Effective damping of r-modes with hyperons

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Workshop on the QCD EoS in dense matter HIC and Astrophysics
MAGIC23
Kovalam - India

March 31, 2023

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- Results are presented in:
  Hyperon bulk viscosity and r-modes of neutron stars;
  O. P. Jyothilakshmi, P. E. Sravan Krishnan, Prashant Thakur,
  V. Sreekanth and T. K. Jha,
  Mon. Not. Roy. Astron. Soc. 516 (2022) 3, 3381-3388.
  arXiv:2208.14436 [astro-ph.HE]

Introduction

- Neutron stars are formed from the collapse of supergiants of mass 8 - 30 $M_\odot$. They are extremely dense objects and they withstand gravitational collapse largely by neutron degenaracy pressure.
- Neutron stars may be subjected to various instabilities associated with unstable oscillating modes.
- A rapidly rotating neutron star produces fluid motions or currents called *r-modes*, which are similar to ocean currents on Earth. These currents produce gravitational waves.
- Gravitational waves are disturbances that propagate in the fabric of spacetime, and were predicted by Einstein's general theory of relativity. These waves carry energy and travel at the speed of light. The first detection of these gravitational waves occurred in 2015 (LIGO).

- These gravitational waves can cause the amplitude of r-modes to grow, thereby generating even more gravitational radiation (GR) and leading to an instability through the Chandrasekhar-Friedman-Schutz (CFS) mechanism.
- This instability can get damped due to viscous effects like shear and bulk viscosities.
- If the GR time scale is shorter than the timescale due to such dissipative processes, then the r-mode will be unstable and a rotating neutron star will dissipate some of its rotational energy through GR.
- The calculation of these timescales require the use of equations from relativistic hydrodynamics.

Objective

- In this work, we study a hyperon neutron star modelled using an effective chiral model invoking $\sigma - \rho$ cross coupling.
- We calculate the hyperonic bulk viscosity coefficient due to nonleptonic weak interactions and calculate the associated timescales.
- We also calculate the timescales associated with other dissipative effects, including viscosity due to modified URCA processes and shear viscosity.
- We then calculate the timescale of gravitation radiation to finally obtain the r-mode instability window.

Model

- The Lagrangian density in an effective chiral model with cross coupling effects under consideration is :

$$\mathcal{L} = \bar{\psi}_B\Big[\Big(i\gamma_\mu\partial^\mu - g_\omega\gamma_\mu\omega^\mu - \frac{1}{2}g_\rho\vec{\rho_\mu}.\vec{\tau}\gamma^\mu\Big) - g_\sigma(\sigma + i\gamma_5\vec{\tau}.\vec{\pi})\Big]\psi_B + \frac{1}{2}(\partial_\mu\vec{\pi}.\partial^\mu\vec{\pi}$$
$$+ \partial_\mu\sigma\partial^\mu\sigma) - \frac{\lambda}{4}\left(x^2 - x_0^2\right)^2 - \frac{\lambda b}{6m^2}(x^2 - x_0^2)^3 - \frac{\lambda c}{8m^4}(x^2 - x_0^2)^4$$
$$- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g_\omega^2x^2\left(\omega_\mu\omega^\mu\right) - \frac{1}{4}\vec{R_{\mu\nu}}.\vec{R^{\mu\nu}} + \frac{1}{2}m_\rho'^{\,2}\vec{\rho_\mu}.\vec{\rho^\mu} + \eta_1\left(\frac{1}{2}g_\rho^2x^2\vec{\rho_\mu}.\vec{\rho^\mu}\right)$$

- Here,
  - ψ_B is the nucleon isospin doublet interacting with different mesons σ, ω and ρ, with the respective coupling strengths g_i, with $i = \sigma, \omega$ and ρ.
  - The b and c are the strength for self couplings of scalar fields. Potential for the scalar fields (π, σ) are written in terms of a chiral invariant field x given by $x^2 = \pi^2 + \sigma^2$.
  - The final term contains cross-coupling terms between ρ and ω and also between ρ and σ. The coupling strength for $\sigma - \rho$ is given by $\eta_1 g_\rho^2$.

