A brief view of Particle Physics: an introduction to Field Theories

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The matter's LEGO©

From the ancient Greece and India philosophers until John Dalton's theory about the matter and the modern atom model, the humanity always sought for the answer to the primordial question: *what are we made of*? The world atom comes from the greek analogous ἄτομος, "indivisible", as that is what we think they were at the time of their discovery, and are the most abundant constituents of matter. They are made of parts, given by a nucleus (made by protons and neutrons, which in turn, are hadrons) encircled by the electron cloud, and are classified by their number of protons.

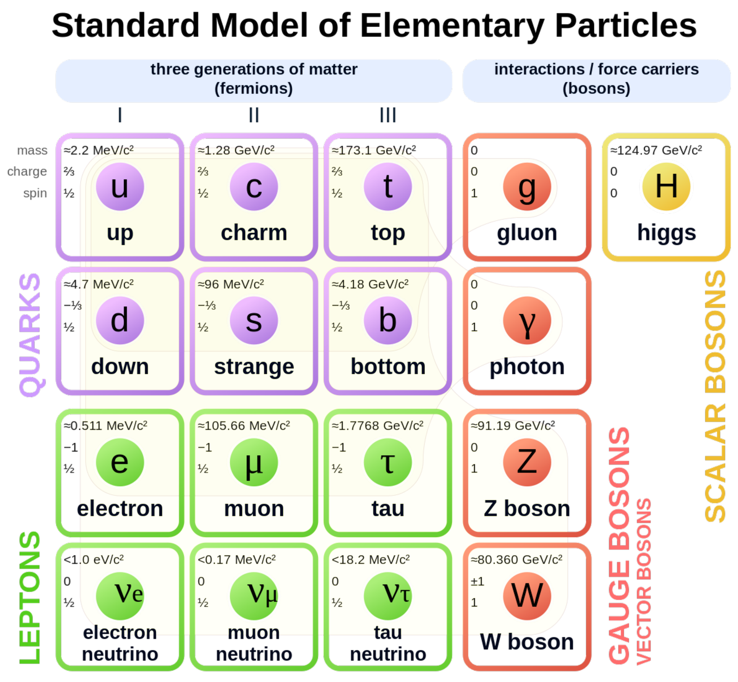

Figure 1. The Standard Model of Particle Physics, with the three fermion generations and the scalar and gauge bosons.

Latterly, with the development of the quantum mechanics, we have a better understanding about them, and it's definitely not that simple. Protons and neutrons are not the final stage, but are not the *fundamental* particles, but are made of them. Quarks (with their six flavors), gluons, the electron cousins, muon and tau, bosons $W^{\pm}$ and Z^0... there is an entire "Particle Zoo" popularized by R. Oppenheimer and others particle physicists who lived during that golden age.

We finally formulated in the 70s, trying to unify Quantum Mechanics and the Relativity Theory of A. Einstein, the Standard Model of particle physics, dividing the matter in two groups, one of them named bosons, which follow the Bose-Einstein statistics and mediate the fundamental forces between the matter, made of fermions, which follow the Fermi-Dirac statistics.

Nowadays, we still have some open questions in this model, like the CP (charge conjugation parity symmetry) violation and the asymmetry between matter and anti-matter, the nature of the dark energy and dark matter and the existence of supersymmetric particles partners. Most of these questions may be answered in the next decades, with the development of new technologies that can provide us enough data which we will use to validate these theories that are *beyond* the standard model.

Quantum Mechanics Rules

The mathematical formulation of quantum mechanics can be briefly described by four postulates, which have shown very useful and unbreakable until today:

- A quantum mechanical state can be fully described by a state vector $|\psi\rangle$, called ket, who lives in a state vector space $\mathscr{H}$, called Hilbert space, implying in a linear algebra mathematical formulation.
- All the physical observables can be described by an operator $\hat{O}$, which acts on those kets, immediately changing or not them (eigenstate are not changed under this action).
- A measurement can be represented by the action of an operator on one state vector, it will return one of its eigenvalues, with a certain probability, and change the state to the respectively eigenstate.
- A quantum system evolves in time can be described by the famous Schrödinger Equation, which is generally statemented by

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = \hat{H}\psi(\mathbf{r},t),$$

and $\psi(\mathbf{r},t)$ represents the state vector projected on the position orthonormal basis.

