Study of f-mode oscillations of hyperonic star within an effective chiral model

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Workshop on the QCD equation of state in dense matter HIC and astrophysics - MAGIC23

March 29, 2023

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Introduction

- Neutron stars are formed upon the collapse of supergiant stars of masses 10 - $25 M_{\odot}$ and they have radii of 10-12 km.
- The properties of neutron stars like its mass and radius depend heavily on their internal composition.
- Neutron stars are subject to various fluid oscillations, each of which can be used to constrain the properties of neutron star matter.
- In particular, the f-mode oscillations are a potential candidate for detection by GW emission.
- This could provide us valuable information about the internal composition of the neutron star.

Model

The Lagrangian density in an effective chiral model with cross coupling effects under consideration is:

$$\mathcal{L} = \bar{\psi}_B \Big[\Big(i\gamma_\mu \partial^\mu - g_\omega \gamma_\mu \omega^\mu - \frac{1}{2} g_\rho \vec{\rho_\mu}.\vec{\tau}\gamma^\mu \Big) - g_\sigma (\sigma + i\gamma_5 \vec{\tau}.\vec{\pi}) \Big] \psi_B + \frac{1}{2} (\partial_\mu \vec{\pi}.\partial^\mu \vec{\pi} + \partial_\mu \sigma \partial^\mu \sigma) - \frac{\lambda}{4} \left(x^2 - x_0^2 \right)^2 - \frac{\lambda b}{6m^2} (x^2 - x_0^2)^3 - \frac{\lambda c}{8m^4} (x^2 - x_0^2)^4 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g_\omega^2 x^2 \left(\omega_\mu \omega^\mu \right) - \frac{1}{4} \vec{R_{\mu\nu}}.\vec{R^{\mu\nu}} + \frac{1}{2} {m_\rho'}^2 \vec{\rho_\mu}.\vec{\rho^\mu} + \eta_1 \left(\frac{1}{2} g_\rho^2 x^2 \vec{\rho_\mu}.\vec{\rho^\mu} \right)$$

[Malik et. al., Phys. Rev. C 96, 035803]

Model

- ψ_B is the nucleon isospin doublet interacting with different mesons σ, ω and ρ, with the respective coupling strengths g_i, with $i = \sigma, \omega$ and ρ.
- The b and c are the strength for self couplings of scalar fields.
- The γ^μ are the Dirac matrices and τ are the Pauli matrices.
- Potential for the scalar fields (π, σ) are written in terms of a chiral invariant field x given by $x^2 = \pi^2 + \sigma^2$.
- The term in blue colour contains cross-coupling terms between ρ and ω and also between ρ and σ. The coupling strength for $\sigma - \rho$ is given by $\eta_1 g_\rho^2$.

Field Equations

- Given a Lagrangian, it is possible to calculate the equation of state using the relativistic mean field approximation.
- The equations of motion for each of the fields φ can be obtained from the Euler–Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial \varphi} = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right).$$

- The field equations can be calculated as:

i. The vector field (ω):

$$\sum_B g_{\omega B} \rho_B = m_\omega^2 Y^2 \omega_0,$$

Field Equations

ii. The Isovector field (ρ_3^0)

$$\sum_B I_{3B} g_{\rho B} \rho_B = m_\rho^2 \Big[1 - \eta_1 (1 - Y^2) C_\rho / C_\omega \Big] \rho_3^0,$$

iii. The Scalar field (σ)

$$\sum_B \frac{2 C_{\sigma B} \rho_{sB}}{m_B Y} = (1 - Y^2) - \frac{b}{m^2 C_\omega} (1 - Y^2)^2 + \frac{c}{m^4 C_\omega^2} (1 - Y^2)^3 + \frac{2 C_{\sigma N} m_\omega^2 \omega_0^2}{m_N^2} + \frac{2 \eta_1 C_{\sigma N} C_\rho m_\rho^2 (\rho_3^0)^2}{C_\omega m_N^2}.$$

