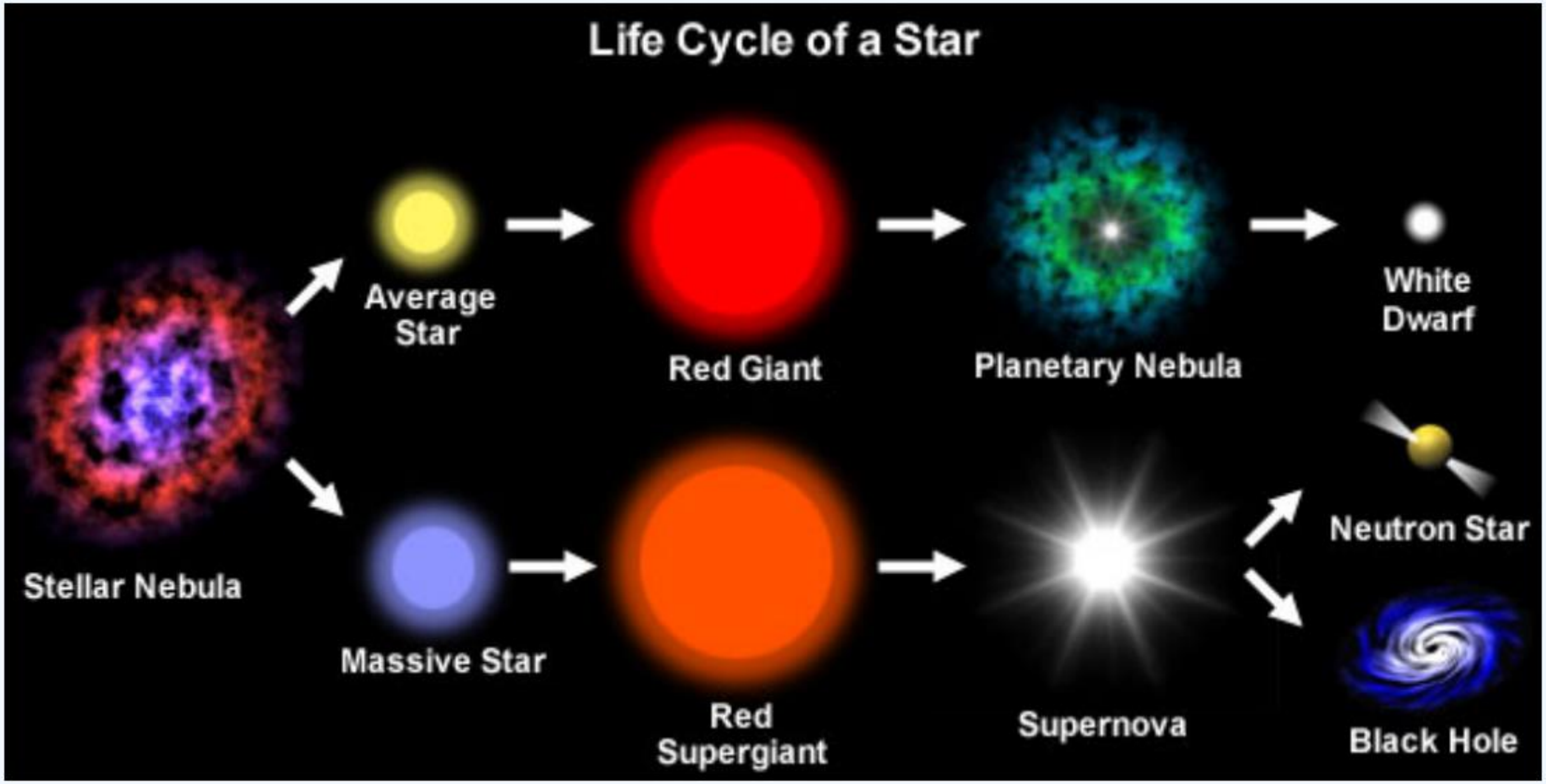
P ⟶ **uud**
N ⟶ **udd**

u + d ⟶ **u + s**

Figs: Dexheimer et al., Universe (2019)

Neutron Stars: Birth

Formation of a neutron star (NS)



Credit: NASA

Neutron Stars: Properties

Properties of NS

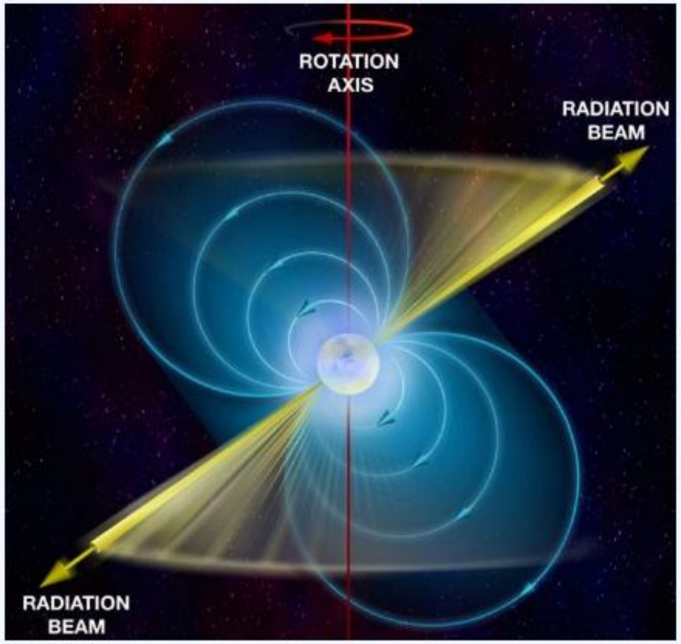
Mass 1.2 - 2.4 solar mass

Radius 10 – 15 km

Period ms – sec

Density at core 10^{14} – 10^{15} gm/cc

Magnetic field 10^{15} G (max)



Credits: earthsky.org

Solving Einstein Equation

Ideal Fluid

$$\underbrace{G_{\mu\nu}}_{\text{Spacetime}} = \frac{8\pi G}{c^2} \underbrace{T_{\mu\nu}}_{\text{Matter}}$$

$$\frac{dP}{dr} = -G\frac{M(r)\rho(r)}{r^2}\frac{\left[1+\frac{P(r)}{\rho(r)c^2}\right]\left[1+\frac{4\pi r^3 P(r)}{M(r)c^2}\right]}{\left[1-\frac{2GM(r)}{rc^2}\right]}, \qquad M(r) = 4\pi\int_0^r \rho(r)r^2 dr$$

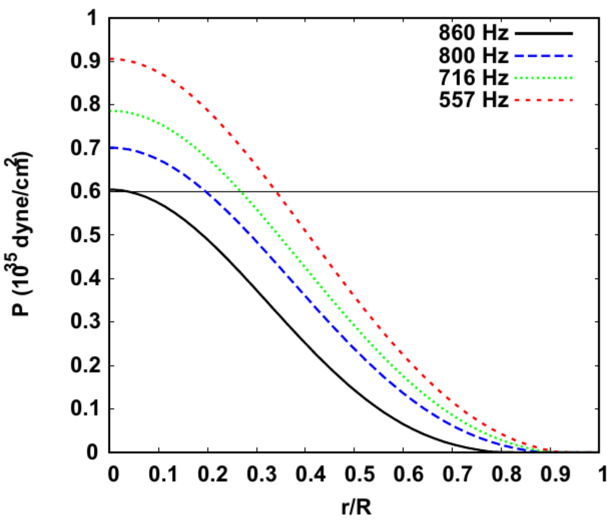
Phase transition in isolated stars

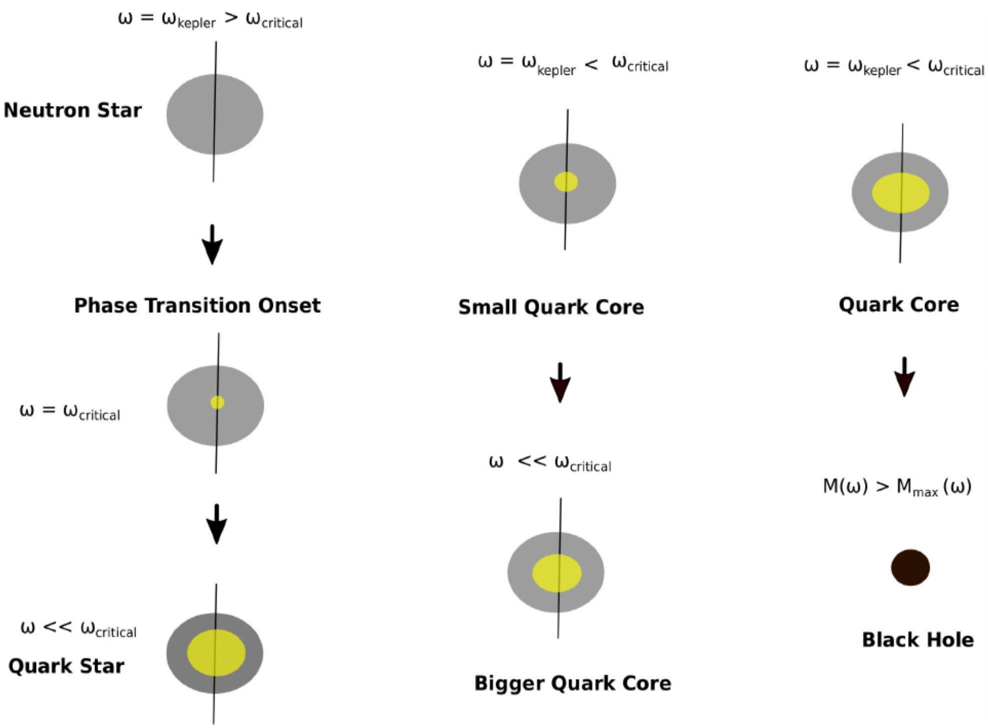
Seeding of the quark core:

The seeding happens as the star slows down.

Once the critical density is crossed the quark seed forms.

The seed grows as the star slows down further.





4 scenarios:

1. Low-mass star never attains quark core
2. Core develops during lifetime
3. Core grows during lifetime
4. Core grows and ultimately star collapse to BH

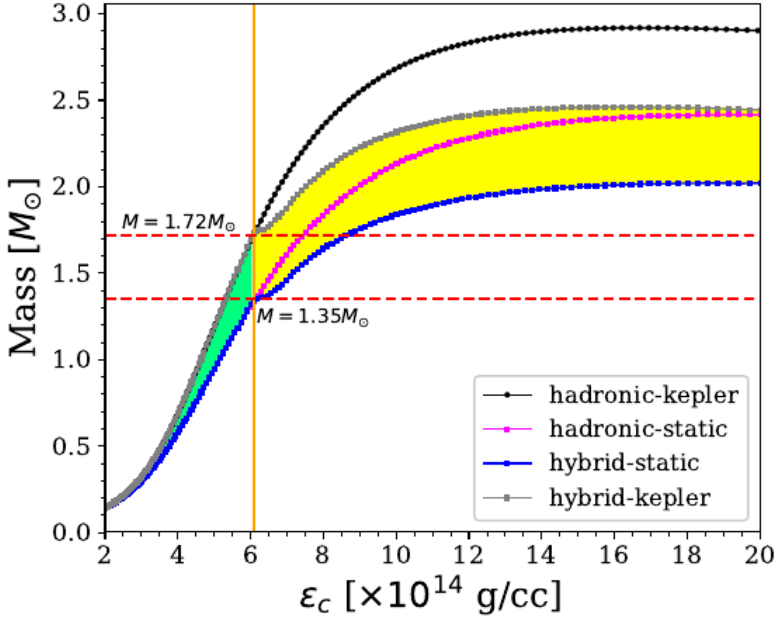
Prasad & Mallick, MNRAS 516, 1127 (2022)

