1-Loop correction to the Muon $g-2$ in Supersymmetric QED: Why we look at the Muon

IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

Francisco N. C. Dahab [1] Advisor: Ricardo D'Elia Matheus [2]

[1]Institute of Physics (USP)/ Institute of Theoretical Physics (IFT/UNESP) [2]Institute of Theoretical Physics (IFT/UNESP)

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Abstract

Whenever a photon and fermions coexist in a theory, the need to preserve gauge invariance forces us to build a gauge covariant derivate, which in turn requires the photon and fermion to couple in a very specific form. For just one fermion (say the muon) the resulting Lagrangian then has the known form

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \bar{\psi}(i\not{D} - m)\psi \tag{1}$$

This Lagrangian has many testable predictions, but perhaps one of the most known is that it implies that the Muon must have an intrinsic magnetic dipole moment $\propto g\vec{S}$ ($\vec{S}$ being the spin). At tree level, one can readily verify that $g = 2$ exactly, however, as one might expect, renormalization due to loops leads us to find a correction (the $g-2$), the value of which, for the electron, is one of the best measured quantities in all of physics.

The aim here is to consider an extension of the Standard Model, particularly, the Supersymmetry extension to the QED sector. Because this extension couples to the original SM, new processes are, in principle, possible, which in turn should contribute with new corrections to the $g-2$. We can then measure difference between the experimental value and the value predicted by the SM, then working under the assumption that the difference is due to SUSY, estimate bounds for where the SUSY scale is.

Finding g in standard QED

In order to understand the SUSY correction, it's important to stress where the correction comes from in usual QED. To see it, notice that the equation of motion for the quantum field, obtained from the Lagrangian in (1), $(i\not{D} - m)\psi = 0$, can be written as $(\not{D}^2 + m^2)\psi = 0$. Now, remember that the gauge covariant derivative is given by

$$D_\mu = \partial_\mu + ieA_\mu$$

Which in turn allows us to write

$$\not{D}^2 = D_\mu^2 + \frac{e}{2}F_{\mu\nu}\sigma^{\mu\nu}$$

where $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$. The second term, coupling $F_{\mu\nu}$ to $\sigma^{\mu\nu}$ is where the magnetic moment comes from, the Lorentz generators in the (Dirac) spinor representation are given by $S^{\mu\nu} = \frac{1}{2}\sigma^{\mu\nu}$, this term then expresses the fact that the magnetic field couples to rotations in spinor space, this is exactly what an intrinsic magnetic moment does.

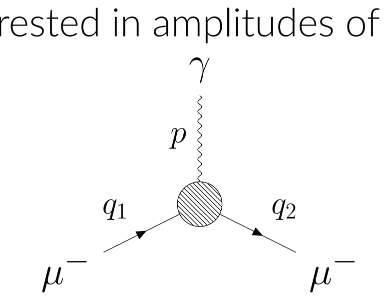
To be explicit, we can then use the expression above into $(\not{D}^2 + m^2)\psi = 0$, and find, after some manipulation

$$\frac{(H - eA_0)^2}{2m} = \left[\frac{m}{2} + \frac{(\vec{p} - e\vec{A})^2}{2m} - \mathbf{2}\frac{e}{2m}\vec{B}\cdot\vec{S} \pm i\vec{E}\cdot\vec{S}\right]\psi$$

Indeed, we find that $g = 2$ exactly. But notice that if loops contribute with terms proportional to $F_{\mu\nu}\sigma^{\mu\nu}$, then renormalization of our theory will give new contributions to g.

Now, corrections to the $g-2$ are really corrections to how photons interact with spinors, and hence, we need to look, in general, at *off-shell* S-matrix elements involving two spinors and a photon. In this particular case however, because we are particularly interested in muons in experiments, the two spinors are really just an incoming muon and one outgoing muon, which we allow to be *on-shell*. We are then interested in amplitudes of the form

$i\mathcal{M}^\mu \quad =$



There is, however, a strong restriction on what this amplitude can depend on. Due to Lorentz covariance, it can, in its most general form be given by

$$i\mathcal{M}^\mu = \bar{u}(q_2)\left(a\gamma^\mu + bp^\mu + cq_1^\mu + dq_2^\mu\right)u(q_1)$$

