Bayesian inference of the EoS of dense nuclear matter from heavy-ion collision data

Manjunath Omana Kuttan, Jan Steinheimer, Kai Zhou, Horst Stöcker

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research
SPONSORED BY THE Federal Ministry of Education and Research
ErUM-Data IDT
GSI
FIAS Frankfurt Institute for Advanced Studies
GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Density dependent EoS in UrQMD

EPJ C 82.5 (2022): 1-12,
EPJ C 82.10 (2022): 1-12

- A density dependent potential enters the QMD equations: $\dot{\mathbf{r}}_i = \frac{\partial \mathbf{H}}{\partial \mathbf{p}_i}, \quad \dot{\mathbf{p}}_i = -\frac{\partial \mathbf{H}}{\partial \mathbf{r}_i}.$
  - The potential energy term of the Hamiltonian $\mathbf{H}$ is density dependent.

- The potential energy $V(n_B)$ is related to the pressure as:

$$P(n_B) = P_{\mathrm{id}}(n_B) + \int_0^{n_B} n' \frac{\partial U(n')}{\partial n'} dn' \,, \; U(n_B) = \frac{\partial (n_B \cdot V(n_B))}{\partial n_B}$$

$U(n_B) \Rightarrow$ single particle potential, $P_{id}(n_B) \Rightarrow$ pressure of an ideal Fermi gas of baryons

constraining the potential energy $V(n_B) \Rightarrow$ constraining the EoS

Parameterisation of the potential energy

- Upto $2n_0$, EoS reasonably constrained by
  - flow data at moderate energies *P. Danielewicz, Et al Science 298, 1592 (2002), H. Kruse, Et al. Phys. Rev. Lett. 54, 289 (1985)*
  - nuclear incompressibility data *Y. Wang, Et al. PLB 778, 207 (2018)*
  - bayesian analysis *S. Huth et al., Nature 606, 276 (2022)*

- Upto $2n_0$, CMF model-fit is used *A. Motornenko et al., PRC 103.5 (2021)* , *A. Motornenko et al., PRC* 101.3 (2020)

- We constrain the $V(n_B) > 2n_0$
  - 7^{th} degree polynomial is used
    - h=-22.07 MeV to match CMF at $2n_0$

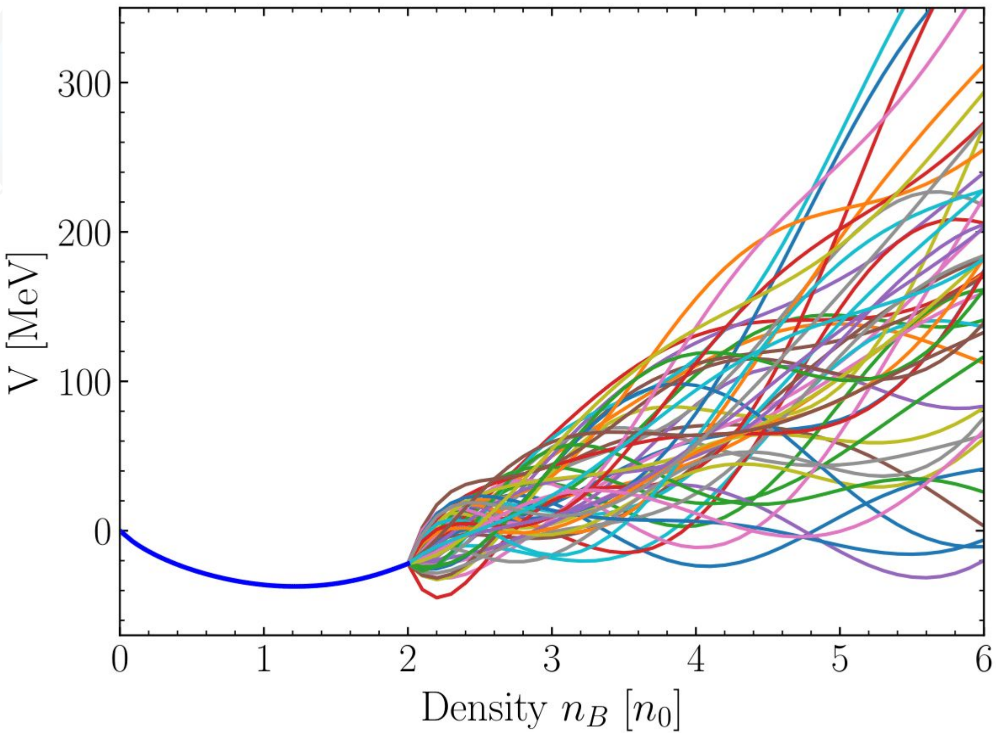
CMF

$$V(n_B) = \sum_{i=1}^{7} \theta_i \left(\frac{n_B}{n_0} - 2\right)^i + h$$

0- $2n_0$

> $2n_0$

Few examples…



$$V(n_B) = \sum_{i=1}^{7} \theta_i \left(\frac{n_B}{n_0} - 2\right)^i + h$$

We constrain the polynomial coefficients

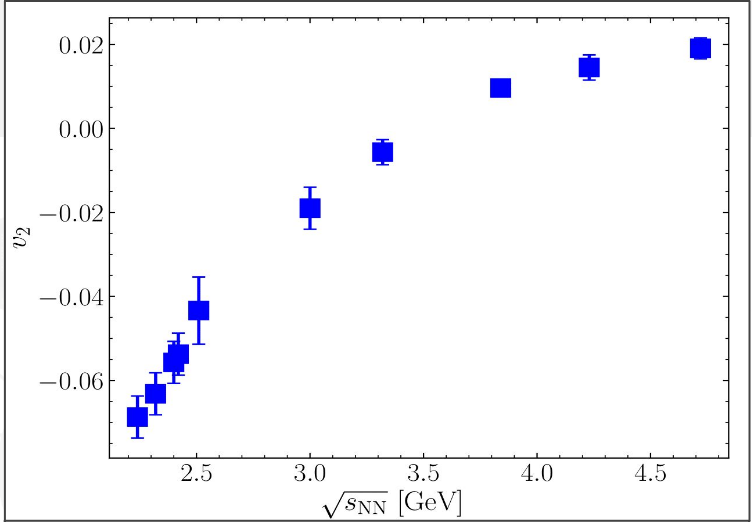
$$\boldsymbol{\theta} = \{\theta_1, \theta_2, \ldots, \theta_7\}$$