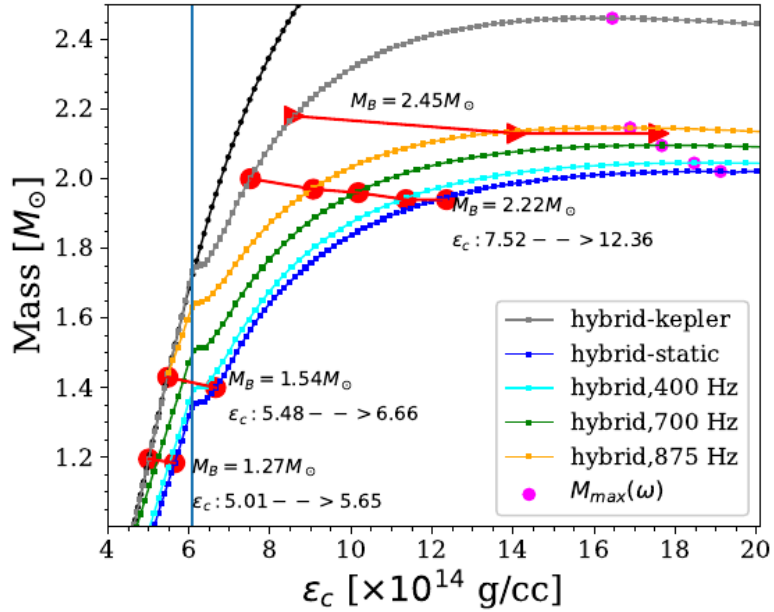
Phase transition in isolated stars

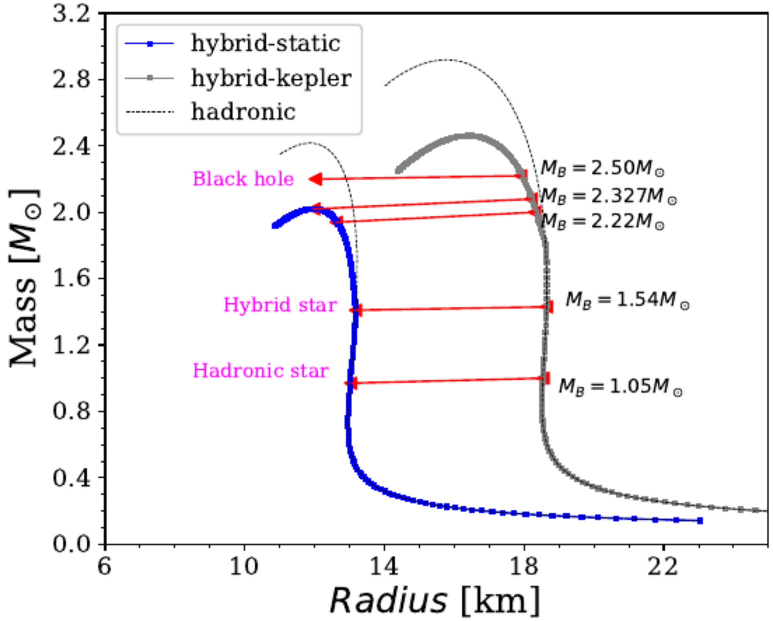
A handful of stars:

The range of stars which attains a quark core depends on the EoS.

Changing the EoS the quantitative nature changes but qualitatively they remain same.





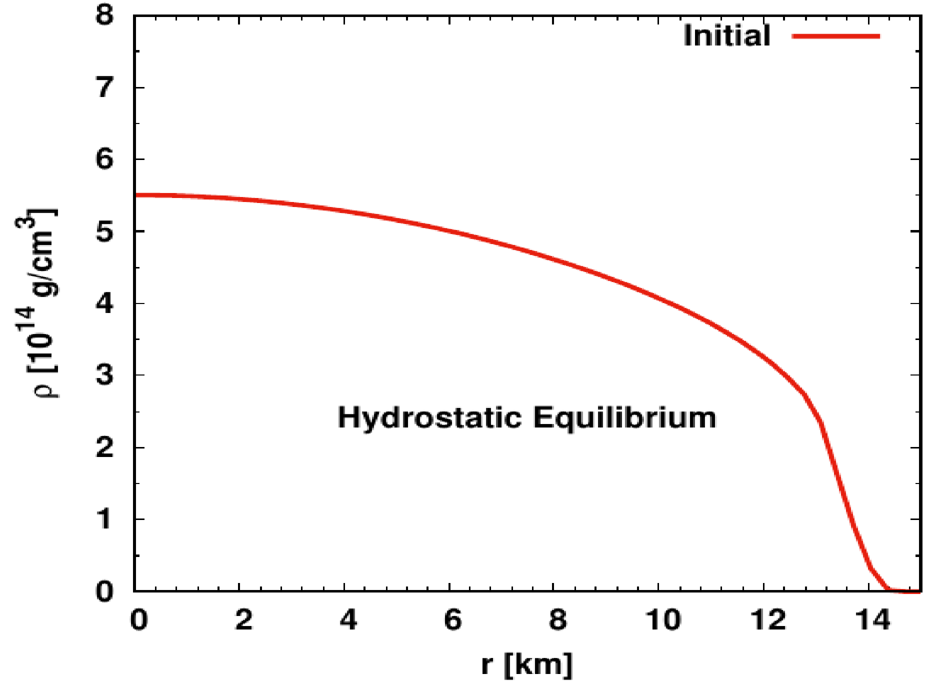


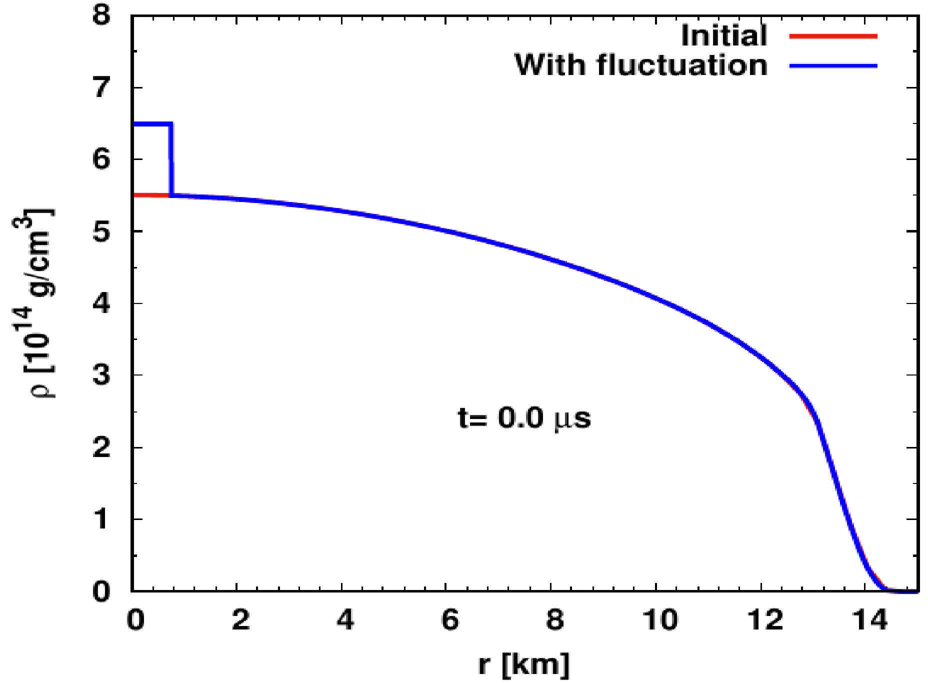
Phase transition in isolated stars

Initial configuration:

A density fluctuation at the centre of the star initiates a shock discontinuity.

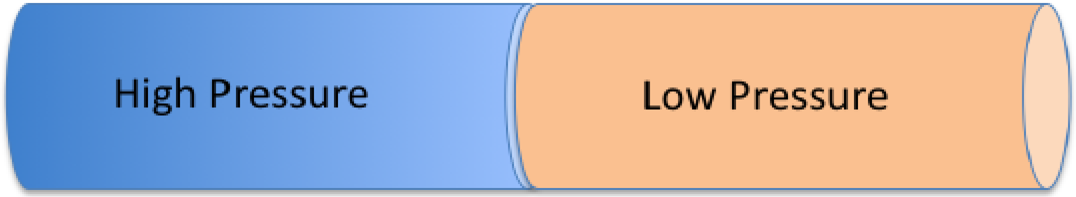
As the shock propagates out deconfinement from HM to QM happens.





Phase transition in isolated stars

Deconfinement transition:



- **Conservation of mass**

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho \vec{u}) = 0$$

- **Conservation of momentum**

$$\frac{\partial (\rho \vec{u})}{\partial t} + \vec{\nabla}.(\rho \vec{u})\vec{u} + P = \rho \vec{a}$$

- **Conservation of energy**

$$\frac{\partial (\rho E)}{\partial t} + \vec{\nabla}.(\rho E \vec{u} + P \vec{u}) = \rho \vec{u}.\vec{a}$$

Riemann-Problem

$$U_t + [F(U)]_x = 0$$

$$U = \begin{pmatrix} \rho \\ \rho u \\ \rho E \end{pmatrix} \qquad F(U) = \begin{pmatrix} \rho u \\ \rho u u + p \\ \rho u E + u p \end{pmatrix}$$

Phase transition in isolated stars

Deconfinement transition: GR1D code

The evolution equations can be obtained using - $\nabla_\mu J^\mu = 0, \qquad \nabla_\mu T^{\mu\nu} = 0$

In the coordinate frame where $u^\mu = (W/\alpha, Wv/X, 0, 0)$, $W = \sqrt{\frac{1}{1-v^2}}$ is the Lorentz factor and v is the physical radial velocity. **(O'Connor and D. Ott, 2010)**

- **Evolution equation of rest mass density**

$$\partial_t(D) + \frac{1}{r^2}\partial_r\left(\frac{\alpha r^2}{X}Dv\right) = 0$$

where D is the conserved variable. $D = X\rho W$ with $X(r,t) = \left(1 - \frac{2m(r,t)}{r}\right)^{-1/2}$.

- **Evolution equation of momentum**

$$\partial_t(S^r) + \frac{1}{r^2}\partial_r\left[\frac{\alpha r^2}{X}(S^r v + P)\right] = \alpha X\left[(S^r v - \tau - D)\left(8\pi r P + \frac{m}{r^2}\right) + \frac{Pm}{r^2} + \frac{2P}{X^2 r}\right]$$

where $S^r = \rho h W^2 v$ and conserved variable $\tau = \rho h W^2 - P - D$.

- **Evolution equation of energy**

$$\partial_t(\tau) + \frac{1}{r^2}\partial_r\left[\frac{\alpha r^2}{X}(S^r - vD)\right] = 0$$

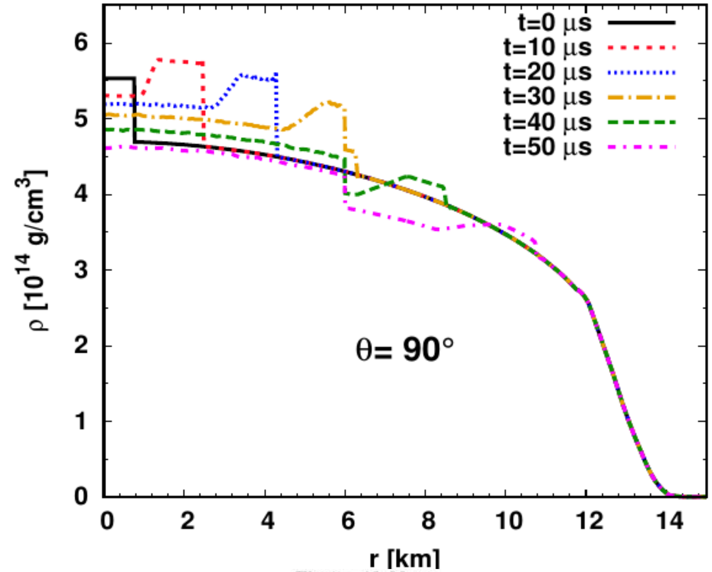
The conserved variables are function of primitive variables ρ, e , v and P.

