Hydrodynamic attractor and thermal particles from heavy ion collisions

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- Results are presented in:
  Hydrodynamical attractor and thermal particle production in heavy-ion collision, [arXiv:2107.08791 [hep-ph]],
  under review in Physical Review C.

Heavy Ion Collisions

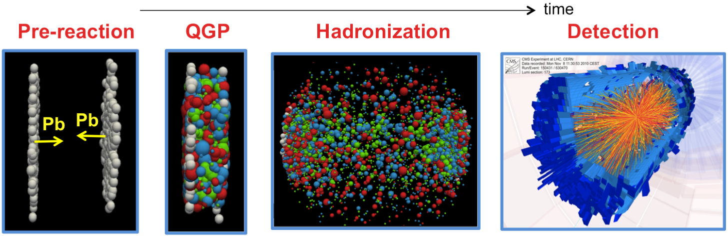

Figure: Schematic sketch of relativistic heavy ion collisions.[http://wl33.web.rice.edu/research.html]

- Relativistic hydrodynamic simulations have been extremely successful in describing the space-time evolution of QGP formed in the early stages of collisions.
- Relativistic viscous hydrodynamics exhibits considerable agreement with the experimental data.

Rel. dissipative hydrodynamics

- Energy-momentum tensor for a viscous fluid :

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - P\Delta^{\mu\nu} + \pi^{\mu\nu},$$

u^μ - fluid 4−velocity
$\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$ - projection operator orthogonal to u^μ
$\pi^{\mu\nu}$ - shear stress tensor

- We consider the conformal case, $\epsilon = 3P$
- Evolution equations for ϵ and u^μ :

$$\begin{aligned} u_\nu \partial_\mu T^{\mu\nu} = 0 \implies \dot{\epsilon} + (\epsilon + P)\theta - \pi^{\mu\nu}\sigma_{\mu\nu} &= 0, \\ \Delta^\alpha_\nu \partial_\mu T^{\mu\nu} = 0 \implies (\epsilon + P)\,\dot{u}^\alpha - \nabla^\alpha P + \Delta^\alpha_\nu \partial_\mu \pi^{\mu\nu} &= 0, \end{aligned}$$

where, $\sigma_{\mu\nu} \equiv \frac{1}{2}(\nabla_\mu u_\nu + \nabla_\nu u_\mu) - \frac{1}{3}\theta\Delta_{\mu\nu}$, $\nabla^\mu \equiv \Delta^{\mu\alpha}\partial_\alpha$

Rel. dissipative hydro. (Contd.)

- Form of $\pi^{\mu\nu}$ need to be specified.
- Relativistic Navier-Stokes theory gives the simplest form of $\pi^{\mu\nu}$:

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu},$$

  η - coefficient of shear viscosity
- Rel. Navier Stokes theory (first order in gradients) shows acausal behaviour
- Causality can be restored by considering higher order gradient corrections
- Minimal causal theory -simplest way to conserve causality : Maxwell-Cattaneo law
  [J. C. Maxwell (1867); C. Cattaneo (1948)]

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}$$

  τ_π - shear relaxation time-scale

Dissipative hydro. from kinetic theory

- Hydrodynamic equations can be derived from kinetic theory

$$T^{\mu\nu}(x) = \int dp p^\mu p^\nu f(x,p), \quad \pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta} \int dp p^\alpha p^\beta \delta f,$$

$f = f_0 + \delta f$

- Rel. Boltzmann equation in the relaxation time approximation is solved iteratively :

$$p^\mu \partial_\mu f = -(u \cdot p) \frac{f - f_{eq}}{\tau_R}$$

- $f = f_0 + \delta f^{(1)} + \delta f^{(2)} + ...$

$$\delta f^{(1)} = -\frac{\tau_R}{u \cdot p} p^\mu \partial_\mu f_0 \qquad \delta f^{(2)} = \frac{\tau_R}{u \cdot p} p^\mu p^\nu \partial_\mu \left(\frac{\tau_R}{u \cdot p} \partial_\nu f_0 \right) ...$$

Second order hydrodynamics

- Substituting $f = f_0 + \delta f^{(1)} + \delta f^{(2)}$, [A. Jaiswal, PRC 87, 051901 (2013)]

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi\sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \frac{10}{7}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma} - \frac{4}{3}\pi^{\mu\nu}\theta.$$

  DNMR theory : [G. S. Denicol, H. Niemi, E. Molnar, D. H. Rischke, PRD 85, 114047 (2012)]

- Minimal causal theory : Muller Israel Stewart (MIS) theory

$$\tau_\pi\dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \frac{4}{3}\tau_\pi\pi^{\mu\nu}\theta.$$