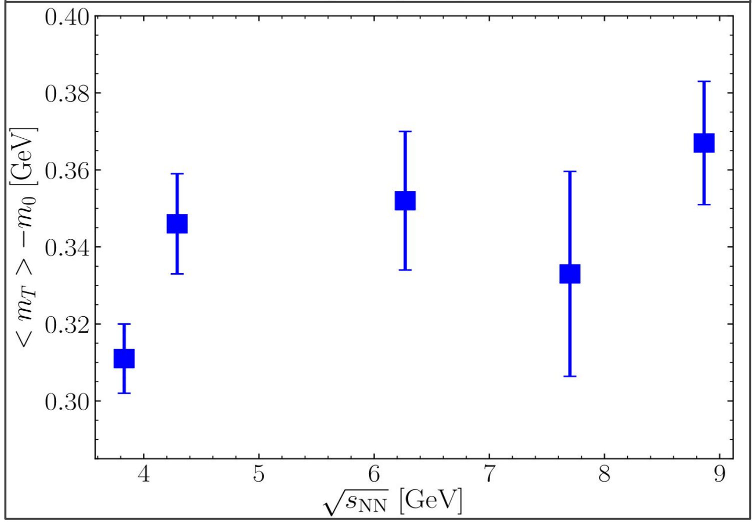
The experimental data

- Proton observables (mid rapidity)
  - Elliptic flow : 10 data points
    - *E895, CERES, FOPI, STAR, HADES*
    - Mid-central collisions
  - Transverse kinetic energy: 5 data points
    - *E802, NA49, STAR*
    - Central collisions

The data, $\mathbf{D} = \{v_2^{exp}, \langle m_T \rangle^{exp} - m_0\}$

is used to constrain the parameters $\boldsymbol{\theta}$.





Fast emulators are necessary

- Bayesian inference involves numerous UrQMD simulations
  - UrQMD ~100 s/event

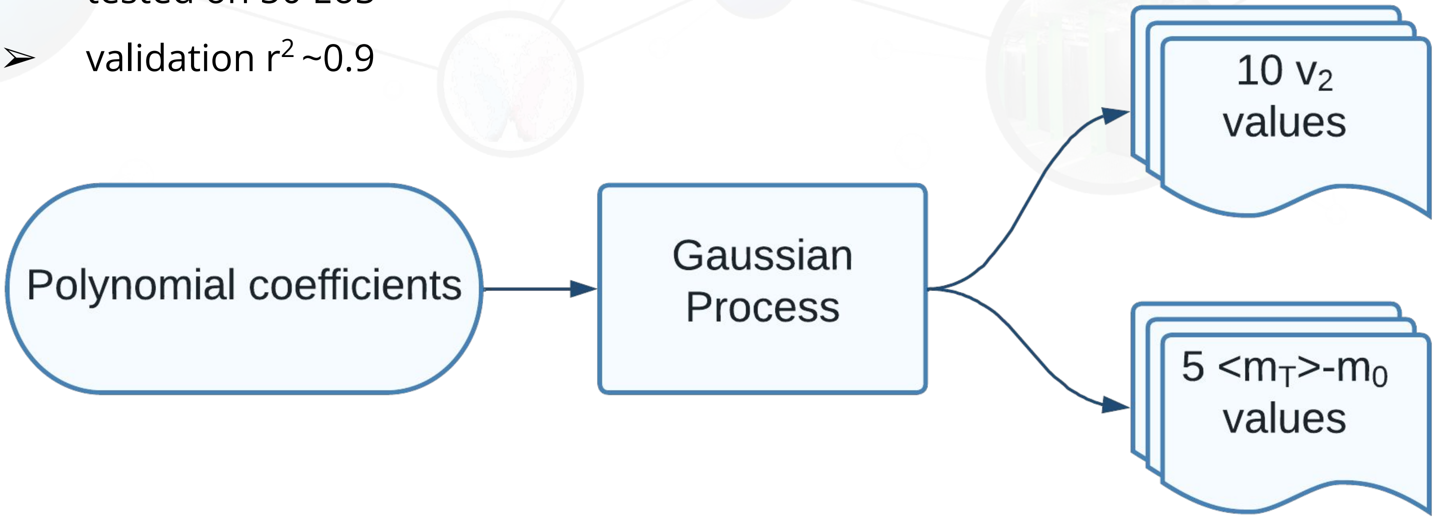
observable	***No of beam energies***	***No of events per beam energy***
v_2	10	12000
$\langle m_T \rangle - m_0$	5	1000