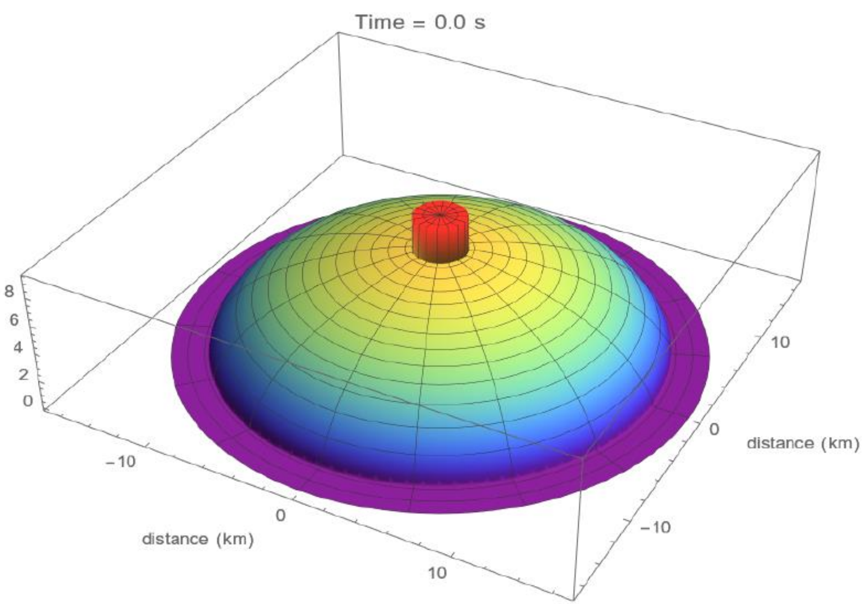
Phase transition in isolated stars

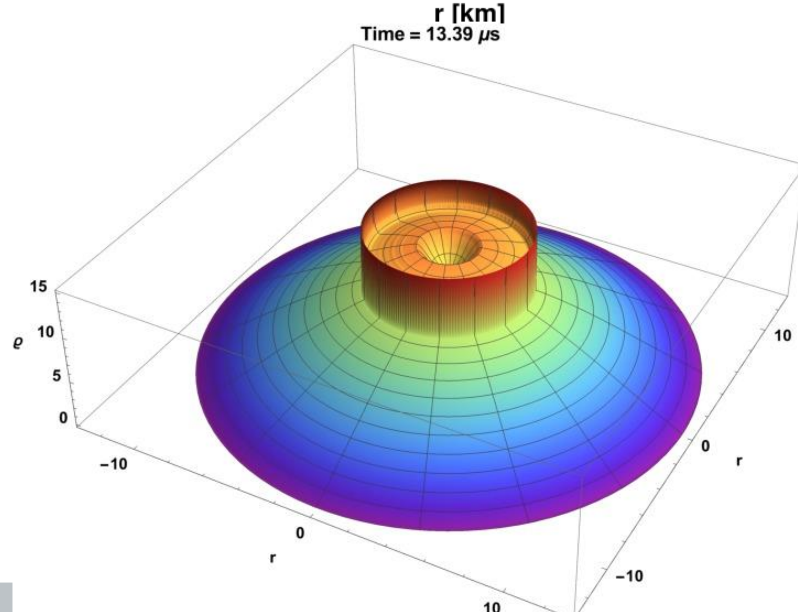
Evolution of the shock leading to deconfinement

The shock velocity is some fraction of velocity of light

The deconfinement takes about 50 microseconds







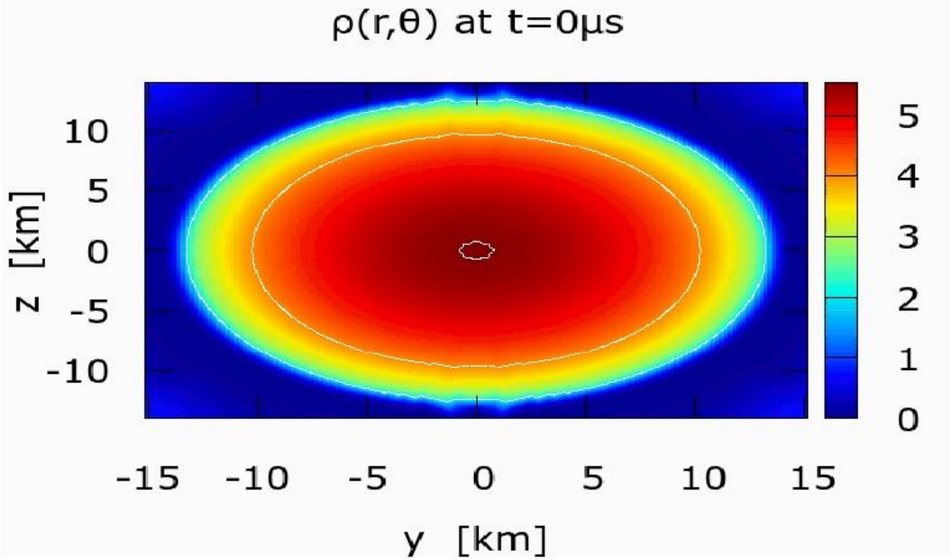
Prasad & Mallick, ApJ, 859, 57 (2018)

Phase transition in isolated stars

Gravitational wave generation

The change in the density profile brings about a change in the mass quadrupole moment

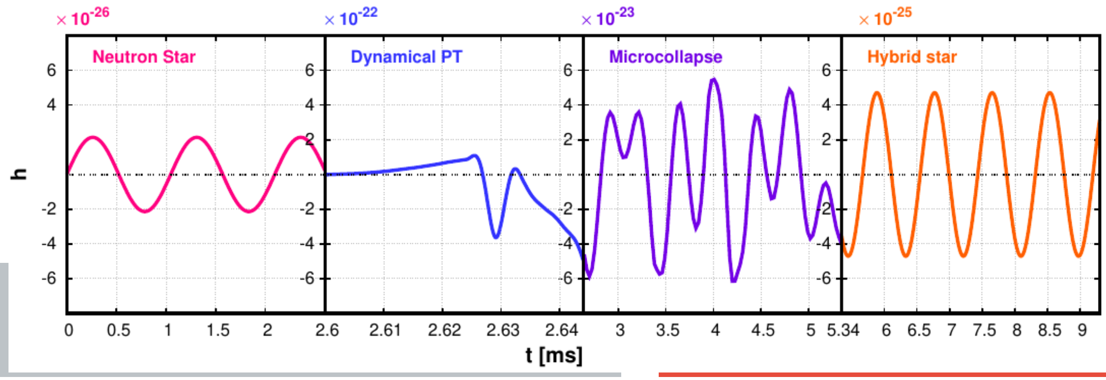
Results in the emission of GW



$$h^{TT}_{\theta\theta} = \frac{1}{8}\sqrt{\frac{15}{\pi}}\sin^2\theta\frac{A^{E2}_{20}}{r}, \qquad A^{E2}_{20} = \frac{d^2}{dt^2}\left(k\int\rho\left(\frac{3}{2}z^2-\frac{1}{2}\right)r^4 drdz\right)$$

where θ is the angle between the symmetry axis and the line of sight of the observer.

$z = \cos\theta$ and $k = \frac{16\pi^{3/2}}{\sqrt{15}}$.



Prasad & Mallick, ApJ, 893, 151 (2020)

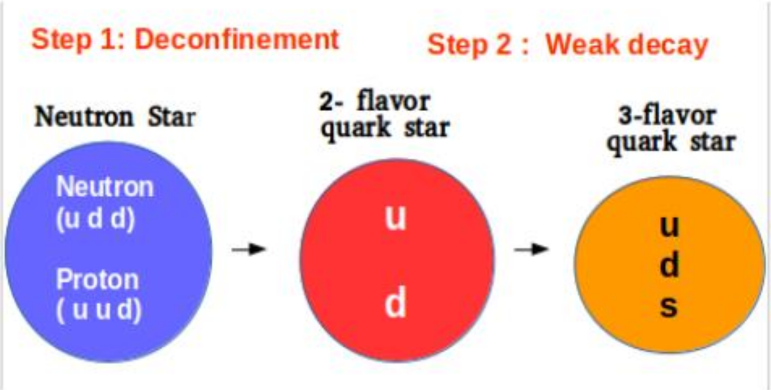
Phase transition in isolated stars

Weak decay and Stability

$$d \rightarrow u + e^- + \nu_{e^-},$$

$$s \rightarrow u + e^- + \nu_{e^-},$$

$$d + u \quad \rightarrow \quad s + u.$$



$$\boxed{\frac{d^2a}{dr^2} + \left[\frac{1}{\sqrt{|g|}}\frac{d}{dr}(\sqrt{|g|}) + \frac{1}{g^{rr}}\frac{d}{dr}(g^{rr}) - \gamma_g \frac{1}{(g^{rr})^2}\frac{v}{D}\right]\frac{da}{dr} - \frac{R(a)}{Dg^{rr}} = 0}$$

$$D \propto \left(\frac{\mu_b}{T}\right)^2 \text{ and } R(a) = \frac{a^3}{\tau}$$

$$a(r) = \frac{n_k^{2f}(r) - n_k^{3f}(r)}{2n_b(r)}$$

$$n_k = \frac{1}{2}(n_d - n_s)$$

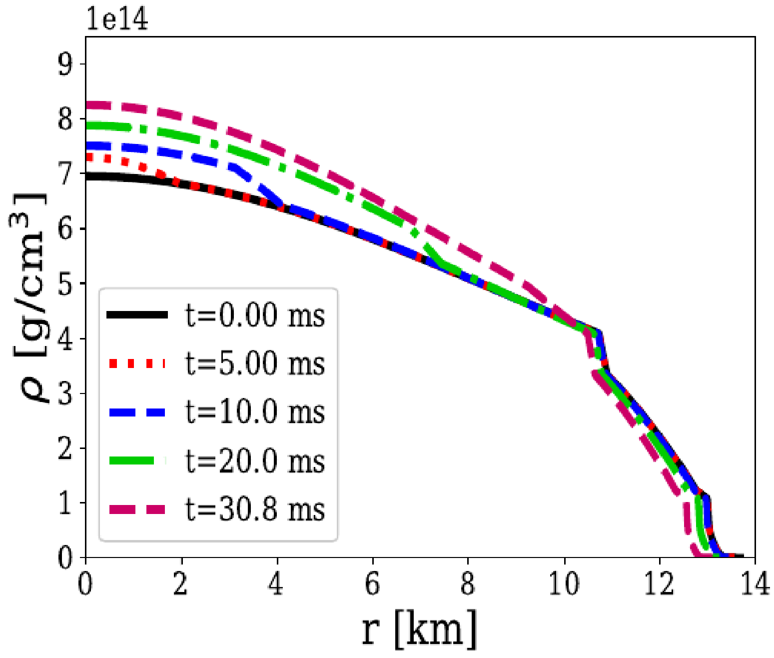
$$a'' - Xa' - R'(a) = 0$$

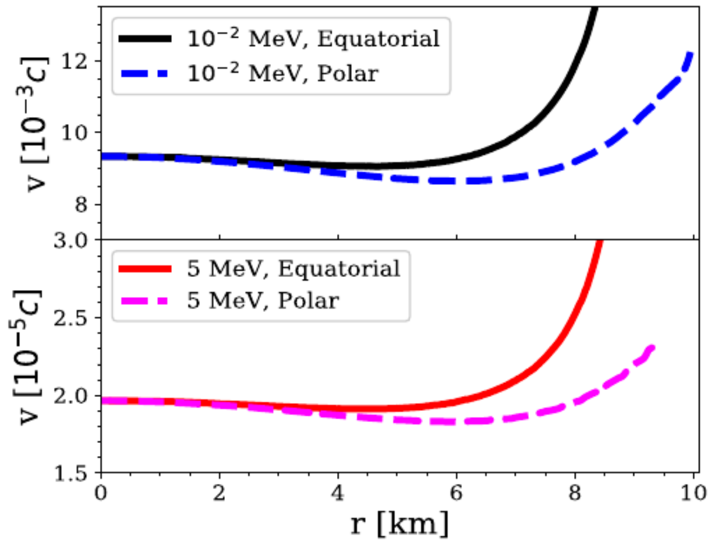
Mallick, Singh & Prasad, MNRAS, 507, 1318 (2021)