"Relative" Theory

A large set of physicist and mathematicians from the 20th century was responsible for the formulation of how the space-time and the matter (energy) interact themselves, postulating and formulating fundamental laws that govern these interactions, like the speed limit imposed by the nature: the speed of light in the vacuum c. After Galileo's space-time, given by $(\mathbb{R}^3, \delta_{\mu\nu})$, and his principle of relativity, which says that different inertial observers can measure the same observables (implying in a privileged observer), the Minkowski space-time consists in a four-dimensional smooth manifold aimed with the analogous named metric, $(\mathbb{R}^{1,3}, \eta_{\mu\nu})$, where we have three spacial dimensions and one guided by the time ("space-time"), and a point at this space is called an *event*, (t, x, y, z), and a sequence of events infinitesimally connected evolving in time is an *worldline*.

At this manifold, we have a group of transformations known as Poincarè's Group, a semi-direct product between the group of translations and Lorentz Group, $\mathcal{P} = SO(3,1) \oslash T^{3,1}$, where the last consists in Lorentz transformations, very important to relativistic change of inertial frame and create phenomena like time dilation and length contraction.

As vectors transform under rotations and preserve their modules, we have four-vectors, which transform under Lorentz transformations and preserve their modules (using that new metric), like the important four-gradient $\partial_\mu = (\partial_t, \nabla)$. Lorentz boosts preserve the spacetime interval between two events, a scalar product given by

$$\begin{aligned} ds^2 &= \eta_{\alpha\beta}dx^\alpha dx^\beta \\ &= \pm c^2(t_2 - t_1)^2 \mp (x_2 - x_1) \\ &\quad \mp (y_2 - y_1) \mp (z_2 - z_1) = ds'^2 \end{aligned} \tag{1}$$

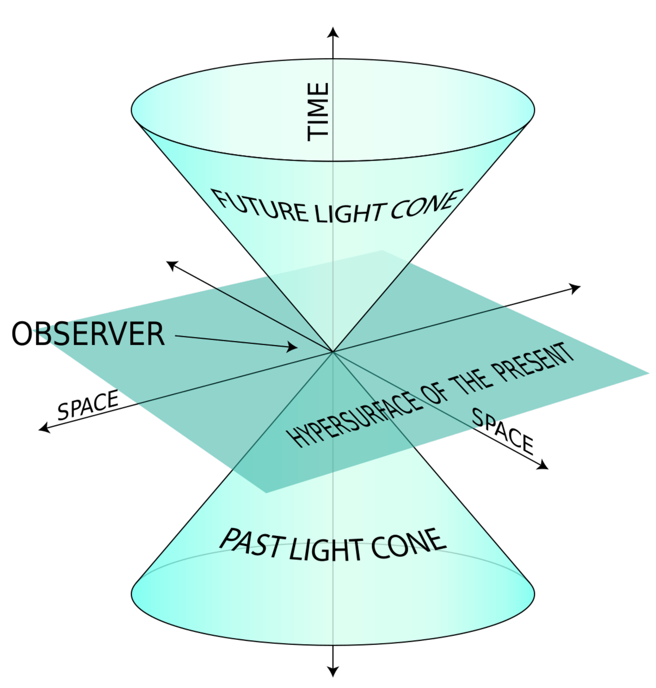

Figure 2. The light cone, a surface constructed by the pseudo-distance (1) in Minkowski space-time.

due the metric signature $(\pm, \mp, \mp, \mp)$, which if null, matches with the equation of a surface known as the *light cone*, a three-dimensional cone hyper-surface,

Given a event's light cone (where it is at the origin), we can have three types of spacetime intervals ds, and each one can naturally give us some relations between the events (for $+ - - -$):

- space-like, given by $ds < 0$, when the other event is outside the observable's light cone,
- and light-like, referring to $ds = 0$, so events like this lies on the light cone,
- time-like, when $ds > 0$, establishing the idea of *causality connected* events,

where the term *causality* emerges from the idea of which point can affect another, so, if two points are not causality connected, they will never affect each other.

Introducing Fields

A field can be described as a mathematical function entity that returns quantities of different natures, as scalars, vectors, tensors, etc, and in our case, they permeate all space-time, mapping one of these entities for each event. The temperature in a room can be given as a number to each point of this little space, while the electromagnetic field created by an electron needs not only a module but also a direction which it points at space.

Fields are extremely important for the description of our universe, and they appear naturally when you try to describe a system with a huge (going to infinity) number of degrees of freedom: taking N coupled harmonic oscillators going to infinity and using the classical mechanical formalism, we can define a Lagrangian $\mathcal{L}$ and Hamiltonian $\mathcal{H}$ densities, which we can use to get the equations of motion of our continuous system, and still have the quantity named *Action* that now can be given by

$$S = \int \mathrm{d}^4x\, \mathcal{L}(\phi, \partial_\mu\phi),$$

which by the Principle of Least Action, always tends to be extremized, what means $\delta S = 0$.