[Malik et al. PRC 96, 035803 2017]

Field Equations

- Given a Lagrangian, it is possible to calculate the equation of state using the relativistic mean field approximation.
- The field equations can be calculated as:

i. The vector field (ω):

$$\sum_B g_{\omega B}\rho_B = m_\omega^2 Y^2 \omega_0,$$

ii. The Isovector field (ρ_3^0)

$$\sum_B I_{3B} g_{\rho B} \rho_B = m_\rho^2 \Big[1 - \eta_1 (1 - Y^2) C_\rho / C_\omega \Big] \rho_3^0,$$

iii. The Scalar field (σ) in terms of $Y = x/x_0$, with $x^2 =< \sigma^2 + \pi^2 >$,

$$\sum_B \frac{2 C_{\sigma B} \rho_{sB}}{m_B Y} = (1 - Y^2) - \frac{b}{m^2 C_\omega} (1 - Y^2)^2 + \frac{c}{m^4 C_\omega^2} (1 - Y^2)^3 + \frac{2 C_{\sigma N} m_\omega^2 \omega_0^2}{m_N^2} + \frac{2 \eta_1 C_{\sigma N} C_\rho m_\rho^2 (\rho_3^0)^2}{C_\omega m_N^2}.$$

$$
\begin{aligned}
\epsilon &= \frac{1}{\pi^2}\sum_B \int_0^{k_{F_B}} k^2\sqrt{k^2+m^{*2}}dk + \frac{m^2}{8C_{\sigma B}}(1-Y^2)^2 \\
&-\frac{b}{12C_{\sigma B}C_{\omega B}}(1-Y^2)^3 + \frac{c}{16m^2C_{\sigma B}C_{\omega B}^2}(1-Y^2)^4 \\
&+\frac{1}{2}m_\rho^2\Big[1-\eta_1(1-Y^2)(C_{\rho B}/C_{\omega B})\Big](\rho_{3B}^0)^2 \\
&+\frac{1}{2}m_\omega^2\omega_0^2Y^2 + \frac{1}{\pi^2}\sum_L \int_0^{kF_L} k^2\sqrt{k^2+m^{*2}}dk, \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
p &= \frac{1}{3\pi^2}\sum_B \int_0^{k_{F_B}} \frac{k^4}{\sqrt{k^2+m^{*2}}}dk - \frac{m^2}{8C_{\sigma B}}(1-Y^2)^2 \\
&+\frac{b}{12C_{\sigma B}C_{\omega B}}(1-Y^2)^3 - \frac{c}{16m^2C_{\sigma B}C_{\omega B}^2}(1-Y^2)^4 \\
&+\frac{1}{2}m_\rho^2\Big[1-\eta_1(1-Y^2)(C_{\rho B}/C_{\omega B})\Big](\rho_{3B}^0)^2 \\
&+\frac{1}{2}m_\omega^2\omega_0^2Y^2 + \frac{1}{3\pi^2}\sum_L \int_0^{kF_L} \frac{k^4}{\sqrt{k^2+m^{*2}}}dk. \qquad (2)
\end{aligned}
$$

TABLE: The model parameters such as the meson-nucleon coupling constants $C_{\sigma N}, C_{\omega N}, C_{\rho N}$, and the higher order scalar field couplings B and C is in the top row along with η_1, the $\sigma - \rho$ mesonic cross coupling. The corresponding saturation properties such as the saturation density ρ_0, the nucleon effective mass, the nuclear matter incompressibility K, energy per particle e_0, symmetry energy J_0 and its slope L_0 for the model are enlisted.

$C_{\sigma N}$ (fm^2)	$C_{\omega N}$ (fm^2)	$C_{\rho N}$ (fm^2)	η_1	B (fm^2)	C (fm^4)
8.81	2.16	13.00	-0.85	-12.08	-36.44

ρ_0 (fm^{-3})	$m^\star/m$	K (MeV)	e_0 (MeV)	J_0 (MeV)	L_0 (MeV)
0.151	0.85	211	-15.8	32.5	61

Equation of State

- The equation of state is obtained from the given Lagrangian density using the relativistic mean field approximation.

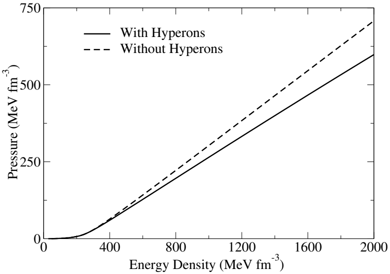

Figure: Pressure as a function of energy density of the model under consideration.

Particle Number

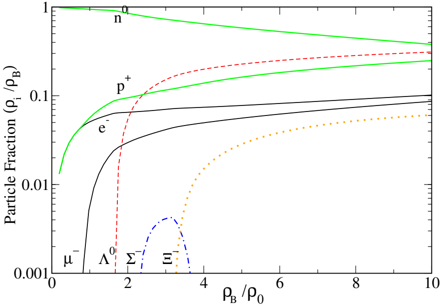

Figure: Particle fraction of various octet of baryons (as marked in the figure) plotted as a function of normalized baryon number density.