- **For one parameter set** ~125000 events* 100 s = **~3500 hrs**

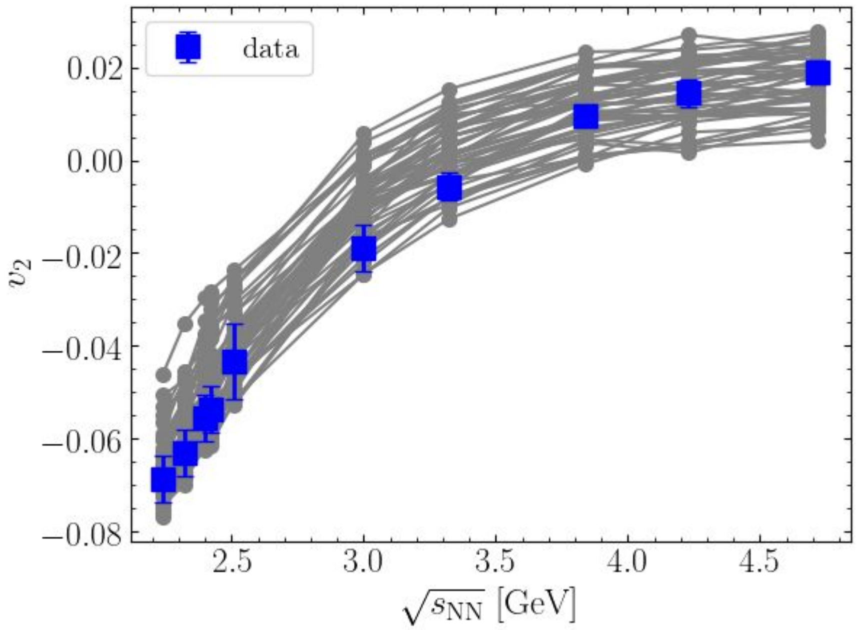
- MCMC sampling requires random walk through 1000s of parameter sets
  - **not feasible to run UrQMD for MCMC !**

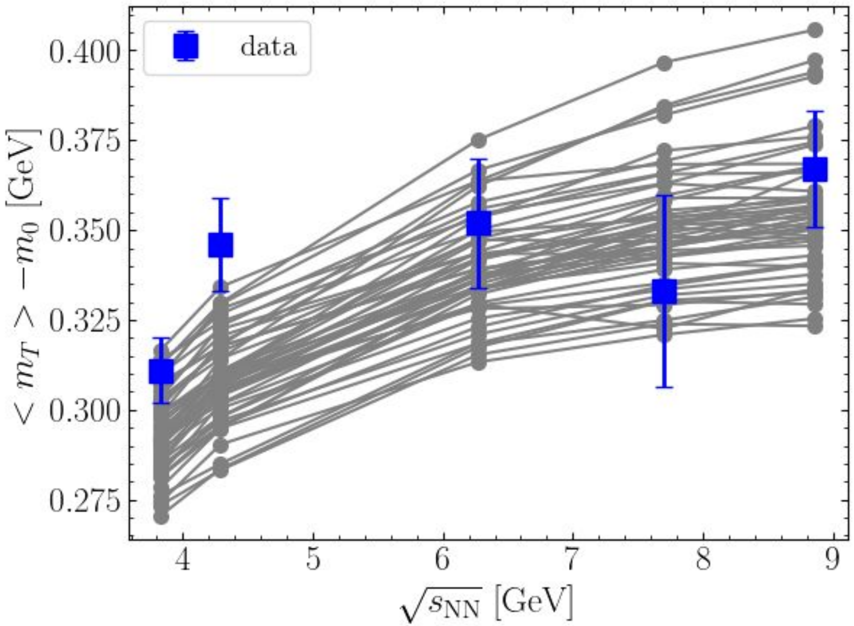
Training the GP models

- Gaussian Process models are trained as fast emulators
  - trained on 200 randomly generated EoSs
  - tested on 50 EoS
  - validation r^2 ~0.9