Evaluating a tiny variation $\delta\phi \to \phi + \delta\phi$ on our field, keeping the lagrangian unchanged so that it has a symmetry, we can get that

$$\delta S = \int \mathrm{d}^4x \left\{ \left[\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right]\delta\phi - \partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta\phi\right]\right\} = 0,$$

implying that when the principle applies, and so we are dealing with a real physical system, there is a *conserved Noether current* and its charge, which are given by

$$J_\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta\phi \quad , \quad Q = \int \mathrm{d}^3x\, J_0,$$

respectively. Some of the most canonical conserved currents and its charges are the Enery-Momentum Tensor $T^\mu{}_\nu$ conserving the energy density (Hamiltonian H) and the Total Angular Momentum Tensor, a sum of Orbital and Spin, $\mathcal{J}^\mu{}_{\nu\rho} = \mathcal{M}^\mu{}_{\nu\rho} + \mathcal{S}^\mu{}_{\nu\rho}$, conserving spin and angular momentum.

Gotta Quantize 'Em All

Now, we must try to establish a connection between this posterior quantum and the classical mechanics, besides to get the same results when approaching to the classical limit. At a first quantization approach try, we can take one of those harmonic oscillators which we already know the well-defined solutions $x(t) = C_1e^{i\omega t} + C_2e^{-i\omega t}$ and Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2,$$

and promote the position and momentum observables to quantum operators (just like every observable), imposing the canonical commutation relation $[\hat{x}, \hat{p}] = i\hbar$; also changing the main variables and getting a new set of powerful tools given by

$$\hat{a} = \sqrt{\frac{m\omega}{2}}\left(\hat{x} + \frac{i\hat{p}}{m\omega}\right) \quad , \quad \hat{a}^\dagger = \sqrt{\frac{m\omega}{2}}\left(\hat{x} - \frac{i\hat{p}}{m\omega}\right) \quad , \quad [a^\dagger, a] = 1 \quad , \quad H = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right),$$

we now have the number operator $\hat{N} = \hat{a}^\dagger\hat{a}$ which give us the discrete energy levels modes of the system given by $E = \hbar\omega$, interpreted as the particle's energy level.

At a second try we take the simplest Lorentz-invariant field's equation of motion, $\Box\phi = 0$, and take its most general plane waves solutions

$$\phi(x,t) = \frac{1}{(2\pi)^3}\int \mathrm{d}^3p\, \left(a_pe^{-ip^\mu x_\mu} + a_p^*e^{ip^\mu x_\mu}\right)$$

and, as the first try, introduce to each $\mathbf{p}$ a creation $\hat{a}^\dagger$ and annihilation $\hat{a}$ operators, which over all wavenumbers results in

$$H = \frac{1}{(2\pi)^3}\int \mathrm{d}^3p\, \omega_p\left(\hat{a}_p^\dagger\hat{a}_p + \frac{1}{2}\right),$$

which talks about the number of particles (interpreted as excitations) of each $E = \hbar\omega$ mode. Now, if we consider the commutator for equal times $[a_p, a_k^\dagger] = (2\pi)^3 \cdot \delta^3(\mathbf{p} - \mathbf{k})$ and know how $a_p^\dagger$ acts on moment states $|\mathbf{p}\rangle$, we can finally describe quantum fields as operators, defined by integrals over the creation and annihilation operators for each wavenumber over the Fock space $\mathcal{F} = \oplus_n\mathcal{H}_n$,

$$\phi_0(\mathbf{x}) = \frac{1}{(2\pi)^3}\int \frac{\mathrm{d}^3p}{\sqrt{2\omega_p}}\left(a_p^\dagger\, e^{-i\mathbf{p}\mathbf{x}} + a_p\, e^{i\mathbf{p}\mathbf{x}}\right),$$

which can be interpreted, for example, by the action on the vacuum state that creates a particle at $\mathbf{x}$, $\phi_0(\mathbf{x})\,|0\rangle = |\mathbf{x}\rangle$, and now, we have a simple quantum description of the field theory.

Conclusions and Expectations

For the next months, I plan to deepen my knowledge in more requirements of particle physics, introducing myself to groups theory and more fundamental concepts about quantum mechanics, as perturbation and scattering theories, while following a guided reading of quantum theory field books as Schwartz[3] and Peskin[2].

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