Static Stars - Tolman-Oppenheimer-Volkoff Equations

- The Tolman-Oppenheimer-Volkoff (TOV) equations are a set of two coupled differential equations in the variables $p(r)$ and $m(r)$.

$$\frac{dp}{dr} = -\frac{[\varepsilon + p]\left[m + 4\pi r^3 p\right]}{r(r-2m)},$$
$$\frac{dm}{dr} = 4\pi r^2 \varepsilon,$$

- The solution of the TOV equations for a specific EoS $\varepsilon = \varepsilon(p)$ is determined by two boundary conditions. One is the central pressure p_c (or equivalently ε_c). The other is the mass at the center which is taken to be zero.
- With a specified central density, the TOV equations can be integrated outwards starting at the center. The radius R of the star is defined as the distance at which the pressure drops to zero, i.e. $p(r = R) = 0$. The mass of the star is then given by $M = m(R)$.

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Rapidly Rotating Neutron Stars

- The Hartle method is based on approximations of slow rotation. More accurate results can be obtained by direct numerical integration of the Einstein Field equations.
- This is implemented in `RNS`, a well-known code written by Nikolaos Stergioulas (1995). It is based on the KEH method (Komatsu, Eriguchi & Hachisu, 1989) and it is available at the website gravity.phys.uwm.edu/rns.
- A model is defined uniquely by specifying two parameters, the central density ε_c and any one of mass, rest mass, angular velocity, angular momentum, or the axes ratio.
- We use the `RNS` code to calculate the global properties for our EoS.

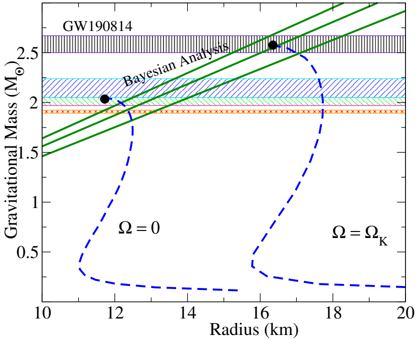


FIGURE: Gravitational Mass plotted as a function of equatorial radius of the star for both static ($\Omega = 0$) and when it is rotating at Kepler's frequency ($\Omega = \Omega_K$) obtained using the RNS code, compared with the mass range of massive known pulsars (1.88 - 2.24) solar masses and the inferred mass range of the secondary component of the Gravitational wave 'GW190814' data. The compactness parameter estimate from Bayesian analysis (Riley 2019) is also indicated.

Hyperon Bulk Viscosity

- Bulk viscosity arises when a system is driven out of equilibrium due to a change in the volume of a fluid element. The equilibrium is then restored by various internal processes, each of which act on a characteristic timescale known as the relaxation timescale 'τ'.
- It has been shown that non-leptonic processes involving hyperons contribute very significantly towards bulk viscosity at temperatures relevant to neutron stars.
- The damping due to bulk viscosity is quantified by the real part of the coefficient of bulk viscosity ζ, which relates the perturbed pressure p and the thermodynamic pressure $\tilde{p}$ to the expansion of the fluid as

$$p - \tilde{p} = -\zeta \nabla \cdot \mathbf{v},$$

  where $\mathbf{v}$ is the velocity of the fluid element.

- A relativistic expression for the real part of ζ within a relaxation time approximation is given as

$$\mathrm{Re}[\zeta] = \frac{p\,(\gamma_\infty - \gamma_0)\,\tau}{1 + (\omega\tau)^2},$$

where ω is the angular frequency of the perturbation in a co-rotating frame and τ is the net microscopic relaxation time.

- The expression for $\gamma_\infty - \gamma_0$ is

$$\gamma_\infty - \gamma_0 = -\frac{{\rho_B}^2}{p}\frac{\partial p}{\partial \rho_n}\frac{d\tilde{x}_n}{d\rho_B}.$$

Here $\tilde{x}_n = \rho_n/\rho_B$ is the neutron fraction where ρ_n is the neutron number density and ρ_B is the total baryon number density.