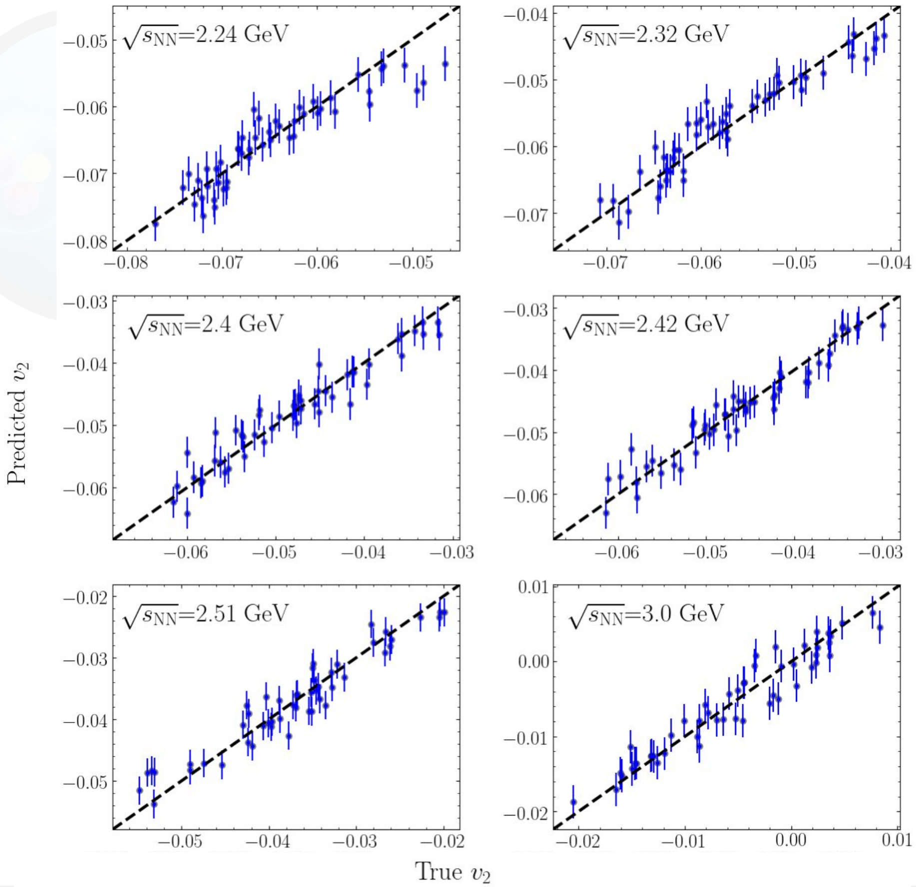


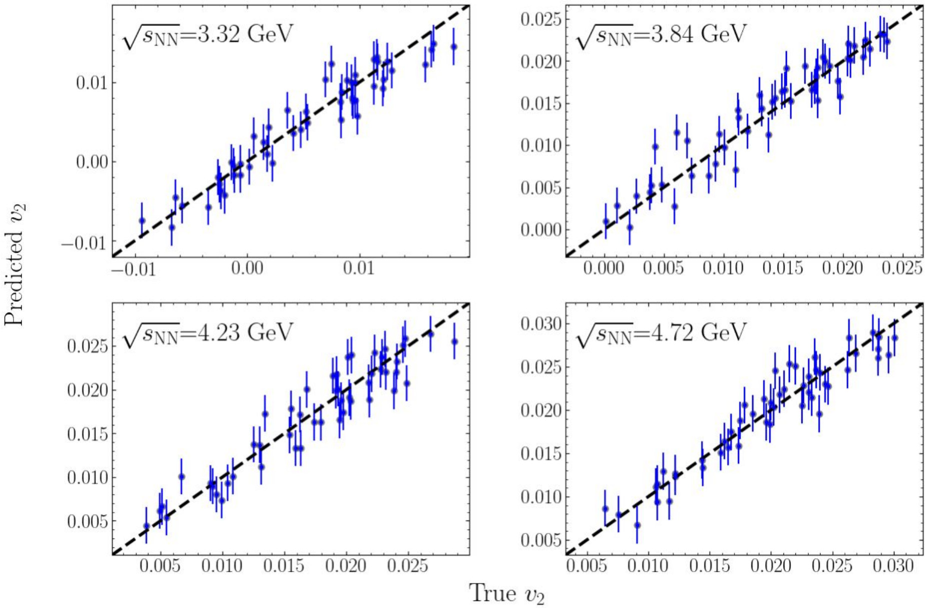
Potentials for training the GP models



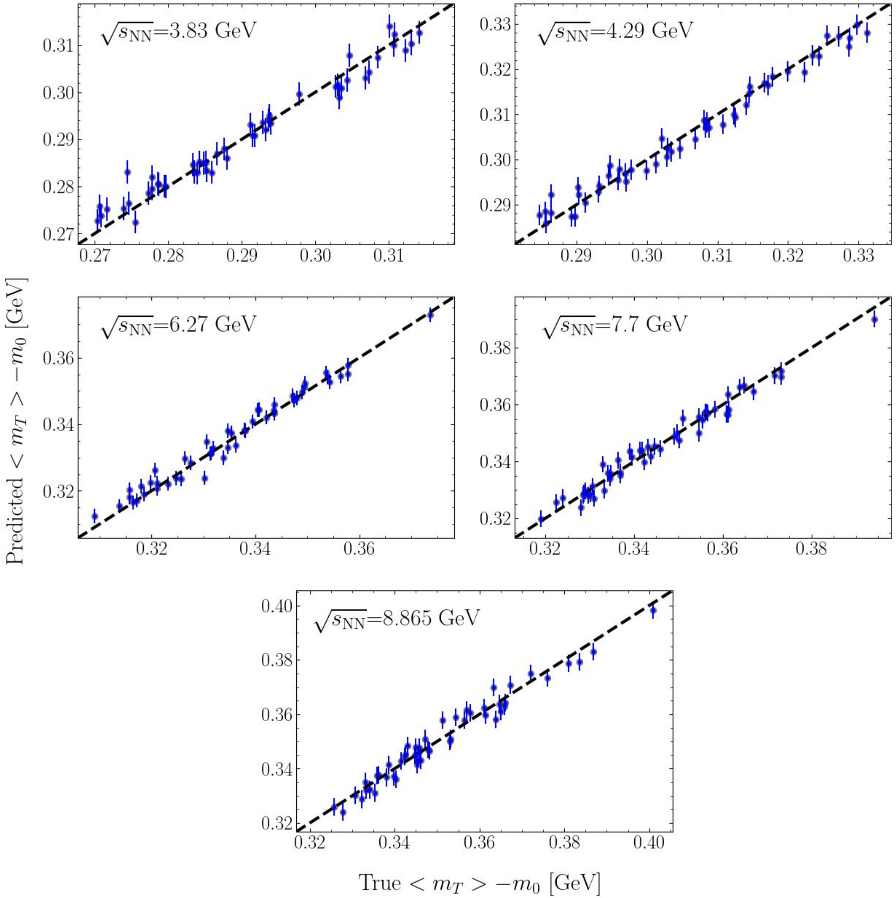


GP models: performance





GP models: performance



Bayesian inference

Posterior

- probability that the parameters $\boldsymbol{\theta}$ explains the data $\mathbf{D}$

$$P(\boldsymbol{\theta}|\mathbf{D}) \propto P(\mathbf{D}|\boldsymbol{\theta})P(\boldsymbol{\theta})$$

Log-likelihood

$$\ln P(\mathbf{D}|\boldsymbol{\theta}) = -\frac{1}{2}\sum_i \left[\frac{(x_i^{\boldsymbol{\theta}} - d_i)^2}{\sigma_i^2} + (\ln(2\pi\sigma_i^2))\right]$$