  [I Muller, Z. Phys. 198, 329 (1967); W. Israel , J. M. Stewart, Annals Phys. 118, 341 (1979)]

Third order hydro. formalism

Third order hydro. is obtained by extending the Chapman-Enskog like iterative solution to one higher order [A. Jaiswal, PRC 88, 021903 (2013)]

$$
\begin{aligned}
\dot{\pi}^{\langle\mu\nu\rangle} =& -\frac{\pi^{\mu\nu}}{\tau_\pi} + 2\beta_\pi\sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \frac{10}{7}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma} - \frac{4}{3}\pi^{\mu\nu}\theta \\
&+ \frac{25}{7\beta_\pi}\pi^{\rho\langle\mu}\omega^{\nu\rangle\gamma}\pi_{\rho\gamma} - \frac{1}{3\beta_\pi}\pi_\gamma^{\langle\mu}\pi^{\nu\rangle\gamma}\theta - \frac{38}{245\beta_\pi}\pi^{\mu\nu}\pi^{\rho\gamma}\sigma_{\rho\gamma} \\
&- \frac{22}{49\beta_\pi}\pi^{\rho\langle\mu}\pi^{\nu\rangle\gamma}\sigma_{\rho\gamma} - \frac{24}{35}\nabla^{\langle\mu}\left(\pi^{\nu\rangle\gamma}\dot{u}_\gamma\tau_\pi\right) \\
&+ \frac{4}{35}\nabla^{\langle\mu}\left(\tau_\pi\nabla_\gamma\pi^{\nu\rangle\gamma}\right) - \frac{2}{7}\nabla_\gamma\left(\tau_\pi\nabla^{\langle\mu}\pi^{\nu\rangle\gamma}\right) \\
&+ \frac{12}{7}\nabla_\gamma\left(\tau_\pi\dot{u}^{\langle\mu}\pi^{\nu\rangle\gamma}\right) - \frac{1}{7}\nabla_\gamma\left(\tau_\pi\nabla^\gamma\pi^{\langle\mu\nu\rangle}\right) \\
&+ \frac{6}{7}\nabla_\gamma\left(\tau_\pi\dot{u}^\gamma\pi^{\langle\mu\nu\rangle}\right) - \frac{2}{7}\tau_\pi\omega^{\rho\langle\mu}\omega^{\nu\rangle\gamma}\pi_{\rho\gamma} \\
&- \frac{2}{7}\tau_\pi\pi^{\rho\langle\mu}\omega^{\nu\rangle\gamma}\omega_{\rho\gamma} - \frac{10}{63}\tau_\pi\pi^{\mu\nu}\theta^2 + \frac{26}{21}\tau_\pi\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma}\theta.
\end{aligned}
$$

Björken Flow

- Björken's prescription assumes that the evolution of QGP is only along the initial beam direction. (z-direction)
  [J. D. Björken, PRD 27, 140-151 (1983)]
- Convenient to parameterize the coordinates in terms of proper time, $\tau = \sqrt{t^2 - z^2}$ and space-time rapidity, $\eta_s = \frac{1}{2} \ln \frac{t+z}{t-z}$:

$$
\begin{aligned}
t &= \tau \cosh \eta_s \\
z &= \tau \sinh \eta_s .
\end{aligned}
$$

- Under Björken expansion, hydrodynamic quantities $(\epsilon, P, \pi^{\mu\nu})$ depends only on the proper time (τ).

Longitudinal expansion of QGP

- Relativistic viscous hydrodynamic equations under 1D Björken expansion are obtained as (generic form)

$$\frac{d\epsilon}{d\tau} = -\frac{1}{\tau}\left(\frac{4}{3}\epsilon - \pi\right),$$

$$\frac{d\pi}{d\tau} = -\frac{\pi}{\tau_\pi} + \frac{1}{\tau}\left[\frac{4}{3}\beta_\pi - \left(\lambda + \frac{4}{3}\right)\pi - \chi\frac{\pi^2}{\beta_\pi}\right],$$

	β_π	a	λ	χ	γ
MIS	$4P/5$	$4/15$	0	0	$4/3$
DNMR	$4P/5$	$4/15$	$10/21$	0	$4/3$
Third-order	$4P/5$	$4/15$	$10/21$	$72/245$	$412/147$

Table: Coefficients appearing in Bjorken flow evolution equation of shear stress tensor for the three theories considered in this work.