Phase transition in isolated stars

Weak decay and Stability

$$v = \sqrt{\frac{D}{\tau Y_s}\frac{1}{\gamma_g g_{rr}^{3/2}} + \frac{D}{\gamma_g g_{rr}^2}\left[\frac{1}{\sqrt{|g|}}\frac{\mathrm{d}}{\mathrm{d}r}\left(\sqrt{|g|}\right) + g_{rr}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{g_{rr}}\right)\right]}$$



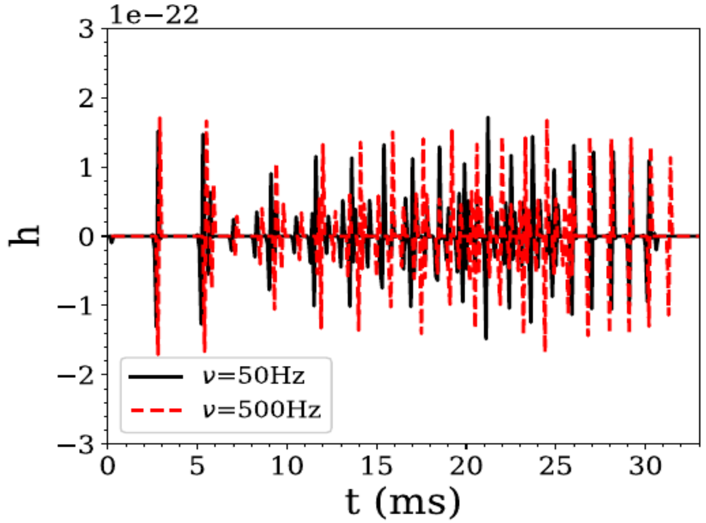


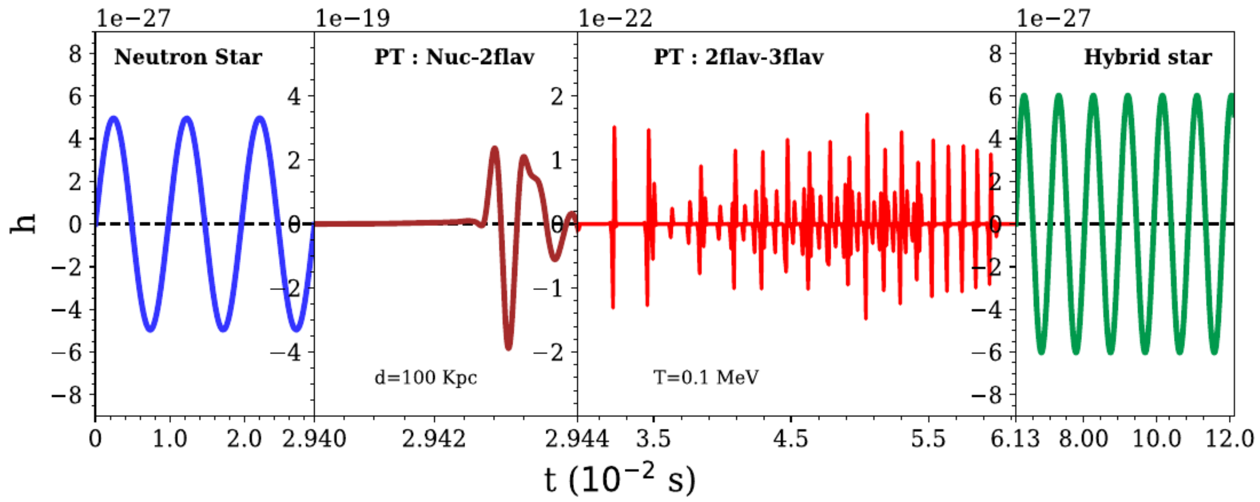
Phase transition in isolated stars

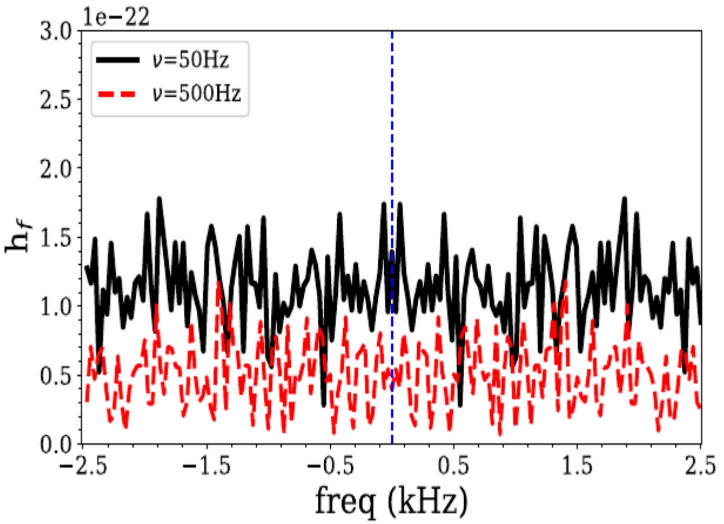
Weak decay and GW

For a NS: Mass = 2.0 solar mass
Frequency = 50 Hz
Distance = 100 Kpc
Temp = 0.1 MeV

Bursts type signal



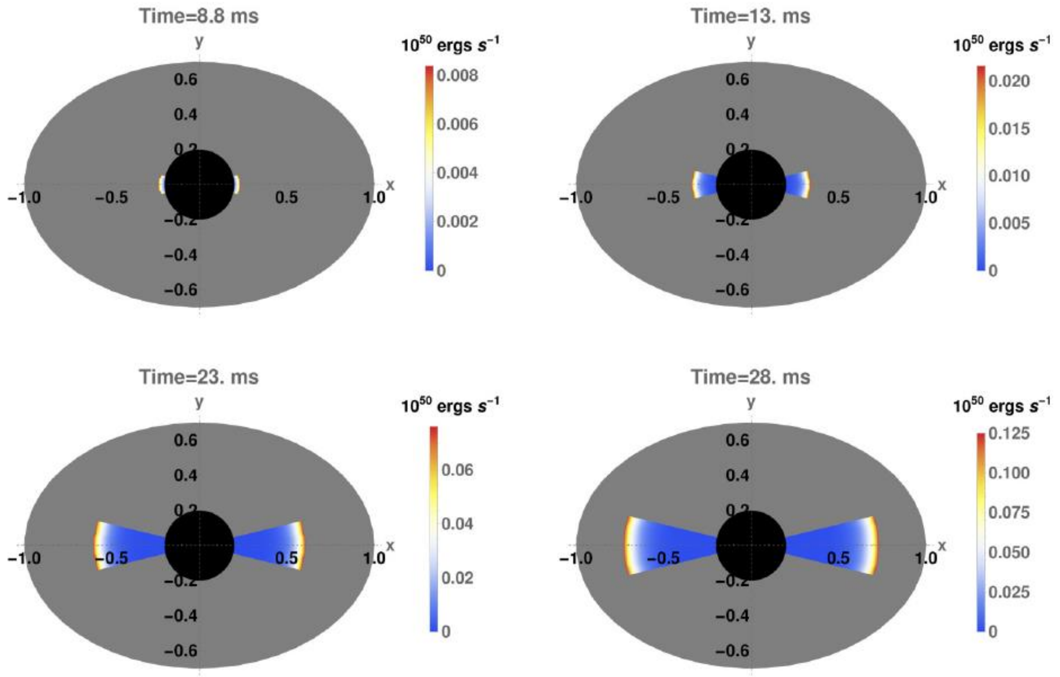


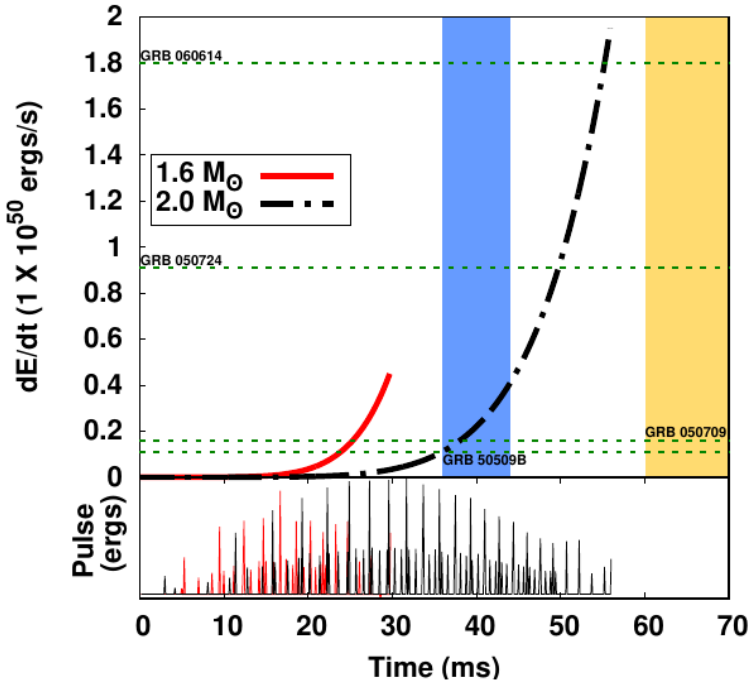


Phase transition in isolated stars

Multitude signal from Phase transition

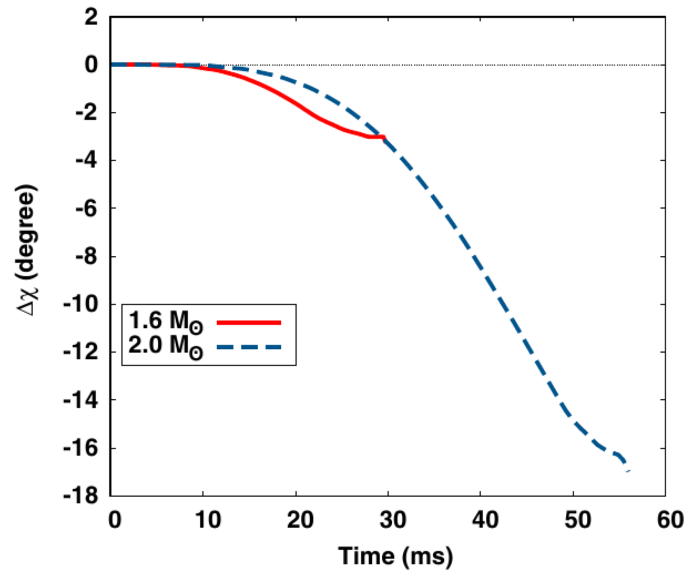
1. Neutrino generation: energy deposided at the surface $10^{49} – 10^{50}$
   Energy budget same as **Gamma Ray Bursts**





Kuzur et al., PRC, 105, 065807 (2022)