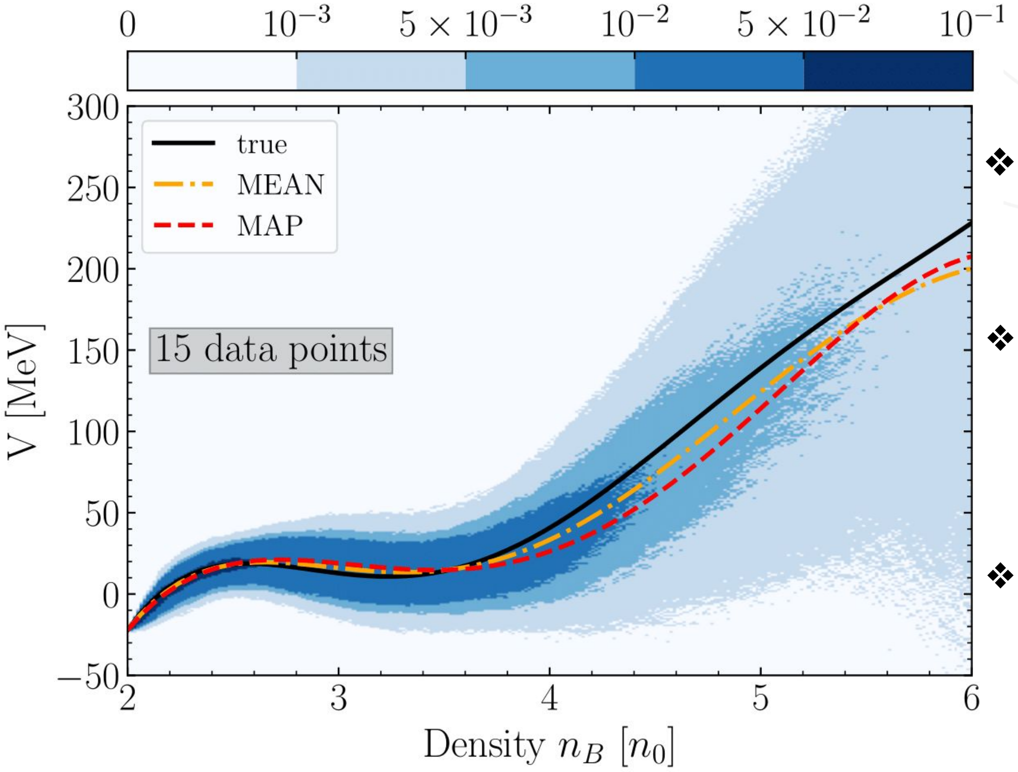
Includes uncertainty from experiment & GP model

- gaussian priors are used
- μ, σ from GP training data

Closure test

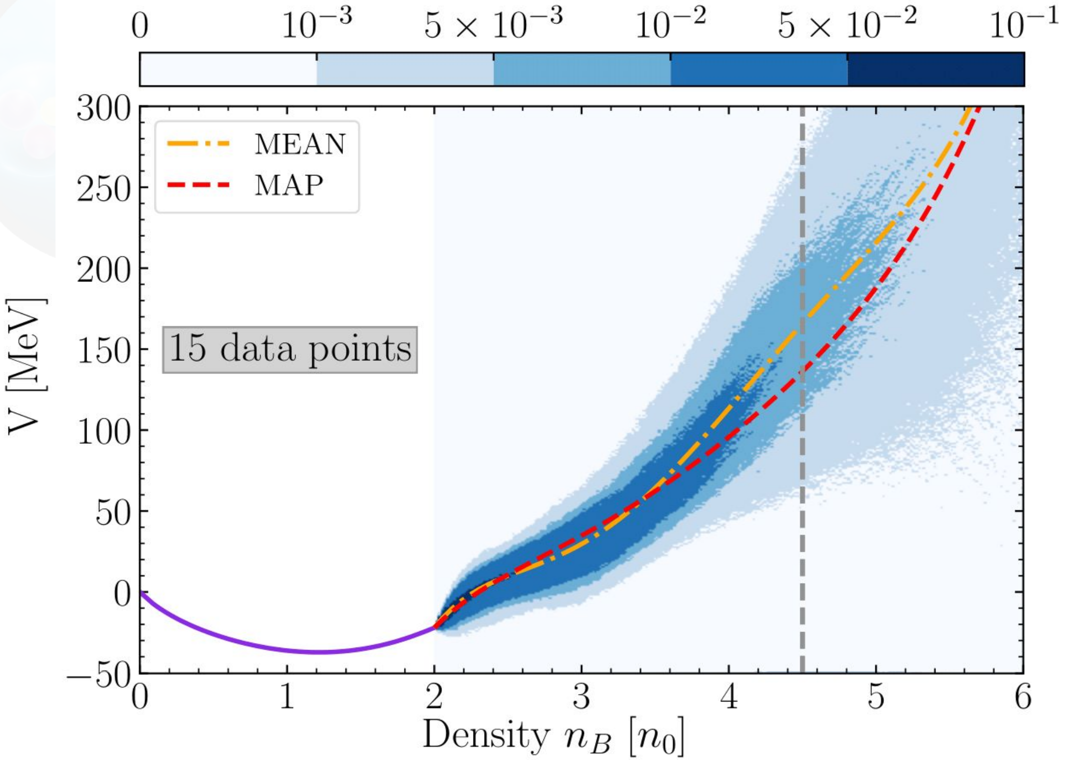
1. Consider a random EoS as “ground-truth”
2. Calculate observables from UrQMD
   a. 10 values of v_2 and 5 values of $\langle m_T \rangle - m_0$
3. Assume the UrQMD observables for this EoS to be the “data”
   a. assume uncertainty similar to experimental data
4. Construct the posterior
5. Compare with the “ground-truth”