[S. Jaiswal et al., PRC 100, 034901 (2019)]

Longitudinal expansion of QGP

- Relativistic viscous hydrodynamic equations under 1D Björken expansion are obtained as

$$
\begin{aligned}
\frac{1}{\epsilon\tau^{4/3}}\frac{d(\epsilon\tau^{4/3})}{d\tau} &= \frac{4}{3}\frac{\bar{\pi}}{\tau}, \\
\frac{d\bar{\pi}}{d\tau} &= -\frac{\bar{\pi}}{\tau_\pi} + \frac{1}{\tau}\left(a - \lambda\bar{\pi} - \gamma\bar{\pi}^2\right),
\end{aligned}
$$

where $\beta_\pi = 4P/5$ and $\bar{\pi} = \pi/(\epsilon + P)$.

Longitudinal expansion of QGP

- The equations can be analytically solved for certain approximations of τ_π.
- For conformal system $\epsilon \propto T^4$ and from kinetic theory $T\tau_\pi = 5(\eta/s) =$const.
- Three cases are considered with $\tau_\pi \propto 1/T$, where T is either a const. or has proper time evolution of ideal hydro. or Navier-Stokes hydrodynamic solutions

Approx. analytical solutions

ϵ and $\bar{\pi}$ evolution are obtained in the generic form :

$$\bar{\pi}(\bar{\tau}) = \frac{(k+m+\frac{1}{2})M_{k+1,m}(w) - \alpha\, W_{k+1,m}(w)}{\gamma|\Lambda|\,[M_{k,m}(w) + \alpha\, W_{k,m}(w)]},$$

$$\epsilon(\bar{\tau}) = \epsilon_0 \left(\frac{w_0}{w}\right)^{\frac{4}{3}\left(|\Lambda|-\frac{k}{\gamma}\right)} e^{-\frac{2}{3\gamma}(w-w_0)} \left(\frac{M_{k,m}(w) + \alpha\, W_{k,m}(w)}{M_{k,m}(w_0) + \alpha\, W_{k,m}(w_0)}\right)^{\frac{4}{3\gamma}},$$

where, $\bar{\tau} \equiv \tau/\tau_\pi$ is the scaled proper-time variable, and $M_{k,m}(w)$ and $W_{k,m}(w)$ are Whittaker functions.

$T(\tau)$	w	Λ	k	m
const.	$\bar{\tau}$	-1	$-\frac{1}{2}(\lambda+1)$	$\frac{1}{2}\sqrt{4a\gamma+\lambda^2}$
ideal	$\frac{3}{2}\bar{\tau}$	$-\frac{3}{2}$	$-\frac{1}{4}(3\lambda+2)$	$\frac{3}{4}\sqrt{4a\gamma+\lambda^2}$
NS	$\frac{3}{2}\left(\bar{\tau}+\frac{a}{2}\right)$	$-\frac{3}{2}$	$-\frac{1}{4}\left[3\left(\lambda-\frac{a}{2}\right)+2\right]$	$\frac{3}{4}\sqrt{4a\gamma+\left(\lambda-\frac{a}{2}\right)^2}$

[S. Jaiswal et al., PRC 100, 034901 (2019)]

Hydrodynamic attractor

- Attractor : set of states toward which a system tend to evolve for a variety of initial conditions
- The procedure for identifying the hydrodynamic attractor was to look for the value α_0 at which the following quantity diverges :

$$\psi(\alpha_0) \equiv \lim_{\bar{\tau}\to 0} \frac{\partial \bar{\pi}}{\partial \alpha}\bigg|_{\alpha=\alpha_0}$$

  This holds true for $\alpha = 0 \implies$ corresponds to attractor
- Equation for repulsor curve is obtained by considering $\bar{\tau} = \infty$ and look for the value α_0 at which ψ diverges $\implies \alpha = \infty$

Hydrodynamic attractor

- Equations can be decoupled by introducing the variables : $\bar{\pi} = \pi/(\epsilon + P)$ and $\bar{\tau} = \frac{\tau}{\tau_\pi}$:

$$\left(\frac{\bar{\pi}+2}{3}\right)\frac{d\bar{\pi}}{d\bar{\tau}} = -\bar{\pi} + \frac{1}{\bar{\tau}}\left(a - \lambda\,\bar{\pi} - \gamma\,\bar{\pi}^2\right),$$

- Initial condition for attractor is obtained by imposing the boundary condition $\bar{\pi}$ and $d\bar{\pi}/d\bar{\tau}$ remain finite as $\bar{\tau} \to 0$.
- Quadratic eqn. : $\gamma\bar{\pi}^2 + \lambda\bar{\pi} - a = 0$ for initial value of $\bar{\pi}$
- Has two solutions : positive root is stable and corresponds to the attractor initial condition and the negative root corresponds to repulsor