Equation of State

- The energy density (ε) and pressure (p) for a given baryon density (in terms of $Y = m^*/m$) in this model is obtained as

$$
\begin{aligned}
\varepsilon &= \frac{1}{\pi^2}\sum_{k_n,k_p}\int_0^{k_F} k^2\sqrt{k^2+m^{*2}}dk + \frac{m^2}{8C_\sigma}(1-Y^2)^2 \\
&- \frac{b}{12C_\sigma C_\omega}(1-Y^2)^3 + \frac{c}{16m^2C_\sigma C_\omega^2}(1-Y^2)^4 + \frac{1}{2}m_\omega^2\omega_0^2Y^2 \\
&+ \frac{1}{2}m_\rho^2\Big[1-\eta_1(1-Y^2)(C_\rho/C_\omega)\Big](\rho_3^0)^2, \\
p &= \frac{1}{3\pi^2}\sum_{k_n,k_p}\int_0^{k_F} \frac{k^4}{\sqrt{k^2+m^{*2}}}dk - \frac{m^2}{8C_\sigma}(1-Y^2)^2 \\
&+ \frac{b}{12C_\sigma C_\omega}(1-Y^2)^3 - \frac{c}{16m^2C_\sigma C_\omega^2}(1-Y^2)^4 + \frac{1}{2}m_\omega^2\omega_0^2Y^2 \\
&+ \frac{1}{2}m_\rho^2\Big[1-\eta_1(1-Y^2)(C_\rho/C_\omega)\Big](\rho_3^0)^2.
\end{aligned}
$$

Equation of State

- The neutron star matter is in β-equilibrium and also charge neutral and therefore one needs to impose the following conditions:

$$\sum_B Q_B \rho_B + \sum_l Q_l \rho_l = 0,$$
$$\mu_B = \mu_n - Q_B \mu_e,$$

- The subscript l denotes leptons (electron, muon) and Q_B and Q_l are electric charges. μ denotes chemical potentials.
- The two conditions above determine when the said baryons appear in the core of the star and also its concentration. This determines the EoS of the model.

Result: EoS

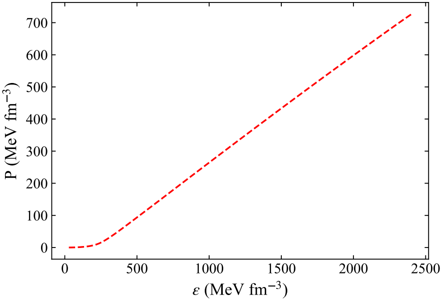


Figure: Pressure as a function of energy density of the model under consideration.

[Malik et. al., Phys. Rev. C 96, 035803]

Static Stars - Tolman-Oppenheimer-Volkoff (TOV) Equations

- We may consider the most general form of the metric of a static, spherically symmetric spacetime:

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right).$$

- The Tolman-Oppenheimer-Volkoff (TOV) equations are a set of two coupled differential equations in the variables $p(r)$ and $m(r)$.

$$\frac{dp}{dr} = -\frac{[\varepsilon + p]\left[m + 4\pi r^3 p\right]}{r(r-2m)},$$
$$\frac{dm}{dr} = 4\pi r^2 \varepsilon.$$

Static stellar structure

- The solution of the TOV equations for a specific EoS $\varepsilon = \varepsilon(p)$ is determined by two boundary conditions. One is the central pressure p_c (or equivalently ε_c). The other is the mass at the center which is taken to be zero.
- With a specified central density, the TOV equations can be integrated outwards starting at the center. The radius R of the star is defined as the distance at which the pressure drops to zero, i.e. $p(r = R) = 0$. The mass of the star is then given by $M = m(R)$.

Result: Stellar Structure

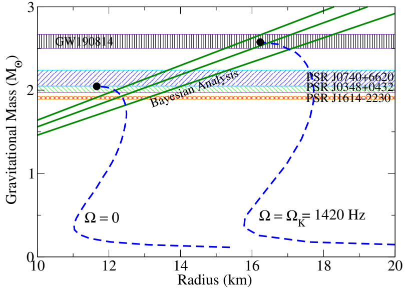


Figure: Gravitational mass vs. equatorial radius of the star obtained using the RNS code, compared with the mass range of massive known pulsars (1.88 - 2.24$M_\odot$) and the inferred mass range of the secondary component of the GW190814 data. The compactness parameter estimate from Bayesian analysis (Riley et al. 2019) is also indicated.

Our results are in agreement with some of the massive pulsars such as the millisecond pulsar $PSR\ J0740+6620$ (Cromartie et al. 2019) $PSR\ J0348+0432$ (Antoniadis et al. 2013)and $PSR\ J1614-2230$ (Demorest et al. 2010).