Closure test: results



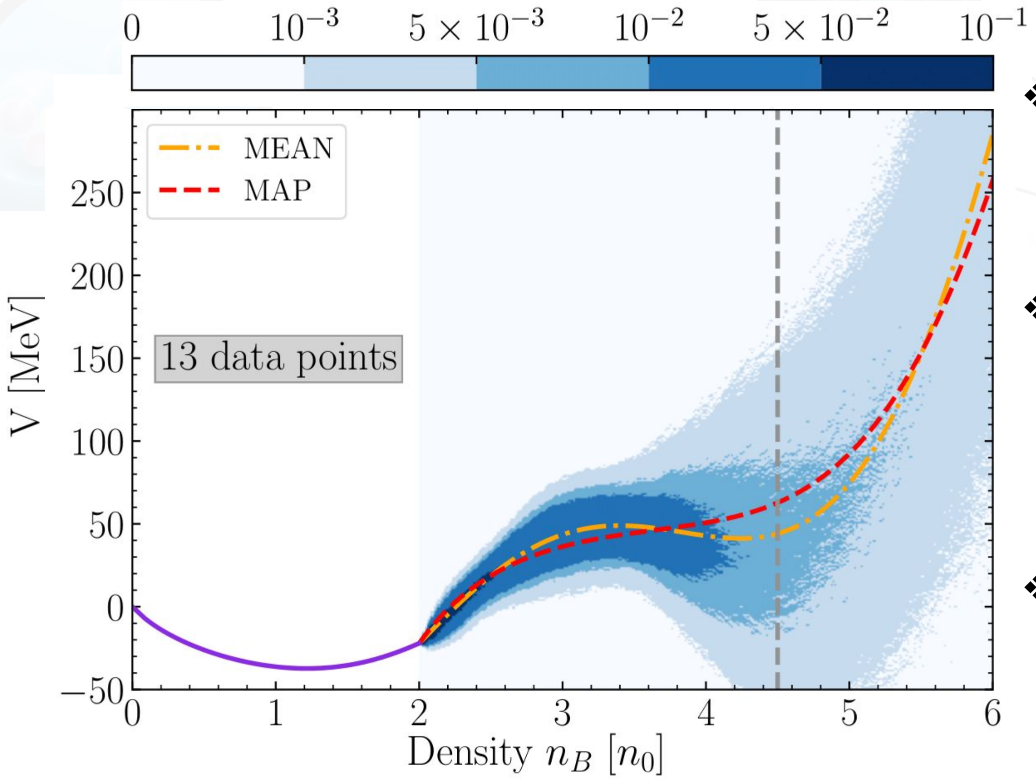
- Tight constraints up to $4n_0$
  - large uncertainty above $4n_0$
  - yet mean closely follows "ground-truth"
- Two curves extracted:
  - "MAP": mode of posterior
  - "MEAN": mean of posterior
- MEAN and MAP closely follows "ground-truth" upto $6n_0$

Result from experimental data



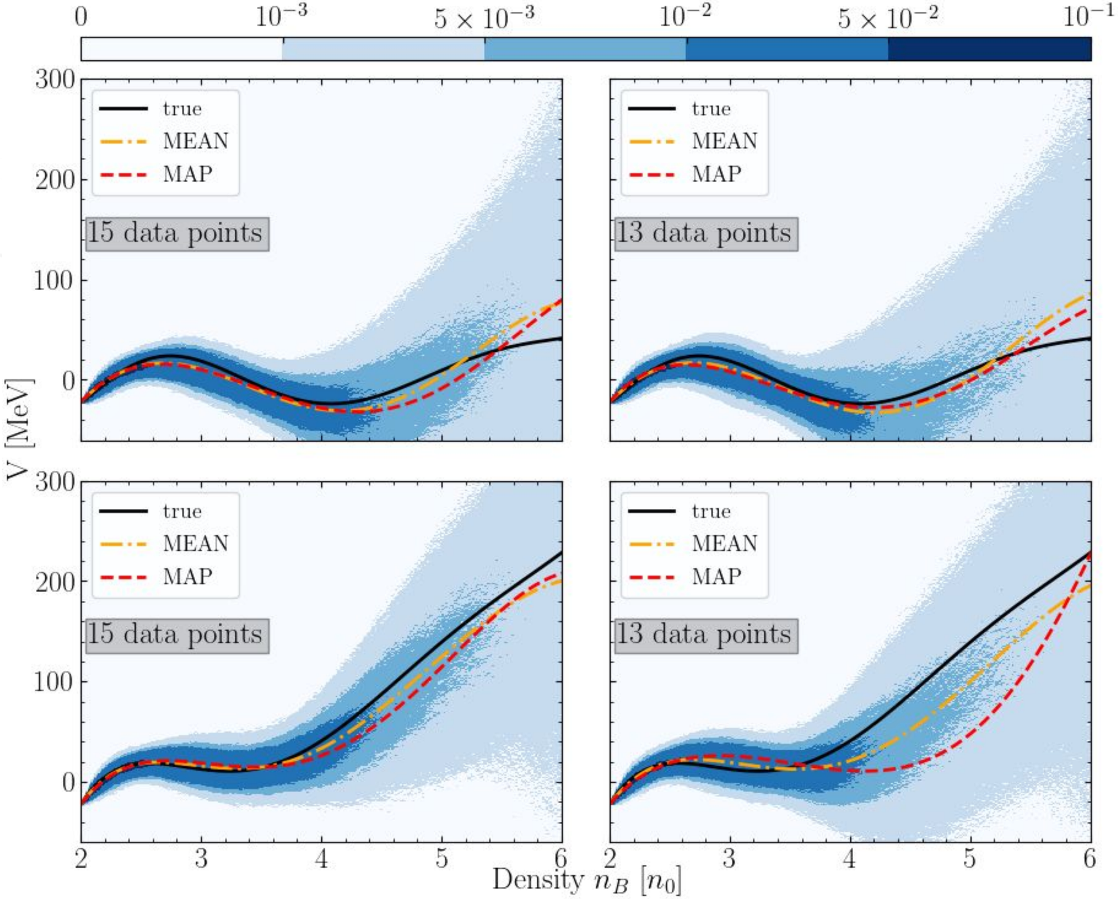
- ❖ Posterior from experimental data
  - ➢ 10 measurements of v_2
  - ➢ 5 measurements of $\langle m_T \rangle - m_0$
- ❖ Tight constraints upto $4n_0$
  - ➢ MEAN, MAP suggests stiff EoS
  - ➢ No phase transition