[Lindblom & Owen, 2002]

- The expression for relaxation timescale at temperature T in the presence of both Σ^- and Λ^0 hyperons is given by,

$$\frac{1}{\tau} = \frac{(k_B T)^2}{192\pi^3} \left(k_\Sigma \left\langle |\mathcal{M}_\Sigma^2| \right\rangle + k_\Lambda \left\langle |\mathcal{M}_\Lambda^2| \right\rangle \right) \frac{\delta\mu}{\rho_B \delta x_n}.$$

Here

- k_B is Boltzmann's constant, k_Λ and k_Σ denote Fermi momenta of Λ and Σ hyperons,
- $\left\langle |\mathcal{M}^2| \right\rangle$ are the angle averaged, squared and summed over initial spinors matrix elements of the reactions calculated from Feynman diagrams and
- $\delta\mu$ is the chemical potential imbalance, δx_n is the difference between the perturbed and equilibrium values of neutron fraction. $\delta\mu/\rho_B \delta x_n$ is calculated using charge neutrality and baryon number conservation

[Lindblom & Owen, PRD 65, 063006 (2002)]

- Thre relevant non leptonic reactions generating bulk viscosity,

$$\begin{aligned} n+n &\longleftrightarrow p+\Sigma^-, \\ n+p &\longleftrightarrow p+\Lambda^0, \\ n+n &\longleftrightarrow n+\Lambda^0. \end{aligned}$$

- Need to determine the factor $\frac{\delta\mu}{n_B \delta x_n}$. Some constrains are imposed on these reactions by charge neutrality and baryon number conservation. They are:
- The electric charge neutrality:

$$\delta x_p - \delta x_\Sigma = 0.$$

- The baryon number conservation:

$$\delta x_n + \delta x_\Lambda + \delta x_p + \delta x_\Sigma = 0.$$

- Chemical potential imbalance relation:

$$\delta\mu \equiv \delta\mu_n - \delta\mu_\Lambda = 2\delta\mu_n - \delta\mu_p - \delta\mu_\Sigma.$$

$$\beta_i = \alpha_{ni} + \alpha_{\Lambda i} - \alpha_{pi} - \alpha_{\Sigma i} \quad \& \quad \alpha_{ij} = \left(\frac{\partial\mu_i}{\partial n_j}\right)_{n_k, k\neq j}.$$

- We obtain the expression for $\delta\mu/(n_B \delta x_n)$ when both the Σ^- and Λ^0 hyperons are present,

$$\frac{\delta\mu}{n_B\delta x_n} = \alpha_{nn} + \frac{(\beta_n - \beta_\Lambda)(\alpha_{np} - \alpha_{\Lambda p} + \alpha_{n\Sigma} - \alpha_{\Lambda\Sigma})}{2\beta_\Lambda - \beta_p - \beta_\Sigma} - \alpha_{\Lambda n} - \frac{(2\beta_n - \beta_p - \beta_\Sigma)(\alpha_{n\Lambda} - \alpha_{\Lambda\Lambda})}{2\beta_\Lambda - \beta_p - \beta_\Sigma}.$$

- For a certain range of densities there are Σ^- hyperons present in equation of state, but no Λ^0. In that case the variable δx_Λ remains zero. So, we get a more simpler expression,

$$\frac{2\delta\mu}{n_B\delta x_n} = 4\alpha_{nn} - 2(\alpha_{pn} + \alpha_{\Sigma n} + \alpha_{np} + \alpha_{n\Sigma}) + \alpha_{pp} + \alpha_{\Sigma p} + \alpha_{p\Sigma} + \alpha_{\Sigma\Sigma}.$$

[Lindblom & Owen, PRD 65, 063006 (2002)]

- Also, if we consider the region where only Λ^0 hyperons are present,

$$\frac{\delta\mu}{n_B\delta x_n} = \alpha_{nn} - \alpha_{\Lambda n} - \alpha_{n\Lambda} + \alpha_{\Lambda\Lambda}.$$

[Chatterjee & Bandyopadhyay PRD 74, 023003 (2006)]

- The general expression for α_{ij}'s using EoS

$$\alpha_{ij} = \frac{m_i^* mi}{\sqrt{k_{Fi}^2 + m_i^{*2}}} \frac{\partial Y}{\partial n_j} + \frac{\pi^2 \delta_{ij}}{k_{Fi}\sqrt{k_{Fi}^2 + m_i^{*2}}} + \frac{g_{\omega i}}{g_{\omega j}} \frac{1}{(Yx_0)^2}$$
$$- \frac{2 g_{\omega i} x_0}{(Yx_0)^3} \Big(\sum_B \frac{\rho_B}{g_{\omega B}} \Big) \frac{\partial Y}{\partial n_j}$$
$$+ \frac{I_{3i} I_{3j} g_{\rho i} g_{\rho j}}{m_\rho^2 [1 - \eta_1 (1 - Y^2) c_\rho / c_\omega]} - \frac{2 \eta_1 m_\rho^2 I_{3i} g_{\rho i} \rho_3^0 Y c_\rho / c_\omega}{m_\rho^2 [1 - \eta_1 (1 - Y^2) c_\rho / c_\omega]} \frac{\partial Y}{\partial n_j}.$$