Hence, through manipulations of these terms, using the *on-shell* condition for the muons and Ward identities, one can arrive at a completely general expression for the form of this amplitude

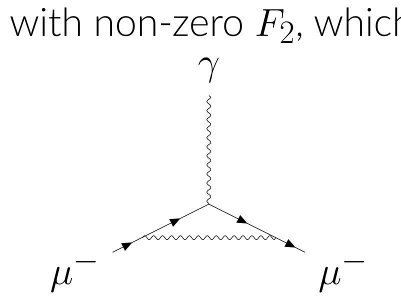
$$i\mathcal{M}^\mu = (-ie)\bar{u}(q_2)\left[F_1\left(\frac{p^2}{m^2}\right)\gamma^\mu + \frac{i\sigma^{\mu\nu}}{2m}p_\nu F_2\left(\frac{p^2}{m^2}\right)\right]u(q_1) \tag{2}$$

The F_2 term has precisely the form we are looking for. The momentum becomes a derivative in position space which will (after some algebra), generate the $F_{\mu\nu}$ term. Now, since the original value, at tree level, of g was 2, and because measurements of g are done at non-relativistic energies (where $p^2/m^2 \to 0$), we can write

$$\boxed{g - 2 = 2F_2(0)} \tag{3}$$

Notice how at tree-level, the contribution to the amplitude is the known $-ie\bar{u}(q_2)\gamma^\mu u(q_1)$, which comparing to (2), gives $F_1 = 1$ and $F_2 = 0$, recovering $g = 2$.

Now, we must consider loops. At 1-loop, thankfully there's only one diagram that will contribute with non-zero F_2, which is the following



$$= -e^3\bar{u}(q_2)\int\frac{d^4k}{(2\pi^4)}\frac{\gamma^\nu(\not{p} + \not{k} + m)\gamma^\mu(\not{k} + m)\gamma_\nu}{\left[(k-q_1)^2 + i\epsilon\right]\left[(p+k)^2 + i\epsilon\right]\left[k^2 - m^2 + i\epsilon\right]}u(q_1)$$

One can then show, that this will yield [2]

$$F_2(p^2) = \frac{\alpha}{\pi}m^2\int_0^1 \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\delta(x+y+z-1)\frac{z(1-z)}{(1-z)^2m^2 - xyp^2} \quad \Rightarrow \quad F_2(0) = \frac{\alpha}{2\pi}$$

where $\alpha = e^2/4\pi$

The QED correction

With this result, we can then find using (3), the beautiful 1-loop correction to the $g-2$ due to QED

$$\boxed{(g-2)_{\text{QED}} = \frac{\alpha}{\pi}} \tag{4}$$

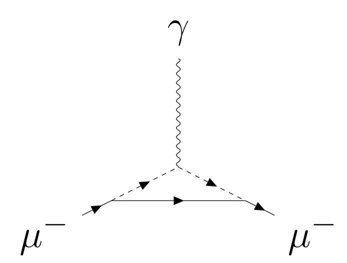
Crucially, this correction does not differentiate between electrons and muons (or taus for that matter), there is no mass dependency, this only depends on their electric charge, which is the same for the three charged leptons.

Onwards to Supersymmetry

Supersymmetry is a candidate extension to the Standard Model. Strictly speaking we're really extending the traditional Poincaré Algebra upon which the Standard Model is built, into a Super-Poincaré Algebra, but let's not go that far. For what we're interested in, it's enough to understand that the SUSY QED sector (we're ignoring the tau for simplicity) adds three new particles, partners, to every pre-existing particle. The electron gets a scalar partner, the *selectron* ($\tilde{e}$), the muon, a scalar *smuon* ($\tilde{\mu}$), and the photon, a fermionic *photino* ($\tilde{A}$). The new Lagrangian, now adopting λ as the electric charge so as to avoid confusion, then becomes [2]