Sensitivity to choice of observables

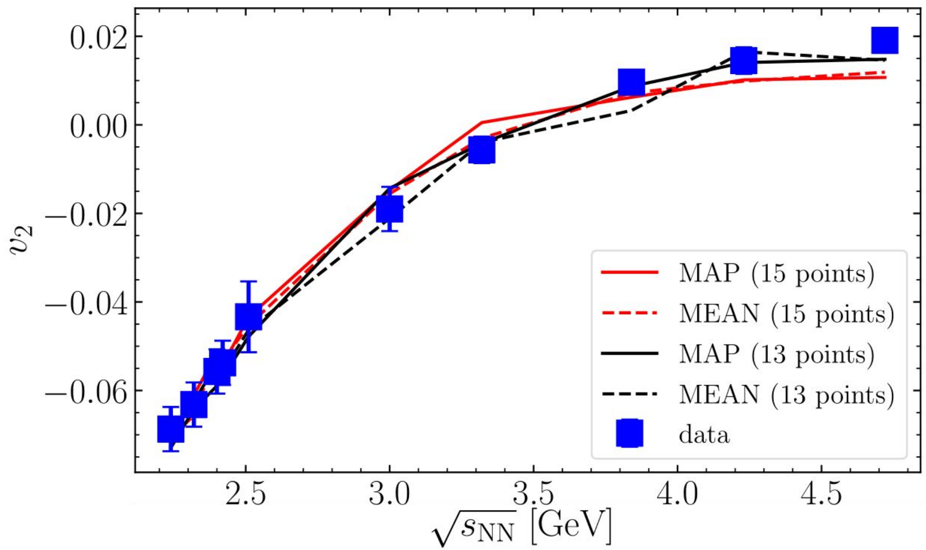


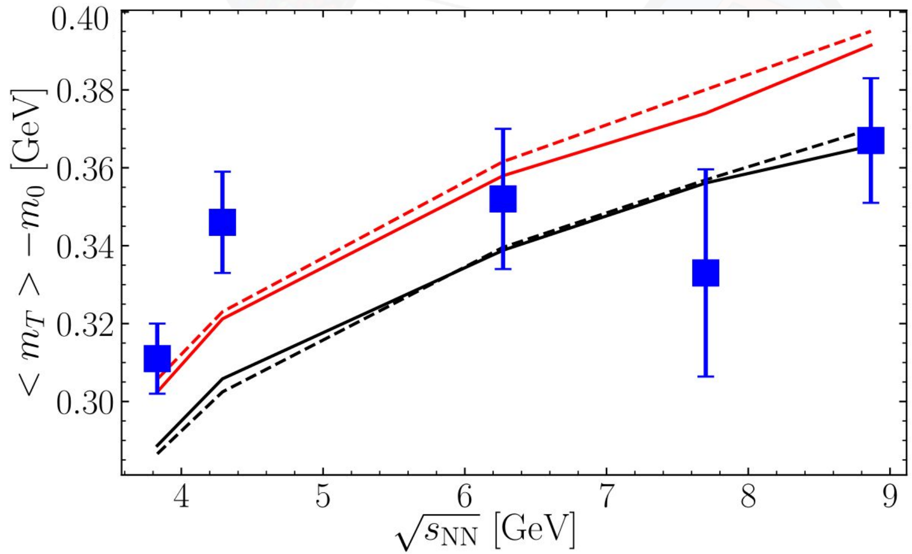
- Only 13 data points are used
  - $\langle m_T \rangle - m_0$ at 3.83, 4.29 GeV not used
- Significant differences in posterior
  - softening at 3- 5n_0
  - phase transition
- Beyond 3n_0 strong dependence to choice of observables