Hydrodynamic attractor

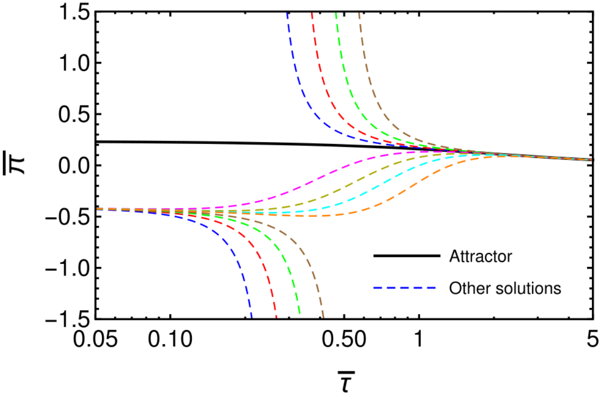

Figure: Attractor behaviour of approximate analytical solution for third order theory.

[S. Jaiswal et al., PRC 100, 034901 (2019)]

Hydrodynamic attractor

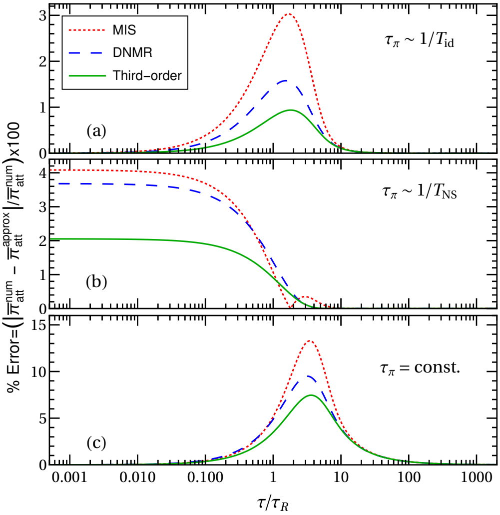

Figure: Comparison of numerical and analytical attractors for different theories.

Thermal particles

- We study the effect of hydrodynamic attractors of causal boost invariant hydrodynamics on thermal particle production from heavy ion collisions
- Thermal particles (dileptons & photons) emitted from QGP act as the most prominent tools to provide information about the hot fireball.
- Viscosity of the QGP can be studied by analysing the thermal particle emission. [J. R. Bhatt et al., Nucl.Phys.A 875 (2012) 181-196; J. R. Bhatt et al., JHEP 11 (2010) 106]

Thermal dilepton production

- From kinetic theory, the dilepton production rate for $q\bar{q}$ annihilation can be written as

$$\frac{dN_{l^+l^-}}{d^4x d^4p} = g^2 \int \frac{d^3\mathbf{p}_1}{(2\pi)^3}\frac{d^3\mathbf{p}_2}{(2\pi)^3} f(E_1, T) f(E_2, T) \times v_{rel}\sigma(M^2)\delta^4(p - p_1 - p_2).$$

- Effect of viscosity enters through distribution function, $f = f_0 + \delta f$.
- Form of distribution function is derived from Chapman-Enskog method

$$\delta f = \frac{f_0 \beta}{2\beta_\pi (u \cdot p)} p^\alpha p^\beta \pi_{\alpha\beta},$$

where $\beta = 1/T$ and $\beta_\pi = (\epsilon + P)/5$. [R. S. Bhalerao et al., PRC 89, 054903 (2014).]

Thermal dilepton production

- $$\frac{dN_{l^+l^-}}{d^4x d^4p} = \frac{dN^0_{l^+l^-}}{d^4x d^4p} + \frac{dN^\pi_{l^+l^-}}{d^4x d^4p},$$

  with

  $$\begin{aligned}
  \frac{dN^0_{l^+l^-}}{d^4x d^4p} &= \frac{1}{2}\frac{M^2 g^2 \sigma\left(M^2\right)}{(2\pi)^5} e^{-u\cdot p/T}\,, \\
  \frac{dN^\pi_{l^+l^-}}{d^4x d^4p} &= \frac{dN^0_{l^+l^-}}{d^4x d^4p}\left\{\frac{5\beta}{2\left[(u\cdot p)^2 - M^2\right]^{5/2}}\right. \\
  &\times\left[\frac{(u\cdot p)\sqrt{(u\cdot p)^2 - M^2}}{2}\left(2(u\cdot p)^2 - 5M^2\right)\right. \\
  &\left.\left. + \frac{3}{4}M^4 \ln\left(\frac{u\cdot p + \sqrt{(u\cdot p)^2 - M^2}}{u\cdot p - \sqrt{(u\cdot p)^2 - M^2}}\right)\right]\right\} \times p^\alpha p^\beta \bar{\pi}_{\alpha\beta},
  \end{aligned}$$

  where $\bar{\pi}_{\alpha\beta} = 5\pi_{\alpha\beta}/\beta_\pi$.