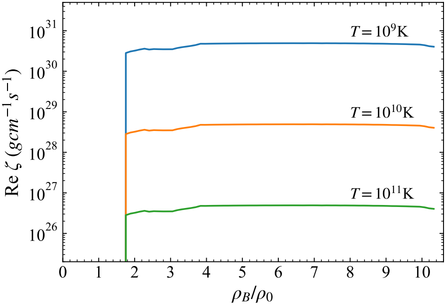


FIGURE: The hyperonic bulk viscosity coefficient Re[ζ] plotted against normalized baryon densities ρ_B/ρ_0

R-mode damping due to Hyperon Bulk Viscosity

- In order to study the nature of damping of the r-modes we need to calculate the time scales associated with the dissipative processes and GR.
- These dissipative effects can damp out the instabilities if the timescales of these effects are comparable to the timescales of gravitational radiation.
- The overall r-mode timescale τ_r is defined in terms of the timescales of the various processes under consideration, namely hyperonic bulk viscosity (B), bulk viscosity due to Urca processes (U), shear viscosity (η) and GR

$$\frac{1}{\tau_r(\Omega,T)} = \frac{1}{\tau_{GR}(\Omega)} + \frac{1}{\tau_B(\Omega,T)} + \frac{1}{\tau_U(\Omega,T)} + \frac{1}{\tau_\eta(\Omega,T)}.$$

- With time scales for each process calculated, we calculate the net r-mode time scale τ_r.
- We solve for the equation $1/\tau_r(\Omega_C,T) = 0$ where Ω_C is defined as the critical angular velocity for a star with temperature T. A star with angular velocity greater than Ω_C will be unstable and subject to GR emission, while one with angular velocity less than Ω_C will be stable.

Timescales

- The time scale of gravitational radiation is given by

$$\frac{1}{\tau_{GR}} = - \frac{32\pi G\Omega^{2l+2}}{c^{2l+3}} \frac{(l-1)^{2l}}{[(2l+1)!!]^2} \left(\frac{l+2}{l+1}\right)^{2l+2} \times \int_0^R \rho(r) r^{2l+2} dr.$$

- The timescale associated with shear viscosity is given by

$$\frac{1}{\tau_\eta} = \frac{(l-1)(2l+1)}{\int_0^R dr \rho(r) r^{2l+2}} \int_0^R dr \eta r^{2l}.$$

Here η is calculated from the prominent nn scattering and is given by

$$\eta = 2 \times 10^{18} \rho_{15}^{9/4} T_9^{-2} \ \mathrm{gcm}^{-1} \ \mathrm{s}^{-1},$$

where $\rho_{15} = \rho/(10^{15}\ \mathrm{g/cm}^3)$ and $T_9 = T/(10^9\ \mathrm{K})$ are the dimensionless density and temperature respectively.

Timescales

Timescale associated with bulk viscosities are given as

$$\frac{1}{\tau_{H,U}} = \frac{4\pi}{690}\left(\frac{\Omega^2 R^{l-4}}{\pi G\bar{\rho}}\right)^2 \left[\frac{\int_0^R \mathrm{Re}[\zeta_{H,U}(r)] \left[1+0.86\left(\frac{r}{R}\right)^2\right] r^8 dr}{\int_0^R \rho(r) r^{2l+2} dr}\right].$$

Bulk viscosity due to modified Urca processes

$$\zeta_U = 146\ \rho(r)^2 \omega^{-2} \left[\frac{k_B T}{1MeV}\right]^6\ \mathrm{gcm}^{-1}\mathrm{s}^{-1},$$

Stellar-Profile

- The Hartle-Thorne approximation is a perturbative approach to determine the structure of slowly rotating stars.

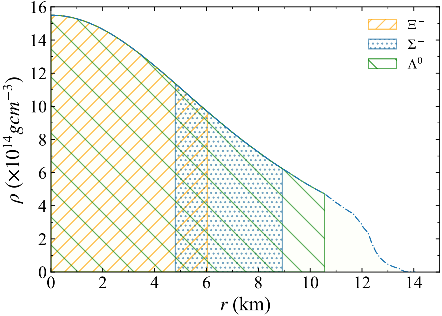


FIGURE: Density profile of the rotating star with threshold radii for hyperons indicated.