Phase transition in isolated stars

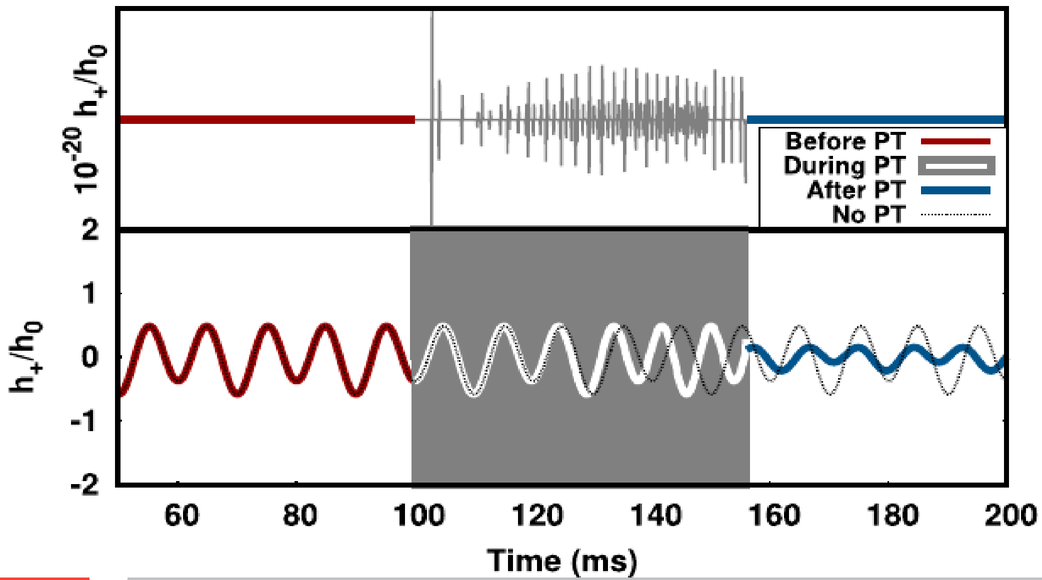


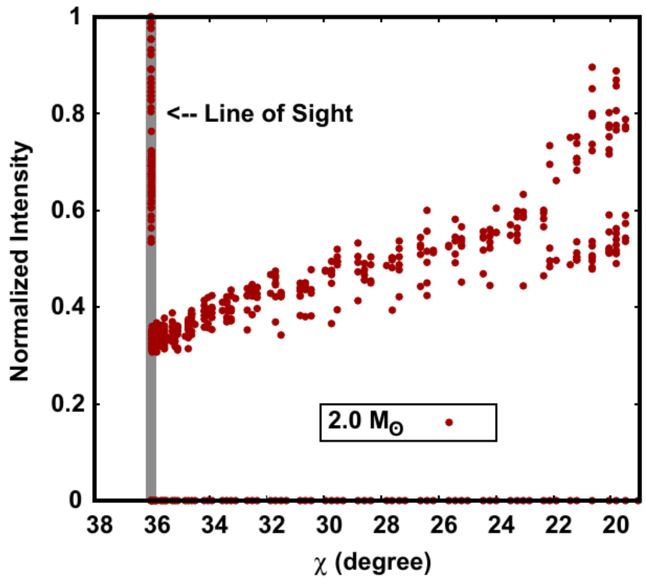
Multitude signal from Phase transition

2. Tilt angle evolution: Can evolve upto 20 degrees
   A sudden evolution (not slow)

Pulsar can suddently go out of line of sight
Some may also suddently emerge

3. Change in the continious GW signal





Phase transition in isolated stars

Summary:

1. Slow down of a neutron star can induce a quark core seed at the centre of the star

2. A shock discontinuity develops at the core and propagates outwards.

3. A two-step conversion process: Deconfinement of nucleons to 2-f quark. Weak decay of 2-f quark matter to 3-f quark matter.

4. Deconfinement transition happens in microseconds and have bursts type strong GW Signal. Frequency of the signal on the higher side.

5. Weak decay transition takes few 100 of milliseconds, also bursts type GW signal. Frequency on the signal in the present detector capacity range.

6. PT can also have other type of signals: Neutrino generation, tilt angle evolution.

Phase transition in Binary NS

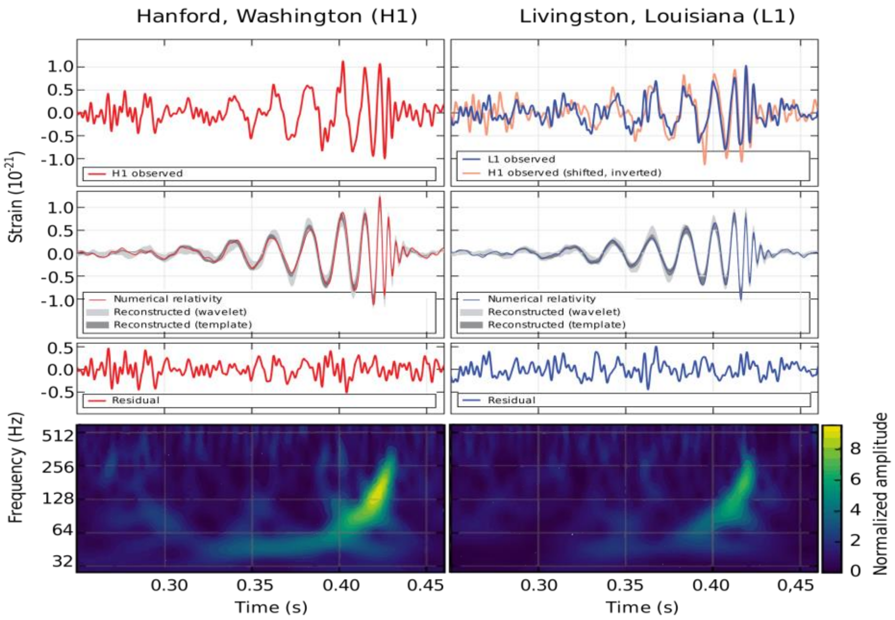
Problem of Astrophysical Phase transition:

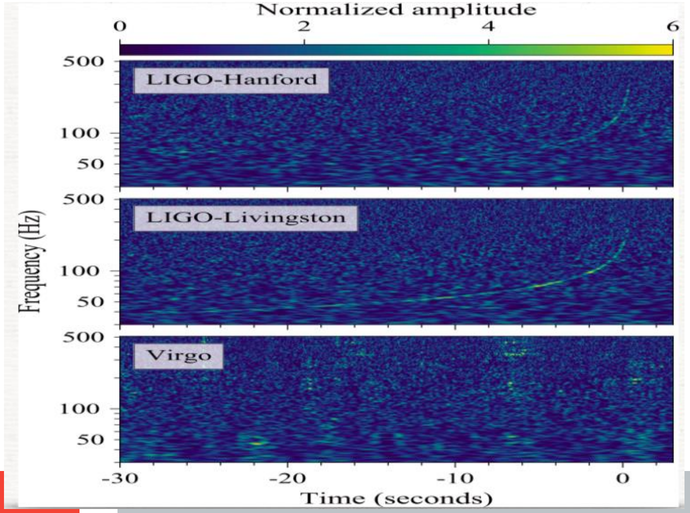
One of the most important discovery of recent times

BH-BH merger GW150914

BH + BH $\xrightarrow{GW}$ *BH*

(Abbott et. al., PRL, 2016)





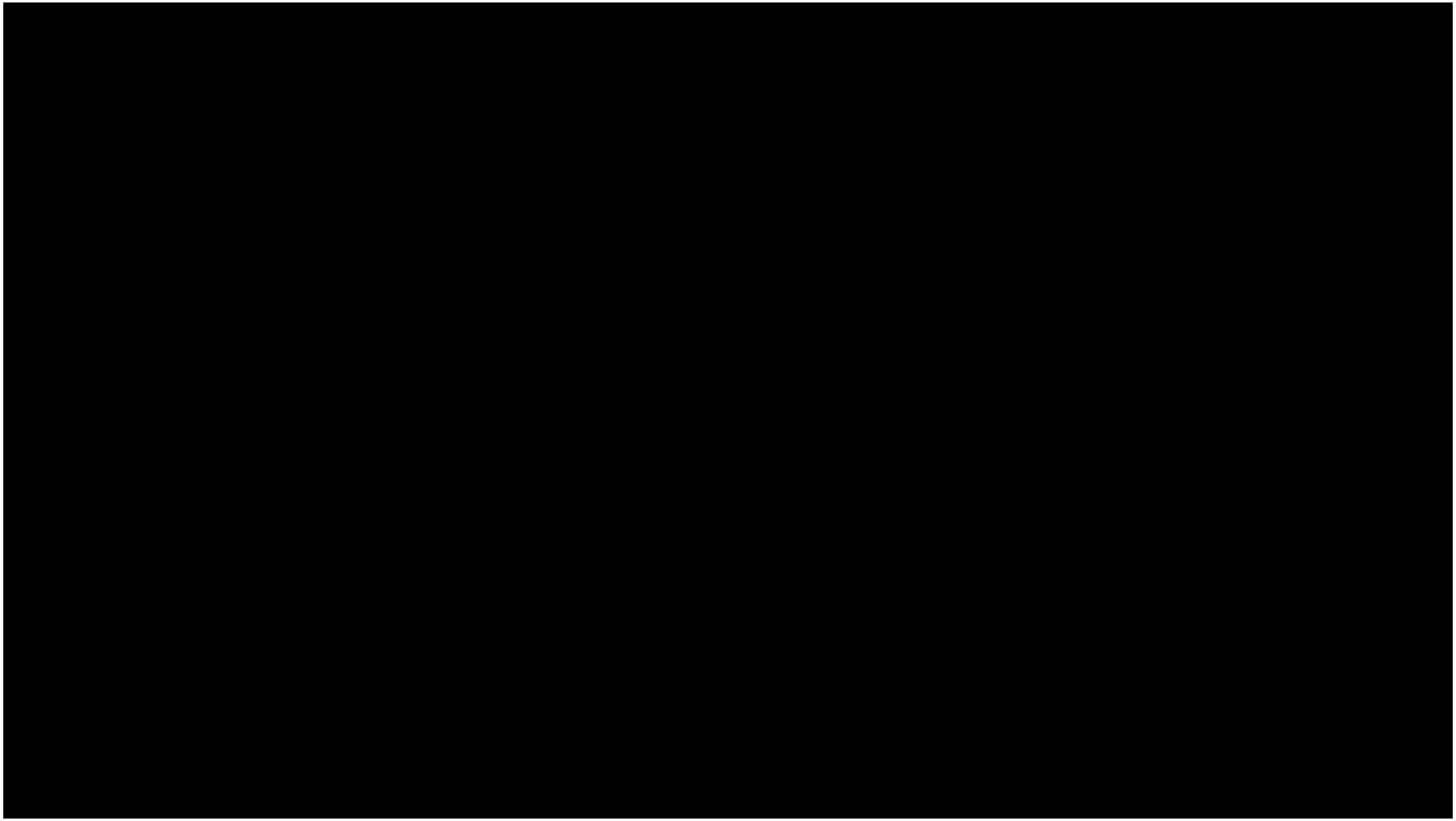
Shortly after NS-NS Binary merger was detected

GW170817

NS + NS $\xrightarrow{GW}$ ***HMNS*** $\xrightarrow{GW}$ ***BH***

Figs: Abbott et al., PRL 119 (2017)

Phase transition in Binary NS



Credit: NASA Visualization Studio

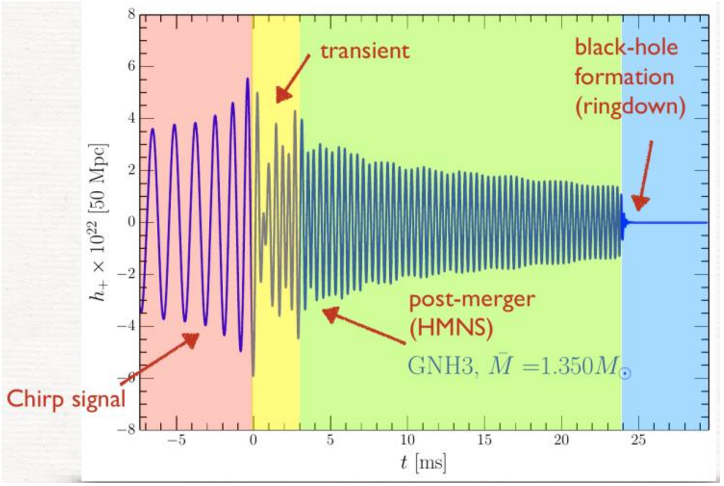
Phase transition in Binary NS

Problem of Astrophysical Phase transition:

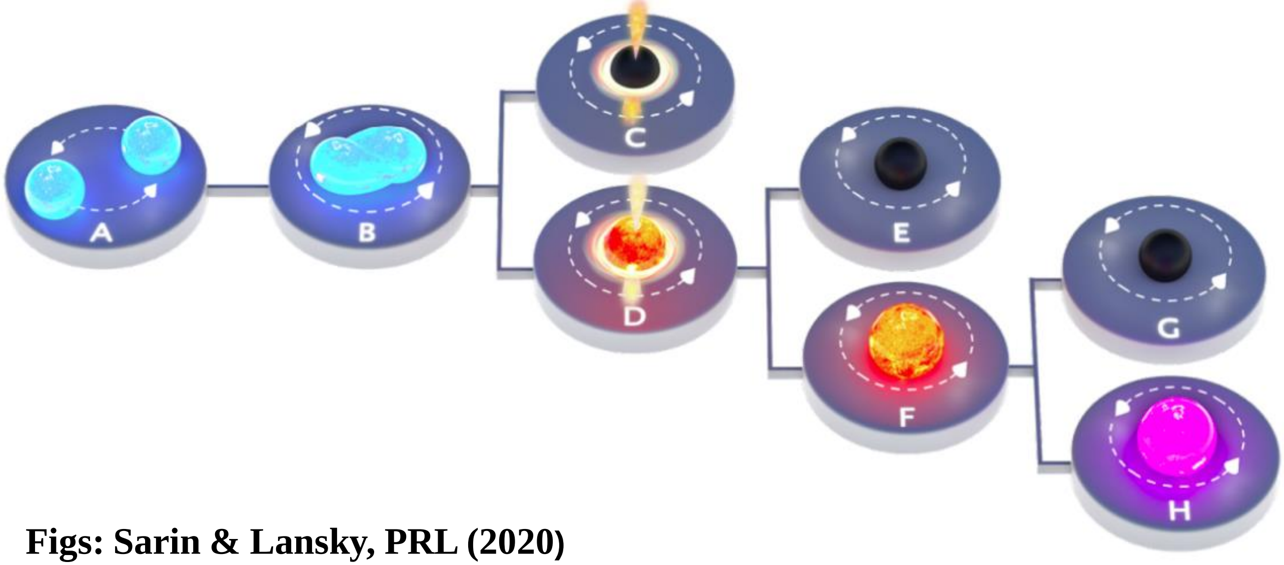
Detection of the inspiral part, before the merger

Not only **GW** but also **sGRB** and other **electromagnetic signal**
Truely **Multi-messenger** signal

Post-merger signal not detected, expected to have more rich physics

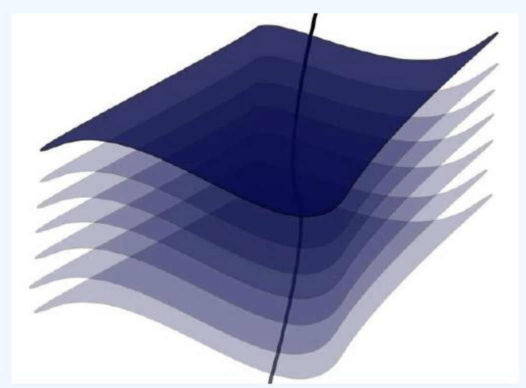


Takami et al., PRL (2014)



Figs: Sarin & Lansky, PRL (2020)

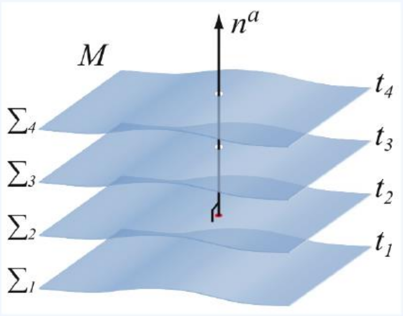
Phase transition in Binary NS



Einstein Equation and Numerical relativity

$$\underbrace{G_{\mu\nu}}_{\text{Spacetime}} = \frac{8\pi G}{c^2} \underbrace{T_{\mu\nu}}_{\text{Matter}}$$

- Foliations of spacetime:



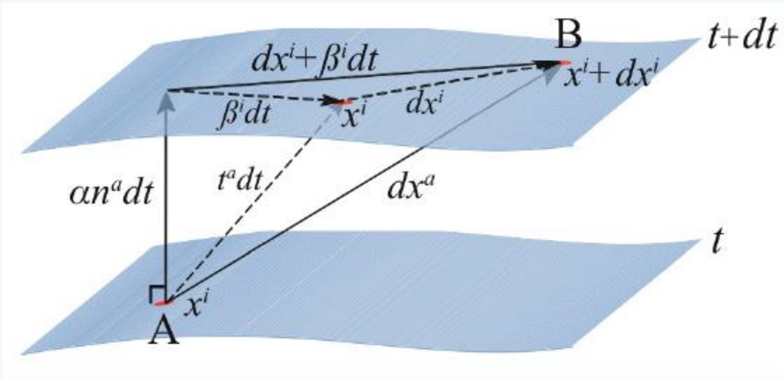
Numerical Relativity: 3+1 Formalism

Foliate 4-d space-time ➡ 3-d spacelike hypersurface

$$\gamma_{ab} = g_{ab} + n_a n_b \implies \textit{spatial metric}$$
$$n^a \implies \textit{normal vector}$$
$$\beta^i \implies \textit{shift vector}$$
$$\alpha \implies \textit{lapse function}$$

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}\left(dx^i + \beta^i dt\right)\left(dx^j + \beta^j dt\right)$$

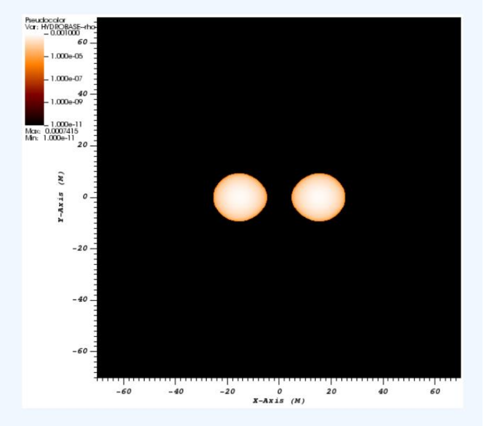
$$g_{ab} = \begin{pmatrix} -\alpha^2 + \beta_l \beta^l & \beta_i \\ \beta_j & \gamma_{ij} \end{pmatrix} = \gamma_{ab} - n_a n_b$$

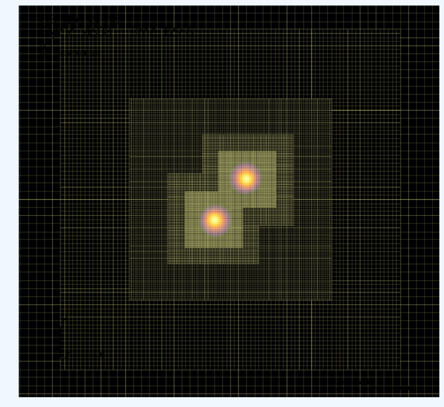


Phase transition in Binary NS

Initial Setup

LORENE code: Binary star code
Solves the constraints equation on a hypersurface





Evolution

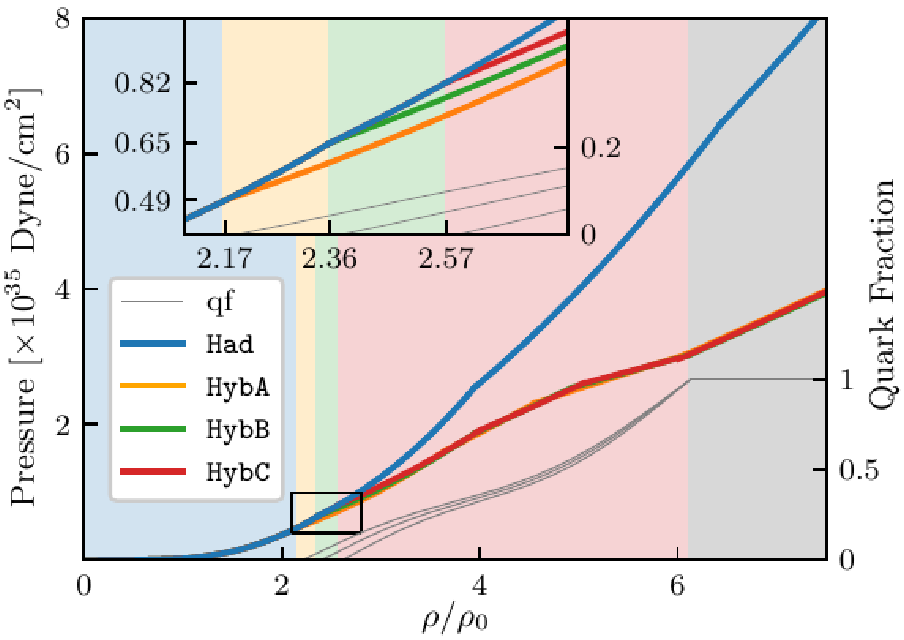
Einstein Toolkit: solves the evolution equations
GW extraction

Equation of State

Hadronic: DD-ME2 Quark: MIT bag model

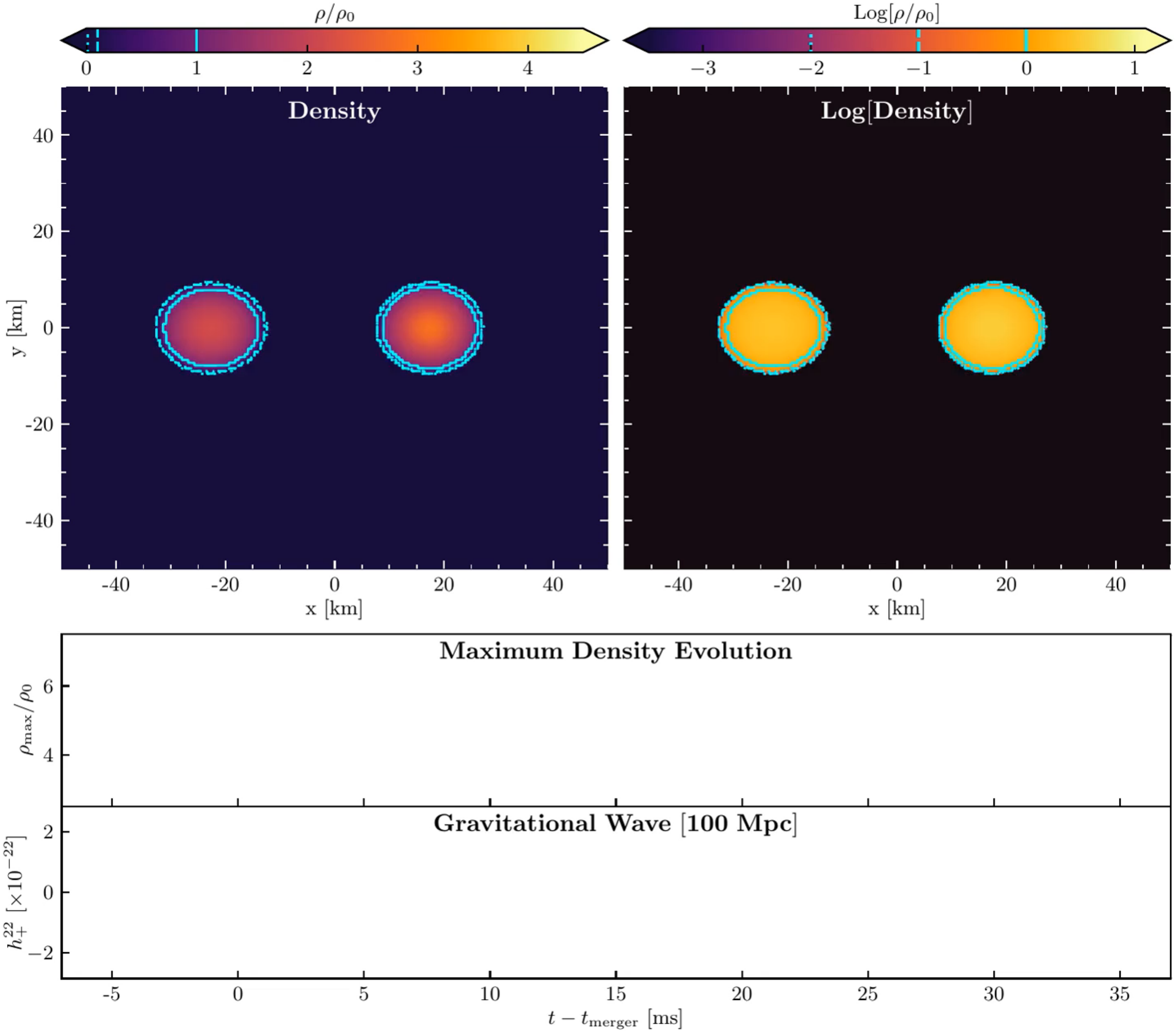
Mixed phase, Polytropic Fit

3-different onset point
(point where mixed phase starts)