Photon production rate

- Total photon production rate including the Compton scattering and $q\bar{q}$ annihilation is

$$
\begin{aligned}
E\frac{dN_\gamma}{d^4xd^3p} &= E\frac{dN_\gamma^0}{d^4xd^3p} + E\frac{dN_\gamma^\pi}{d^4xd^3p} \\
E\frac{dN_\gamma^0}{d^4xd^3p} &= \frac{5}{9}\frac{\alpha_e\alpha_s}{2\pi^2}T^2 e^{-u\cdot p/T}\left[\ln\left(\frac{12(u\cdot p)}{g^2T}\right) + \frac{C_{ann}+C_{Comp}}{2}\right], \\
E\frac{dN_\gamma^\pi}{d^4xd^3p} &= E\frac{dN_\gamma^0}{d^4xd^3p}\left\{\frac{5\beta}{2(u\cdot p)}\right\}p^\alpha p^\beta \bar{\pi}_{\alpha\beta},
\end{aligned}
$$

where $g = \sqrt{4\pi\alpha_s}$, $C_{ann} = -1.9163$, $C_{Comp} = -0.41613$

Thermal particle yield

Once we obtain the temperature profile of the medium, thermal dilepton and photon yields can be computed by integrating the rates over space-time history of the QGP expansion.

$$
\begin{aligned}
\frac{dN_{l^+l^-}}{dM^2 d^2p_T dy} &= \pi R_A^2 \int_{\tau_0}^{\tau_f} d\tau \; \tau \int_{-\infty}^{\infty} d\eta_s \left(\frac{1}{2} \frac{dN_{l+l-}}{d^4x d^4p} \right), \\
\frac{dN_\gamma}{d^2p_T dy} &= \pi R_A^2 \int_{\tau_0}^{\tau_f} d\tau \; \tau \int_{-\infty}^{\infty} d\eta_s \left(E \frac{dN_\gamma}{d^3p d^4x} \right).
\end{aligned}
$$

Thermal dilepton yield

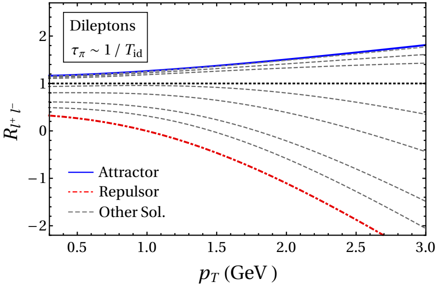

Figure: Ratio of viscous to ideal dilepton spectra ($\tau_\pi \sim 1/T_{id}$).

$$R_{l^+l^-} = \left(\frac{dN_{l^+l^-}}{dM^2 d^2 p_T dy}\right) \Big/ \left(\frac{dN^i_{l^+l^-}}{dM^2 d^2 p_T dy}\right)$$

- Dilepton invariant mass, $M = 1$ GeV
- The yield corresponding to attractor gets the maximum enhancement and the one corresponding to repulsor suffers maximum suppression.

Thermal photon yield

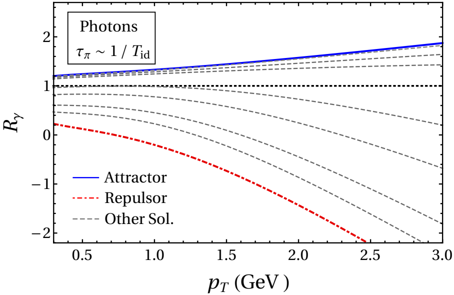

Figure: Ratio of viscous to ideal photon spectra ($\tau_\pi \sim 1/T_{id}$).

$$R_\gamma = \left(\frac{dN_\gamma}{d^2 p_T dy}\right) \Big/ \left(\frac{dN^i_\gamma}{d^2 p_T dy}\right)$$

Summary

- We analyzed the analytical attractor solutions of relativistic viscous hydrodynamics for 1D expansion.
- We studied the effect of hydrodynamic attractor of causal dissipative hydrodynamics on thermal particle emission in the context of heavy ion collisions
- Thermal particle production rates are calculated in the presence of viscous modified distribution functions
- Thermal particle spectra is studied by using hydrodynamic attractors for the evolution of the plasma
- The evolution corresponding to attractor solution leads to maximum production of thermal particles.

THANK YOU