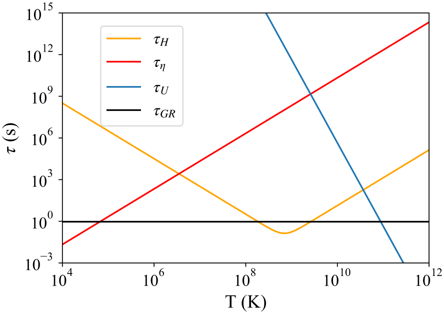


FIGURE: The temperature dependence of damping time scales (in seconds) due to hyperonic bulk viscosity τ_ζ, *modified* Urca bulk viscosity τ_B and shear viscosity τ_η. τ_{GR} represents the temperature independent gravitational radiation time scale. (The star is considered to be rotating with the $Kepler$ frequency Ω_K here).

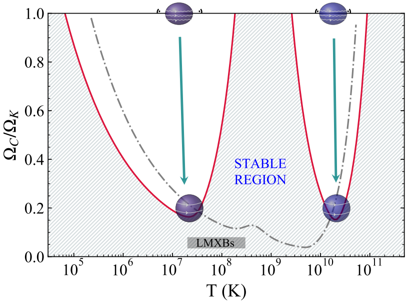


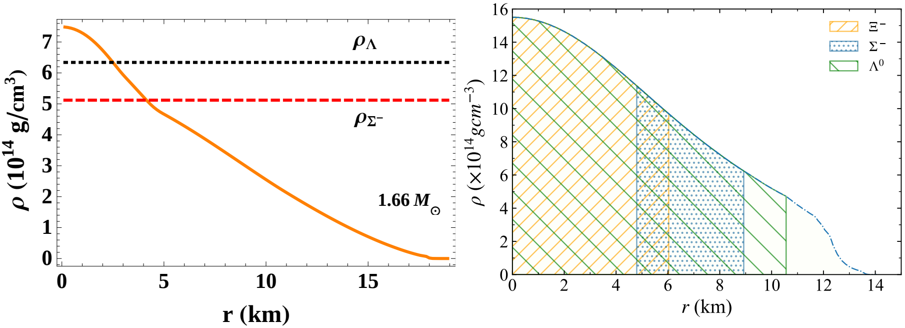
FIGURE: Critical angular velocities in units of Kepler frequency as a function of the core temperature T. The shaded box shows the observed Low Mass X-ray Binaries (LMXBs).

- Here we obtain two instability windows, one in the low temperature and other in the high temperature region.
- Both are separated by a stable region formed by the complete damping of the r-modes due to hyperonic bulk viscosity.

Conclusion

- A model of a rotating neutron star with a hyperon core is considered in the present study using an effective chiral model with cross coupling.
- We obtain a mass of $2.56 M_\odot$ for the pulsar rotating at its Kepler frequency $\Omega_K = 1420$ Hz and the corresponding equatorial radius is 16.5 km.
- We report that there are two separate instability windows for the model considered.
- It is seen that between temperatures of 1.8×10^8K and 2.67×10^9K, the instability is completely suppressed due to hyperon bulk viscosity.
- The minima of the two windows occur at $\sim 0.15\Omega_K$. We also find that the instability can reduce the angular velocity of the star up to $0.3\Omega_K$ as it crosses the first window, and it doesn't lose any angular momentum as it crosses the second window.

THANK YOU



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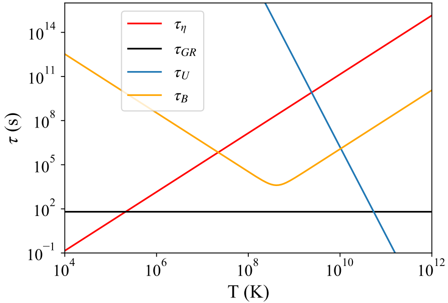

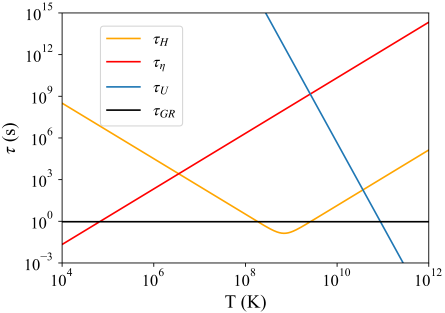

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