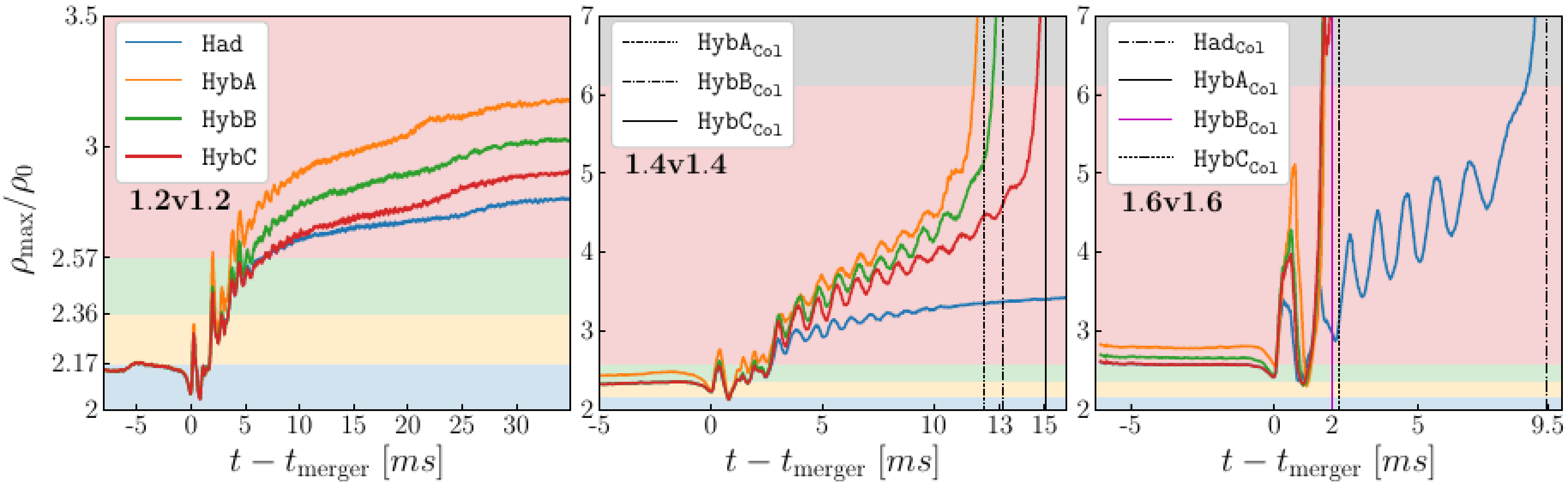
Shamim et al, ArXiv:2208. (2022)

Phase transition in Binary NS



Phase transition in Binary NS

Equal Mass binaries



Small binary stars made entirely of hadrons: after merging gives stable configuration

Intermediate mass binaries: hadronic star stable
hybrid HMHS ultimately collapses

Heavy mass binaries: Ultimately HMNS/HMHS every star collapses

Phase transition in Binary NS

Equal Mass binaries: 1.2 +1.2

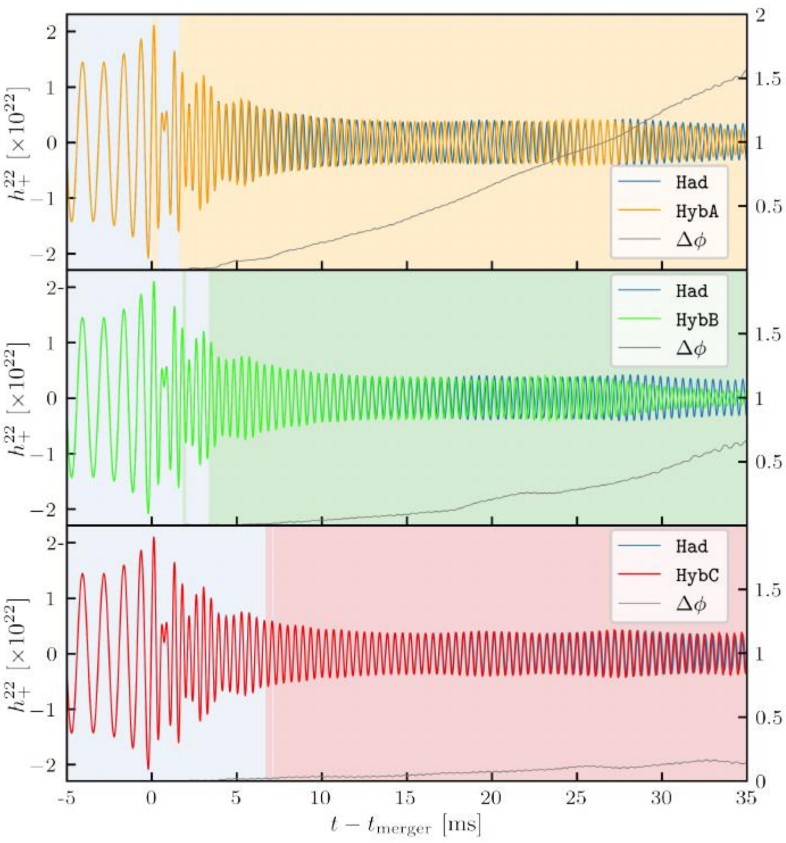
Results shown for equal mass binary of 1.2 + 1.2 solar mass binaries

Difference in the GW signal depending on whether the merger product is HMNS or HMHS

Difference is maximum for stars where mixed phase appears earlier

If mixed phase appears at higher density, GW signal of HMNS and HMHS is almost same

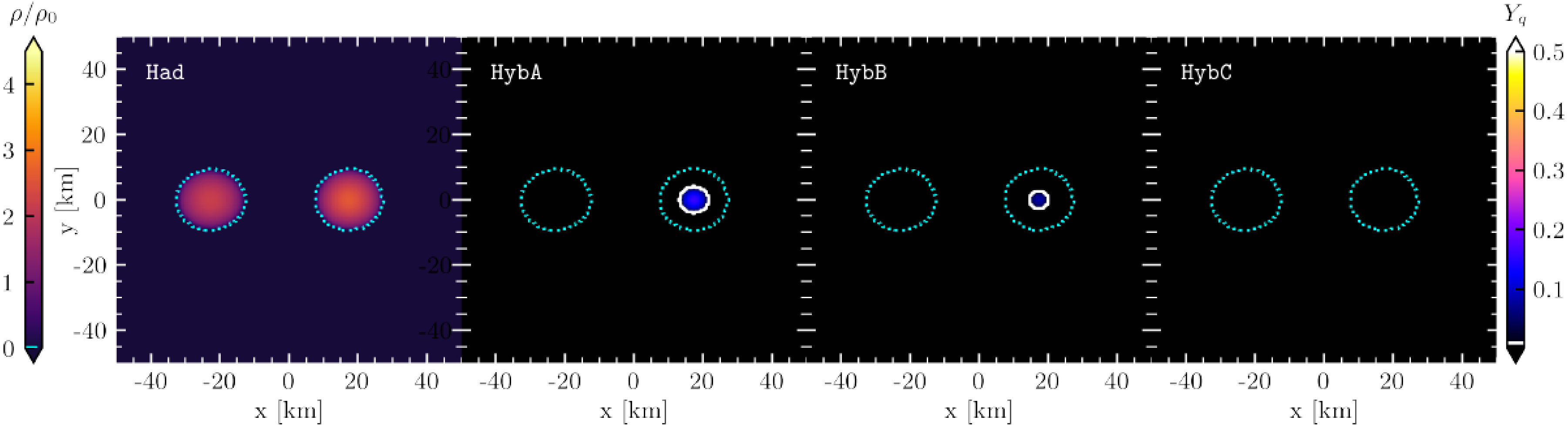
The two peak frequency for all configuration appears at almost same frequency



Phase transition in Binary NS

Unequal Mass binaries: 1.2 + 1.6

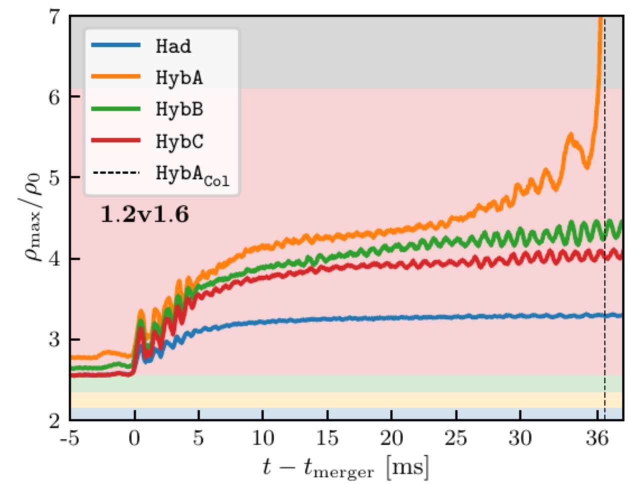
Initial configuration: Appearance of mixed phase region even before merging