F-mode Instability

- Neutron stars are subjected to various oscillations, which are in general classified according to the restoring force acting to bring back the system to equilibrium position.
- These oscillation modes can be damped by the GWs emmission and such modes of oscillations are called quasinormal modes (QNMs).
- QNMs are classified into polar and axial modes. The polar modes include the fundamental (f) modes, pressure (p) modes, and gravity (g) modes. The axial modes include r-mode and spacetime (w) modes.
- These modes provides the information about the internal structure of neutron stars.

- The f-modes are the fundamental modes of oscillation. The f-mode frequencies are much lower than other modes. Thus they maybe detectable with the current generation of GW detectors.
- We aim to obtain the f-mode frequency using Cowling approximation.

$$\frac{dW(r)}{dr} = \frac{d\epsilon}{dp}\left[\omega^2 r^2 e^{\Lambda(r)-2\phi(r)} V(r) + \frac{d\Phi(r)}{dr} W(r)\right] - l(l+1)e^{\Lambda(r)} V(r),$$

$$\frac{dV(r)}{dr} = 2\frac{d\Phi(r)}{dr} V(r) - \frac{1}{r^2} e^{\Lambda(r)} W(r).$$

- Where $\frac{d\Phi(r)}{dr} = \frac{-1}{\epsilon(r)+p(r)} \frac{dp}{dr}$ and $e^{2\Lambda} = \frac{r}{r-2m(r)}$.
- The W and V are the perturbation functions. We have to solve these differential equations to obtain W and V as a function of r.

[Bikram Keshari Pradhan and Debarati Chatterjee, Phys. Rev. C 103, 035810]

- Boundary conditions (BC) are as follows:
  Towards center of the star ($r = 0$) ,

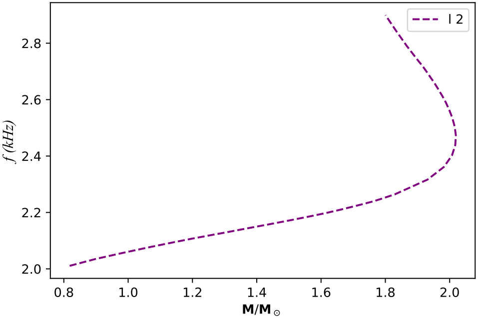
$$W(r) = Ar^{l+1},\ V(r) = -\frac{A}{l}r^{l}.$$

  Here, A is an arbitrary constant and l can take values 2,3 or 4. The differential equations are to be solved with some initial guess for ω^2.
- ω^2 need to satisfy the surface BC

$$\omega^2 e^{\Lambda(R)-2\Phi(R)}V(R) + \frac{1}{R^2}\frac{d\Phi(r)}{dr}\Big|_{r=R}W(R) = 0.$$

- The equations need to be integrated from center to surface (r to R) and we try to match the surface BC.
- After each integration the initial guess of ω^2 is to be changed and the calculations are repeated until the surface BC is satisfied.

Results: Cowling Approximation



We find that our lower limit of f -mode frequency is in agreement with the limit obtained from observations of GW170817. G. Pratten et. al *Nature Commun. 11, 2553 (2020)*

The f-mode analysis of $Sly4$ EoS is given in Hong-Bo Li et. al *Mon. Not. Roy. Astron. Soc. 516 (2022) 4, 6172-6179.*

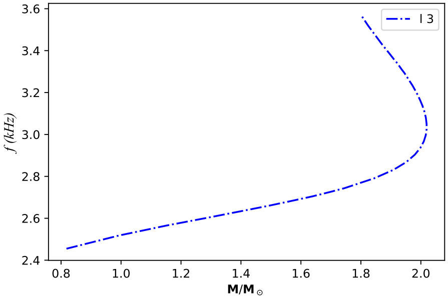


FIGURE: f-mode frequencies as a function of Neutron star Mass for $l = 3$

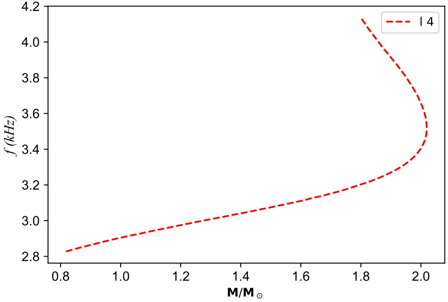


FIGURE: f-mode frequencies as a function of Neutron star Mass for $l = 4$

Conclusion

- We considered a hyperon NS in an effective chiral model with cross coupling effects.
- We studied the stellar structure for the obtained EoS. Our results were found to agree well with some of the massive pulsars.
- Then we studied the f-mode oscillation using the Cowling approximation for model considered.

THANK YOU