More closure test results



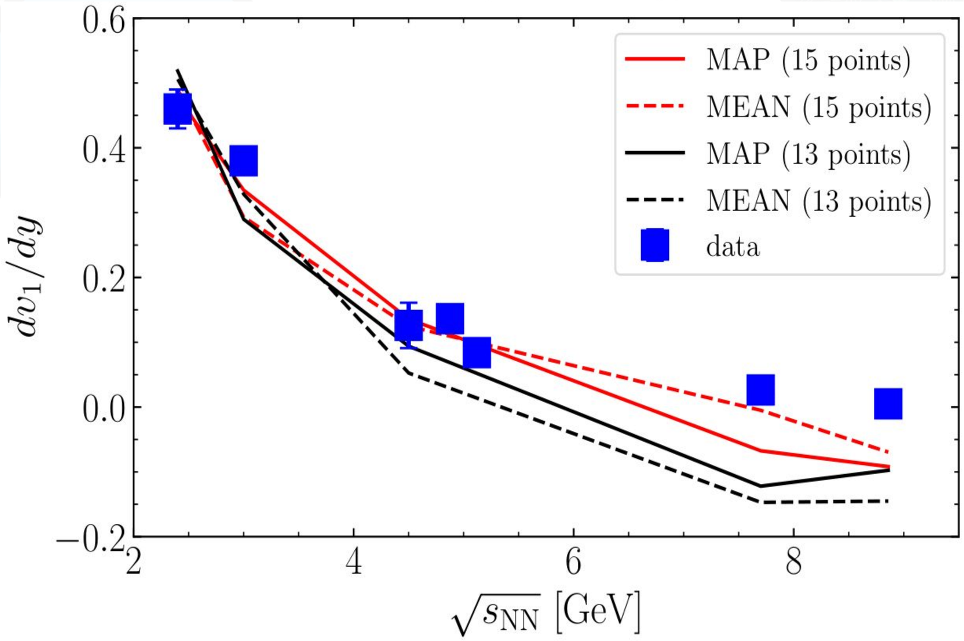
Reconstructed EoSs: v_2, $<m_T>-m_0$





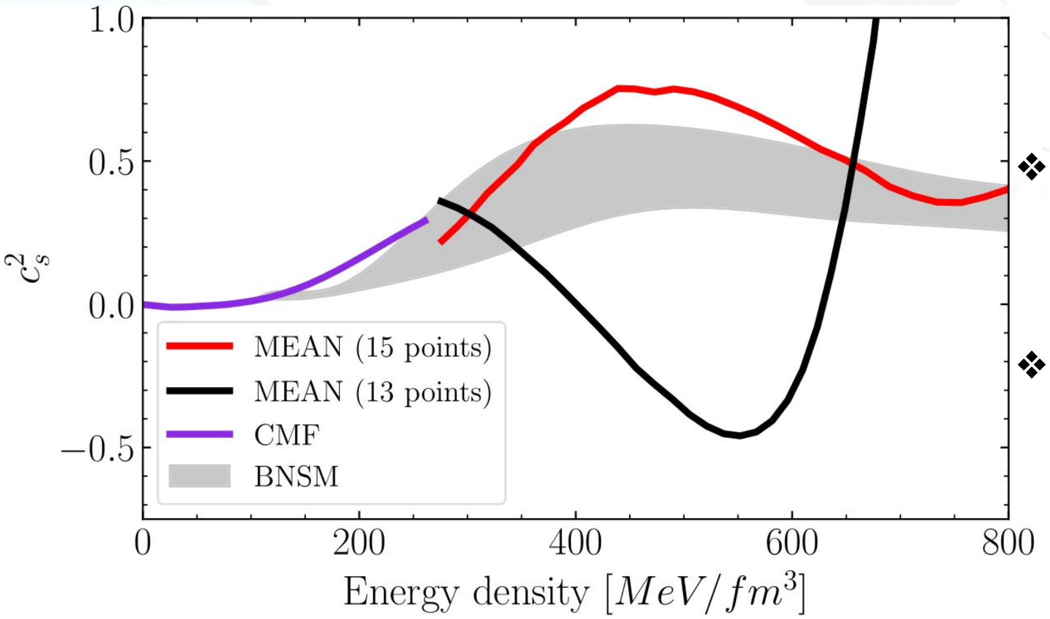
- better v_2 predictions at high energies when 2 data points are removed
  - but also results in lower $<m_t>-m_0$
- large $<m_t>-m_0$ values for the stiff EoS (extracted using all data points)
- possible tension in data at ~4 GeV!
  - Measurement uncertainty? or limitation of the model?

Reconstructed EoSs: dv_1/dy



- dv_1/dy data was **not used for inference**
  - yet consistent with reconstructed EoSs
    - especially with all 15 data points

Reconstructed EoSs: c_s^2



- 15 points, predicts a stiff EoS
  - consistent with astrophysical constraints
    arXiv:2203.14974 (2022)
    - broad peak structure
- 13 points, drastic drop in c_s^2
  - first order phase transition

Summary

- Bayesian inference using polynomial parameterization of the density dependence of EoS
  - v_2, $<m_t>-m_0$ of protons are used for inference
- inference using all 15 data points:
  - constraints the EoS upto $4n_0$
  - stiff EoS upto $4n_0$, no phase transition
  - consistent with BNSM constraint, dv_1/dy data
- strong dependence on choice of observables for $> 3n_0$
- tension in data at ~4 GeV
  - measurement uncertainty or model limitation?

https://arxiv.org/abs/2211.11670

For stricter, robust constraints on the EoS below $4n_0$, significant improvements and consistency in flow measurements are necessary for E_lab = 2-10 A GeV

Backup slides

Microscopic transport with density dependent potential

- Non-equilibrium MD part of UrQMD is used
- UrQMD:
  - Propagation of hadrons on classic trajectories
    - stochastic binary scattering , color string formation, resonance excitation and decays
  - Imaginary part of interactions:
    - geometric interpretation of cross section
      - Experiment, detailed balance
  - Hadronic cascade
    - effective EoS of HRG with respective dof
- Real part of interactions in UrQMD
  - QMD + density dependent potential
    - Unlike other mean field models, QMD is an n-body theory of interactions between n nucleons

Microscopic transport with density dependent potential

A density dependent potential enters QMD equations

$$\dot{\mathbf{r}}_i = \frac{\partial \mathbf{H}}{\partial \mathbf{p}_i}, \quad \dot{\mathbf{p}}_i = -\frac{\partial \mathbf{H}}{\partial \mathbf{r}_i}.$$

The total hamiltonian function is sum over all hamiltonians of the i baryons

$$\mathbf{H} = \sum_i H_i, \quad H_i = E_i^{kin} + V_i$$

This include KE and total potential energy **V**

$$\mathbf{V} = \sum_i V_i \equiv \sum_i V(n_B(r_i))$$

The local interaction density n_B at r_k is calc by assuming each particle as gaussian wave packet

$$n_B(r_k) = n_k = \sum_{j, j\neq k} n_{j,k}$$

$$= \left(\frac{\alpha}{\pi}\right)^{3/2} \sum_{j, j\neq k} B_j \exp\left(-\alpha(\mathbf{r_k} - \mathbf{r_j})^2\right)$$

α=1/2L, L= 2 fm^2

The change in momentum for baryon 'i' is then

$$\dot{\mathbf{p}}_i = -\frac{\partial \mathbf{H}}{\partial \mathbf{r}_i} = -\frac{\partial \mathbf{V}}{\partial \mathbf{r_i}} \qquad n_{\{i,j\}} \equiv n_B(r_{\{i,j\}})$$

$$= -\left(\frac{\partial V_i}{\partial n_i} \cdot \frac{\partial n_i}{\partial \mathbf{r_i}}\right) - \left(\sum_{j\neq i} \frac{\partial V_j}{\partial n_j} \cdot \frac{\partial n_j}{\partial r_i}\right)$$

Force on i^{th} baryon depends on change in potential energy at point r_i due to local gradient of $n_B(r_i)$ and change in potential at positions r_j of all baryons j due to change in r_i

-solved in timestep 0.2fm/c

$$P(n_B) = P_{\mathrm{id}}(n_B) + \int_0^{n_B} n' \frac{\partial U(n')}{\partial n'} dn' \,, \; U(n_B) = \frac{\partial\left(n_B \cdot V(n_B)\right)}{\partial n_B}$$

$$\mu'_B(n_B) = \mu_B^{id}(n_B) + U(n_B)$$

$$\epsilon(n_B) = -P(n_B) + \mu'_B n_B + sT$$