With hybrid EoS one star has quark core one does not for at least two Hyb EoS

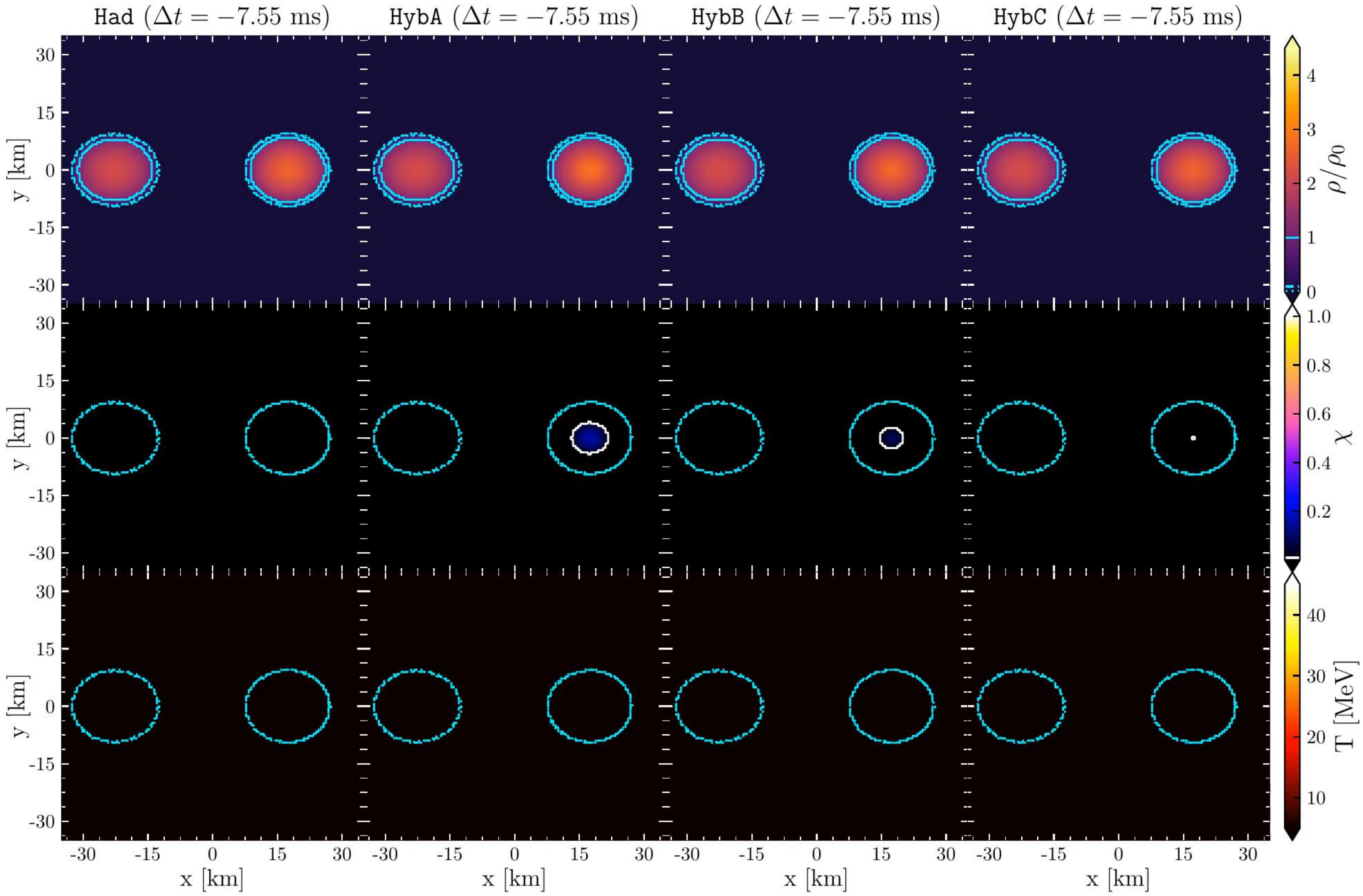
The HMHS where mixed phase appears collapses early

The HMNS reamins stable for the longest time



Phase transition in Binary NS

Unequal Mass binaries: 1.2 + 1.6



Phase transition in Binary NS

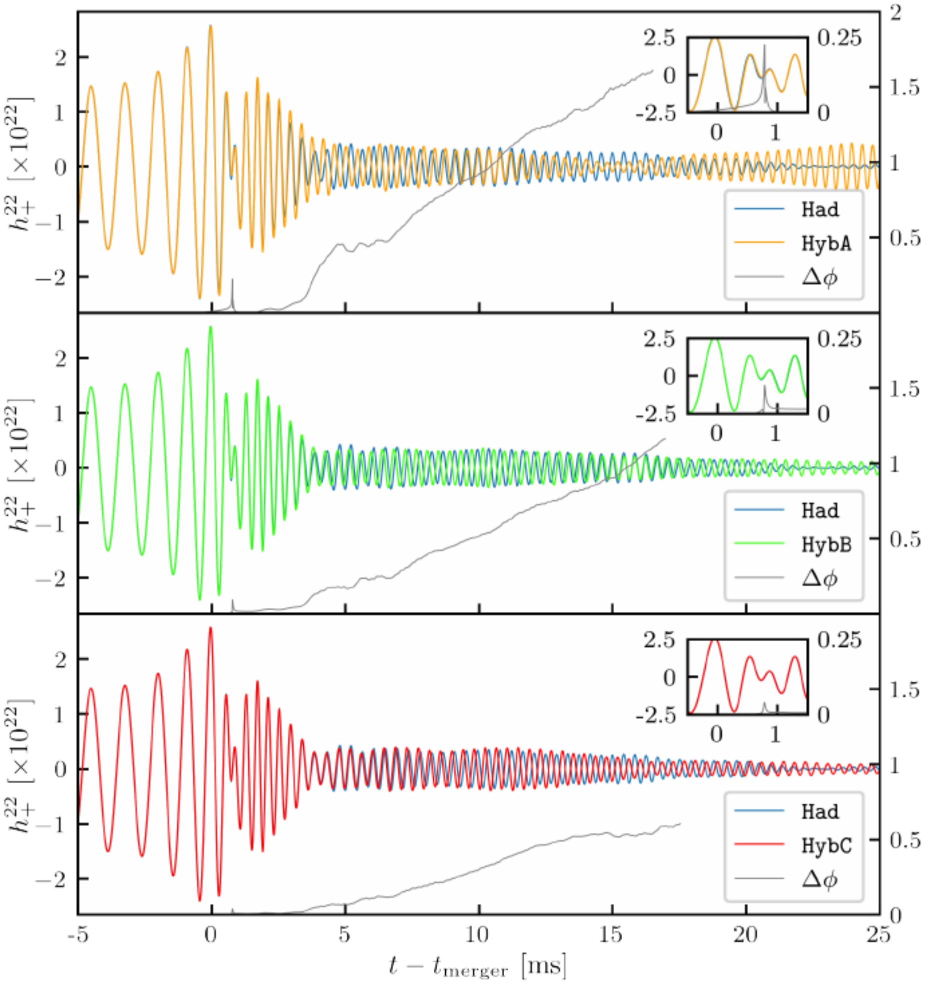
Unequal Mass binaries: 1.2 + 1.6

Have same baryonic mass as that of 1.4+1.4 equal mass binary merger

Difference in the GW signal depending on whether the merger product is HMNS or HMHS

Difference is maximum for stars where mixed phase appears earlier

At the moment of first contact the phase difference spikes momentarily

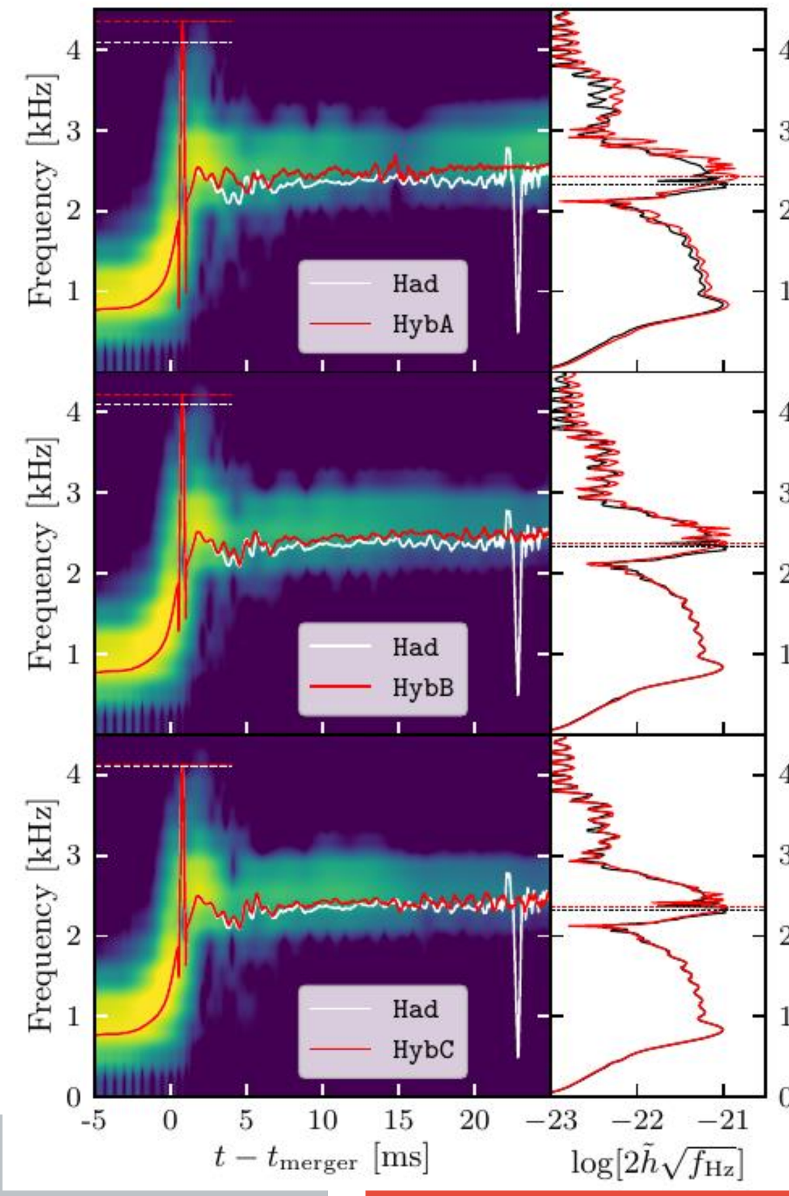


Phase transition in Binary NS

Unequal Mass binaries: 1.2 + 1.6

The peak frequency is different for HMNS and HMHS

The rotational frequency of HMHSs is higher than that of HMNS



Phase transition in Binary NS

Summary and Conclusion:

1. Numerical Relativity is needed for simulating BNSM

2. Difference in the GW signature depending on whether merging stars are NS or HS

3. After the merger the GW differs between HMNS and HMHS

4. Difference is prominent if the quark appearance is at low density

5. The onset point (of mixed phase) and the stiffness of the EoS can be gauged by having several observation of the post-merger phase of BNSM for different binaries.

Research Group

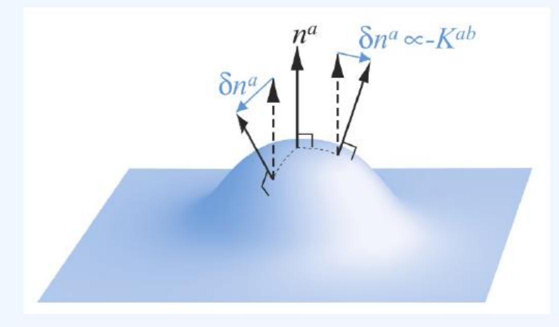
Thank You All

Phase transition in Binary NS

Einstein Equation and Numerical relativity

Extrinsic Curvature

$$\begin{aligned} K_{ab} &= -\gamma_a{}^c \gamma_b{}^d \nabla_c n_d \\ &= -\nabla_a n_b - n_a a_b \\ &= -\frac{1}{2}\mathcal{L}_{\mathbf{n}}\gamma_{ab} \end{aligned}$$



Source term

$$\begin{aligned} \rho &= n_a n_b T^{ab} \\ S^i &= -\gamma^{ij} n^a T_{aj} \\ S_{ij} &= \gamma_{ia}\gamma_{jb} T^{ab} \\ S &= \gamma^{ij} S_{ij} \end{aligned}$$

Constraints Equations

Hamiltonian

$$R + K^2 - K_{ij}K^{ij} = 16\pi\rho$$

Momentum

$$D_j\left(K^{ij} - \gamma^{ij}K\right) = 8\pi S^i$$

Evolution Equation

Spatial metric

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + D_i\beta_j + D_j\beta^i$$

Extrinsic curvature

$$\begin{aligned} \partial_t K_{ij} = &-D_i D_j \alpha + \alpha\left(R_{ij} - 2K_{ik}K^k{}_j + KK_{ij}\right) - 8\pi\alpha\left(S_{ij} - \frac{1}{2}\gamma_{ij}(S-\rho)\right) \\ &+ \beta^k D_k K_{ij} + K_{ik}D_j\beta^k + K_{kj}D_i\beta^k \end{aligned}$$