$$\mathcal{L}_{\text{SUSY}} = \mathcal{L}_{\text{QED}} + (\partial_\mu\tilde{e} + i\lambda A_\mu\tilde{e})^\star(\partial_\mu\tilde{e} + i\lambda A_\mu\tilde{e}) + m_{\tilde{e}}^2|\tilde{e}|^2 + \lambda\bar{e}\tilde{e}\tilde{A} + \lambda\tilde{e}^\star\bar{\tilde{A}}e$$
$$+(\partial_\mu\tilde{\mu} + i\lambda A_\mu\tilde{\mu})^\star(\partial_\mu\tilde{\mu} + i\lambda A_\mu\tilde{\mu}) + m_{\tilde{\mu}}^2|\tilde{\mu}|^2 + \lambda\bar{\mu}\tilde{\mu}\tilde{A} + \lambda\tilde{\mu}^\star\bar{\tilde{A}}\mu + \bar{\tilde{A}}(\not{\partial} + m_{\tilde{A}})\tilde{A}$$

This might look intimidating, but for our purposes it isn't. For the muon $g-2$, this only adds one new process at 1-loop, which is completely analogous to the previous one, but where the virtual particles are now smuons and photinos



$$= \int\frac{d^4k}{(2\pi^4)}\frac{-\lambda^3\bar{u}(q_2)(2k^\mu + p^\mu)u(q_1)}{[(k-q_1)^2 - m_{\tilde{A}}^2 + i\epsilon][(p+k)^2 - m_{\tilde{\mu}}^2 + i\epsilon][k^2 - m_{\tilde{\mu}}^2 + i\epsilon]}$$

This can be evaluated to find F_2, but the result is not quite as nice as in standard QED, we find

$$F_2(0) = \frac{\alpha}{2\pi}m_\mu\int_0^1 \mathrm{d}z\frac{(1-z)\left[z(1-z)m_\mu + m_{\tilde{A}}z\right]}{(1-z)(m_{\tilde{\mu}}^2 - m_\mu^2) - (z-1)^2m_\mu^2 + zm_{\tilde{A}}^2}$$

The supersymmetry correction

With this result have now found the SUSY QED correction to the $g-2$, it is

$$\boxed{(g-2)_{\text{SUSY}} = \frac{\alpha}{\pi}\int_0^1 \mathrm{d}z\frac{m_\mu(1-z)\left[z(1-z)m_\mu + m_{\tilde{A}}z\right]}{(1-z)(m_{\tilde{\mu}}^2 - m_\mu^2) - (z-1)^2m_\mu^2 + zm_{\tilde{A}}^2}} \tag{5}$$

Crucially, there is one key difference with this result, **it is sensitive to the mass of the muon**, if we had done the same calculation with the electron, we would obtain a **different** correction. As we shall see, for the regime we're interested in, the correction is really proportional to the mass of the muon, and hence, the effects of new physics, in this case, are ~ 200 times stronger for the muon than they are for the electron.

Effects like this are often very common whenever we have new particles due to Beyond the Standard Model physics, and hence, this is why we look at the Muon, we have a better chance of finding discrepancies should they exist

Estimating the SUSY scale

Now imagine we make a supposition, that the difference between the experimental value of the muon $g-2$, and the theoretical value due to the full Standard Model (there are other contributions due to the weak sector and QCD sector that were not covered here), can be accounted by the supersymmetry correction obtained. This analysis is not simple if we use the full form given in (5), thanfully, we don't have to. Using that SUSY must be broken, we don't see supersymmetric partners in our daily lives, and as such we must take $m_\mu \ll m_{\tilde{A}} \sim m_{\tilde{\mu}} \equiv m_{\text{SUSY}}$, we find that (5) becomes

$$(g-2)_{\text{SUSY}} = \frac{\alpha m_\mu}{6\pi m_{\text{SUSY}}}$$

Now under the supposition made, this should account for the difference between g_{exp} and g_{SM}. Defining then the difference between theory and experiment, $\Delta g = g_{\text{exp}} - g_{\text{SM}}$, in order for our hypothesis to be justified, we must have

$$m_{\text{SUSY}} > \frac{\alpha m_\mu}{6\pi\Delta g}$$

Using recent values, obtained from FermiLab, as well as the most recent full Standard Model calculation to find Δg, we find a lower bound for the SUSY scale at $\sim 8\,\text{TeV}$

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fran.ncd@usp.br